# Expectation-Maximization Based Defense Mechanism for Distributed Model Predictive Control

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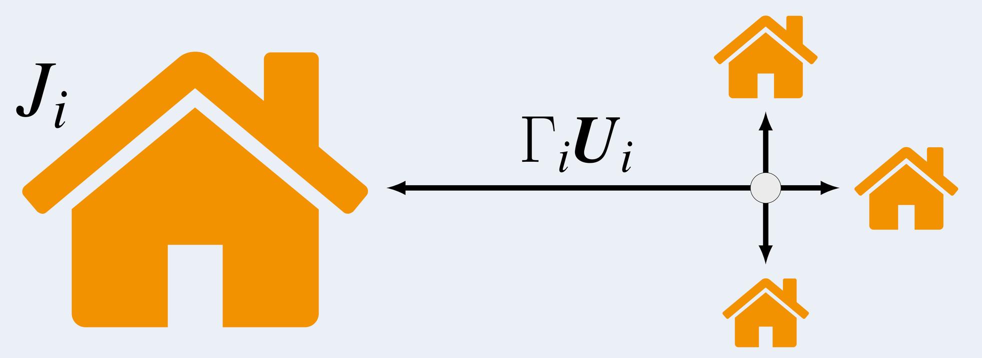
## 1. Challenge - False Data injection in dMPC exchange

- Decomposable quadratic objective $\sum_{i=1}^{M} J_i$
- Coupling constraint $\sum_{i=1}^{M} \Gamma_i \boldsymbol{U}_i[k] \leq \boldsymbol{U}_{\max}$




Solution →

Primal Decomposition based distributed MPC

**Agent I**

$$\begin{aligned}\min_{\boldsymbol{U}_I[k]}\ & \tfrac{1}{2}\|\boldsymbol{U}_I[k]\|_{H_I} + f_I[k]^T \boldsymbol{U}_I[k] \\ \text{s.t.}\ & \boldsymbol{U}_I[k] \succeq \boldsymbol{0} \\ & \bar{\Gamma}_I \boldsymbol{U}_I[k] \preceq \boldsymbol{\theta}_I[k] : \boldsymbol{\lambda}_I[k]\end{aligned} \quad \text{(A)}$$

$\cdots$

**Agent M** (A)

$\boldsymbol{\theta}_I[k]$ ↑ ↓ $\boldsymbol{\lambda}_I[k]$ $\boldsymbol{\theta}_M[k]$ ↑ ↓ $\boldsymbol{\lambda}_M[k]$

**Coordinator**

Update $\boldsymbol{\theta}_i$ using past $\boldsymbol{\theta}_i$ and all $\boldsymbol{\lambda}_i$

Coordinator allocates $\boldsymbol{\theta}_i$
Agent has dissatisfaction $\boldsymbol{\lambda}_i$

What happens if an agent lies about $\boldsymbol{\lambda}_i$?

## 2. Attack and consequences

- $\boldsymbol{\lambda}_i$ is the dissatisfaction of $i$ to allocation $\boldsymbol{\theta}_i$
- Attacker increases $\boldsymbol{\lambda}_i$ using function $\gamma(\cdot)$
- ↑ dissatisfaction == ↑ allocation

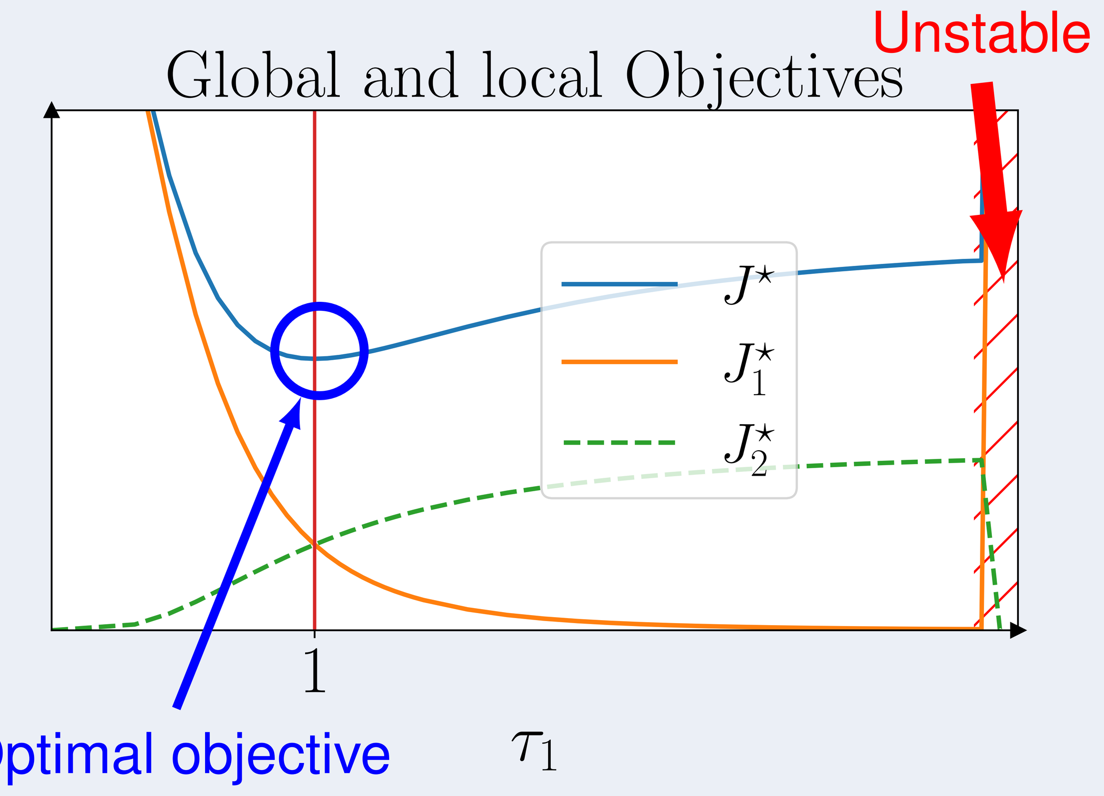
***Remark***
Attacker says it is satisfied **only when it is**

***Assumption***
Attacker chooses an *invertible linear function*

$$\tilde{\boldsymbol{\lambda}}_i = \gamma_i(\boldsymbol{\lambda}_i) = T_i[k]\boldsymbol{\lambda}_i,$$

- Effects of cheating matrix $T_i[k]$
  - Increase on global objective
  - Destabilization

### Example $T_1[k] = \tau_1 I$




### Can we mitigate the effects?

YES! If we estimate $T_i[k]$ and invert it
But how?

## 3. Estimating cheating matrix $T_i[k]$

Local problems (A) are **QP**
**Explicit Solution** with **PWA form w.r.t** $\boldsymbol{\theta}_i$:

$$\boldsymbol{\lambda}_i[k] = -P_i^n \boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^n[k], \text{ if } G_i^n[k]\boldsymbol{\theta}_i[k] \preceq \boldsymbol{b}_i^n[k] \quad \text{(B)}$$

with $n \in \{1:N\}$. $G_i^n[k]$ and $\boldsymbol{b}_i^n[k]$ define regions.

***Remark***
Sensibilities $P_i^n$ are **time invariant**.

***Another assumption***
In Region 1 **local constraints are active**:

$$\boldsymbol{\lambda}_i[k] = -P_i^1 \boldsymbol{\theta}_i[k] - \boldsymbol{s}_i^1[k], \text{ if } G_i^1[k]\boldsymbol{\theta}_i[k] \preceq \boldsymbol{b}_i^1[k] \quad \text{(C)}$$

and $\boldsymbol{\theta}_i = \boldsymbol{0}$ belongs to it

Attacker **modifies sensibility** $\tilde{P}_i[k] = T_i[k]\bar{P}_i$ and $\tilde{\boldsymbol{s}}_i[k] = T_i[k]\boldsymbol{s}_i[k]$

If we can know **nominal** $\bar{P}_i^1$,
by estimating $\tilde{P}_i[k]$, we can find $T_i[k]^{-1}$:

$$\widehat{T_i[k]^{-1}} = \bar{P}_i^1 \widehat{\tilde{P}_i^1[k]}^{-1} \quad \text{(D)}$$

### But how can we estimate the $\tilde{P}_i^1[k]$?

Enter Expectation Maximization
- Classify data in regions (latent variables)
- Estimates parameters using weighted LS

EM needs minimally excited inputs $\boldsymbol{\theta}_i$ and $\tilde{\boldsymbol{\lambda}}_i$.
- ~~During negotiation~~ (time dependence)
- Solution: estimate in a separate phase
  - Generate independent points near $\boldsymbol{\theta}_i = \boldsymbol{0}$ Artificial Scarcity Sampling

## 4. Expectation Maximization

- Regions are indexed by $z \in \mathcal{Z} = \{1:Z\}$
- Gaussian mixture (mean (B) and $\Sigma \to 0$)
- Parameters $\mathcal{P} = \{\mathcal{P}^z \mid z \in \mathcal{Z}\}$, with $\mathcal{P}^z = (\tilde{P}^z, \tilde{\boldsymbol{s}}^z, \pi^z)$.
- Observations $o \in \mathcal{O} = \{1:O\}$ of $(\boldsymbol{\theta}_i, \boldsymbol{\lambda}_i)$ stacked as $(\underline{\Theta}, \underline{\Lambda})$ with corresponding $\underline{Z}$

**Algorithm 1:** Expectation Maximization
Initialize parameters $\mathcal{P}_{\text{new}}$
**repeat**
  $\mathcal{P}_{\text{cur}} \leftarrow \mathcal{P}_{\text{new}}$
  **E step:**
   Evaluate $\zeta_{zo}(\mathcal{P}_{\text{cur}}) = \mathbb{P}(\underline{z}_o = z | \underline{\boldsymbol{\lambda}}_o, \underline{\boldsymbol{\theta}}_o; \mathcal{P}_{\text{cur}})$
  **M step:**
   Reestimate parameters using:
   $\mathcal{P}_{\text{new}} = \arg\max_{\mathcal{P}} \mathbb{E}_{\zeta_{zo}(\mathcal{P}_{\text{cur}})} [\ln \mathbb{P}(\underline{\Theta}, \underline{\Lambda}, \underline{Z}; \mathcal{P})]$
**until** $\mathcal{P}_{\text{cur}}$ *converges*

## 5. Secure dMPC

Modified negotiation (some additional steps):

1. **Detection Phase**

 1.1 Estimate sensibility $\widehat{\tilde{P}_i^1}[k]$
  - Artificial Scarcity Sampling + EM

 1.2 Detect attack if $\|\widehat{\tilde{P}_i^1}[k] - \bar{P}_i^1\|_F \geq \epsilon_P$

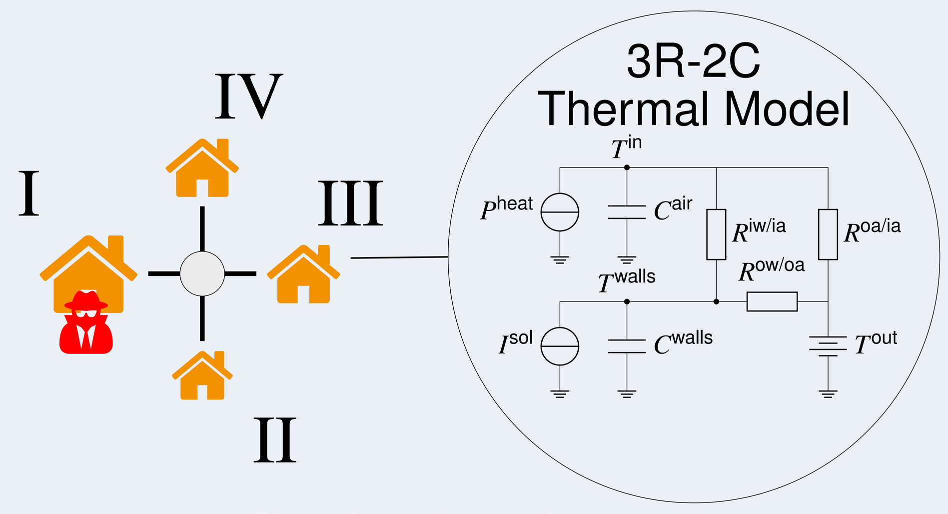
2. **Negotiation Phase**

 2.1 If detected reconstruct $\boldsymbol{\lambda}_i$

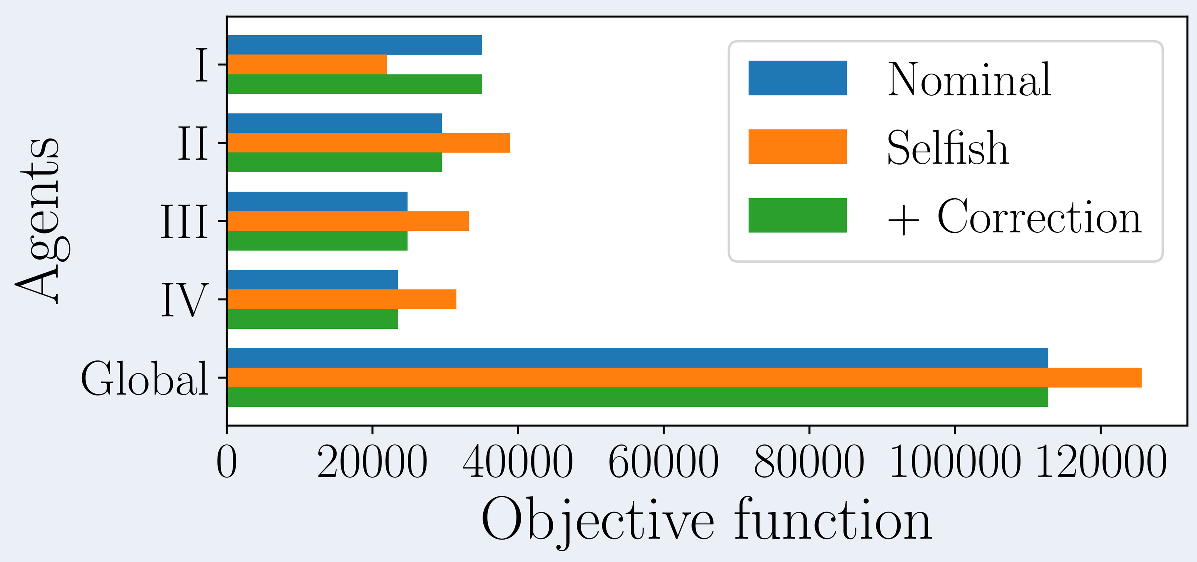
$$\boldsymbol{\lambda}_{i\text{rec}} = \widehat{T_i[k]^{-1}} \tilde{\boldsymbol{\lambda}}_i \quad \text{(E)}$$

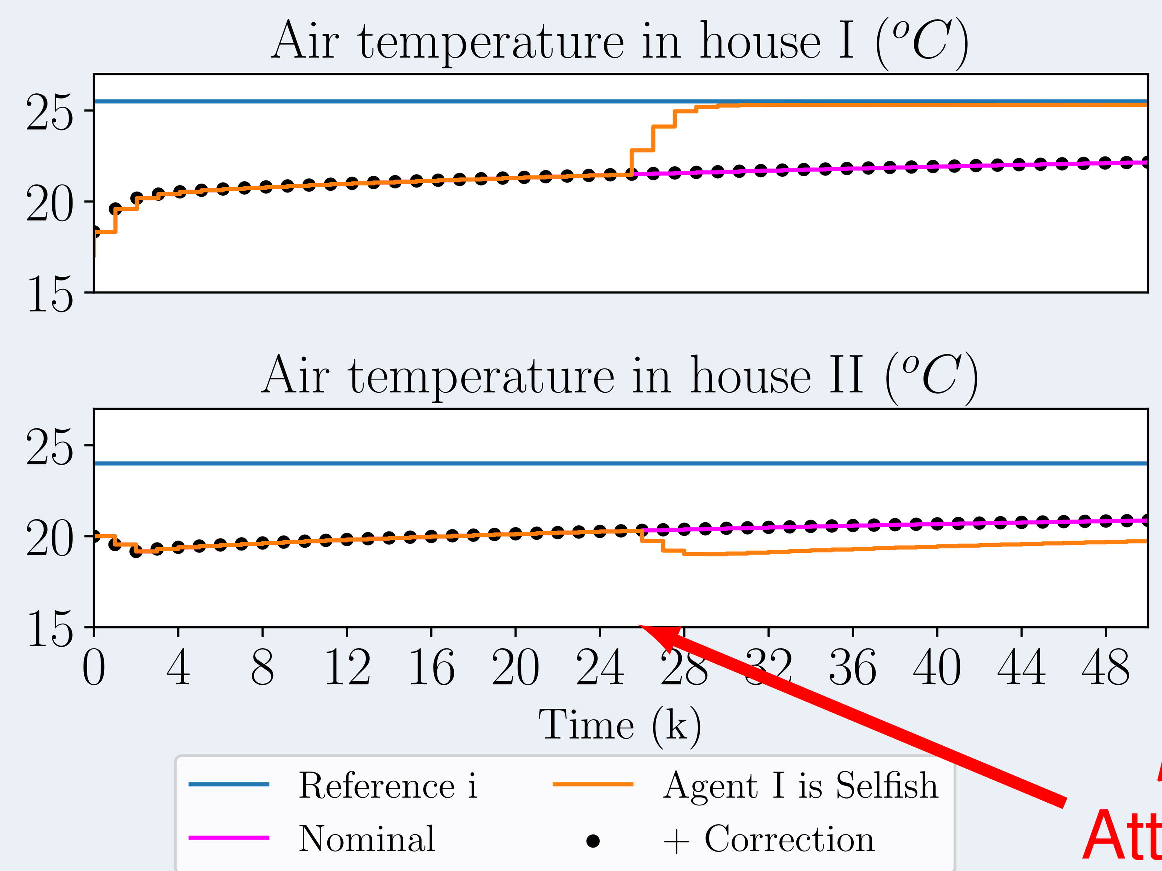
 2.2 Use adequate $\lambda_i$ to update $\theta_i$

## 6. Example: Control of a heating network under power scarcity - 3 Scenarios (Nominal, Selfish, + Correction)



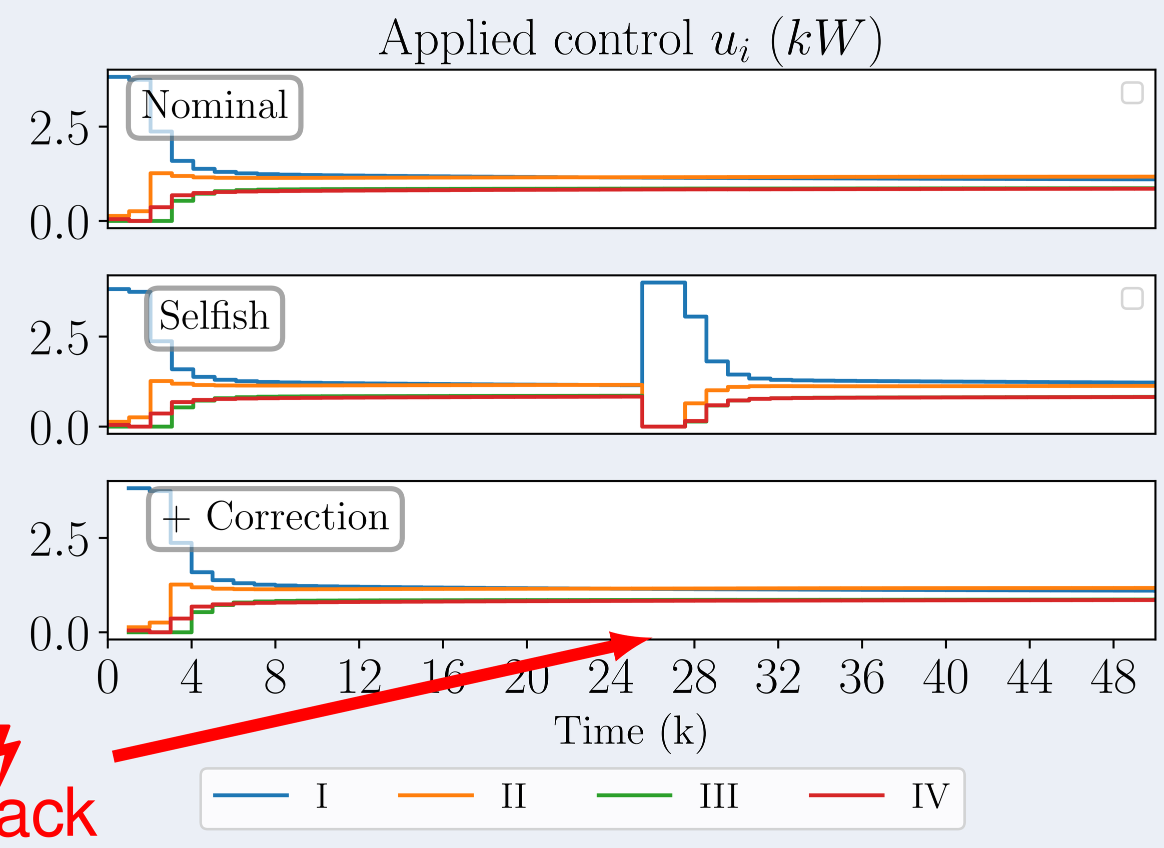

District with 4 houses




Air temperature in houses I and II.

Attack




Control applied in all houses for the 3 scenarios.

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