Modern analysis of axisymmetric shells with AQUINAS: a MATLAB finite element toolbox

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Abstract

AQUINAS is a specialised MATLAB finite element (FE) toolbox for the nonlinear analysis of axisymmetric thin-walled shell systems, conceived by the Authors to recover a research functionality once widely available but now largely lost in the age of supposedly 'general' 3D FE software. AQUINAS exploits the powerful built-in sparse linear algebra routines of MATLAB with efficient matrix assembly in pre-compiled and parallelised C++ code via the MEX functionality, as well as MATLAB's extensive visualisation and quality-of-life features. In addition to making nonlinear bifurcation buckling calculations of axisymmetric shells more accessible to a wider research community, the toolbox is designed to facilitate the ongoing active development of the structural Eurocode EN 1993-1-6 governing the design of metal civil engineering shells through its capability to automate the accurate computation of nominal resistance capacity curves for reference axisymmetric shell systems.

Keywords: FEM, axisymmetric shells, buckling, Eurocode development

Current code version

Nr.	Code metadata description	Code metadata value
C1	Current code version	0.10.0
C2	Link to repository	*https://github.com/AchilleasF/AQUINAS−FE*
C3	Legal Code License	BSD 3-Clause License
C4	Code versioning system used	git (GitHub)
C5	Software code languages and tools	MATLAB, C++, OpenMP 4.5, Maple
C6	Compilation requirements, operating environments & dependencies	MATLAB R2022a, MinGW-w64 6.3 for OpenMP 4.5 support
C7	Link to developer documentation/manual	*https://github.com/AchilleasF/AQUINAS−FE*
C8	Support email for questions	a.filippidis@imperial.ac.uk a.sadowski@imperial.ac.uk

1. Motivation and significance

The computational buckling analysis of axisymmetric thin-walled shell systems under axisymmetric pre-buckling stress states presents a significant modelling challenge to the structural analyst. The complete circumferential uniformity of the pre-buckling stress state theoretically offers an infinite number of possible locations for an asymmetric bifurcation buckle, an uncomfortable prospect for classical computers working in finite precision. Yet despite their 'traditional' position in the structural mechanics literature, axisymmetric shells of revolution correspond to important reference structural forms in the structural, mechanical, aeronautical and nuclear industries primarily in the form of cylindrical, conical, spherical or hyperboloid geometries and continue to be the subject of research interest (e.g. see [15, 28, 27, 20]).

A representative list of specialised software known to have once been widely used for the nonlinear elastic-plastic stability analysis of axisymmetric shells with a Fourier series representation of the unsymmetrical response in the circumferential dimension is as follows:

- The BOSOR (**B**ucking **o**f **S**hells **o**f **R**evolution) suite of advanced finite difference-based programs developed by D. Bushnell and his research group, widely used for computational shell buckling studies in the aerospace sector for many decades starting in the 1960s (e.g. see [3, 4, 5]). The source code appears to have been available for download until about 2021 from its author's personal website (*www.shellbuckling.com*), but is now no longer publicly obtainable or supported.

- The FELASH (**F**inite **El**ement **A**nalysis of Axisymmetric **Sh**ells) suite of advanced finite element-based programs developed by J.G. Teng and J.M. Rotter [22, 23] in the late 1980s, and in particular the NEPAS (**N**onlinear **E**lastic-**P**lastic **A**nalysis for Bifurcation **S**tability) component that was primarily used in civil engineering shells research (e.g. see [16, 26]). It does not appear to be publicly available.

- The INCA software developed by A. Combescure in the 1980s, used for stability analyses of cooling towers for the nuclear industry until the 2000s (e.g. see [1, 9, 12]). It, too, does not appear to be publicly available.

There are doubtless many others that were developed by individual researchers for internal use, never having been intended for public release. The reader may note that the use of custom-written research-oriented specialised finite element software appears to have largely waned in the past 20 years in structural mechanics research. This appears to correspond to the widespread adoption of powerful commercial and 'general' FE software packages such as ABAQUS or ANSYS, which benefit from a steady licensing income stream, active development and extensive validation. In the present context, however, while these 'general' FE packages do offer axisymmetric shell elements as part of their finite element library (e.g. SAX1 or SAX2 in ABAQUS, and SHELL208 or SHELL209 in ANSYS), these elements are typically not enhanced with the ability to represent arbitrary unsymmetrical deformation modes through the use of a circumferential Fourier series expansion. ABAQUS offers the SAXA1n or SAXA2n axisymmetric shell elements which support circumferentially asymmetric loading and deformation in the form of a circumferential Fourier harmonic n, but only up to $n = 4$. Similarly, ANSYS offers the SHELL61 'harmonic' axisymmetric shell element supporting loading and deformation into arbitrary n, but only for linear elastic stress analyses.

The reader is invited to see a fuller discussion of the background to the above given in Sadowski et al. [20]. This reference includes a workaround implemented in ABAQUS to allow the detection of the critical unsymmetrical bifurcation buckling mode on the basis of the so-called 'panel analysis' technique, where conditions of circumferential symmetry are applied to a 3D shell panel that is designed to have an angular spread sympathetic to the target circumferential mode into which buckling is being triggered. Yet this workaround is only partially successful at countering the fundamental limitation of having to use a 3D modelling space for axisymmetric shell systems, namely that the need for an explicit discretisation of the circumferential dimension is parasitic and detrimental to the quality of the analysis, and its application is tedious to the extreme.

2. Software description

AQUINAS is an open source MATLAB toolbox that permits efficient stress, small/large displacement theory plastic collapse and non-symmetric bifurcation buckling analyses of geometrically nonlinear elastic-plastic axisymmetric shells. The doubly-curved axisymmetric shell finite element formulation presented by Teng and Rotter [22, 23] was adopted for AQUINAS, with additional details sourced from Jumikis [13] and the Authors' own derivations. This formulation was chosen in preference over others because it offers very complete strain-displacement relations, an independent definition of the meridional curvature of the shell and an established track record of successful use in scientific publications [22, 23, 13, 26, 16, 21, 15, 25, 24] for the purposes of software verification. Torsional or non-symmetric loading has not been implemented at present. Full details of the formulation may be found in the accompanying documentation, as well as in the source references.

2.1. Software Architecture

Usage of AQUINAS is exposed to the programmer through an object-oriented API. The user instantiates a suite of objects representing the geometry, loads, constraints, materials, solver settings and output requests, and submits these as a `varargin` variable-length argument list to the `AQUINAS_Protocol.m` function which acts as a central control pipeline through the software (Fig. 1). Assembly of the finite element system is accelerated through the use of compiled C++ code via the MEX functionality (the release includes standard Windows `.mexw64` and Linux `.mexa64` 64-bit MATLAB executables) which invokes OpenMP threaded parallelism for additional performance. A native MATLAB-only serial execution is possible, but it is much slower. The modular and open source nature of AQUINAS makes the source code accessible to anyone who wishes to add specific functionality, though any changes made to the C++ accelerated matrix assembly will require re-compilation with a GCC 6.3 compiler or later (to support OpenMP 4.5 features such as array reductions).

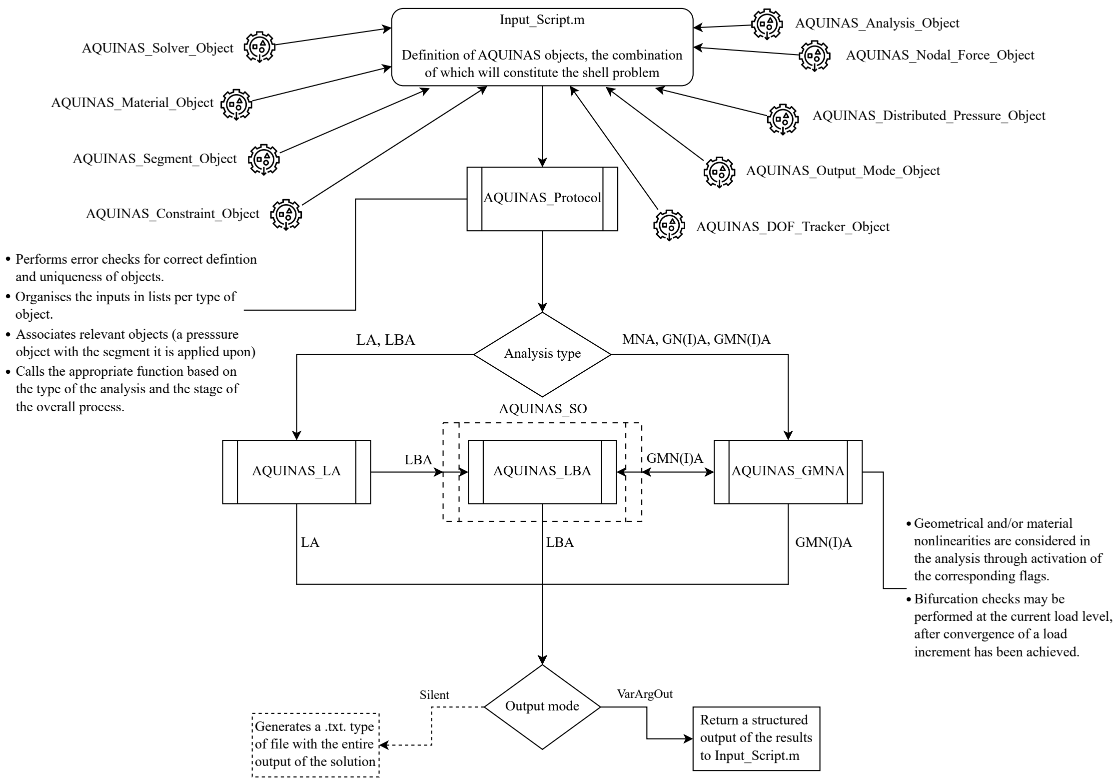

Figure 1: Flowchart of the functionality of AQUINAS.

2.2. *Software Functionalities*

The AQUINAS toolbox is presently capable of carrying out the following computational structural analyses as defined by the Eurocode prEN 1993-1-6 [7] on the strength and stability of metal shells:

- LA: Linear elastic analysis to obtain the complete primary (membrane) and secondary (bending) stress state;
- LBA: Linear bifurcation analysis to obtain the critical and near-critical bifurcation buckling eigenvalues and associated eigenmodes;
- MNA: Materially nonlinear analysis to obtain the small-displacement plastic collapse mechanism;
- GNA: Geometrically nonlinear elastic analysis to obtain the deformation and stress state assuming large deflections, and to detect axisymmetric limit point and / or potentially non-symmetric bifurcation buckling;
- GMNA: Geometrically and materially nonlinear analysis which extends a GNA to include plasticity;
- GNIA: Geometrically nonlinear elastic analysis which extends a GNA to include imperfections (primarily unintended geometric deviations of the shell midsurface);
- GMNIA: Geometrically and materially nonlinear analysis which extends a GMNA to include imperfections.

By convention, LAs, LBAs, MNAs, GNAs and GMNAs are performed on a reference shell geometry defined by a 'perfect' midsurface assuming no imperfections of any kind, although a number of recent research works have undertaken 'LBIAs' [20] and 'MNIAs' [29] under special circumstances. All (G)(M)N(I)As are implemented in AQUINAS as incremental path-tracing analyses using either load steps of constant magnitude or the versatile arc-length algorithm [10, 14]. Every bifurcation buckling check assumes an axisymmetric pre-buckling stress state.

2.3. Verification and validation library

The AQUINAS git repository contains an extensive library of over 30 example `.m` MATLAB 'input' files which are intended to illustrate usage on how to perform each one of the above specialised FE analyses on a selection of classical axisymmetric shells of revolution (cylinders, cones, spheres, ellipsoids, ogivals and more complex custom shapes), while acting as a comprehensive record of verification and validation of the various capabilities of AQUINAS against algebraic predictions from classical shell membrane and / or bending theory (where available), ABAQUS simulations (where appropriate) and physical experiments (where possible). This library is in an of itself an important resource that may be used for the verification and validation of models of thin-walled shells created by other FE software as is now required by prEN 1993-1-6 [7] and prEN 1993-1-14 [6] for structural steel design assisted by FE, and more examples will be added periodically.

3. Illustrative Examples

Two representative examples are presented here to illustrate the main functionalities of the AQUINAS toolbox, both on what is simultaneously the most important and the most difficult reference system of a thin cylindrical shell under uniform meridional compression.

3.1. Example 1 - GNAs of a length-dependent system

In the first example (Fig. 2), a thin cylinder with a radius to thickness r/t ratio of 1000 is analysed at multiple different lengths l as defined by the Batdorf dimensionless length group $Z = l^2\sqrt{(1-\nu^2)}/(rt)$. Very short cylinders exhibit plate-like behaviour dominated by compatibility bending, very long cylinders exhibit Euler column-like buckling behaviour in the $n = 1$ mode, while intermediate 'medium-length' cylinders exhibit local buckling at a resistance that is approximately invariant with length in this region. The cylinder geometries were perfect and the response was elastic but the analyses were geometrically nonlinear (GNA).

The top of this figure shows the solution by AQUINAS, where a prediction of the normalised buckling resistances R_{GNA}/R_{cl} (where R_{cl} is the classical elastic critical buckling resistance) corresponding to every trial circumferential wave number $n \in [0, 30]$ per Z is obtained at the critical bifurcation load level and plotted as a 3D surface, together with a black curve which identifies the global minimum for each Z. The data is clean, complete and was generated in less than an hour on an 8-core CPU (with threaded parallelisation to accelerate matrix assembly). The bottom of the figure shows a 2018 attempt at recreating the same surface using 3D shell elements in ABAQUS and the laborious 'panel analysis' technique [20]. The data is messy (due to numerical convergence problems associated with mesh economisation), incomplete (certain 3D panels simply would not be forced to buckling into particular modes) and required several days' worth of runtime.

3.2. Example 2 - GMNAs of a slenderness-dependent system

A second, more complex, example concerns the careful generation of nominal 'capacity curves', safety-critical algebraic constructs that underpin structural resistance rules in prEN 1993-1-6 [7]. These are relationships between the normalised resistance R_{GMNA}/R_{MNA} of a system against either its dimensionless slenderness $\lambda = \sqrt{R_{MNA}/R_{LBA}}$ (the 'traditional' form Fig. 3a) or against an alternatively normalised resistance R_{GMNA}/R_{LBA} (the 'modified' form Fig. 3b as conceived by Rotter [17]). Since the system exhibits different failure modes at different slendernesses (plastic collapse at low λ characterised by extensive yielding in membrane and bending action, elastic-plastic bifurcation buckling at intermediate λ into low n and elastic bifurcation buckling at high λ into high n) and each failure mode

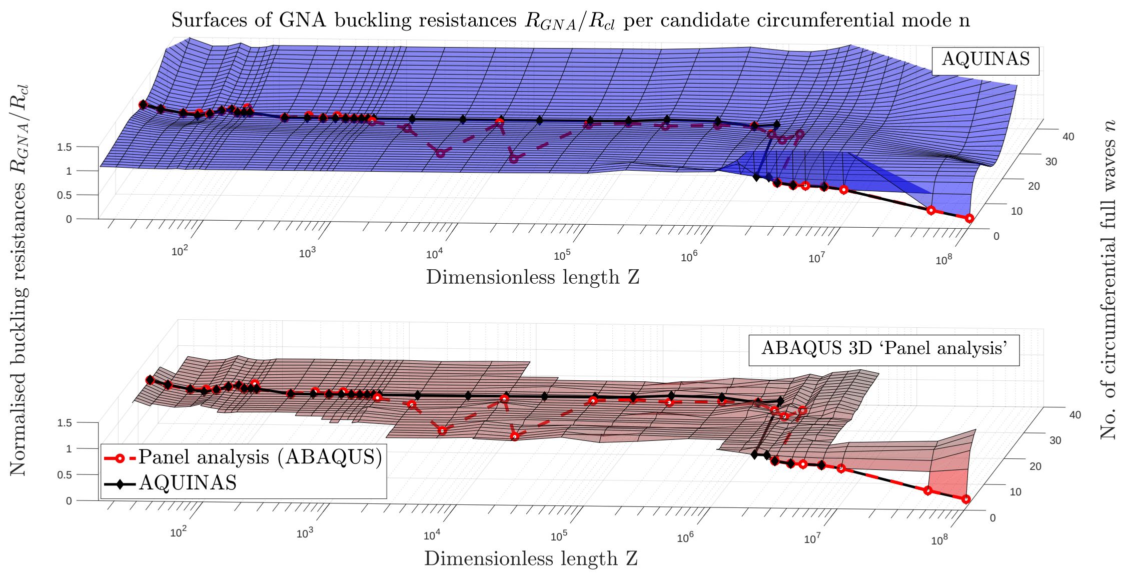

Figure 2: Automated identification of the critical geometrically nonlinear GNA bifurcation buckling load for a thin cylindrical shell under uniform meridional compression as a function of the Batdorf dimensionless length parameter Z, with comparison against the 3D 'panel models' from ABAQUS.

requires a different termination criterion [19], it is a challenging benchmark test for the capabilities of a nonlinear structural FE solver.

The curves shown in Fig. 3 were generated by AQUINAS over a 24-hour period and include GMNAs computed with automatic termination regardless of first encountered failure mode, GMNAs computed with automatic detection of non-axisymmetric bifurcations only, GNAs performed under the same conditions to identify the slenderness at which plasticity begins to degrade resistance, and axisymmetric GMNAs performed by ABAQUS (which does *not* detect unsymmetric bifurcations) for comparison. A 3D ABAQUS computation of a capacity curve using the 'panel analysis' method has never been attempted but it is unlikely to ever be an attractive option from the point of view of runtime or solution quality, and the case for why the classical modelling of nominally axisymmetric systems in a 3D space should be avoided entirely is made in Sadowski et al. [20].

4. Impact

The study of the mechanics of metal shell systems and their characterisation for the purposes of structural design in the framework of the 'capacity curve' device of prEN1993-1-6 [7] has been the focus of significant research endeavours over

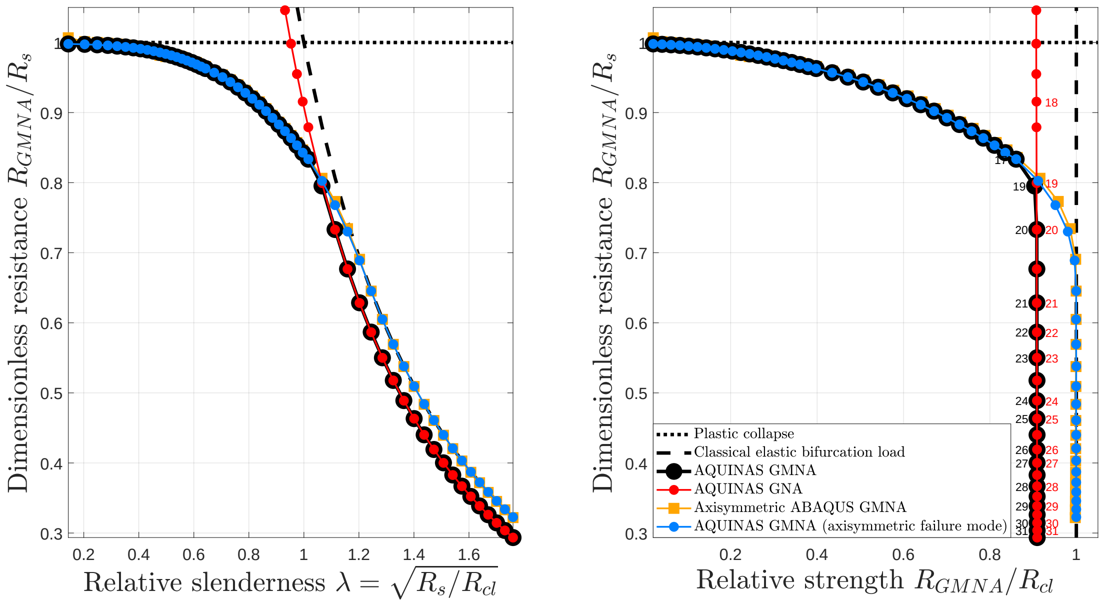

Figure 3: Automated generation of a GMNA capacity curve for the reference system of a thin cylindrical shells under uniform meridional compression, with comparison against axisymmetric models from ABAQUS.

the past decade [8, 11, 30, 29], and will continue to be so until an exhaustive repository of the main systems of industrial importance has become established and accepted into modern design practice. This manner of research is very computationally demanding since large parameter hyperspaces must be explored for an exhaustive treatment using both linear and nonlinear analysis of varying degrees of computational expense. The most comprehensive treatment to date by Wang et al. [29] of cylindrical shells under uniform bending required almost 30 thousand nonlinear MN(I)As and G(M)N(I)As, and while various strategies have been suggested to automate aspects of the procedure [20, 18] the issue of computational expense cannot be avoided entirely.

The use of general 3D FE tools such as ABAQUS or ANSYS will continue to be widely applied to the characterisation of *unsymmetrical* systems, however it is vital that the reference axisymmetric systems to which these will be directly compared to be established using the highest-quality analyses available. Unfortunately, for reasons explored above, 3D FE modelling does not appear to offer an optimal or highest-quality solution, and the reintroduction of a specialised axisymmetric FE solver for shells such as AQUINAS into the research community is long overdue. It is stressed that AQUINAS is not intended to be a ‘rewrite of legacy science software’ (though such an endeavour is in and of itself valuable if it makes obsolete or undocumented software available again for use by scientists [2]) but rather a modernisation of it and, to the Authors' knowledge, this new MATLAB toolbox is currently the *only* publicly-available documented computational tool that can natively perform nonlinear unsymmetrical elastic-plastic bifurcation buckling analyses of axisymmetric shells.

5. Conclusions

This paper briefly outlines the case for and capabilities of AQUINAS, a newly-developed MATLAB toolbox for the natively axisymmetric analysis of axisymmetric shells systems with the capability of performing unsymmetrical linear and nonlinear bifurcation analyses. The toolbox is capable of performing a full suite of modern quasi-static analyses types (LA, LBA, MNA, GNA, G(M)N(I)A etc.) and its modular and object-oriented structure will permit more functionality to be added as required. This software release is accompanied by a `GitHub` repository containing the entirety of the code, pre-compiled C++ routines for the acceleration of matrix assembly, and an extensive verification and validation library of over 30 example systems. The toolbox is being actively used in the study of reference axisymmetric shell systems of industrial importance and their characterisation into the 'capacity curve' structural design framework of the Eurocode EN 1993-1-6.

Conflict of Interest

We wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

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