How to design for robustness and reliability?

Paper helicopter project

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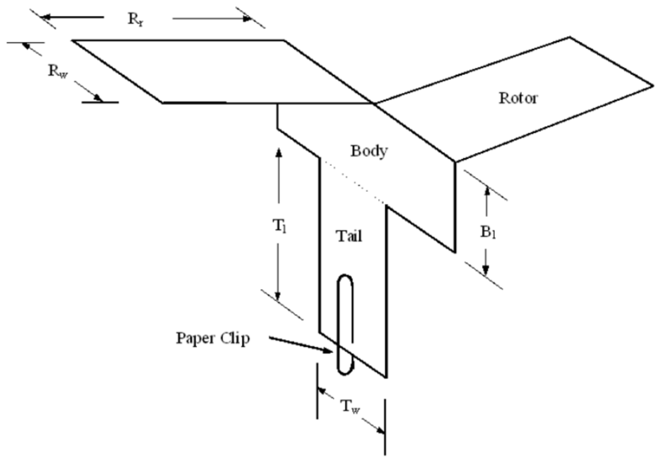
1 General introduction

Helicopters rely on a phenomenon called auto-rotation to slow their descent to the ground when they lose power. The air-flow past the rotors generated by the downward speed causes the rotor to spin and generate drag that slows down the fall. The paper helicopter is a simple construction that shares this property of auto-rotation when falling to the ground, and the aim of the project is to build a paper helicopter based on a A4 sheet of paper (see below) that takes the longest to fall to the ground from a given height and quantify uncertainties associated with the falling time.



Model of paper helicopter

The values of Bl and Tl are fixed and chosen to be 4cm and 5cm respectively and in addition, to help with the stability, two paper clips need to be used.

1.1 Drag coefficient

As the paper helicopter rotates and falls to the ground, the upward force (drag D) on it is given approximately as: $D = \frac{1}{2}\rho_{air}V^2SC_D$, where $S = \pi R_r^2$.
For dimensions and paper clip weights that keep the paper helicopter stable, the drag coefficient C_D can be expressed as: $C_D = \frac{2(W_{helicopter}+2W_{paperclip})}{\rho_{air}V^2\pi R_r^2}$, where $W_{helicopter}$ is the weight of the paper used to build the helicopter and $W_{paperclip}$ is the weight of a paperclip.

2 Approximation of the helicopter drag coefficient

The objective of the first part of this project is to find experimentally an expression for C_D in terms of three dimensions of the helicopter, R_r, R_w, and T_w. The drag coefficient is to be deduced from experimental measurements of the falling time by dropping the helicopter a number of times and measuring the time it takes it to fall to the ground.

2.1 Hands-on experience

We built 15 helicopters of different dimensions and each of these helicopter was dropped 15 times at first. We obtained the following results of falling time and C_D depending on the different helicopters:

	Dimensions (cm)			C_D
N° hélicoptère	Tw	Rr	Rw	
1	2,2	11,96	5,79	0,0801292
2	2,31	13,1	8,28	0,0663021
3	5,89	10,83	7,59	0,0491338
4	5,08	14,51	4,79	0,0659904
5	4,15	13,97	5,44	0,0663115
6	3,88	8,42	5,44	0,1018018
7	3,13	10,2	3,63	0,102219
8	2,91	12,2	5,19	0,0865038
9	3,62	9,51	4,18	0,0927684
10	3,47	13,22	5,19	0,0631564
11	5,42	8,09	2,94	0,0825433
12	4,85	7,16	3,81	0,0866448
13	4,59	6,41	4,2	0,0850991
14	2,65	7,26	2,66	0,1129098
15	5,72	9,7	4,26	0,1024623

But because each drop was slightly different due to the way of dropping, the times will differ, and the mean will have a random component to it (the uncertainty in the mean is considered to be the standard error, i.e. the standard deviation of the individual samples divided by the square root of the number of samples). So we need to drop each helicopter enough times so that the estimate of the mean fall time is accurate to about 5%. Unfortunately for most helicopters the number of optimal drops is really high:

	Nb lancers optimal
N° hélicoptère	
1	378
2	324
3	182
4	137
5	180
6	58
7	21
8	111
9	67
10	165
11	57
12	24
13	17
14	94
15	35

So we decided to stop at 15 drops each.

2.2 Maximization of the Drag coefficient

Then from the results obtained, we constructed thanks to Python, a polynomial response surface approximation of the mean drag coefficient, as a function of the three design variables (dimensions of the paper helicopter R_r, R_w, and T_w). So we used these results to calculate the meta-model which we will then use to find the dimensions of the optimal helicopter. We decided to carry out the meta-model on the falling times.

3 Helicopter design optimization and uncertainty quantification

The objective of the second part of the project is to determine the optimum design using the response surface approximation built. The optimum design is considered to be the one that has the longest falling time, which will be considered equivalent to maximizing the drag coefficient.
Finally, we want to estimate also the uncertainty in the predicted falling time of the optimal helicopter design. This part is composed of two steps: The first one concerns uncertainty quantification of the various sources of uncertainty that affect the falling time and the second one is Monte Carlo simulation (MCS) in order to predict the uncertainty in the falling time for a given design.

3.1 Design of the optimal helicopter

Thanks to the model built in the previous part and by defining the objective function such that is maximizes the drag coefficient, we found thanks to Python, the dimensions of the optimal helicopter:

```
[0.02  0.14  0.045] [0.02  0.06  0.085]

[Done] exited with code=0 in 0.725 seconds
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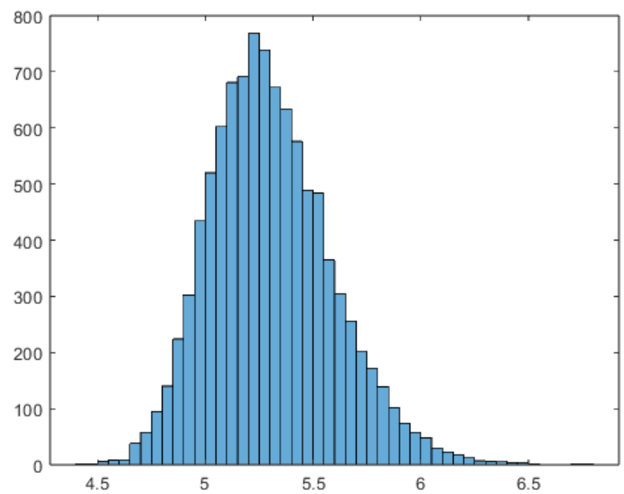
The first three values correspond to the falling time meta-model and the other three to the drag coefficient meta-model. We have chosen to use the lengths given by the falling time meta-model $T_w = 2cm, R_r = 14cm$ and $R_w = 4,5cm$.

We built this helicopter and then carried out 15 drops and found an average fall time of $6,003s$ and a $C_D = 0,081$. The drop time is therefore the greatest but the drag coefficient is not the best. In fact, with the formula: $C_D = \frac{2(W_{helicopter}+2W_{paperclip})}{\rho_{air}V^2\pi R_r^2}$, the helicopter with the best fall time is not the on with the best C_D, as it is an approximation. As for us the optimal helicopter is the one having the greatest fall time.
We then built two other helicopters with the same dimensions to gain access to the construction uncertainties. Here are our average times and Cd for these three helicopters:

	Moyenne temps de chute (s)	C_D
N° hélicoptère		
1	6,003333333	0,081295916
2	5,514666667	0,06859972
3	4,152666667	0,038898957

3.2 Quantification of the uncertainties

It can be seen that the uncertainties are very large: for small differences in size, there are large differences in drop time and drag coefficient. The mean time of $4,15s$ for the third helicopter can be explained by the fact that when the helicopter was launched it did not rotate for the entire duration of the fall, unlike the other two helicopters.
Thanks to these experiments with the optimal helicopter, we calculated thanks to Matlab the standard deviation of the fall time due to uncertainties in mass, fall height and dimensions. We made 10000 drops with the parameters of mass, fall height and dimensions varying and we computed a histogram of the fall time:



We deduce then that the average fall time follows a normal distribution with the following parameters:

- mean: $\mu = 5,3043$.
- standard deviation: $\sigma = 0,0804$.

From this model of the drop time, we can determine if the model is coherent by performing a MCS: We computed the reliability of the optimal design, defined as the probability of failure $PF = P(T_{fall} < T_{fallsecondbest})$ where $T_{fallsecondbest}$

is the second best falling time obtained experimentally in part 1. We obtain that $PF = 0,49$. So it means that there is 49% of chance that the fall time of the optimal helicopter is less than the second best helicopter of the first part. So the model of the optimal helicopter is not so good and in fact we can explain this result because the second best helicopter has these dimensions: $T_w = 5,08cm, R_r = 14,51cm$ and $R_w = 4,79cm$, which are very similar to the dimensions of the optimal helicopter ($T_w = 2cm, R_r = 14cm$ and $R_w = 4,5cm$).