

# COMPUTING TRAJECTORIES USING OPENCL

DTS

ABSTRACT. This tool demonstrates how to execute a simple game physics engine for computing trajectory of a projectile within a framework of OpenCL kernels.

## 1 INTRODUCTION

To compute projectile trajectory coordinates using parametric representation, when target and launch point are at the same level, we use,

$$\begin{aligned} x(t) &= (v_0 \cos(\theta)) t \\ y(t) &= (v_0 \sin(\theta)) t - \frac{gt^2}{2} \\ v_x(t) &= v_0 \cos(\theta) \\ v_y(t) &= v_0 \sin(\theta) - gt \\ \|\mathbf{v}(t)\|_2 &= \sqrt{v_0^2 - 2gtv_0 \sin(\theta) + (gt)^2} \end{aligned} \tag{1}$$

where  $v_0$  is the initial speed. When a projectile is dropped from a moving system,

$$\begin{aligned} x(t) &= v_0 t \\ y(t) &= h_0 - \frac{gt^2}{2} \\ v_x(t) &= v_0 \\ v_y(t) &= -gt \\ \|\mathbf{v}(t)\|_2 &= \sqrt{v_0^2 + (gt)^2} \end{aligned} \tag{2}$$

where  $h_0$  is the initial height.

## 2 COMPUTE KERNELS

To implement an OpenCL kernel for computing trajectories of a projectile, we will use parametric representations of a trajectory. For this tool there will be two OpenCL kernels: one will implement parametric set of equations in (1) and the other implements the set in (2).

The inputs to both kernels are the constants of initial time  $t_0$ , initial velocity  $v_0$ , and time difference  $\Delta t$ . The first kernel also takes as an input the initial angle of launch  $\theta_0$ , whilst the second kernel takes the initial height  $h_0$ .

For both kernels, the total number of points along a timeline is computed according to,

$$(3) \quad n = \frac{|t_n - t_0|}{\Delta t_m},$$

where  $|t_n - t_0|$  is the duration of computation in seconds, with  $t_0 < t_1 < \dots < t_n$  the time interval, and time change  $\Delta t_m = t_m - t_{m-1}$  for all  $m = 1, \dots, n$ . Since we are considering equally spaced time intervals (3), all  $\Delta t_m$  will be equal, and as such will be denoted by  $\Delta t$ .

Furthermore, the outputs for both kernels are:

- position vector  $\mathbf{r}(t) = x(t)\hat{\mathbf{e}}_x + y(t)\hat{\mathbf{e}}_y$
- velocity vector  $\mathbf{v}(t) = v_x(t)\hat{\mathbf{e}}_x + v_y(t)\hat{\mathbf{e}}_y$
- speed  $\|\mathbf{v}(t)\|_2 = \sqrt{v_x(t)^2 + v_y(t)^2}$

Lastly, the set of equations in (1) is implemented in a compute kernel as,

$$(4) \quad f_k(t_0, \Delta t, v_0, \theta; \mathbf{r}(t), \mathbf{v}(t), \|\mathbf{v}(t)\|_2) = \begin{cases} t_1 = t_0 + k\Delta t \\ v_1 = gt_1 \\ v_2 = v_0 \cos(\theta) \\ v_3 = v_0 \sin(\theta) \\ v_4 = 2v_3 \\ v_5 = v_4 - v_1 \\ v_6 = v_0^2 - v_1v_5 \\ x(t) = v_2t_1 \\ y(t) = \frac{1}{2}v_5t_1 \\ v_x(t) = v_2 \\ v_y(t) = v_3 - v_1 \\ \|\mathbf{v}(t)\|_2 = \sqrt{v_6} \end{cases}$$

with the set of equations in (2) implemented in a compute kernel as,

$$(5) \quad f_k(t_0, \Delta t, v_0, h_0; \mathbf{r}(t), \mathbf{v}(t), \|\mathbf{v}(t)\|_2) = \begin{cases} t_1 = t_0 + k\Delta t \\ v_1 = gt_1 \\ v_2 = v_0^2 + v_1^2 \\ x(t) = v_0t_1 \\ y(t) = h_0 - \frac{1}{2}v_1t_1 \\ v_x(t) = v_0 \\ v_y(t) = -v_1 \\ \|\mathbf{v}(t)\|_2 = \sqrt{v_2} \end{cases}$$

for  $k = 0, \dots, n$ . Note that, one may recover the equations in (1) and (2) by elementary algebraic substitutions.

### 3 BUILD REQUIREMENTS

To successfully build the Xcode project, containing the sources and the kernel, use the following:

- Mac OS X v10.6 or later.
- Xcode v3.2 or later.

## 4 RUNTIME REQUIREMENTS

On Mac OS X v10.6 or later, to use NVidia GPU as a compute device, use one of the following hardware configurations:

- MacBook Pro w/NVidia GeForce 8600M
- Mac Pro w/NVidia GeForce 8800GT

## 5 PACKAGING LIST

- ReadMe.txt
- Docs
  - Trajectories.pdf
- Sources
  - Kernel
    - \* TrajectoriesKernel.cl
  - Main
    - \* Trajectories.cpp
  - OpenCL
    - \* Headers
      - OpenCLBuffer.h
      - OpenCLFile.h
      - OpenCLKernel.h
      - OpenCLKit.h
      - OpenCLProgram.h
    - \* Sources
      - OpenCLBuffer.cpp
      - OpenCLFile.cpp
      - OpenCLKernel.cpp
      - OpenCLProgram.cpp
  - Trajectory
    - \* Trajectory.cpp
    - \* Trajectory.h
- Trajectories.xcodeproj