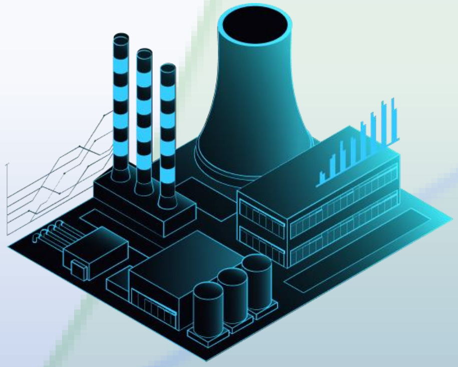
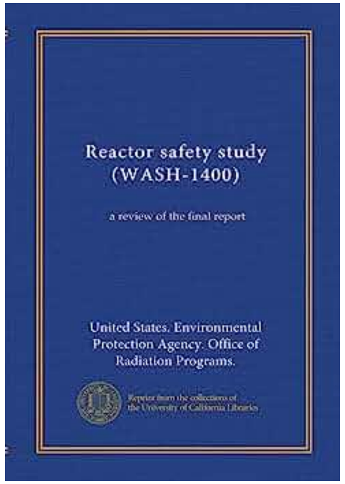
How Uncertainty Quantification and Probabilistic Safety Assessment for Nuclear Safety go Hand-in-Hand?

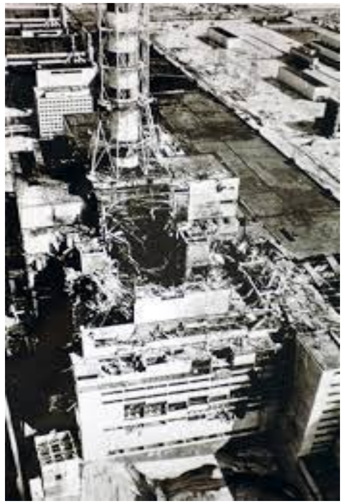


Speaker: Adolphus Lye, Research Fellow

Date: 18-July-2023

PSA in Nuclear Safety and What do we know?





Conceptual Summary:

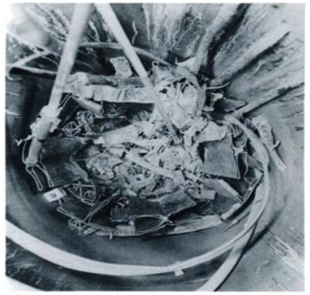
- Started with the Rasmussen Report *a.k.a* WASH-1400;
- A data-based stochastic risk modelling approach to study a severe accident;
- A quantitative risk assessment reflecting the probability of occurrence of an event;
- Main objectives:
  - ✓ Identify the different combinations of events leading to the severe accident;
  - ✓ Assessing the corresponding probability of occurrence for each event combination; and
  - ✓ Evaluating the associated consequences for each event combination

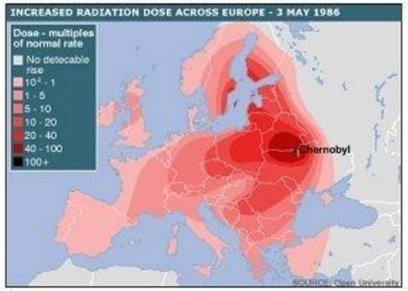
PSA in Nuclear Safety and What do we know?

Conceptual Summary:

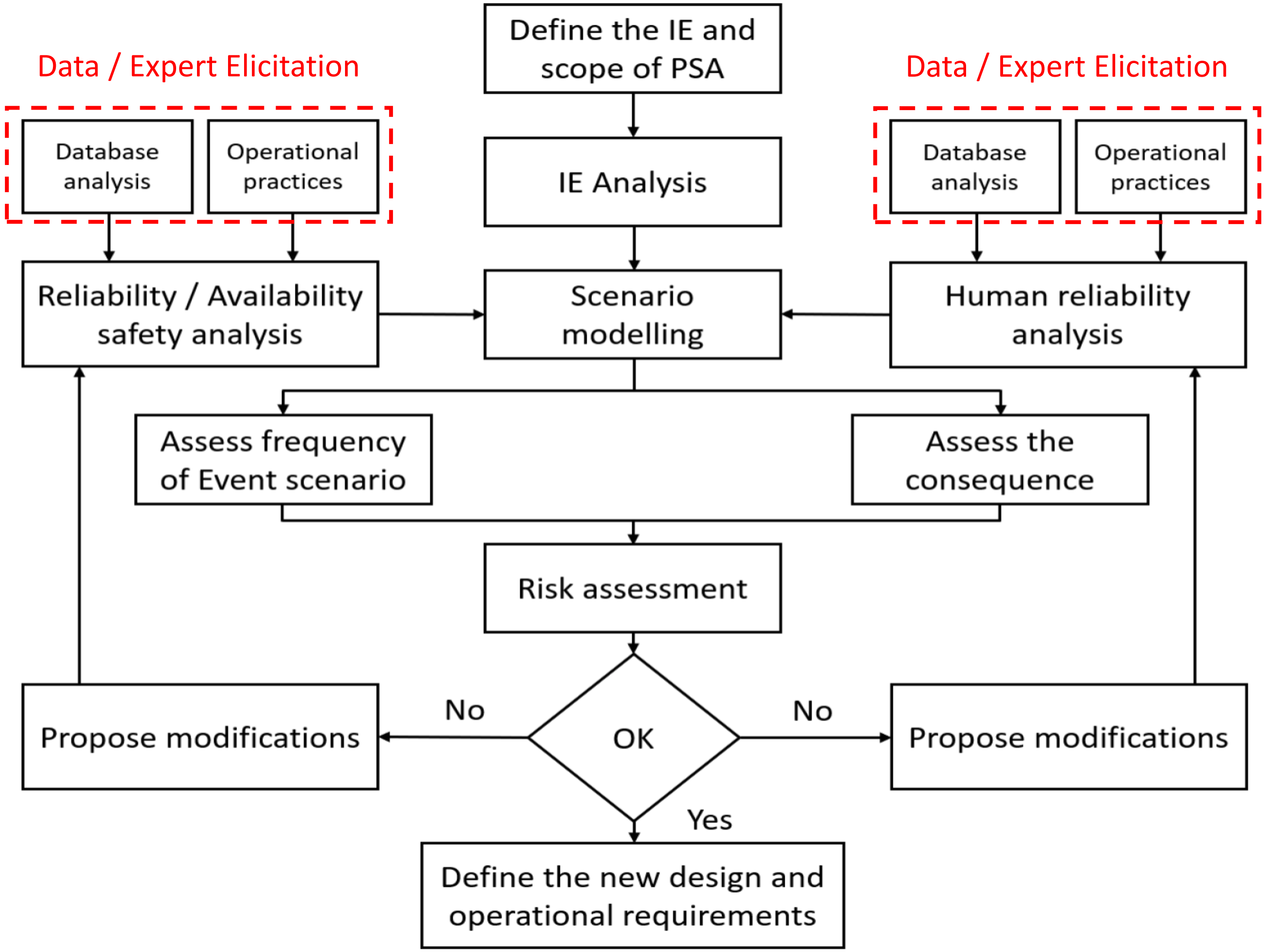
- Scope of Analysis:

Level 1:	Level 2:	Level 3:
Probability of core degradation	Probability of radioactive releases	Probability of impact on public and environment





PSA in Nuclear Safety and What do we know?



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Advances in System Reliability Engineering
2019, Pages 43-75

Chapter 2 - Deterministic and Probabilistic Safety Analyses

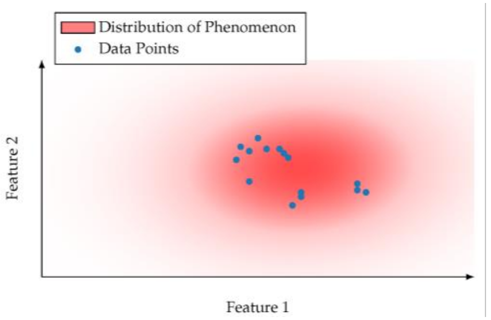
Vanderley de Vasconcelos, Wellington Antonio Soares, Antônio Carlos Lopes da Costa, Amanda Laureano Raso

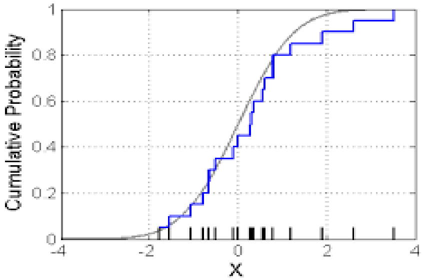
Tools employed for PSA	
ETA	Event Tree Analysis
FTA	Fault Tree Analysis
BN	Bayesian Network Analysis
PN	Petri Net Analysis
BMU	Bayesian Model Updating
RBA	Reliability-based Analysis

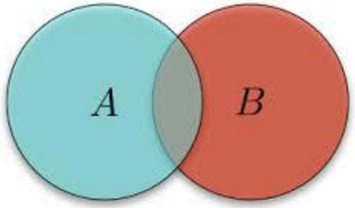
Challenges with PSA in Nuclear Safety

- Key selling point in PSA is its ability to quantify the risk of a severe accident based on data - e.g. component failure/reliability data or expert judgement;

- However, such technique falls short under the following:
  - ❑ Scarce data / information;
  - ❑ Independence assumption;
  - ❑ Distribution model assumption.



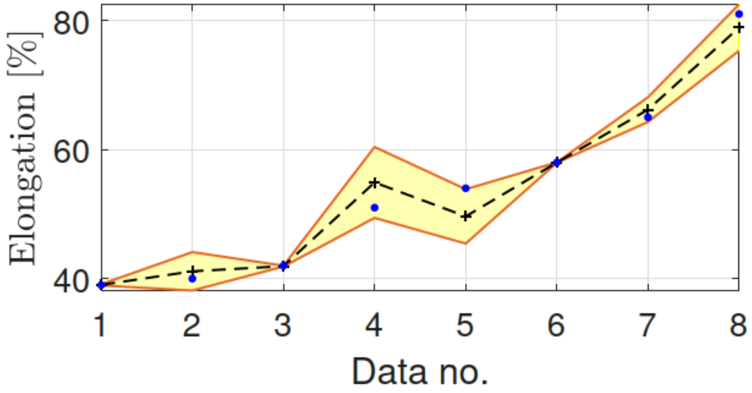


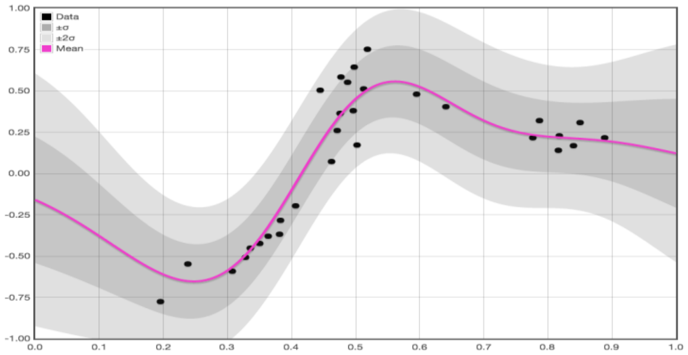


Challenges with PSA in Nuclear Safety

- **Challenge 1 – Scarce data / information:**
  - Attributed to the high costs, complexity, or the time required to perform experiments – e.g. failure data of a valve or pump;
  - This introduces uncertainty due to lack of knowledge over the probability estimates of root events or component failure;
  - Resulting estimates on the Top event probability would not be meaningful using estimates from the scarce data set (i.e. FTA);

❖ Question: Can we quantify and propagate the uncertainty due to limited data into the resulting probability estimates?





Solutions and Tools

- **Solutions to Challenge 1:**

1) Bayesian Model Updating

✓ A stochastic approach towards inferring uncertain parameters – e.g. Failure rate parameter;

✓ Based on Bayes' Rule:

$$P(\boldsymbol{\theta}|D, M) = \frac{P(\boldsymbol{\theta}|M) \cdot P(D|\boldsymbol{\theta}, M)}{P(D|M)}$$

Mechanical Systems and Signal Processing
Volume 159, October 2021, 107760

Sampling methods for solving Bayesian model updating problems: A tutorial

Adolphus Lye [a], Alice Cicirello [a b], Edoardo Patelli [a c]

✓ Estimates are obtained numerically via sampling: Markov Chain Monte Carlo or Sequential Monte Carlo.

2) Interval Arithmetic

✓ To represent uncertain parameters numerically as intervals instead of point values;

✓ Mathematical operations with Intervals:

- $[a, b] \pm [c, d] = [a \pm c\,,\ \ b \pm d]$
- $[a, b] \times [c, d] = [\min\{ac, ad, bc, bd\},\ \max\{ac, ad, bc, bd\}]$
- $[a, b] \div [c, d] = [a, b] \times \frac{1}{[c,d]}$

$$\frac{1}{[y_1, y_2]} = \left[\frac{1}{y_2}, \frac{1}{y_1}\right] \qquad \text{if } 0 \notin [y_1, y_2]$$

$$\frac{1}{[y_1, 0]} = \left[-\infty, \frac{1}{y_1}\right]$$

$$\frac{1}{[0, y_2]} = \left[\frac{1}{y_2}, \infty\right]$$

$$\frac{1}{[y_1, y_2]} = \left[-\infty, \frac{1}{y_1}\right] \cup \left[\frac{1}{y_2}, \infty\right] \subseteq [-\infty, \infty] \qquad \text{if } 0 \in (y_1, y_2)$$

Application Example

- **Solution to Challenge 1: Bayesian Model Updating**

Parameter:	Prior:	Likelihood:
Demand failure probability, θ	$P(\theta) = Beta(\theta; 23, 6717)$	$P(X = 7 \mid \theta) = Bin(7; 290, \theta)$
Operational failure rate, λ	$P(\lambda) = Gamma(22, 2920.63)$	$P(Y = 1 \mid \lambda) = Po(579.37\ \lambda)$

Bayesian Model Updating of Reliability Parameters using Transitional Markov Chain Monte Carlo with Slice Sampling

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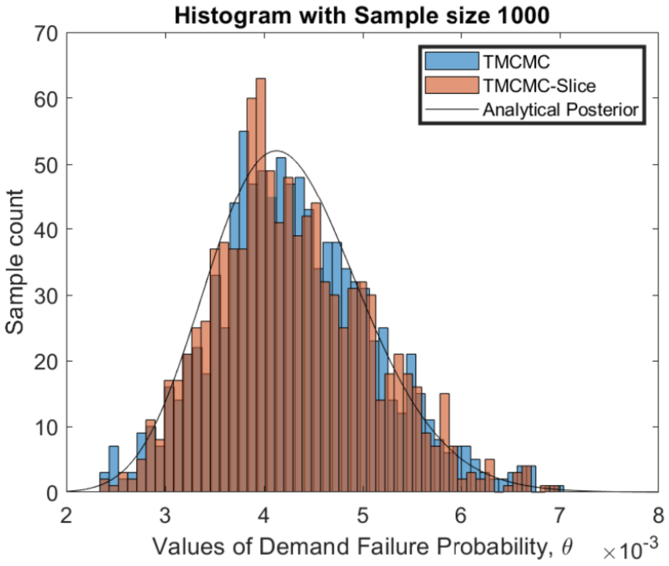
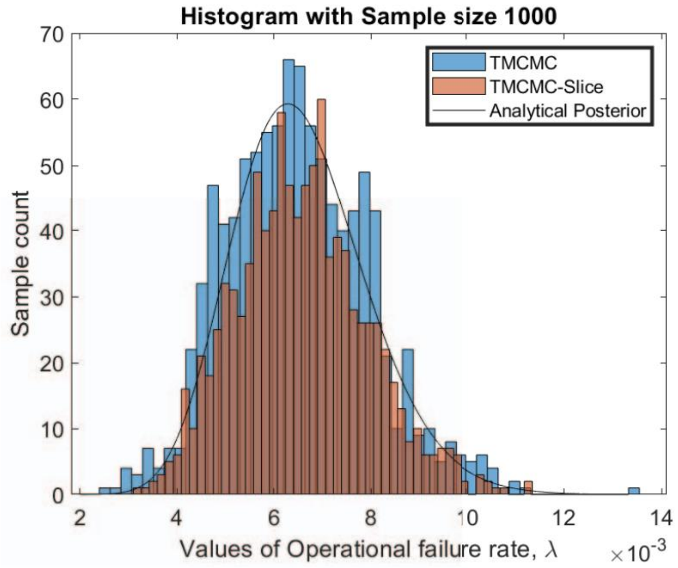
Alice Cicirello

Dynamics, Vibration and Uncertainty (DVU) Lab, University of Oxford, United Kingdom.
E-mail: alice.cicirello@eng.ox.ac.uk

Edoardo Patelli

Centre for Intelligent Infrastructure, Civil and Environmental Engineering, University of Strathclyde, United Kingdom.
E-mail: edoardo.patelli@strath.ac.uk





Parameter:	TMCMC:	TMCMC-Slice:
θ	4.32×10^{-3} (8.16×10^{-4})	4.29×10^{-3} (7.92×10^{-4})
λ	6.51×10^{-3} (1.48×10^{-3})	6.57×10^{-3} (1.34×10^{-3})

Illustrative Example

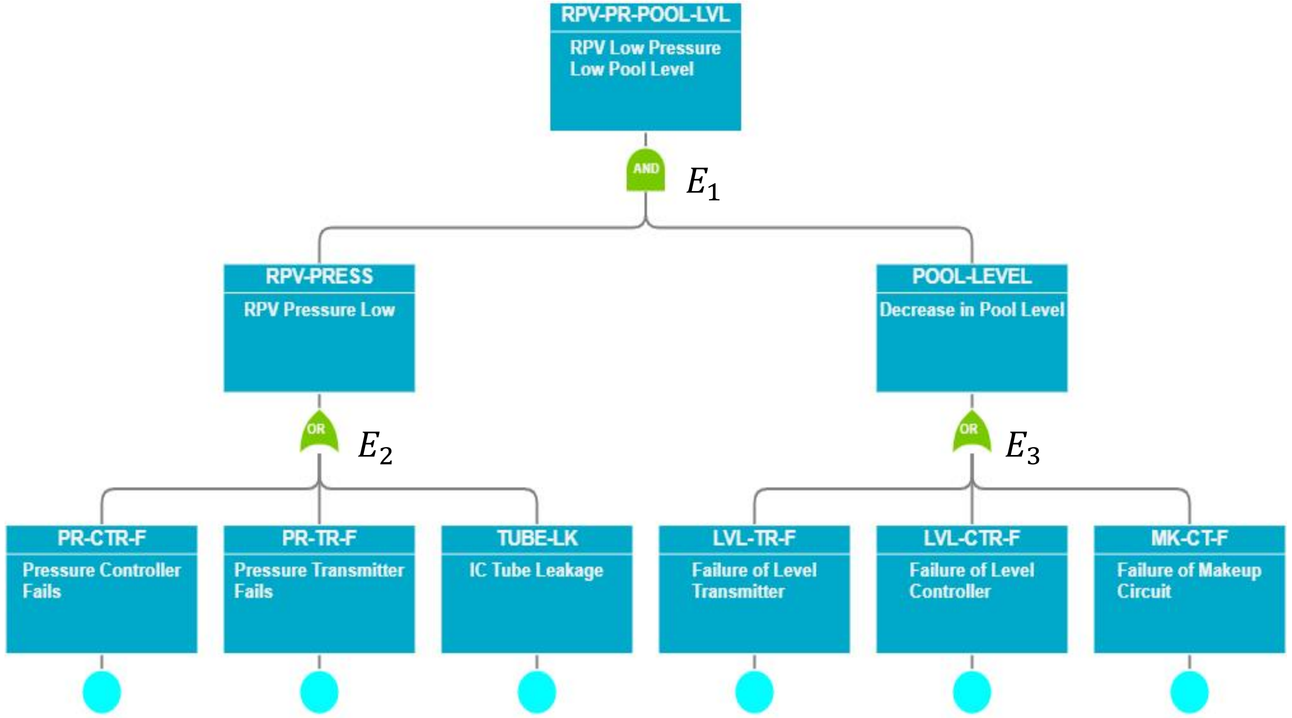
- **Solution to Challenge 1: Interval Arithmetic**

juliacon

ProbabilityBoundsAnalysis.jl: Arithmetic with sets of distributions

Ander Gray[1], Scott Ferson[1], and Edoardo Patelli[1, 2]

[1]Institute for Risk and Uncertainty, University of Liverpool
[2]Centre for Intelligent Infrastructure, University of Strathclyde



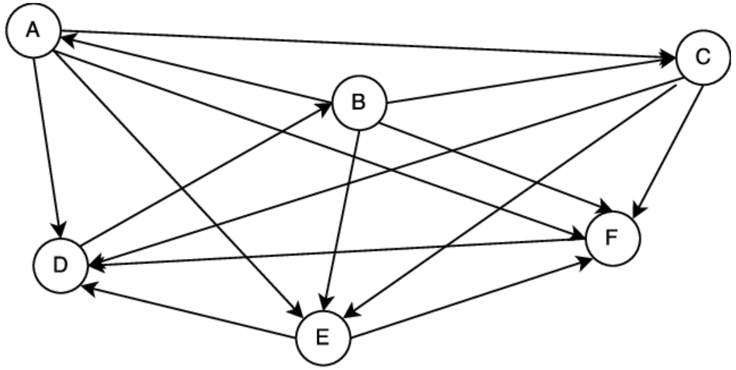
Component:	Imprecise Probabilities:
PR-CTR-F	$[5.11, 6.25] \times 10^{-6}$
PR-TR-F	$[4.61, 5.63] \times 10^{-6}$
TUBE-LK	$[1.44, 1.76] \times 10^{-4}$
LVL-TR-F	$[7.92, 9.68] \times 10^{-6}$
LVL-CTR-F	$[5.11, 6.25] \times 10^{-6}$
MK-CT-F	$[0.90, 1.10] \times 10^{-4}$

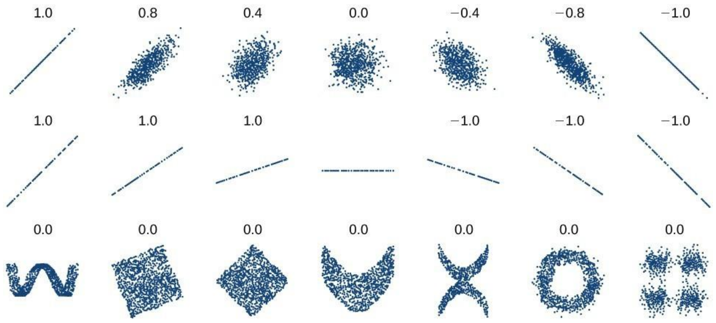
Event:	Case 1: Independence
E_1	$[2.85, 4.25] \times 10^{-8}$
E_2	$[1.54, 1.88] \times 10^{-4}$
E_3	$[1.85, 2.26] \times 10^{-4}$

Challenges with PSA in Nuclear Safety

- **Challenge 2 – Independence assumption:**
  - A common assumption made that the inter-event dependencies can be neglected – this is done to simplify calculations;
  - However, this does not reflect the reality where such dependencies between events may exist – esp. for multi-unit PSA;
  - Such dependencies are usually not known explicitly and present a source of uncertainty;

❖ Question: Can we generalise the probability computation by accounting for the uncertain dependencies?





Solutions and Tools

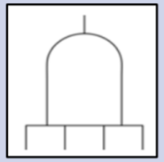
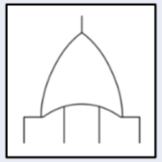
- **Solution to Challenge 2:**

  1) Probability Bounds Analysis with Fuzzy Logic & Fréchet Bounds

Correlated Boolean Operators for Uncertainty Logic

Enrique Miralles-Dolz[1,2(✉)], Ander Gray[1,2(✉)], Edoardo Patelli[3], and Scott Ferson[1]

[1] Institute for Risk and Uncertainty, University of Liverpool, Liverpool, UK
{enmidol,akgray,ferson}@liverpool.ac.uk
[2] United Kingdom Atomic Energy Authority, Abingdon, UK
[3] Centre for Intelligent Infrastructure, University of Strathclyde, Glasgow, UK
edoardo.patelli@strath.ac.uk

Logic Type:		Formula:
	AND	$\max\left(0, \sum_{k=1}^{n} P(A_k) - (n-1)\right) \leq P\left(\bigcap_{k=1}^{n} A_k\right) \leq \min_k\{P(A_k)\}$
	OR	$\max_k\{P(A_k)\} \leq P\left(\bigcup_{k=1}^{n} A_k\right) \leq \min\left(1, \sum_{k=1}^{n} P(A_k)\right)$

- General Formulation:

  ✓ The subset $S = [\underline{\rho_{AB}}, \overline{\rho_{AB}}] \in [-1, 1]$ are determined as follows:

$$\underline{\rho_{AB}} = \frac{\max(P(A)+P(B)-1, 0) - P(A)\cdot P(B)}{\sqrt{P(A)P(A')P(B)P(B')}}, \qquad \overline{\rho_{AB}} = \frac{\min\big(P(A), P(B)\big) - P(A)\cdot P(B)}{\sqrt{P(A)P(A')P(B)P(B')}}$$

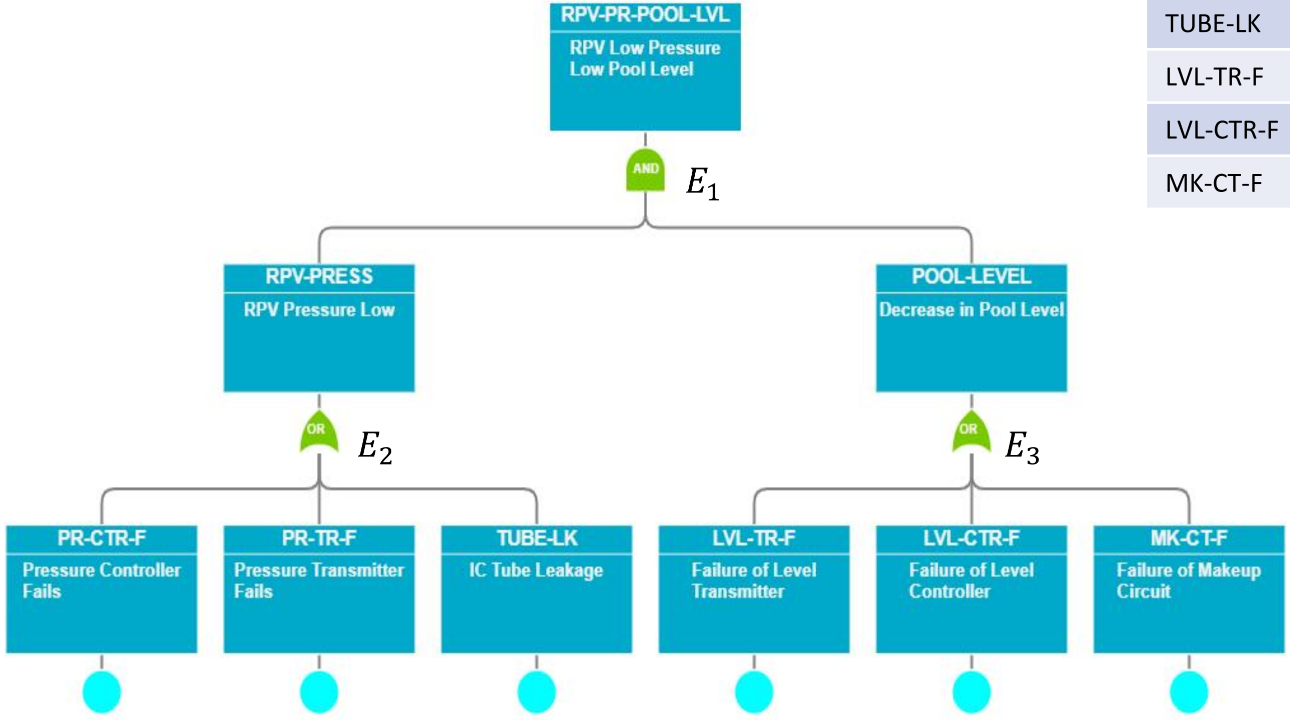
  ✓ As such, to ensure that the answers for $P(A \cap B) \in [0, 1]$:

$$P(A \cap B) \begin{cases} \max(P(A)+P(B)-1, 0)\text{, if } \rho_{AB} \leq \underline{\rho_{AB}} \\ \min\big(P(A), P(B)\big)\text{, if } \rho_{AB} \geq \overline{\rho_{AB}} \\ P(A)P(B) + \rho_{AB}\cdot\sqrt{P(A)P(A')P(B)P(B')}\text{, otherwise} \end{cases}$$

$$P(A \cup B) = 1 - P(A' \cap B')$$

Illustrative Example

- **Solution to Challenge 2: Probability Bounds Analysis with Fuzzy Logic & Fréchet Bounds**



Component:	Imprecise Probabilities:
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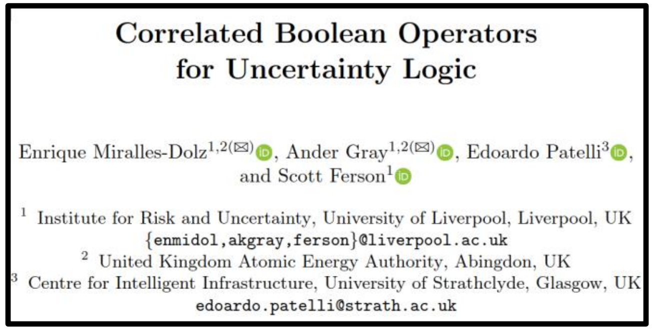

Correlated Boolean Operators for Uncertainty Logic

Enrique Miralles-Dolz[1,2](✉), Ander Gray[1,2](✉), Edoardo Patelli[3], and Scott Ferson[1]

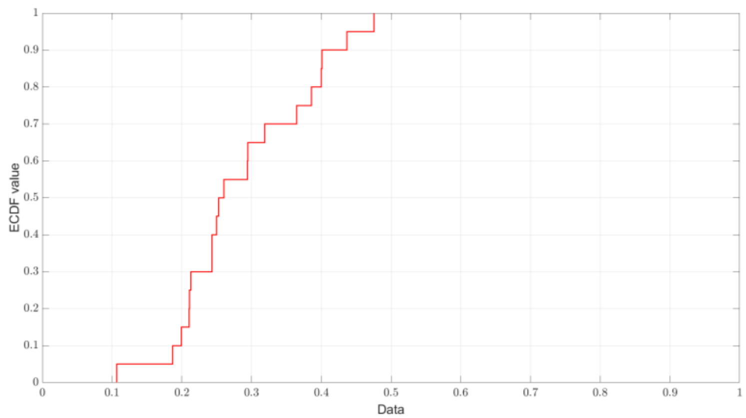
[1] Institute for Risk and Uncertainty, University of Liverpool, Liverpool, UK
{enmidol,akgray,ferson}@liverpool.ac.uk
[2] United Kingdom Atomic Energy Authority, Abingdon, UK
[3] Centre for Intelligent Infrastructure, University of Strathclyde, Glasgow, UK
edoardo.patelli@strath.ac.uk

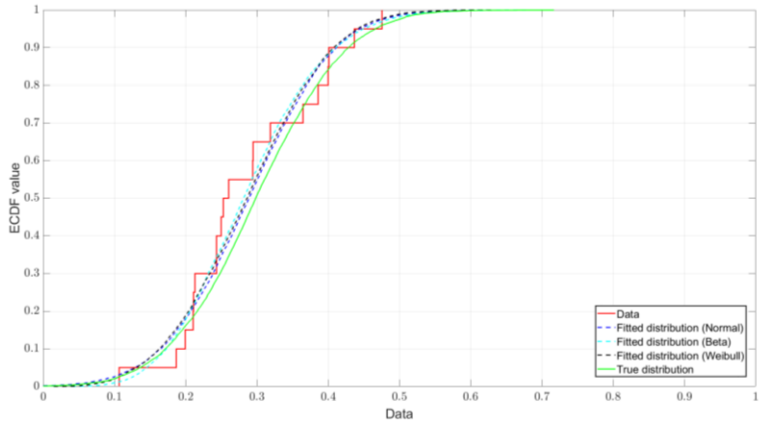
Event:	Case 2: Mixed Dependence	Case 3: Uncertain Dependence
E_1	$\rho = 0$	$\rho = [-1, 1]$
E_2	$\rho = [-0.3, 0.1]$	
E_3	$\rho = 0.15$	

Event:	Case 2: Mixed Dependence	Case 3: Uncertain Dependence
E_1	$[2.50, 3.86] \times 10^{-8}$	$[0.00, 1.88] \times 10^{-4}$
E_2	$[1.48, 1.88] \times 10^{-4}$	$[1.44, 1.88] \times 10^{-4}$
E_3	$[1.68, 2.06] \times 10^{-4}$	$[0.00, 1.88] \times 10^{-4}$

Challenges with PSA in Nuclear Safety

- **Challenge 3 – Distribution model assumption:**
  - Presently, precise distribution models have been fitted on a given data-set, even when the data-set is small;
  - Beta distribution to model Demand-based failure; Gamma distribution to model Failure rate; and Exponential distribution to model Time-based failure;
  - Such assignments are done in a deterministic manner;
  - ❖ Question: Can we loosen the precision or assumption over the distribution of the failure probability?





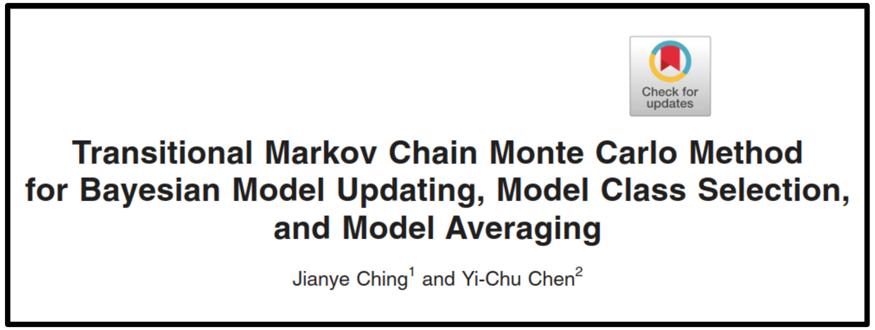
Solutions and Tools

- **Solutions to Challenge 3:**

1) Bayesian Model Selection

✓ Based on Bayes' Rule:

$$P(\boldsymbol{\theta}|D,M) = \frac{P(\boldsymbol{\theta}|M)\cdot P(D|\boldsymbol{\theta},M)}{P(D|M)}$$


Transitional Markov Chain Monte Carlo Method for Bayesian Model Updating, Model Class Selection, and Model Averaging

Jianye Ching[1] and Yi-Chu Chen[2]

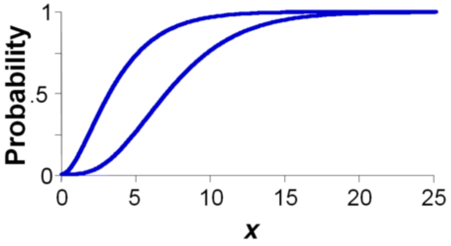
✓ $P(D|M)$ is obtained numerically to rank the choice of model: Transitional Markov Chain Monte Carlo.

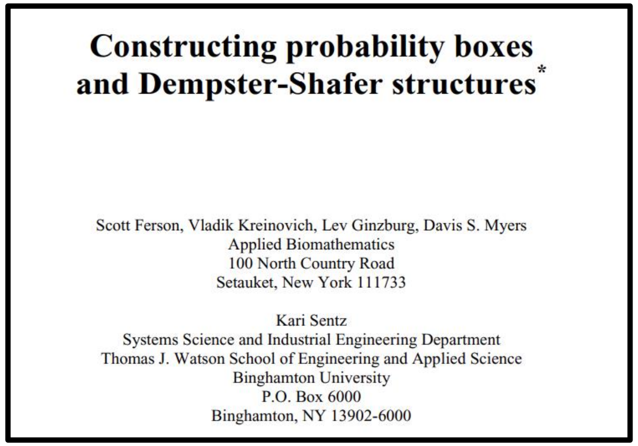
2) Probability boxes

✓ Imprecise CDF expressed as distributional bounds;

✓ Mathematically expressed as:

$$\underline{F}(x) \leq F(x) \leq \bar{F}(x)$$




Constructing probability boxes and Dempster-Shafer structures*

Scott Ferson, Vladik Kreinovich, Lev Ginzburg, Davis S. Myers
Applied Biomathematics
100 North Country Road
Setauket, New York 111733

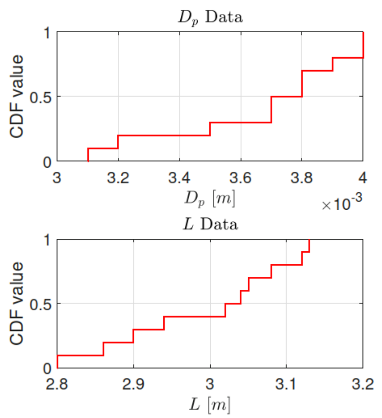
Kari Sentz
Systems Science and Industrial Engineering Department
Thomas J. Watson School of Engineering and Applied Science
Binghamton University
P.O. Box 6000
Binghamton, NY 13902-6000

✓ Mainly categorised into two distinct types:

- ❑ Distributional – constructed from a known distribution family, e.g. Normal, or Beta
- ❑ Distribution-free – constructed from empirical distributions or via matching moments

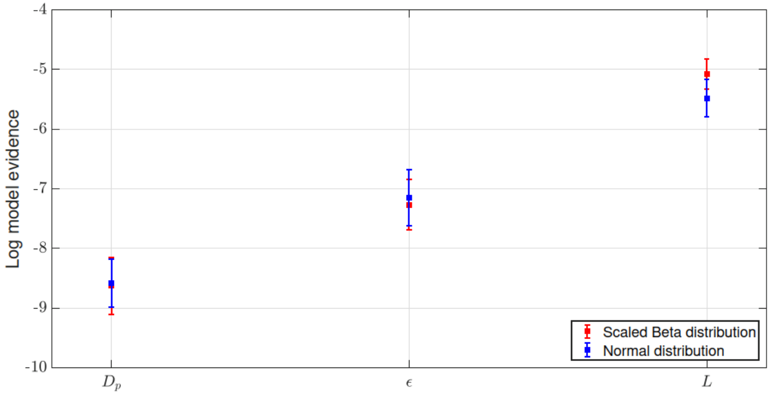
Illustrative Example

- **Solutions to Challenge 3: Bayesian Model Selection**

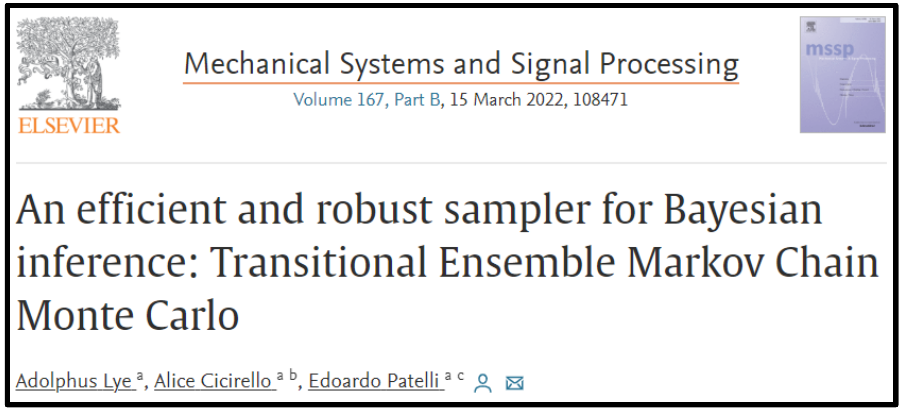


Variable:	Distribution candidates:	
D_p	Normal (μ, σ)	$0.0005 \times$ Beta (α, β)
ε	Normal (μ, σ)	$0.5 \times$ Beta (α, β)
L	Normal (μ, σ)	$5 \times$ Beta (α, β)

Sample 1000 samples with the TEMCMC sampler and repeat this 100 times.

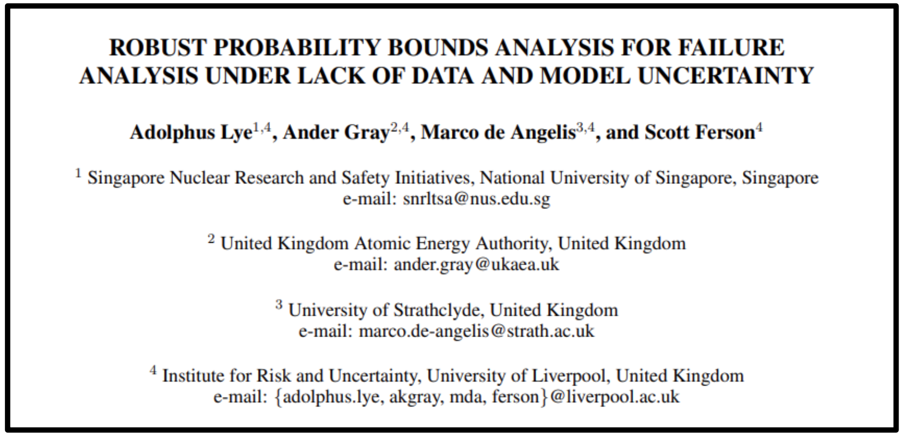


Variable:	Normal	Beta
D_p	-8.5844 (0.4011)	-8.6268 (0.4808)
ε	-7.1504 (0.4707)	-7.2679 (0.4195)
L	-5.4810 (0.3116)	-5.0708 (0.2526)


Mechanical Systems and Signal Processing
Volume 167, Part B, 15 March 2022, 108471

An efficient and robust sampler for Bayesian inference: Transitional Ensemble Markov Chain Monte Carlo

Adolphus Lye [a], Alice Cicirello [a b], Edoardo Patelli [a c]


ROBUST PROBABILITY BOUNDS ANALYSIS FOR FAILURE ANALYSIS UNDER LACK OF DATA AND MODEL UNCERTAINTY

Adolphus Lye[1,4], Ander Gray[2,4], Marco de Angelis[3,4], and Scott Ferson[4]

[1] Singapore Nuclear Research and Safety Initiatives, National University of Singapore, Singapore
e-mail: snrltsa@nus.edu.sg

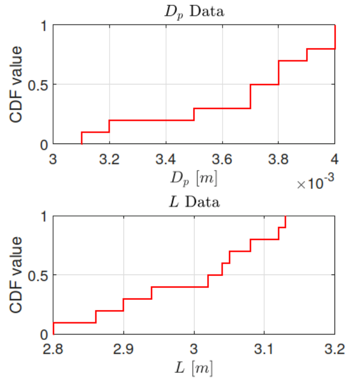
[2] United Kingdom Atomic Energy Authority, United Kingdom
e-mail: ander.gray@ukaea.uk

[3] University of Strathclyde, United Kingdom
e-mail: marco.de-angelis@strath.ac.uk

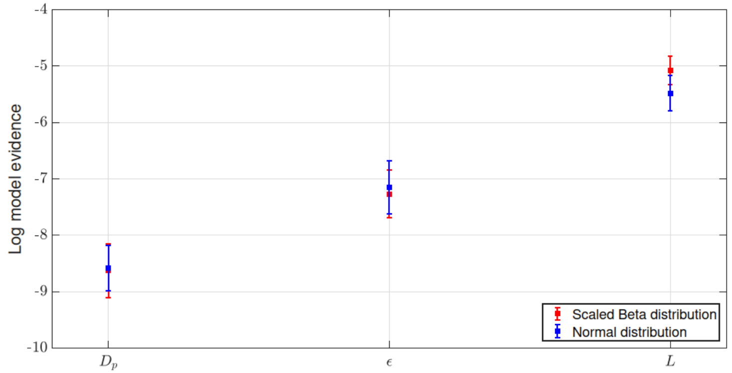
[4] Institute for Risk and Uncertainty, University of Liverpool, United Kingdom
e-mail: {adolphus.lye, akgray, mda, ferson}@liverpool.ac.uk

Illustrative Example

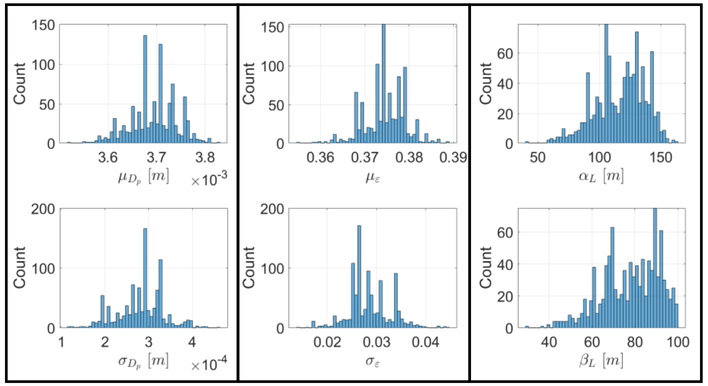
- **Solution to Challenge 3: Probability Boxes** (Distributional)



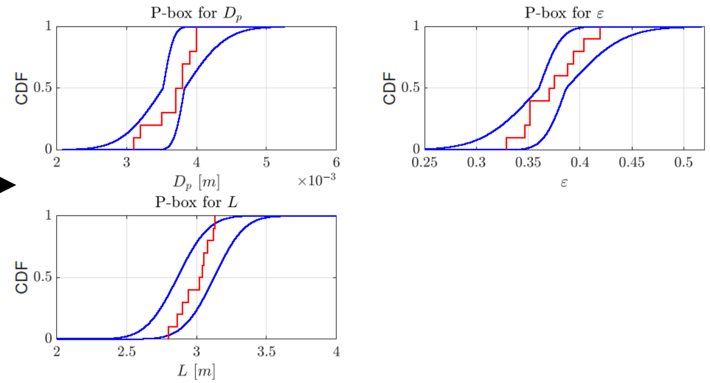
Bayesian model selection



Bayesian model updating



D_p ε L



juliacon

ProbabilityBoundsAnalysis.jl: Arithmetic with sets of distributions

Ander Gray[1], Scott Ferson[1], and Edoardo Patelli[1, 2]

[1]Institute for Risk and Uncertainty, University of Liverpool
[2]Centre for Intelligent Infrastructure, University of Strathclyde

ROBUST PROBABILITY BOUNDS ANALYSIS FOR FAILURE ANALYSIS UNDER LACK OF DATA AND MODEL UNCERTAINTY

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[1] Singapore Nuclear Research and Safety Initiatives, National University of Singapore, Singapore
e-mail: snrltsa@nus.edu.sg

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e-mail: ander.gray@ukaea.uk

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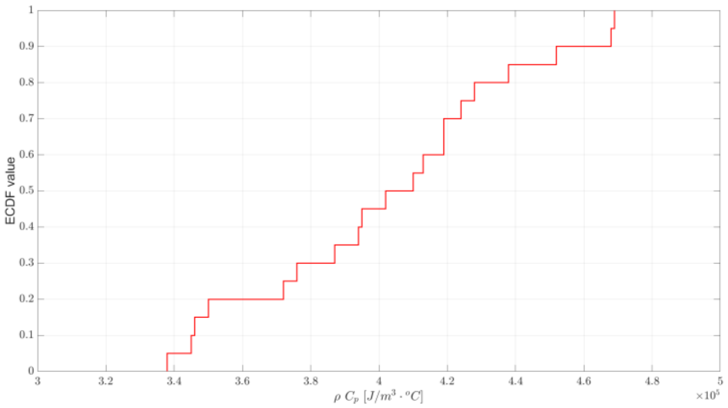
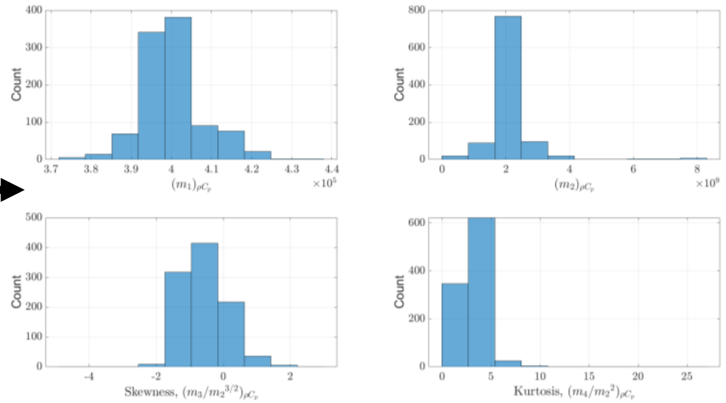
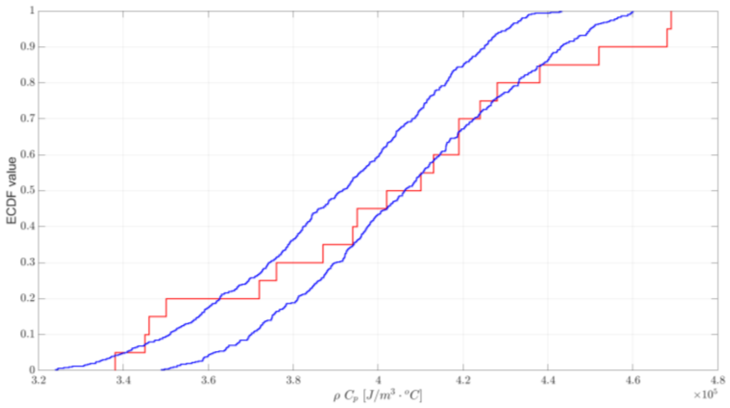
[4] Institute for Risk and Uncertainty, University of Liverpool, United Kingdom
e-mail: {adolphus.lye, akgray, mda, ferson}@liverpool.ac.uk

Illustrative Example

- **Solution to Challenge 3: Probability Boxes** (Distribution-free)

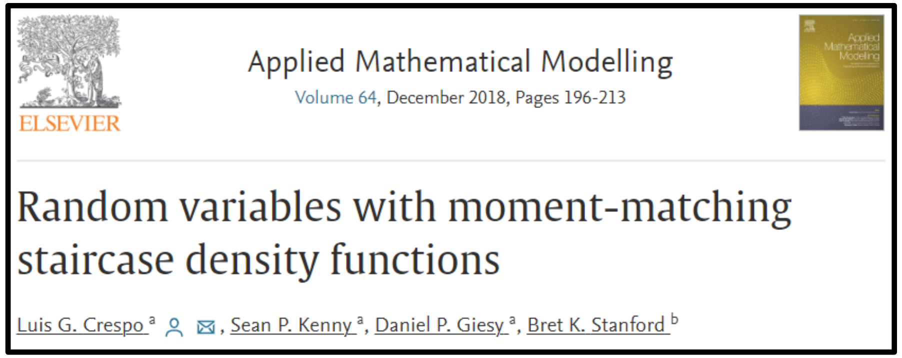






✓ The Staircase density function is a meta-model to model distributions using the method of matching moments:

$$m_r = \int (z-\mu)^r \cdot f(z) \cdot dz \;,\; r = 0, 1, 2, 3, 4$$



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Random variables with moment-matching staircase density functions

Luis G. Crespo [a], Sean P. Kenny [a], Daniel P. Giesy [a], Bret K. Stanford [b]

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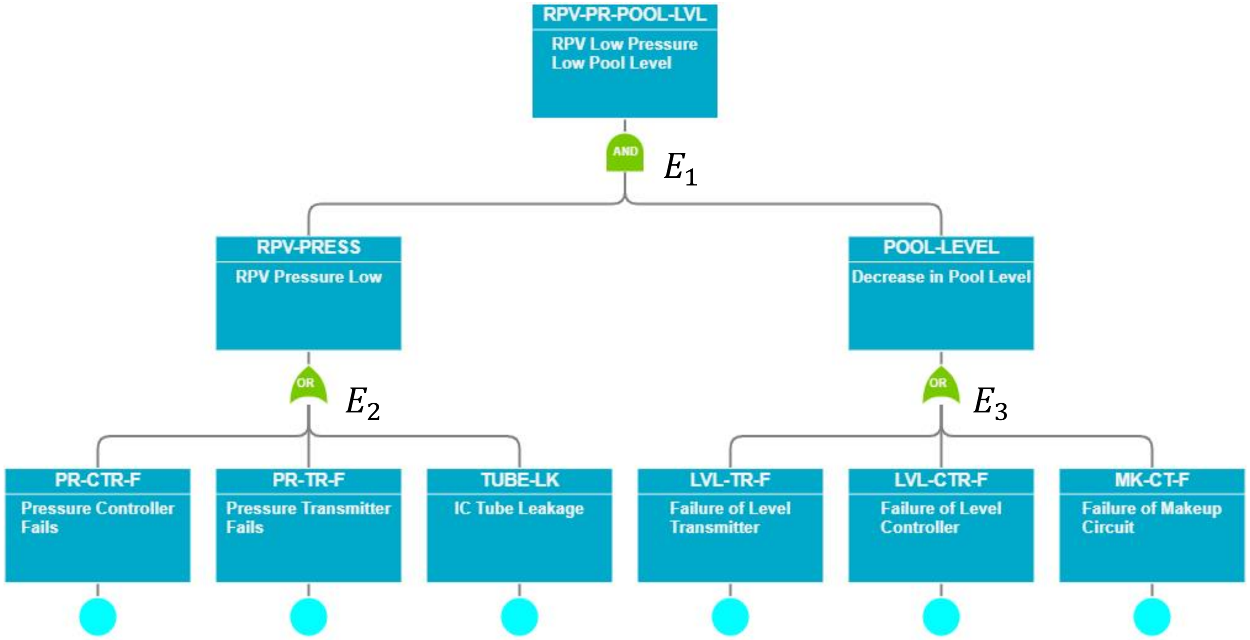
Robust optimization of a dynamic Black-box system under severe uncertainty: A distribution-free framework

Adolphus Lye [a 1], Masaru Kitahara [b 1], Matteo Broggi [b], Edoardo Patelli [c]

Illustrative Example

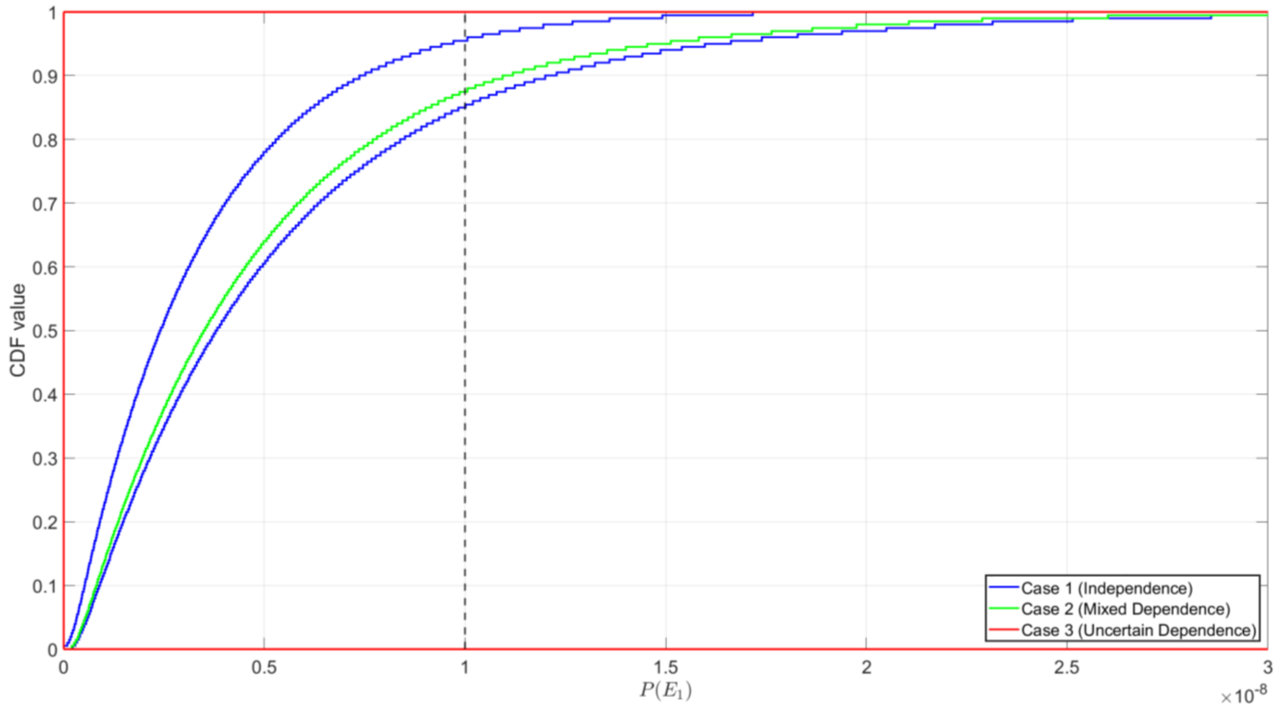
- **Solution to Challenge 3: Probability Boxes** (Probabilistic Risk Analysis)



Component:	Imprecise Distributions (P-box):
PR-CTR-F	$\exp([6.39, 7.81] \times 10^{-7})$
PR-TR-F	$\exp([5.76, 7.04] \times 10^{-7})$
TUBE-LK	$\exp([1.80, 2.20] \times 10^{-5})$
LVL-TR-F	$\exp([0.99, 1.21] \times 10^{-6})$
LVL-CTR-F	$\exp([6.39, 7.81] \times 10^{-7})$
MK-CT-F	$[0.90, 1.10] \times 10^{-4}$

Illustrative Example

- **Solution to Challenge 3: Probability Boxes** (Probabilistic Risk Analysis)



Event:	Case 1: Independence	Case 2: Mixed Dependence	Case 3: Uncertain Dependence
E_1	$\rho = 0$	$\rho = [-0.01, 0]$	$\rho = [-1, 1]$
E_2		$\rho = [-0.3, 0.1]$	
E_3		$\rho = 0.15$	

- Consider the case where the required failure probability of the tank be no more than 10^{-8} such that $P(E_1) \leq 10^{-8}$:

Event:	Case 1: Independence	Case 2: Mixed Dependence	Case 3: Uncertain Dependence
E_1	$[0.850, 0.955]$	$[0.875, 1.00]$	$[0.00, 1.00]$

Summary

Challenge 1 – Scarce data / information

- Can we quantify and propagate the uncertainty due to limited data into the resulting probability estimates?

→

Solutions to Challenge 1:

- Bayesian Model Updating
- Interval arithmetic

Challenge 2 – Independence assumption

- Can we generalise the probability computation by accounting for the uncertain dependencies?

→

Solution to Challenge 2:

- Probability Bounds Analysis with Fuzzy Logic & Fréchet bounds

Challenge 3 – Distribution model assumption

- Can we loosen the precision or assumption over the distribution of the failure probability?

→

Solutions to Challenge 3:

- Bayesian model selection
- Probability boxes (Distributional or Distribution-free)

Summary

Solutions to Challenge 1:

- Bayesian Model Updating
- Interval arithmetic

Tools available:

- Transitional Ensemble Markov Chain Monte Carlo (https://github.com/Adolphus8/Transitional_Ensemble_MCMC.git)
- Probability Bounds Analysis Julia Package (https://github.com/AnderGray/ProbabilityBoundsAnalysis.jl.git)

Solution to Challenge 2:

- Probability Bounds Analysis with Fuzzy Logic & Fréchet bounds

Tools available:

- Uncertain Boolean Logic Julia Package (https://github.com/Institute-for-Risk-and-Uncertainty/UncLogic.jl.git)
- Probability Bounds Analysis Julia Package (https://github.com/AnderGray/ProbabilityBoundsAnalysis.jl.git)

Solutions to Challenge 3:

- Bayesian model selection
- Probability boxes (Distributional or Distribution-free)

Tools available:

- Transitional Ensemble Markov Chain Monte Carlo (https://github.com/Adolphus8/Transitional_Ensemble_MCMC.git)
- Staircase Density Function (https://doi.org/10.1016/j.apm.2018.07.029)

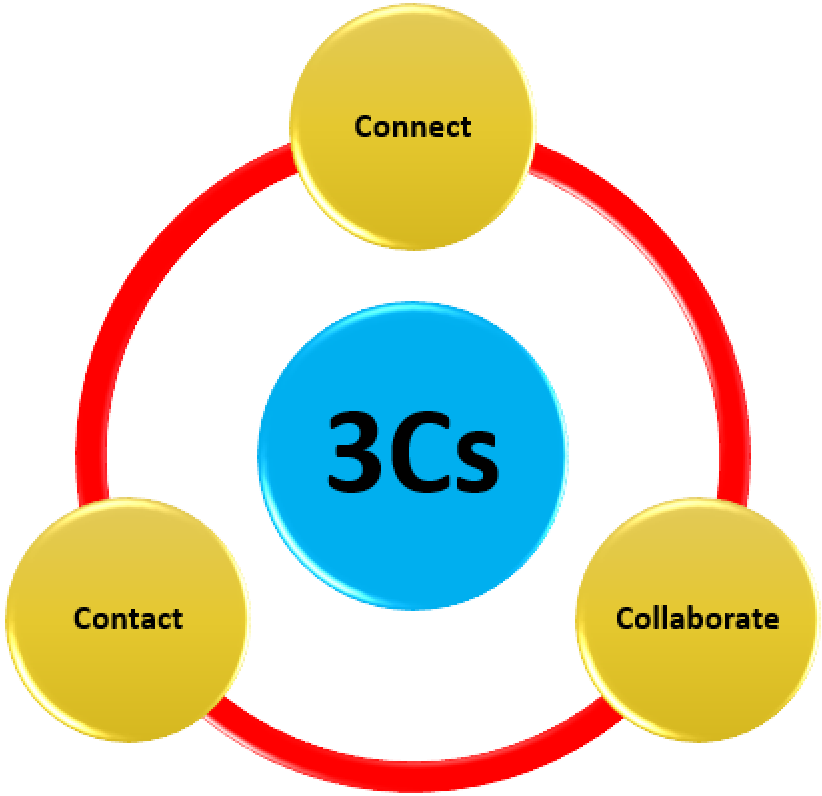
Conclusion

- As useful as PSA is in quantifying the risk of a severe accident, its capabilities are limited by the data size and the underlying assumptions made in the calculations involved;
- These challenges bring forth the need for UQ tools to address the problems highlighted and to loosen the underlying assumptions;
- In doing so, it provides for a more realistic and robust PSA framework as the uncertainty is quantified and reflected in resulting risk estimates;
- There remains opportunities to extend current methods in twinning PSA with UQ to bring forth “fuzzy” yet meaningful bounds on the risk estimates and a distribution-free approach towards the risk analysis.

References:

- **Bayesian Model Updating:**
  - ✓ Adolphus Lye, Alice Cicirello, and Edoardo Patelli (2021). Sampling methods for solving Bayesian model updating problems: A tutorial. *Mechanical Systems and Signal Processing,* ***159***, 107760. doi: 10.1016/j.ymssp.2021.107760
  - ✓ Adolphus Lye, Alice Cicirello, and Edoardo Patelli (2022). An efficient and robust sampler for Bayesian inference: Transitional Ensemble Markov Chain Monte Carlo. *Mechanical Systems and Signal Processing,* ***167***, 108471. doi: 10.1016/j.ymssp.2021.108471
  - ✓ Adolphus Lye, Luca Marino, Alice Cicirello, and Edoardo Patelli (2023). Sequential Ensemble Monte Carlo Sampler for On-Line Bayesian Inference of Time-Varying Parameter In Engineering Applications. *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems Part B: Mechanical Engineering*, ***9***, 031202. doi: 10.1115/1.4056934
- **Probability Bounds Analysis:**
  - ✓ Ander Gray, Scott Ferson, and Edoardo Patelli (2021). ProbabilityBoundsAnalysis. jl: Arithmetic with sets of distributions. In the *Proceedings of JuliaCon,* ***1***.
  - ✓ Adolphus Lye, Ander Gray, Marco De Angelis, Scott Ferson (2021). Robust Probability Bounds Analysis for Failure Analysis under Lack of Data and Model Uncertainty. In the *Proceedings of the 5th ECCOMAS Thematic Conference on Uncertainty Quantification in Computational Sciences and Engineering,* ***1***.
- **Fuzzy Logic:**
  - ✓ Enrique Miralles-Dolz, Ander Gray, Edoardo Patelli, and Scott Ferson (2022). Correlated Boolean operators for uncertainty logic. International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems, **1**, 798-811. doi: 10.1007/978-3-031-08971-8_64
- **P-boxes and Distribution-free Risk Analysis:**
  - ✓ Scott Ferson, Vladik Kreinovick, Lev Ginzburg, and Fari Sentz (2002). Constructing Probability Boxes and Dempster-Shafer Structures. *Sandia National Laboratories,* ***1***. doi: 10.2172/809606
  - ✓ Luis Crespo, Sean Kenny, Daniel Giesy, and Bret Stanford (2018). Random variables with moment-matching staircase density functions. *Applied Mathematical Modelling,* ***64***, 196-213. doi: 10.1016/j.apm.2018.07.029
  - ✓ Adolphus Lye, Masaru Kitahara, Matteo Broggi, and Edoardo Patelli (2022). Robust optimisation of a dynamic Black-box system under severe uncertainty: A Distribution-free framework. *Mechanical Systems and Signal Processing,* ***167***, 108522. doi: 10.1016/j.ymssp.2021.108522
  - ✓ Ander Gray, Scott Ferson, Vladik Kreinovich, and Edoardo Patelli (2022). Distribution-free risk analysis. *International Journal of Approximate Reasoning,* ***146***, 133-156. doi: 10.1016/j.ijar.2022.04.001

3C Policy: Connect, Contact, Collaborate



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