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Software

COSSAN TRAINING COURSE
on UNCERTAINTY QUANTIFICATION

Bayesian Model Updating: Part I

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Software

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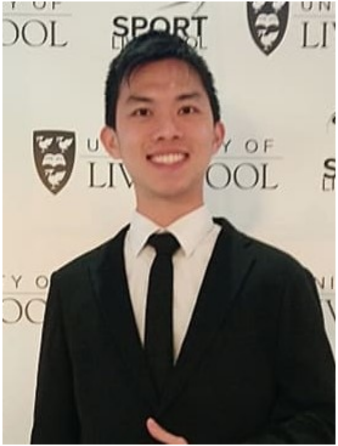
Introduction



- PhD Student, 2^{nd} Year of Study
- Affiliated with Singapore Nuclear Research and Safety Initiatives (SNRSI)
- Research Interests:
  - Bayesian Model Updating;
  - Mathematical Modelling;
  - Nuclear Energy;
  - Predictive Maintenance;
  - Probabilistic Safety Assessment

Pre-requisites

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There are no pre-requisites for this part of the course.

Overview

Model Update

Conceptual Introduction and Motivation

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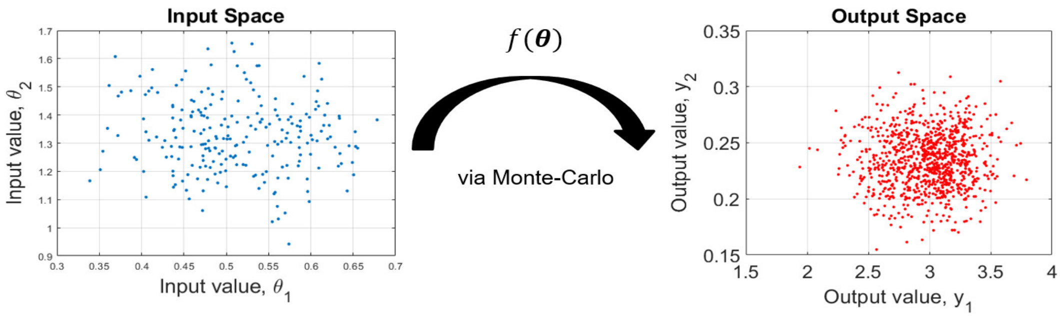
Definition of the Forward problem:



Model Update

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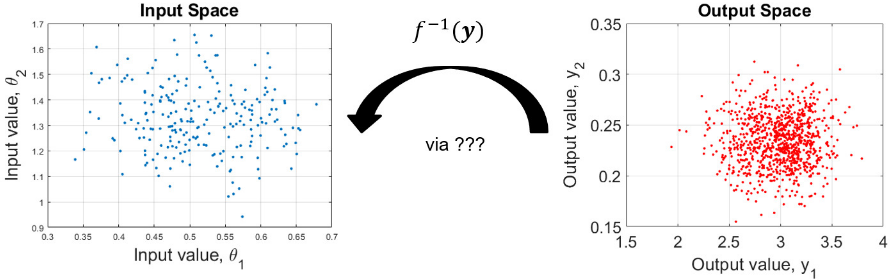
Conceptual Introduction and Motivation

Definition of the Inverse problem:

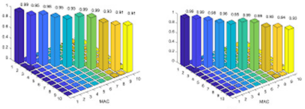


The Answer: Model Updating

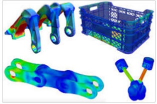
Model Update

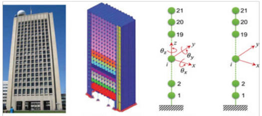
Conceptual Introduction and Motivation

- Mathematical models are used to describe virtual behaviours of engineering structures under operational and extreme conditions;
- For the model to produce output representative of the structure's response, there is a need to update the model's input parameter(s);
- This seeks to minimise the difference between the model output and the measured response of the system.



(a) (b)





(a) Green Building (b) Full scale model (c) 126 DOFs (d) 42 DOFs

Model Update

Conceptual Introduction and Motivation

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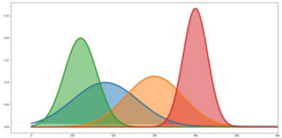

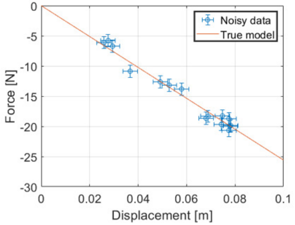

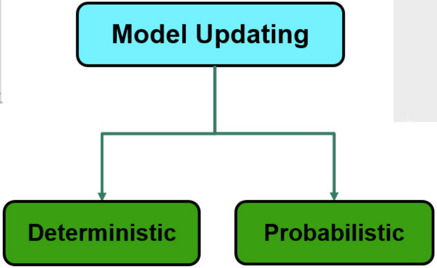

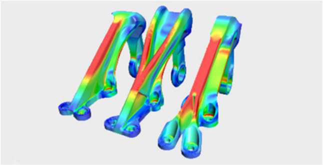

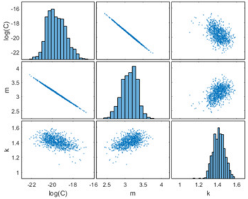

Deterministic Model Update

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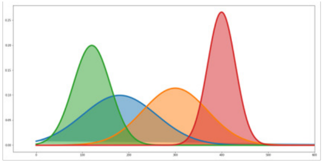
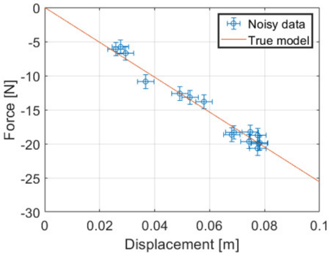
Conceptual Introduction

- Involves the calibration of model based on one data-set;
- Provides point estimates of θ;
- Assumes one definite model to describe the data-set;
- E.g. Maximum Likelihood Estimates, Least-squares method.





Deterministic Model Update

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Conceptual Introduction

There are 4 main problems in performing Deterministic Model Update:

- Mathematical model is assumed to fully describe the Physics of the problem;
- Does not account the factor of “noise” that exists in measurements;
- Does not account for variation of the measured response of nominal identical structures even under the same loading/condition due to manufacturing and material variability;
- Point estimates may not represent the entire set of all possible solutions to θ.

Deterministic Model Update

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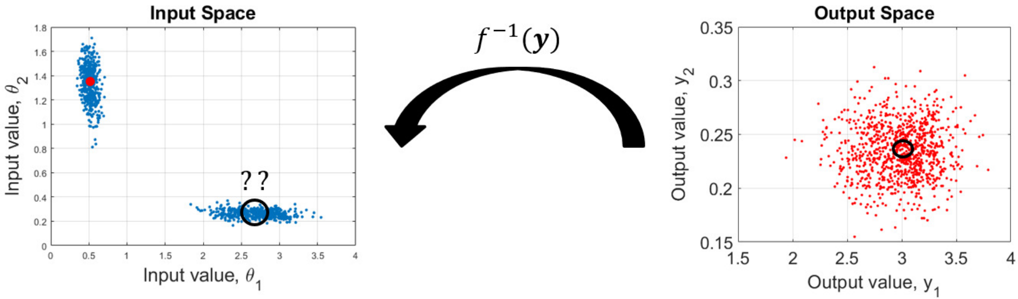
Conceptual Introduction



Model Update

Conceptual Introduction and Motivation

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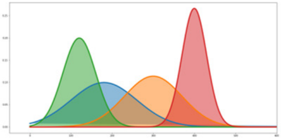
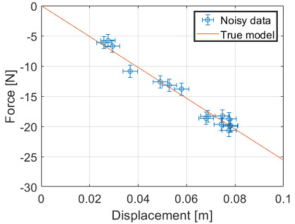
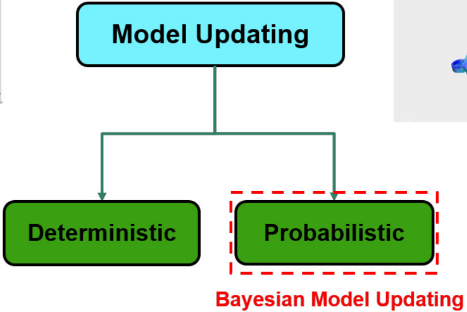
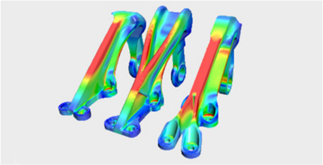
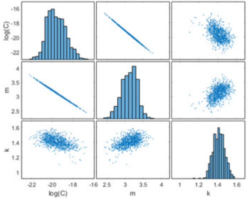










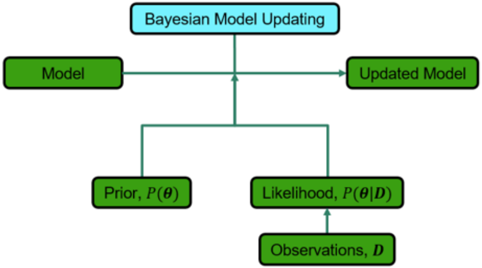
Bayesian Model Update

Conceptual Introduction and Work-flow

A Probabilistic Model Updating technique based on Bayes' Inference:

$$P(\theta|\boldsymbol{D}) = \frac{P(\theta) \cdot P(\boldsymbol{D}|\theta)}{P(\boldsymbol{D})} \qquad (1)$$

where $P(\theta)$ is the Prior; $P(\boldsymbol{D}|\theta)$ is the Likelihood function; $P(\theta|\boldsymbol{D})$ is the Posterior; $P(\boldsymbol{D})$ is the Evidence. θ denotes vector of epistemic parameters; $\boldsymbol{D}$ denotes vector of observations.



Bayesian Model Update

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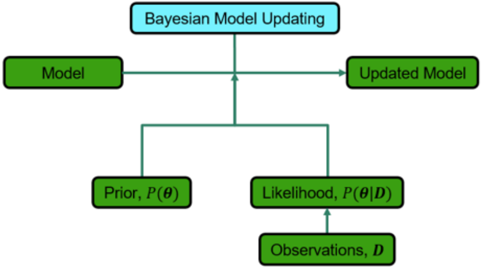
Conceptual Introduction and Work-flow

$P(\boldsymbol{D})$ is a normalisation factor which is independent of θ and, thus, a numerical constant.

Equation (1) can therefore be re-expressed as shown below:

$$P(\theta|\boldsymbol{D}) \propto P(\theta) \cdot P(\boldsymbol{D}|\theta)$$



Bayesian Model Update

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Techniques and Tools

- To generate samples from a distribution, a standard tool would be **Monte-Carlo** sampling;
- HOWEVER...recall that:

$$P(\theta|\boldsymbol{D}) \propto P(\theta) \cdot P(\boldsymbol{D}|\theta)$$

- Standard Monte-Carlo technique is <u>**unable**</u> to sample from un-normalised distribution function;
- We need advanced sampling techniques to do so ... ...

Bayesian Model Update

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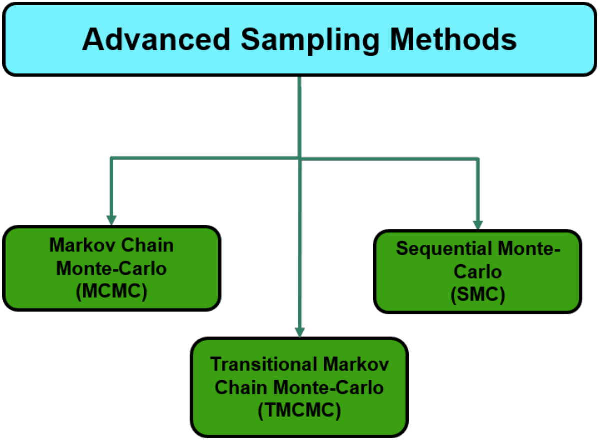
Techniques and Tools



Conclusion

Summary

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- Overview of Model Updating concept and the need for it;
- Deterministic vs Probabilistic Model Updating;
- Understanding of Bayesian Model Updating concept;
- Familiarisation with Advanced Sampling Techniques and the need for them.

Conclusion

Follow-up

What's next

- Read Lecture Notes: Bayesian Model Updating [Part II]
- Watch Lecture Video: Bayesian Model Updating [Part II]