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Software

COSSAN TRAINING COURSE on UNCERTAINTY QUANTIFICATION

Bayesian Model Updating: Part II

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Introduction

Facilitator

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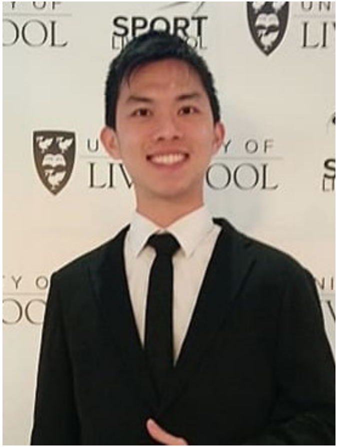

- PhD Student, 2^{nd} Year of Study
- Affiliated with Singapore Nuclear Research and Safety Initiatives (SNRSI)
- Research Interests:
  - Bayesian Model Updating;
  - Mathematical Modelling;
  - Nuclear Energy;
  - Predictive Maintenance;
  - Probabilistic Safety Assessment

Pre-requisites

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Participants/Students should have watched/read the following:

- Lecture Note: Bayesian Model Updating [Part I]
- Lecture Video: Bayesian Model Updating [Part I]

Overview

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1. Re-cap of Bayesian Model Updating
2. Motivation behind Advanced Sampling Techniques
3. Detailed description of Advanced Sampling Techniques:
   - Markov Chain Monte-Carlo (MCMC)
   - Transitional Markov Chain Monte-Carlo (TMCMC)
   - Sequential Monte-Carlo (SMC)
4. Tutorials

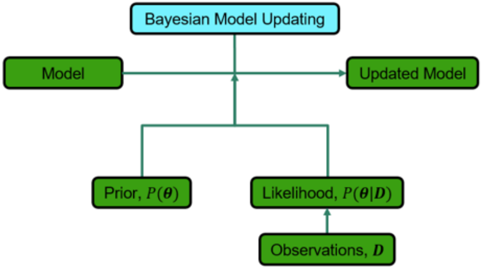
Bayesian Model Update

Re-cap

A Probabilistic Model Updating technique based on Bayes' Inference:

$$P(\theta|\boldsymbol{D}) = \frac{P(\theta) \cdot P(\boldsymbol{D}|\theta)}{P(\boldsymbol{D})} \qquad (1)$$

where $P(\theta)$ is the Prior; $P(\boldsymbol{D}|\theta)$ is the Likelihood function; $P(\theta|\boldsymbol{D})$ is the Posterior; $P(\boldsymbol{D})$ is the Evidence. θ denotes vector of epistemic parameters; $\boldsymbol{D}$ denotes vector of observations.



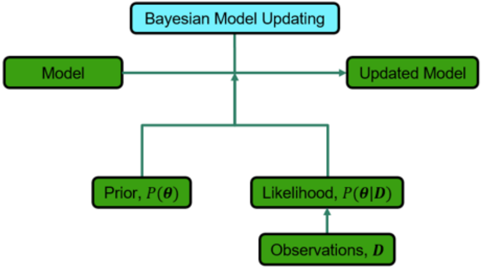
Bayesian Model Update

Re-cap

$P(\boldsymbol{D})$ is a normalisation factor which is independent of $\boldsymbol{\theta}$ and, thus, a numerical constant.

Equation (1) can therefore be re-expressed as shown below:

$$P(\boldsymbol{\theta}|\boldsymbol{D}) \propto P(\boldsymbol{\theta}) \cdot P(\boldsymbol{D}|\boldsymbol{\theta})$$



Bayesian Model Update

Re-cap

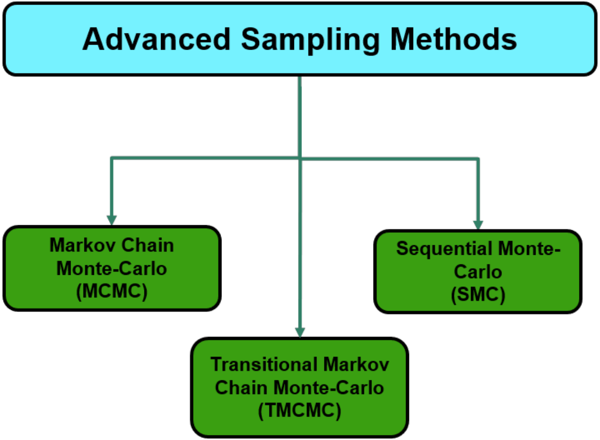
- To generate samples from a distribution, a standard tool would be **Monte-Carlo** sampling;
- HOWEVER...recall that:

$$P(\theta|\boldsymbol{D}) \propto P(\theta) \cdot P(\boldsymbol{D}|\theta)$$

- Standard Monte-Carlo technique is **unable** to sample from un-normalised distribution function;
- We need advanced sampling techniques to do so ... ...

Bayesian Model Update

Re-cap



MCMC Sampler

Conceptual Introduction

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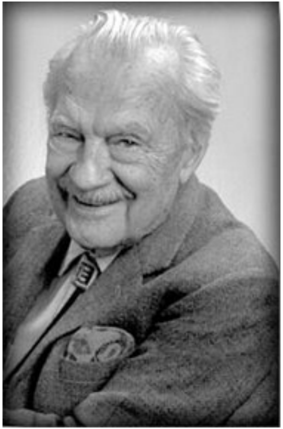
- Conceptualised by Nicholas Metropolis;
- Adopts the use of Markov Chains to generate samples;
- New samples are generated based on current sample via a Proposal distribution;
- The chain will run until it approaches stationary distribution (Posterior);
- Accept-Reject algorithm: Metropolis-Hastings.



MCMC Sampler

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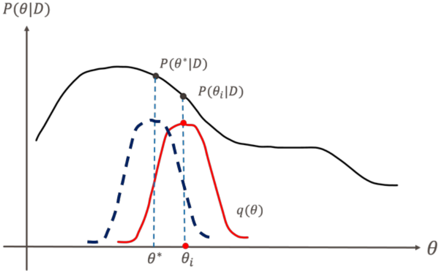
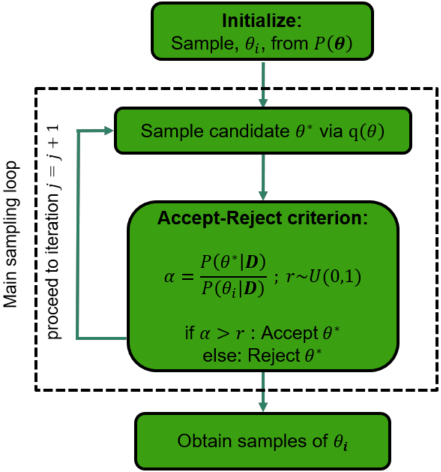
Work-flow of the MCMC sampler



Note: Should θ^* be accepted, the Proposal distribution will shift from its current location (in red), to the new one represented by the blue dotted curve. Otherwise, the Proposal distribution remains in its current location.



TMCMC Sampler

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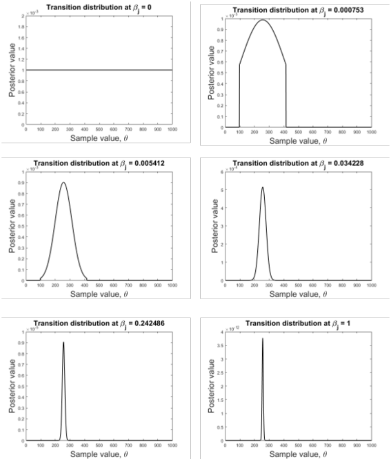
Conceptual Introduction

- Based on Adaptive Metropolis-Hastings (AMH) algorithm;
- Adopts the use "transitional" distributions, P^j:

$$P^j = P(\boldsymbol{D}|\boldsymbol{\theta})^{\beta_j} \cdot P(\boldsymbol{\theta})$$

where $j = 1, \dots, m$ denotes the iteration number, and β_j is such that
$\beta_0 = 0 < \beta_1 <, \dots, < \beta_{m-1} < \beta_m = 1$

- Change in β_j has to be small to ensure smooth, gradual transition;
- Performs parallel sampling: N samples obtained per iteration.



TMCMC Sampler

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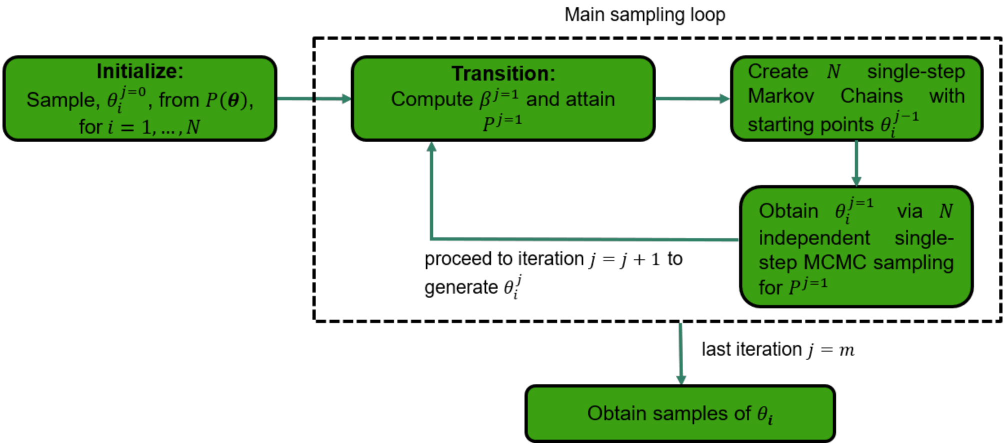
Work-flow of the TMCMC sampler



SMC Sampler

Conceptual Introduction

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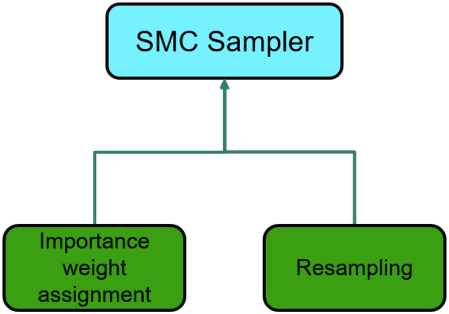
- Based on the Sequential Importance Resampling (SIR) Particle-Filter algorithm;
- Adopted for systems identification and to sample from dynamic posteriors in a sequential manner;
- Recursive algorithm.



SMC Sampler

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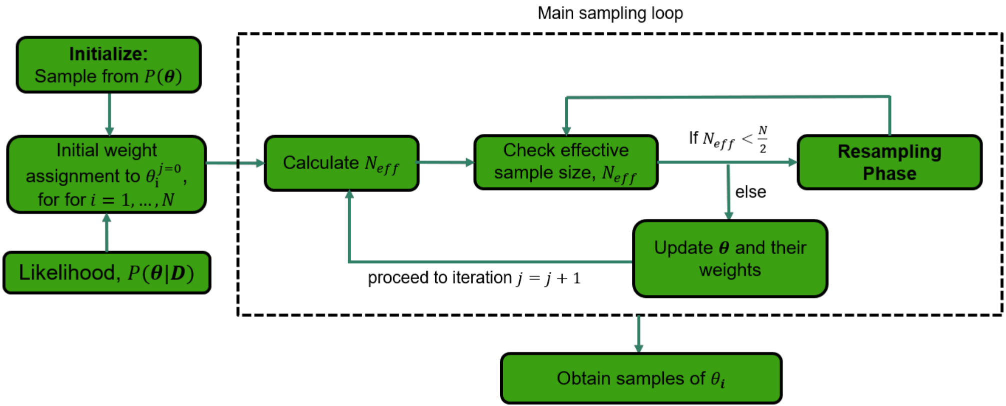
Work-flow of the SMC sampler



Relevant References

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For More Information

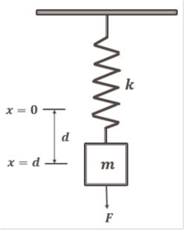
Technique:	References:
MCMC	W. K. Hastings (1970). Monte Carlo Sampling Methods using Markov Chains and their Applications, Biometrika **57**, 97-109. doi: `10.1093/biomet/57.1.97`
TMCMC	J. Y. Ching, and Y. C. Chen (2007). Transitional Markov Chain Monte Carlo Method for Bayesian Model Updating, Model Class Selection, and Model Averaging, Journal of Engineering Mechanics **133**. doi: `10.1061/(ASCE)0733-9399(2007)133:7(816)`
SMC	P. D. Moral, A. Doucet, and A. Jasra (2006). Sequential Monte Carlo Samplers, Journal of the Royal Statistical Society. Series B (Statistical Methodology) **68**, 411-436. N. Chopin (2002). A Sequential Particle Filter Method for Static Models, Biometrika **89**, 539-552. doi: `10.1093/biomet/89.3.539`

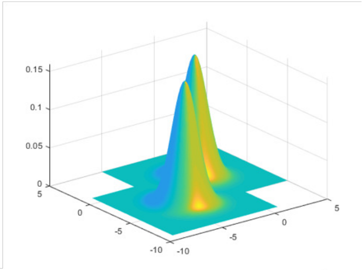
Bayesian Model Update Tutorial Problems

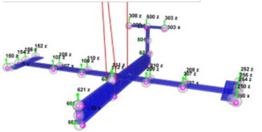
Tutorials on the use of Advanced Monte Carlo Sampling methods

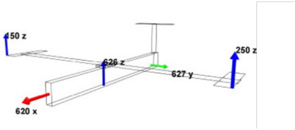
- 4 sets of tutorials are available on OpenCOSSAN:
  - 1-D Linear Static Spring-Mass System
  - 1-D Simple Harmonic Oscillator System
  - 2-D Inverse Eigen-value Problem
  - 18-D DLR-AIRMOD

- Each tutorial presents the implementation of 3 advanced sampling techniques:
  - MCMC Sampler
  - TMCMC Sampler
  - SMC Sampler









Conclusion

Summary

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- Overview of the different Advanced Sampling Techniques;
- Detailed description of the workings of each Advanced Sampling Technique
- Understanding of the difference between each Advanced Sampling Technique;

Conclusion

Follow-up

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What's next

- Read up / watch lecture series on the implementation of Bayesian Model Updating on OpenCOSSAN.