آرین شکر - ۴۰۰۲۳۴۹۴ تکلیف اول درس شبکه های عصبی

Hagan E2.1:

i. Linear: $wp+b = 1.6 \Rightarrow 1.3p + 3.0 = 1.6 \Rightarrow p \approx -1.077$

positive Linear: $wp+b = 1.6 > 0 \longrightarrow p \approx -1.077$

ii. Hard lim: $wp+b \geq 0 \Rightarrow 1.3p + 3.0 \geq 0 \Rightarrow p \geq -2.31$

Symmetrical Hard limit: $wp+b \geq 0 \Rightarrow p \geq -2.31$

Linear: $wp+b = 1.0 \Rightarrow 1.3p + 3.0 = 1.0 \Rightarrow p \approx -1.54$

Saturating Linear: $wp+b \geq 1 \Rightarrow 1.3p + 3.0 \geq 1 \Rightarrow p \geq -1.54$

Symmetrical Saturating Linear: $wp+b \geq 1 \Rightarrow p \geq -1.54$

Log-Sigmoid: Cannot be exactly 1.0 but it can tend to it

$$\rightarrow \frac{1}{1+e^{-n}} = 1 \Rightarrow e^{-n} = 0 \Rightarrow -n \ln e = -\infty \Rightarrow n \rightarrow +\infty$$
$$\Rightarrow wp+b \rightarrow +\infty \Rightarrow p \rightarrow +\infty$$

Hyperbolic Tangent Sigmoid: same as above.

$$\rightarrow \frac{e^{n}-e^{-n}}{e^{n}+e^{-n}} = 1 \rightarrow \cancel{e^{n}} - e^{-n} = \cancel{e^{n}} + e^{-n} \Rightarrow e^{-n} + e^{-n} = 0 \Rightarrow e^{-n} = 0$$
$$\longrightarrow -n \ln e = -\infty \Rightarrow n \rightarrow +\infty \Rightarrow wp+b \rightarrow +\infty$$
$$\Rightarrow p \longrightarrow +\infty$$

positive Linear: $wp+b = 1.0 \geq 0 \Rightarrow p \approx -1.54$

iii. Linear: $wp+b = 0.9963 \Rightarrow 1.3p + 3.0 = 0.9963 \Rightarrow p \approx -1.541$

Saturating Linear: $wp+b = 0.9963 \leq 1 \longrightarrow p \approx -1.541$

Symmetrical saturating Linear: $wp+b = 0.9963 \leq 1 \rightarrow p \approx -1.541$

Log-sigmoid: $\frac{1}{1+e^{-n}} = 0.9963 \rightarrow n \approx 5.6 \rightarrow 1.3p + 3.0 = 5.6$
$\rightarrow p = 2$

Hyperbolic Tangent Sigmoid: $\tanh(n) = 0.9963 \rightarrow n \approx 3.145$
$\rightarrow 1.3p + 3.0 = 3.145 \rightarrow p \approx 0.1$

positive Linear: $1.3p + 3.0 = 0.9963 \geq 0 \rightarrow p \approx -1.541$

iv. Symmetrical Hard Limit: $1.3p + 3.0 < 0 \rightarrow p < -2.31$

Linear: $1.3p + 3.0 = -1 \rightarrow p \approx 1.54$

Symmetrical saturating Linear: $1.3p + 3.0 \leq -1.0 \rightarrow p < 1.53$

Hyperbolic Tangent Sigmoid: $\tanh(n) = -1 \rightarrow n \rightarrow -\infty \rightarrow p \rightarrow -\infty$

Hagan E2.3. There are many such combinations, for instance assumming a Linear transfer function: $n = 0.5 \rightarrow [3. \ \ 2.] \begin{bmatrix} -5 \\ 7 \end{bmatrix} + b = 0.5 \rightarrow -1 + b = 0.5$
$\rightarrow b = 1.5$

i. if bias is zero $\rightarrow n = wp + b = -1$ → none of the transfer functions produce output 0.5 for $n = -1$.

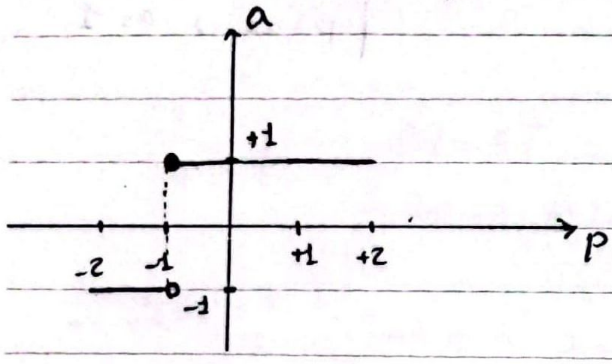
ii. Indeed as explained earlier setting $b = 1.5$ will produce the desired output

iii. $\frac{1}{1+e^{-n}} = 0.5 \rightarrow n = 0 \rightarrow wp + b = 0 \rightarrow -1 + b = 0 \rightarrow b = 1$

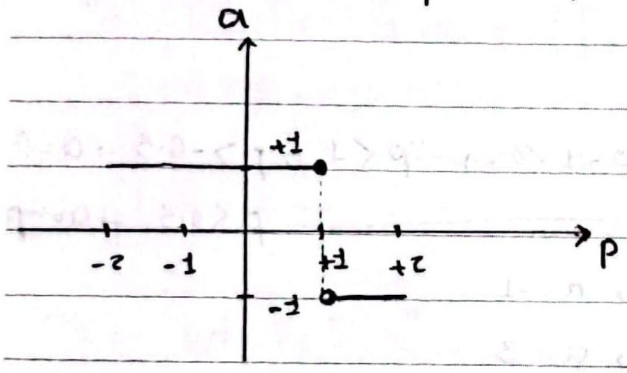
iv. No, because the Symmetrical Hard Limit transfer function can only produce ± 1.

Hagan E 2.5.

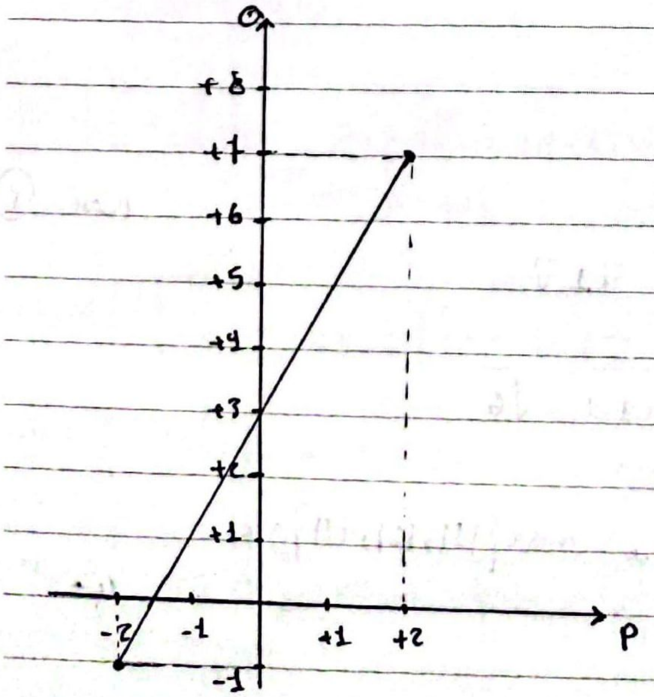
i. $w=1$, $b=1$, hardlims | Solve $wp+b=0 \Rightarrow p+1=0 \Rightarrow p=-1 \rightarrow \begin{cases} a=-1, & a<-1 \\ a=1, & a \geq -1 \end{cases}$



ii. $w=-1$, $b=1$, hardlims | Solve $wp+b=0 \Rightarrow -p+1=0 \Rightarrow p=1 \longrightarrow \begin{cases} a=-1, & a>1 \\ a=1, & a \leq 1 \end{cases}$



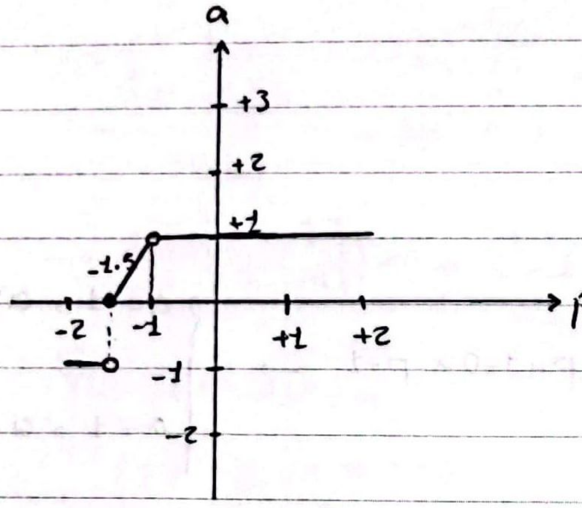
iii. $w=2$, $b=3$, purelin | line eq.: $a=wp+b \Rightarrow a=2p+3 \longrightarrow \begin{cases} p=2, & a=7 \\ p=-2, & a=-1 \end{cases}$



iii. $w=2,\ b=3,$ satlins | $wp+b<0 \rightarrow 2p+3<0 \rightarrow 2p<-3 \rightarrow p<-1.5$

$wp+b>1 \rightarrow 2p+3>1 \rightarrow 2p>-2 \rightarrow p>-1$

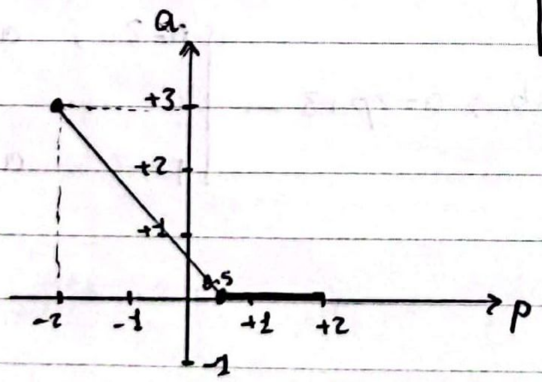
for $-1.5 \leq p \leq -1 \rightarrow a=wp+b \rightarrow \begin{cases} p=-1.5\ ,\ a=0 \\ p=-1\ ,\ a=1 \end{cases}$



iv. $w=-2,\ b=-1,$ poslin | $wp+b<0 \rightarrow -2p-1<0 \rightarrow -2p<1 \rightarrow p>-0.5 \rightarrow a=0$

$p<0.5 \rightarrow a=wp+b$

$\begin{cases} p=0.5\ ,\ a=0 \\ p=-2\ ,\ a=3 \end{cases}$



(۲) الف.

$\vec{u}\cdot\vec{v} = (1\times(-1)) + (2\times 0) + (1\times 1) = 0 \Rightarrow \vec{u} \perp \vec{v}$

$\|\vec{v}\|_2 = \sqrt{1+0+1} = \sqrt{2}$, $\|\vec{u}\|_2 = \sqrt{1+4+1} = \sqrt{6}$

$\|\vec{v}\|_\infty = \max\{|-1|, |0|, |1|\} = 1$ $\quad \|\vec{u}\|_\infty = \max\{|1|, |2|, |1|\} = 2$

ب) $\vec{u}\cdot\vec{v} = (1\times 3) + (2\times(-1)) + (1\times 1) = 2 \neq 0 \rightarrow \vec{u} \not\perp \vec{v}$

$\|\vec{u}\|_2 , \|\vec{u}\|_\infty \rightarrow$ مشابه است $\quad \|\vec{u}\|_2 = \sqrt{9+1+1} = \sqrt{11}$

$\|\vec{v}\|_\infty = \max\{|3|, |-1|, |1|\} = 3$

۳) الف)

$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad A\vec{v} = \lambda\vec{v} \rightarrow (A-\lambda I)\vec{v} = 0$

$\rightarrow \det(A-\lambda I) = 0 \Rightarrow \det\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) = 0$

$\rightarrow \det\left(\begin{bmatrix} -\lambda & 1 \\ 1 & -\lambda \end{bmatrix}\right) = 0 \Rightarrow \lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$

$\vec{v}_1 : (A-\lambda_1 I)\vec{v}_1 = 0 \Rightarrow \left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0$

$\Rightarrow \begin{cases} -x_1 + y_1 = 0 \rightarrow x_1 = y_1 \\ x_1 - y_1 = 0 \end{cases} \rightarrow$ solution set : $a\begin{bmatrix} 1 \\ 1 \end{bmatrix}$; $\forall a \in \mathbb{R}$

$\vec{v}_2 : (A-\lambda_2 I)\vec{v}_2 = 0 \Rightarrow \left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\right)\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = 0$

$\Rightarrow \begin{cases} x_2 + y_2 = 0 \rightarrow x_2 = -y_2 \\ y_2 + x_2 = 0 \end{cases} \longrightarrow$ solution set: $a\begin{bmatrix} 1 \\ -1 \end{bmatrix}$; $\forall a \in \mathbb{R}$

ب) $A = \begin{bmatrix} 10 & -9 \\ 4 & -2 \end{bmatrix} \qquad A\vec{v} = \lambda\vec{v} \rightarrow (A-\lambda I)\vec{v} = 0$

$\xrightarrow{\vec{v}\neq 0} \det(A-\lambda I) = 0 \Rightarrow \det\left(\begin{bmatrix} 10 & -9 \\ 4 & -2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) = 0$

$\rightarrow \det\left(\begin{bmatrix} 10-\lambda & -9 \\ 4 & -2-\lambda \end{bmatrix}\right) = 0 \Rightarrow (10-\lambda)(-2-\lambda) - 4\times(-9) = 0 \Rightarrow \lambda^2 - 8\lambda + 16 = 0$

$\rightarrow (\lambda-4)^2 = 0 \Rightarrow \lambda_1 = 4 \rightarrow v_1 , (A-\lambda_1 I)\vec{v}_1 = 0 \Rightarrow \left(\begin{bmatrix} 10 & -9 \\ 4 & -2 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0$

$\rightarrow \begin{bmatrix} 6 & -9 \\ 4 & -6 \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0 \Rightarrow \begin{cases} 6x_1 - 9y_1 = 0 \rightarrow x_1 = \frac{3}{2}y_1 \\ 4x_1 - 6y_1 = 0 \end{cases}$

solution set : $a\begin{bmatrix} 1 \\ \frac{2}{3} \end{bmatrix}$; $\forall a \in \mathbb{R}$

ب)

$$A=\begin{bmatrix}0&1&0\\0&0&2\\0&0&0\end{bmatrix} \qquad A\vec{v}=\lambda\vec{v} \to (A-\lambda I)\vec{v}=0 \xrightarrow{\vec{v}\neq 0} \det(A-\lambda I)=0$$

$$\to \det\left(\begin{bmatrix}0&1&0\\0&0&2\\0&0&0\end{bmatrix}-\begin{bmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{bmatrix}\right)=\det\left(\begin{bmatrix}-\lambda&1&0\\0&-\lambda&2\\0&0&\lambda\end{bmatrix}\right)=0$$

$$\xrightarrow[\text{ضرب درایه‌های قطر اصلی}]{\text{دترمینان ماتریس بالامثلثی}} -\lambda^3=0 \Rightarrow \lambda_1=0$$

$$v_1:\ (A-\lambda_1 I)\vec{v}=0 \longrightarrow \begin{bmatrix}0&1&0\\0&0&2\\0&0&0\end{bmatrix}\begin{bmatrix}x_1\\y_1\\z_1\end{bmatrix}=0 \Rightarrow \begin{cases}y_1=0\\2z_1=0 \to z_1=0\\0=0\end{cases}$$

$$\text{Solution set: } a\begin{bmatrix}x_1\\0\\0\end{bmatrix};\ \forall a\in\mathbb{R}$$

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$$A=\begin{bmatrix}2&-2\\-1&3\end{bmatrix} \longrightarrow A\vec{v}=\lambda\vec{v} \to (A-\lambda I)\vec{v}=0 \xrightarrow{\vec{v}\neq 0} \det(A-\lambda I)=0$$

$$\to \det\left(\begin{bmatrix}2&-2\\-1&3\end{bmatrix}-\begin{bmatrix}\lambda&0\\0&\lambda\end{bmatrix}\right)=\det\begin{pmatrix}2-\lambda&-2\\-1&3-\lambda\end{pmatrix}=0$$

$$\Rightarrow (2-\lambda)(3-\lambda)-(-2)(-1)=0 \Rightarrow \lambda^2-5\lambda+4=0 \Rightarrow (\lambda-4)(\lambda-1)=0 \to \begin{cases}\lambda_1=4\\\lambda_2=1\end{cases}$$

$$\vec{v}_1:\ (A-\lambda_1 I)\vec{v}_1=\left(\begin{bmatrix}2&-2\\-1&3\end{bmatrix}-\begin{bmatrix}4&0\\0&4\end{bmatrix}\right)\begin{bmatrix}x_1\\y_1\end{bmatrix}=0 \Rightarrow \begin{bmatrix}-2&-2\\-1&-1\end{bmatrix}\begin{bmatrix}x_1\\y_1\end{bmatrix}=0$$

$$\to \begin{cases}-2x_1-2y_1=0\\-x_1-y_1=0\end{cases} \to x_1=-y_1 \to \text{Solution set: } a\begin{bmatrix}1\\-1\end{bmatrix};\ \forall a\in\mathbb{R}$$

$$\vec{v}_2:\ (A-\lambda_2 I)\vec{v}_2=\left(\begin{bmatrix}2&-2\\-1&3\end{bmatrix}-\begin{bmatrix}1&0\\0&1\end{bmatrix}\right)\begin{bmatrix}x_2\\y_2\end{bmatrix}=0 \Rightarrow \begin{bmatrix}1&-2\\-1&2\end{bmatrix}\begin{bmatrix}x_2\\y_2\end{bmatrix}=0$$

$$\to \begin{cases}x_2-2y_2=0\\-x_2+2y_2\end{cases} \to y_2=\frac{1}{2}x_2 \to \text{Solution set: } a\begin{bmatrix}1\\\frac{1}{2}\end{bmatrix};\ \forall a\in\mathbb{R}$$

$$\longrightarrow V=\left[\vec{v}_1\,\middle|\,\vec{v}_2\right]=\begin{bmatrix}1&1\\-1&\frac{1}{2}\end{bmatrix},\quad V^{-1}=\overset{\frac{2}{3}}{\frac{1}{\frac{1}{2}+1}}\begin{bmatrix}\frac{1}{2}&-1\\1&1\end{bmatrix}=\begin{bmatrix}\frac{1}{3}&-\frac{2}{3}\\\frac{2}{3}&\frac{2}{3}\end{bmatrix}$$

$$A_{diag} = V^{-1} A V = \begin{bmatrix} \frac{1}{3} & -\frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \overset{\lambda_1}{4} & 0 \\ 0 & \underset{\lambda_2}{1} \end{bmatrix}.$$

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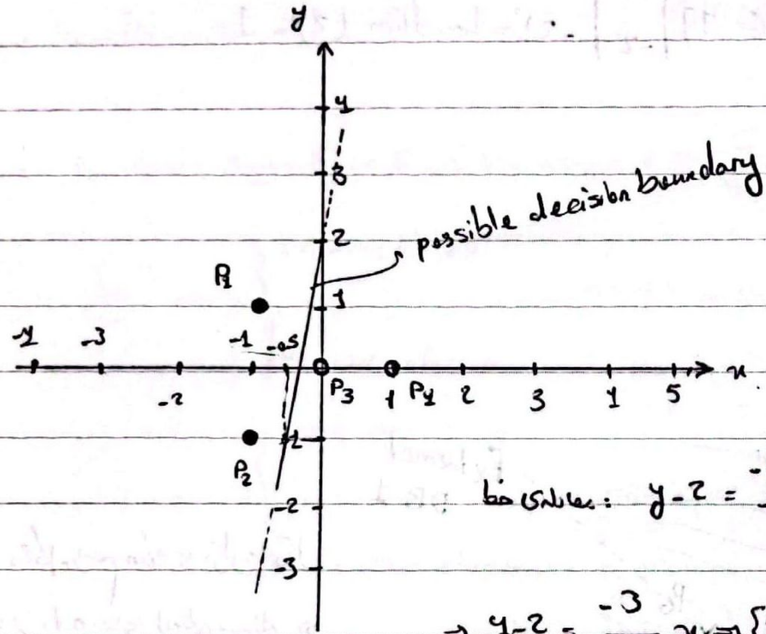
Hogan E4.2. $t=1$ black $t=0$ white

i.



معادله خط: $y-2 = \dfrac{-1-2}{-0.5-0}(x-0)$

$$\Rightarrow y-2 = \frac{-3}{-\frac{1}{2}} x \Rightarrow \begin{cases} y-6x-2=0 \\ -y+6x+2=0 \end{cases} \rightarrow W = \begin{cases} \begin{bmatrix} -6 \\ 1 \end{bmatrix}, & b=-2 \\ \begin{bmatrix} 6 \\ -1 \end{bmatrix}, & b=+2 \end{cases}$$

$$w_1 x + w_2 y + b = 0$$

همانطور که در نمودار مشخص است بردار $\begin{bmatrix} 6 \\ -1 \end{bmatrix}$ عمود بر decision boundary اما در جهت خلاف صفر است بنابراین مشخصات perceptron تک نورون ما شکل روبرو رو خواهد بود:

$$W = \begin{bmatrix} -6 \\ 1 \end{bmatrix}, \quad b = -2$$

ii. $P_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \rightarrow a = \text{hardlim}(W^T p + b) = \text{hardlim}\left(\begin{bmatrix} -6 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} - 2\right) = \text{hardlim}(5) = 1$

$P_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \Rightarrow a = \text{hardlim}\left(\begin{bmatrix} -6 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} - 2\right) = \text{hardlim}(3) = 1$

$P_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow a = \text{hardlim}\left(\begin{bmatrix} -6 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} - 2\right) = \text{hardlim}(-2) = 0$

$P_4 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow a = \text{hardlim}\left(\begin{bmatrix} -6 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 2\right) = \text{hardlim}(-8) = 0$

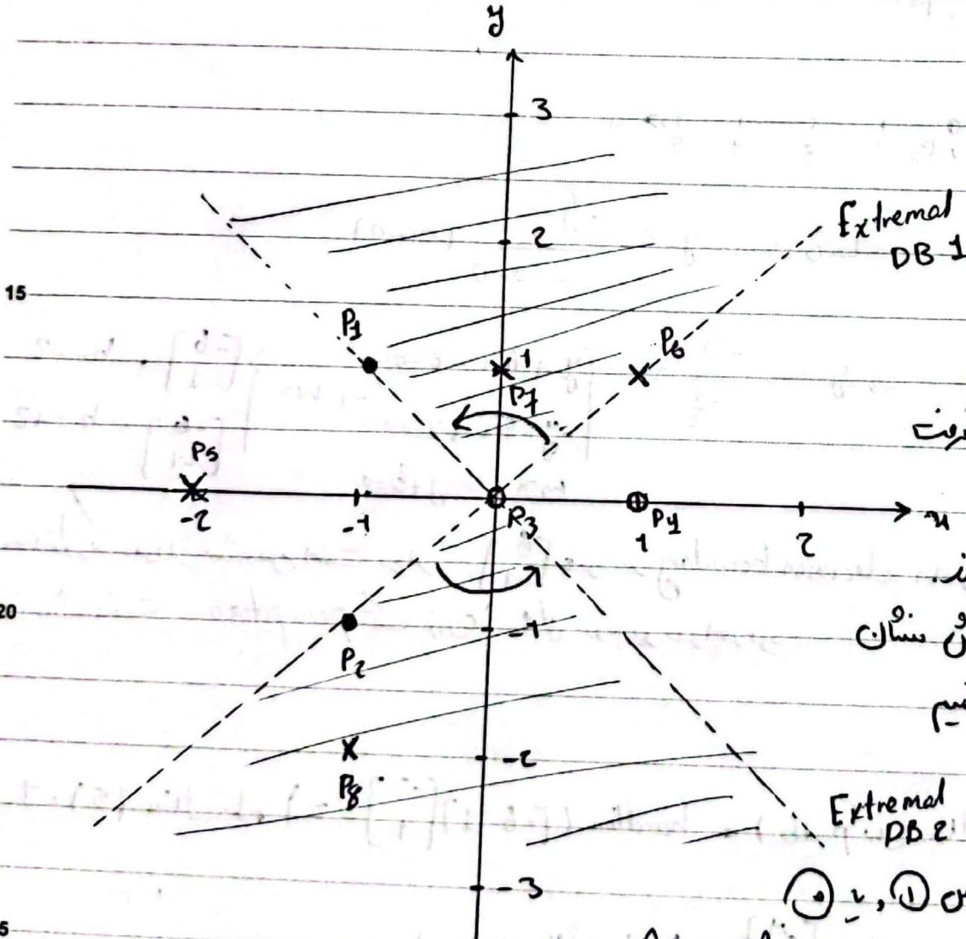
iii. $P_5 = \begin{bmatrix} -2 \\ 0 \end{bmatrix} \rightarrow a = \text{hardlim}\left(\begin{bmatrix} -6 & 1 \end{bmatrix}\begin{bmatrix} -2 \\ 0 \end{bmatrix} - 2\right) = \text{hardlim}(10) = 1$

$P_6 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \rightarrow a = \text{hardlim}\left(\begin{bmatrix} -6 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} - 2\right) = \text{hardlim}(-7) = 0$

$P_7 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow a = \text{hardlim}\left(\begin{bmatrix} -6 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} - 2\right) = \text{hardlim}(-1) = 0$

$P_8 = \begin{bmatrix} -1 \\ -2 \end{bmatrix} \rightarrow a = \text{hardlim}\left(\begin{bmatrix} -6 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ -2 \end{bmatrix} - 2\right) = \text{hardlim}(2) = 1$

iv.



با توجه به شکل رسم شده می توان گفت
با استفاده از داده های اصلی مسئله دو
مرز تصمیم Extremal برای مسئله در نظر گرفت
که خطوط غیر از خطوط بین این دو
به عنوان یک مرز تصمیم قابل قبول نیستند.
از آنجایی که تمام خطوط در جهت چرخش نشان
داده شده برای این مسئله مرزهای تصمیم
مناسبی هستند، هر نقطه در قسمت
هاشور زده شده می تواند نسبت
به مرز تصمیم گیری انتخاب شده در کلاس 1 و یا 0
قرار بگیرد. ولی نقاط خارج از قسمت هاشور زده شده همیشه
یک کلاس مشخص خواهند داشت ←

$P_5 \rightarrow$ کلاس مشخص (همیشه 1)

$P_6, P_7, P_8 \rightarrow$ کلاس متغیر

(نقطه P_6 روی Extremal DB 1 قرار دارد و به دلیل
استفاده از تابع hardlim، این نقطه نیز می تواند هر دو کلاس
باشد)

Scanned with CamScanner

Hagan E4.5. $a = w^T p + b$, $w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$

$\rightarrow \text{hardlim}(a_1) = t_1 = 1 \Rightarrow \text{hardlim}\left(\begin{bmatrix} w_1 & w_2 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} + b\right) = 1 \Rightarrow -w_1 + w_2 + b \geq 0$ (I)

$\rightarrow \text{hardlim}(a_2) = t_2 = 0 \Rightarrow -w_1 - w_2 + b < 0$ (II)

$\rightarrow \text{hardlim}(a_3) = t_3 = 1 \Rightarrow w_1 - w_2 + b \geq 0$ (III)

$\rightarrow \text{hardlim}(a_4) = t_4 = 0 \Rightarrow w_1 + w_2 + b < 0$ (IV)

$$\text{(I)}, \text{(III)} \rightarrow \begin{cases} w_1 - w_2 + b \geq 0 \\ w_1 + w_2 + b \geq 0 \end{cases} \rightarrow 2b \geq 0 \Rightarrow b \geq 0$$

$$\text{(II)}, \text{(IV)} \rightarrow \begin{cases} -w_1 - w_2 + b < 0 \\ -w_1 + w_2 + b < 0 \end{cases} \rightarrow 2b < 0 \Rightarrow b < 0$$

$\rightarrow$ تناقض

$\Leftarrow$ چنین perceptron تک نرونی وجود ندارد.