EL-GY 9343 Homework 6

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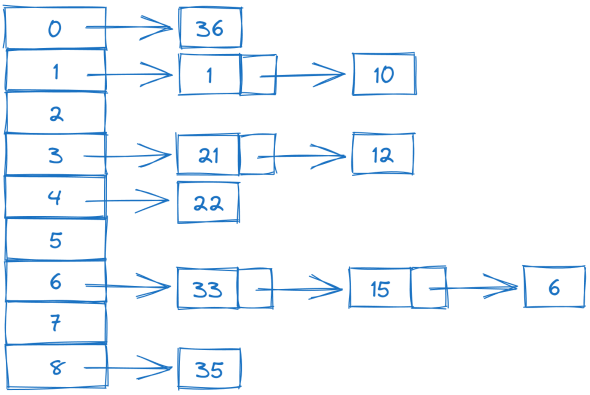
1. Let the table have 9 slots, and let the hash function be $h(k) = k \bmod 9$. Demonstrate what happens when we insert the keys 10, 22, 35, 12, 1, 21, 6, 15, 36, 33 into a hash table with collisions resolved by chaining.



2. Suppose we use a hash function h to hash n distinct keys into an array T of length m. Assuming simple uniform hashing, what is the expected number of collisions?

- The expected number of collisions can be calculated as $n - \mathbb{E}[k]$, where k is the number of slots that will be occupied.
- To compute $\mathbb{E}[k]$, we need to know the probability that one slot will be occupied, which is $1 - p$, where p is the probability that one slot will not be occupied eventually.
- One slot being not occupied means that all n keys go to the other $m - 1$ slots, whose probability is $(\frac{m-1}{m})^n$.
- Therefore, $\mathbb{E}[k] = m \cdot [1 - (\frac{m-1}{m})^n]$ because all slot are independent.
- Finally, The expected number of collisions is $\underline{n - m \cdot [1 - (\frac{m-1}{m})^n]}$.

3.(a) Suppose you have two sequences, {24, 19, 12, 6, 24, 36, 40, 39} and {6, 12, 24, 19, 39, 40, 36, 24}. You know the first one is generated from some BST A by pre-order tree walk, and the second one is generated from some BST B by post-order tree work. Please draw all the possible BST A that can generate the sequence. Repeat that for BST B.

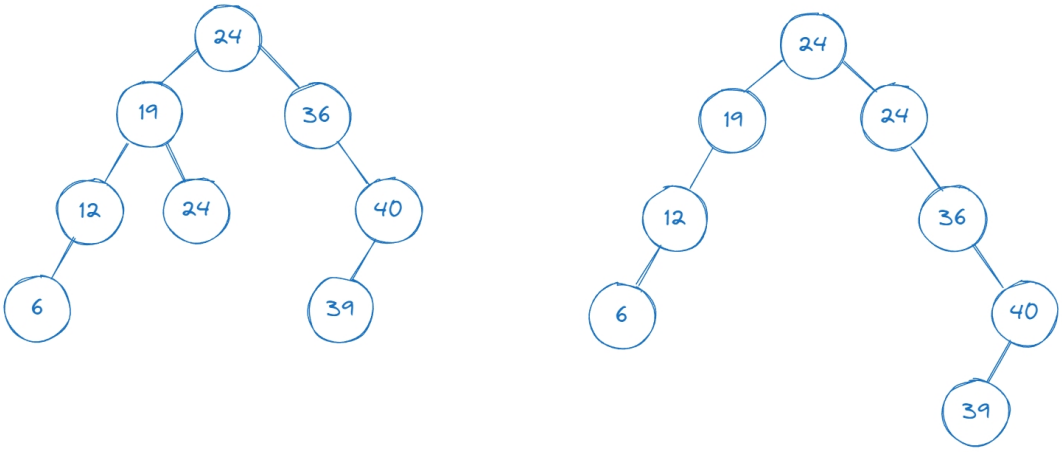


Figure 1: 3.(a) Two possibilities of BST A

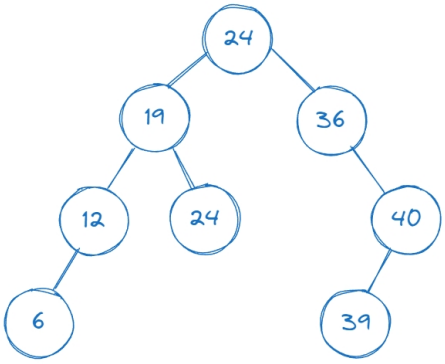
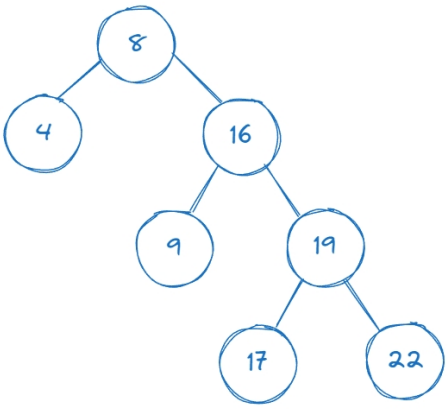


Figure 2: 3.(b) The only possibility of BST B

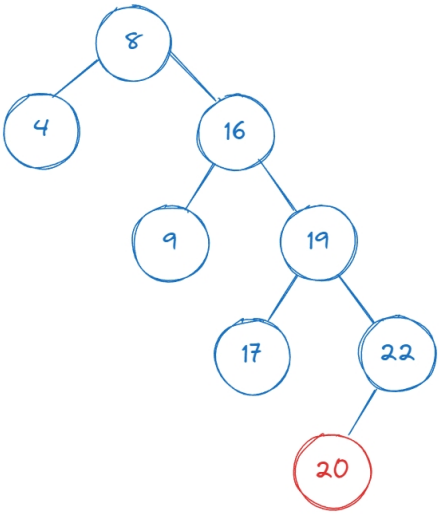
(b) If all the keys in a BST are distinct, can you draw a unique BST when only given its pre-order tree walk? If yes, please describe why; if no, find a counter-case.

Yes. The first element in the pre-order traversal is always the root. And as we move through the pre-order traversal, we can compare each element with the previous element. If it is smaller, then it belongs to the left sub-tree; vice versa, it belongs to the right sub-tree of it's parent.

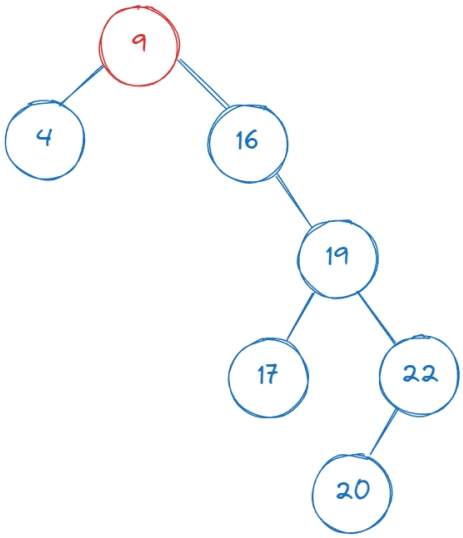
4. For the binary search tree (BST) in pre-order as {8, 4, 16, 9, 19, 17, 22}. **Please first draw the BST**, then show the result of following operations (each operation is carried out on the result of the previous operation):



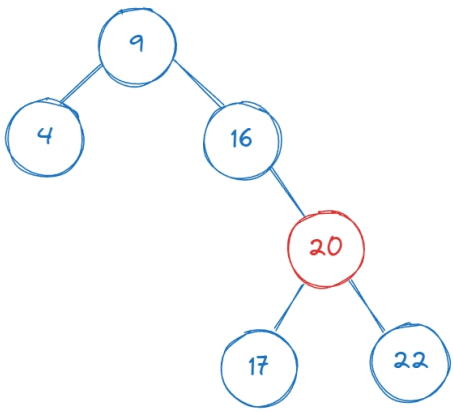
(a) Insert key 20;



(b) Then, delete key 8;



(c) Then, delete key 19;



(d) Finally, delete key 16.