ECE GY 9343 Final Exam (Summer, 2021)

Name: **ID:**

August 21, 2021

Write all answers on your own blank answer sheets, scan and upload answer sheets to newclasses at the end of exam, keep your answer sheets until the grading is finished.

Multiple choice questions may have multiple correct answers. You will get partial credits if you only select a subset of correct answers, and will get zero point if you select one or more wrong answers.

1. (**24 points, 2 points each**) **True or False**

   (a) **T or F:** If adjacency matrix is used to represent a graph, the time complexity to determine whether an edge (u, v) exists is $\theta(1)$.

   (b) **T or F:** Given an adjacency-list representation of a directed graph $G = (V, E)$, it takes $O(V)$ time to compute the in-degree of all vertices.

   (c) **T or F:** Dynamic programming algorithms can always be solved in polynomial time.

   (d) **T or F:** Dynamic programming can be solved in either top-down or bottom-up fashions.

   (e) **T or F:** Let T be a minimum spanning tree of G. Then, for any pair of vertices s and t, the shortest path from s to t in G is the path from s to t in T.

   (f) **T or F:** In a connected undirected graph, breadth-first search starting from any vertex will put all vertices into the same BFS tree.

   (g) **T or F:** Depth-first search of a graph is asymptotically slower than breadth-first search.

   (h) **T or F** In a Depth-First-Search tree of a Directed Acyclic Graph, there cannot be an edge going from a vertex to another vertex at a higher level of the tree.

   (i) **T or F:** The rod-cutting problem is solved using dynamic programming.

   (j) **T or F:** Given a directed graph $G = (V, E)$, if you reverse all the edges in G, the resulting graph G^T has the same strongly connected components as the original graph G.

   (k) **T or F:** For a directed graph with a negative cycle, there is no pair of vertices that has a finite cost shortest path.

   (l) **T or F:** Floyd-Warshall algorithm solves the all-pairs shortest-paths problem using using dynamic programming.

Multiple choice questions: You will get all points if all correct answers are chosen and no wrong answers are chosen, two points if you choose a subset of correct answers, zero point if you choose any wrong answers.

2. (**4 points**) Which of the following algorithms can be used to find shortest paths in a graph where some links have negative weights? (Circle all that apply)

A: Dijkstra; **B:** Bellman Ford; **C:** Floyd-Warshall; **D:** BFS; **E:** Topological Sort.

3. (**4 points**) Which of the following kind of edges could NOT exist in a Depth-First-Search tree of an undirected Graph? (Circle all that apply)

A: Tree edges
B: Forward edges
C: Back edges
D: Cross edges

4. (**4 points**) Which of the following code can be huffman code for four symbols? (Circle all that apply)
A: 0, 10, 110, 1110
B: 0, 10, 100, 101
C: 111, 110, 10, 1
D: 00, 01, 100, 11
E: None of the above

5. (**4 points**) Which of the following statements about depth-first search (DFS) are true?
A: Complexity of DFS on a dense graph is $\theta(E)$;
B: DFS can generate shortest path in a graph;
C: In a DAG, if there is an edge (u, v), then $f(u) > f(v)$;
D: For an undirected graph, DFS tree has no cross edges;
E: None of the above.

6. (**8 points**) For the AVL Tree in Figure 1, when a key with value 17 is inserted, how many rotations are needed to restore the AVL tree property? what is the restored AVL tree with the inserted key?

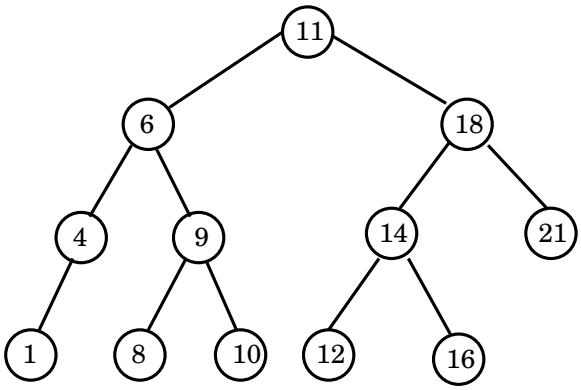

Figure 1: AVL Tree for Question 6

7. (**10 points**) Please use dynamic programming to find the Longest Common Subsequence between String "ABCMDFNK" and "BLCADFEK". Show the final $c[i, j]$ and $b[i, j]$ matrices.

8. (**10 points**) Use Bellman Ford algorithm to find the shortest paths from vertex D to all the other vertices in Figure 2, after each iteration, plot the graph and put the d values in vertices, and use shaded edges to illustrate the predecessors. The order for edge relaxation is AB, AC, BD, CS, DA, DB, DC, SA, SD

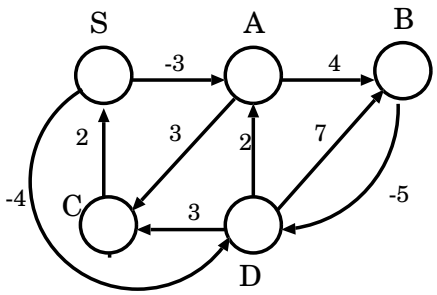

Figure 2: Bellman Ford Algorithm for Question 8

9. (**10 points**) In a directed graph, we define a vertex v to be a source, if there is no edge coming into v, and a sink if there is no edge going out of v. Please prove that in a directed acyclic graph (or DAG), there is at least one source and at least one sink.

10. (**10 points**) Prove the correctness of the *Earliest-Finish-Time-First* algorithm for the activity selection problem,

(a) (**5 points**) describe and prove the *Optimal Substructure* property;

(b) (**5 points**) describe and prove the *Greedy Choice* property.

11. (**12 points**) Design a dynamic programming approach to finding a longest weighted simple path from a vertex s to another vertex t in a weighted directed acyclic graph $G = (V, E)$. Write down the pseudo-code, and analyze the complexity of your algorithm.

++++++++++ **End of Exam** ++++++++++