EL9343 Midterm Exam (summer, 2020)

Name: **ID:**

July 10, 2020

Write all answers on your own answer sheets.
Multiple choice questions may have multiple correct answers. You will get partial credits if you only select a subset of correct answers, and will get zero point if you select one or more wrong answers.

1. (**18 points**) **True or False**

(a) **(3 points) T or F** $n^2 + 3n = \omega(n^2 + 2n)$;
(b) **(3 points) T or F** Insertion-sort follows the divide-and-conquer design;
(c) **(3 points) T or F** All comparison sorting algorithms have $\Omega(n \log n)$ complexity;
(d) **(3 points) T or F** The complexity of building a max-heap with n nodes is $O(n \log n)$;
(e) **(3 points) T or F** Merge-sort follows the divide-and-conquer design;
(f) **(3 points) T or F** Quick-sort is not stable;

2. (**4 points**) If $f(n) = \Theta(g(n))$, and $g(n) = O(z(n))$, which of the following are true?

(a) $z(n) = \Omega(g(n))$;
(b) $f(n) = \Omega(g(n))$;
(c) $f(n) = \Omega(z(n))$;
(d) $f(n) = O(z(n))$;
(e) None of the above

3. (**2 points**) In QuickSort, if the array has n elements, the running time for PARTITION procedure is:
(a) $\Theta(1)$; (b) $\Theta(\log n)$; (c) $\Theta(n)$; (d) $\Theta(n \log n)$; (e) $\Theta(n^2)$.

4. (**4 points**) Given a max-heap with height 10, if all the elements are distinct, the 4-th largest element can be at height of:
(a) 1; (b) 6; (c) 7; (d) 9; (e) none.

5. (**4 points**) Which of the following sorting algorithms are in-place?

(a) InsertionSort;
(b) MergeSort;
(c) QuickSort;
(d) HeapSort;
(e) None of the above

6. (**4 points**) Which of the following statements about sorting algorithms are true?

(a) QuickSort runs in best-case time $O(n)$;
(b) HeapSort runs in worst-case time $O(nlogn)$;
(c) Radix sort is a comparison sort algorithm;
(d) MergeSort runs in worst-case time $O(nlogn)$;
(e) None of the above

7. (**15 points**) Solve the following recurrences:

   (a) (**4 points**) Use the iteration method to solve $T(n) = T(\frac{n}{6}) + T(\frac{n}{4}) + T(\frac{n}{3}) + n$;

   (b) (**4 points**) Use the substitution method to verify your solution for Question 7a;

   (c) (**4 points**) Solve the recurrence of $T(n) = 9T(n/3) + n\sqrt{n}$.

8. (**6 points**) Illustrate the operation of insertion sort on the array [13, 20, 71, 25, 60, 80, 40, 30]

9. (**6 points**) Show the steps of radix sort on the following array $[4812, 8012, 6231, 3112, 1699, 4803, 2390, 3108]$

10. (**12 points**) Three-way merge sort first divides an array into three segments with equal length, recursively sort each segment, then combine the three sorted segments into the final sorted array.

    (a) (**8 points**) write down the pseudo-code of three-way merge sort algorithm;

    (b) (**4 points**) analyze the time complexity of three-way merge sort.

11. (**12 points**) For Bubble sort:

    (a) (**8 points**) prove that it is correct (using the Loop-invariant method);

    (b) (**4 points**) prove or disprove that it is stable.

12. (**13 points**) We have a hash table with m slots with collisions resolved by chaining. Suppose n distinct keys are inserted into the table sequentially. Each key is equally likely to be hashed to each slot.

    (a) (**5 points**) what is the probability that the number of keys inserted into a given slot is larger than K?

    (b) (**5 points**) what is the expected number of elements examined when searching for the first key inserted into the hash table? What is the expected number of elements examined when searching for the last key inserted into the hash table?

    (c) (**3 points**) what is the expected number of elements examined when searching for a key not in the hash table?