

EL-GY9343

Data Structure and Algorithm

Lecture 1: Syllabus, Introduction, and Asymptotic notation

Instructor: Pei Liu

Welcome

- ▶ Instructor: Pei Liu
 - ▶ Contact: peiliu@nyu.edu
- ▶ TA: Xiaotian Ki
 - ▶ Email: xl3399@nyu.edu
- ▶ All office hours will be virtual over Zoom.
 - ▶ Link and time can be found on Brightspace.

Class Recordings

- ▶ All lectures are in-person
 - ▶ Videos from previous semesters are on Brightspace, which could be outdated;
 - ▶ Make sure you check the slides for the latest announcements such as homework, exam schedule.
- ▶ Whiteboard will be available online
 - ▶ <https://1drv.ms/u/s!AhGPBmraRihnkGD8UnfdP2pau5Qp>

Why taking this course

- ▶ You can learn
 - ▶ Classic algorithms for classic problems, mathematical insights
 - ▶ Techniques for analyzing performance of various algorithms
 - ▶ How to design good algorithms for solving real-world problems
- ▶ Improving programming skills
 - ▶ Then you can rock in classes, job interviews, etc.
- ▶ It's also fun!

Prerequisites

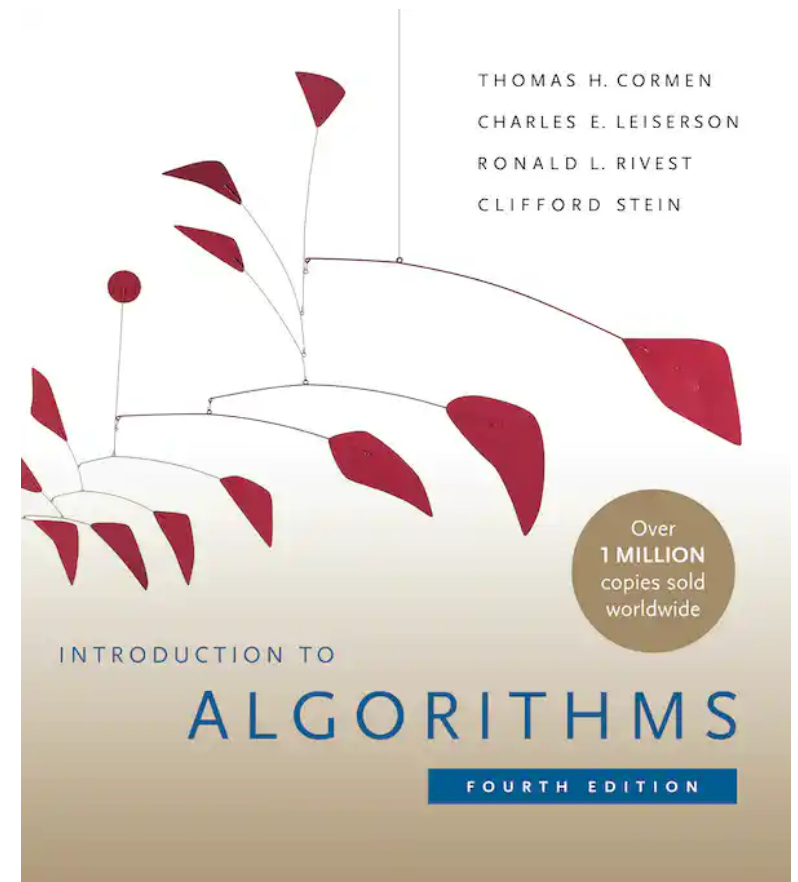
- ▶ Basic knowledge of fundamental data structures
 - ▶ stacks, queues, heaps, ...
- ▶ Some programming experience
 - ▶ C, C++, Python, Java, etc.
- ▶ Discrete mathematics, probabilities, ...
- ▶ Should not take this course if you have taken a similar course e.g. CS6033 with a B or better grade.

Textbook

Introduction to Algorithms, 4th Edition, by Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest and Clifford Stein, MIT Press, 2022; ISBN-13: 9780262046305. It is known as CLRS.

Free access to CLRS (on the library web site <https://library.nyu.edu>, search the book name under “Books & More”).

If you have 3rd edition of the book, make sure you check the homework questions with the latest book.



Grading policy

- ▶ Your final grade will be calculated as:

Homework	10%
Midterm	40%
Final	50%

- ▶ No extra work to improve grade!

Homework

- ▶ Key component to mastering the course material
 - ▶ Very good exercise and practice
 - ▶ Will not do well on exams if you have not done the hw
 - ▶ Will be assigned weekly or biweekly
- ▶ Submission guidelines
 - ▶ Homework is due in one week after it is released
 - ▶ You need to download an APP called “Adobe Scan” on your phone, and use it to scan your homework.
 - ▶ Please submit in a single PDF file, instead of multiple files.

Exams

- ▶ One mid-term, one final exam
 - ▶ Check syllabus for the exam date
- ▶ Close-book and two page cheatsheets
- ▶ Attendance at exams is mandatory

Remember: If you miss an exam without a valid excuse (need documents to prove), you will receive a **grade of zero** on that exam.

What is an algorithm?

- ▶ An *algorithm* is any well-defined computational procedure that takes some values as *input* and produces some values as *output*.
- ▶ Provide a step-by-step method for solving a computational problem. (Like cooking recipes)
- ▶ Not dependent on a particular programming language, machine, system, or compiler. (Unlike programs)

Example on sorting

- ▶ **Problem:** Sorting
 - ▶ **Input:** sequence of n numbers (a_1, a_2, \dots, a_n)
 - ▶ **Output:** a permutation $(a'_1, a'_2, \dots, a'_n)$ of the input instance such that $a'_1 \leq a'_2 \leq \dots \leq a'_n$

For example:

Input: 53, 12, 35, 21, 59, 15

Output: 12, 15, 21, 35, 53, 59

- ▶ **Algorithms:** Insertion sort, merge sort, quick sort, . . .

Issues in algorithm analysis & design

- ▶ Two fundamental issues: **correctness** and **efficiency**
- ▶ Steps to analyze and design an algorithm
 - ▶ Formally define a problem
 - ▶ Clearly describe an algorithm
 - ▶ Prove correctness of the algorithm
 - ▶ Analyze the efficiency of the algorithm

Efficiency of algorithms

▶ Goal

- ▶ To compare algorithms mainly in terms of running time but also in terms of other factors (e.g., memory requirement, programmer's effort).

▶ Running time analysis

- ▶ Determine how running time increases as the **size** of the problem increases.

How do we compare algorithms?

- ▶ Need to define a number of **objective measures**

(1) Compare execution time?

Not good: time is specific to a particular computer !!

(2) Count the number of statements executed?

Not good: number of statements vary with the programming language as well as the style of the individual programmer.

- ▶ Ideal solution

- ▶ Express running time as a function of the input size n (i.e., $f(n)$).
- ▶ Compare different functions corresponding to running time.
- ▶ Such an analysis is independent of machine time, programming style, etc.

Random-Access Machine (RAM)

- ▶ A computational model
 - ▶ All memory equally expensive to access
 - ▶ No concurrent operations
 - ▶ All reasonable instructions take unit time
 - ▶ Except, of course, function calls

Example

- ▶ Associate a "cost" with each statement.
- ▶ Find the "total cost" by finding the total number of times each statement is executed.

<i>Algorithm</i>	<i>Cost</i>
sum = 0;	c_1
for(i=0; i<N; i++)	c_2
for(j=0; j<N; j++)	c_2
sum += arr[i][j];	c_3

$c_1 + c_2 \times (N+1) + c_2 \times N \times (N+1) + c_3 \times N^2$	

Input size (number of elements in the input)

- ▶ How we characterize input size is problem-specific:
 - ▶ Sorting: number of input items
 - ▶ Multiplication: total number of bits
 - ▶ Graph algorithms: number of nodes & edges
 - ▶ ...

Common orders of magnitude

Table 2.1 The running times (rounded up) of different algorithms on inputs of increasing size, for a processor performing a million high-level instructions per second. In cases where the running time exceeds 10^{25} years, we simply record the algorithm as taking a very long time.

	n	$n \log_2 n$	n^2	n^3	1.5^n	2^n	$n!$
$n = 10$	< 1 sec	< 1 sec	< 1 sec	< 1 sec	< 1 sec	< 1 sec	4 sec
$n = 30$	< 1 sec	< 1 sec	< 1 sec	< 1 sec	< 1 sec	18 min	10^{25} years
$n = 50$	< 1 sec	< 1 sec	< 1 sec	< 1 sec	11 min	36 years	very long
$n = 100$	< 1 sec	< 1 sec	< 1 sec	1 sec	12,892 years	10^{17} years	very long
$n = 1,000$	< 1 sec	< 1 sec	1 sec	18 min	very long	very long	very long
$n = 10,000$	< 1 sec	< 1 sec	2 min	12 days	very long	very long	very long
$n = 100,000$	< 1 sec	2 sec	3 hours	32 years	very long	very long	very long
$n = 1,000,000$	1 sec	20 sec	12 days	31,710 years	very long	very long	very long

Types of analysis

- ▶ Worst case
 - ▶ Provides an **upper bound** on running time
 - ▶ An absolute **guarantee** that the algorithm would not run longer
- ▶ Best case
 - ▶ Provides a **lower bound** on running time
 - ▶ Input is the one for which the algorithm runs the fastest

$$\textit{Lower Bound} \leq \textit{Running Time} \leq \textit{Upper Bound}$$

- ▶ Average case
 - ▶ Provides a **prediction** about the running time
 - ▶ Very useful, but treat with care: what is “average”?
 - ▶ Random (equally likely) inputs
 - ▶ Real-life inputs

Asymptotic Analysis

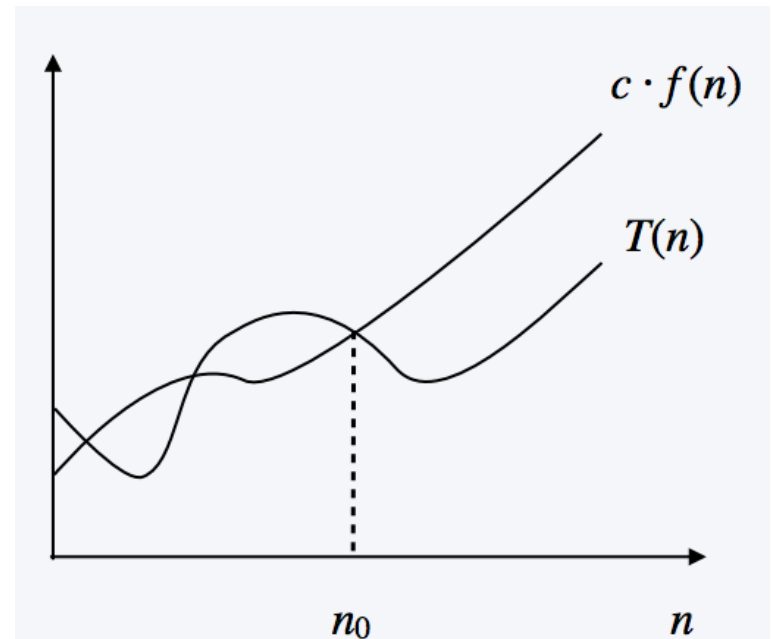
- ▶ To compare two algorithms with running times $f(n)$ and $g(n)$, we need a **rough measure** that characterizes **how fast each function grows**.
- ▶ Simplifications
 - ▶ Ignore actual and abstract statement costs
 - ▶ Order of growth: **highest-order term is what counts**
 - ▶ Remember, we are doing asymptotic analysis
 - ▶ As the input size grows larger, the high order term dominates
- ▶ For example: $5n^3 + 100n^2 + 10n + 50 \sim n^3$

Asymptotic notation: Big-Oh notation

- ▶ **Upper bounds.** $T(n)$ is $O(f(n))$ if there exist constants $c > 0$ and $n_0 \geq 0$ such that $T(n) \leq c \cdot f(n)$ for all $n \geq n_0$.

Ex. $T(n) = 32n^2 + 17n + 1$.

- ▶ $T(n)$ is $O(n^2)$. ← choose $c=50, n_0=1$
- ▶ $T(n)$ is also $O(n^3)$.
- ▶ $T(n)$ is neither $O(n)$ nor $O(n \log n)$.

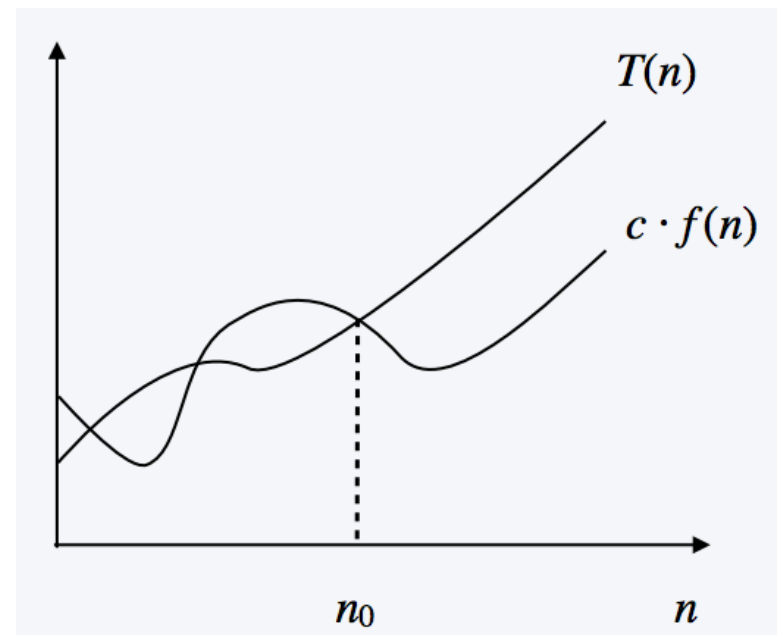


Asymptotic notation: Big-Omega notation

- ▶ **Lower bounds.** $T(n)$ is $\Omega(f(n))$ if there exist constants $c > 0$ and $n_0 \geq 0$ such that $T(n) \geq c \cdot f(n)$ for all $n \geq n_0$.

Ex. $T(n) = 32n^2 + 17n + 1$.

- ▶ $T(n)$ is $\Omega(n^2)$. ← choose $c=32, n_0=1$
- ▶ $T(n)$ is also $\Omega(n)$.
- ▶ $T(n)$ is neither $\Omega(n^3)$ nor $\Omega(n^3 \log n)$.

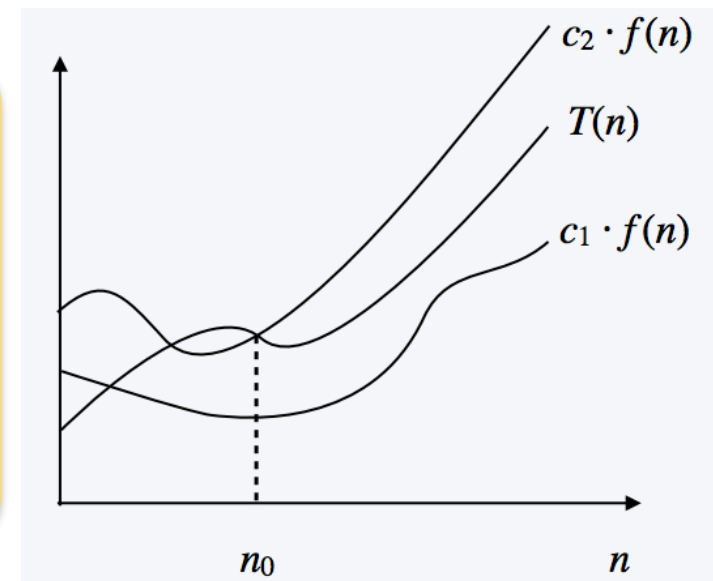


Asymptotic notation: Big-Theta notation

- ▶ **Tight bounds.** $T(n)$ is $\Theta(f(n))$ if there exist constants $c_1 > 0$, $c_2 > 0$ and $n_0 \geq 0$ such that $c_1 \cdot f(n) \leq T(n) \leq c_2 \cdot f(n)$ for all $n \geq n_0$.

Ex. $T(n) = 32n^2 + 17n + 1$.

- ▶ $T(n)$ is $\Theta(n^2)$. ← choose $c_1=32, c_2=50, n_0=1$
- ▶ $T(n)$ is neither $\Theta(n)$ nor $\Theta(n^3)$.



Asymptotic notation: little-o, and little- ω

- ▶ **Little-o:** $T(n)$ is $o(f(n))$ if for any constant $c > 0$, there exists $n_0 \geq 0$ such that $T(n) < c \cdot f(n)$ for all $n \geq n_0$
- ▶ **Little- ω :** $T(n)$ is $\omega(f(n))$ if for any constant $c > 0$, there exists $n_0 \geq 0$ such that $T(n) > c \cdot f(n)$ for all $n \geq n_0$.
- ▶ Intuitively
 - ▶ $o()$ is like $<$ $O()$ is like \leq
 - ▶ $\omega()$ is like $>$ $\Omega()$ is like \geq
 - ▶ $\Theta()$ is like $=$

Properties

- ▶ *Theorem:*

$$f(n) = \Theta(g(n)) \Leftrightarrow f = O(g(n)) \text{ and } f = \Omega(g(n))$$

- ▶ **Transitivity:**

- ▶ $f(n) = \Theta(g(n))$ and $g(n) = \Theta(h(n)) \Rightarrow f(n) = \Theta(h(n))$
- ▶ Same for O and Ω

- ▶ **Reflexivity:**

- ▶ $f(n) = \Theta(f(n))$
- ▶ Same for O and Ω

- ▶ **Symmetry:**

- ▶ $f(n) = \Theta(g(n))$ if and only if $g(n) = \Theta(f(n))$

- ▶ **Transpose symmetry:**

- ▶ $f(n) = O(g(n))$ if and only if $g(n) = \Omega(f(n))$

What's next...

- ▶ Mathematical foundations
 - ▶ Review basic math functions
- ▶ Solving recurrence (Divide-and-Conquer)
 - ▶ Substitution, Iteration, Master method
 - ▶ Read CLRS Chapter 4