EL9343 Homework 6

Due: Oct. 20th 11:00 a.m.

1. Let the table have 9 slots, and let the hash function be $h(k) = k \mod 9$. Demonstrate what happens when we insert the keys 10, 22, 35, 12, 1, 21, 6, 15, 36, 33 into a hash table with collisions resolved by chaining.

   Solution:

$h(k)$	keys
0	36
1	$1 \to 10$
2	
3	$21 \to 12$
4	22
5	
6	$33 \to 15 \to 6$
7	
8	35

2. Suppose we use a hash function h to hash n distinct keys into an array T of length m. Assuming simple uniform hashing, what is the expected number of collisions?

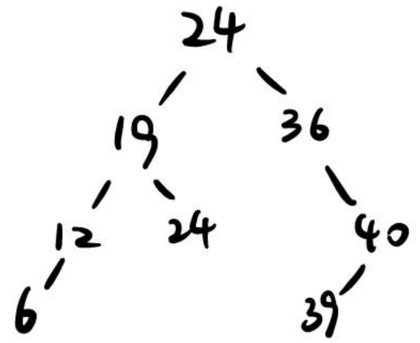
   Solution:

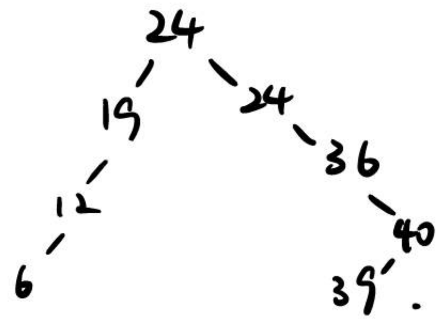
   Suppose the keys are $k_1, k_2, \ldots, k_n$. Let X_i be the number of $j > i$ with $h(k_j) = h(k_i)$. Under the assumption of simple uniform hashing, $\mathbb{E}(X_i) = \sum_{j>i} \Pr(h(k_j) = h(k_i)) = \sum_{j>i} \frac{1}{m} = \frac{n-i}{m}$. Then, by linearity of expectation, the number of collisions C is the sum of X_i for all i, which makes $\mathbb{E}(C) = \mathbb{E}(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} \mathbb{E}(X_i) = \sum_{i=1}^{n} \frac{n-i}{m} = \frac{n^2-n}{2m}$.

3. (a) Suppose you have two sequences, $\{24, 19, 12, 6, 24, 36, 40, 39\}$ and $\{6, 12, 24, 19, 39, 40, 36, 24\}$. You know the first one is generated from some BST A by pre-order tree walk, and the second one is generated from some BST B by post-order tree work. Please draw all the possible BST A that can generate the sequence. Repeat that for BST B.

   Solution:

   There are two possible BST A that can generate the first pre-order tree walk sequence, as shown below.





   The BST B can only be the left one.

   (b) If all the keys in a BST are distinct, can you draw a unique BST when only given its pre-order tree walk? If yes, please describe why; if no, find a counter-case.

   Solution:

   Yes. Given that the keys are distinct, there will not be occasions (like in 3a) when an element equals to the current root node and it can be placed in the left sub-tree as well as in the right sub-tree.
   Therefore, an algorithm could be designed: for a sequence (or sub-sequence), the first element is the value of the root node. Then we should be able to find an unique point, where all the elements ahead of it are smaller than the root while all the elements beyond it are larger. (If such point doesn't exist, this sequence cannot be pre-order traversal of a BST.) All the elements between the first and this point forms a sub-sequence, and the resulting BST from this sub-sequence is the left

sub-tree of the root. All the elements between this point and the end forms a sub-sequence, and the resulting BST from this sub-sequence is the right sub-tree of the root. Thus the BST we find is unique.

Note 1: This statement is also true if we substitute "pre-order" with "post-order".

Note 2: The given condition that all the keys are distinct is actually a very strong one. The post-order traversal sequence given in 3a could also lead to an unique BST. It is whether the separation point in the algorithm described above is unique or not.

4. For the binary search tree (BST) in pre-order as $\{8, 4, 16, 9, 19, 17, 22\}$. **Please first draw the BST**, then show the result of following operations (each operation is carried out on the result of the previous operation):

   (a) Insert key 20;

   (b) Then, delete key 8;

   (c) Then, delete key 19;

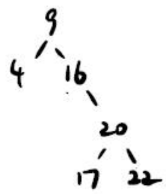
   (d) Finally, delete key 16.

   Solution:

   Original: 

   After a: 

   After b: 

   After c: 

   After d: