Mathematical Formulae
A Book of High School and Engineering Common Core Mathematical Formulae

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Contents

I Algebra **2**

1 Logarithm **3**

1.1 Basic Formulae 3
1.2 Series 3

2 Complex Numbers **4**

2.1 Basic Formulae 4
2.2 Arithmetic Operation of Complex Number 4
2.3 Euler's Formula 4
2.4 Trigonometric Ratios in Complex Form 5
2.5 De Moivre's Formula 5
2.6 Application of Euler's and De Moivre's Formula 5
2.7 Roots of Unity 5
2.8 Important Relations of Complex Numbers 6

Part I
Algebra

Chapter 1
Logarithm

1.1 Basic Formulae

For $a^x = b$:

$$\log_a x, \forall x \leq 0 \text{ is undefined} \tag{1.1}$$

$$\log_a b = x, bax \neq 1, a \neq 1 \tag{1.2}$$

$$\log_b a^m = m \log_b a, \text{ for } a^m = b \tag{1.3}$$

$$a^{\log_a x} = x \tag{1.4}$$

$$a^{\log_b c} = c^{\log_b a} \tag{1.5}$$

$$\frac{1}{\log_a b} = \log_b a \tag{1.6}$$

$$\log_c(ab) = \log_c a + \log_c b \tag{1.7}$$

$$\log_c \left(\frac{a}{b}\right) = \log_c a - \log_c b \tag{1.8}$$

$$|\log_a x| = \begin{cases} -\log_a x, & \text{if } 0 < x < 1 \\ \log_a x, & \text{if } 1 \leq x < \infty \end{cases} \tag{1.9}$$

1.2 Series

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \infty = \lim_{n \to \infty} \sum_{i=0}^{n} \frac{x^i}{i!} \tag{1.10}$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \infty = \lim_{n \to \infty} \sum_{i=1}^{n} (-1)^{(i-1)} \frac{x^i}{i} \tag{1.11}$$

Chapter 2

Complex Numbers

2.1 Basic Formulae

For $z = x + iy$,

$$|z| = \sqrt{x^2 + y^2} \tag{2.1}$$

$$\tan\theta = \frac{y}{x} \tag{2.2}$$

$$\bar{z} = x - iy \tag{2.3}$$

2.2 Arithmetic Operation of Complex Number

For two complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$:

$$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2) \tag{2.4}$$

$$z_1 z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1) \tag{2.5}$$

$$|z_1 z_2| = |z_1| \cdot |z_2| \tag{2.6}$$

$$\frac{z_1}{z_2} = \frac{(x_1 x_2 + y_1 y_2) + i(x_2 y_1 - x_1 y_2)}{{a_2}^2 + {b_2}^2} \tag{2.7}$$

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} \tag{2.8}$$

2.3 Euler's Formula

$$z = re^{i\theta}, \text{ where} \tag{2.9}$$

$$r = |z| \tag{2.10}$$

$$e^{i\theta} = \cos\theta + i\sin\theta \tag{2.11}$$

$$\theta = \arctan\left(\frac{y}{x}\right) \tag{2.12}$$

2.4 Trigonometric Ratios in Complex Form

$$e^{i\theta} + e^{-i\theta} = 2\cos\theta \tag{2.13}$$
$$\Rightarrow \cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2} \tag{2.14}$$
$$e^{i\theta} - e^{-i\theta} = 2\sin\theta \tag{2.15}$$
$$\Rightarrow \sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2} \tag{2.16}$$

2.5 De Moivre's Formula

According to DeMoivre's Formula:

$$(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta) \tag{2.17}$$

Proof

$$\begin{aligned} \cos\theta + i\sin\theta &= e^{i\theta} \\ \Rightarrow (\cos\theta + i\sin\theta)^n &= e^{n(i\theta)} \\ &= \cos(n\theta) + i\sin(n\theta) \\ &\text{Q.E.D.} \end{aligned}$$

2.6 Application of Euler's and De Moivre's Formula

For $z_1 = |r_1|\, e^{i\theta_1}$ and $z_2 = |r_2|\, e^{i\theta_2}$

$$z_1 z_2 = (r_1 r_2)\, e^{i(\theta_1+\theta_2)} \tag{2.18}$$
$$\frac{z_1}{z_2} = \left(\frac{r_1}{r_2}\right) e^{i(\theta_1-\theta_2)} \tag{2.19}$$

2.7 Roots of Unity

$$\sqrt[n]{1} = e^{i\dfrac{2k\pi}{n}}, \text{ where } k \in [0, n-1] \tag{2.20}$$

2.8 Important Relations of Complex Numbers

$$|z_1 + z_2| \leq |z_1| + |z_2| \tag{2.21}$$
$$|z_1 - z_2| \leq |z_1| + |z_2| \tag{2.22}$$
$$|z_1 - z_2| \geq |z_1| - |z_2| \tag{2.23}$$
$$|z_1 + z_2| \geq ||z_1| - |z_2|| \tag{2.24}$$
$$|z_1 + z_2|^2 = 2\left(|z_1|^2 + |z_2|^2\right) \tag{2.25}$$