Mathematical Formulae
A Book of High School and Engineering Common Core Mathematical Formulae

Agnij Mallick

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Part I
Algebra

Chapter 1
Logarithm

1.1 Basic Formulae

For $a^x = b$:

$$\log_a x, \forall x \leq 0 \text{ is undefined} \tag{1.1}$$
$$\log_a b = x, bax \neq 1, a \neq 1 \tag{1.2}$$
$$\log_b a^m = m \log_b a, \text{ for } a^m = b \tag{1.3}$$
$$a^{\log_a x} = x \tag{1.4}$$
$$a^{\log_b c} = c^{\log_b a} \tag{1.5}$$
$$\frac{1}{\log_a b} = \log_b a \tag{1.6}$$
$$\log_c(ab) = \log_c a + \log_c b \tag{1.7}$$
$$\log_c \left(\frac{a}{b}\right) = \log_c a - \log_c b \tag{1.8}$$
$$|\log_a x| = \begin{cases} -\log_a x, & \text{if } 0 < x < 1 \\ \log_a x, & \text{if } 1 \leq x < \infty \end{cases} \tag{1.9}$$

1.2 Series

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \infty = \lim_{n \to \infty} \sum_{i=0}^{n} \frac{x^i}{i!} \tag{1.10}$$
$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \infty = \lim_{n \to \infty} \sum_{i=1}^{n} (-1)^{(i-1)} \frac{x^i}{i} \tag{1.11}$$

Chapter 2
Complex Numbers

2.1 Basic Formulae

For $z = x + iy$,

$$|z| = \sqrt{x^2 + y^2} \tag{2.1}$$

$$\tan\theta = \frac{y}{x} \tag{2.2}$$

$$\bar{z} = x - iy \tag{2.3}$$

2.2 Arithmetic Operation of Complex Number

For two complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$:

$$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2) \tag{2.4}$$

$$z_1 z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1) \tag{2.5}$$

$$|z_1 z_2| = |z_1| \cdot |z_2| \tag{2.6}$$

$$\frac{z_1}{z_2} = \frac{(x_1 x_2 + y_1 y_2) + i(x_2 y_1 - x_1 y_2)}{{a_2}^2 + {b_2}^2} \tag{2.7}$$

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} \tag{2.8}$$

2.3 Euler's Formula

$$z = re^{i\theta}, \text{ where} \tag{2.9}$$

$$r = |z| \tag{2.10}$$

$$e^{i\theta} = \cos\theta + i\sin\theta \tag{2.11}$$

$$\theta = \arctan\left(\frac{y}{x}\right) \tag{2.12}$$

2.4 Trigonometric Ratios in Complex Form

$$e^{i\theta} + e^{-i\theta} = 2\cos\theta \tag{2.13}$$

$$\Rightarrow \cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2} \tag{2.14}$$

$$e^{i\theta} - e^{-i\theta} = 2\sin\theta \tag{2.15}$$

$$\Rightarrow \sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2} \tag{2.16}$$

2.5 De Moivre's Formula

According to DeMoivre's Formula:

$$(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta) \tag{2.17}$$

Proof

$$\begin{aligned} \cos\theta + i\sin\theta &= e^{i\theta} \\ \Rightarrow (\cos\theta + i\sin\theta)^n &= e^{n(i\theta)} \\ &= \cos(n\theta) + i\sin(n\theta) \\ &\text{Q.E.D.} \end{aligned}$$

2.6 Application of Euler's and De Moivre's Formula

For $z_1 = |r_1|\, e^{i\theta_1}$ and $z_2 = |r_2|\, e^{i\theta_2}$

$$z_1 z_2 = (r_1 r_2)\, e^{i(\theta_1+\theta_2)} \tag{2.18}$$

$$\frac{z_1}{z_2} = \left(\frac{r_1}{r_2}\right) e^{i(\theta_1-\theta_2)} \tag{2.19}$$

2.7 Roots of Unity

$$\sqrt[n]{1} = e^{i\frac{2k\pi}{n}}, \text{ where } k \in [0, n-1] \tag{2.20}$$

2.8 Important Relations of Complex Numbers

$$|z_1 + z_2| \leq |z_1| + |z_2| \tag{2.21}$$

$$|z_1 - z_2| \leq |z_1| + |z_2| \tag{2.22}$$

$$|z_1 - z_2| \geq |z_1| - |z_2| \tag{2.23}$$

$$|z_1 + z_2| \geq ||z_1| - |z_2|| \tag{2.24}$$

$$|z_1 + z_2|^2 = 2\left(|z_1|^2 + |z_2|^2\right) \tag{2.25}$$

Chapter 3

Progression

3.1 Arithmetic Progression (A.P.)

An arithmetic sequence is $a, a+n, a+2n, ...\infty$ or $t_n = a + (n-1)d$, where a is the first term, d is the common difference, and n is the n^{th}-term.
An arithmetic series is $a + (a+d) + (a+2d) + ...\infty$.

3.1.1 Sum of A.P. Series

$$\begin{aligned} S_n &= a + (a+d) + ... + (a + \overline{n-2}d) + (a + \overline{n-1}d) \\ S_n &= (a + \overline{n-1}d) + (a + \overline{n-2}d + ... + (a+d) + a \\ \Rightarrow 2S_n &= n(2a + \overline{n-1}d) \\ \Rightarrow S_n &= \frac{n}{2}(2a + \overline{n-1}d) \end{aligned} \tag{3.1}$$

3.1.2 Important Relation

If the three terms a, b, c are in A.P., then

$$2b = a + c \tag{3.2}$$

3.2 Geometric Progression (G.P.)

An geometric sequence is $a, ar, ar^2, ...\infty$ or $t_n = ar^{n-1}$, where a is the first term, r is the common ratio, and n is the n^{th}-term. An geometric series is $a + ar + ar^2 + ...\infty$.

3.2.1 The Value of 'r'

$$r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = ... = \frac{t_n}{t_{n-1}} \tag{3.3}$$

3.2.2 Sum of a G.P. Series

For a definite G.P. series, where there are n terms in the series, the sum of the series is:

$$S_n = \frac{a|r^n - 1|}{|r-1|} \tag{3.4}$$

For an infite G.P. series the sum of the series is defined for $r < 1$. Sum of such a series is:

$$S_\infty = \frac{a}{1-r} \tag{3.5}$$

3.2.3 Important relations

If the three terms a, b, c are in G.P., then:

$$b^2 = ac \tag{3.6}$$

3.3 Harmonic Progression (H.P.)

If a, b, c are terms of an H.P. then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$$\frac{2}{b} = \frac{1}{a} + \frac{1}{c} \tag{3.7}$$

$$\Rightarrow b = \frac{2ac}{a+c} \tag{3.8}$$

3.4 Arithmetico-Geometric Progression (A.G.P.)

Sequence $a, (a+d)r, (a+2d)r^2, ..., (a+\overline{n-1}d)r^{n-1}$, where a $\rightarrow$first term of A.G.P., d $\rightarrow$common difference, and r $\rightarrow$common ratio.

3.4.1 Sum of A.G.P.:

For an infinite A.G.P. series, the sum is defined for $r < 1$:

$$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2} \tag{3.9}$$

3.5 Special Series

For $n \in \mathbb{N}$

$$1 + 2 + 3 + ... + (n-1) + n = \frac{n(n-1)}{2} \tag{3.10}$$

$$1^2 + 2^2 + 3^2 + ... + (n-1)^2 + n^2 = \frac{n(n+1)(2n+1)}{6} \tag{3.11}$$

$$1^3 + 2^3 + 3^3 + ... + (n-1)^3 + n^3 = [\frac{n(n-1)}{2}]^2 \tag{3.12}$$

3.5.1 Riemann Zeta Function

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \tag{3.13}$$

3.5.2 Riemann's Infinite Series as an Integration

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=r_1}^{r_2} f(\frac{i}{n}) = \int_{\frac{r_1}{n}}^{\frac{r_2}{n}} f(x)dx \tag{3.14}$$