Chapter 1

Progression

1.1 Arithmetic Progression (A.P.)

An arithmetic sequence is $a, a+n, a+2n, ...\infty$ or $t_n = a + (n-1)d$, where a is the first term, d is the common difference, and n is the n^{th}-term.
An arithmetic series is $a + (a+d) + (a+2d) + ...\infty$.

1.1.1 Sum of A.P. Series

$$\begin{aligned} S_n &= a + (a+d) + ... + (a+\overline{n-2}d) + (a+\overline{n-1}d) \\ S_n &= (a+\overline{n-1}d) + (a+\overline{n-2}d + ... + (a+d) + a \\ \Rightarrow 2S_n &= n(2a+\overline{n-1}d) \\ \Rightarrow S_n &= \frac{n}{2}(2a+\overline{n-1}d) \end{aligned} \tag{1.1}$$

1.1.2 Important Relation

If the three terms a, b, c are in A.P., then

$$2b = a + c \tag{1.2}$$

1.2 Geometric Progression (G.P.)

An geometric sequence is $a, ar, ar^2, ...\infty$ or $t_n = ar^{n-1}$, where a is the first term, r is the common ratio, and n is the n^{th}-term. An geometric series is $a + ar + ar^2 + ...\infty$.

1.2.1 The Value of 'r'

$$r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = ... = \frac{t_n}{t_{n-1}} \tag{1.3}$$

1.2.2 Sum of a G.P. Series

For a definite G.P. series, where there are n terms in the series, the sum of the series is:

$$S_n = \frac{a|r^n - 1|}{|r - 1|} \tag{1.4}$$

For an infite G.P. series the sum of the series is defined for $r < 1$. Sum of such a series is:

$$S_\infty = \frac{a}{1 - r} \tag{1.5}$$

1.2.3 Important relations

If the three terms a, b, c are in G.P., then:

$$b^2 = ac \tag{1.6}$$

1.3 Harmonic Progression (H.P.)

If a, b, c are terms of an H.P. then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$$\frac{2}{b} = \frac{1}{a} + \frac{1}{c} \tag{1.7}$$

$$\Rightarrow b = \frac{2ac}{a + c} \tag{1.8}$$

1.4 Arithmetico-Geometric Progression (A.G.P.)

Sequence $a, (a+d)r, (a+2d)r^2, ..., (a+\overline{n-1}d)r^{n-1}$, where a $\rightarrow$first term of A.G.P., d $\rightarrow$common difference, and r $\rightarrow$common ratio.

1.4.1 Sum of A.G.P.:

For an infinite A.G.P. series, the sum is defined for $r < 1$:

$$S_\infty = \frac{a}{1 - r} + \frac{dr}{(1 - r)^2} \tag{1.9}$$

1.5 Special Series

For $n \in \mathbb{N}$

$$1 + 2 + 3 + ... + (n - 1) + n = \frac{n(n - 1)}{2} \tag{1.10}$$

$$1^2 + 2^2 + 3^2 + ... + (n - 1)^2 + n^2 = \frac{n(n + 1)(2n + 1)}{6} \tag{1.11}$$

$$1^3 + 2^3 + 3^3 + ... + (n - 1)^3 + n^3 = [\frac{n(n - 1)}{2}]^2 \tag{1.12}$$

1.5.1 Riemann Zeta Function

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \tag{1.13}$$

1.5.2 Riemann's Infinite Series as an Integration

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=r_1}^{r_2} f(\frac{i}{n}) = \int_{\frac{r_1}{n}}^{\frac{r_2}{n}} f(x)dx \tag{1.14}$$