# 计算几何基础与应用

yuguanyi@megvii.com

2022/7/15

# 写在前面

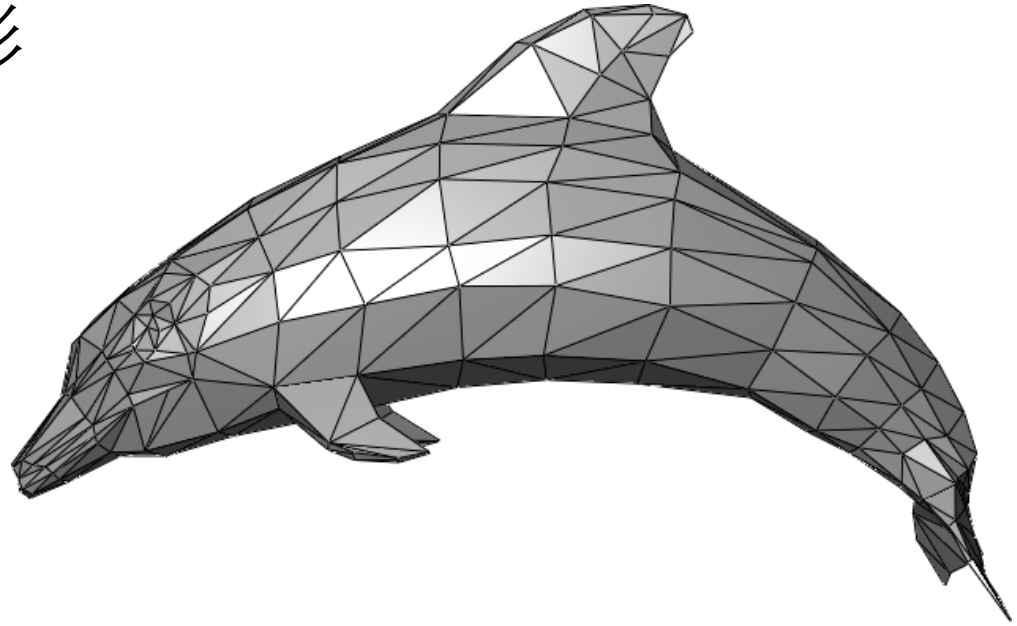
- What's 计算几何?
- Why 计算几何?
- 规约: 向量表示 (非解析表示), Slides & PA都使用NumPy
- PA: 3选1, Jupyter代码填空, ~5 blanks/task

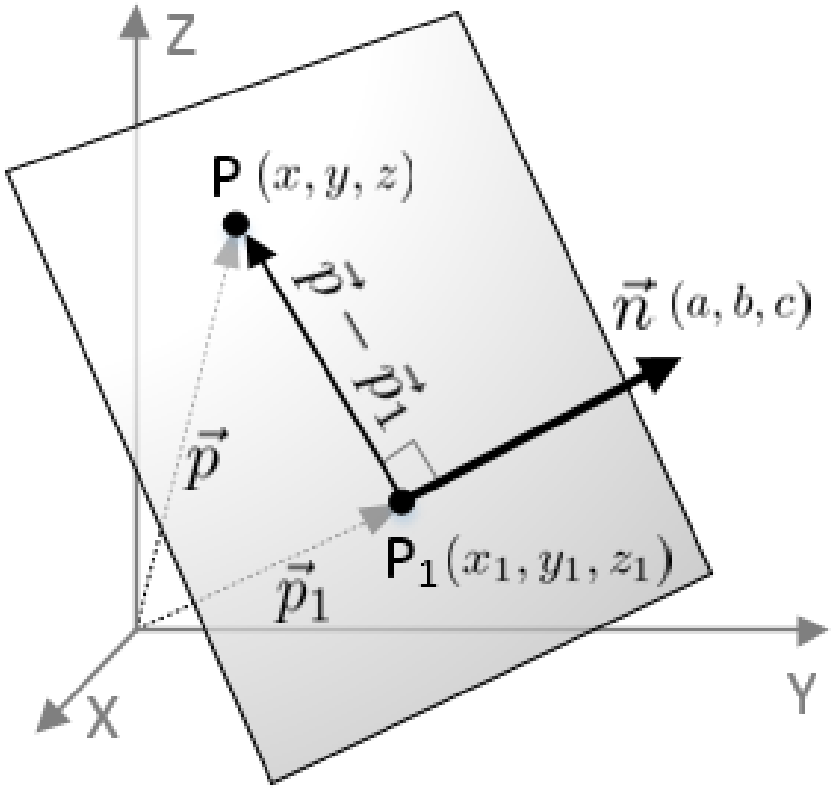
# 大纲

- 计算几何基础
  - 计算机中的几何表示
  - 点与直线位置关系
  - 线段求交
  - 点与多边形位置关系
  - 多边形面积
- 计算几何应用
  - 扫描线——Poly2mask
  - 凸包——3D BBox
  - 半平面交——3D IOU (yaw)

# 计算机中的几何表示

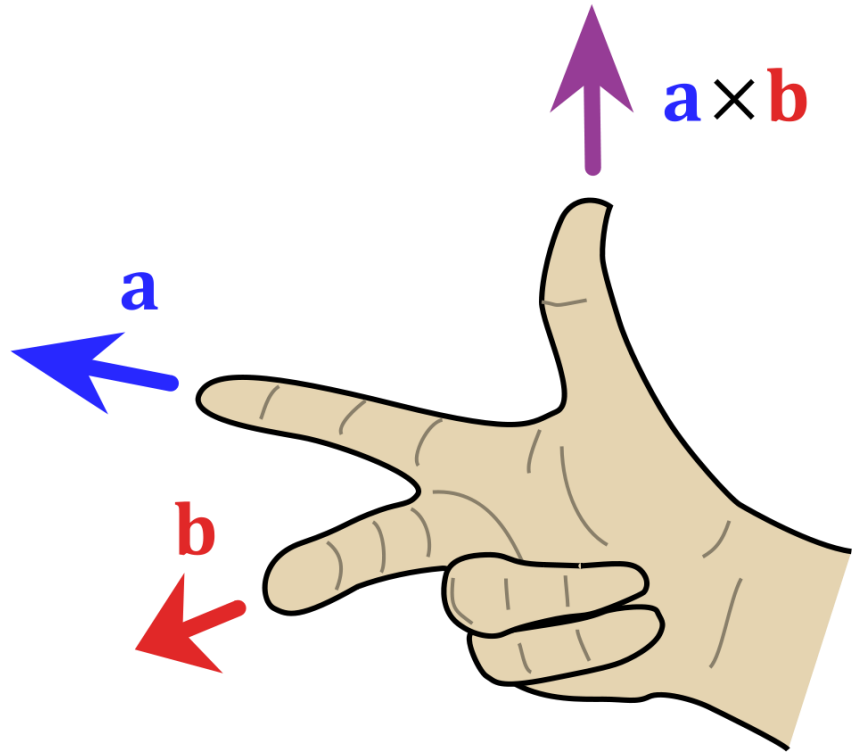
- 2D/3D 点/向量
- 2D/3D 线: 直线/射线/线段
- 平面多边形
- 三维图形





# 点与直线位置关系

- 叉积, 右手定则

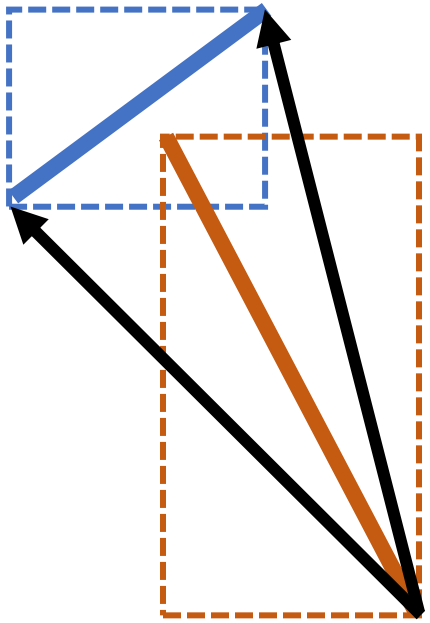



$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

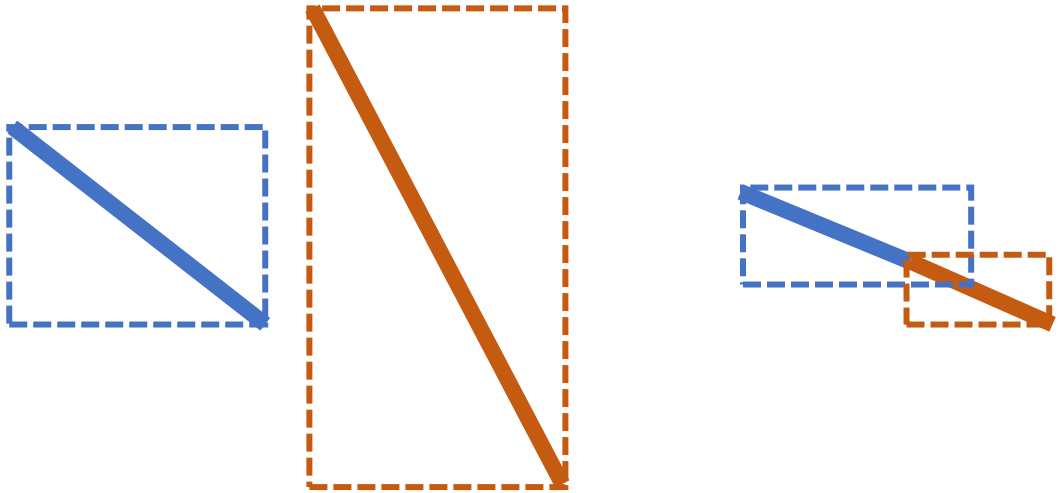
$$\begin{aligned} \mathbf{u} \times \mathbf{v} &= (u_2 v_3 \mathbf{i} + u_3 v_1 \mathbf{j} + u_1 v_2 \mathbf{k}) - (u_3 v_2 \mathbf{i} + u_1 v_3 \mathbf{j} + u_2 v_1 \mathbf{k}) \\ &= (u_2 v_3 - u_3 v_2)\mathbf{i} + (u_3 v_1 - u_1 v_3)\mathbf{j} + (u_1 v_2 - u_2 v_1)\mathbf{k} \end{aligned}$$

# 线段求交

- 判断相交+直线求交

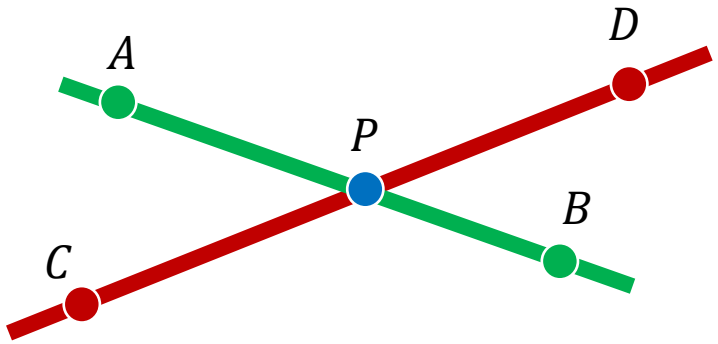


跨立试验



快速排斥试验 (共线)

# 直线求交

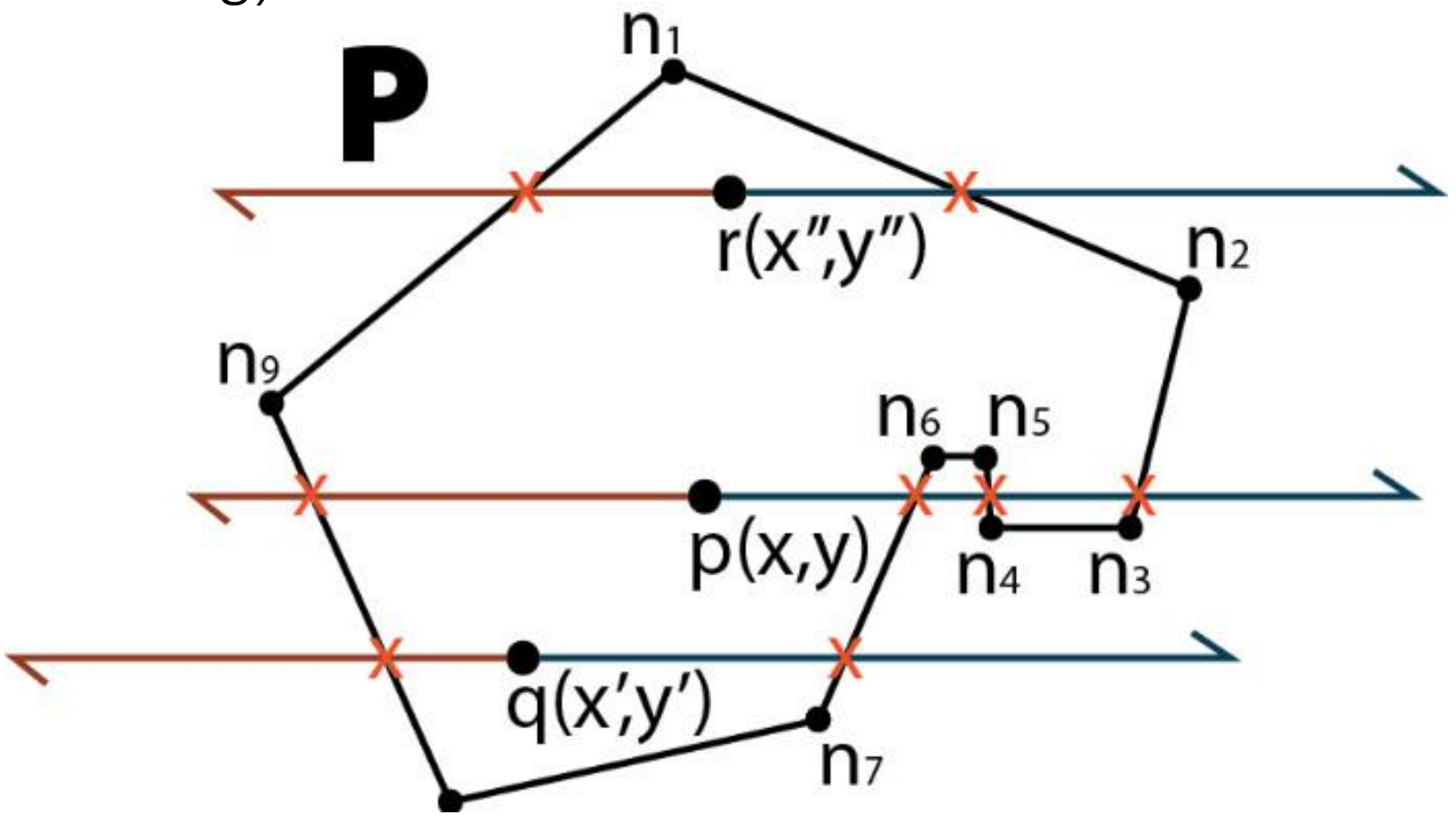



$$\begin{cases} P = A + t \cdot \overrightarrow{AB} \\ \overrightarrow{PC} \times \overrightarrow{CD} = 0 \end{cases} \Rightarrow t = \frac{\overrightarrow{AC} \times \overrightarrow{CD}}{\overrightarrow{AB} \times \overrightarrow{CD}}$$

```
def line_inter(A, B, C, D):
    return A + (B - A) * np.cross(C - A, D - C) / np.cross(B - A, D - C)
```

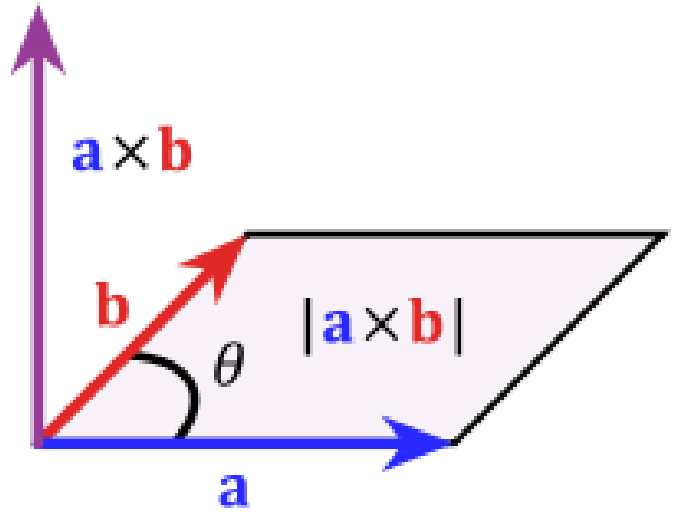
# 点与多边形位置关系 (Point in polygon)

- 射线法(Ray-casting)

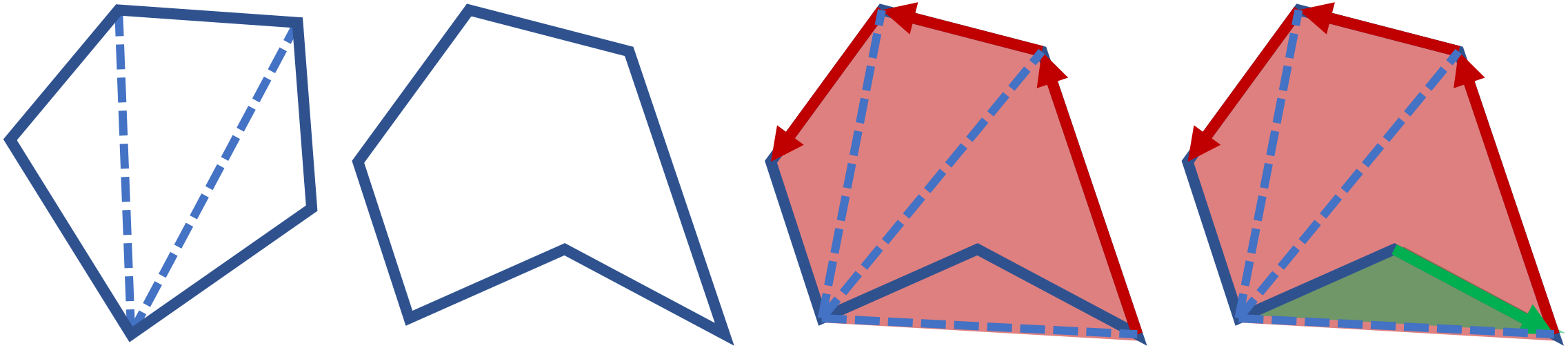


# 多边形面积

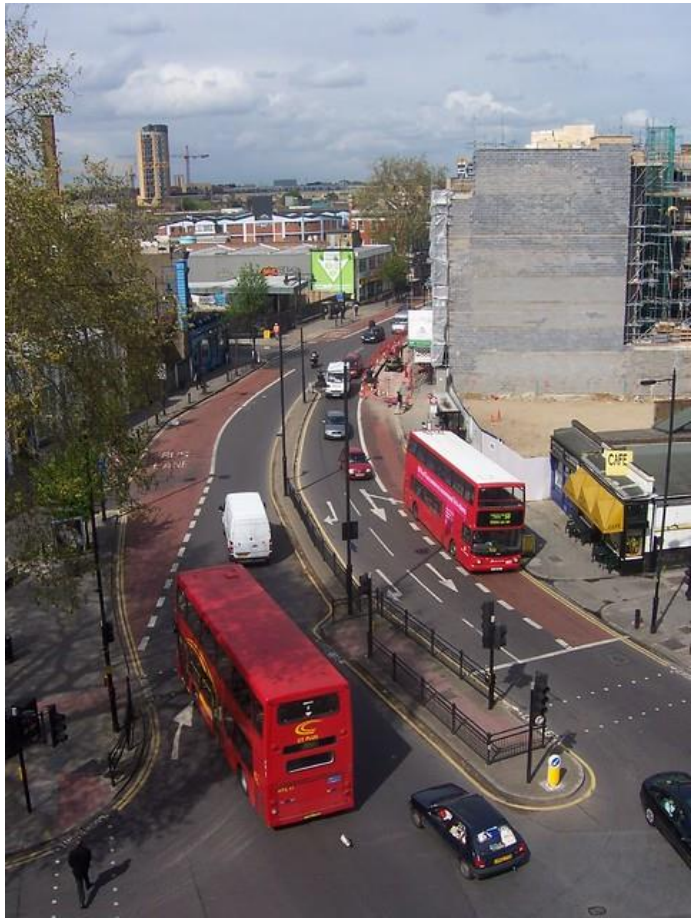
- 叉积几何含义, 三角剖分, 有向面积

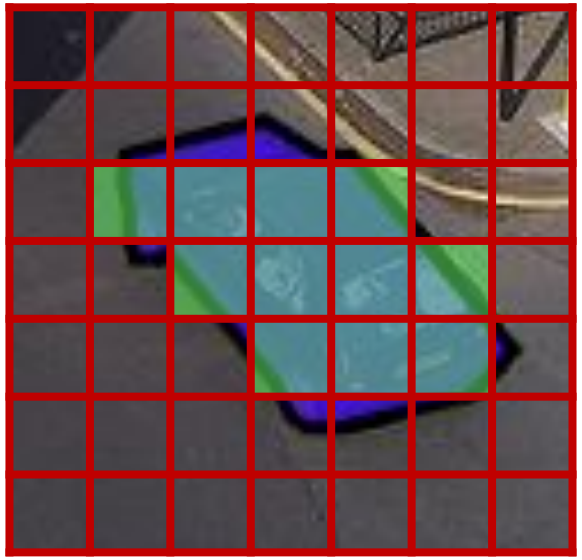



$$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\| \, \|\mathbf{b}\| \sin\theta.$$

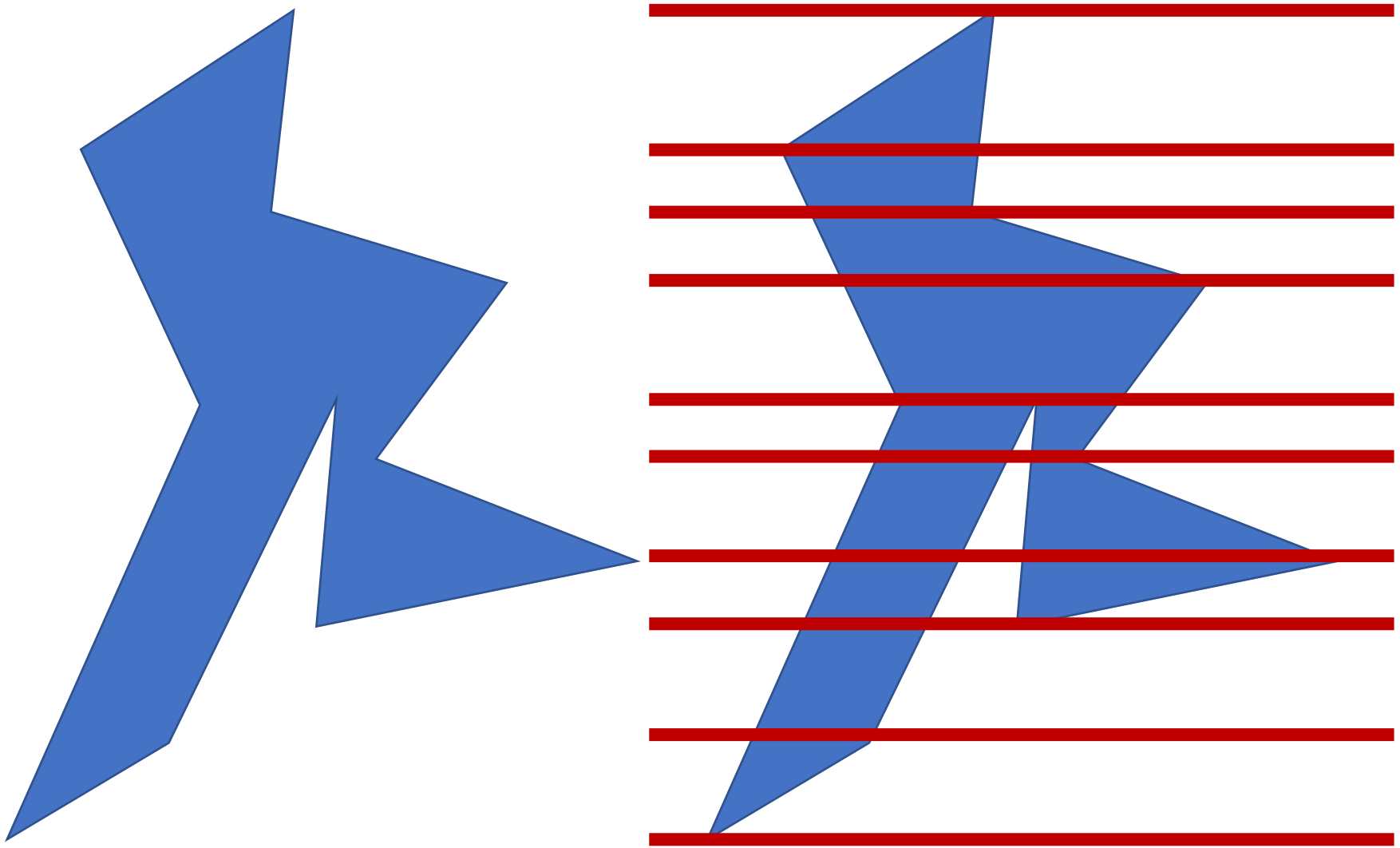


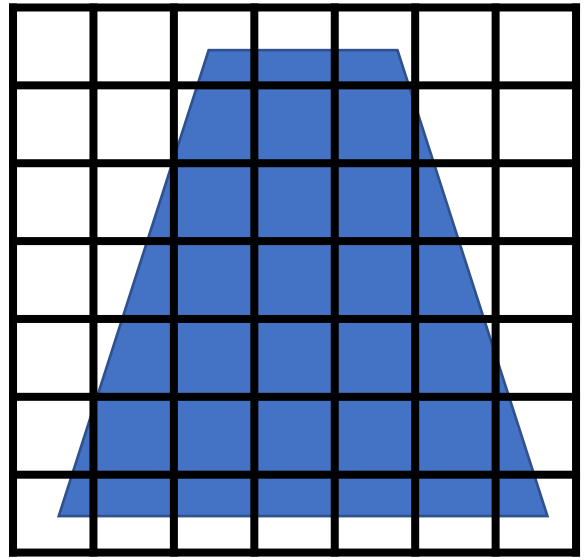
# Poly2mask



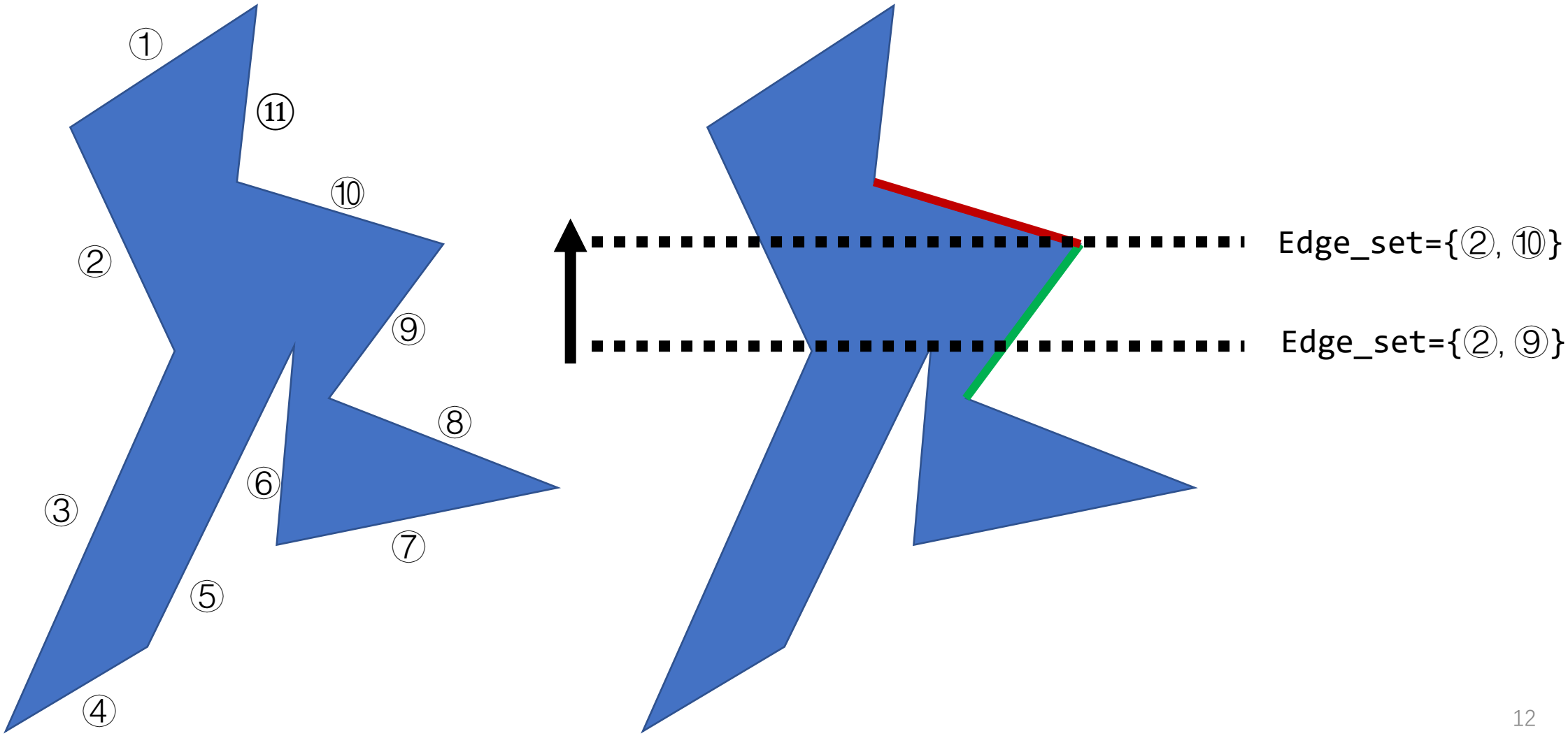


# Poly2mask



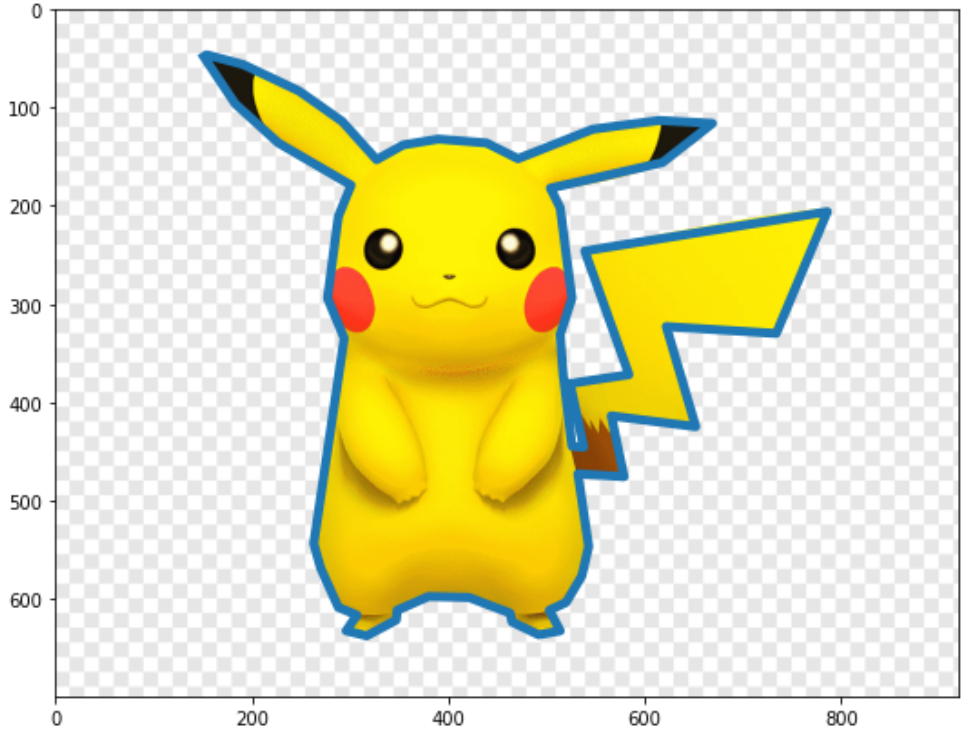


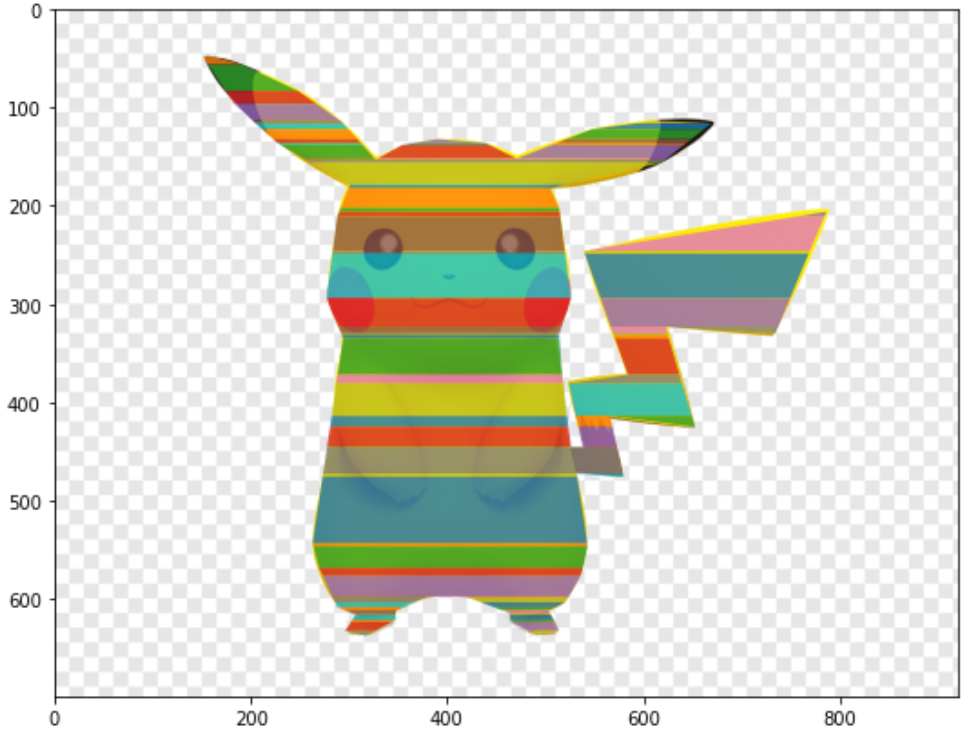
# 扫描线 (Sweep Line)



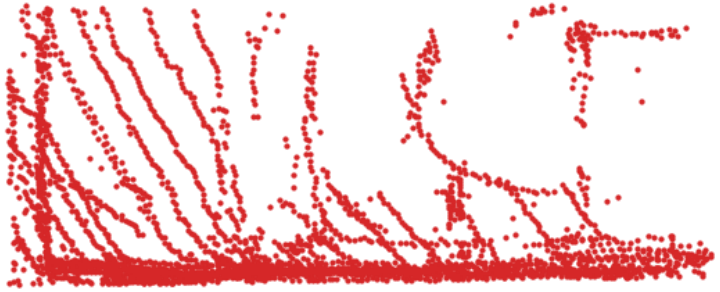
# PA1: Poly2mask

- 实现扫描线将给定的多边形转变为梯形mask, 辅助函数和整体框架已实现, 填补poly2mask函数中缺失的部分, 最终效果如下图:



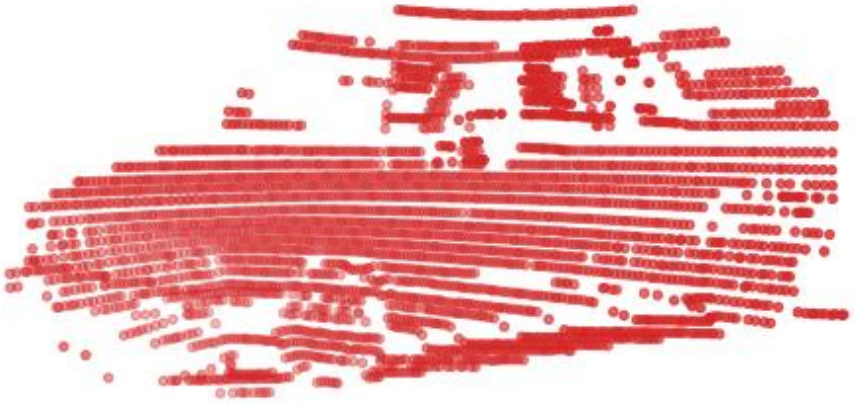



# 3D Bounding Box



Car(BEV)

Pedestrian

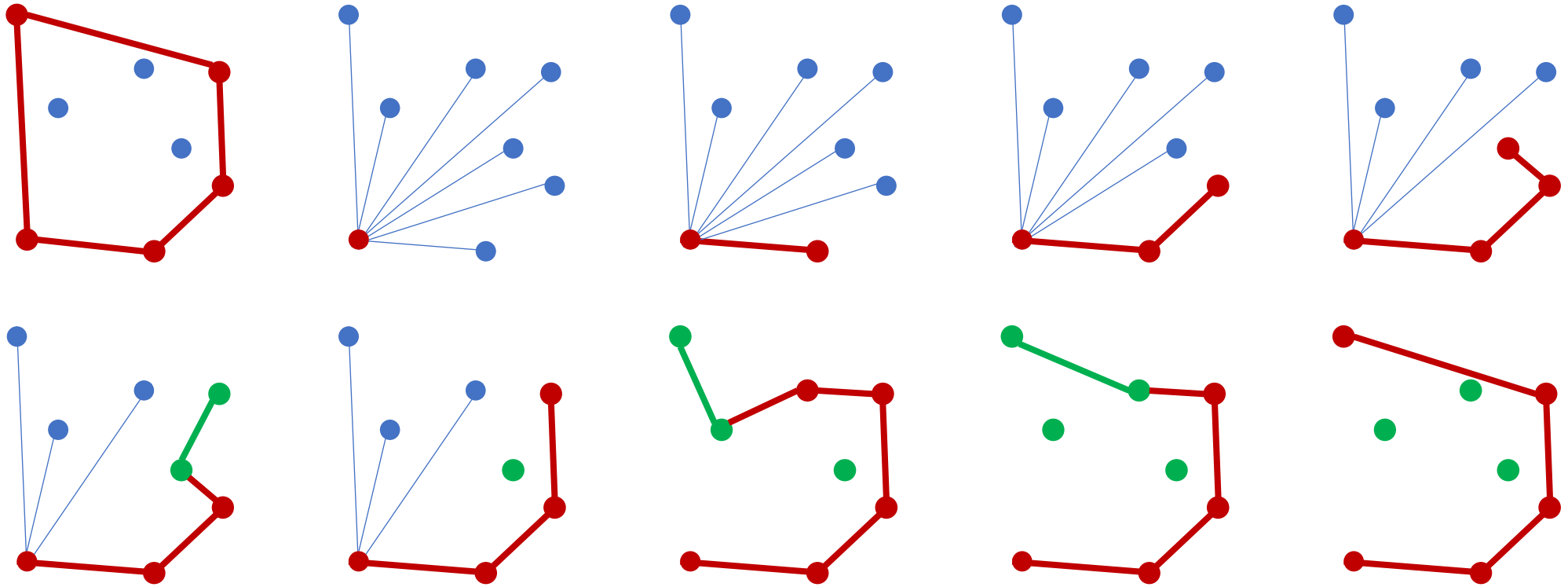


Car

Cyclist

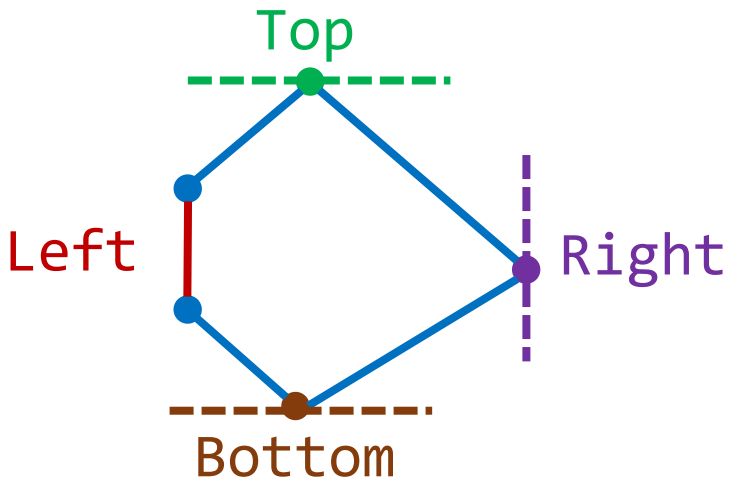
# 凸包 (Convex Hull)

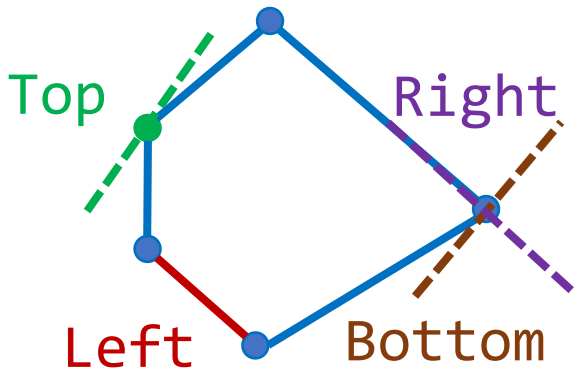
- 包含所有点的最小凸集; Graham's scan O(nlogn)



# 旋转卡壳 (Rotating Calipers)

- 结论：凸包与最小外接矩形必有重合边
- 凸性: 单调性

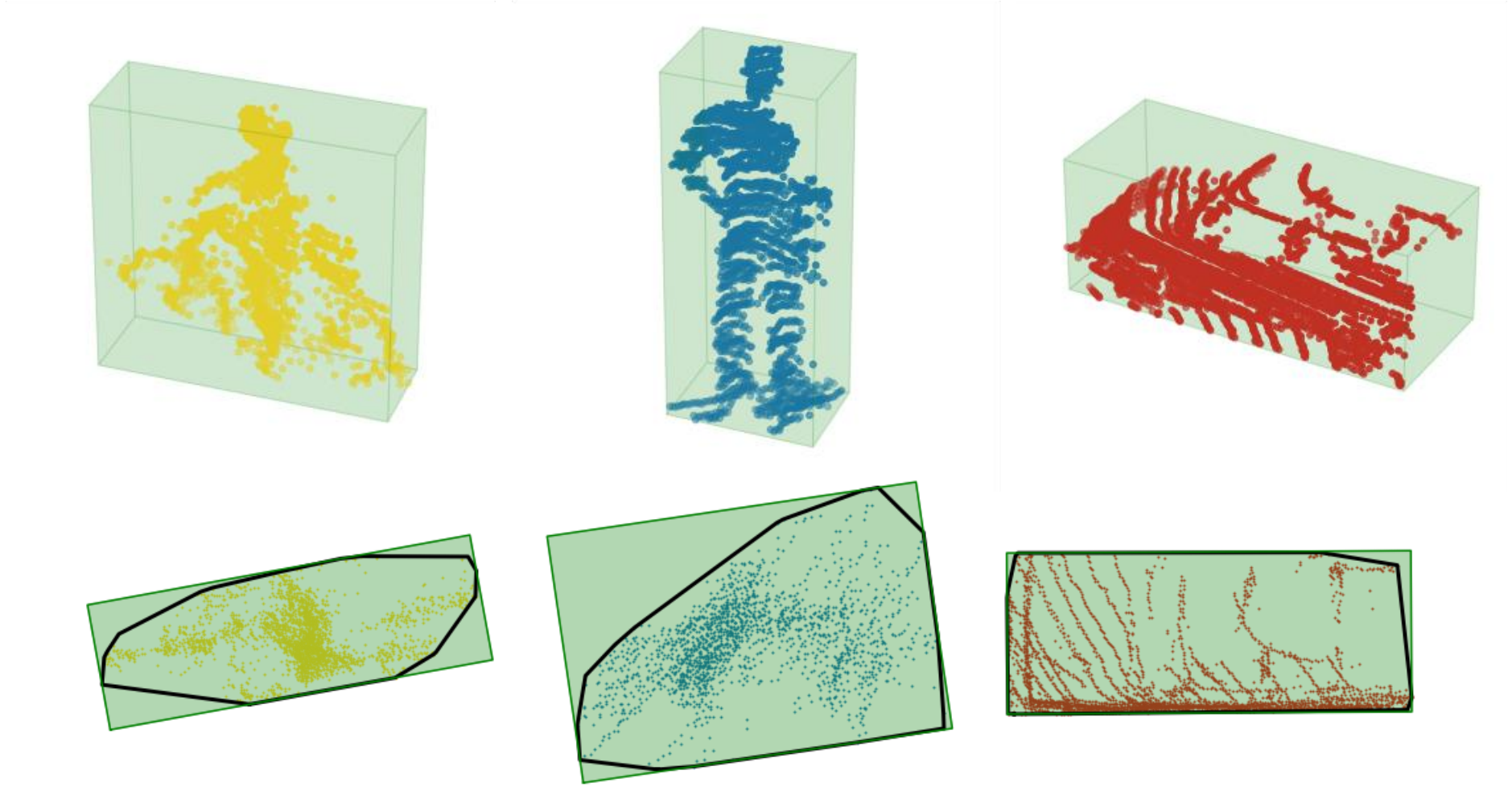



```
for Left in ...:
    for Bottom in ...:
        for Right in ...:
            for Top in ...:
                ...
```

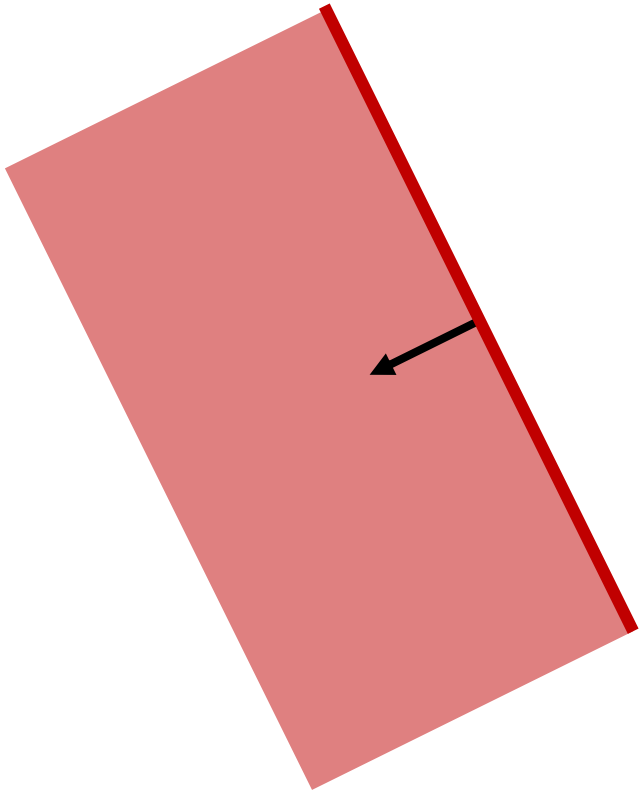
```
for Left in ...:
    while ...: Bottom += 1
    while ...: Right += 1
    while ...: Top += 1
```

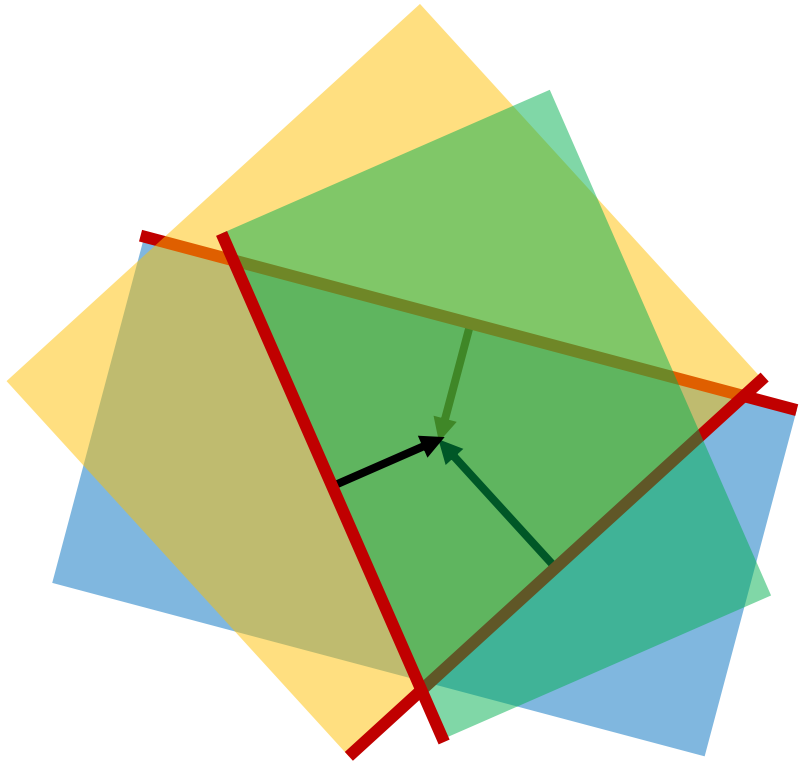
# PA2: 3D Bounding Box



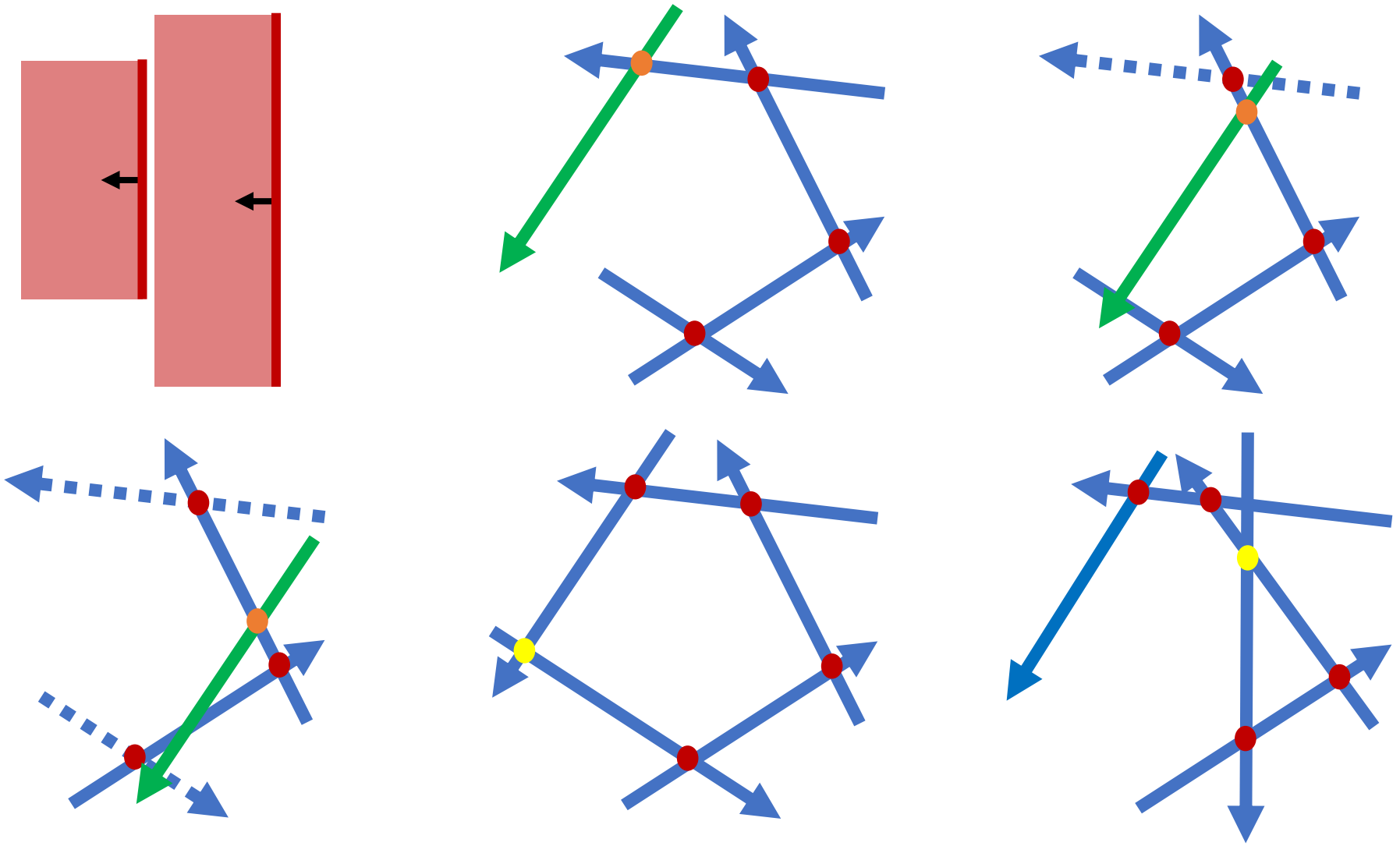
# 半平面交 (Half-plane intersection)

- 凸集: 凸集的交仍是凸集

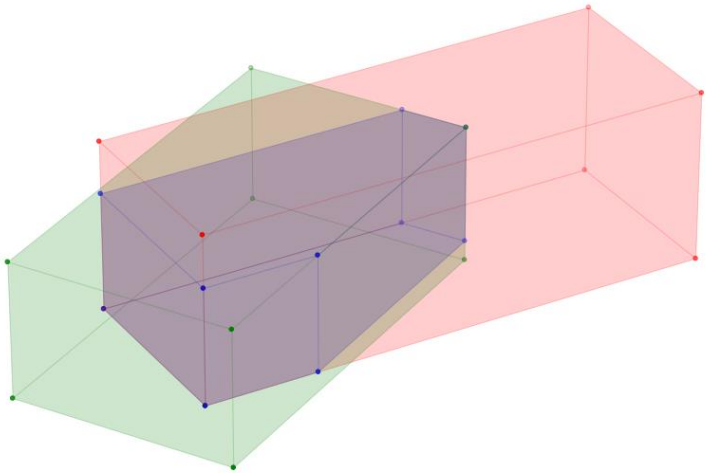


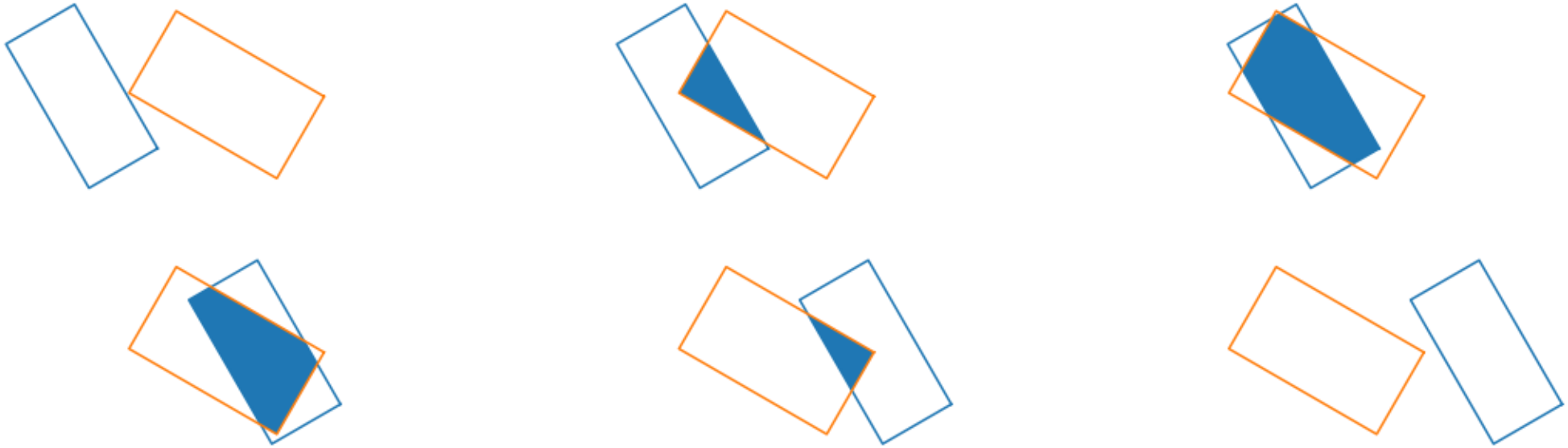


# 半平面交 (Half-plane intersection)



# PA3: 3D IOU (yaw)





# 总结

- 计算几何基础
- 全量枚举→动态维护增量
- 凸性(单峰性)/单调性优化枚举
- 有时通解比特解更简洁
- PA & Slides: github.com/Aguin/Computational-Geometry-Tutorial