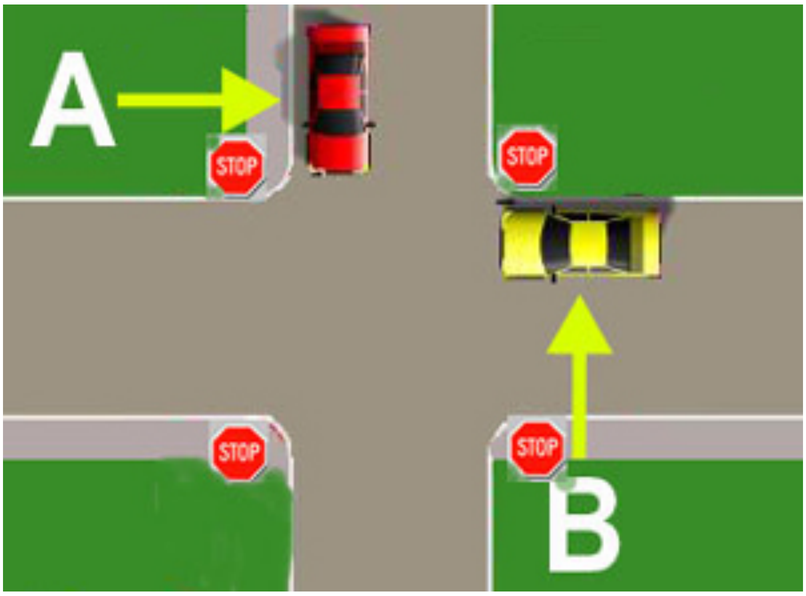
Cars and Conditional Probability

So far you have gotten a glimpse of how probability is used in self-driving cars when events are independent from each other. However, in many cases, the previous event helps shape the probability of the current event. In this case, these events are said to be conditional, meaning they are affected by another (usually previous) event. This is a bit of a tricky nuance so let us spend some time trying to build some intuition.

If you are already comfortable with conditional probability, please feel free to skip these exercises.



Here is a real-problem that plagues many self-driving cars. Suppose your self-driving car and another car arrive at a four-way stop. The rules of the road state that the car that arrives at the stop sign first has the right of way to continue through the intersection before the other car. Unfortunately, sometimes this is hard to determine because two cars appear to get to the stop sign at the same time. Additionally, sometimes, the car that arrives second will actually go before the car that arrives first (thus breaking the law).

At this point you know have several scenarios. You could arrive first, or you could arrive second. If you arrive first, you could go first or second, and similar for arriving second. However, the action of going is dependent on whether you arrived first or second.

This is called conditional probability, and can be denoted as P(X|Y)
$P(X|Y)$ (read as "the probability of X given Y").

Now, let’s calculate the conditional probability of some events. In order to do this we need the following formula:

$P(A|B)=P(A \cap B)/P(B)$

What this means in English is given an event B, find all the events shared with A, and divide by the probability of event B happening by itself. The ∩ symbol for intersection represents these events shared between A & B.

When dealing with these probabilities, the basic rules of algebra apply so we manipulate the equation by multiplying both sides by P(B)

$P(B)$ to also look like this:

$P(A \cap B)=P(A|B)*P(B)$

Other formulations are also valid provided you adhere to the algebraic rules, but these two equations should be sufficient for many of your needs.

Law of Total Probability

A useful law when dealing with conditional probability is the Law of Total Probability.

If $B_1, B_2, B_3, ...$ is a partition of a sample space S, then for any event A:

$P(A) = \sum_i P(A \cap B_i) = \sum_i P(A|B_i) * P(B_i)$

The formal definition is a bit "mathy", but think of it as calculating the sum of all probabilities necessary to ensure all scenarios for specified event are included. This is actually quite similar to the equation from before, but now we calculate it for each i of B, and then sum all of these together.