Uncertainty in Robotics

Using the word **certain** the way we'll use it in this Nanodegree, *nothing* in the previous question is ever certain. Let me explain.

1. **What other traffic will do**: People are impossible to predict with certainty!
2. **Where you are**: It may seem like you know where you are when you drive, but you don't. At least not with *complete* certainty. You may know where you are with *sufficient* certainty, but if I asked you how many millimeters away from the center lane you were, you wouldn't know.
3. **How fast you're going**: The same reasoning applies to knowing your speed. You can get a good idea of how fast you're going by looking at your speedometer (which measures your speed), but these measurements are never perfect.
4. **What will happen when you turn the wheel**: A car is an imperfect mechanical system. If you turned the steering wheel by the same amount 100 times, the car would turn a slightly different amount every time.

As humans we solve these problems in a variety of ways. Number **2** and **3** we solve by saying "ehh, I actually don't need to know exactly where I am or how fast I'm going, I just need to know those quantities with a high degree of certainty". Number **4** we solve by using our brain as a high performance adaptive controller. And number **1**... who knows how we handle number 1...? (just kidding, you'll learn more about this in the machine learning course at the end of this curriculum).

Probabilistic "Events" in Robotics

You might be wondering what coins have to do with robotics.

A coin flip is a perfect example of a **probabilistic event**: a set of **outcomes** to some experiment where each outcome has a **probability**.

With a coin, the outcomes are clear: heads or tails, and the probabilities are simple: 0.5 and 0.5.

A self-driving car makes *hundreds* of calculations about probabilistic events every second, but the events are not as clean as a coin flip. For example:

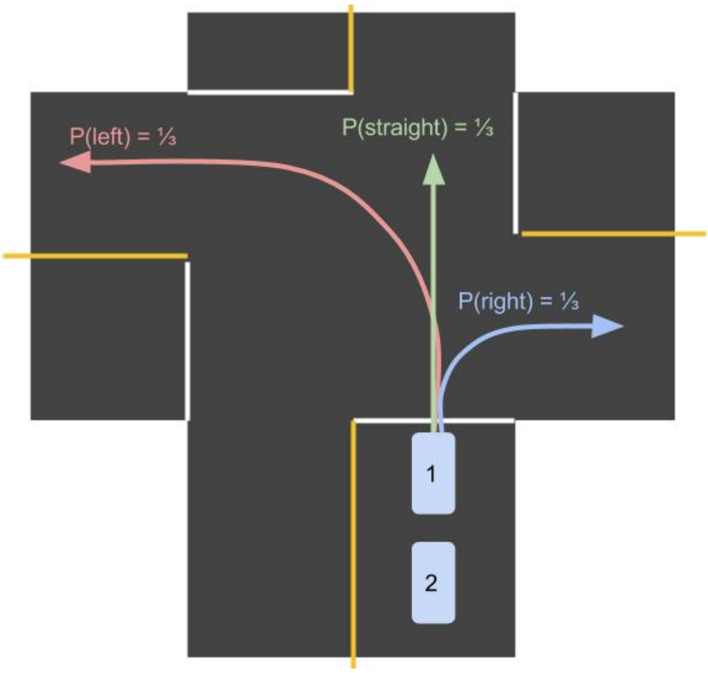
- What is the probability that this sensor measurement is accurate to within 5 centimeters? What about 1 centimeter?
- What is the probability that some other vehicle will turn left at this intersection? Go straight? Turn right? What if they just sit there forever?
- The radar and lidar measurements seem to disagree! What's the probability that the range finder somehow became detached from the roof?

These examples are all much more interesting than "heads or tails?" but they are also less straightforward, which makes it much harder to learn probability theory from them.

Two Cars

The following questions deal with two cars that have pulled up to an intersection. It is equally likely than any individual car will turn left (L), go straight (S), or turn right (R).

In the notation we are using, P(S, L) means "probability that car one goes straight (S) and car two turns left (L)."



Question 1

For this question, assume the probabilities of left, straight, and right are all equal. That is:

$P(L) = \frac{1}{3}$

$P(S) = \frac{1}{3}$

$P(R) = \frac{1}{3}$

Question 2

For this question assume that P(L) is known to be 1/2. The other two quantities are unknown

$P(L) = \frac{1}{2}$

$P(S) = ?$

$P(R) = ?$

Question 3

Now assume we know the following:

$P(L) = 0.5$

$P(S) = 0.3$

$P(R) = ?$

Question 4

Match the mathematical calculation to the vehicle behavior.

Assume the following probabilities:

$P(L) = 0.5$

$P(S) = 0.3$

$P(R) = 0.2$

Question 5

In the questions with two coin flips, each of which had two possible outcomes, you filled out a truth table with four rows (one each for "H,H", "H,T", "T,H" and "T,T").

Two flips. Two outcomes per flip. Four rows in the truth table.

FLIP-1	FLIP-2	
H	H	0.36
H	T	0.24
T	H	0.24
T	T	0.16

(Optional) Managing Complexity

*Note: this section is optional and only here for those who are interested. You will not be expected to remember anything presented in any section labeled **optional**.*

Roboticists will often use the term **state space** to describe the set of all possible outcomes for a probabilistic **event**.

For a coin the state space for a "flip" event can be written mathematically as:

$$\{\mathrm{H}, \mathrm{T}\}$$

And for a car at an intersection the state space for a "turn" event can be written mathematically as:

$$\{\mathrm{L}, \mathrm{S}, \mathrm{R}\}$$

Coins and cars may seem differently, but we can treat them in similar ways when we think in terms of events and state spaces.

In the last question you saw that calculating a truth table for 2 coin flips requires 4 calculations while calculating the truth table for 2 car turns at an intersection requires 9 calculations.

We can make these statements more broadly applicable:

1. When calculating the truth table for **2 events** which each have a **state space size of 2**, we need to make **4 calculations**.
2. When calculating the truth table for **2 events** which each have a **state space size of 3**, we need to make **9 calculations**.

And in fact, there's a mathematical pattern here that can be expressed algebraically:

3. When calculating the truth table for N **events** which each have a **state space size of** x, we need to make x^N **calculations**

x^N gets very big very fast as x or N get bigger.

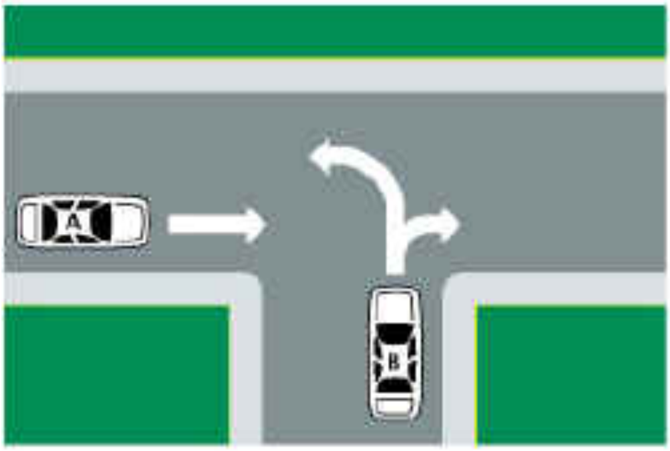
You will see later this Nanodegree how this **exponential complexity growth** can really slow down the performance of the code running inside of a self driving car.

Cars and Probability

Before you start programming your own self-driving car, having a solid foundation in probability will be very important. Probability is used to analyze sensor data, predict future events, and make decisions. Below we will explore the foundation of probability to aid you on your journey to be a self-driving car engineer!

If you are already comfortable with basic probability, please feel free to skip these exercises.

Combinatorics (i.e. counting)



Probability

Now that you have built your sample space and understand the mechanics, it is time to move into some probability.

Probability can typically be thought of as:

\Large \frac {\textrm {Number of ways an event can occur}}{\textrm{Total number of events that could occur}}

Total number of events that could occur Number of ways an event can occur

An "event" is defined as some type of state that can happen. For example, turning "left" or "right" are both events. Similarly, pressing the "gas" or "brake" are also events. All probabilities will be between 0 and 1 inclusive.

If a probability is ever negative or larger than one, there is something wrong with the calculation, so please double-check the answer.

Often, the notation P(X) is used. This typically means the probability of "X" occuring. "X" is just a placeholder, but a more specific example will be provided below.

Compliments

Often in probability, there will be a time when you want to know the result of a specific event. Unfortunately, sometimes, this event can be really tricky to calculate. However, it may be really easy to calculate all other events *except* this event. By doing this, we can exploit the following property:

\textrm {Specific outcome = 1 - (The sum of probability of all other events)}

$$\textrm{Specific outcome = 1 - (The sum of probability of all other events)}$$

This will yield the probability for the event in question.

Multiplying Probabilities (i.e. using "and")

When dealing with probabilities, sometimes outcomes can be independent of each other - meaning, a previous outcome does not change the probability for the current outcome. Usually, when dealing two more events occurring, the word “and” is used. If “and” is used, this implies that the probabilities are multiplied instead of added.

All Outcomes

It has been shown that independent events can be multiplied together. We will build on this concept below.