Recursive Least Squares (RLS) of ARX (Transfer Function) Systems Based on Equation Error

Equations

Transfer function model

$$A(z)y(t) = z^{-d}B(z)u(t) + e(t)$$

Where

$$A(z) = z^{na} + a_1 z^{na-1} + \cdots + a_{na}\,, \quad B(z) = b_0 z^{nb} + b_1 z^{nb-1} + \cdots + b_{nb}$$
$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{na} z^{-na}\,, \quad B(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_{nb} z^{-nb}$$

$e(t)$ is a white noise

Writing the difference equation, z is a shift operator ($zy(t) = y(t+1), \;\; z^{-1}y(t) = y(t-1)$)

$$(1 + a_1 z^{-1} + \cdots + a_{na} z^{-na})y(t) = z^{-d}(b_0 + b_1 z^{-1} + \cdots + b_{nb} z^{-nb})u(t)$$
$$y(t) = -a_1 y(t-1) - a_2 y(t-2) - \cdots - a_{na} y(t-na) + b_1 u(t-d-1) + b_2 u(t-d-2) + \cdots$$
$$+ b_{nb} u(t-d-nb)$$
$$y(t) = \varphi^T(t-1)\theta^0$$

$$\varphi^T(t-1) = [-y(t-1) \ldots - y(t-na) \;\; u(t-d-1) \ldots \;\; u(t-d-nb)], \qquad \theta^0 = \begin{bmatrix} a_1 \\ a_2 \\ \cdot \\ \cdot \\ \cdot \\ a_{na} \\ b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ b_{nb} \end{bmatrix}$$

Algorithm (note replace each $\varphi(t)$ in the standard RLS algorithm with $\varphi(t-1)$ for the transfer function model)

At $t = 0, P(0) = \alpha I$, where I is the identity matrix $n \times n$ and α is large number. $\theta(0) = 0$. For t > 0 do the following

1. $\varphi^T(t-1) = [-y(t-1) \ldots - y(t-na) \;\; u(t-1) \ldots \;\; u(t-nb)]$ Regressors
2. $P(t) = P(t-1) - \frac{P(t-1)\varphi(t-1)}{\left(I+\varphi^T(t-1)P(t-1)\varphi(t-1)\right)}\varphi^T(t-1)P(t-1) = \left(I - K(t)\varphi^T(t-1)\right)P(t-1)$ Covariance matrix
3. $K(t) = P(t)\varphi(t) = \frac{P(t-1)\varphi(t-1)}{\left(1+\varphi^T(t-1)P(t-1)\varphi(t-1)\right)}$ Weighting (scaling) factor
4. $\varepsilon(t) = y(t) - \varphi^T(t-1)\hat{\theta}(t-1)$ Predicted error
5. $\hat{\theta}(t) = \hat{\theta}(t-1) + K(t)\varepsilon(t)$ Estimation
6. $e(t) = y(t) - \varphi^T(t-1)\hat{\theta}(t)$ Residual error (not used here)

Where $\left(I + \varphi^T(t)P(t-1)\varphi(t)\right)$, is scalar for single output system.

Matlab file name = RLS_”**na**”_”**nb**”_”**n**”

na: order of the **numerator**.

nb: order of the **denominator**.

d: **delay**.

nu=na+nb+1: number of **parameters** to be estimated.