Adaptive Control: Gain Scheduling

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Introduction

In many situations it is known how the dynamics of a process change with the operating conditions of the process. One source for the change in dynamics may be non-linearities that are known. It is then possible to change the parameters of the controller by monitoring the operating conditions of the process. This idea is called ***gain scheduling***, since the scheme was originally used to accommodate changes in process gain only. Gain scheduling is a nonlinear feedback of special type; it has a linear controller whose parameters are changed as a function of operating conditions in a pre-programmed way. The idea of relating the controller parameters to auxiliary variables is old, but the hardware needed to implement it easily was not available until recently. To implement gain scheduling with analog techniques, it is necessary to have function generators and multipliers. Such components have been quite expensive to design and operate. Gain scheduling has thus been used only in special cases. such as in autopilots for high performance aircraft. Gain scheduling is easy to implement in computer-controlled systems, provided that there is support in the available software. Gain scheduling based on measurements of operating conditions of the process is often a good way to compensate for variations in process parameters or known non-linearities of the process. It is controversial whether a system with gain scheduling should be considered an adaptive system or not, because the parameters are changed in an open-loop or pre-programmed fashion. If we use the informal definition of adaptive controllers given in Section L1, gain scheduling can be regarded as an adaptive controller. Gain scheduling is a very useful technique for reducing the effects of parameter variations. In fact it is the foremost method for handling parameter variations in flight control.

The Principle

It is sometimes possible to find auxiliary variables that correlate well with the changes in process dynamics. It is then possible to reduce the effects of parameter variations simply by changing the parameters of the controller as functions of the auxiliary variables. Gain scheduling can thus be viewed as a feedback control system in which the feedback gains are adjusted by using feed forward compensation. The concept of gain scheduling originated in connection with the development of flight control systems. In this application the Mach number and the dynamic pressure are measured by air data sensors and used as scheduling variables.

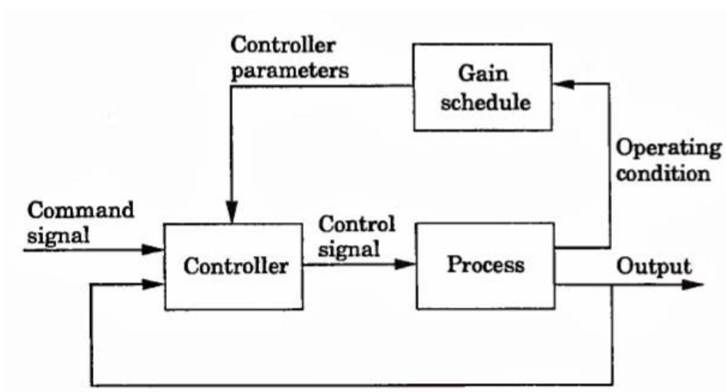


Figure 1: Block diagram of a system in which influences of parameter variations are reduced by gain scheduling.

When scheduling variables have been determined, the controller parameters are calculated at a number of operating conditions by using some suitable design method. The controller is thus tuned or calibrated for each operating condition. The stability and performance of the system are typically evaluated by simulation; particular attention is given to the transition between different operating conditions. The number of entries in the scheduling tables is increased if necessary. Notice, however, that there is no feedback from the performance of the closed-loop system to the controller parameters. It is sometimes possible to obtain gain schedules by introducing nonlinear transformations in such a way that the transformed system does not depend on the operating conditions. The auxiliary measurements are used together with the process measurements to calculate the transformed variables. The transformed control variable is then calculated and re transformed before it is applied to the process. The controller thus obtained can be regarded as being composed of two nonlinear transformations with a linear controller in between. Sometimes the transformation is based on variables that are obtained indirectly through state estimation.

One drawback of gain scheduling is that it is an open-loop compensation. There is no feedback to compensate for an incorrect schedule. Another draw- back of gain scheduling is that the design may be time-consuming. The controller parameters must be determined for many operating conditions, and the performance must be checked by extensive simulations. This difficulty is partly avoided if scheduling is based on nonlinear transformations.

Gain scheduling has the advantage that the controller parameters can be changed very quickly in response to process changes. Since no estimation of parameters occurs, the limiting factors depend on how quickly the auxiliary measurements respond to process changes.

Design of Gain Scheduling Controllers

It is difficult to give general rules for designing gain-scheduling controllers. The key question is to determine the variables that can be used as scheduling variables. It is clear that these auxiliary signals must reflect the operating conditions of the plant. Ideally, there should be simple expressions for how the controller parameters relate to the scheduling variables. It is thus necessary to have good insight into the dynamics of the process if gain scheduling is to be used. The following general ideas can be useful:

- Linearization of nonlinear actuators,
- Gain scheduling based on measurements of auxiliary variables,
- Time scaling based on production rate, and
- Nonlinear transformations.

Non-Linear Valve

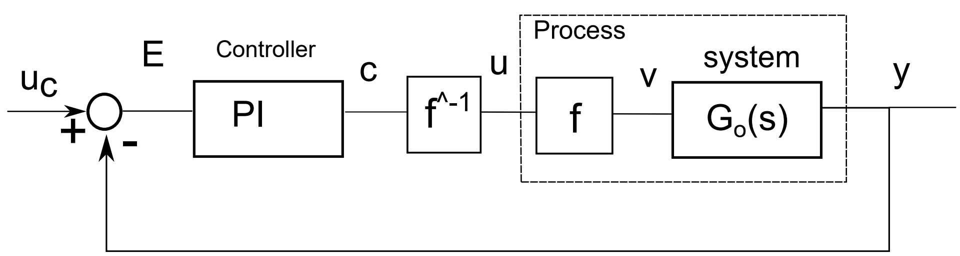


Figure 2: Compensation of a nonlinear actuator using an approximate inverse.

Consider the system with a nonlinear valve. The nonlinearity is assumed to be

$$v = f(u) = u^4, \qquad u >= 0 \tag{1}$$

Let $\hat{f^{-1}}$ be an approximation of the inverse of the valve characteristic. To compensate for the nonlinearity, the output of the controller is fed through this function before it is applied to the valve (see Fig.2). This gives the relation

$$v = f(u) = f(\hat{f^{-1}}(c)) \tag{2}$$

where c is the output of the PI controller. The function $f(\hat{f^{-1}}(c))$ should have less variation in gain than f. If $\hat{f^{-1}}$ is the exact inverse, then $v = c$. In the next part, the nonlinear relation between c and u will be approximated by 2, 3 line segments and with a total inverse to show to show the effect on the system time response as the number of segments increases. A SIMULINK model was used for simulation (see Fig.3). First, a step input with different amplitudes (1, 3 and 5) was used to show the different time response of the system as seen in Fig.4. We see that there is difference between the time responses because the effect of the nonlinearity of the valve.

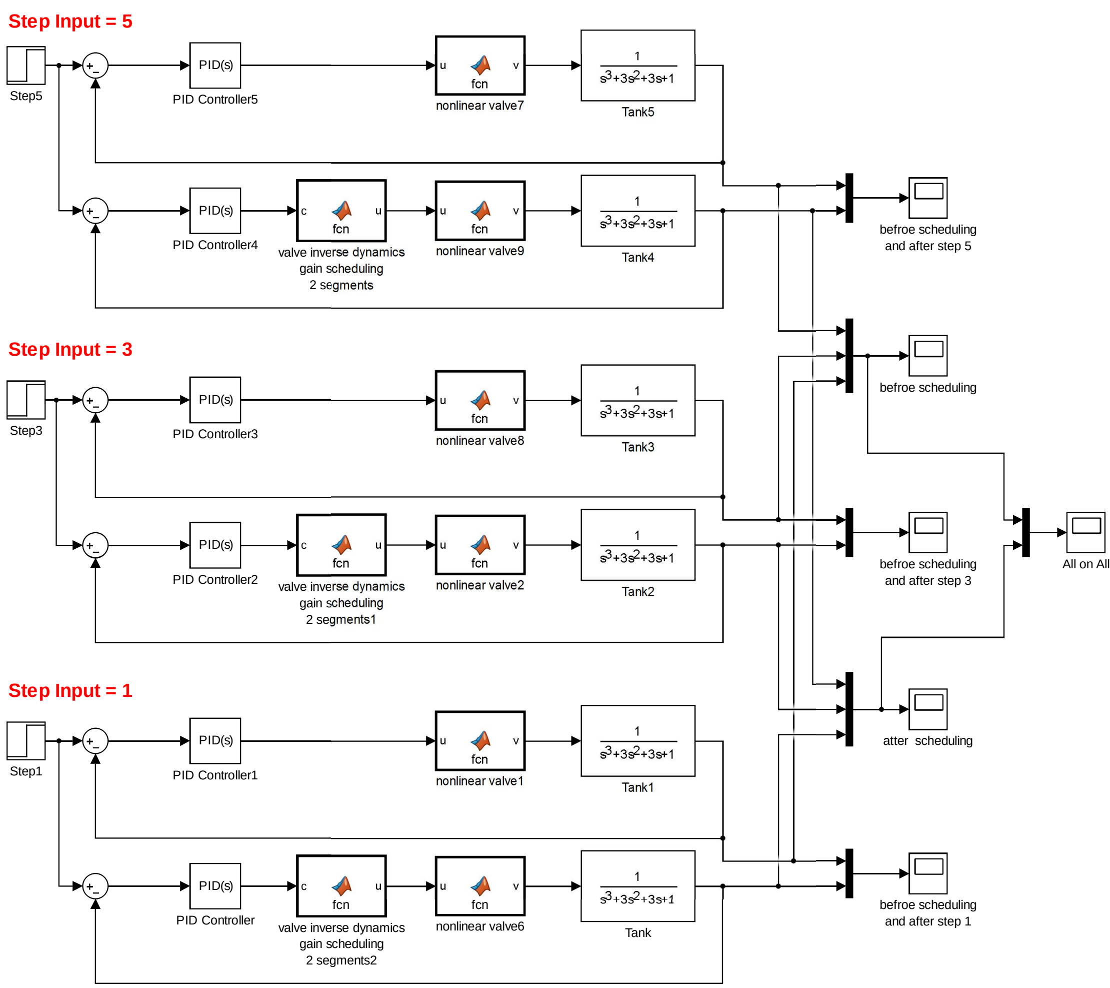


Figure 3: SIMULINK Model.

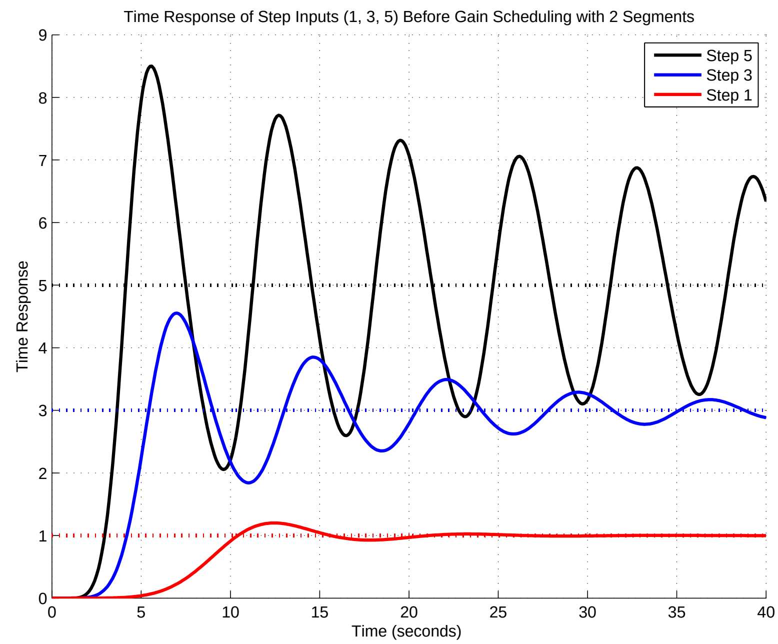


Figure 4: Time Response of the System without Gain Scheduling (different responses due to valve nonlinearity).

Scheduling with Tow Line Segments Approximation

The line segments of the inverse of the nonlinearity of the valve was divided by a created MATLAB function that requires the equation and the points at which you want to linearize at as the following

```
% This function is made by Ahmed ElTahan
%{
    This function is inteded to divide any equation curve that mainly contains
    nonlinearity to user defined linear segments based on the points that
    are chosen to linearize on

    Function inputs
    1)y : the equation that to be segmentally linarized in a symbolic form
    2)Points : vector that contains the x-values of distinct lines
    Function output
    1)Regions of segments and their equations
    2)The coefficient of the different line segments
    A figure is shown at the end to show the original cuve and the
    linearized curves.
%}
function [C] = Linearization( y, Points )
syms x
n = length(Points);
xx = Points;
C = cell(n, 1);
yy =  subs(y, x, xx);
disp(['number of line segments is ', num2str(n-1)])
for i = 1 : n-1
    syms aa bb
    [a(i), b(i)] = solve(yy(i) == aa*xx(i) + bb, yy(i+1) == aa*xx(i+1) + bb, aa, bb);
    disp(['Region ', num2str(i), ' of ', num2str(xx(i)), ' < x < ', num2str(xx(i+1)),' ...
```

```
        equation is'])
    disp(['y = ', num2str(double(a(i))), ' x + ', num2str(double(b(i)))])
    C{i} = [double(a(i)), double(b(i))];
end
C = cell2mat(C);
xf = min(xx):0.01:max(xx);
yf = subs(y, x, xf);
figure;
hold all
plot(xf, yf, 'color', 'blue', 'LineWidth', 2);
for i = 1:n-1
    hold all
    line([xx(i) xx(i+1)], [yy(i) yy(i+1)], 'color', 'black', 'LineWidth', 2)
end
xlabel('x')
ylabel('y')
title('Linearization of Non-linear Curve')
legend('Non-Linear Curve', 'Linearized Curve', 'Location', 'best')
grid on
end
```

For two line segments between the points $x = 0$ and $x = 16$, the lines are

$$\begin{aligned} y &= 0.43869x && ;0 < x < 3 \\ y &= 0.05261x + 1.1582x && ;3 < x < 16 \end{aligned}$$

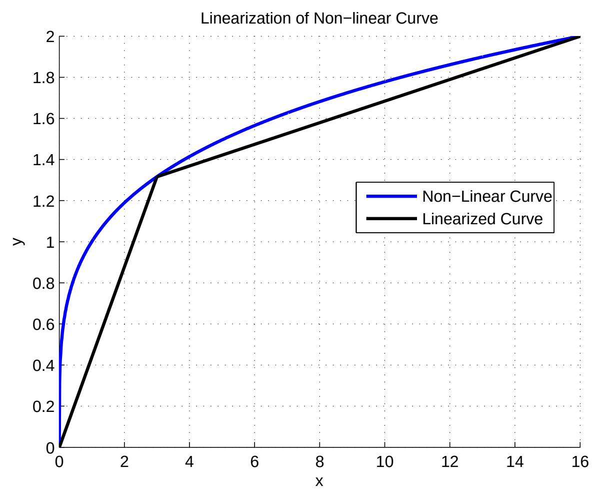
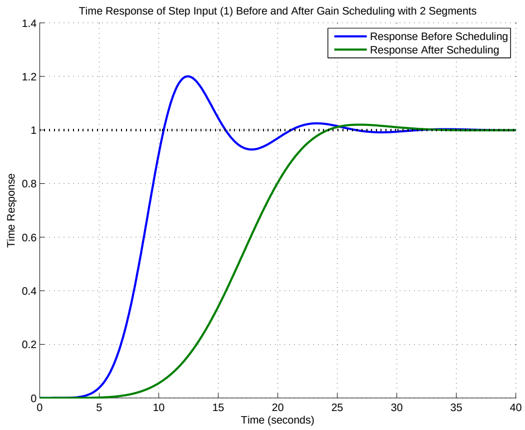
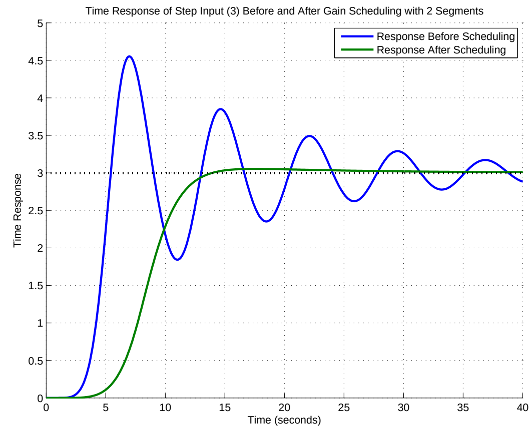


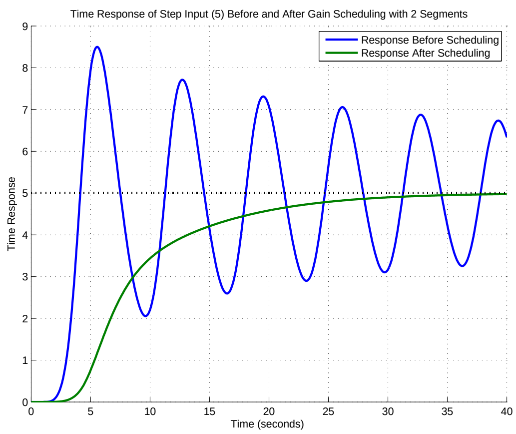
Figure 5: Two Segments of the Nonlinear Inverse Valve Dynamic Function



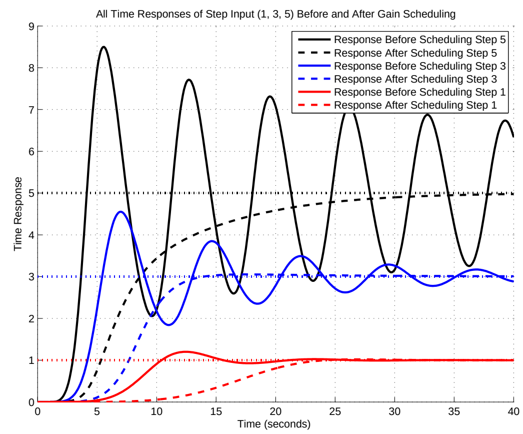
(a) Step input 1



(b) Step input 3



(c) Step input 5



(d) Total Responses

Figure 6: Time Response of the System After Gain Scheduling with 3 Line Segments

Scheduling with Three Line Segments Approximation

For Three line segments between the points $x = 0$, $x = 0.5$,$x = 5$ and $x = 16$, the lines are

$$
\begin{aligned}
y &= 1.6818x & ;0 < x < 0.5 \\
y &= 0.14543x + 0.76818 & ;0.5 < x < 5 \\
y &= 0.045877x + 1.266 & ;5 < x < 16
\end{aligned}
$$

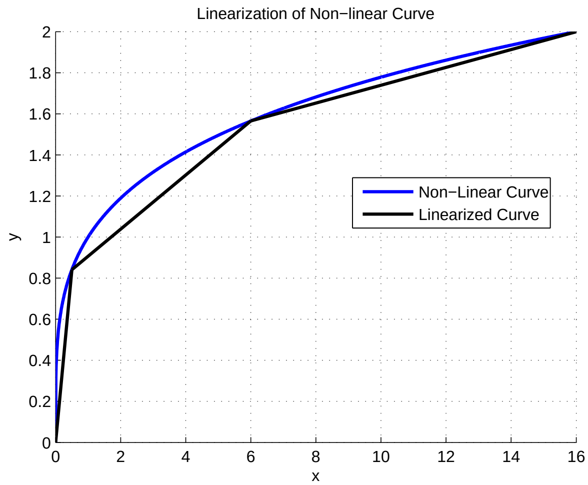
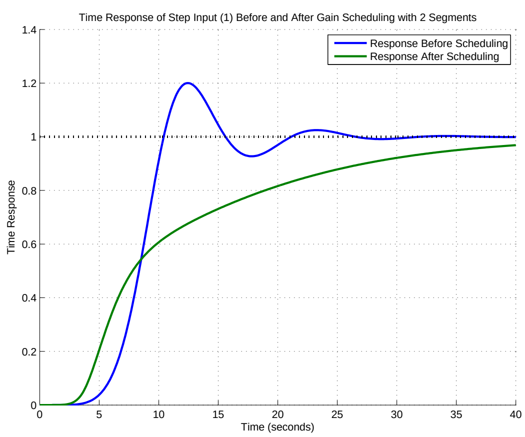
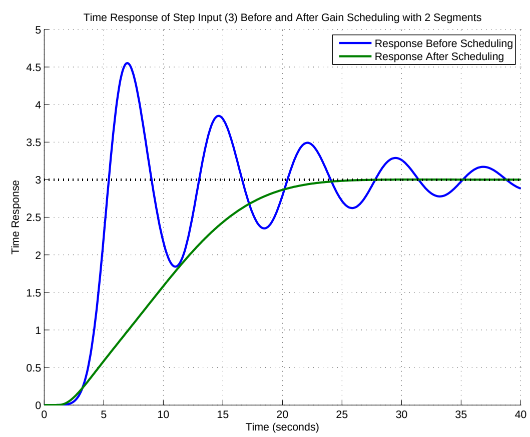


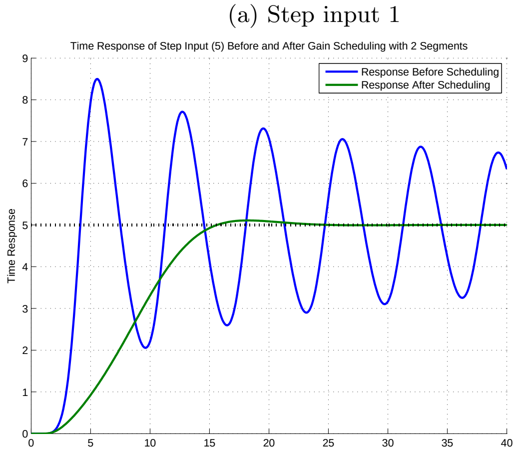
Figure 7: Three Segments of the Nonlinear Inverse Valve Dynamic Function



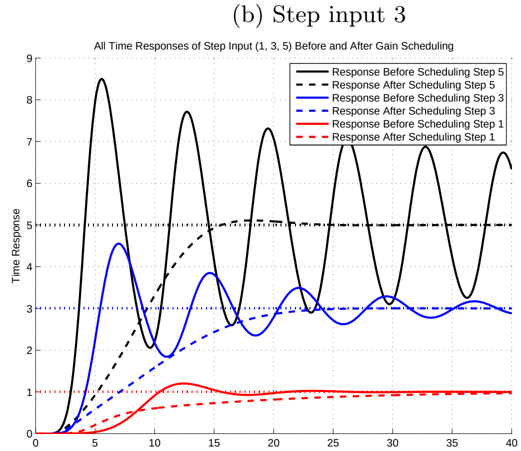
(a) Step input 1



(b) Step input 3



(c) Step input 5



(d) Total Responses

Figure 8: Time Response of the System After Gain Scheduling with 2 Line Segments

Scheduling with Total Inverse Approximation

For total inverse dynamics of the nonlinear valve (see Fig.9)the response will be linear(i.e. no change in the shape of the response just the magnitude) as shown in Fig.10

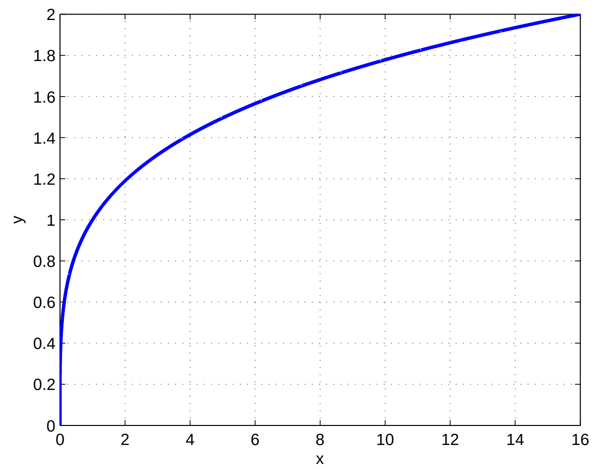
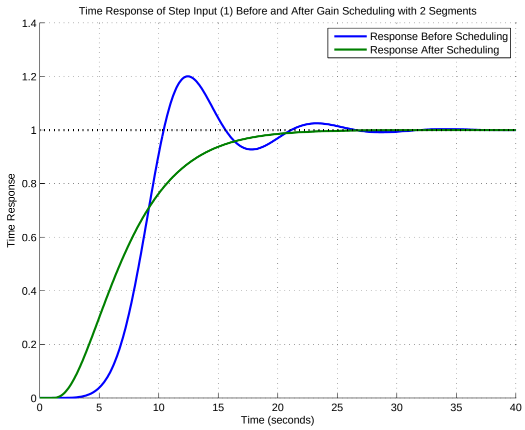
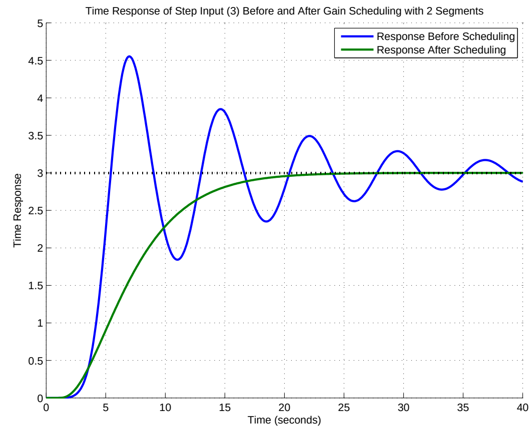


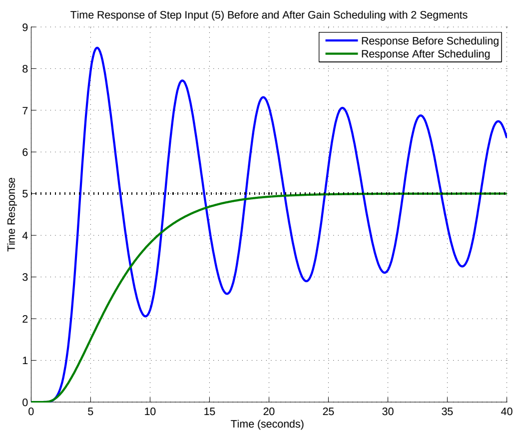
Figure 9: Three Segments of the Nonlinear Inverse Valve Dynamic Function



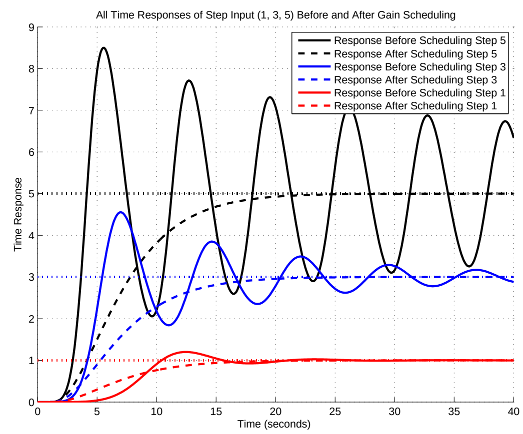
(a) Step input 1



(b) Step input 3



(c) Step input 5



(d) Total Responses

Figure 10: Time Response of the System After Gain Scheduling Total Inverse of Valve Dynamics

Tank Level Control

Regular Tank

Regular tank means that the tank cross section area is constant along the tank height. The simple fluid dynamics equations can give

$$q_{in} - q_{out} = A\frac{\mathrm{d}h}{\mathrm{d}t} \tag{3}$$

where q_{in} is the volume flow rate that enters the tank, q_{out} is the volume flow rate that leaves the tank at the bottom of the tank, A is the cross section area of the tank and h is the level of the fluid in the tank measured from the center line of the outlet. We can assume a liner relation between the out volume flow rate and the height of the fluid and the resistance R of the outlet (analogous to Ohm's low in electricity) as follows

$$q_{out} = \frac{h}{R} \tag{4}$$

Then we can write

$$q_{in} - \frac{h}{R} = A\frac{\mathrm{d}h}{\mathrm{d}t} \tag{5}$$

Applying Laplace transform to Eqn.5 to become

$$Q_{in}(s) - \frac{H(s)}{R} = AH(s)s \tag{6}$$

then the final transfer function of the regular tank is

$$\boxed{\frac{H(s)}{Q_{in}(s)} = \frac{R}{ARs+1}} \tag{7}$$

Variable Area Tank

Suppose that the tank cross section area is variable with the height of the tank.

$$d\forall = A(x)\mathrm{d}x \tag{8}$$

$$\forall = \int_0^h A(x)\mathrm{d}x \tag{9}$$

$$q_{in} - q_{out} = \frac{\mathrm{d}\forall}{\mathrm{d}t} = \frac{\mathrm{d}\forall}{\mathrm{d}h}\frac{\mathrm{d}h}{\mathrm{d}t} = A(h)\frac{\mathrm{d}h}{\mathrm{d}t} \tag{10}$$

$$q_{out} = a\sqrt{2gh} \tag{11}$$

$$q_{in} - a\sqrt{2gh} = A(h)\frac{\mathrm{d}h}{\mathrm{d}t} \tag{12}$$

Using small disturbance theory to linearize the equation

$$h = h_o + \Delta h, \qquad q_{in} = q_{in_o} + \Delta q_{in} \tag{13}$$

Assume that

$$A(h_o + \Delta h) \cong A(h_o), \qquad \frac{\mathrm{d}h_o}{\mathrm{d}t} \cong 0 \tag{14}$$

$$q_{in_o} + \Delta q_{in} - a\sqrt{2g(h_o + \Delta h)} = A(h_o)\frac{\mathrm{d}\Delta h}{\mathrm{d}t} \tag{15}$$

Let

$$L = \sqrt{2g(h_o + \Delta h)} = \sqrt{2gh_o(1 + \frac{\Delta h}{h_o})} = \sqrt{2gh_o}(1 + \frac{\Delta h}{h_o})^{0.5} \cong \sqrt{2gh_o}(1 + \frac{\Delta h}{2h_o}) = \sqrt{2gh_o} + \frac{\Delta h}{2h_o}\sqrt{2gh_o} \tag{16}$$

The steady state solution is

$$q_{in} - a\sqrt{2gh_o} = 0 \tag{17}$$

Substituting L in Eqn.15 and let $\Delta q_{in} \equiv q_{in}$, $\Delta h \equiv h$ and $A(h_o) \equiv A_o$ we get

$$q_{in} - a\frac{h}{2h_o}\sqrt{2gh_o} = A_o\frac{\mathrm{d}h}{\mathrm{d}t} \tag{18}$$

Applying Laplace transform to Eqn.18 we get the final transfer function for the tank system as

$$\frac{H(s)}{Q_{in}(s)} = \frac{\beta}{s+\alpha} \tag{19}$$

Where

$$\alpha = \frac{a\sqrt{2gh_o}}{2h_oA_o}, \qquad \beta = \frac{1}{A_o} \tag{20}$$

Note that the auxiliary variables used to schedule the controller parameters are h_o and hence $A_o = A(h_o)$. When calculating the dc gain of the tank system the result is not 1 and as the tank system here is 1st order the steady state error is given by $e_{ss} = \frac{1}{1+k_p}$ where k_p is the dc gain. So, the steady state error is not zero. We shall use a PI controller to compensate the steady state error. The block diagram of the system will be as in Fig.11

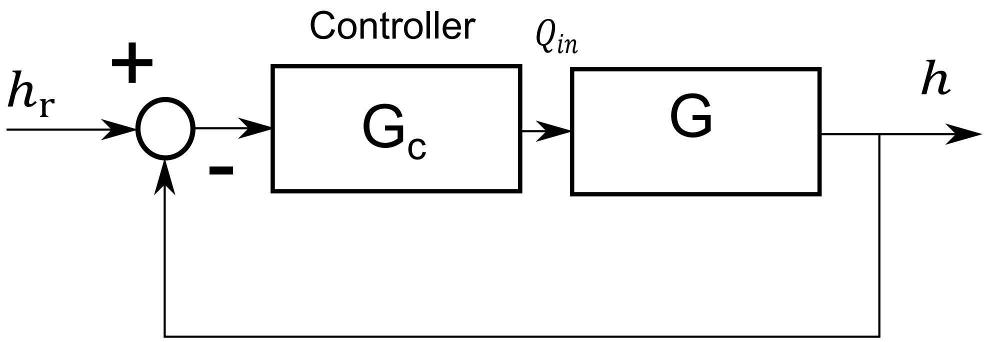


Figure 11: Block of Tank with the Controller

The PI controller is given by

$$Gc(s) = \frac{k(1 + 1/T_i)}{s} \tag{21}$$

The closed loop system is given by

$$\frac{H(s)}{H_c(s)} = \frac{G_c G}{1 + GcG} \tag{22}$$

Substituting

$$\frac{H(s)}{H_c(s)} = \frac{\frac{k(1+1/T_i)}{s}\frac{\beta}{s+\alpha}}{1 + \frac{k(1+1/T_i)}{s}\frac{\beta}{s+\alpha}} \tag{23}$$

Simplifying

$$\frac{H(s)}{H_c(s)} = \frac{k(s + 1/T_i)\beta}{s^2 + (\alpha + k\beta)s + \frac{k\beta}{T_i}} \tag{24}$$

We want to specify the dynamics of the tank to be such as

$$\frac{H(s)}{H_c(s)} = \frac{k(s + 1/T_i)\beta}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{25}$$

Comparing coefficients, we get

$$\alpha + k\beta = 2\zeta\omega_n \to k = \frac{2\zeta\omega_n\alpha}{\beta} \to \boxed{k = A_o(2\zeta\omega_n - \alpha)} \tag{26}$$

$$\frac{k\beta}{T_i} = \omega_n^2 \to T_i = \frac{\omega_n^2}{k\beta} \to \boxed{T_i = \frac{2\zeta\omega_n - \alpha}{\omega_n^2}} \tag{27}$$

Notice that if $\alpha << 2\zeta\omega_n$ we will get

$$\boxed{k = 2\zeta\omega_n A_o} \quad and \quad \boxed{T_i = \frac{2\zeta}{\omega_n}} \tag{28}$$

We see form Eqn.28 that T_i does not depend on the tank parameters while k depends on the cross section area at h_o which in this case will be our scheduling variable given the tank cross section area as a function of its height.

Example:Variable Area Tank

Suppose the tank has the following data

A_b	$0.5\ m^2$
$A(h)$	$(A_b + h^2)\ m^2$
ζ	0.7
ω_n	10

We can assume here that $\alpha << 2\zeta\omega_n$, so $T_i = 0.14$ and $k = 14(0.1 + h^2)$. Using a SIMULINK model as in Fig.12

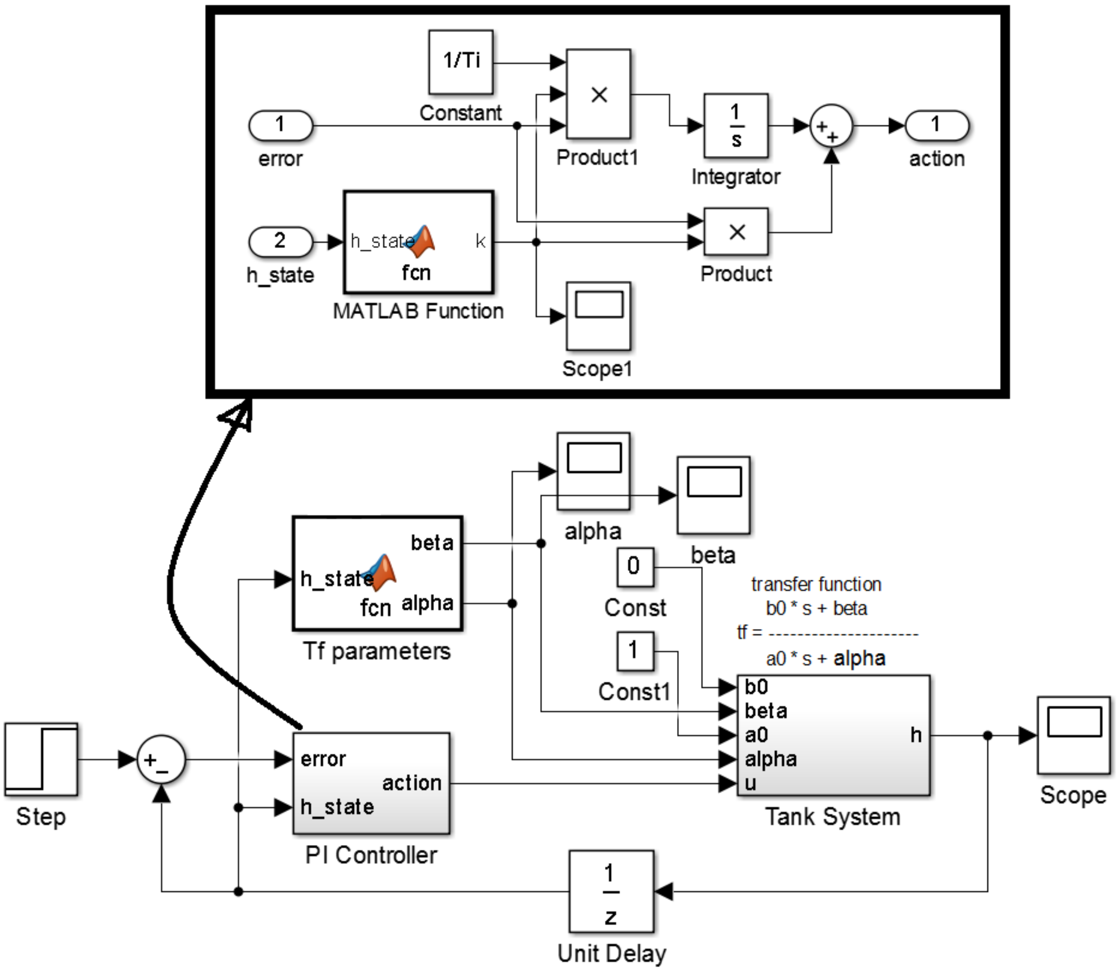


Figure 12: SIMULINK Model for the Tank System Including the PI Controller Block above the Model

The time Response and the gain k values are given in Fig.13 and Fig.14 , respectively

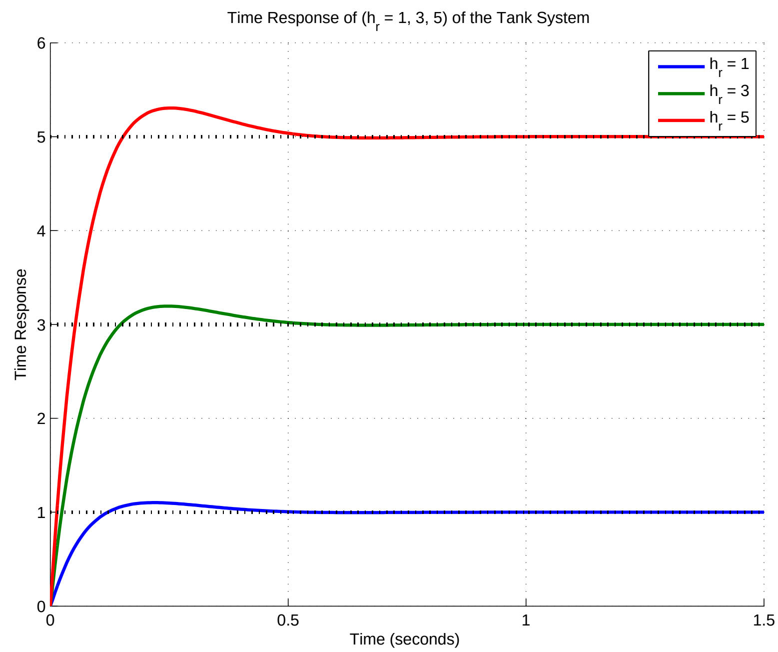


Figure 13: Tank Height Time Response to Step Input

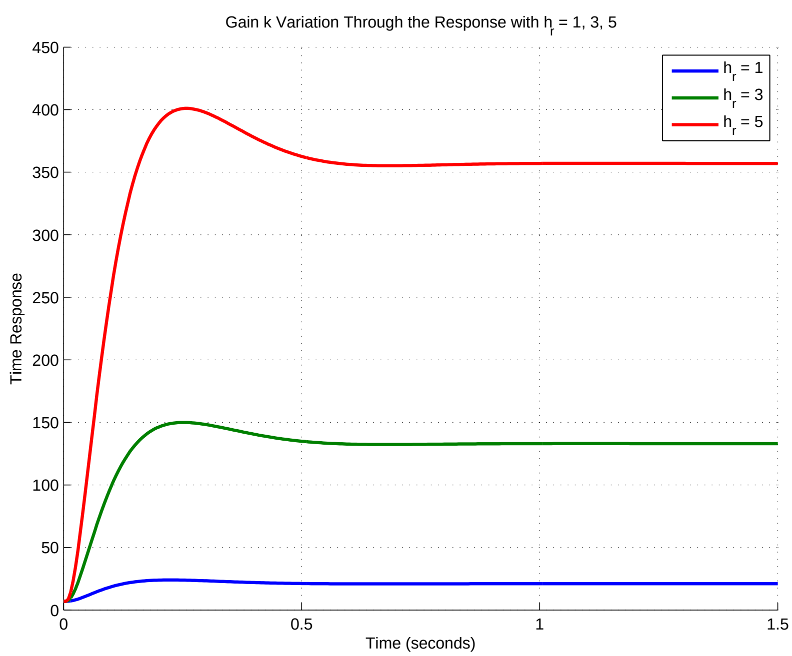


Figure 14: Gain k Values Through the Response

The values of α and β are plotted in Fig.15 and Fig.16, respectively.

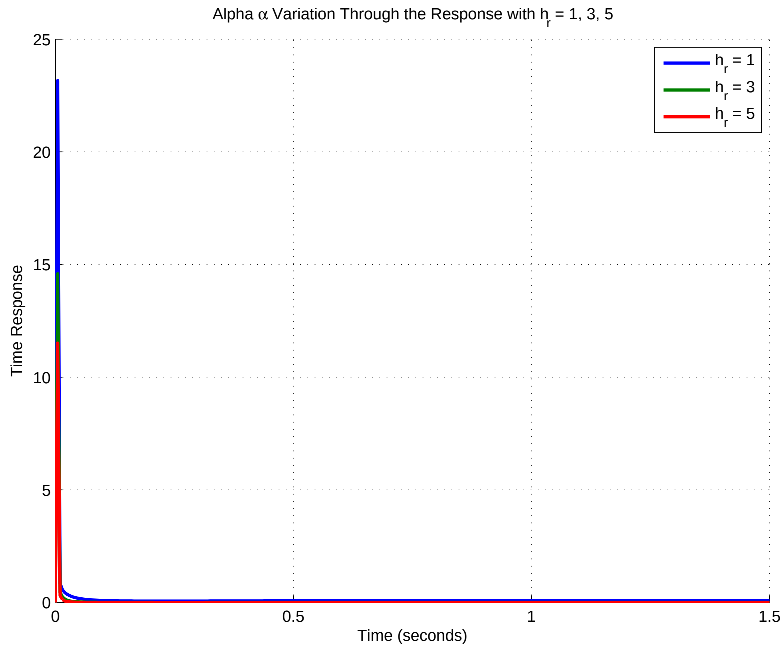


Figure 15: Gain k Values Through the Response

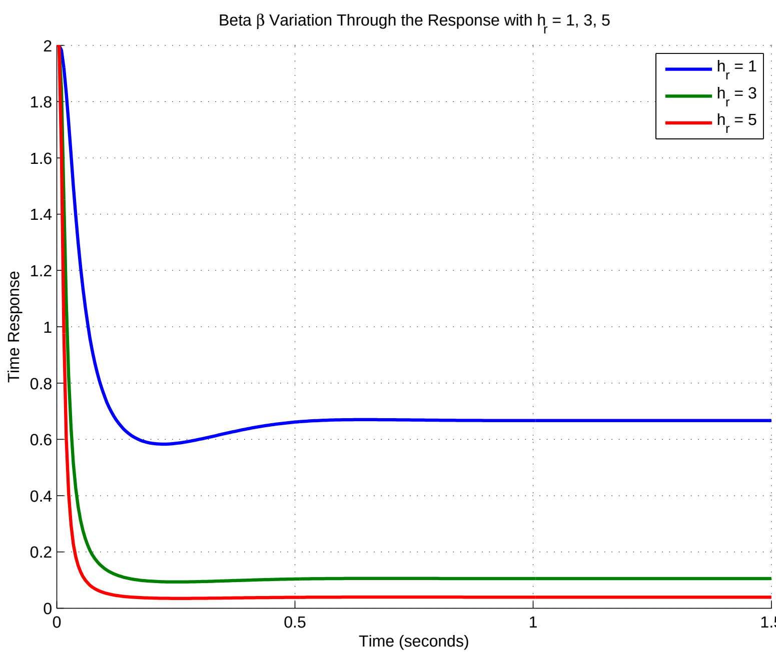


Figure 16: Gain k Values Through the Response

This example illustrates that it can sometimes be sufficient to measure one or two variables in the process and use them as inputs to the gain schedule. Often, it is not as easy as in this case to determine the controller parameters as a function of the measured variables. The design of the controller must then be redone for different working points of the process. Some care must also be exercised if the measured signals are noisy. They may have to be filtered properly before they are used as scheduling variables.

References

[1] Astrom, Karl J. and Bjorn Wittenmark. *Adaptive Control.*