- General Circulation Models (GCMs) when used for extremes of : [1]
  - at the regional scale, are still limited by the rarity of events
  - For uncertainty quantification larger multi-model ensembles wanted

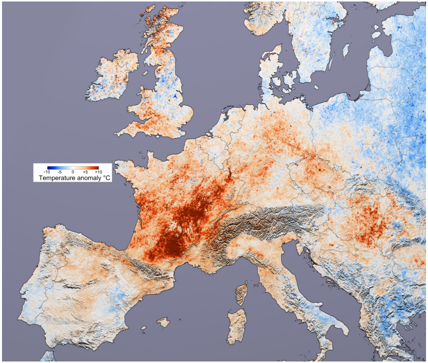

Figure: European heat wave 2003

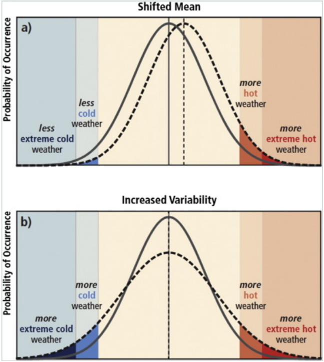

Figure: Changes in temperatures[2]

[1] S. Seneviratne et al., A Special Report of Working Groups I and II of the IPCC (2012)
[2] S. E. Perkins, Atmospheric Research (2015)

Outline

Scandinavian blocking: HW onset

- Rossby wave breaking and blocking
- Advection: persistent anticyclonic anomaly

$$V = \frac{\hat{k}}{f} \times \nabla z \qquad (1)$$

$$z(p) = R \int_p^{p_s} \frac{T}{g} \frac{dp}{p} \qquad (2)$$

Coriolis parameter

500 mbar geopotential height

- Dry soil contributes to heating due to lack of latent heat

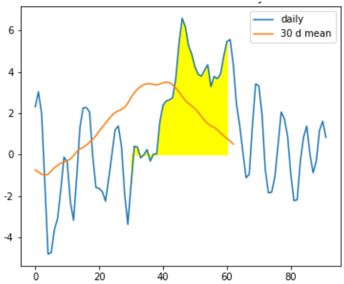

Figure: Scandinavia: Average temperature

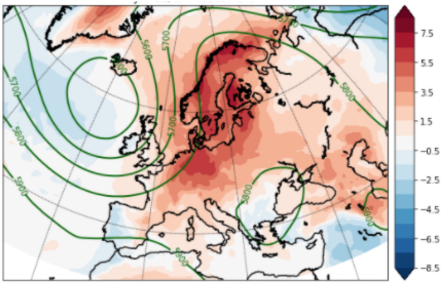

Figure: Temperature, geopotential (ECMWF)

Summer HWs over France: definition

- HW: extreme of space-time averaged temperature anomalies:

$$A_T(t) = \frac{1}{\underset{\uparrow}{T}} \int_t^{t+T} \frac{1}{\underset{\uparrow}{|\mathcal{D}|}} \int_D \left(T_{2m} - \mathbb{E}\left(T_{2m}\right)\right)(\vec{r}, u)\, \mathrm{d}\vec{r}\mathrm{d}u \quad (3)$$

Duration: $T = 14$ days

Area D - "France"

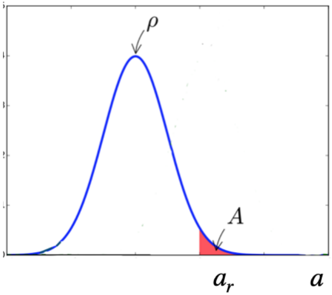

Figure: Temperature fluctuations

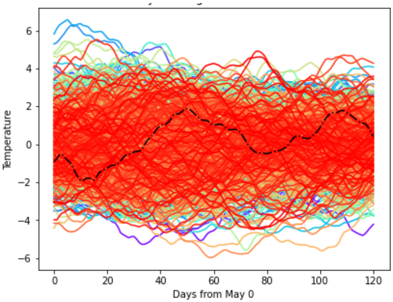

Figure: 1000 years of $A(t)$

Plasim: Planet Simulator, HWs in France

- Intermediate complexity model allows long simulation (8000 years)
- SST and the ice cover is repeated cyclically every year
- Resolution: 2.8 by 2.8 degrees. 10 vertical atmospheric levels

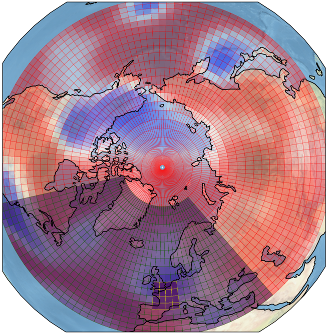

Figure: Plasim gridpoints

- dynamical core - primitive equations for
  - vorticity
  - temperature
  - surface pressure...
- Moisture is included by transport of water vapor
- Parametrized unresolved processes:
  - radiation
  - interactive clouds

[2] https://www.mi.uni-hamburg.de/en/arbeitsgruppen/theoretische-meteorologie/modelle/plasim.html

Evaluating the performance of predictions

The goal of inference: find committor function $P(Y|X)$

$$\mathbb{P}\left(X = x \text{ and } Y = y\right) = P(x, y) = P(Y|X)P(X). \tag{4}$$

[3] R. Benedetti, Monthly Weather Review (2010)

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Logarithmic (a.k.a, cross-entropy) score is suitable for rare events[3]

$$-S\left[\hat{p}_Y(X)\right] = -\sum_{k=0}^{K-1} Y_k \log\left[\hat{p}_k(x)\right], \quad K = 2 \text{ for binary} \tag{5}$$

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Normalized Skill Score (NSS): subtract climatological prediction

$$\text{NSS}=\frac{-\sum_i \overline{p}_i \log \overline{p}_i-\mathbb{E}\left\{S\left[\hat{p}_Y(X)\right]\right\}}{-\sum_i \overline{p}_i \log \overline{p}_i} \tag{7}$$

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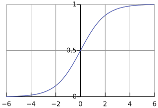
Probabilistic prediction: softmax output

- Soft-max (sigmoid) bounds to $(0,1)$ range [4][5]

$$P\left(Y_n = k \mid \boldsymbol{x}_n, \{w_{k'}\}_{k'=0}^{K-1}\right) = \frac{\mathrm{e}^{-x_n^T w_k}}{\sum_{k'=0}^{K-1} \mathrm{e}^{-x_n^T w_{k'}}}, \qquad (8)$$



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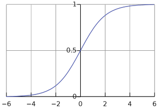

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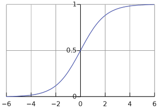

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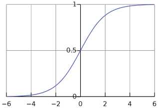

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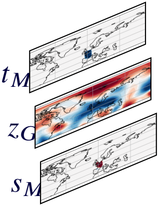

Figure: Possible field inputs

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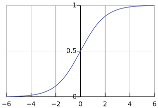

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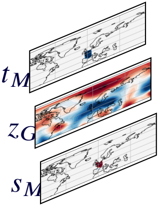

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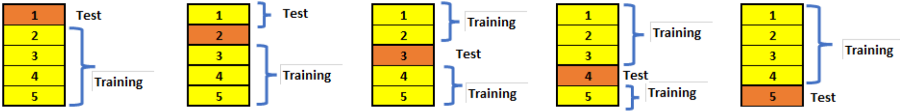

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