Exercise for Machine Learning (SoSe 2020)

Assignment 0: Introduction (Theory)

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3 Error Measures

1 Definitions

$$MSE = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n}$$

$$MAE = \frac{\sum_{i=1}^{n} |y_i - \hat{y}_i|}{n}$$

2 Differences

Since in MSE the difference between the true and the predicted value is squared, MSE penalizes larger derivations much more than MSE does. Here, the simple euclidean distance is taken into account.

3 Binary Classificatoin

Assuming the labels are “1” for true and “0” for false and that they can be subtracted from one another, MSE and MAE represent the negative accuracy as it divides the number of wrong predictions by the total number of predictions. MSE and MAE are the same thing in this case, because squaring 0 or 1 doesn't change the result.

Machine Learning Sheet 1

Excercise 1)

a) $XA + A^T = I \quad \Big| - A^T$

$XA = I - A^T \quad \Big| \cdot A^{-1}$

$X = A^{-1} - (A^T A^{-1})$

b) $X^T C = [2A(X+B)]^T$

$X^T C = (2AX + 2AB)^T$

$X^T C = (2AX)^T + (2AB)^T \quad \Big| - (2AX)^T$

$X^T C - (2AX)^T = (2AB)^T$

$X^T C - 2X^T A^T = 2B^T A^T$

$X^T \cdot (C - 2A^T) = 2B^T A^T \quad \Big| \cdot (C - 2A^T)^{-1}$

$X^T = 2B^T A^T \cdot (C - 2A^T)^{-1}$

$X = (C^T - 2A)^{-1} \cdot 2AB$

c) $(Ax - y)^T A = 0_n^T$

$(x^T A^T - y^T) A = 0_n^T$

$x^T A^T A - y^T A = 0_n^T \quad \Big| + y^T A$

$x^T A^T A = y^T A \quad \Big| \cdot (A^T A)^{-1}$

$x^T = y^T A \cdot (A^T A)^{-1}$

$x = (A^T A^T)^{-1} \cdot A^T y$

d) $(Ax - y)^T A + x^T B = 0_n^T \quad \Big|$ analog zu c)

$x^T (A^T A + B) = y^T A$

Da $A^T A$ p. semidefinit und B p. definit, gilt $x^T (A^T A + B) x$
$= x^T (A^T A) x + x^T B x > 0$, also $(A^T A + B)$ invertierbar

$$X^T = y^T A \cdot (A^T A + B)^{-1}$$

$$X = \left[(A^T A + B)^{-1}\right]^T \cdot (y^T A)^T$$

$$X = \left((A^T A)^T + B^T\right)^{-1} \cdot A^T y$$

$$X = (A^T A + B^T) \cdot A^T y$$