04. Bias, Correspondence & Outliers, YaoGeFAD.

Q1: Define the augmented state as:

$$x'_{k-1} = \begin{bmatrix} x_{k-1} \\ \bar{v}_{k-1} \end{bmatrix}$$

$$\begin{bmatrix} x_k \\ \bar{v}_k \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_{k-1} \\ \bar{v}_{k-1} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_k + \begin{bmatrix} w_k \\ s_k \end{bmatrix}$$

$\therefore$

$$d_k = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ \bar{v}_k \end{bmatrix}$$

$\therefore$ $$A' = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$c' = [c, 0] = [1, 0]$$

$$N + U = 1 + 1 = 2$$

$\because$ $$O' = \begin{bmatrix} c' \\ c'A' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$ $\text{rank } O' = 2 = N + U$

$\therefore$ bias $\bar{v}$ is observable

$\therefore$ Q.E.D.

Q2: Define the augmented state as:

$$x'_k = \begin{bmatrix} x_k \\ v_k \\ \bar{d}_k \end{bmatrix} \qquad u_k = a_k$$

$$\therefore x'_k = \begin{bmatrix} x_k \\ v_k \\ \bar{d}_k \end{bmatrix} = \begin{bmatrix} x_{k-1} + v_{k-1} \\ v_{k-1} \\ \bar{d}_{k-1} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} a_k + \begin{bmatrix} 0 \\ 0 \\ S_k \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x'_{k-1} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u_k + \begin{bmatrix} 0 \\ 0 \\ S_k \end{bmatrix}$$

$$\therefore y'_k = \begin{bmatrix} d_{1,k} \\ d_{2,k} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} x'_k$$

$$\therefore A' = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad C' = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \qquad N+U=3$$

$$\therefore O' = \begin{bmatrix} C' \\ C'A' \\ C'A'^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ & C'A'^2 & \end{bmatrix}$$

$\because \operatorname{rank} O' = 3 = N + U$

$\therefore$ bias $\bar{d}_k$ is observable

$\therefore$ Q.E.D

Q 3.

$$k \geq \left\lceil \frac{\ln(1-p)}{\ln(1-w^n)} \right\rceil = \lceil 6904.3 \rceil = 6905$$

$\therefore$ at least 6905 iterations.