6.6.1

ANS:

$$u^{\wedge}v = \begin{bmatrix} 0 & -u_z & u_y \\ u_z & 0 & -u_x \\ -u_y & u_x & 0 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} -u_z v_y + u_y v_z \\ u_z v_x - u_x v_z \\ -u_y v_x + u_x v_y \end{bmatrix}$$

$$= \begin{bmatrix} -(-v_z)u_y + (-v_y)u_z \\ (-v_z)u_x - (-v_x)u_z \\ -(-v_y)u_x + (-v_x)u_y \end{bmatrix} = \begin{bmatrix} 0 & v_z & -v_y \\ -v_z & 0 & v_x \\ v_y & -v_x & 0 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = -v^{\wedge}u$$

6.6.2.

ANS:

$\because C = \cos\theta I + (1-\cos\theta) a a^T + \sin\theta a^\wedge$

$\therefore C^T = \cos\theta I + (1-\cos\theta) a a^T - \sin\theta a^\wedge$

$\therefore C C^T = \cos^2\theta I + (\cos\theta - \cos^2\theta) a a^T - \cos\theta \sin\theta a^\wedge +$

$(\cos\theta - \cos^2\theta) a a^T + (1-\cos\theta)^2 a a^T + 0 +$

$\cos\theta \sin\theta a^\wedge + 0 - \sin^2\theta a^\wedge a^\wedge$

$= \cos^2\theta \cdot I + \sin^2\theta \cdot a a^T - \sin^2\theta \; a^\wedge a^\wedge$

$= \cos^2\theta \cdot I + \sin^2\theta \cdot a a^T - \sin^2\theta \cdot a a^T + \sin^2\theta \cdot I$

$= (\cos^2\theta + \sin^2\theta) I$

$= I$

$\therefore C^{-1} = C^T$

$\therefore$ Q.E.D

6.6.3:

$$\because (Cv)^\wedge = \begin{bmatrix} r_1^T v \\ r_2^T v \\ r_3^T v \end{bmatrix}^\wedge = \begin{bmatrix} 0 & -r_3^T v & r_2^T v \\ r_3^T v & 0 & -r_1^T v \\ -r_2^T v & r_1^T v & 0 \end{bmatrix}$$

$$\because Cv^\wedge C^T = \begin{bmatrix} r_1^T \\ r_2^T \\ r_3^T \end{bmatrix} v^\wedge \begin{bmatrix} r_1, r_2, r_3 \end{bmatrix}$$

$$= \begin{bmatrix} r_1^T \\ r_2^T \\ r_3^T \end{bmatrix} \begin{bmatrix} v\times r_1, & v\times r_2, & v\times r_3 \end{bmatrix}$$

$$= \begin{bmatrix} r_1^T(v\times r_1) & r_1^T(v\times r_2) & r_1^T(v\times r_3) \\ r_2^T(v\times r_1) & r_2^T(v\times r_2) & r_2^T(v\times r_3) \\ r_3^T(v\times r_1) & r_3^T(v\times r_2) & r_3^T(v\times r_3) \end{bmatrix}$$

However: $r_1^T(v\times r_2) = \det(v|r_2|r_1) = -\det(r_1|r_2|v)$

$$\therefore Cv^\wedge C^T = -\begin{bmatrix} (r_1\times r_1)^T v & (r_1\times r_2)^T v & (r_1\times r_3)^T v \\ (r_2\times r_1)^T v & (r_2\times r_2)^T v & (r_2\times r_3)^T v \\ (r_3\times r_1)^T v & (r_3\times r_2)^T v & (r_3\times r_3)^T v \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -r_3^T v & r_2^T v \\ r_3^T v & 0 & -r_1^T v \\ -r_2^T v & r_1^T v & 0 \end{bmatrix} = (Cv)^\wedge \quad \therefore Q.E.D$$

6.6.4.