$$\frac{dRp}{dR}$$

lie algebra

$$\lim_{\varphi \to 0} \frac{\exp[(\phi+\varphi)^\wedge]p - \exp[(\phi)^\wedge]p}{\varphi} = \lim_{\varphi \to 0} \frac{\exp[(\phi)^\wedge]\exp[(J_r(\phi)\varphi)^\wedge]p - \exp[(\phi)^\wedge]p}{\varphi}$$

$$= \lim_{\varphi \to 0} \frac{R(J_r(\phi)\varphi)^\wedge p}{\varphi} = \lim_{\varphi \to 0} - \frac{Rp^\wedge J_r(\phi)\varphi}{\varphi} = -Rp^\wedge J_r(\phi)$$

perturbation:

$$\lim_{\varphi \to 0} \frac{\exp(\phi^\wedge)\exp(\varphi^\wedge)p - \exp(\phi^\wedge)p}{\varphi} = \lim_{\varphi \to 0} \frac{R(\varphi)^\wedge p}{\varphi} = \lim_{\varphi \to 0} - \frac{Rp^\wedge \varphi}{\varphi}$$

$$= -Rp^\wedge$$

$$\frac{d\ln(R_1R_2)^\vee}{dR_2} = \lim_{\varphi\to 0} \frac{\ln(R_1R_2\exp\varphi^\wedge)^\vee - \ln(R_1R_2)^\vee}{\varphi}$$

$$= \lim_{\varphi\to 0} \frac{\ln(R_1R_2)^\vee + J_r(\phi)^{-1}\varphi - \ln(R_1R_2)^\vee}{\varphi}$$

$$= J_r(\phi)^{-1} \quad , \phi = \ln(R_1R_2)^\vee$$

$$\frac{d\ln(R_1R_2)^\vee}{dR_1} = \lim_{\varphi\to 0} \frac{\ln(R_1\exp\varphi^\wedge R_2)^\vee - \ln(R_1R_2)^\vee}{\varphi}$$

$$= \lim_{\varphi\to 0} \frac{\ln(R_1R_2R_2^T\exp(\varphi^\wedge R_2))^\vee - \ln(R_1R_2)^\vee}{\varphi}$$

$$= \lim_{\varphi\to 0} \frac{\ln(R_1R_2\exp(R_2^T\varphi)^\wedge)^\vee - \ln(R_1R_2)^\vee}{\varphi}$$

$$= \lim_{\varphi\to 0} \frac{\ln(R_1R_2)^\vee + J_r(\phi)^{-1}R_2^T\varphi - \ln(R_1R_2)^\vee}{\varphi}$$

$$= J_r(\phi)^{-1}R_2, \qquad \phi = \ln(R_1R_2)^\vee$$

$$\frac{dR^{-1}p}{dR}$$

$$\lim_{\varphi \to 0} \frac{[\exp(\phi^\wedge)\exp(\varphi^\wedge)]^{-1}p - [\exp(\phi^\wedge)]^{-1}p}{\varphi}$$

$$= \lim_{\varphi \to 0} \frac{\exp(-\varphi^\wedge)\exp(-\phi^\wedge)p - \exp(-\phi^\wedge)p}{\varphi}$$

$$= \lim_{\varphi \to 0} - \frac{\varphi^\wedge R^{-1}p}{\varphi} = \lim_{\varphi \to 0} \frac{(R^{-1}p)^\wedge \varphi}{\varphi} = (R^{-1}p)^\wedge$$

$$\frac{d\ln(R_1R_2^{-1})^\vee}{dR_2} = \lim_{\varphi\to 0}\frac{\ln[R_1(R_2\exp(\varphi^\wedge))^{-1}]^\vee - \ln(R_1R_2^{-1})^\vee}{\varphi}$$

$$= \lim_{\varphi\to 0}\frac{\ln[R_1\exp(-\varphi^\wedge)R_2^{-1}]^\vee - \ln(R_1R_2^{-1})^\vee}{\varphi}$$

$$= \lim_{\varphi\to 0}\frac{\ln[R_1R_2^{-1}R_2\exp(-\varphi^\wedge)R_2^{-1}]^\vee - \ln(R_1R_2^{-1})^\vee}{\varphi}$$

$$= \lim_{\varphi\to 0}\frac{\ln(R_1R_2^{-1})^\vee + J_r(\phi)^{-1}(-R_2\varphi) - \ln(R_1R_2^{-1})^\vee}{\varphi}$$

$$= -J_r(\phi)R_2, \qquad \phi := \ln(R_1R_2^{-1})^\vee$$