

$$\frac{dR_p}{dR}$$

He algebra

$$\begin{aligned} \lim_{\varphi \rightarrow 0} \frac{\exp[(\phi + \varphi)^\wedge] p - \exp[\phi^\wedge] p}{\varphi} &= \lim_{\varphi \rightarrow 0} \frac{\exp[\phi^\wedge] \exp[(J_R(\phi) \varphi)^\wedge] p - \exp[\phi^\wedge] p}{\varphi} \\ &= \lim_{\varphi \rightarrow 0} \frac{R(J_R(\phi) \varphi)^\wedge p}{\varphi} = \lim_{\varphi \rightarrow 0} - \frac{R p^\wedge J_R(\phi) \varphi}{\varphi} = -R p^\wedge J_R(\phi) \end{aligned}$$

Perturbation:

$$\begin{aligned} \lim_{\varphi \rightarrow 0} \frac{\exp(\phi^\wedge) \exp(\varphi^\wedge) p - \exp(\phi^\wedge) p}{\varphi} &= \lim_{\varphi \rightarrow 0} \frac{R(\varphi)^\wedge p}{\varphi} = \lim_{\varphi \rightarrow 0} - \frac{R p^\wedge \varphi}{\varphi} \\ &= -R p^\wedge \end{aligned}$$

$$\begin{aligned}
 \frac{d \ln(R, R_2)^V}{d R_2} &= \lim_{\varphi \rightarrow 0} \frac{\ln(R, R_2 \exp \varphi^A)^V - \ln(R, R_2)^V}{\varphi} \\
 &= \lim_{\varphi \rightarrow 0} \frac{\ln(R, R_2)^V + J_r(\phi)^{-1} \varphi - \ln(R, R_2)^V}{\varphi} \\
 &= J_r(\phi)^{-1}, \quad \phi = \ln(R, R_2)^V
 \end{aligned}$$

$$\begin{aligned}
 \frac{d \ln(R, R_2)^V}{d R_1} &= \lim_{\varphi \rightarrow 0} \frac{\ln(R, \exp \varphi^A R_2)^V - \ln(R, R_2)^V}{\varphi} \\
 &= \lim_{\varphi \rightarrow 0} \frac{\ln(R, R_2 R_2^T \exp(\varphi^A R_2))^V - \ln(R, R_2)^V}{\varphi} \\
 &= \lim_{\varphi \rightarrow 0} \frac{\ln(R, R_2 \exp(R_2^T \varphi^A))^V - \ln(R, R_2)^V}{\varphi} \\
 &= \lim_{\varphi \rightarrow 0} \frac{\ln(R, R_2)^V + J_r(\phi)^{-1} R_2^T \varphi - \ln(R, R_2)^V}{\varphi} \\
 &= J_r(\phi)^{-1} R_2, \quad \phi = \ln(R, R_2)^V
 \end{aligned}$$

$$\frac{dR^{-1}P}{dR}$$

lie algebra:

$$\lim_{\varphi \rightarrow 0} \frac{\exp[-(\phi + \varphi^\wedge)]P - \exp[-\phi^\wedge]P}{\varphi}$$

$$= \lim_{\varphi \rightarrow 0} \frac{\exp(-\phi^\wedge) \exp[(-J_r(-\phi)\varphi^\wedge)]P - \exp[-\phi^\wedge]P}{\varphi}$$

$$= \lim_{\varphi \rightarrow 0} \frac{R^{-1}(-J_r(-\phi)\varphi^\wedge)P}{\varphi} = \lim_{\varphi \rightarrow 0} \frac{R^{-1}P^\wedge J_r(-\phi)\varphi}{\varphi} = R^{-1}P^\wedge J_r(-\phi)$$

perturbation:

$$\lim_{\varphi \rightarrow 0} \frac{\exp(-\phi) \exp(-\varphi)P - \exp(-\phi)P}{\varphi} = \lim_{\varphi \rightarrow 0} \frac{R^{-1}(-\varphi)^\wedge P}{\varphi}$$

$$= R^{-1}P^\wedge$$

$$\frac{d \ln(R_1 R_2^{-1})^\vee}{d R_2} = \lim_{\varphi \rightarrow 0} \frac{\ln[R_1 R_2^{-1} \exp(-\varphi)^\vee]^\vee - \ln(R_1 R_2^{-1})^\vee}{\varphi}$$

$$= \lim_{\varphi \rightarrow 0} \frac{\ln(R_1 R_2^{-1})^\vee + \text{Tr}(\phi)^\top (-\varphi) - \ln(R_1 R_2^{-1})^\vee}{\varphi}$$

$$= -\text{Tr}(\phi)^\top, \quad \phi = \ln(R_1 R_2^{-1})^\vee$$