$$\frac{dRp}{dR}$$

lie algebra

$$\lim_{\varphi\to 0} \frac{\exp[(\phi+\varphi)^\wedge]p - \exp[(\phi)^\wedge]p}{\varphi} = \lim_{\varphi\to 0} \frac{\exp[(\phi)^\wedge]\exp[(J_r(\phi)\varphi)^\wedge]p - \exp[(\phi)^\wedge]p}{\varphi}$$

$$= \lim_{\varphi\to 0} \frac{R(J_r(\phi)\varphi)^\wedge p}{\varphi} = \lim_{\varphi\to 0} - \frac{Rp^\wedge J_r(\phi)\varphi}{\varphi} = -Rp^\wedge J_r(\phi)$$

perturbation:

$$\lim_{\varphi\to 0} \frac{\exp(\phi^\wedge)\exp(\varphi^\wedge)p - \exp(\phi^\wedge)p}{\varphi} = \lim_{\varphi\to 0} \frac{R(\varphi)^\wedge p}{\varphi} = \lim_{\varphi\to 0} - \frac{Rp^\wedge\varphi}{\varphi}$$

$$= -Rp^\wedge$$

$$\frac{d\ln(R_1R_2)^\vee}{dR_2} = \lim_{\varphi\to 0} \frac{\ln(R_1R_2\exp\varphi^\wedge)^\vee - \ln(R_1R_2)^\vee}{\varphi}$$

$$= \lim_{\varphi\to 0} \frac{\ln(R_1R_2)^\vee + J_r(\phi)^{-1}\varphi - \ln(R_1R_2)^\vee}{\varphi}$$

$$= J_r(\phi)^{-1} \quad , \phi = \ln(R_1R_2)^\vee$$

$$\frac{d\ln(R_1R_2)^\vee}{dR_1} = \lim_{\varphi\to 0} \frac{\ln(R_1\exp\varphi^\wedge R_2)^\vee - \ln(R_1R_2)^\vee}{\varphi}$$

$$= \lim_{\varphi\to 0} \frac{\ln(R_1R_2R_2^T\exp(\varphi^\wedge R_2)^\vee - \ln(R_1R_2)^\vee}{\varphi}$$

$$= \lim_{\varphi\to 0} \frac{\ln(R_1R_2\exp(R_2^T\varphi)^\wedge)^\vee - \ln(R_1R_2)^\vee}{\varphi}$$

$$= \lim_{\varphi\to 0} \frac{\ln(R_1R_2)^\vee + J_r(\phi)^{-1}R_2^T\varphi - \ln(R_1R_2)^\vee}{\varphi}$$

$$= J_r(\phi)^{-1}R_2, \qquad \phi = \ln(R_1R_2)^\vee$$

$$\frac{dR^{-1}p}{dR}$$

lie algebra:

$$\lim_{\varphi \to 0} \frac{\exp[-(\phi+\varphi)^\wedge]p - \exp[-\phi^\wedge]p}{\varphi}$$

$$= \lim_{\varphi \to 0} \frac{\exp(-\phi^\wedge)\exp[(-J_r(-\phi)\varphi)^\wedge]p - \exp[-\phi^\wedge]p}{\varphi}$$

$$= \lim_{\varphi \to 0} \frac{R^{-1}(-J_r(-\phi)\varphi)^\wedge p}{\varphi} = \lim_{\varphi \to 0} \frac{R^{-1}p^\wedge J_r(-\phi)\varphi}{\varphi} = R^{-1}p^\wedge J_r(-\phi)$$

perturbation:

$$\lim_{\varphi \to 0} \frac{\exp(-\phi)\exp(-\varphi)p - \exp(-\phi)p}{\varphi} = \lim_{\varphi \to 0} \frac{R^{-1}(-\varphi)^\wedge p}{\varphi}$$

$$= R^{-1}p^\wedge$$

$$\frac{d\ln(R_1R_2^{-1})^\vee}{dR_2} = \lim_{\varphi\to 0}\frac{\ln[R_1R_2^{-1}\exp(-\varphi)^\wedge]^\vee - \ln(R_1R_2^{-1})^\vee}{\varphi}$$

$$= \lim_{\varphi\to 0}\frac{\ln(R_1R_2^{-1})^\vee + J_r(\phi)^{-1}(-\varphi) - \ln(R_1R_2^{-1})^\vee}{\varphi}$$

$$= -J_r(\phi)^{-1}, \quad \phi = \ln(R_1R_2^{-1})^\vee$$