

Perturbation for Derivative.

$$\frac{\partial (Rp)}{\partial \varphi}, \text{ left perturbation,}$$

$$\text{def: } \varphi = J(\phi)^T \varphi'$$

$$\therefore \varphi + \phi = J(\phi)^T \varphi' + \phi$$

$$\therefore \varphi' \rightarrow 0, \ln[\exp(\varphi'^\wedge) \exp(\phi^\wedge)]^\vee \doteq J(\phi)^T \varphi' + \phi$$

$$\therefore \exp(\phi + \varphi) = \exp[\phi + J(\phi)^T \varphi'] \doteq \exp(\varphi'^\wedge) \exp(\phi^\wedge)$$

$$\doteq [I + (\varphi')^\wedge] R$$

$$\therefore \lim_{\varphi \rightarrow 0} \frac{\exp(\phi + \varphi) p - \exp \phi^\wedge p}{\varphi} = \lim_{\varphi' \rightarrow 0} \frac{[I + (\varphi')^\wedge] R p - R p}{\varphi'}$$

$$= \lim_{\varphi' \rightarrow 0} \frac{(\varphi')^\wedge R p}{\varphi'} = \lim_{\varphi' \rightarrow 0} - (R p)^\wedge \varphi'$$

$$= - (R p)^\wedge$$

$\frac{\partial \langle R \rangle}{\partial \phi}$, right perturbation

$$\therefore \ln[\exp(\phi^\wedge) \exp(\varphi'^\wedge)]^\vee \doteq \phi + J(-\phi)^T \varphi', \quad \varphi' \rightarrow 0$$

$$\therefore \varphi := J(-\phi)^T \varphi'$$

$$\begin{aligned} \therefore \lim_{\varphi \rightarrow 0} \frac{\exp[(\phi + \varphi)^\wedge] p - \exp[\phi^\wedge] p}{\varphi} &= \lim_{\varphi' \rightarrow 0} \frac{\exp(\phi^\wedge) \exp(\varphi'^\wedge) p - \exp(\phi^\wedge) p}{\varphi'} \\ &= \lim_{\varphi' \rightarrow 0} \frac{R \varphi'^\wedge p}{\varphi'} = -R p^\wedge \end{aligned}$$

$$\frac{d \ln(R_1 R_2)^V}{d R_2}$$

$$\text{def: } \phi := \ln(R_1 R_2)^V$$

$$\varphi := J(-\phi)^{-1} \varphi'$$

$$\begin{aligned} \therefore \lim_{\varphi' \rightarrow 0} \frac{\ln[R_1 R_2 \exp(\varphi'^\wedge)]^V - \ln[R_1 R_2]^V}{\varphi'} &= \lim_{\varphi' \rightarrow 0} \frac{\ln[R_1 R_2]^V + J(-\phi)^T \varphi' - \ln[R_1 R_2]^V}{\varphi'} \\ &= J(-\phi)^T \end{aligned}$$

$$\begin{aligned} \therefore \frac{d \ln(R_1 R_2)^V}{d R_1} &= \lim_{\varphi \rightarrow 0} \frac{\ln[R_1 \exp(\varphi^\wedge) R_2]^V - \ln[R_1 R_2]^V}{\varphi} \\ &= \lim_{\varphi \rightarrow 0} \frac{\ln[R_1 R_2 R_2^T \exp(\varphi^\wedge) R_2]^V - \ln[R_1 R_2]^V}{\varphi} \\ &= \lim_{\varphi \rightarrow 0} \frac{\ln[R_1 R_2 \exp(R_2^T \varphi^\wedge)]^V - \ln[R_1 R_2]^V}{\varphi} \\ &= \lim_{\varphi \rightarrow 0} J(-\phi)^T R_2^T \end{aligned}$$

$$\therefore \frac{d(R^{-1}p)}{dR} = \lim_{\varphi \rightarrow 0} \frac{\exp[-(\phi + \varphi^{\wedge})]p - \exp[-\phi^{\wedge}]p}{\varphi} \quad \begin{aligned} \varphi &= J[-(-\phi)]^{-1} \varphi' \\ &= J(\phi)^{-1} \varphi' \end{aligned}$$

$$= \lim_{\varphi' \rightarrow 0} \frac{\exp(-\phi^{\wedge}) \exp(-\varphi'^{\wedge})p - \exp(-\phi^{\wedge})p}{\varphi'}$$

$$= \lim_{\varphi' \rightarrow 0} \frac{\exp(-\phi^{\wedge})(I - \varphi'^{\wedge})p - \exp(-\phi^{\wedge})p}{\varphi'}$$

$$= \lim_{\varphi' \rightarrow 0} \frac{\exp(-\phi^{\wedge})(-\varphi'^{\wedge})p}{\varphi'} = \lim_{\varphi' \rightarrow 0} \frac{\exp(-\phi^{\wedge})p^{\wedge} \varphi'}{\varphi'}$$

$$= R^{-1}p^{\wedge}$$

$$\frac{d \ln(R_1 R_2^{-1})^\vee}{d R_2} = \lim_{\varphi \rightarrow 0} \frac{\ln[R_1 R_2^{-1} \exp(-\varphi^\vee)]^\vee - \ln[R_1 R_2^{-1}]^\vee}{\varphi}$$

$$= \lim_{\varphi \rightarrow 0} \frac{-\text{Tr}(\phi)^\vee \varphi}{\varphi} = -\text{Tr}(\phi)^\vee, \quad \phi = \ln(R_1 R_2^{-1})^\vee$$