Of Pose-and-Point Estimation Problem.

Prove that:

$$T_{op,k}P_{op,j} + \varepsilon_k^{\wedge} T_{op,k}P_{op,j} + T_{op,k}D\xi_j$$

$$+ \frac{1}{2}\varepsilon_k^{\wedge}\varepsilon_k^{\wedge} T_{op,k}P_{op,j} + \varepsilon_k^{\wedge} T_{op,k}D\xi_j$$

$$= z(x_{op,jk}) + Z_{jk}\,\delta x_{jk} + \frac{1}{2}\sum_i 1_i\,\delta x_{jk}^T \mathcal{Z}_{jk}\,\delta x_{jk}$$

ANS:

1. for 0th-order term:

$$z(x_{op,jk}) = T_{op,k}P_{op,j}$$

2. for 1st-order term:

$$\because \varepsilon_k^{\wedge} T_{op,k}P_{op,j} = (T_{op,k}P_{op,j})^{\odot}\varepsilon_k$$

$$\therefore \varepsilon_k^{\wedge} T_{op,k}P_{op,j} + T_{op,k}D\xi_j = \left[(T_{op,k}P_{op,j})^{\odot},\ T_{op,k}D\right]\begin{bmatrix}\varepsilon_k\\ \xi_j\end{bmatrix}$$

$$= Z_{jk}\,\delta x_{jk}$$

3. for 2nd-order term:

$$\because \text{i-th elem. } 1_i^T\varepsilon_k^{\wedge}\varepsilon_k^{\wedge} T_{op,k}P_{op,j} = \varepsilon_k^T 1_i^{\circledcirc}(T_{op,k}P_{op,j})^{\odot}\varepsilon_k$$

$$1_i^T\varepsilon_k^{\wedge} T_{op,k}D\xi_j = \varepsilon_k^T 1_i^{\circledcirc} T_{op,k}D\xi_j$$

$$= \xi_j^T(1_i^{\circledcirc} T_{op,k}D)^T\varepsilon_k$$

$$\therefore \frac{1}{2}1_i^T(\varepsilon_k^{\wedge}\varepsilon_k^{\wedge} T_{op,k}P_{op,j} + \varepsilon_k^{\wedge} T_{op,k}D\xi_j)$$

$$= [\varepsilon_k^T, \xi_j^T]\begin{bmatrix}1_i^{\circledcirc}(T_{op,k}P_{op,j})^{\odot}, & 1_i^{\circledcirc}T_{op,k}D\\ (1_i^{\circledcirc}T_{op,k}D)^T, & 0\end{bmatrix}\begin{bmatrix}\varepsilon_k\\ \xi_j\end{bmatrix}$$

$$= \frac{1}{2}\delta x_{jk}^T \mathcal{Z}_{jk}\,\delta x_{jk}$$

$\therefore$ Q.E.D