Perturbation for Derivative.

$\frac{\partial(Rp)}{\partial\varphi}$, left perturbation,

def: $\varphi = J(\phi)^{-1}\varphi'$

$$\therefore \varphi + \phi = J(\phi)^{-1}\varphi' + \phi$$

$$\therefore \varphi' \to 0, \quad \ln[\exp(\varphi'^{\wedge})\exp(\phi^{\wedge})]^{\vee} \doteq J(\phi)^{-1}\varphi' + \phi$$

$$\therefore \exp(\phi+\varphi) = \exp[\phi + J(\phi)^{-1}\varphi'] \doteq \exp(\varphi'^{\wedge})\exp(\phi^{\wedge})$$

$$\doteq [I + (\varphi')^{\wedge}]R$$

$$\therefore \lim_{\varphi\to 0}\frac{\exp(\phi+\varphi)p - \exp\phi^{\wedge}p}{\varphi} = \lim_{\varphi'\to 0}\frac{[I+(\varphi')^{\wedge}]Rp - Rp}{\varphi'}$$

$$= \lim_{\varphi'\to 0}\frac{(\varphi')^{\wedge}Rp}{\varphi'} = \lim_{\varphi'\to 0} -(Rp)^{\wedge}\cdot\varphi'$$

$$= -(Rp)^{\wedge}$$

$\frac{\partial(Rp)}{\partial\varphi}$, right perturbation

$\because \ln[\exp(\phi^\wedge)\exp(\varphi'^\wedge)]^\vee \doteq \phi + J(-\phi)^{-1}\varphi'$, $\varphi' \to 0$

$\therefore \varphi := J(-\phi)^{-1}\varphi'$

$$\therefore \lim_{\varphi\to 0} \frac{\exp[(\phi+\varphi)^\wedge]p - \exp[\phi^\wedge]p}{\varphi} \doteq \lim_{\varphi'\to 0} \frac{\exp(\phi^\wedge)\exp(\varphi'^\wedge)p - \exp(\phi^\wedge)p}{\varphi'}$$

$$\doteq \lim_{\varphi'\to 0} \frac{R\varphi'^\wedge p}{\varphi'} = -Rp^\wedge$$

$$\frac{d\ln(R_1R_2)^\vee}{dR_2}$$

def: $\phi := \ln(R_1R_2)^\vee$

$\varphi := J(-\phi)^{-1}\varphi'$

$$\therefore \lim_{\varphi'\to 0}\frac{\ln[R_1R_2\exp(\varphi'^\wedge)]^\vee - \ln[R_1R_2]^\vee}{\varphi'} = \lim_{\varphi'\to 0}\frac{\ln[R_1R_2]^\vee + J(-\phi)^{-1}\varphi' - \ln[R_1R_2]^\vee}{\varphi'}$$

$$= J(-\phi)^{-1}$$

$$\therefore \frac{d\ln(R_1R_2)^\vee}{dR_1} = \lim_{\varphi\to 0}\frac{\ln[R_1\exp(\varphi^\wedge)R_2]^\vee - \ln[R_1R_2]^\vee}{\varphi}$$

$$= \lim_{\varphi\to 0}\frac{\ln[R_1R_2R_2^T\exp(\varphi^\wedge)R_2]^\vee - \ln[R_1R_2]^\vee}{\varphi}$$

$$= \lim_{\varphi\to 0}\frac{\ln[R_1R_2\exp(R_2^T\varphi^\wedge)]^\vee - \ln[R_1R_2]^\vee}{\varphi}$$

$$= \lim_{\varphi\to 0} J(-\phi)^{-1}R_2^T$$

$$\therefore \frac{d(R^{-1}p)}{dR} = \lim_{\varphi \to 0} \frac{\exp[-(\phi+\varphi)^\wedge]p - \exp[-\phi^\wedge]p}{\varphi}$$

$$\varphi = J[-(-\phi)]^{-1}\varphi'$$
$$= J(\phi)^{-1}\varphi'$$

$$= \lim_{\varphi' \to 0} \frac{\exp(-\phi^\wedge)\exp(-\varphi'^\wedge)p - \exp(-\phi^\wedge)p}{\varphi'}$$

$$= \lim_{\varphi' \to 0} \frac{\exp(-\phi^\wedge)(I - \varphi'^\wedge)p - \exp(-\phi^\wedge)p}{\varphi'}$$

$$= \lim_{\varphi' \to 0} \frac{\exp(-\phi^\wedge)(-\varphi'^\wedge)p}{\varphi'} = \lim_{\varphi' \to 0} \frac{\exp(-\phi^\wedge)p^\wedge\varphi'}{\varphi'}$$

$$= R^{-1}p^\wedge$$

$$\frac{d\ln(R_1R_2^{-1})^\vee}{dR_2} = \lim_{\varphi\to 0} \frac{\ln[R_1R_2^{-1}\exp(-\varphi^\wedge)]^\vee - \ln[R_1R_2]^\vee}{\varphi}$$

$$= \lim_{\varphi\to 0} \frac{-J_r(\phi)^{-1}\varphi}{\varphi} = -J_r(\phi)^{-1}, \quad \phi = \ln(R_1R_2^{-1})^\vee$$