Math381, Computer Homework on Solution of Linear Systems
(Due date: 27 December 2022)

Rules on submission of Homework:

- HW is a 5 points bonus assignment. You are free not to submit it.

- You can submit the HW in groups by writing the name and student ID of each group member.

- Submit your HW in a form of well-written report which contains your computer codes and the results of the questions.

- The HWs should be clearly written, you can type them in Latex or Word or by hand.

Questions

1. Write a computer program in a language of your choice (e.g. Matlab, Phyton, C, ext..) to solve a system of n linear equations and n unknowns using:

   (a) the Jacobi iterative method

   (b) the Gauss-Seidel method.

   Input data to the program should be:

   (a) The number of equations n

   (b) The augmented matrix

   (c) The starting vector.

   Output should consist of the first 10 iterations as a table of which columns contain:

   (a) the iteration number k

   (b) the vector solution $x^{(k)}$

   (c) the relative error between two successive iteration, i.e. $Rel_e = \dfrac{\left\| x^{(k)} - x^{(k-1)} \right\|_\infty}{\left\| x^{(k)} \right\|_\infty}$

   Test your program to solve the system

$$
\begin{aligned}
2x - y + 2z - w &= -1 \\
2x + y - 2z - 2w &= -2 \\
-x + 2y - 4z + w &= 1 \\
3x - 3w &= -3
\end{aligned}
$$

2. Given the system

$$
\begin{aligned}
x + z &= 2 \\
x - y &= 0 \\
x + 2y - 3z &= 0
\end{aligned}
$$

   Use the programs in Question 1 to check that the Jacobi method converges whereas the Gauss-Seidel method diverges. Use the tolerance (ϵ) between two successive iterations is $\epsilon = 10^{-3}$ and start with $x^{(0)} = 0$-vector. That is, stop the iteration when the relative error between two successive iteration $(Rel_e = \dfrac{\left\| x^{(k)} - x^{(k-1)} \right\|}{\left\| x^{(k)} \right\|})$ is less than tolerance, i.e.

$$
Rel_e = \frac{\left\| x^{(k)} - x^{(k-1)} \right\|_\infty}{\left\| x^{(k)} \right\|_\infty} < 10^{-3} = \epsilon .
$$