ENGR 2340 Dynamics
Free Response and Step Response

Consider the single degree-of-freedom, mass-spring-damper system

$$\ddot{x}(t) + 2\zeta\omega_n \dot{x}(t) + \omega_n^2 x(t) = f(t) \text{ with IC's } x(0) = x_0 \text{ and } \dot{x}(0) = v_0$$

You can use the MATLAB codes in the course folder or write your own.

1) a) Generate and plot the free response (f(t) = 0) for systems that are undamped, overdamped, critically damped, and underdamped. Try various IC's (including values < 0). Do the plots look as you expect?
b) Hold ω_n constant and vary ζ. Explain how the change effects the response.
c) Hold ζ constant and vary ω_n . Explain how the change effects the response.

2) Now set f(t) = A (constant).
a) Generate and plot the response to a step function, f(t) = Au(t), for systems that are undamped, overdamped, critically damped, and underdamped. Try various IC's (including values < 0). Do the plots look as you expect?
b) Hold ω_n constant and vary ζ. Explain how the change effects the response.
c) Hold ζ constant and vary ω_n . Explain how the change effects the response. Note that you have to normalize the amplitude of the response to make a direct comparison.

3) a) Now consider a system with natural frequency, $\omega_n = 4$, subject to a unit step input and IC's = 0, determine a value for the (underdamped) damping factor such that

- the maximum percent overshoot is as small as possible no greater than 35%
- the rise time should be as small as possible but no greater than 0.68 sec

b) Include an addition requirement of the setting time to be no greater than 5.7 sec.

4) Think of an example in which the settling time would be the most important design requirement.