GENG-8030 Computational Methods and Modeling for Engineering Applications

Summer 2023

Tutorial 9

All questions were taken from the course textbook:

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Author	William J. Palm
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Chapter 9: Numerical Methods for Calculus and Differential Equations

5. A certain object's acceleration is given by $a(t) = 5t \sin 8t$ m/s^2. Compute its velocity at $t = 20$ s if its initial velocity is zero.

10.* A rocket's mass decreases as it burns fuel. The equation of motion for a rocket in vertical flight can be obtained from Newton's law, and it is

$$m(t)\frac{dv}{dt} = T - m(t)g$$

where T is the rocket's thrust and its mass as a function of time is given by $m(t) = m_0(1 - rt/b)$. The rocket's initial mass is m_0, the burn time is b, and r is the fraction of the total mass accounted for by the fuel.
Use the values $T = 48{,}000$ N, $m_0 = 2200$ kg, $r = 0.8$, $g = 9.81$ m/s^2, and $b = 40$ s. Determine the rocket's velocity at burnout.

21. Use the `diff` function to estimate the derivative of

$$y = e^{-2x} \frac{\sin(4x)}{x^2 + 3}$$

at the point $x = 0.6$.

29. The equation describing the water height h in a spherical tank with a drain at the bottom is

$$\pi \left(2rh - h^2\right) \frac{dh}{dt} = -C_d A \sqrt{2gh}$$

Suppose the tank's radius is $r = 3$ m and the circular drain hole has a radius of 2 cm. Assume that $C_d = 0.5$ and that the initial water height is $h(0) = 5$ m. Use $g = 9.81$ m/s^2.

a. Use an approximation to estimate how long it takes for the tank to empty.

b. Plot the water height as a function of time until $h(t) = 0$.

32. The following equation describes the motion of a certain mass connected to a spring, with viscous friction on the surface

$$3\ddot{y} + 18\dot{y} + 102y = f(t)$$

where $f(t)$ is an applied force. Suppose that $f(t) = 0$ for $t < 0$ and $f(t) = 10$ for $t \geq 0$.

a. Plot $y(t)$ for $y(0) = \dot{y}(0) = 0$.

b. Plot $y(t)$ for $y(0) = 0$ and $\dot{y}(0) = 10$. Discuss the effect of the nonzero initial velocity.

44. The following state model describes the motion of a certain mass connected to a spring, with viscous friction on the surface, where $m = 1, c = 2$, and $k = 5$.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f(t)$$

a. Use the `initial` function to plot the position x_1 of the mass, if the initial position is 5 and the initial velocity is 3.

b. Use the `step` function to plot the step response of the position and velocity for zero initial conditions, where the magnitude of the step input is 10. Compare your plot with the figure shown below.

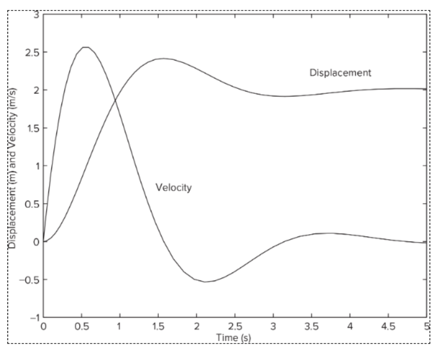

Figure 9.5–1 Displacement and velocity of the mass as a function of time.

45. Consider the following equation.

$$5\ddot{y} + 2\dot{y} + 10y = f(t)$$

a. Plot the free response for the initial conditions $y(0) = 10, \dot{y}(0) = -5$.

b. Plot the unit-step response (for zero initial conditions).

c. The *total response* to a step input is the sum of the free response and the step response. Demonstrate this fact for this equation by plotting the sum of the solutions found in parts *a* and *b* and comparing the plot with that generated by solving for the total response with $y(0) = 10, \dot{y}(0) = -5$.