MATLAB

for Engineering Applications

Fifth Edition

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Chapter 10

- **Simulink**

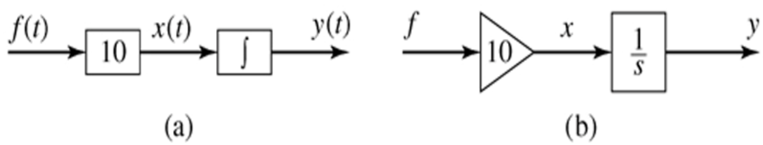
Simulink:

Simulink is a graphical programming environment for modeling, simulating and analyzing systems.

Graphical Blocks : Its primary interface is a graphical block diagramming tool and a customizable set of block libraries.

Simulink is widely used in automatic control and digital signal processing for simulation and Model-Based Design.

$$dy/dt = 10 f(t)$$



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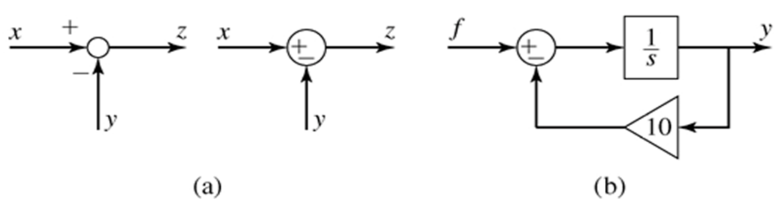
Block diagram elements:

(a) The summer element.

(b) Simulation diagram for

$$dy/dt = f(t) - 10y$$



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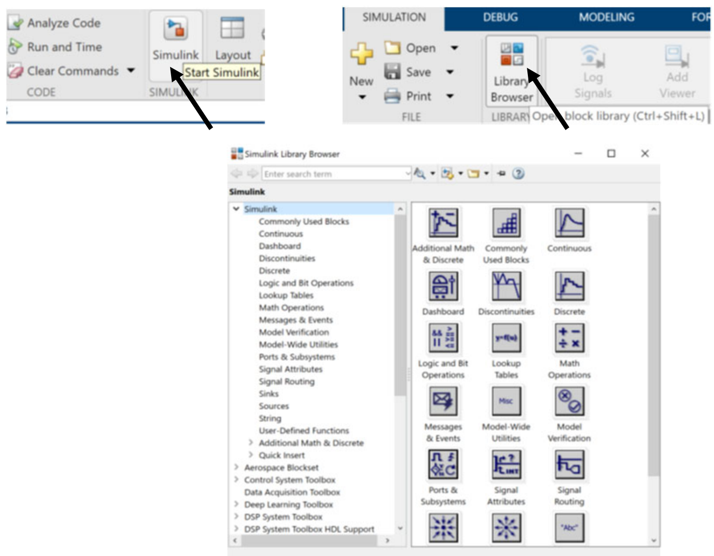
The Simulink library browser:



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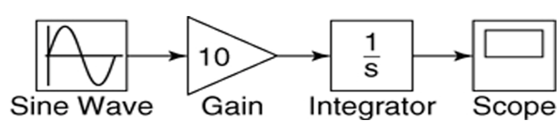
Simulink model for a differential equation:

Use Simulink to solve the following problem for $0 \leq t \leq 10$.

$$dy/dt = 10sint \quad y(0) = 0$$

The exact solution is $y(t) = 10(1 - cost)$.



Set the Initial condition to 0

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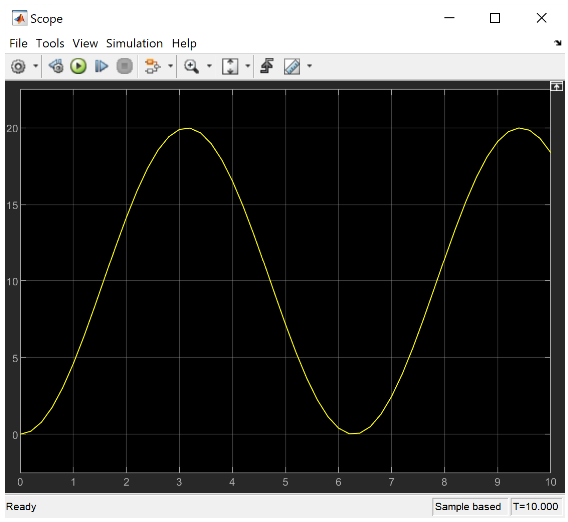
Simulink model for a differential equation:



$$y(t) = 10(1 - cost)$$

Block parameters window:

Note that blocks have a Block Parameters window that opens when you double-click on the block.

This window contains several items, the number and nature of which depend on the specific type of block.

In general, you can use the default values of these parameters, except where we have explicitly indicated that they should be changed.

You can always click on **Help** within the Block Parameters window to obtain more information.

Note that most blocks have default labels.

You can edit text associated with a block by clicking on the text and making changes.

Block parameters window:

You can save the Simulink model as an .slx file by selecting **Save** from the **File** menu in Simulink.

The model file can then be reloaded at a later time.

You can also print the diagram by selecting **Print** on the **File** menu.

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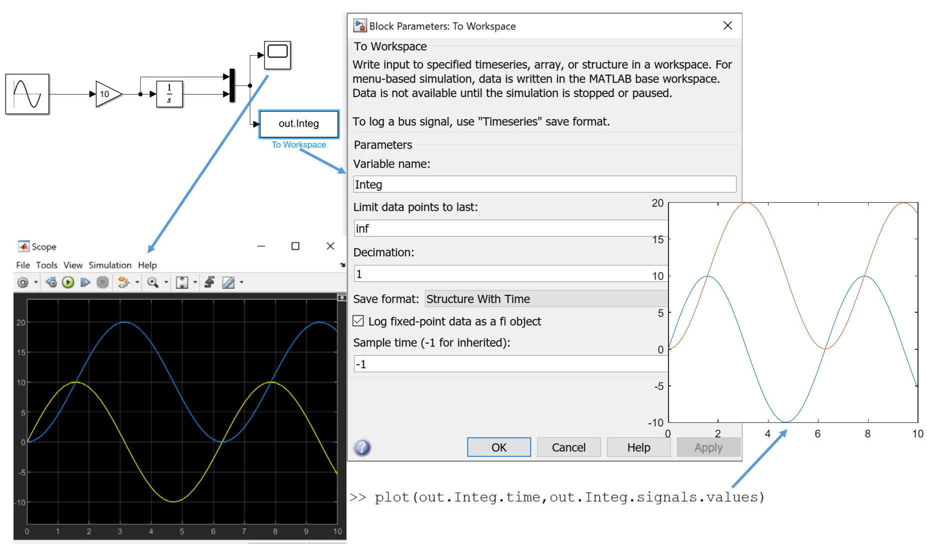
Simulink model with "Mux" and "To workspace" blocks:




```
>> plot(out.Integ.time,out.Integ.signals.values)
```

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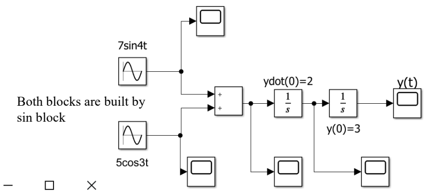
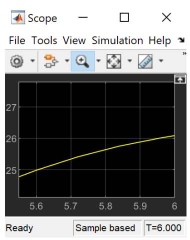
Question:

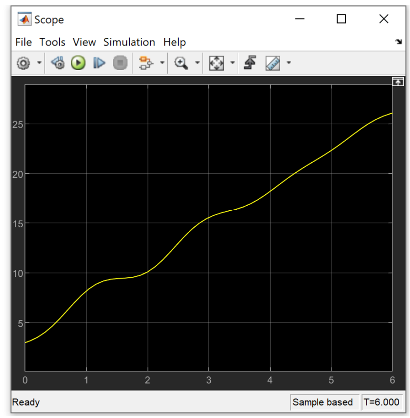
Q1-Create a Simulink model to plot the solution of the following equation for

$0 \le t \le 6.\ \ddot{y} = 7\sin 4t + 5\cos 3t \quad \dot{y}(0) = 2 \quad y(0) = 3$

Y at t=6s is equal to:







Question:

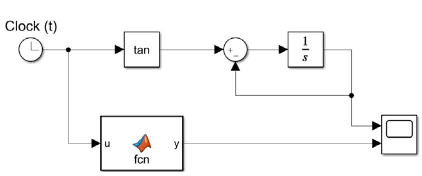
Q2-The following equation has no analytical solution even though it is linear.

$$\dot{x} + x = \tan t \qquad x(0) = 0$$

The approximate solution, which is less accurate for larger values of t, is

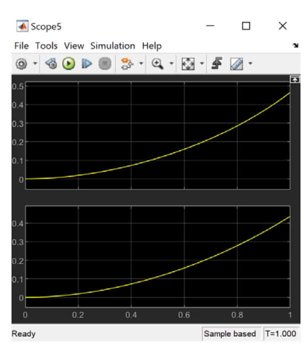
$$x(t) = \frac{1}{3}t^3 - t^2 + 3t - 3 + 3e^{-t}$$

Create a Simulink model to solve this problem, and compare its solution with the approximate solution over the range $0 \le t \le 1$



MATLAB Function*

```
function y = fcn(u)
y=(u.^3)/3-u.^2+3*u-3+3*exp(-u);
end
```



Linear state variables model:

The following are the equations of motion of the two-mass suspension model shown below.

$$m_1\ddot{x}_1 = k_1(x_2 - x_1) + c_1(\dot{x}_2 - \dot{x}_1)$$

$$m_2\ddot{x}_2 = -k_1(x_2 - x_1) - c_1(\dot{x}_2 - \dot{x}_1) + k_2(y - x_2)$$

$$y_1 = x_1$$

$$y_2 = x_2$$

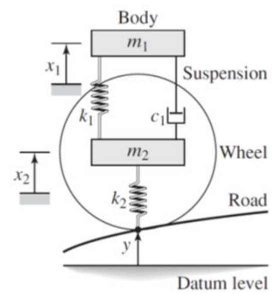
Assuming $z_1 = x_1, \quad z_2 = \dot{x}_1,$

$$z_3 = x_2, \quad z_4 = \dot{x}_2,$$

We have

$$\dot{z}_1 = z_2, \qquad \dot{z}_2 = \frac{1}{m_1}(-k_1 z_1 - c_1 z_2 + k_1 z_3 + c_1 z_4)$$

$$\dot{z}_3 = z_4, \qquad \dot{z}_4 = \frac{1}{m_2}[k_1 z_1 + c_1 z_2 - (k_1 + k_2) z_3 - c_1 z_4 + k_2 y]$$



Two-mass suspension model

Linear state variables model:

$$\dot{z}_1 = z_2, \qquad \dot{z}_2 = \frac{1}{m_1}(-k_1 z_1 - c_1 z_2 + k_1 z_3 + c_1 z_4)$$

$$\dot{z}_3 = z_4, \qquad \dot{z}_4 = \frac{1}{m_2}[k_1 z_1 + c_1 z_2 - (k_1 + k_2) z_3 - c_1 z_4 + k_2 y]$$

$$\dot{\boldsymbol{z}} = \boldsymbol{Az} + \boldsymbol{B}y(t)$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -a_1 & -a_2 & a_1 & a_2 \\ 0 & 0 & 0 & 1 \\ a_3 & a_4 & -a_6 & -a_4 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ a_5 \end{bmatrix}$$

$$a_1 = \frac{k_1}{m_1} \qquad a_2 = \frac{c_1}{m_1}$$

$$a_3 = \frac{k_1}{m_2} \qquad a_4 = \frac{c_1}{m_2}$$

$$a_5 = \frac{k_2}{m_2} \qquad a_6 = a_3 + a_5$$

$$y = \boldsymbol{Cz} + \boldsymbol{D}u(t)$$

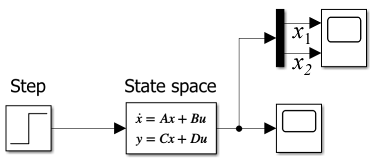
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

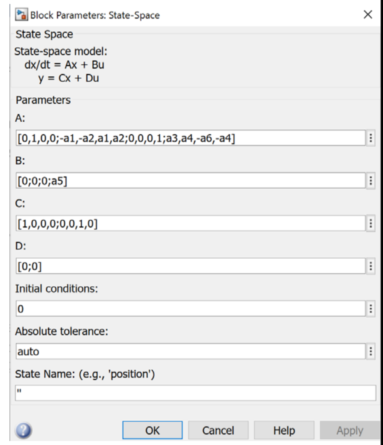
Question:

Q3-The input $y(t)$ is a unit step function, and the initial conditions are zero. Use the following values to determine the response for a two-mass suspension model.

$m_1 = 250\ kg, \quad m_2 = 40\ kg,$
$k_1 = 1.5 \times 10^4 \frac{N}{m}, \quad k_2 = 1.5 \times 10^5\ N/m$, and $\quad c_1 = 1917\ N\ s/m.$

```
>> m1=250; m2=40; k1=1.5e+4; k2=1.5e+5;c1=1917;
a1=k1/m1; a2=c1/m1; a3=k1/m2; a4=c1/m2; a5=k2/m2; a6=a3+a5;
```





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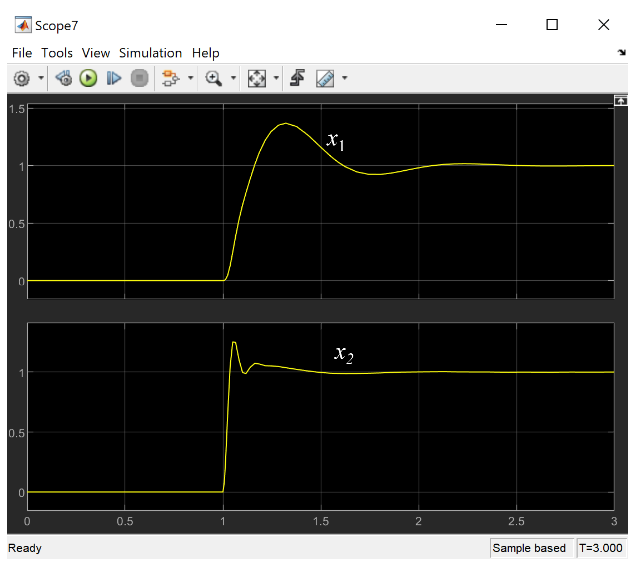
Question:



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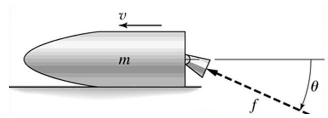
Simulink model example of a rocket-propelled sled:

The rocket thrust initially is horizontal, but the engine accidentally pivots during firing and rotates with an angular acceleration of $\ddot{\theta} = \frac{\pi}{50}\ rad/s$. Compute the sled's velocity v for $0 \le t \le 10$ if $v(0) = 0$. The rocket thrust is 4000 N and the sled mass is 450 kg.



$$F = ma \qquad 4000 \cos\theta\,(t) = 450\,\dot{v}$$

$$\theta(t) = \iint_0^t \ddot{\theta} dt = \iint_0^t \frac{\pi}{100} dt = \frac{\pi}{100} t^2$$

$$4000 cos\left(\frac{\pi}{100} t^2\right) = 450\,\dot{v}$$

$$v(t) = \frac{80}{9}\int_0^t \cos\left(\frac{\pi}{100} t^2\right) dt$$

No closed-form solution is available for the integral, which is called *Fresnel's cosine integral*.

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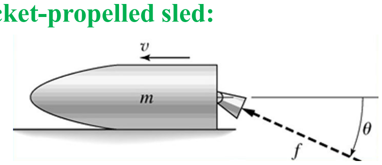
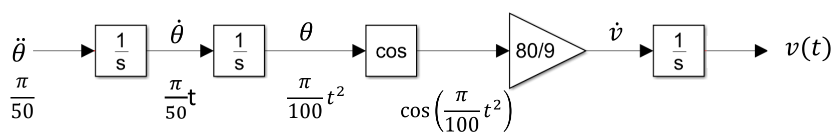
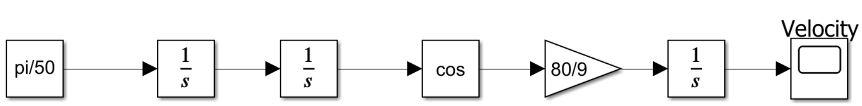
Simulink model example of a rocket-propelled sled:

Model of a Rocket-propelled Sled

$$v(t) = \frac{80}{9}\int_0^t \cos\left(\frac{\pi}{100} t^2\right) dt$$







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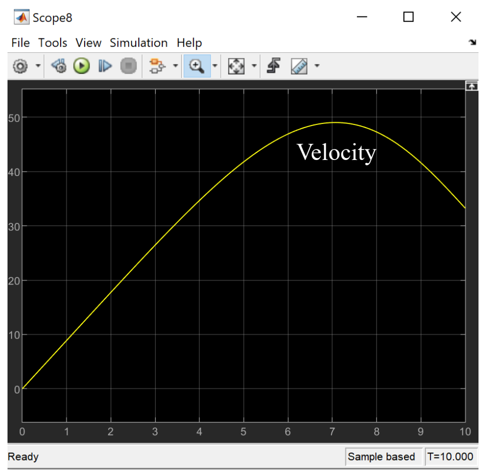
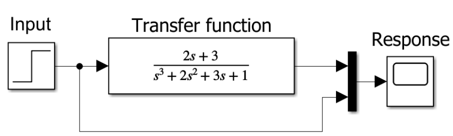
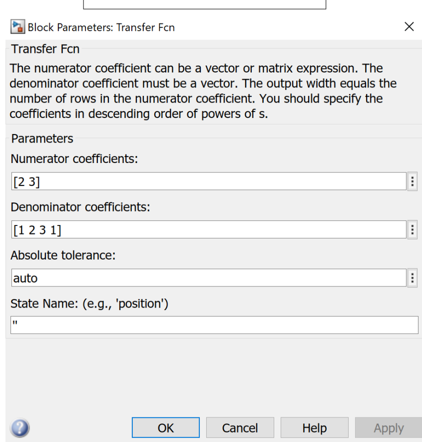
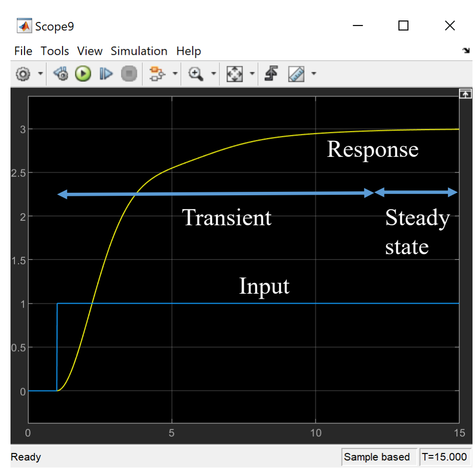
Simulink model example of a rocket-propelled sled:



Transfer function response:



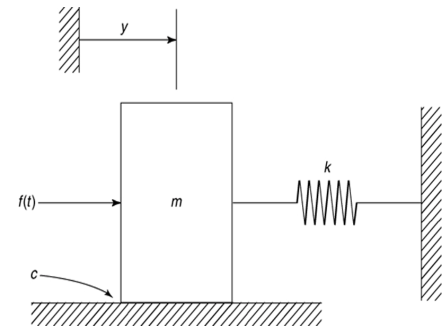




Transfer function response and PID controller:

A mass and spring with viscous surface friction. Its equation of motion is

$$m\ddot{y} + c\dot{y} + ky = f(t)$$



Transfer function response and PID controller:

Taking the Laplace transform of the governing equation, we get

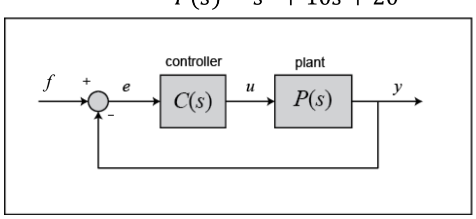
$$ms^2Y(s) + csY(s) + kY(s) = F(s)$$

The transfer function between the input force *F(s)* and the output displacement *Y(s)* then becomes

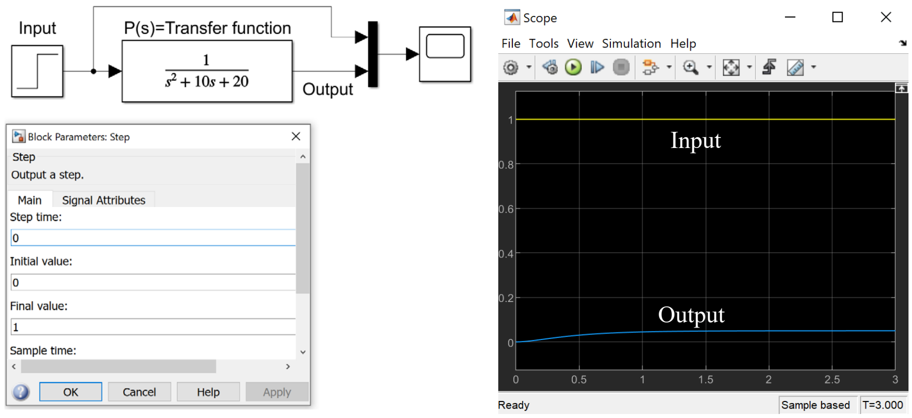
$$\frac{Y(s)}{F(s)} = \frac{1}{ms^2 + cs + k}$$

For instance if m=1 kg, c= 10 N s/m, k=20 N/m, then

$$P(s) = \frac{Y(s)}{F(s)} = \frac{1}{s^2 + 10s + 20}$$



Transfer function response and PID controller:



Warning: Output is not following the input

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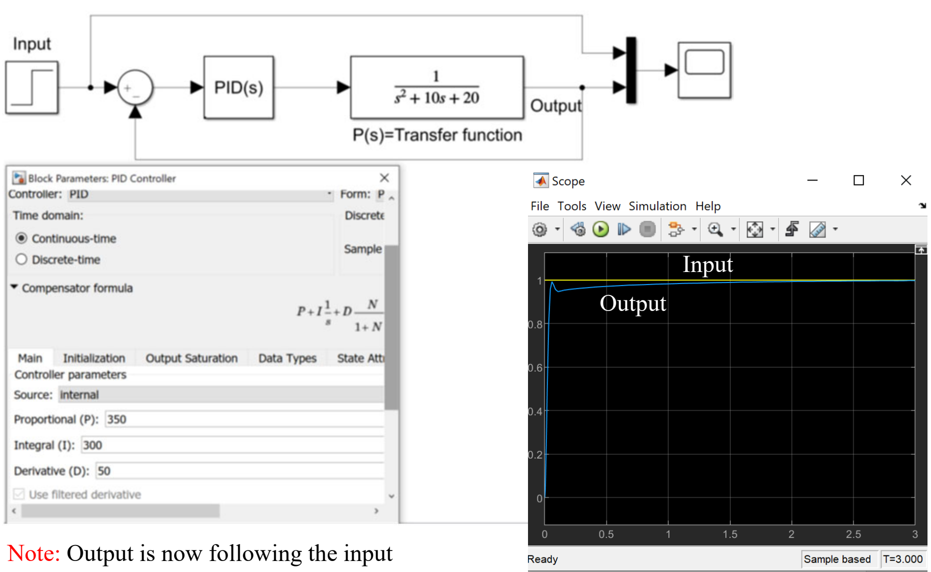
Transfer function response and PID controller:



Note: Output is now following the input

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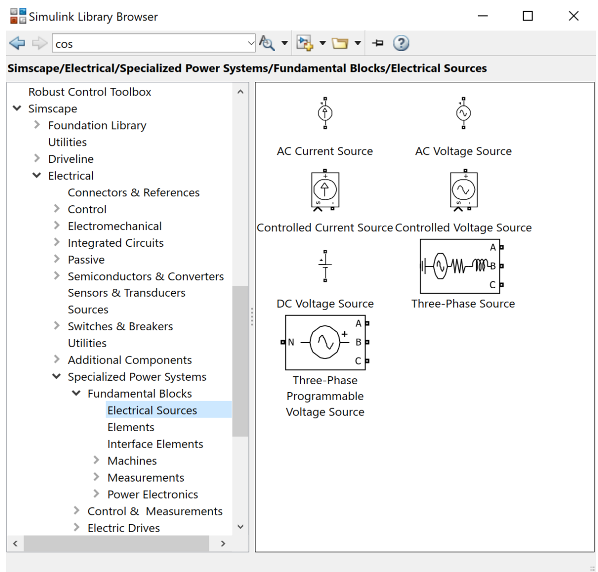
Electric circuits response:



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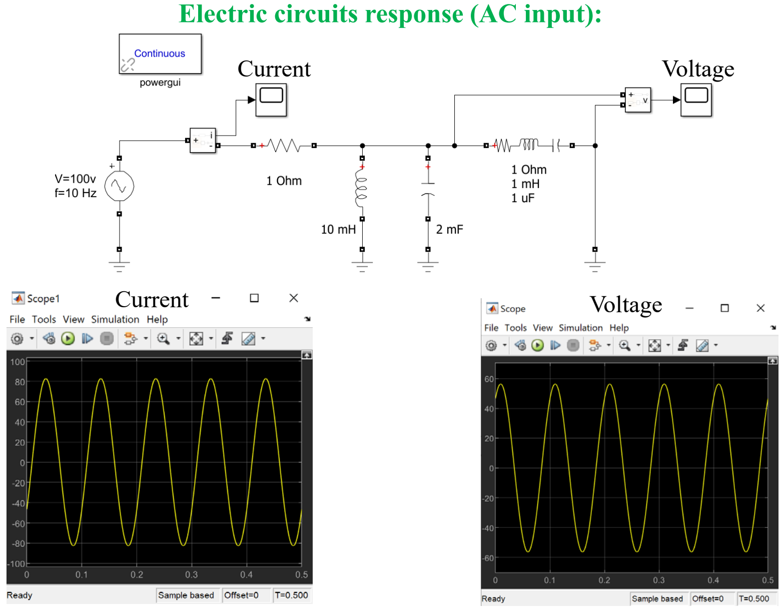
Electric circuits response (AC input):



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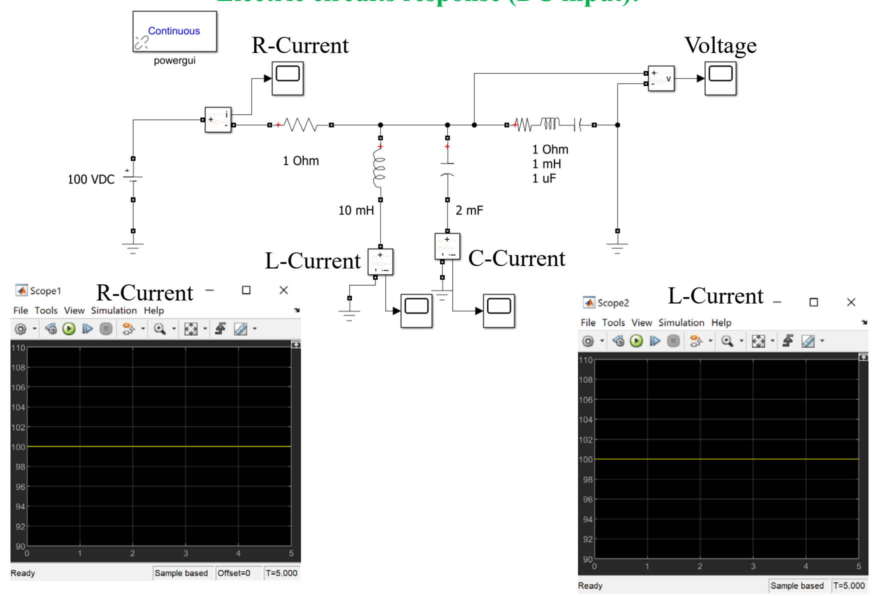
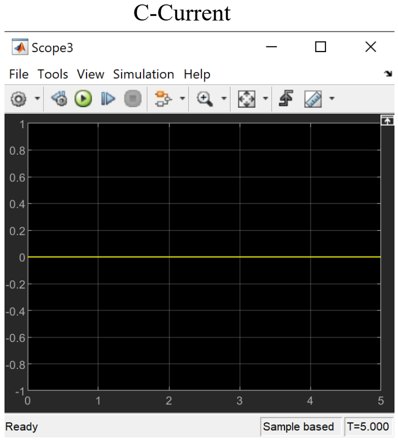
Electric circuits response (DC input):

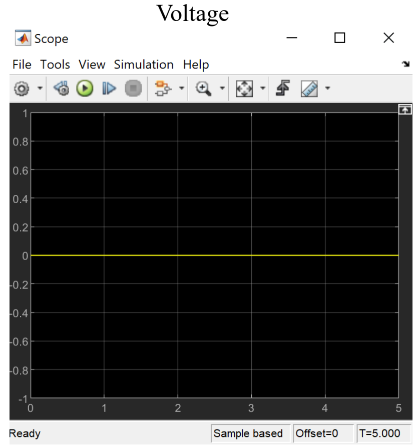


Electric circuits response (DC input):

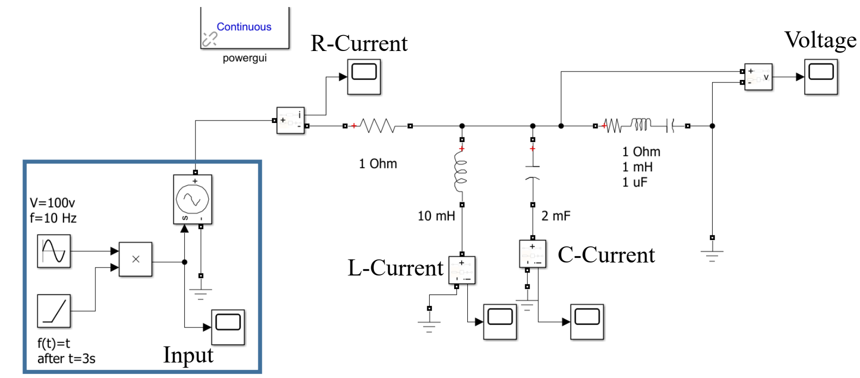
C-Current



Voltage



Electric circuits response (Non-linear input):



$V(t) = 100\sin(20\pi t) \quad 0 \leq t < 3$

Input

$V(t) = t \times 100\sin(20\pi t) \quad 3 \leq t$

Electric circuits response (Non-linear input):