

Problem 1:

- a) If the resistivity of Poly-Silicon is $10^{-4} \Omega \cdot m$ and thickness of Poly-Silicon layer is $1 \mu m$, Calculate the sheet resistance R_{\square}
- b) If the distance between two metal layers is $100 nm$ and the space between the two metals is filled with silicon oxide ($\epsilon = 3 \times 10^{-11} F/m$), Calculate the Capacitance Per Unit area

Solution

$$a) \quad R = \frac{\rho L}{A} = \frac{\rho L}{t * W} = R_{\square} * \frac{L}{W}$$

sheet resistance $\leftarrow R_{\square} = \frac{\rho}{t} = \frac{10^{-4} \Omega \cdot m}{1 \mu m} = 100 \Omega / \square$

$$b) \quad C = \frac{\epsilon A}{t}$$

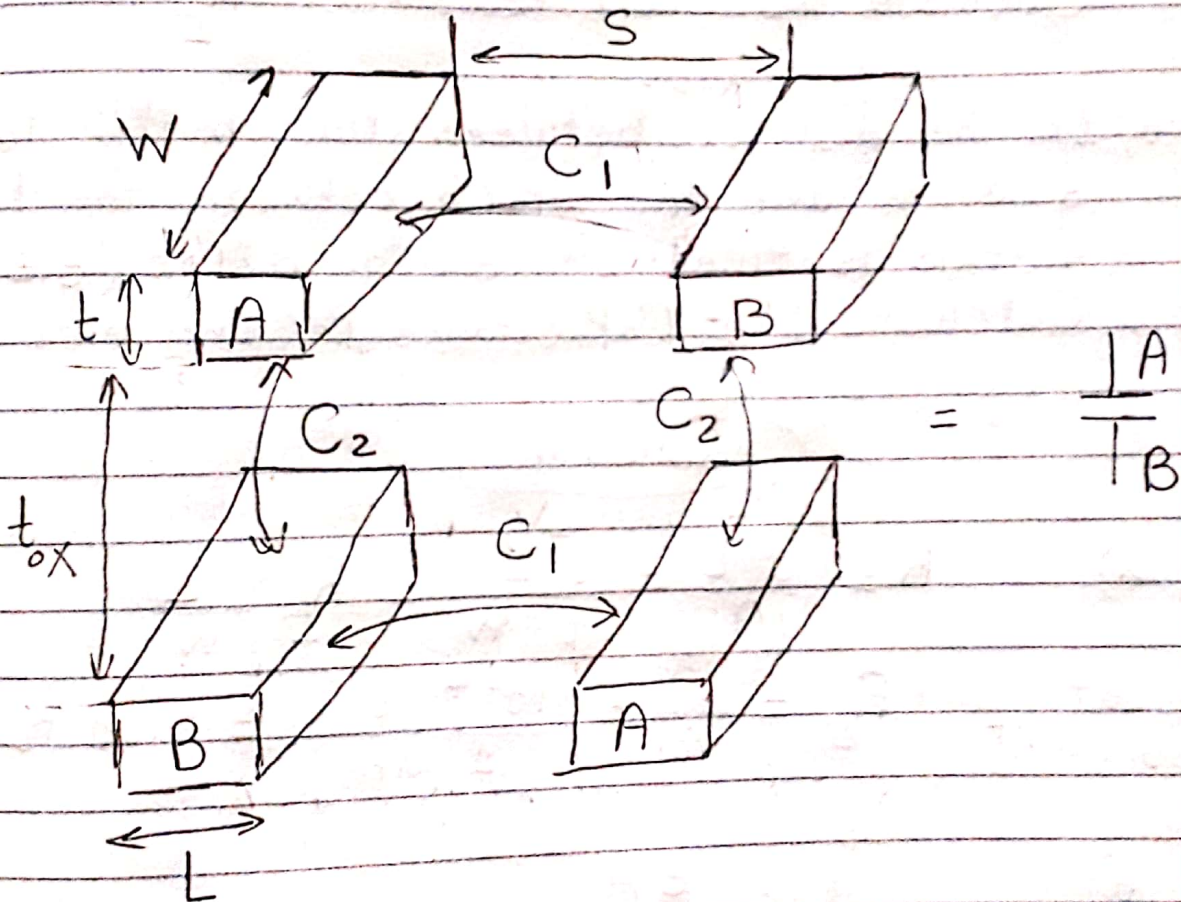
Capacitance Per Unit area $\leftarrow \frac{C}{A} = \frac{\epsilon}{t} = \frac{3 \times 10^{-11} F/m}{100 nm} = 0.3 fF / \mu m^2$

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Problem 2:

Find the Value of the Capacitor shown below
for $W = 300 \mu\text{m}$, $S = t = t_{ox} = L = 1 \mu\text{m}$, and
 $\epsilon = 3 \times 10^{-11} \text{ F/m}$



$$C_{AB} = 2C_1 + 2C_2$$

$$C_1 = \epsilon \frac{Wt}{S} \quad \& \quad C_2 = \epsilon \frac{WL}{t_{ox}}$$

$$\therefore C_{AB} = 2\epsilon \left(\frac{Wt}{S} + \frac{WL}{t_{ox}} \right)$$

$$= 4\epsilon * W = 4 * 3 * 10^{-2} \text{ fF}/\mu\text{m} * 300 \mu\text{m}$$

$$= 36 \text{ fF}$$

Problem 3:

Sketch V_{out} as a function of V_{in} for the circuit shown. With $V_{DD} = 3V$, $\mu_n C_{ox} = 50 \mu A/V^2$, $V_{th0} = 0.8V$, $2\phi_f = 0.7V$, and $\gamma = 0.4 V^{0.5}$

Solution

① $V_{out} = V_{DD} - I_{DS} R_1 = 3 - I_{DS} R_1$

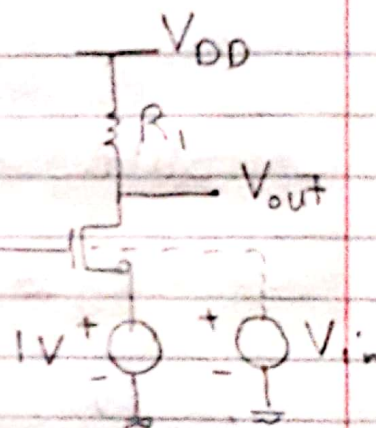
Assume that the transistor operates in sat. region

②
$$I_{DS} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th})^2$$

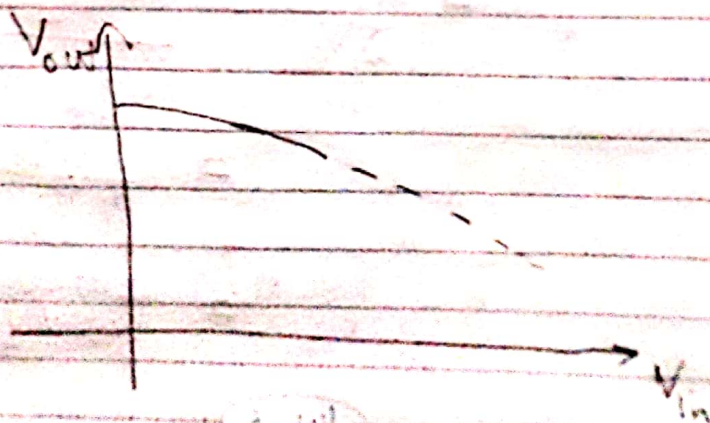
$$= \frac{1}{2} * 50 \mu A/V^2 * \frac{W}{L} (1V - V_{th})^2$$

③
$$V_{th} = V_{th0} + \gamma (\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f})$$

$$= 0.8 + 0.4 (\sqrt{0.7 + 1 - V_{in}} - \sqrt{0.7})$$



$V_{in} \uparrow$ $V_{th} \downarrow$ $I_{DS} \uparrow$ $V_{out} \downarrow$



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Problem 4:

An NMOS device operating in the sub-threshold region has $\eta = 1.5$. What variation in V_{GS} results in a tenfold change in I_D ? If $I_D = 10 \mu A$, Calculate g_m

Solution

In subthreshold region, I_D can be written as

$$I_D = I_0 \frac{W}{L} e^{\frac{(V_{GS} - V_{TH})}{\eta V_T}}$$

$$\therefore V_{GS} = V_{TH} + \eta V_T \ln \left(\frac{I_D}{I_0 \frac{W}{L}} \right)$$

$$V_{GS_1} = V_{TH} + \eta V_T \ln \left(\frac{I_{D_1}}{I_0 \frac{W}{L}} \right)$$

$$V_{GS_2} = V_{TH} + \eta V_T \ln \left(\frac{I_{D_2}}{I_0 \frac{W}{L}} \right)$$

$$\therefore V_{GS_2} - V_{GS_1} = \eta V_T \ln \left(\frac{I_{D_2}}{I_{D_1}} \right) = 1.5 \times 26 \text{ mV} \times \ln(10) = 89.8 \text{ mV}$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{I_0 \frac{W}{L} e^{\frac{(V_{GS} - V_{TH})}{\eta V_T}}}{\eta V_T}$$

$$= \frac{I_D}{\eta V_T}$$

$$= \frac{10 \mu A}{1.5 \times 26 \text{ mV}} = 0.256 \text{ mS}$$

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Problem 5:

(a) Show that the transit frequency of a Mos device is given by:

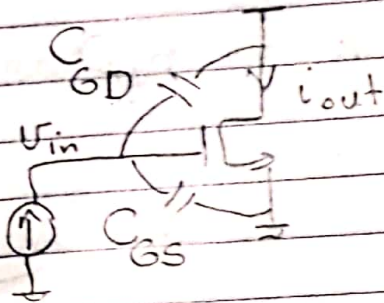
$$f_T = \frac{g_m}{2\pi(C_{GD} + C_{GS})}$$

(b) Calculate the transit frequency for a device in sub-threshold region (weak inversion)

Solution

(a)
$$i_{out} = g_m V_{in}$$

$$= g_m \frac{i_{in} \cdot 1}{s(C_{GD} + C_{GS})}$$



$$\therefore \left| \frac{i_{out}}{i_{in}} \right| = \frac{g_m}{\omega(C_{GD} + C_{GS})}$$

@ $f_T \rightarrow \left| \frac{i_{out}}{i_{in}} \right| = 1$

$$\therefore f_T = \frac{g_m}{2\pi(C_{GD} + C_{GS})}$$

(b) In sub-threshold: $g_m = \frac{I_D}{\eta V_T}$
 $C_{GD} = C_{GS} = \eta C_{ov}$

$$\therefore f_T = \frac{I_D / \eta V_T}{4\pi \eta C_{ov}} = \frac{I_D}{4\pi \eta V_T \eta C_{ov}}$$

النسبة $\propto \frac{1}{L}$

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Problem 6:

For the source follower shown, M_1 & M_2 are identical
 $\mu_n C_{ox} = 50 \mu A/V^2$, $(\frac{W}{L}) = \frac{2 \mu m}{1 \mu m}$, $V_{Th} = 0.8V$, $V_b = 2V$
 $\lambda = 0.02 V^{-1}$, find:

(a) Gain $(\frac{V_{out}}{V_{in}})$ and G_m if $\lambda = 0$ & $\lambda = 0.4$

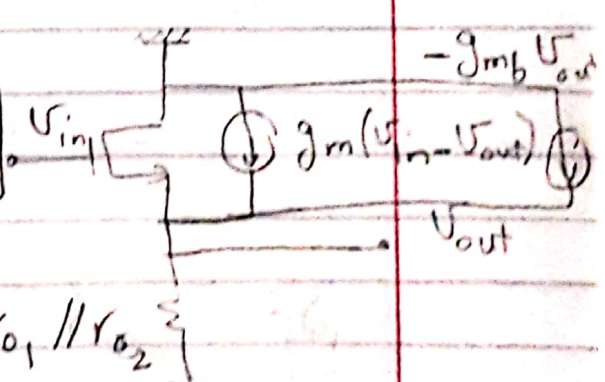
(b) R_{out} if $\lambda = 0$ & $\lambda = 0.4$

(c) Poles at the input and output. $C_{ox} = 10 fF/\mu m^2$
 and $C_{ov} = 1 fF/\mu m$

Assume $R_s = 100 \Omega$ and $C_L = 10 pF$. Consider C_{gs} and C_{gd} only and use Miller.

Solution

(a) Gain:

$$V_{out} = (r_{o1} // r_{o2}) [g_m V_{in} - V_{out} (g_m + g_{mb})]$$


$$V_{out} [1 + (g_m + g_{mb}) (r_{o1} // r_{o2})] = g_m (r_{o1} // r_{o2}) V_{in}$$

$$\therefore \text{Gain} = \frac{V_{out}}{V_{in}} = \frac{g_m (r_{o1} // r_{o2})}{1 + (g_m + g_{mb}) (r_{o1} // r_{o2})}$$

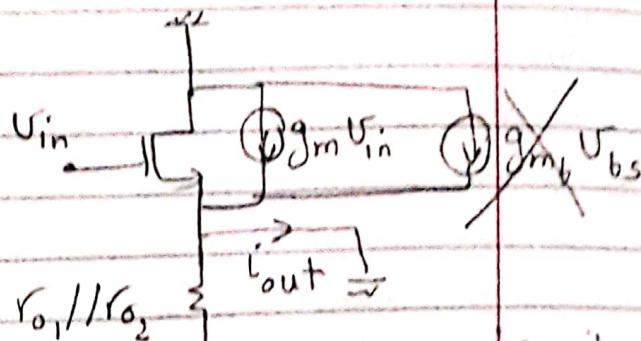
$$= \frac{g_m}{g_m + g_{mb} + \frac{1}{(r_{o1} // r_{o2})}} = \frac{g_m}{g_m + g_{mb} + g_{ds1} + g_{ds2}}$$

Gm: $i_{out} = g_m V_{in}$

$$\therefore G_m = \frac{i_{out}}{V_{in}} = g_m$$

Body-effect doesn't

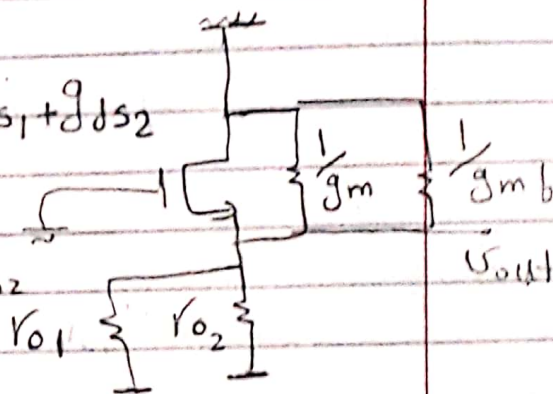
affect the overall transconductance of the circuit



(b) R_{out}:

$$G_{out} = g_m + g_{mb} + g_{ds1} + g_{ds2}$$

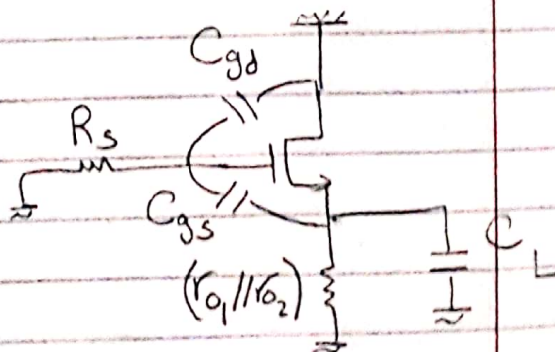
$$\therefore R_{out} = \frac{1}{g_m + g_{mb} + g_{ds1} + g_{ds2}}$$



(c)

$$W_{in} \approx \frac{1}{R_S C_{in}}$$

$$W_{out} \approx \frac{1}{R_{out} C_L}$$



$$C_{in} = C_{gd} + C_{gs} (1 - A_v)$$

$$C_{in} = C_{gd} + C_{gs} \times \frac{g_{mb} + g_{ds1} + g_{ds2}}{g_m + g_{mb} + g_{ds1} + g_{ds2}}$$