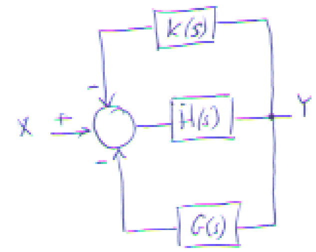
Problem 13.2

$$\frac{Y}{X}(s) = \frac{H(s)}{1 + H(s)\left[K(s) + G(s)\right]}$$



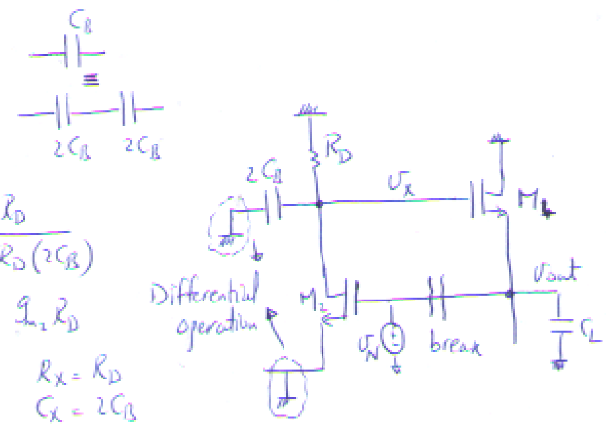
For this system to oscillate $-ve$ FB $\rightarrow$ $+ve$ FB

$1 + H(s)\left[K(s) + G(s)\right] = 0$ Let $K(s) + G(s) = N(s)$

$H(s)\,N(s) = -1$ $\quad |H(j\omega_{osc})| \cdot |N(j\omega_{osc})| \geq 1$

$\angle H(j\omega_{osc})\,N(j\omega_{osc}) = \angle H(j\omega_{osc}) + \angle N(j\omega_{osc}) = 180^\circ$

Problem 13.6



Loop Gain $= LG = \dfrac{U_{out}}{U_N} = \dfrac{U_{out}}{U_x} \cdot \dfrac{U_x}{U_N}$

$$\frac{U_x}{U_N} = -g_{m_2}\left[R_D \,/\!/\, \frac{1}{s(2C_B)}\right] = \frac{-g_{m_2} R_D}{1 + sR_D(2C_B)}$$

$$= -\frac{A_0}{1 + \frac{s}{\omega_{p_1}}}$$

$A_0 = DC$ gain $= g_{m_2} R_D$

$\omega_{p_1} = \dfrac{1}{R_x C_x}$ $\quad R_x = R_D$, $\quad C_x = 2C_B$

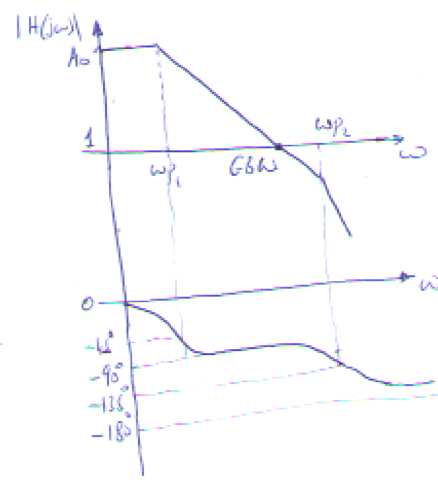
$$\frac{U_{out}}{U_x} = \frac{g_{m_4} Z_L}{1 + g_{m_4} Z_L} = \frac{g_{m_4}\frac{1}{sC_L}}{1 + g_{m_4}\frac{1}{sC_L}} = \frac{g_{m_4}}{g_{m_4} + sC_L} = \frac{1}{1 + \frac{sC_L}{g_{m_4}}}$$

$$= \frac{1}{1 + \frac{s}{\omega_{p_2}}}$$

$1 = DC$ gain (source follower)

$\omega_{p_2} = \dfrac{1}{R_{out} C_{out}}$ $\quad R_{out} = \dfrac{1}{g_{m_4}}$ $\quad C_{out} = C_L$

$$LG = -\frac{A_0}{\left(1 + \frac{s}{\omega_{p_1}}\right)\left(1 + \frac{s}{\omega_{p_2}}\right)} = -H(s)$$



For the system to oscillate

$\angle H(j\omega) = -180$ $\quad$ happens at $\omega = \infty$

$|H(j\omega)| \geq 1$ $\quad$ where $|H(j\omega)| \simeq 0$ (< 1)

Thus the system cannot oscillate

However we need to calculate & design for good PM ($\geq 60^\circ$)

$$\omega_{p_1} = \frac{1}{R_D(2C_B)} = \omega_d = \frac{1}{10K \times 1p} = 100M\,rad/sec \quad (\text{dominant pole})$$

$$\omega_{p_2} = \frac{g_{m4}}{C_L} = \omega_{nd} = \frac{10mS}{2p} = 5000M\,rad/sec \quad (\text{non-dominant pole})$$

$$GBW = A_0 \cdot \omega_d = \frac{g_{m2}}{2C_B} = \frac{1m}{1p} = 1000M\,rad/sec = 1G\,rad/sec$$

$$\underline{\lfloor H(jGBW)} = -\tan^{-1}\left(\frac{GBW}{\omega p_1}\right) - \tan^{-1}\left(\frac{GBW}{\omega p_2}\right) = -90^\circ - \tan^{-1}\left(\frac{1G}{5G}\right) = -101^\circ$$

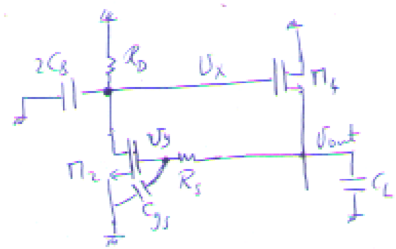
$$PM = 180 - 101 = 79^\circ$$

For $PM > 60^\circ$ $\quad \omega_{nd} > 2\,GBW \quad \frac{g_{m4}}{C_L} > 2\,Grad/sec \quad C_L < 5pF$

* In this problem, ω_d is fixed and ω_{nd} is at the output (C_L)
Thus large C_L would result in small ω_{nd} (GBW is fixed depend on C_B) and bad PM

Problem 13.7

A 3^{rd} pole is created in the loop gain or $H(s)$



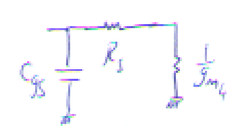
$$\omega_{p_3} \cong \frac{1}{R_s C_{gs}} = \frac{1}{10k \times 100f} = 1G\,rad/sec$$

$$\underline{\lfloor H(jGBW)} = -\tan^{-1}\left(\frac{GBW}{\omega p_1}\right) - \tan^{-1}\left(\frac{GBW}{\omega p_2}\right) - \tan^{-1}\left(\frac{GBW}{\omega p_3}\right)$$

$$= -90 - 11 - \tan^{-1}\left(\frac{1G}{1G}\right) = -146^\circ$$

$$PM = 180 - 146 = 34^\circ \quad (\text{bad PM})$$

PM reduces for increased number of poles



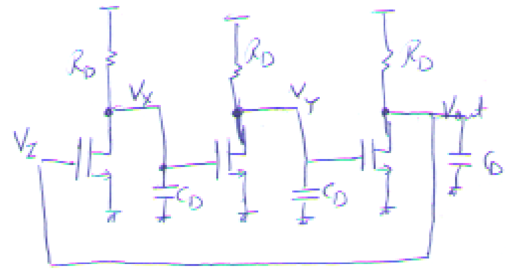
$$R_s + \frac{1}{g_{m4}} \simeq R_s$$

C_L is considered open when calculating Resistance to ⊥ of C_{gs} at node V_y

Problem 13.8 & 13.9

$A_0 = g_m R_D$

$H_{unit} = \frac{V_X}{V_Z} = -\frac{g_m R_D}{\left(1+\frac{s}{\omega_p}\right)}$ $\quad \omega_p = \frac{1}{R_D C_D}$



$H(s) = H_{unit}^3 = \left(\frac{-A_0}{1+\frac{s}{\omega_p}}\right)^3 = -\frac{A_0^3}{\left(1+\frac{s}{\omega_p}\right)^3}$

Edge of Oscillation $\quad H(s) = 1$

$\frac{-A_0^3}{(1+sR_DC_D)^3} = 1 \quad + A_0^3 + 1 + 3sR_DC_D + 3(sR_DC_D)^2 + (sR_DC_D)^3 = 0$

put $s = j\omega_{osc}$

Real $= 1 + A_0^3 - 3\omega_{osc}^2 (R_DC_D)^2 = 0 \Rightarrow A_0 = 2$ (min gain)

Imaginary $= 3\omega_{osc} R_D C_D - \omega_{osc}^3 R_D^3 C_D^3 = 0 \Rightarrow \omega_{osc} = \frac{\sqrt{3}}{R_D C_D}$

start-up Condition $\quad A_0 \geq 2 \quad g_m R_D \geq 2$

Oscillation freq $\quad \omega_{osc} = \frac{\sqrt{3}}{R_D C_D}$

$R_D * 2 \quad A_0 * 2 \quad$ (better startup) $\quad \frac{\omega_{osc}}{2}$

$C_D * 2 \quad A_0$ same $\quad$ (no change in startup) $\quad \frac{\omega_{osc}}{2}$

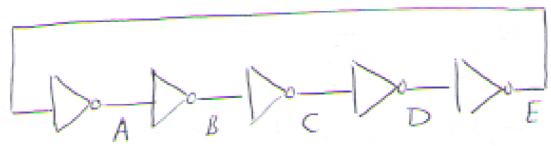
Problem 13.12 $\quad H(s) = H_{unit}^5 = -\frac{A_0^5}{\left[1+\frac{s}{\omega_0}\right]^5} \quad A_0 = g_m R_D$ & $\omega_0 = \frac{1}{R_D C_D}$

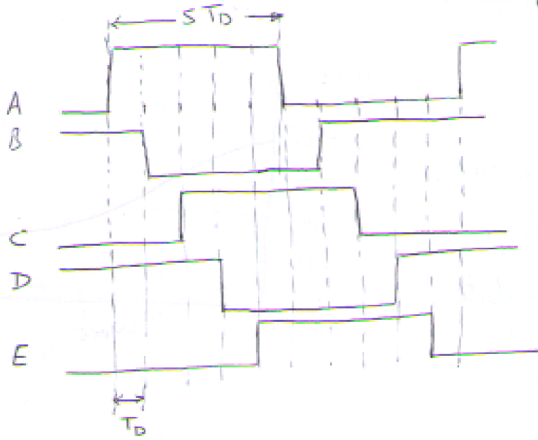
$\angle H(j\omega_{osc}) = -180 - 5\tan^{-1}\left(\frac{\omega_{osc}}{\omega_0}\right) = -360°$ (no other multiple of 360°)

$\tan^{-1}\left(\frac{\omega_{osc}}{\omega_0}\right) = \frac{180°}{5} = 36° \Rightarrow \omega_{osc} = 0.73\,\omega_0$

startup condition $\quad |H(j\omega_{osc})| \geq 1 \quad \frac{A_0^5}{\left[\sqrt{1+\left(\frac{\omega_{osc}}{\omega_0}\right)^2}\right]^5} \geq 1 \Rightarrow A_0 \geq 1.53$

Problem 13.16





Assume delay of each inverter is T_D

$T_{period} = 10\ T_D = 2 * \#stages * T_D$

$f_{osc} = \frac{1}{T_{period}}$