# Computational Physics : PH-452:
# Week 6 (Differential equations -1)

Solve the differential equation $\mathbf{dy/dx = (y^2 + 1)}$ from the interval 0 to 1.56 using Euler method, Modified and Improved Euler methods with dx =0.001. Carry out the same calculation using RK4 method with dx=0.01. At x=0, y =0 (the initial condition). Save the data in 4 different files.

The solution to the differential is y = tan(x). The actual value of y (x=1.56) = $y_A$ = 92.620. The solutions obtained using Euler, Modified Euler, Improved Euler and RK4 methods will be referred to as $y_E$, $y_{ME}$ $y_{IE}$ and $y_{RK4}$ respectively. Now answer the following questions.

Q0. Plot the solutions solved by the 4 methods. Zoom in to point out the differences in solution.

Q1. The value of the difference $y_A - y_E$ at x=1.56 is: (Marks 1)
(a) 22.248 (b) 3.248 (c) 35.248 (d) 25.811

Q2. The value of the difference $y_A - y_{ME}$ at x=1.56 is: (Marks 1)
(a) 0.538 (b) 1.077 (c) 0.877 (d) 0.13

Q3. The value of the difference $y_A - y_{IE}$ at x=1.56 is : (Marks 1)
(a) 0.177 (b) 0.367 (c) 0.73 (d) 0.402

Q4. The value of the difference $y_A - y_{RK4}$ at x=1.56 is : (Marks 1)
(a) 0.517 (b) 0.129 (c) 0.0015 (d) 0.63

Q5. If you use dx=0.001 for Euler, Modified Euler, and Improved Euler cases, but continue to use dx=0.01 for RK4: then replot the solutions and also provide zoomed-in plot to show differences between the 4 cases for relevant values of x, as you deem appropriate.

Q6. Solve the differential equation $\mathbf{d^2x / dt^2 = -sin(x)}$ with initial values $x_0$ =0.1 and $v_0$ =2.1 at time t=0 with RK4. The integration interval dt =0.01. Run the calculation for 5000 iterations, i.e. for elapsed time =50. The value of x at the end of 5000 iterations is : (Marks: 2)

(a) 2.403 (b) 63.913 (c) 41.235 (d) -81.235

Q7. For the previous problem if the initial conditions were changed to $x_0$ =0.1 and $v_0$ = 1.0, then the value of x at the end of 5000 iterations is :

(a) -8.333 (b) 5.333 (c) 0.493 (d) 1.333

Q8. Compare the solution(s) of $\mathbf{d^2x / dt^2 = -sin(x)}$ with the solution(s) of $\mathbf{d^2x / dt^2 = -x}$. Initial condition: $x_0$=0 at t=0, but choose different values of $v_0$ = 0.03, -0.1, 0.3 and show differennces appearing as you increase the value of $v_0$.
If you choose $x_0$=0 at t=0 for solving $\mathbf{d^2x / dt^2 = -sin(x)}$, but keep increasing $V_0$ , why does the nature of solution change ?