Integration: Assignment 1 **Due date: September 6, 2021**

Instructions:

- *You need to submit your codes. You will not be given any marks if codes are not submitted.*
- *Name the file containing your code such that the filename can be associated with the question number.*
- *Include your name in the filename of each of the files that you submit.*
- *All plots should be properly labelled.*

(A) Consider the following integral: (10 marks)

$$I = 4\int_0^1 \frac{1}{1+x^2}\,dx$$

The answer to this is π. In Fortran, π=2 $\mathrm{Sin}^{-1}(1)$.

1) Write a code (in double precision) to evaluate the above integral using the composite trapezoidal rule.

2) Compute the error as a function of number of grid points (n) between 0 and 1. Your code should print out in a file log(n) vs log(abs(error)). Start with n=1, increase n in multiples of 5 and compute the error for each value of n. Repeat this for n < 1000000000.

3) Plot log(n) vs log(abs(error)). What do you observe? Can you fit your plot or a part of it to some equation? If so what type of eqn.? Explain your answer in detail.

(B) Change the integrand function to sin(x) and the limits of integration between 0 and π (double precision). Choose suitable values of grid size and convince yourself that the integration scheme works. (2 marks)

(C) Now take a normalized Gaussian function $\frac{1}{\sqrt{2\pi}}e^{-x^2/2}$ with standard deviation of 1. If you analytically integrate this function from -∞ to ∞, what do you expect? In order to compute the same numerically on a computer, you need to consider finite values of the lower and upper limit of the integral. Write a program that performs the job. Integrate between (i) -3 and +3 and (ii) -5 and +5. What are the values of the integral for each case? Comment on the accuracy of the integral.

(3 marks)

(D) Using the composite trapezoidal integration rule write a program that will generate data that can be used to plot the error function (erf(x)) for -3.0 < x < 3.0. Write the explicit functional form of the error function that you are using. Show the plot. (10 marks)