Integration: Assignment 2 **Due date: September 15, 2021**

Instructions:

- *You need to submit your codes. You will not be given any marks if codes are not submitted.*
- *Name the file containing your code such that the filename can be associated with the question number.*
- *Include your name in the filename of each of the files that you submit.*
- *All plots should be properly labelled.*

1) Write a code that can generate random numbers with the following distribution function starting from a uniform distribution of random numbers lying between 0 and 1:

$$p(x) = e^{-2x}$$

(a) What will be the allowed range of *x*? (1)

(b) What is the mean and standard deviation of this distribution? Derive your answer. (4)

(c) Test your random number generator by (i) plotting a histogram (ii) computing the mean and standard deviation (iii) plotting the scatter plot and (iv) plotting the auto correlation function. Perform these tests using 100, 10000 and 1000000 random numbers. Please submit nicely labelled plots. (10)

2) Compute the following integral using (a) trapezoidal method and (b) acceptance rejection method:

$$I = \int_0^3 e^x dx$$

For (a) start with 1 grid point and go up to 100000000. Similarly, for (b) start with one random number and go up to 100000000. Increase the grid points/no. of random numbers (N) in multiples of 10. For each of the above methods compute and compare the error as a function of grid points/no. of random numbers. The error can be computed w.r.t. the analytical value of the integral. What do you observe? Explain your observation.

Compute the same integral using importance sampling Monte-Carlo integration. Compute the error as a function of N. How do the error changes? Compare with the error obtained in the previous two methods and comment on your observation. (10)

3) Write a program that computes the following multi-dimensional integral using the (a) brute force and (b) importance sampling Monte Carlo integration methods:

$$I = \int_{-\infty}^{\infty} d\vec{x} d\vec{y}\, g(\vec{x}, \vec{y}),$$

where $g(\vec{x}, \vec{y}) = exp(-\vec{x}^2 - \vec{y}^2 - 0.5(\vec{x} - \vec{y})^2)$

Suppose you do not know the exact value of the integral. How do you compute the error? Compare the efficiency of the two methods. (10)