Computational Physics : Phy 453: Semester Aug-Dec 2021
Assignment 6: Last date for submission 8th November.

ODE (4 marks) + 1d-Boundary value (2 marks) + PDE (8).
(Previous ODE assignment -assignment no.-5, had 6 marks. Total ODE+ PDE=10+10 marks)

1. Suppose you have 50 particles in a circular ring of radius 5.0 placed on the x-z plane. There is periodic boundary conditions such that the *50*-th particle has particle number *1* and *49* as its neighbours. The particles can only be displaced from the x-z plane along the y-direction. The spring constant kappa =1, and the mass of each of the particles is 1, such that kappa/m = km =1. The force acting on particle *i* in the y direction is $\mathbf{f_i^y = km *(y_{i+1} + y_{i-1} - 2\, y_i).}$

The initial conditions are (i) velocity v_i (t=0) = 0 for all the particles.
(ii) y_i (t=0) = 0 for all *i*, except for two particles, i.e. 1 and 26: $y_1\ (t=0) = y_{26}\ (t=0)$ = 0.8d0 .

Using RK4, with dt =0.02, find y_i as a function of time for all *i*. The position of the 4-th particle after 2000 iterations (i.e. at time t=40) is: **(4 marks)**

2. Solve the differential equation **y'' - 5 y' + 10y = 10x** using Gauss Seidel method and with the Boundary conditions y(x=0) =0.d0 and y (x=1.0) = 40.0d0 with dx =0.01 and convergence condition as 0.0001. Use double precision (real*8) for all real variables. Plot solution.

The boundary condition y (x=1.0d0) = 40.0 implies the value of y =40.0d0 at x= 1.0.

The value of y at x=0.78 is **(2 marks).**

Solving PDE by boundary conditions (4+4 marks): plot Temperature profiles.

Q3. Consider a 2-d plate of size 34x34. Solve the Laplaces equation on the lattice (assuming no time dependence of temperature T) to find out the temperature profile at different points on the lattice as discussed in the lectures. The boundary conditions at the edges of the plate are as follows:

At x =1, T = 3.7 for all y. At x=34, T =0.4 for all y

At y=1, T decreases linearly from 3.7 to 0.4 as x changes from 1 to 34.

At y=34, T decreases linearly from 3.7 to 0.4 as x changes from 1 to 34.

The limit of convergence is 0.0001. Plot the temperature profile, labelling axes properly.

The tempertaure at point (20,20) is :

Q4. Consider a 2-d plate of size 34x34. Solve the Laplaces equation on the lattice (assuming no time dependence of temperature T) to find out the temperature profile at different points on the lattice as discussed in the lectures. The Neumann boundary conditions at the edges of the plate are as follows: Limit of convergence= 0.00001d0

The temperature at (1,1) =2000.
A = dT/dx = -70 ; B = dT/dx = -40;
C = dT/dy = 20 ; D = dT/dy = -10

What is the temperature at (10,10):