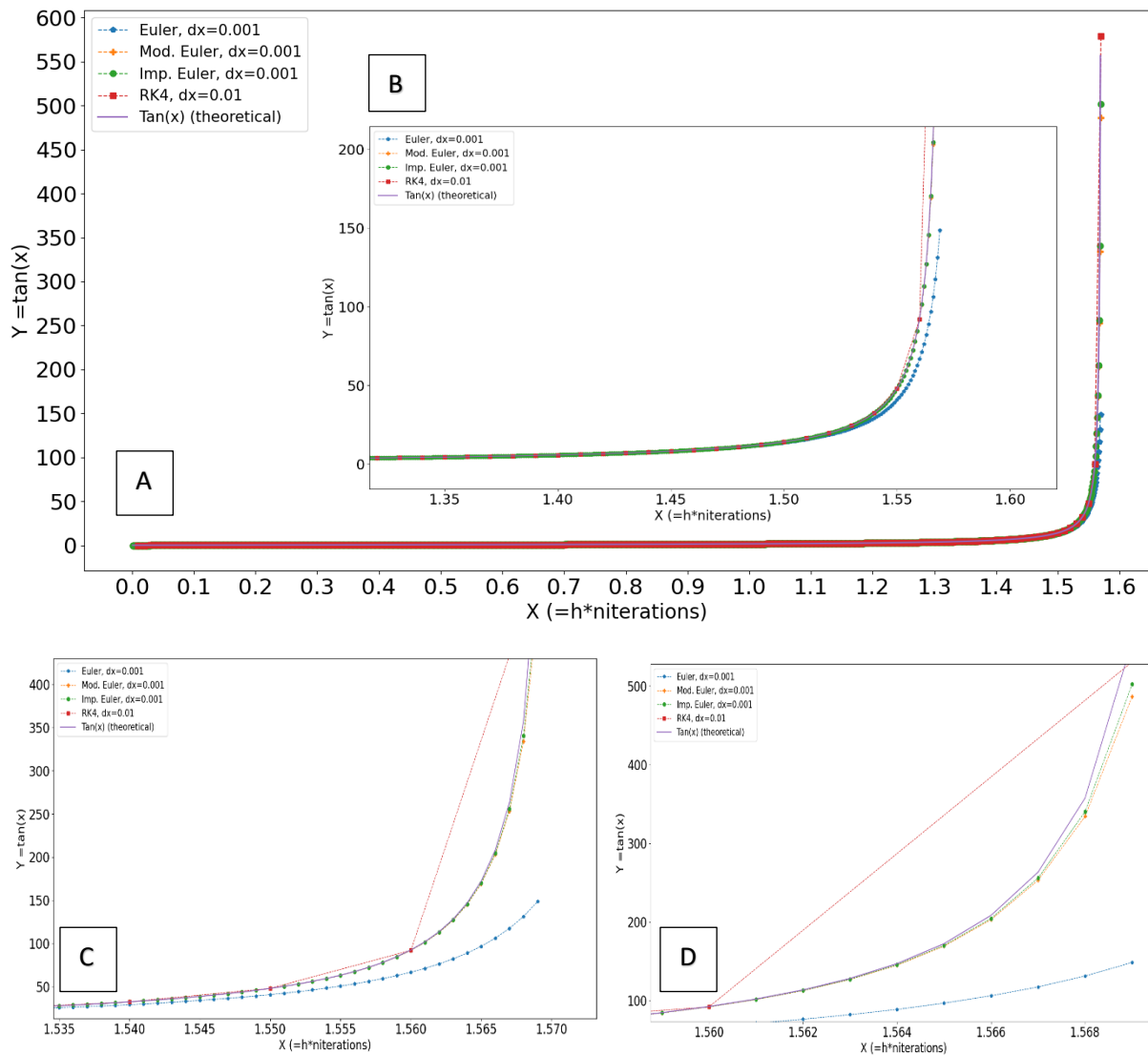


PHY453 Computational Physics | Assignment 5

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27 Oct 2021 (Differential Equations)

Q1.



- We have plotted the results for the solution of the differential equation $\frac{dy}{dx} = 1 + y^2$
- Initially in FigA, all the curves seem to superimpose in the range (0,1.4). In the zoomed-in plots (B,C,D), it is evident that the Euler method is the least accurate among the methods. Since RK4 uses a step size of 0.01 and the others use a step size of 0.001, the number of data

points from former method are lesser. From Fig.C and Fig.D, it is evident that the Improved Euler method performs slightly better than the Modified Euler Method.

- At $x_0 = 1.56$,

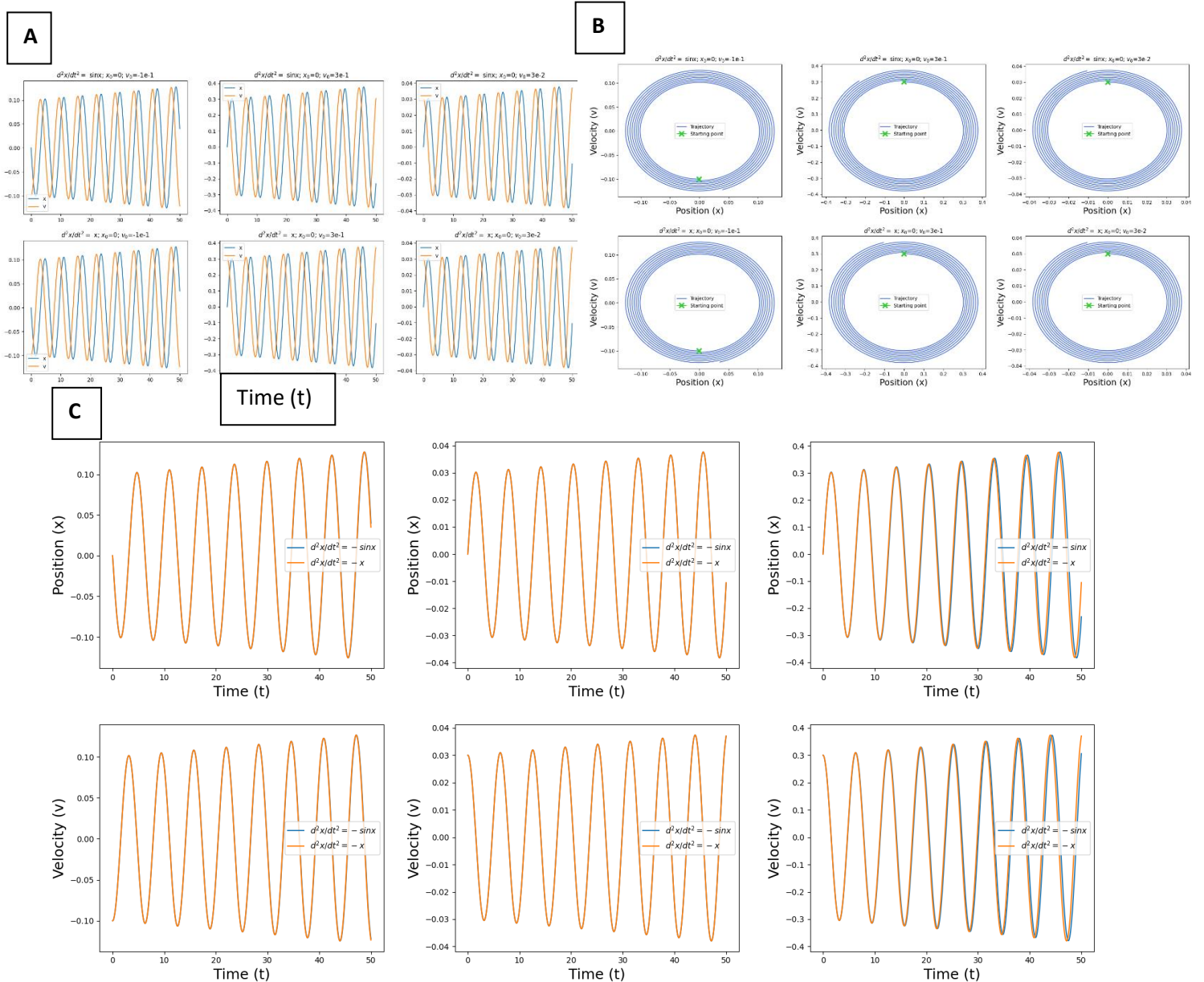
Method	$Y(x_0)$	$Y_A - Y_{x_0}$
Theoretical	$Y_A = 93.60$	0
Euler Method	$Y_e = 66.809$	$26.79 \sim 25.811$
Modified Euler Method	$Y_{me} = 92.083$	$0.517 \sim 0.538$
Improved Euler Method	$Y_{ie} = 92.253$	$0.347 \sim 0.367$
Runge-Kutta Order 4	$Y_{RK4} = 92.104$	$0.496 \sim 0.517$

So the answers to (Q1, Q2, Q3, Q4) are respectively: (D, A, B, A)

Q6. 69.936 ~ 63.913 (Option B) (code available in folder Q2)

Q7. 0.493 (Option D)

Q8



Here, we have compared the solutions of $d^2y/dt^2 = -\sin(x)$ and $d^2y/dt^2 = -x$ and plotted the resulting x and $dx/dt (=v)$ for different initial conditions using the RK4 integration method. From Fig.C in Q8, it is evident that as we increase v_0 from -0.1 to 0.03 to 0.3, the two superimposed curves of " v " for the different potentials slightly differ after 30s. This can be attributed to the fact that the potential containing only " x " considers the small angle approximation where only oscillations are allowed. However, the " $\sin(x)$ " potential considers the general case where both large oscillations and revolutions about the point of suspension are allowed. In such a potential, the x value can rapidly increase if the particle has an initial kick (high velocity v_0), but the total energy of this system is always conserved. (We have a Hamiltonian system).
