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Abstract

The abstract paragraph should be indented $\frac{1}{2}$ inch (3 picas) on both the left- and right-hand margins. Use 10 point type, with a vertical spacing (leading) of 11 points. The word **Abstract** must be centered, bold, and in point size 12. Two line spaces precede the abstract. The abstract must be limited to one paragraph.

1 Introduction

Recent advances have allowed reinforcement learning (RL) [27] to achieve impressive results in a wide variety of complex tasks, ranging from Atari [20], to the game of Go [26], to the control of sophisticated robotics systems [12, 18, 17]. The main limitation is that RL algorithms still require an enormous amount of experience samples before successfully learning such complicated tasks. One of the most promising solutions is transfer learning, which focuses on reusing past knowledge available to the agent in order to reduce the sample-complexity for learning new tasks. In the typical settings of transfer in RL [29], the agent is assumed to have already solved a set of *source tasks* generated from some unknown distribution. Then, given a *target task* drawn from the same distribution, or a slightly different one, the agent can rely on knowledge from the source tasks to speed-up the learning process. This constitutes a significant advantage over plain RL, where the agent learns each new task from scratch independently of previous learning experience. Several algorithms have been proposed in the literature to transfer experience samples [16, 28], policies/options [8, 14], rewards [13], features [3], parameters [7, 11], and so on. We refer the reader to [29] for a thorough survey on transfer in RL.

Under the assumption that tasks follow a certain distribution, an intuitive choice for designing a transfer algorithm is to attempt at characterizing the uncertainty over the target task. Then, an ideal algorithm would leverage prior knowledge from the source tasks to interact with the target task in such a way that this uncertainty is reduced as quickly as possible. This simple intuition makes Bayesian methods appealing approaches for transfer in RL, and many previous works have been proposed in this direction. [31] assume tasks share similarities in their dynamics and rewards and propose a hierarchical Bayesian model for the distribution of these two elements. Similarly, [15] assume tasks are similar in their value functions and design a different hierarchical Bayesian model for transferring such information. More recently, [7], and its extension [11], consider tasks whose dynamics are governed by some hidden parameters, and propose efficient Bayesian models for quickly learning such parameters in new tasks. However, all these algorithms require specific, and sometimes restrictive, assumptions (e.g., on the distributions involved or the function approximators adopted), which might limit their practical applicability. The importance of having transfer algorithms that alleviate the need of strong assumptions and can be easily adapted to different contexts motivates us to take a more general approach.

Similarly to [15], we assume tasks to share similarities in their value functions and use the given source tasks to learn a distribution over such functions. Then, we use this distribution as a prior for learning the target task and we propose an efficient variational approximation of the corresponding posterior. Leveraging on recent ideas from randomized value functions [21], we design a Thompson sampling-based algorithm which efficiently explores the target task by repeatedly sampling from the posterior

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and acting greedily w.r.t. (with respect to) the sampled value function. We show that our approach is very general, in the sense that it does not require any specific choice of function approximator or prior/posterior distribution models. We propose a finite-sample analysis that theoretically validates our approach, while empirically evaluating its performance on three domains with increasing level of difficulty.

2 Preliminaries

We consider a distribution \mathcal{D} over tasks, where each task is modeled as a discounted Markov decision process (MDP). We define an MDP as a tuple $\mathcal{M} = \langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, p_0, \gamma \rangle$, where \mathcal{S} is the state-space, \mathcal{A} is a finite set of actions, $\mathcal{P}(\cdot|s, a)$ is the distribution of the next state s' given that action a is taken in state s , $\mathcal{R} : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$ is the reward function, p_0 is the initial-state distribution, and $\gamma \in [0, 1)$ is the discount factor. We assume the reward function to be uniformly bounded by a constant $R_{max} > 0$. A deterministic policy $\pi : \mathcal{S} \rightarrow \mathcal{A}$ is a mapping from states to actions. At the beginning of each episode of interaction, the initial state s_0 is drawn from p_0 . Then, the agent takes the action $a_0 = \pi(s_0)$, receives a reward $\mathcal{R}(s_0, a_0)$, transitions to the next state $s_1 \sim \mathcal{P}(\cdot|s_0, a_0)$, and the process is repeated. The goal is to find the policy maximizing the long-term return over a possibly infinite horizon: $\max_{\pi} J(\pi) \triangleq \mathbb{E}[\sum_{t=0}^{\infty} \gamma^t r_t | \mathcal{M}, \pi]$. To this end, we define the optimal value function $Q^*(s, a)$ as the expected return obtained by taking action a in state s and following an optimal policy thereafter. Then, an optimal policy π^* is a policy that is greedy with respect to the optimal value function, i.e., $\pi^*(s) = \operatorname{argmax}_a Q^*(s, a)$ for all states s . It can be shown (e.g., [22]) that Q^* is the unique fixed-point of the optimal Bellman operator T defined by $TQ(s, a) = \mathcal{R}(s, a) + \gamma \mathbb{E}_{\mathcal{P}}[\max_{a'} Q(s', a')]$ for any value function Q . From now on, we adopt the term Q -function to denote any plausible value function, i.e., any function $Q : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$ uniformly bounded by $\frac{R_{max}}{1-\gamma}$.

Should we provide a more transfer-oriented definition?

We consider a parametric family of Q -functions, $\mathcal{Q} = \{Q_{\mathbf{w}} : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R} \mid \mathbf{w} \in \mathbb{R}^K\}$, and we assume each function in \mathcal{Q} to be uniformly bounded by $\frac{R_{max}}{1-\gamma}$. When learning the optimal value function, a quantity of interest is how close a given function $Q_{\mathbf{w}}$ is to the fixed-point of the Bellman operator. A possible measure is its Bellman error (or Bellman residual), defined by $B(\mathbf{w}) \triangleq TQ_{\mathbf{w}} - Q_{\mathbf{w}}$. Notice that $Q_{\mathbf{w}}$ is optimal if, and only if, $B(\mathbf{w})(s, a) = 0$ for all s, a . If we assume the existence of a distribution ν over $\mathcal{S} \times \mathcal{A}$, an appealing objective is to directly minimize the squared Bellman error of $Q_{\mathbf{w}}$ under ν , denoted by $\|B(\mathbf{w})\|_{\nu}^2$. Unfortunately, it is well-known that an unbiased estimator of this quantity requires two independent samples of the next state s' for each s, a (e.g., [19]). In practice, the Bellman error is typically replaced by the TD error $b(\mathbf{w})$, which approximates the former using a single transition sample, $b(\mathbf{w}) = r + \gamma \max_{a'} Q_{\mathbf{w}}(s', a') - Q_{\mathbf{w}}(s, a)$. Finally, given a dataset $D = \langle s_i, a_i, r_i, s'_i \rangle_{i=1}^N$ of N samples, the squared TD error is computed as $\|B(\mathbf{w})\|_D^2 = \frac{1}{N} \sum_{i=1}^N (r_i + \gamma \max_{a'} Q_{\mathbf{w}}(s'_i, a') - Q_{\mathbf{w}}(s_i, a_i))^2 = \frac{1}{N} \sum_{i=1}^N b_i(\mathbf{w})^2$. Whenever the distinction is clear from the context, with slight abuse of terminology, we refer to the squared Bellman error and squared TD error as Bellman error and TD error, respectively.

Should we define norms before?

3 Variational Transfer Learning

In this section, we describe our variational approach to transfer in RL. In Section 3.1, we start by introducing our algorithm from a high-level perspective, in such a way that it can be used for any choice of prior and posterior distributions. Then, in Sections 3.2 and 3.3, we propose practical implementations based on Gaussians and mixtures of Gaussians, respectively. We conclude with some considerations on how to optimize the proposed objective in Sec. 3.4.

3.1 Algorithm

We begin with a simple consideration: the distribution \mathcal{D} over tasks clearly induces a distribution over optimal Q -functions. Moreover, for any MDP, learning its optimal Q -function is sufficient for solving the problem. Thus, one can safely replace the distribution over tasks with the distribution over their optimal value functions. In our parametric settings, we reduce the latter to a distribution $p(\mathbf{w})$ over weights.

Should we just make the assumption that Q -functions share knowledge in their weights?

Algorithm 1 Variational Transfer

Require: Target task τ , source Q -function weights \mathcal{W}_s , batch size M

```
1: Estimate prior  $p(\mathbf{w})$  from  $\mathcal{W}_s$ 
2: Initialize variational parameters:  $\xi \leftarrow \operatorname{argmin}_{\xi} KL(q_{\xi} || p)$ 
3: Initialize replay buffer:  $D = \emptyset$ 
4: repeat
5:   Sample initial state:  $s_0 \sim p_0^{(\tau)}$ 
6:   while  $s_h$  is not terminal do
7:     Sample weights:  $\mathbf{w} \sim q_{\xi}(\mathbf{w})$ 
8:     Take action  $a_h = \operatorname{argmax}_a Q_{\mathbf{w}}(s_h, a)$ 
9:     Observe transition  $s_{h+1} \sim \mathcal{P}^{(\tau)}(\cdot | s_h, a_h)$  and collect reward  $r_h = \mathcal{R}^{(\tau)}(s_h, a_h)$ 
10:    Add sample to the replay buffer:  $D \leftarrow D \cup \langle s_h, a_h, r_h, s_{h+1} \rangle$ 
11:    Sample mini-batch  $D' = \langle s_i, a_i, r_i, s'_{i+1} \rangle_{i=1}^M$  from  $D$ 
12:    Estimate the gradient  $\nabla_{\xi} \mathcal{L}(\xi)$  using  $D'$ 
13:    Update  $\xi$  in the direction of  $-\nabla_{\xi} \mathcal{L}(\xi)$  using any stochastic optimizer (e.g., ADAM)
14:  end while
15: until forever
```

87 Assume, for the moment, that we know the distribution $p(\mathbf{w})$ and consider a dataset $D =$
88 $\{(s_i, a_i, s'_i, r_i) \mid i = 1, 2, \dots, N\}$ of samples from some task τ that we want to solve. Then, the
89 posterior distribution over weights given such dataset can be computed by applying Bayes theorem
90 as $p(\mathbf{w} | D) \propto p(D | \mathbf{w}) p(\mathbf{w})$. Unfortunately, this cannot be directly used in practice since we do not
91 have a model of the likelihood $p(D | \mathbf{w})$. In such case, it is very common to make strong assumptions
92 on the MDPs or the Q -functions so as to get tractable posteriors. However, in our transfer settings
93 all distributions involved depend on the family of tasks under consideration, and making such as-
94 sumptions is likely to limit the applicability to specific problems. Thus, we take a different approach
95 to derive a more general, but still well-grounded, solution. Recall that our final goal is move all
96 probability mass over the weights minimizing some empirical loss measure, which in our case is the
97 TD error $\|B(\mathbf{w})\|_D^2$. Then, given a prior $p(\mathbf{w})$, we know from PAC-Bayesian theory that the optimal
98 Gibbs posterior q takes the form (e.g., [6]):

$$q(\mathbf{w}) = \frac{e^{-\Lambda \|B(\mathbf{w})\|_D^2} p(\mathbf{w})}{\int e^{-\Lambda \|B(\mathbf{w}')\|_D^2} p(d\mathbf{w}')} \quad (1)$$

99 for some parameter $\Lambda > 0$. Since Λ is typically chosen to increase with the number of samples
100 N , in the remaining we set it to $\lambda^{-1} N$, for some constant $\lambda > 0$. Notice that, whenever the term
101 $e^{-\Lambda \|B(\mathbf{w})\|_D^2}$ can be interpreted as the actual likelihood of D , q becomes a classic Bayesian posterior.
102 Although we now have an appealing distribution, the integral at the denominator of (1) is intractable
103 to compute even for simple Q -function models. Thus, we propose a variational approximation q_{ξ} by
104 considering a simpler family of distributions parameterized by $\xi \in \Xi$. Then, our problem reduces to
105 finding the variational parameters ξ such that q_{ξ} minimizes the Kullback-Leibler (KL) divergence
106 w.r.t. the Gibbs posterior q . From the theory of variational inference (e.g., [4]), this can be shown to
107 be equivalent to minimizing the well-known (negative) *evidence lower bound* (ELBO):

$$\min_{\xi \in \Xi} \mathcal{L}(\xi) = \mathbb{E}_{\mathbf{w} \sim q_{\xi}} [\|B(\mathbf{w})\|_D^2] - \frac{\lambda}{N} KL(q_{\xi}(\mathbf{w}) || p(\mathbf{w})) \quad (2)$$

108 Intuitively, the approximate posterior trades-off between placing probability mass over those weights
109 \mathbf{w} that have low expected TD error (first term), and staying close to the prior distribution (second
110 term). Assuming that we are able to compute the gradients of (2) w.r.t. the variational parameters ξ ,
111 our objective can be easily optimized using any stochastic optimization algorithm.

112 We now highlight our general transfer procedure in Alg. 1, while deferring a description of specific
113 choices for the distributions involved to the next two sections. Given a set of weights \mathcal{W}_s from
114 the source tasks' optimal Q -functions, we start by estimating the prior distribution (line 1) and we
115 initialize the variational parameters by minimizing the KL divergence w.r.t. such distribution¹ (line

¹If the prior and approximate posterior were in the same family of distributions we could simply set ξ to the prior parameters. However, we are not making this assumption at this point.

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116 2). Then, at each time step of interaction, we re-sample the weights from the current approximate
 117 posterior and act greedily w.r.t. the corresponding Q -function (lines 7,8). After collecting and storing
 118 the new experience (lines 9-10), we draw a mini-batch of samples from the replay buffer (line 11), use
 119 this to estimate the objective function gradient (line 12), and, finally, update the variational parameters
 120 (line 13).

121 The key property of our approach is the weight resampling at line 7. This resembles the well-known
 122 Thompson sampling approach adopted in multi-armed bandits [5] and closely relates to the recent
 123 value function randomization [21]. In some sense, at each time we guess what is the task we are
 124 trying to solve based on our current belief and we act as if such guess were actually true. This allows
 125 an efficient adaptive exploration of the target task. Intuitively, during the first steps of interaction,
 126 the agent is very uncertain about the current task and such uncertainty induces stochasticity in the
 127 chosen actions, allowing rather informed exploration to take place. Consider, for instance, that actions
 128 that are bad on average for all tasks are very unlikely to be sampled, while this cannot happen in
 129 uninformed exploration strategies, like ϵ -greedy, before learning takes place. As the learning process
 130 goes on, the algorithm will quickly figure out which task is being solved, thus moving all probability
 131 mass over the weights minimizing the TD error. From that point, sampling from the posterior is
 132 approximately equivalent to deterministically taking such weights, and no more exploration will be
 133 performed.

134 Finally, notice the generality of the proposed approach: as far as the objective \mathcal{L} is differentiable
 135 in the variational parameters ξ , and its gradients can be efficiently computed, any approximator for
 136 the Q -function and any prior/posterior distributions can be adopted. For the latter, we describe two
 137 practical choices in the next two sections.

138 3.2 Gaussian Variational Transfer

139 We now restrict to a specific choice of the prior and posterior families that makes our algorithm
 140 very efficient and easy to implement. We assume that optimal Q -functions (or better, their weights)
 141 follow a multivariate Gaussian distribution. That is, we model the prior as $p(\mathbf{w}) = \mathcal{N}(\boldsymbol{\mu}_p, \boldsymbol{\Sigma}_p)$
 142 and we learn its parameters from the set of source weights using maximum likelihood estimation
 143 (with small regularization to make sure the covariance is positive definite). Then, our variational
 144 family is the set of all well-defined Gaussian distributions, i.e., the variational parameters are
 145 $\Xi = \{(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \mid \boldsymbol{\mu} \in \mathbb{R}^K, \boldsymbol{\Sigma} \in \mathbb{R}^{K \times K}, \boldsymbol{\Sigma} \succ 0\}$. To prevent the covariance from becoming not pos-
 146 itive definite, we consider its Cholesky decomposition $\boldsymbol{\Sigma} = \mathbf{L}\mathbf{L}^T$ and learn the lower-triangular
 147 Cholesky factor \mathbf{L} instead. In this case, deriving the gradient of the objective is very simple. Both the
 148 KL between two multivariate Gaussians and its gradients have a simple closed-form expression. The
 149 expected log-likelihood, on the other hand, can be easily differentiated by adopting the reparameteri-
 150 zation trick (e.g., [10, 23]). Although these results are well-known in the literature, we report them in
 151 App. ?? to have a more self-contained description.

152 3.3 Mixture of Gaussian Variational Transfer

153 Although the Gaussian assumption of the previous section is very appealing as it allows for a simple
 154 and efficient way of computing the variational objective and its gradients, we believe that such
 155 assumption almost never holds in practice. In fact, even for families of tasks in which the reward and
 156 transition models are Gaussian, the Q -values might be far from it. Depending on the family of tasks
 157 under consideration and, since we are learning a distribution over weights, on the chosen function
 158 approximator, the prior might have arbitrarily complex shapes. When the information loss due to
 159 the Gaussian approximation becomes too severe, the algorithm is likely to fail at capturing any
 160 similarities between the tasks. We now propose a variant to successfully solve this problem, while
 161 keeping the algorithm simple and efficient enough to be applied in practice.

162 Given the source tasks' weights \mathcal{W}_s , we model our estimated prior as a mixture of Gaussians with one
 163 equally weighted isotropic Gaussian centered at each weight: $p(\mathbf{w}) = \frac{1}{|\mathcal{W}_s|} \sum_{\mathbf{w}_s \in \mathcal{W}_s} \mathcal{N}(\mathbf{w}|\mathbf{w}_s, \sigma_p^2 \mathbf{I})$.
 164 This resembles a kernel density estimator [25] and, due to its nonparametric nature, it allows capturing
 165 arbitrarily complex distribution. Consistently with the prior, we model our approximate posterior as a
 166 mixture of Gaussians. However, we allow a different number of components (typically much less
 167 than the prior's) and we adopt full covariances instead of only diagonals, so that our posterior has the
 168 potential to match complex distributions with less components. Using C components, our posterior

169 is $q_{\xi}(\mathbf{w}) = \frac{1}{C} \sum_{i=1}^C \mathcal{N}(\mathbf{w} | \boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i)$, with variational parameters $\xi = (\boldsymbol{\mu}_1, \dots, \boldsymbol{\mu}_C, \boldsymbol{\Sigma}_1, \dots, \boldsymbol{\Sigma}_C)$.
 170 Once again, we learn Cholesky factors instead of full covariances.

171 Although this new model has the potential to capture much more complex distributions, it poses
 172 a major complication: the KL divergence between two mixture of Gaussians has no closed-form
 173 expression. To solve this issue, we rely on an upper bound to such quantity, so that negative ELBO
 174 still upper bounds the KL between the approximate and true posterior. Among the many upper bounds
 175 available in the literature, we adopt the one proposed in [9], which we report here for the sake of
 176 completeness. We refer the reader to the original paper for the proof.

177 **Theorem 1** ([9]). *Let $p = \sum_i c_i^{(p)} f_i^{(p)}$ and $q = \sum_j c_j^{(q)} f_j^{(q)}$ be two mixture of Gaussian distributions,*
 178 *where $f_i^{(p)} = \mathcal{N}(\boldsymbol{\mu}_i^{(p)}, \boldsymbol{\Sigma}_i^{(p)})$ denotes the i -th component of p , $c_i^{(p)}$ denotes its weight, and similarly*
 179 *for q . Introduce two vectors $\chi^{(1)}$ and $\chi^{(2)}$ such that $c_i^{(p)} = \sum_j \chi_{j,i}^{(2)}$ and $c_j^{(q)} = \sum_i \chi_{i,j}^{(1)}$. Then:*

$$KL(p||q) \leq KL(\chi^{(2)}||\chi^{(1)}) + \sum_{i,j} \chi_{j,i}^{(2)} KL(f_i^{(p)}||f_j^{(q)}) \quad (3)$$

180 Our new algorithm replaces the KL with the above-mentioned upper bound. Each time we require
 181 its value, we have to recompute the parameters $\chi^{(1)}$ and $\chi^{(2)}$ that tighten the bound. As shown in
 182 [9], this can be achieved by a simple fixed-point procedure. Finally, both terms in the objective are
 183 now linear combinations of functions of the variational parameters of different components, and their
 184 gradients are easily derived from the ones of the Gaussian case. We report the derivation in App. ??.

185 3.4 Optimizing the TD error

186 From Sections 3.2 and 3.3, we know that differentiating the negative ELBO \mathcal{L} w.r.t. ξ requires
 187 differentiating $\|B(\mathbf{w})\|_D^2$ w.r.t. \mathbf{w} . Unfortunately, the TD error is well-known to be non-differentiable
 188 due to the presence of the max operator. This rarely represents a problem since typical value-based
 189 algorithms are actually semi-gradient methods, i.e., they do not differentiate the targets (see, e.g.,
 190 Chapter 11 of [27]). However, our transfer settings are rather different than common RL. In fact, our
 191 algorithm is likely to start from Q -functions that are very close to an optimum, and the only thing that
 192 needs to be done is to adapt the weights in a direction of lower error (i.e., higher likelihood) so as to
 193 quickly converge to the solution of the task that is being solved. Unfortunately, this property cannot
 194 be guaranteed for most semi-gradient algorithms. Even worse, many online RL algorithms combined
 195 with complex function approximators (e.g., DQNs) are well-known to be unstable, especially when
 196 approaching an optimum, and require many tricks and tuning to work well [24, 30]. This is obviously
 197 an undesirable property in our case, as we only aim at adapting already good solutions. Thus, we
 198 consider using a residual gradient algorithm [2]. In order to differentiate the targets, we replace
 199 the optimal Bellman operator with the mellow Bellman operator introduced in [1], which adopts a
 200 softened version of max called *mellowmax*:

$$\text{mm}_a Q_{\mathbf{w}}(s, a) = \frac{1}{\kappa} \log \frac{1}{|\mathcal{A}|} \sum_a e^{\kappa Q_{\mathbf{w}}(s, a)} \quad (4)$$

201 where κ is a hyperparameter and $|\mathcal{A}|$ is the number of actions. The mellow Bellman operator, which
 202 we denote as \tilde{T} , has several appealing properties that make it suitable for our settings: (i) it converges
 203 to the maximum as $\kappa \rightarrow \infty$, (ii) it has a unique fixed point, and (iii) it is *differentiable*. Denoting
 204 by $\tilde{B}(\mathbf{w}) = \tilde{T}Q_{\mathbf{w}} - Q_{\mathbf{w}}$ the Bellman residual w.r.t. the mellow Bellman operator \tilde{T} , we have that
 205 the corresponding TD error, $\|\tilde{B}(\mathbf{w})\|_D^2$, is now differentiable with respect to \mathbf{w} . Although residual
 206 algorithms have guaranteed convergence, they are typically much slower than their semi-gradient
 207 counterpart. [2] proposed to project the gradient in a direction that achieves higher learning speed,
 208 while preserving convergence. This can be easily done by including a parameter $\psi \in [0, 1]$ in the TD
 209 error gradient such that:

$$\nabla_{\mathbf{w}} \|\tilde{B}(\mathbf{w})\|_D^2 = \frac{2}{N} \sum_{i=1}^N b_i(\mathbf{w}) \left(\gamma \psi \nabla_{\mathbf{w}} \text{mm}_{a'} Q_{\mathbf{w}}(s'_i, a') - \nabla_{\mathbf{w}} Q_{\mathbf{w}}(s_i, a_i) \right) \quad (5)$$

210 where $b_i(\mathbf{w}) = r_i + \gamma \text{mm}_{a'} Q_{\mathbf{w}}(s'_i, a') - Q_{\mathbf{w}}(s_i, a_i)$. Notice that ψ trades-off between the semi-
 211 gradient ($\psi = 0$) and the full residual gradient ($\psi = 1$). A good criterion for choosing such
 212 parameter is to start with values close to zero (to have faster learning) and move to higher values
 213 when approaching the optimum (to guarantee converge).

4 Theoretical Analysis

In this section, we theoretically analyze our variational transfer algorithm...

A first important question that we need to answer is whether replacing max with mellow-max in the Bellman operator constitutes a strong approximation or not. It has been proved [] that the mellow Bellman operator is a contraction under the L_∞ -norm and, thus, has a unique fixed-point. However, how such fixed-point differs from the one of the optimal Bellman operator remains an open question. Since mellow-max monotonically converges to max as $\kappa \rightarrow \infty$, it would be desirable if the corresponding operator also monotonically converged to the optimal one. We confirm that this property actually holds in the following theorem.

Cite MM

Theorem 2. *Let V be the fixed-point of the optimal Bellman operator T , and Q the corresponding action-value function. Define the action-gap function $g(s)$ as the difference between the value of the best action and the second best action at each state s . Let \tilde{V} be the fixed-point of the mellow Bellman operator \tilde{T} with parameter $\kappa > 0$ and denote by $\beta > 0$ the inverse temperature of the induced Boltzmann distribution (as in []). Let ν be a probability measure over the state-space. Then, for any $p \geq 1$:*

Cite MM

$$\|V - \tilde{V}\|_{\nu,p}^p \leq \frac{2R_{max}}{(1-\gamma)^2} \left\| 1 - \frac{1}{1 + |\mathcal{A}| e^{-\beta g}} \right\|_{\nu,p}^p \quad (6)$$

Theorem 3. *Let Q^* be the fixed-point of the optimal Bellman operator T . Define the action-gap function $g(s)$ as the difference between the value of the best action and the second best action at each state s . Let \tilde{Q} be the fixed-point of the mellow Bellman operator \tilde{T} with parameter $\kappa > 0$ and denote by $\beta_\kappa > 0$ the inverse temperature of the induced Boltzmann distribution (as in []). Let ν be a probability measure over the state-action space. Then, for any $p \geq 1$:*

Cite MM

$$\|Q^* - \tilde{Q}\|_{\nu,p}^p \leq \frac{2\gamma R_{max}}{(1-\gamma)^2} \left\| \frac{1}{1 + \frac{1}{|\mathcal{A}|} e^{\beta_\kappa g}} \right\|_{\nu,p}^p \quad (7)$$

5 Related Works

Our approach is mostly related to []. Although we both assume the tasks to share similarities in their value functions, [] consider only linear approximators and adopt a hierarchical Bayesian model of the corresponding weights' distribution, which is assumed Gaussian. On the other hand, our variational approximation allows for more general distribution families and can be combined with non-linear approximators. Furthermore, [] propose a Dirichlet process model for the case where weights cluster into different classes, which relates to our mixture formulation of Sec. 3.3 and proves again the importance of capturing more complicated task distributions. Another related approach is [], where the authors propose a hierarchical Bayesian model for the distribution over tasks. Differently from our approach and [], they consider a distribution over transition probabilities and rewards, rather than value functions. In the same spirit of our method, they consider a Thompson sampling-based procedure which, at each iteration, samples a new task from the posterior and solves it, thus providing efficient exploration via the transferred knowledge. However, [] consider only finite MDPs, which poses a severe limitation in the algorithm's applicability. On the contrary, our approach can handle high dimensional tasks by allowing powerful function approximators.

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The idea of exploration via value function randomization we build upon was first proposed in the RL community by []. The authors extend the well-known LSTD [] by adopting Bayesian linear regression to model the uncertainty over the predicted value function weights, and use that to perform a Thompson sampling-like procedure. Such approach was recently extended by [], where the approximator is replaced by a Bayesian neural network, leading to an algorithm capable of solving much more complicated problems. Both these algorithms rely on the Gaussian assumption and, since they work in plain RL settings, have no informative prior available. On the other hand, our variational approximation allows more complex distributions (e.g., mixtures) to be adopted, while knowledge from the source tasks allows us to learn very informative priors. Finally, the variational approximation technique we consider in this work has been previously adopted in [] to approximate

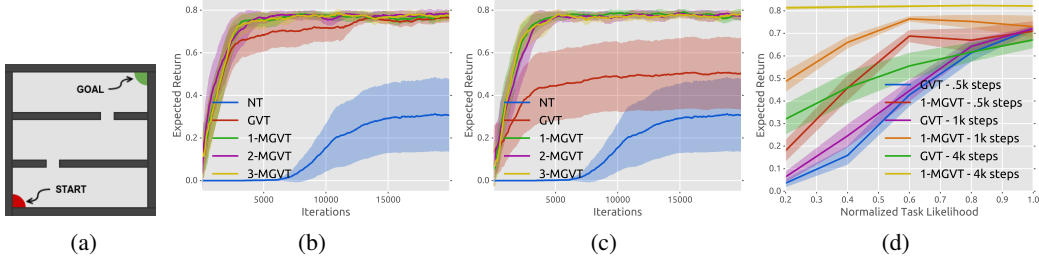


Figure 1: (a) the rooms environment, (b) transfer from 10 source tasks with both doors moving, (c) transfer from 10 source tasks with only one door moving, and (d) transfer performance as a function of how likely the target task is according to the prior.

an intractable posterior over the transition dynamics, which is then used to drive efficient exploration via an information-maximizing procedure.

6 Experiments

In this section, we provide an experimental evaluation of our approach. We begin by analyzing the behavior of our algorithm on a toy problem in Sec. 6.1. Then, in Sec. 6.2, we consider two classic control benchmarks, namely cartpole and mountain car. We conclude with a complex maze navigation task in Sec. 6.3. In all experiments, we compare our Gaussian variational transfer algorithm (GVT) and the version using mixture of Gaussians with c components (c -MGVT) to plain no-transfer RL (NT) with ϵ -greedy exploration. To the best of our knowledge, no existing transfer algorithm is directly comparable to our approach from an experimental perspective.

Any better motivation?

6.1 The Rooms Problem

We consider an agent navigating in the rooms environment depicted in Fig. ?? . The agent starts in the bottom-left corner and must move from one room to another to reach the goal position in the top-right corner. The rooms are connected by small doors whose positions are unknown to the agent. The state-space is modeled as a 10×10 continuous grid, while the action-space is the set of 4 movement directions (up, right, down, left). After each action, the agent moves by 1 in the chosen direction and the final position is corrupted by Gaussian noise with 0.2 standard deviation. In case the agent hits a wall, its position remains unchanged. The reward is 1 when reaching the goal (after which the process terminates) and 0 otherwise, while the discount factor is $\gamma = 0.99$. We consider a distribution over tasks in which doors have a fixed width of 1, while their positions are sampled uniformly in $[0.5, 9.5]$. Since the agent does not know where the doors are located in advance and receives only very sparse feedback, it must efficiently explore the environment to figure out (i) their positions, and (ii) how to reach the goal. While this might be a complicated problem for plain RL, our transfer algorithm should be able to quickly figure out the door positions. In fact, notice that, although different, the optimal Q -functions for all tasks share some similarities. For example, once the agent has passed the last door before the goal, the Q -values are exactly the same in all tasks. This does not hold for positions nearby the start state. However, it is clear that there should be a preference over actions up and right, rather than down and left (which are worse in all tasks). Thus, we expect our algorithm to efficiently explore any target task.

In order to prove that our guesses are correct, we generate a set of 50 source tasks for the three-room environment of Fig. ?? by sampling both door positions uniformly, and solve all of them by directly minimizing the TD error as presented in Sec. 3.4. In order to make sure that their solutions are accurate enough, we allow sampling the initial state uniformly in the environment and run until converge. Then, we use our algorithms to transfer from 10 source tasks sampled from the previously generated set. The average return over the last 50 learning episodes as a function of the number of iterations is shown in Fig. ?? . Each curve is the result of 20 independent runs, each resampling the target and source tasks. 95% confidence intervals are shown. Further details on the parameters adopted in this experiment are given in App. ?? . As expected, the no-transfer algorithm NT fails at learning the task in so few iterations due to the limited exploration provided by an ϵ -greedy policy.

On the other hand, all our algorithms achieve a significant speed-up and are able to converge to the optimal performance in few iterations, with GVT being slightly slower. Interestingly, we notice that there is no advantage in adopting more than 1 component for the posterior in MGVT. This is intuitive since, when the algorithm quickly figures out which task is being solved, it will move all components in the same direction.

To better understand the differences between GVT and MGVT, we now consider transferring from a slightly different distribution than the one from which target tasks are drawn. We generate again 50 source tasks but this time with the bottom door fixed in the center and the other one moving. Then, we repeat the previous experiment, allowing both doors to move when sampling target tasks. The results are shown in Fig. ???. Interestingly, MGVT seems almost unaffected by this change, proving that it has sufficient representation power to generalize to slightly different task distributions. The same does not hold for GVT, which now is not able to solve many of the sampled target tasks (this is the reason for the very high variance). This proves again that assuming Gaussian distributions can pose severe limitations in our transfer settings.

Finally, we analyze the transfer performance as a function of how likely the target task is according to the prior. We consider a two-room version of the environment of Fig. ???. Differently from before, we generate tasks by sampling the door position from a Gaussian with mean 5, and standard deviation 1.8, so that tasks where the door is close to the sides are very unlikely. Fig. ?? shows the performance reached by GVT and MGVT with 1 component at fixed iterations as a function of how likely the target task is according to such distribution. As expected GVT achieves poor performance on very unlikely tasks, even after many iterations. In fact, estimating a single Gaussian distribution definitely implies some information loss, especially about the unlikely tasks. On the other hand, MGVT keeps such information and, consequently, performs much better. Perhaps not surprisingly, MGVT reaches the optimal performance in $4k$ iterations no matter what task is being solved.

6.2 Classic Control

6.3 Maze Navigation

For this setting we propose a maze environment, such as those in Fig. *, for a robotic agent to navigate. We created a set of 20 mazes of size 10×10 with continuous space within which we ensured four groups: each group with the goal position in one of the corners of the maze. In order to simulate a robotic agent, we set an action space that allows to control the linear and angular motion by a fixed distance/angle each time an action is performed: *move forward*, *rotate counter-* and *clockwise*. Moreover, we enhanced the state space to include, in addition to the absolute position and orientation, a set of distance measures to the nearest obstacles from the agent’s perspective with maximum range of 2 and a field of vision from -90° to 90° relative to its orientation. In this way, we provide the agent with information that a robot would normally collect from LIDARs, sonars and other sensors alike. Finally, in the same field of perception, we provide a boolean vector indicating whether the goal is within the range of observation. Regarding rewards, the agent gets 1 whenever it enters the goal area and 0 in any other situation.

The main motivation for this set-up is that a robot is able to exploit locally-sensed information in addition to its estimation of the actual position—as opposed to the Rooms problem presented above—and it is such information that the robotic agent could learn to use and, also, exploit in different maze configurations leveraging its previous experience. Moreover, simple variations in the maze structure could, potentially, produce big differences in the value function with respect to the ones seen in previous tasks which makes the setting very good to assess how robust our algorithm is robust to negative transfer.

These mazes were solved using Deep Q Networks (DQN) [1] with a multilayer perceptron with hidden layers 32×32 and, in order to perform the knowledge transfer, the target maze would be excluded from the source tasks given to the algorithm. Finally, the MGVT algorithm, with one component for the posterior distribution, was used to solve the target tasks shown in Fig. *.

cite DQN?

7 Conclusion

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423 A Proofs

424 **Theorem 2.** *Let V be the fixed-point of the optimal Bellman operator T , and Q the corresponding*
 425 *action-value function. Define the action-gap function $g(s)$ as the difference between the value of*
 426 *the best action and the second best action at each state s . Let \tilde{V} be the fixed-point of the mellow*
 427 *Bellman operator \tilde{T} with parameter $\kappa > 0$ and denote by $\beta > 0$ the inverse temperature of the*
 428 *induced Boltzmann distribution (as in []). Let ν be a probability measure over the state-space. Then,*
 429 *for any $p \geq 1$:*

Cite MM

$$\|V - \tilde{V}\|_{\nu,p}^p \leq \frac{2R_{max}}{(1-\gamma)^2} \left\| 1 - \frac{1}{1 + |\mathcal{A}| e^{-\beta g}} \right\|_{\nu,p}^p \quad (6)$$

430 *Proof.* We begin by noticing that:

$$\begin{aligned} \|V - \tilde{V}\|_{\nu,p}^p &= \|TV - \tilde{T}\tilde{V}\|_{\nu,p}^p \\ &= \|TV - \tilde{T}V + \tilde{T}V - \tilde{T}\tilde{V}\|_{\nu,p}^p \\ &\leq \|TV - \tilde{T}V\|_{\nu,p}^p + \|\tilde{T}V - \tilde{T}\tilde{V}\|_{\nu,p}^p \\ &\leq \|TV - \tilde{T}V\|_{\nu,p}^p + \gamma \|V - \tilde{V}\|_{\nu,p}^p \end{aligned}$$

431 where the first inequality follows from Minkowsky's inequality and the second one from the contrac-
 432 tion property of the mellow Bellman operator. This implies that:

$$\|V - \tilde{V}\|_{\nu,p}^p \leq \frac{1}{1-\gamma} \|TV - \tilde{T}V\|_{\nu,p}^p \quad (8)$$

433 Let us bound the norm on the right-hand side separately. In order to do that, we will bound the
 434 function $|TV(s) - \tilde{T}V(s)|$ point-wisely for any state s . By applying the definition of the optimal
 435 and mellow Bellman operators, we obtain:

$$\begin{aligned} |TV(s) - \tilde{T}V(s)| &= \left| \max_a \{R(s, a) + \gamma \mathbb{E}[V(s')]\} - \min_a \{R(s, a) + \gamma \mathbb{E}[V(s')]\} \right| \\ &= \left| \max_a Q(s, a) - \min_a Q(s, a) \right| \end{aligned}$$

436 Recall that applying the mellow-max is equivalent to computing an expectation under a Boltzmann
 437 distribution with inverse temperature β induced by Q []. Thus, we can write:

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$$\begin{aligned} \left| \max_a Q(s, a) - \min_a Q(s, a) \right| &= \left| \sum_a \pi^*(a|s) Q(s, a) - \sum_a \pi_\beta(a|s) Q(s, a) \right| \\ &= \left| \sum_a Q(s, a) (\pi^*(a|s) - \pi_\beta(a|s)) \right| \\ &\leq \sum_a |Q(s, a)| |\pi^*(a|s) - \pi_\beta(a|s)| \\ &\leq \frac{R_{max}}{1-\gamma} \sum_a |\pi^*(a|s) - \pi_\beta(a|s)| \end{aligned} \quad (9)$$

438 where π^* is the optimal (deterministic) policy w.r.t. Q and π_β is the Boltzmann distribution induced
 439 by Q with inverse temperature β :

$$\pi_\beta(a|s) = \frac{e^{\beta Q(s,a)}}{\sum_{a'} e^{\beta Q(s,a')}}$$

440 Denote by $a_1(s)$ the optimal action for state s under Q . We can then write:

$$\begin{aligned}
\sum_a |\pi^*(a|s) - \pi_\beta(a|s)| &= |\pi^*(a_1(s)|s) - \pi_\beta(a_1(s)|s)| + \sum_{a \neq a_1(s)} |\pi^*(a|s) - \pi_\beta(a|s)| \\
&= |1 - \pi_\beta(a_1(s)|s)| + \sum_{a \neq a_1(s)} |\pi_\beta(a|s)| \\
&= 2 |1 - \pi_\beta(a_1(s)|s)|
\end{aligned} \tag{10}$$

441 Finally, let us bound this last term:

$$\begin{aligned}
|1 - \pi_\beta(a_1(s)|s)| &= \left| 1 - \frac{e^{\beta Q(s, a_1(s))}}{\sum_{a'} e^{\beta Q(s, a')}} \right| \\
&= \left| 1 - \frac{e^{\beta(Q(s, a_1(s)) - Q(s, a_2(s)))}}{\sum_{a'} e^{\beta(Q(s, a') - Q(s, a_2(s)))}} \right| \\
&= \left| 1 - \frac{e^{\beta g(s)}}{\sum_{a'} e^{\beta(Q(s, a') - Q(s, a_2(s)))}} \right| \\
&= \left| 1 - \frac{e^{\beta g(s)}}{e^{\beta g(s)} + \sum_{a' \neq a_1(s)} e^{\beta(Q(s, a') - Q(s, a_2(s)))}} \right| \\
&\leq \left| 1 - \frac{e^{\beta g(s)}}{e^{\beta g(s)} + |\mathcal{A}|} \right| \\
&= \left| 1 - \frac{1}{1 + |\mathcal{A}| e^{-\beta g(s)}} \right|
\end{aligned} \tag{11}$$

442 Combining Eq. (14), (15), and (16), we obtain:

$$\left| \max_a Q(s, a) - \min_a Q(s, a) \right| \leq \frac{2R_{max}}{1 - \gamma} \left| 1 - \frac{1}{1 + |\mathcal{A}| e^{-\beta g(s)}} \right|$$

443 Taking the norm and plugging this into Eq. (12) concludes the proof. \square

444 **Theorem 3.** Let Q^* be the fixed-point of the optimal Bellman operator T . Define the action-gap
445 function $g(s)$ as the difference between the value of the best action and the second best action at
446 each state s . Let \tilde{Q} be the fixed-point of the mellow Bellman operator \tilde{T} with parameter $\kappa > 0$ and
447 denote by $\beta_\kappa > 0$ the inverse temperature of the induced Boltzmann distribution (as in []). Let ν be a
448 probability measure over the state-action space. Then, for any $p \geq 1$:

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$$\left\| Q^* - \tilde{Q} \right\|_{\nu, p}^p \leq \frac{2\gamma R_{max}}{(1 - \gamma)^2} \left\| \frac{1}{1 + \frac{1}{|\mathcal{A}|} e^{\beta_\kappa g}} \right\|_{\nu, p}^p \tag{7}$$

449 *Proof.* We begin by noticing that:

$$\begin{aligned}
\left\| Q^* - \tilde{Q} \right\|_{\nu, p}^p &= \left\| TQ^* - \tilde{T}\tilde{Q} \right\|_{\nu, p}^p \\
&= \left\| TQ^* - \tilde{T}Q^* + \tilde{T}Q^* - \tilde{T}\tilde{Q} \right\|_{\nu, p}^p \\
&\leq \left\| TQ^* - \tilde{T}Q^* \right\|_{\nu, p}^p + \left\| \tilde{T}Q^* - \tilde{T}\tilde{Q} \right\|_{\nu, p}^p \\
&\leq \left\| TQ^* - \tilde{T}Q^* \right\|_{\nu, p}^p + \gamma \left\| Q^* - \tilde{Q} \right\|_{\nu, p}^p
\end{aligned}$$

450 where the first inequality follows from Minkowsky's inequality and the second one from the contrac-
451 tion property of the mellow Bellman operator. This implies that:

$$\left\| Q^* - \tilde{Q} \right\|_{\nu, p}^p \leq \frac{1}{1 - \gamma} \left\| TQ^* - \tilde{T}Q^* \right\|_{\nu, p}^p \tag{12}$$

452 Let us bound the norm on the right-hand side separately. In order to do that, we will bound the
 453 function $|TQ^*(s, a) - \tilde{T}Q^*(s, a)|$ point-wisely for any state s, a . By applying the definition of the
 454 optimal and mellow Bellman operators, we obtain:

$$\begin{aligned} |TQ^*(s, a) - \tilde{T}Q^*(s, a)| &= |R(s, a) + \gamma \mathbb{E} \left[\max_{a'} Q^*(s', a') \right] - R(s, a) - \gamma \mathbb{E} \left[\text{mm}_{a'} Q^*(s', a') \right]| \\ &= \gamma \left| \mathbb{E} \left[\max_{a'} Q^*(s', a') \right] - \mathbb{E} \left[\text{mm}_{a'} Q^*(s', a') \right] \right| \\ &\leq \gamma \mathbb{E} \left[\left| \max_{a'} Q^*(s', a') - \text{mm}_{a'} Q^*(s', a') \right| \right] \end{aligned} \quad (13)$$

455 Thus, bounding this quantity reduces to bounding $|\max_a Q^*(s, a) - \text{mm}_a Q^*(s, a)|$ point-wisely
 456 for any s . Recall that applying the mellow-max is equivalent to computing an expectation under a
 457 Boltzmann distribution with inverse temperature β_κ induced by κ []. Thus, we can write:

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$$\begin{aligned} \left| \max_a Q^*(s, a) - \text{mm}_a Q^*(s, a) \right| &= \left| \sum_a \pi^*(a|s) Q^*(s, a) - \sum_a \pi_{\beta_\kappa}(a|s) Q^*(s, a) \right| \\ &= \left| \sum_a Q^*(s, a) (\pi^*(a|s) - \pi_{\beta_\kappa}(a|s)) \right| \\ &\leq \sum_a |Q^*(s, a)| |\pi^*(a|s) - \pi_{\beta_\kappa}(a|s)| \\ &\leq \frac{R_{max}}{1 - \gamma} \sum_a |\pi^*(a|s) - \pi_{\beta_\kappa}(a|s)| \end{aligned} \quad (14)$$

458 where π^* is the optimal (deterministic) policy w.r.t. Q^* and π_{β_κ} is the Boltzmann distribution induced
 459 by Q^* with inverse temperature β_κ :

$$\pi_{\beta_\kappa}(a|s) = \frac{e^{\beta_\kappa Q^*(s, a)}}{\sum_{a'} e^{\beta_\kappa Q^*(s, a')}}$$

460 Denote by $a_1(s)$ the optimal action for state s under Q^* . We can then write:

$$\begin{aligned} \sum_a |\pi^*(a|s) - \pi_{\beta_\kappa}(a|s)| &= |\pi^*(a_1(s)|s) - \pi_{\beta_\kappa}(a_1(s)|s)| + \sum_{a \neq a_1(s)} |\pi^*(a|s) - \pi_{\beta_\kappa}(a|s)| \\ &= |1 - \pi_{\beta_\kappa}(a_1(s)|s)| + \sum_{a \neq a_1(s)} |\pi_{\beta_\kappa}(a|s)| \\ &= 2 |1 - \pi_{\beta_\kappa}(a_1(s)|s)| \end{aligned} \quad (15)$$

461 Finally, let us bound this last term:

$$\begin{aligned} |1 - \pi_{\beta_\kappa}(a_1(s)|s)| &= \left| 1 - \frac{e^{\beta_\kappa Q^*(s, a_1(s))}}{\sum_{a'} e^{\beta_\kappa Q^*(s, a')}} \right| \\ &= \left| 1 - \frac{e^{\beta_\kappa (Q^*(s, a_1(s)) - Q^*(s, a_2(s)))}}{\sum_{a'} e^{\beta_\kappa (Q^*(s, a') - Q^*(s, a_2(s)))}} \right| \\ &= \left| 1 - \frac{e^{\beta_\kappa g(s)}}{\sum_{a'} e^{\beta_\kappa (Q^*(s, a') - Q^*(s, a_2(s)))}} \right| \\ &= \left| 1 - \frac{e^{\beta_\kappa g(s)}}{e^{\beta_\kappa g(s)} + \sum_{a' \neq a_1(s)} e^{\beta_\kappa (Q^*(s, a') - Q^*(s, a_2(s)))}} \right| \\ &\leq \left| 1 - \frac{e^{\beta_\kappa g(s)}}{e^{\beta_\kappa g(s)} + |\mathcal{A}|} \right| \\ &= \left| \frac{1}{1 + \frac{1}{|\mathcal{A}|} e^{\beta_\kappa g(s)}} \right| \end{aligned} \quad (16)$$

Combining Eq. (14), (15), and (16), we obtain:

$$\left| \max_a Q(s, a) - \min_a Q(s, a) \right| \leq \frac{2R_{max}}{1-\gamma} \left| \frac{1}{1 + \frac{1}{|\mathcal{A}|} e^{\beta_{\kappa} g(s)}} \right|$$

Finally, using Eq. (13) we get:

$$\left| TQ^*(s, a) - \tilde{T}Q^*(s, a) \right| \leq \frac{2\gamma R_{max}}{1-\gamma} \left| \frac{1}{1 + \frac{1}{|\mathcal{A}|} e^{\beta_{\kappa} g(s)}} \right|$$

Taking the norm and plugging this into Eq. (12) concludes the proof. \square

Lemma 1. Let p and ν denote probability measures over Q -functions and state-action pairs, respectively. Assume Q^* is the unique fixed-point of the optimal Bellman operator T . Then, for any $\delta > 0$, with probability at least $1 - \delta$ over the choice of a Q -function Q , the following holds:

$$\|Q - Q^*\|_{\nu}^2 \leq \frac{\mathbb{E}_p [\|B(Q)\|_{\nu}^2]}{(1-\gamma)\delta} \quad (17)$$

Proof. First notice that:

$$\begin{aligned} \|Q - Q^*\| &= \|Q + TQ - TQ - TQ^*\| \\ &\leq \|Q - TQ\| + \|TQ - TQ^*\| \\ &\leq \|Q - TQ\| + \gamma \|Q - Q^*\| \\ &= \|B(Q)\| + \gamma \|Q - Q^*\| \end{aligned}$$

which implies that:

$$\|Q - Q^*\| \leq \frac{1}{1-\gamma} \|B(Q)\|$$

Then we can write:

$$P(\|Q - Q^*\| > \epsilon) \leq P(\|B(Q)\| > \epsilon(1-\gamma)) \leq \frac{\mathbb{E}_p [\|B(Q)\|_{\nu}^2]}{(1-\gamma)\epsilon}$$

Settings the right-hand side equal to δ and solving for ϵ concludes the proof. \square

Corollary 1. Let p and ν denote probability measures over Q -functions and state-action pairs, respectively. Assume \tilde{Q} is the unique fixed-point of the mellow Bellman operator \tilde{T} . Then, for any $\delta > 0$, with probability at least $1 - \delta$ over the choice of a Q -function Q , the following holds:

$$\|Q - \tilde{Q}\|_{\nu}^2 \leq \frac{\mathbb{E}_p [\|\tilde{B}(Q)\|_{\nu}^2]}{(1-\gamma)\delta} \quad (18)$$

Lemma 2. Assume Q -functions belong to a parametric space of functions bounded by $\frac{R_{max}}{1-\gamma}$. Let p and q be arbitrary distributions over the parameter space \mathcal{W} , and ν be a probability measure over $\mathcal{S} \times \mathcal{A}$. Consider a dataset D of N samples and define $v(\mathbf{w}) \triangleq \mathbb{E}_{\nu} [\text{Var}_{\mathcal{P}} [b(\mathbf{w})]]$. Then, for any $\delta > 0$, with probability at least $1 - \delta$, the following two inequalities hold simultaneously:

$$\mathbb{E}_q [\|B(\mathbf{w})\|_{\nu}^2] \leq \mathbb{E}_q [\|B(\mathbf{w})\|_D^2] - \mathbb{E}_q [v(\mathbf{w})] + \frac{\lambda}{N} KL(q||p) + 4 \frac{R_{max}^2}{(1-\gamma)^2} \sqrt{\frac{\log \frac{2}{\delta}}{2N}} \quad (19)$$

479

$$\mathbb{E}_q [\|B(\mathbf{w})\|_D^2] \leq \mathbb{E}_q [\|B(\mathbf{w})\|_{\nu}^2] + \mathbb{E}_q [v(\mathbf{w})] + \frac{\lambda}{N} KL(q||p) + 4 \frac{R_{max}^2}{(1-\gamma)^2} \sqrt{\frac{\log \frac{2}{\delta}}{2N}} \quad (20)$$

480 *Proof.* From Hoeffding's inequality we have:

$$P \left(\left| \mathbb{E}_{\nu, \mathcal{P}} \left[\|B(\mathbf{w})\|_D^2 \right] - \|B(\mathbf{w})\|_D^2 \right| > \epsilon \right) \leq 2 \exp \left(- \frac{2N\epsilon^2}{\left(2 \frac{R_{max}}{1-\gamma} \right)^4} \right)$$

481 which implies that, for any $\delta > 0$, with probability at least $1 - \delta$:

$$\left| \mathbb{E}_{\nu, \mathcal{P}} \left[\|B(\mathbf{w})\|_D^2 \right] - \|B(\mathbf{w})\|_D^2 \right| \leq 4 \frac{R_{max}^2}{(1-\gamma)^2} \sqrt{\frac{\log \frac{2}{\delta}}{2N}}$$

482 Under independence assumptions, the expected TD error can be re-written as:

$$\begin{aligned} \mathbb{E}_{\nu, \mathcal{P}} \left[\|B(\mathbf{w})\|_D^2 \right] &= \mathbb{E}_{\nu, \mathcal{P}} \left[\frac{1}{N} \sum_{i=1}^N (r_i + \gamma \min_{a'} Q_{\mathbf{w}}(s'_i, a') - Q_{\mathbf{w}}(s_i, a_i))^2 \right] \\ &= \mathbb{E}_{\nu, \mathcal{P}} \left[(R(s, a) + \gamma \min_{a'} Q_{\mathbf{w}}(s', a') - Q_{\mathbf{w}}(s, a))^2 \right] \\ &= \mathbb{E}_{\nu} \left[\mathbb{E}_{\mathcal{P}} [b(\mathbf{w})^2] \right] \\ &= \mathbb{E}_{\nu} \left[\text{Var}_{\mathcal{P}} [b(\mathbf{w})] + \mathbb{E}_{\mathcal{P}} [b(\mathbf{w})]^2 \right] \\ &= v(\mathbf{w}) + \|B(\mathbf{w})\|_{\nu}^2 \end{aligned}$$

483 where $v(\mathbf{w}) \triangleq \mathbb{E}_{\nu} [\text{Var}_{\mathcal{P}} [b(\mathbf{w})]]$. Thus:

$$\left| \|B(\mathbf{w})\|_{\nu}^2 + v(\mathbf{w}) - \|B(\mathbf{w})\|_D^2 \right| \leq 4 \frac{R_{max}^2}{(1-\gamma)^2} \sqrt{\frac{\log \frac{2}{\delta}}{2N}} \quad (21)$$

484 From the change of measure inequality [], we have that, for any measurable function $f(\mathbf{w})$ and any
485 two probability measures p and q :

$$\log \mathbb{E}_p \left[e^{f(\mathbf{w})} \right] \geq \mathbb{E}_q [f(\mathbf{w})] - KL(q||p)$$

Find a reference for this

486 Thus, multiplying both sides of (21) by $\lambda^{-1}N$ and applying the change of measure inequality with
487 $f(\mathbf{w}) = \lambda^{-1}N \left| \|B(\mathbf{w})\|_{\nu}^2 + v(\mathbf{w}) - \|B(\mathbf{w})\|_D^2 \right|$, we obtain:

$$\mathbb{E}_q [f(\mathbf{w})] - KL(q||p) \leq \log \mathbb{E}_p \left[e^{f(\mathbf{w})} \right] \leq 4 \frac{R_{max}^2 \lambda^{-1}N}{(1-\gamma)^2} \sqrt{\frac{\log \frac{2}{\delta}}{2N}}$$

488 where the second inequality holds since the right-hand side of (21) does not depend on \mathbf{w} . Finally,
489 we can explicitly write:

$$\mathbb{E}_q \left[\left| \|B(\mathbf{w})\|_{\nu}^2 + v(\mathbf{w}) - \|B(\mathbf{w})\|_D^2 \right| \right] \leq \frac{\lambda}{N} KL(q||p) + 4 \frac{R_{max}^2}{(1-\gamma)^2} \sqrt{\frac{\log \frac{2}{\delta}}{2N}}$$

490 from which the lemma follows straightforwardly. \square

491 **Lemma 3.** Let p be a prior distribution over the parameter space \mathcal{W} , and ν be a probability measure
492 over $\mathcal{S} \times \mathcal{A}$. Assume $\hat{\xi}$ is the minimizer of $ELBO(\xi) = \mathbb{E}_{q_{\xi}} \left[\|B(\mathbf{w})\|_D^2 \right] + \frac{\lambda}{N} KL(q_{\xi}||p)$ for a
493 dataset D of N samples. Define $v(\mathbf{w}) \triangleq \mathbb{E}_{\nu} [\text{Var}_{\mathcal{P}} [b(\mathbf{w})]]$. Then, for any $\delta > 0$, with probability at
494 least $1 - \delta$:

$$\mathbb{E}_{q_{\hat{\xi}}} \left[\|B(\mathbf{w})\|_{\nu}^2 \right] \leq \inf_{\xi \in \Xi} \left\{ \mathbb{E}_{q_{\xi}} \left[\|B(\mathbf{w})\|_{\nu}^2 \right] + \mathbb{E}_{q_{\xi}} [v(\mathbf{w})] + 2 \frac{\lambda}{N} KL(q_{\xi}||p) \right\} + 2 \frac{R_{max}^2}{(1-\gamma)^2} \sqrt{\frac{\log \frac{2}{\delta}}{N}}$$

495 *Proof.* Let us use Lemma 2 for the specific choice $q = q_{\hat{\xi}}$. From Eq. (19), we have:

$$\begin{aligned} \mathbb{E}_{q_{\hat{\xi}}} [\|B(\mathbf{w})\|_{\nu}^2] &\leq \mathbb{E}_{q_{\hat{\xi}}} [\|B(\mathbf{w})\|_D^2] - \mathbb{E}_{q_{\hat{\xi}}} [v(\mathbf{w})] + \frac{\lambda}{N} KL(q_{\hat{\xi}} \| p) + 4 \frac{R_{max}^2}{(1-\gamma)^2} \sqrt{\frac{\log \frac{2}{\delta}}{2N}} \\ &\leq \mathbb{E}_{q_{\hat{\xi}}} [\|B(\mathbf{w})\|_D^2] + \frac{\lambda}{N} KL(q_{\hat{\xi}} \| p) + 4 \frac{R_{max}^2}{(1-\gamma)^2} \sqrt{\frac{\log \frac{2}{\delta}}{2N}} \\ &= \inf_{\xi \in \Xi} \left\{ \mathbb{E}_{q_{\xi}} [\|B(\mathbf{w})\|_D^2] + \frac{\lambda}{N} KL(q_{\xi} \| p) \right\} + 4 \frac{R_{max}^2}{(1-\gamma)^2} \sqrt{\frac{\log \frac{2}{\delta}}{2N}} \end{aligned}$$

496 where the second inequality holds since $v(\mathbf{w}) > 0$, while the equality holds from the definition of $\hat{\xi}$.

497 We can now use Eq. (20) to bound $\mathbb{E}_{q_{\hat{\xi}}} [\|B(\mathbf{w})\|_D^2]$, thus obtaining:

$$\mathbb{E}_{q_{\hat{\xi}}} [\|B(\mathbf{w})\|_{\nu}^2] \leq \inf_{\xi \in \Xi} \left\{ \mathbb{E}_{q_{\xi}} [\|B(\mathbf{w})\|_{\nu}^2] + \mathbb{E}_{q_{\xi}} [v(\mathbf{w})] + 2 \frac{\lambda}{N} KL(q_{\xi} \| p) \right\} + 2 \frac{R_{max}^2}{(1-\gamma)^2} \sqrt{\frac{\log \frac{2}{\delta}}{N}}$$

498 This concludes the proof. \square

499 B Additional Details on the Algorithms

500 B.1 Gaussian Variational Transfer

501 Under Gaussian distributions, all quantities of interest for using Alg. 1 can be computed very easily.
502 The KL divergence between the prior and approximate posterior can be computed in closed-form as:

$$KL(q_{\xi}(\mathbf{w}) \| p(\mathbf{w})) = \frac{1}{2} \left(\log \frac{|\Sigma_p|}{|\Sigma|} + \text{Tr}(\Sigma_p^{-1} \Sigma) + (\boldsymbol{\mu} - \boldsymbol{\mu}_p)^T \Sigma_p^{-1} (\boldsymbol{\mu} - \boldsymbol{\mu}_p) - K \right) \quad (22)$$

503 for $\xi = (\boldsymbol{\mu}, L)$ and $\Sigma = LL^T$. Its gradients with respect to the variational parameters are []:

$$\nabla_{\boldsymbol{\mu}} KL(q_{\xi}(\mathbf{w}) \| p(\mathbf{w})) = \Sigma_p^{-1} (\boldsymbol{\mu} - \boldsymbol{\mu}_p) \quad (23)$$

504

$$\nabla_L KL(q_{\xi}(\mathbf{w}) \| p(\mathbf{w})) = \Sigma_p^{-1} L - (L^{-1})^T \quad (24)$$

505 Finally, the gradients w.r.t. the expected likelihood term of the variational objective (2) can be
506 computed using the reparameterization trick (e.g., []):

$$\nabla_{\boldsymbol{\mu}} \mathbb{E}_{\mathbf{w} \sim \mathcal{N}(\boldsymbol{\mu}, LL^T)} [\|B(\mathbf{w})\|_D^2] = \mathbb{E}_{\mathbf{v} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} [\nabla_{\mathbf{w}} \|B(\mathbf{w})\|_D^2] \text{ for } \mathbf{w} = L\mathbf{v} + \boldsymbol{\mu} \quad (25)$$

507

$$\nabla_L \mathbb{E}_{\mathbf{w} \sim \mathcal{N}(\boldsymbol{\mu}, LL^T)} [\|B(\mathbf{w})\|_D^2] = \mathbb{E}_{\mathbf{v} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})} [\nabla_{\mathbf{w}} \|B(\mathbf{w})\|_D^2 \cdot \mathbf{v}^T] \text{ for } \mathbf{w} = L\mathbf{v} + \boldsymbol{\mu} \quad (26)$$

508 B.2 Mixture of Gaussian Variational Transfer

509 C Additional Details on the Experiments

510 C.1 The Rooms Problem

511 C.2 Classic Control

512 C.3 Maze Navigation

513 Transfer approaches to mention:

- 514 • Taylor 2009: survey
- 515 • Konidaris and Barto transfer shaped reward functions PROs: this allows the agent to
- 516 have a more goal-directed behavior, thus limiting unnecessary random exploration CONS:
- 517 applicable only to simple problems, does not scale

Cite matrix
cookbook

Cite deep-
mind and
another

- 518 • Wilson 2007 propose a hierarchical Bayesian model for the distribution over tasks PROs it
519 explicitly models the uncertainty over which tasks are being solved, thus quickly adapting
520 to new ones and allowing informed exploration decisions to be taken
- 521 • Lazaric 2008 transfer samples PROs select good samples, can scale to continuous domains
522 CONs eps greedy is used
- 523 • Taylor 2008 transfer samples for model-based RL PROs good exploration via model-based
524 RL CONs does not scale, can negatively transfer when tasks are very different
- 525 • Lazaric 2010 propose a hierarchical bayesian model for the distribution over value functions
526 PROs quickly adapts to new tasks, non-parameteric (GP) models CONs scaling, strong
527 assumptions (GPTD)
- 528 • Fernandez and veloso 2006 propose an exploration strategy based on probabilistic policy
529 reuse PROs good exploration (?) CONs Do not try to figure out which policies are good or
530 bad
- 531 • Brunskill propose a method to transfer in model based RL (E3) - PROs theory, exploration
532 CONs scaling
- 533 • Barreto use successor features PROs simple, theory CONs eps-greedy exploration

534 Exploration approaches to mention:

- 535 • Osband 2014 propose a method to efficiently explore via randomized value functions
- 536 • Osband 2016 adapt such algorithm to DQNs
- 537 • Houthoof 2016 use variational inference to approximate the posterior distribution over
538 parameters of the dynamics, and use that to drive exploration
- 539 • Azizzadenesheli 2018 extend Osband 2016 to use Bayesian DQN instead. Still makes
540 Gaussian assumptions though