GuessTuples Project

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Abstract

Notes on GuessTuples project

1 Configuring the nets

1.1 Alice

The input array to guess is $\boldsymbol{x} = (x_j)_{j=0,\ldots,N_{\text{elements}}}$. There should be N_{code} outputs taking values $\boldsymbol{y} = (y_j)_{j=0,\ldots,N_{\text{code}}}$.

Normalise all the rewards so that for each bit j, $r_{j\checkmark} + (N_{\text{code}} - 1)\, r_{j\times} = 0$. In other words

$$r_{jk} \leftarrow r_{jk} - \frac{r_{j\checkmark} + (N_{\text{code}} - 1)\, r_{j\times}}{N_{\text{code}}}. \tag{1}$$

The Q estimate is then taken to be

$$Q(\boldsymbol{x}) = \sum_j b_j y_j \equiv \sum_j \left|y_j\right|, \tag{2}$$

where

$$b_j = \operatorname{sgn}(y_j) \tag{3}$$

is the prediction for the machine value of the jth bit. The loss function is

$$L = \left|Q(\boldsymbol{x}) - r\right|^2. \tag{4}$$

Alternative approaches include:

1. Two outputs for each bit showing the reward for each of 0 and 1. *May reflect negative rewards better?*

2. Combine the rewards from the bits (with either one or two outputs per bit) by something other than addition - e.g. multiplication or via an NN. *The NN option seems quite interesting. Interesting to use* `pytorch`*'s gradients for that.*

3. One outputs for each possible code. *Might work but* $2^{N_{code}}$ *is quite large... not impossibly so if* $N_{code} = 10$.

4. Inspired by Ref. [1], feed $\boldsymbol{x}$ into Alice's 'first' net, to get output $\boldsymbol{y}$, and all possible codes $\boldsymbol{c}$ into her 'second' net, both net's having the same target dimensionality (a hyperparameter). Then the code to use is the one $\boldsymbol{c}(\boldsymbol{x})$ closest to the output of the first net, with the Q being given by the inner product $Q = \langle \boldsymbol{y}, \boldsymbol{c}(\boldsymbol{x}) \rangle$. Ref. [2] might provide an alternative, actor–critic, approach on a similar theme.

5. Ref. [3] suggest sequentialising, which points to a variant of our main approach which does each bit in succession and feeding those results into successive Alice–nets so the Q-estimate for later bits takes account of earlier bits / estimates, with the N_{code}th estimate providing a final code $\boldsymbol{c}$ and Q-estimate for that code.

6. Move away from typical Q-learning. Instead Alice's output is the code $\boldsymbol{c}$ and then when Bob makes his choice $\boldsymbol{x}_{\text{pred}}$ (see below) run that choice through a copy of Alice, to get $\boldsymbol{c}_{\text{Bob}}$ and then the loss function is
$$L = -r(\boldsymbol{c}, \boldsymbol{c}_{\text{Bob}}). \tag{5}$$

1.2 Bob

Bob receives a matrix, $\mathbf{X} = (\boldsymbol{X}_i) = (X_{ij})$ for $0 \leq i < N_{\text{select}}$, $0 \leq j < N_{\text{elements}}$, and a code $\boldsymbol{c} = (c_k)_{k=0,\ldots,N_{\text{code}}}$. Why not makes his outputs be Q-estimates $\boldsymbol{z} = (z_i)_{i=0,\ldots,N_{\text{select}}}$. Bob's prediction is then $\boldsymbol{x}_{\text{pred}} = \boldsymbol{X}_{i_{\text{pred}}}$ where
$$i_{\text{pred}} = \text{argmax}_i(z_i). \tag{6}$$
The loss function is
$$L = \left| z_{i_{\text{pred}}} - r \right|^2 . \tag{7}$$

How do we enforce covariance with respect to the order of $(\boldsymbol{X}_i)$?

1. Covariance will occur naturally and quickly without any specific intervention. *To be determined.*

2. Covariance can be enforced through choosing a set $\{\sigma\} \subseteq S_{n_{\text{code}}}$, which could be generated element–by–element by composing randomly–selected basis transpositions $(j \; j+1)$, and then adding to the loss a term
$$\mu \sum_{\sigma} \left| z - \sigma^{-1} \left[z(\sigma[\mathbf{X}]) \right] \right|^2 \tag{8}$$
for some fixed hyperparameter $\mu > 0$. Note this the term is still run backward through the original $\boldsymbol{x} \mapsto \boldsymbol{z}$ net configuration only. *How effective would that be? How big does $\{\sigma\}$ have to be? And how much time would the permutation and the additions forward passes cost?*

3. Enforce covariance via direct identification of weights in Bob's net. *How?*

4. Something related to set transformers. *?*

5. Adopt a different basic set–up where each (X_i) is fed through the net separately, alongside the code $\boldsymbol{c}$, resulting in a Q-estimate z_i. Then find the loss function as in Eq. 7. *Seems the most straightforward?* **Adopt this for now.**

None of these quite amount to Bob seeks to reproduce the Alice's code vocabulary. However Bob could additionally set up a net in the same basic configuration as Alice's (he doesn't know the weights of course) and train *that* net jointly with his main net.

References

[1] J. He, J. Chen, X. He, J. Gao, L. Li, L. Deng et al., *Deep reinforcement learning with a natural language action space*, *arXiv preprint arXiv:1511.04636* (2015) .

[2] G. Dulac-Arnold, R. Evans, H. van Hasselt, P. Sunehag, T. Lillicrap, J. Hunt et al., *Deep reinforcement learning in large discrete action spaces*, *arXiv preprint arXiv:1512.07679* (2015) .

[3] S. J. Majeed and M. Hutter, *Exact reduction of huge action spaces in general reinforcement learning*, *arXiv preprint arXiv:2012.10200* (2020) .