```
Strand[i, j] represents a strand from index i to index j that can be stitched with other strands into longer strands or circles.

SetAttributes[Strand, Orderless];
```

Strand /: Strand[i , j ] Strand[j , k ] :=

 $Strand /: Strand[i_, i_] := -q^{1/2} - q^{-1/2};$ 

Strand /: Strand  $[ ]^2 := -q^{1/2} - q^{-1/2};$ 

Strand[i, k];