

$\text{Strand}[i, j]$  represents a strand from index  $i$  to index  $j$  that can be stitched with other strands into longer strands or circles.

**SetAttributes[Strand, Orderless];**

**Strand /:**

**Strand[i\_, j\_] Strand[j\_, k\_] := Strand[i, k];**

**Strand /: Strand[i\_, i\_] :=  $-q^{1/2} - q^{-1/2}$ ;**

**Strand /: Strand[\_\_\_]<sup>2</sup> :=  $-q^{1/2} - q^{-1/2}$ ;**