

Colourings[k, m] gives the number of colourings of the knot k,
 which is given in modified DT form onto the permutation group S_m .

```

Colourings[k_MDT, m_Integer] :=
  Colourings[k, m] = If[k === MDT[],
    Length /@ (Sort[Length /@
      (List @@ PermutationCycles@#) [[1]] &
      // GroupBy[Permutations@Range@m, #] &
      // Values]),
    Block[{e = EdgeSequence@k,
      s, v, w = List @@@ List @@ ToPD@k},
      s = SortBy[If[Order[Position[Join @@ e, #[[1]]],
        Position[Join @@ e, #[[3]]] == 1,
        #, Reverse@#] & /@
        (w /. (Max@# -> Min@# & /@ w[[;;, {3, 5}]])) [[
          ;;, 2 ;; 4]],
        Max@Table[Position[Join @@ e, #[[j]]], {j, 2}] &];
      (*In s, the third values are
        to be derived from the first two.*)
      Total /@ Table[If[ValidColouring[
        w, e[[1]], s, g], 1, 0],
        {p, (Length /@ (List @@ PermutationCycles@#) [[1]]
          // Sort) &
          // GroupBy[Permutations@Range@m, #] &
          // Values},
        {g, Tuples[p, Length@e[[1]]]}]]];
  
```