INDIAN INSTITUTE OF INFORMATION TECHNOLOGY, ALLAHABAD

C3 LAB.

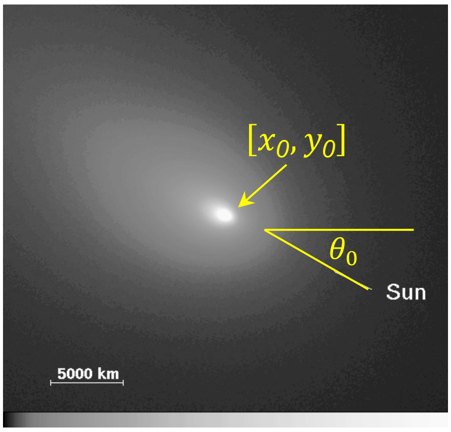
Computational Astrophysics

B.Tech: (IT & ECE) *Elective* *VI – Semester* *QP: PC* **Time - 3hrs.**

Marks - + LAB: {10}

Answers should be brief and to the point. Unnecessary Extra writing will attract negative marks.

1. The figure shows an $(n_1 \times n_2)$ pixel image of a cometary coma. The centroid of the is at $[x_0, y_0]$. The Radial Intensity profile of the coma generally follow an intensity gradient of



$$I[i,j] = I(\rho) = I_0\rho^{-\alpha(\theta)} \qquad (1)$$

Where I_0 is the peak intensity at $[x_0, y_0]$. ρ is the radial distance from the coma centre $[x_0, y_0]$ to the pixel centre $[(i+0.5), (j+0.5)]$ where $[i, j]$ is the pixel coordinates which vary from $[0 \to (n_1 - 1), 0 \to (n_2 - 1)]$ $\alpha(\theta)$ is the exponential gradient of the coma. $\alpha(\theta)$ is close to 1, but varies from maximum exponential gradient in the direction of the sun and a minimum gradient in the anti-sun direction. Since $\alpha(\theta)$ is expected to be cyclic in θ, it can be modeled as

$$\alpha(\theta) = 1 + A\cos(\theta - \theta_0) \qquad (2)$$

Where θ_0 is the angular direction of the sun from the horizontal plane of the image.

a) Write a short pseudo code to determine the $\rho[i,j]$, $\theta[i,j]$ and determine $I(\rho,\theta)$ for all pixel $[i,j]$ and store in a table **T1** with columns $[\,|\, i \,|\, j \,|\, \rho \,|\, \theta \,|\, I(\rho,\theta) \,|\,]$. You may make use of ***ATAN2(y, x)***. ***ATAN2(y, x)*** returns the angle θ between the positive x-axis and the ray from the origin to the point (x, y), confined to (−π, π).

b) The data from **Table T1** will be used to 1st fit ***Equ.(1)*** for different θ. From the $\alpha(\theta)$s obtained for different θ, the ***Equ(2)*** needs to be fitted to determine A and θ_0. Explain the methodology and the plots to determine I_0, $\alpha(\theta)$s, A and θ_0.

Please note I_0 should be the same for all the $\alpha(\theta)$ fits. You may use the function ***T2=SORT(T1,*** ρ**)**, ***T3=SORT(T2,*** θ**)** to sort the table **T1** in ρ and **T2** in θ.

Hint: a Log-Log plot linearizes an exponential function. **[2+8=10 Marks]**

SHOW/SUBMIT ALL THE FITS FOR DIFERENT θ THE LINEAR FITS OF EQ.(1). FIX THE INTERSEPT AND REFIT THE EQ(1).
SHOW/SUBMIT THE FIT OF EQ(2).

12.

a) pseudo code to store in a table T1 with columns $[\,|\,i\,|\,j\,|\,\rho\,|\,\theta\,|\,I(\rho,\theta)\,|\,]$

k = 0

For i =0, to $(n_1 - 1)$ *do {*

For j =0, to $(n_2 - 1)$ *do {*

$$\rho[i,j] = \sqrt{(i + 0.5 - x_0)^2 + (j + 0.5 - y_0)^2}$$

$$\theta[i,j] = ATAN2\big((j + 0.5 - y_0),\ (i + 0.5 - x_0)\big)$$

$\theta[i,j] = \theta[i,j] \times 360/2\pi$ {Converts to degree from Radian}

$T_1[k,\ 1] = i;$

$T_1[k,\ 2] = j;$

$T_1[k,\ 3] = \rho[i,j];$

$T_1[k,\ 4] = \theta[i,j];$

$T_1[k,\ 5] = I[i,j];$

$k = k + 1;\ \ \}\}$

b) $\boxed{I[i,j] = I(\rho) = I_0\rho^{-\alpha(\theta)} \qquad \text{eq(1)}}$

$$\ln(I[i,j]) = ln\big(I(\rho)\big) = ln(I_0) - \alpha(\theta)\cdot ln(\rho)$$

$y = \ln(I[i,j]);\ x = ln(\rho)$ *Intersept =* $b = ln(I_0)$

Strait line plot $y = -\alpha(\theta)x + b$

T₂=SORT(T₁, ρ***);*** ***T₃=SORT(T₂,*** θ***)***

k=0; m=0;

While $\big(k < (n_1 \times n_2)\big)$ *do {*

l=0; $\vartheta[m] = T_3[k\quad 4];\ \ d\vartheta = 1;$

While $T_3[k\quad 4] < (\vartheta + d\vartheta)$ *do {*

$x[l] = ln(T_3[k,\ 3])$

$y[l] = \ln(T_3[k,\ 4]);$

$\boldsymbol{l = l + 1;\quad k = k + 1;}\qquad \}$

$LSQfit(x[m,l],\ y[m,l]) \rightarrow \alpha(\theta)[m],\ b[m]$

$m = m + 1;\qquad \}$

$m_0 = m;$

Since all I_0 *should be the same for all the* $\alpha(\theta)$ *fits. The* intercepts $b[m] = ln(I_0)$ should be equal. $b_0 = AVERAGE(b[m])$; re-fit the Strait line for all m keeping the intercept fixed to b_0

For m = 0 to $(m_0 - 1)$ *do {*

$$LSQfit(x[m,l],\ y[m,l],\ b_0) \rightarrow \alpha(\theta)[m]$$

$$\boxed{\alpha(\theta) = 1 + A\cos(\theta - \theta_0) \qquad \text{Eq.(2)}}$$

$$Z[m] = \alpha(\theta)[m] - 1 = A\cos(\vartheta[m] - \theta_0)$$

$$\boxed{fit\ Z[m]\ vs\ \vartheta[m]}$$

$$A \pm \delta A = Max(Z[m])$$

Determine the phase θ_0 from fitting $\vartheta[m]$ and $Z[m]/A$.

$$\frac{Z[m]}{A} = \cos(\vartheta[m] - \theta_0)$$