GT TUT1

- Notating partite graphs $G = (A \cup B, E)$ where each A, B represent a partite set. For n-partite it could be A, B, C...
- These are also questions from West's book
- Handshaking lemma $\sum_{i=1}^{n} deg(v_i) = 2m$

1. $K_{1,1}$ or K_2 is the only complete bipartite graph which is also a complete graph.
2. G vertex set has all $(0,1), (0,0), (1,0), (1,1)$ has vertices. Two vertices are adjacent if they are 1 unit apart. Therefore it is bipartite. Now, generalize it to k-tuple i.e. $(0,0,0,0,0,1,0,0,0,0,0)$ and solve.
   - We may have an edge between two vertices only when their parity differs. (parity: number of ones)
   - Since parity can either be 0 or 1, (i.e. odd or even), it creates two sets, which are now bipartite. (btw, it's not complete bipartite (000 \noedge 111))
   - [WILD] You can also apply induction, $P(1)$ is true, assume $P(k)$, in case of $P(k+1)$ you'll find that there two sets (one with $k+1^{th}$ element as 0 and other being 1)
3. When G is a simple graph (with m edges and n vertices). Let $G - v_i$ have m_i edges. Then
   - $m = \frac{1}{n-2}\sum_{i=1}^{n} m_i$ because each edge involves two vertices, so each edge will appear exactly $n-2$ types for all $G - v_i$ when $i = 1, \ldots, n$
   - $deg(v_i) = \left(\frac{1}{n-2}\sum_{j=1}^{n} m_j\right) - m_i$ pretty obvious.
   - Or, use handshaking lemma (Long ass tho)
4. Decanting problem: not solved (make all states and join...nothing much)
5. Using rectangular blocks whose entries are all equal, write down an adjacency matrix for $K_{r,s}$.
   0 matr, 1 matr
   1 matr, 0 matr,
   where first is $r \times r$ and 4th is $s \times s$
6. k-regular graphs, pretty self explanatory