We are interested in pricing an American equity option. We assume the following standard, Black-Scholes dynamics for the price of the stock at time t

$$S_t = S_0 e^{\left(r-q-\frac{\sigma^2}{2}\right)t+\sigma W_t},$$

where r represents the risk-free interest rate, q is the borrow rate, σ is a constant and W_t is a Brownian motion.

We assume that we can continuously delta hedge with no transaction costs. We also assume that there are 365 days in a year and that the option can only be exercised at the end of a day (Bermudan option).

PART A

1) Write a function to compute the price of a call or put option with time-to-expiry T. Respect the following API:

   double price(strike, is_call , s_0, r, q, T, sigma, extra_arguments)

   You can use the $extra\_arguments$ variable to specify options related to the numerical precision of your algorithm.

Now, suppose a cash dividend of amount $\$d$ is payed out at time t_{div},

$$S_t = S_0 e^{\left(r-q-\frac{\sigma^2}{2}\right)t+\sigma W_t} - d1\{t \geq t_{div}\},$$

where

$$1\{t \geq t_{div}\} = \begin{cases} 0 \; if \; t < t_{div} \\ 1 \; otherwise \end{cases}.$$

2) Extend the pricing function above to consider the cash dividend. Respect the following API:

   double price(strike, is_call , s_0, r, q, T, sigma, t_div, div, extra_arguments)

Now, suppose an earnings event is scheduled for time t_{ea}:

$$S_t = S_0 e^{\left(r-q-\frac{\sigma^2}{2}\right)t+J1\{t\geq t_{ea}\}+\sigma W_t} - d1\{t \geq t_{div}\},$$

3) Extend the pricing function to consider the earnings event. Assume that the earnings event return, J, is normally distributed with standard deviation σ_{ea}: $J \sim N(\mu, \sigma_{ea})$.

4) Repeat 3) when $J = J_0 + (2\alpha - 1)k$, where $J_0 \sim N(\mu, \sigma_{ea})$, α is a Bernoulli random variable with probability p and k is a constant.

PART B

For this part, we have included three datasets that describe different options (strike/expiry/kind) and their market prices. Datasets are simple tables and are provided in JSON format.

5) Dataset A lists 78 different options – identified by exercise-style/strike/expiry/kind – and 6 different models – specified by the set of parameters $(r, q, \sigma, \sigma_{ea}, d, t_{div}, t_{ea})$. Price the options defined in dataset A using the function developed in 4) when $S_0 = \$170$. As part of your write-up, provide us with dataset A with an additional price column containing the results you computed.

6) The prices in dataset B are computed for American options according to the dynamics specified in 3) with $S_0 = \$170, r = 2\%, q = 0.4\%, t_{div} = \frac{14}{365}, d = \$0.5, t_{ea} = \frac{7}{365}, k = 0$.
Can you estimate what value of σ and σ_{ea} they were computed with? Write an *algorithm* to do this.

7) Using dataset C, repeat 6) to estimate r, q, d, σ and σ_{ea} when you know $S_0 = \$170$, $t_{div} = \frac{14}{365}, t_{ea} = \frac{7}{365}, k = 0$.

NOTES

- You may choose among the following programming languages: Python, Matlab, R, Java and C++. For ease of use, consider using an interpreted language.
- Program design and code quality will be evaluated. Modular, clean, and readable code is preferred.
- Numerical precision and performance are relevant factors. At the same time, you are not expected to write highly performant code. Use common sense to achieve a balance between correctness, error control and performance.
- Datasets are provided in JSON format. The following units are used: times are in year, rate and vols are in natural units (0.20 means 20%).
- You might notice that (in the datasets) time-to-expiry is not a multiple of 1/365. Please, assume that days end (and options can be exercised) at times t_i = timeToExpiry - (i / 365), i=0, 1, 2, 3,… (s.t. t_i > 0).
- For PART B, please provide uncertainties around your estimates. Your implementation might differ from that used to compute such prices.
- The solution should include:
  - Your code and instructions on how to compile and run your code.
  - An auxiliary document containing any comments related to your implementation, example outputs, if any, and the answers to part B (required).
- If completing the entire test is not possible, please prioritize A-2, A-3, B-5 and B-6 (in that order). Certain choices might allow you to cut down on development time considerably -- potentially affecting performance. That might be acceptable if it allows you to complete the task effectively -- simply motivate/explain your choices to provide context.