Image Filtering → Edge Detection → Corner Detection

Corner Detection

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Motivation: Image Matching

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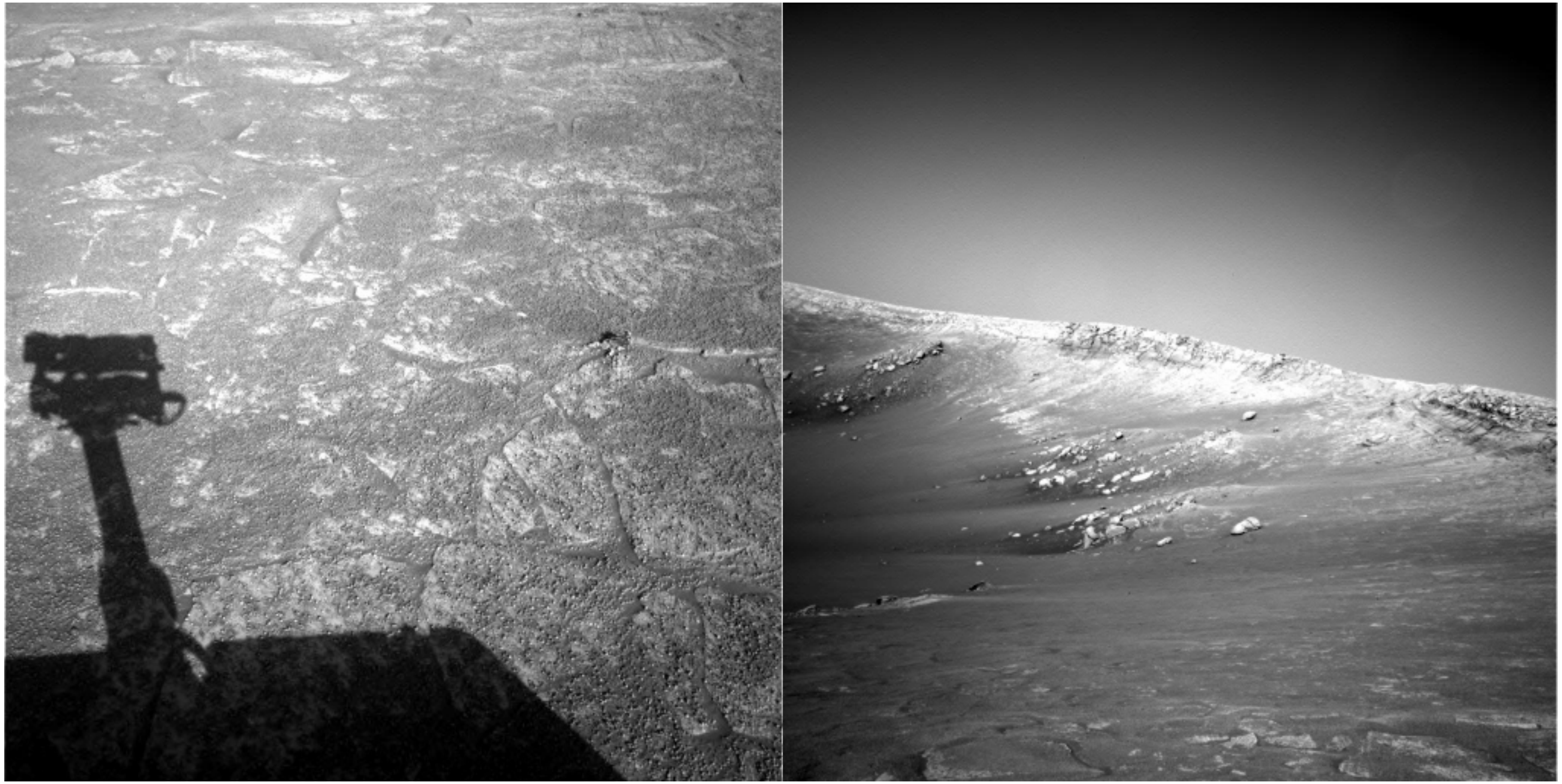

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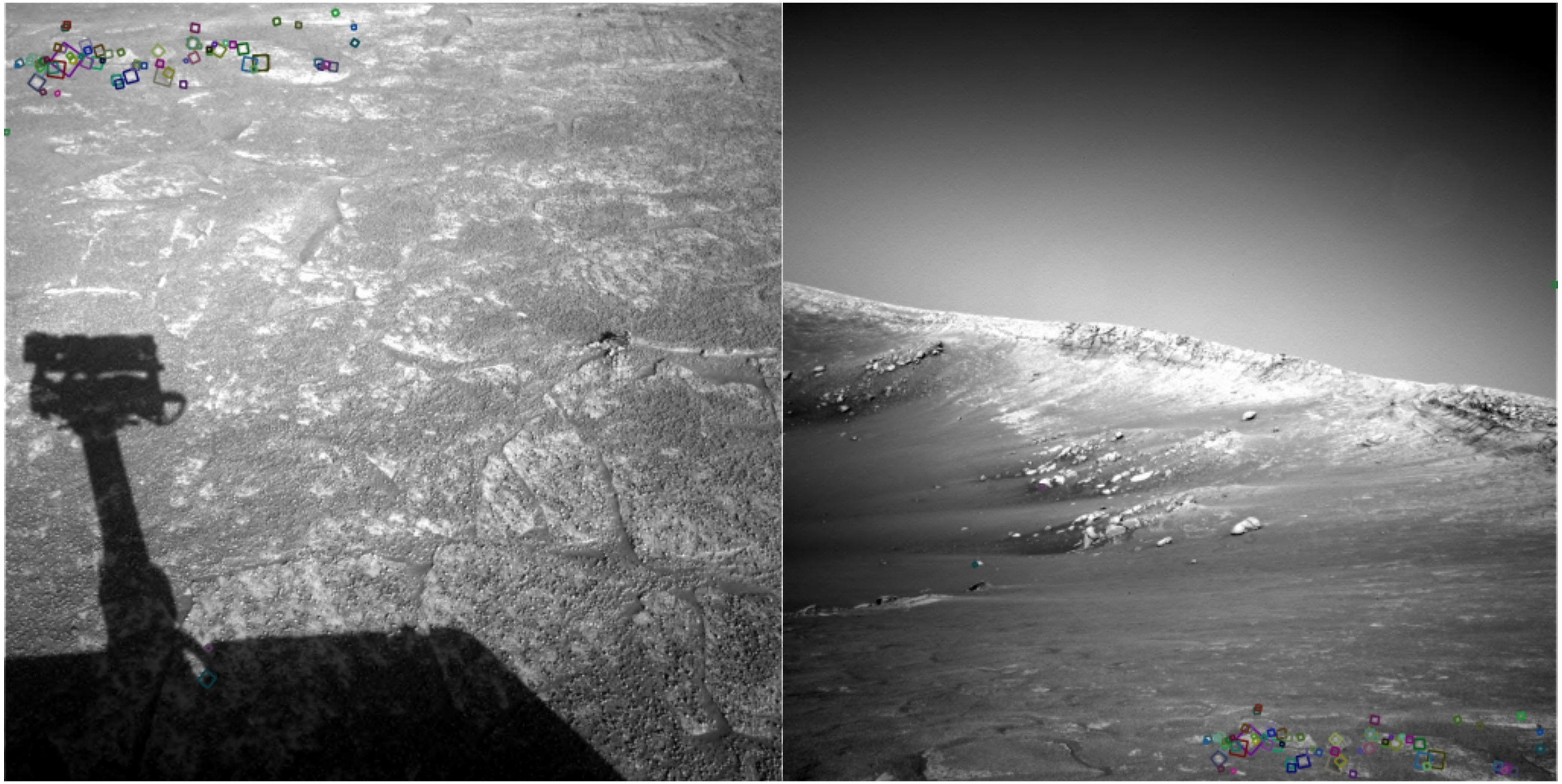

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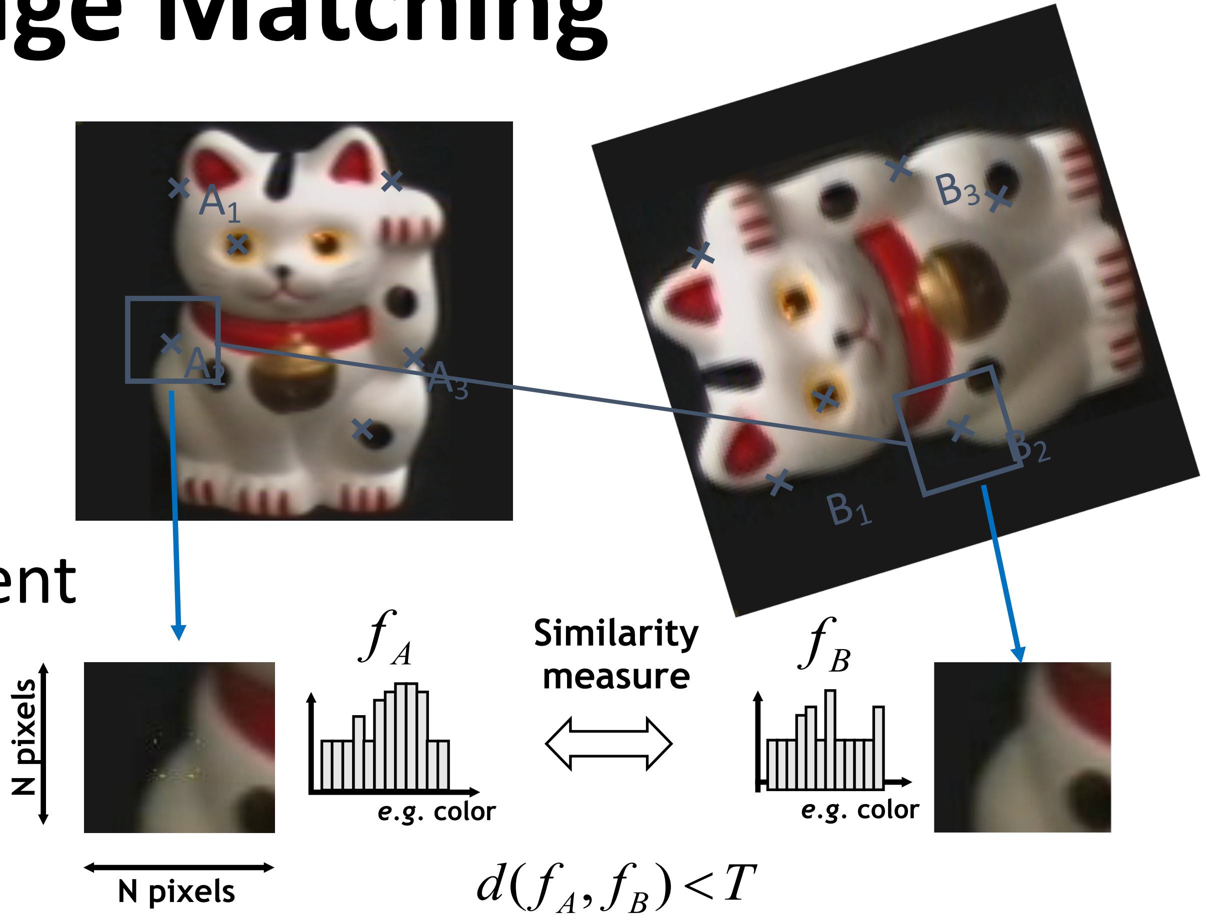
General Approach to Image Matching

- Find a set of distinctive key-points
- Define a region around each key-point
- Extract and normalize the region content
- Compute a local descriptor from the normalized region
- Match local descriptors



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Requirements of Image Matching

Repeatable Detector

- Detect the same point independently in both images

No chance to match!

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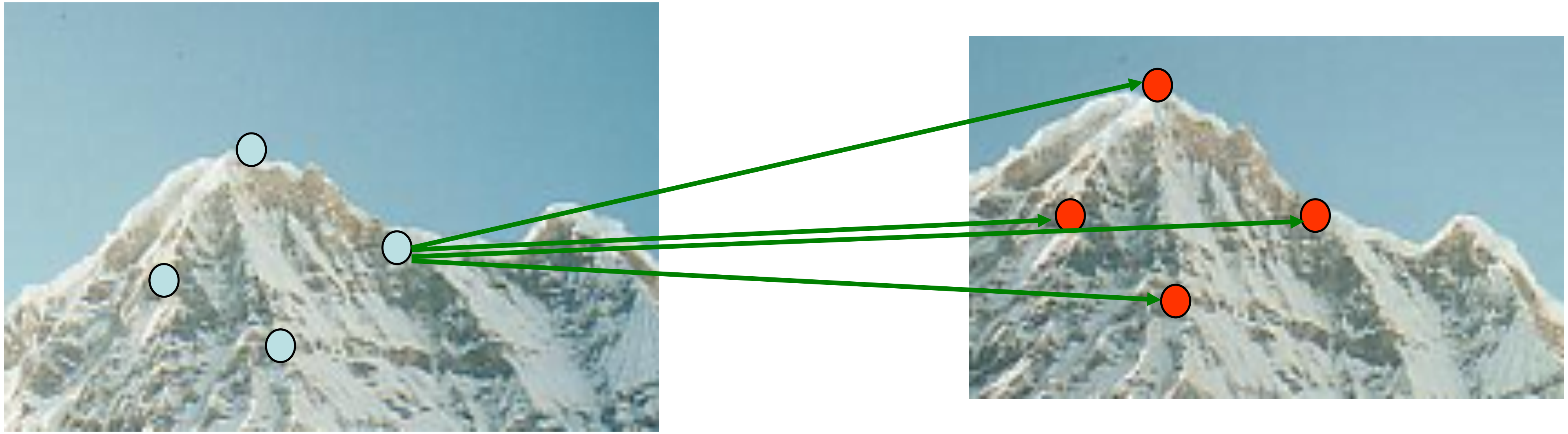
Requirements of Image Matching

Reliable and distinctive descriptor

- For each point correctly recognize the corresponding one



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Requirements of Image Matching

Invariance to Geometric Transformations

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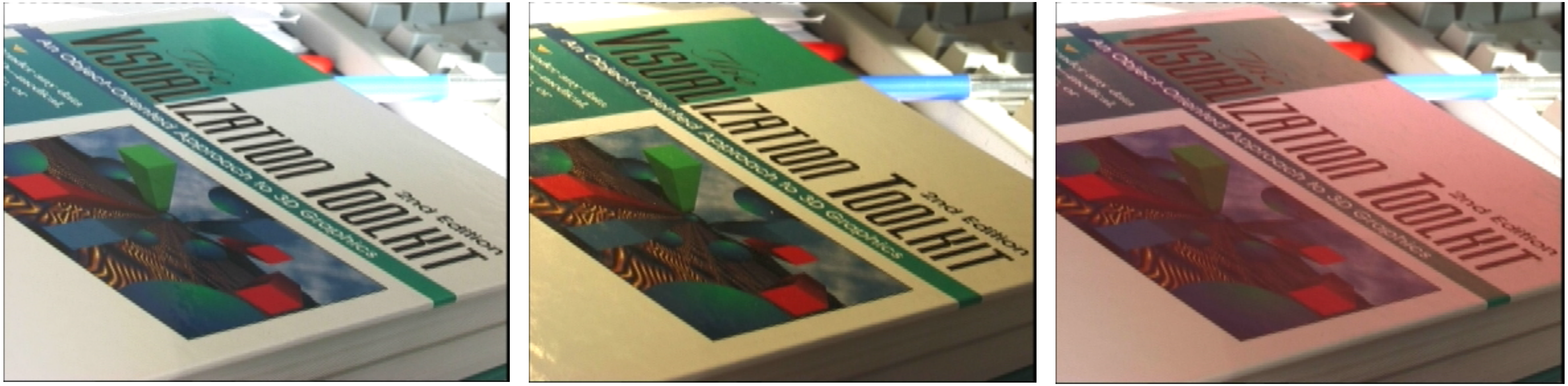
Requirements of Image Matching

Invariance to Photometric Transformations



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Feature Detectors

- Harris Corners
- DoG (SIFT, SURF)
- MSER
- ORB

Corner Detection

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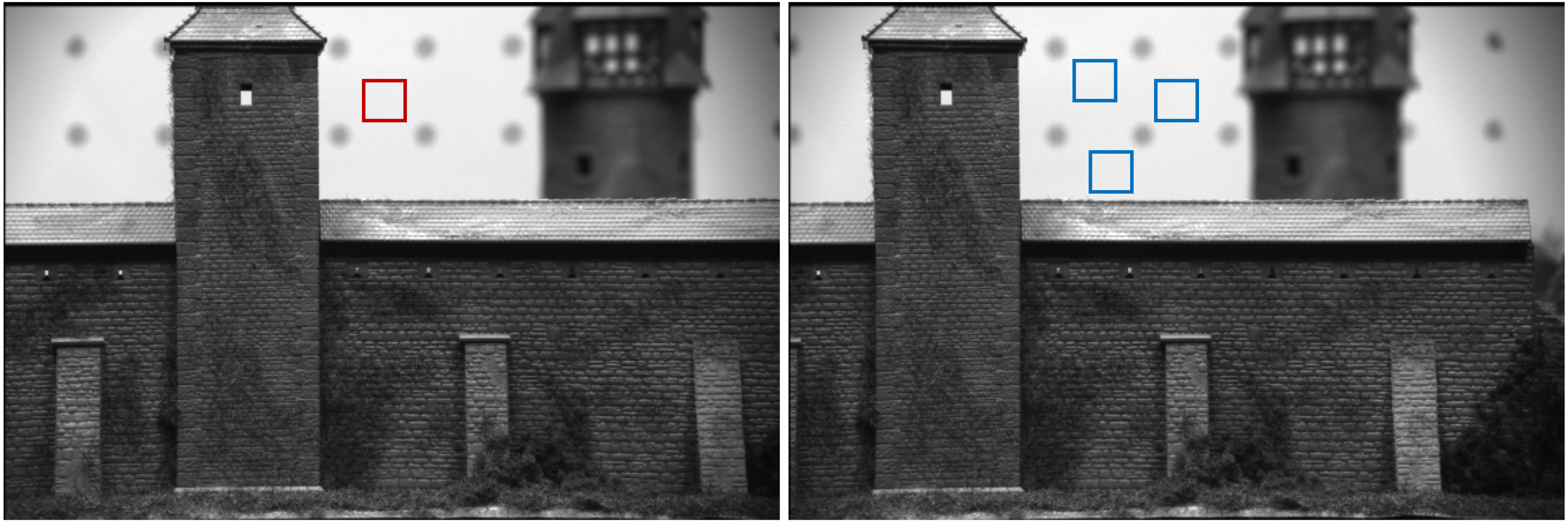
Patch Matching



Task: Find the patch in the right image which is most similar to the red patch in the left image

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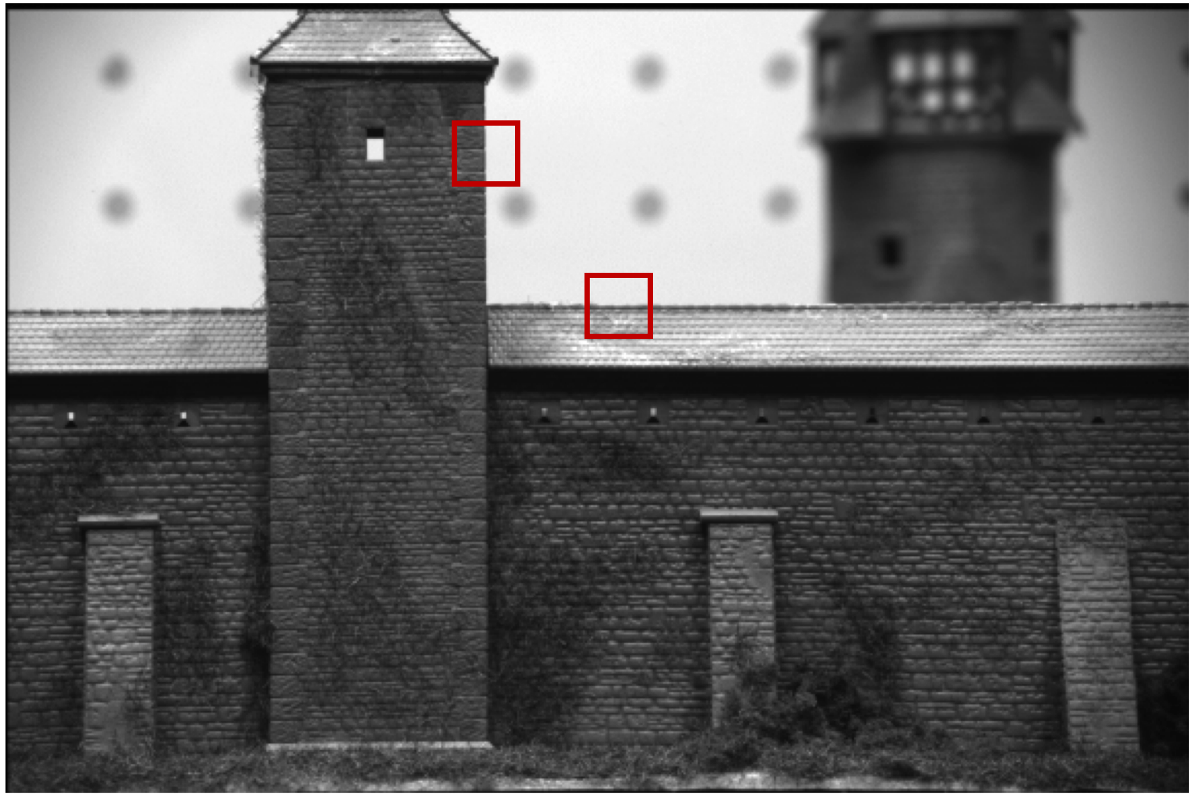
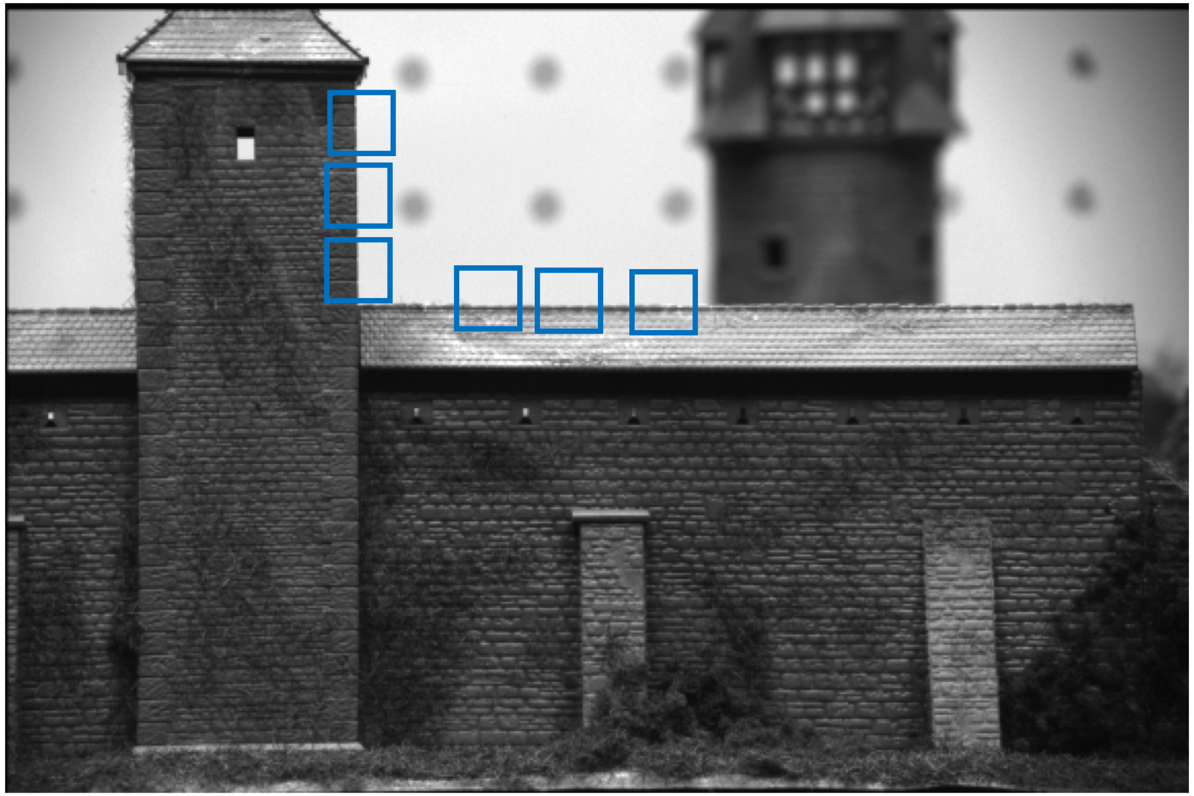
Patch Matching

Task: Find the patch in the right image which is most similar to the red patch in the left image

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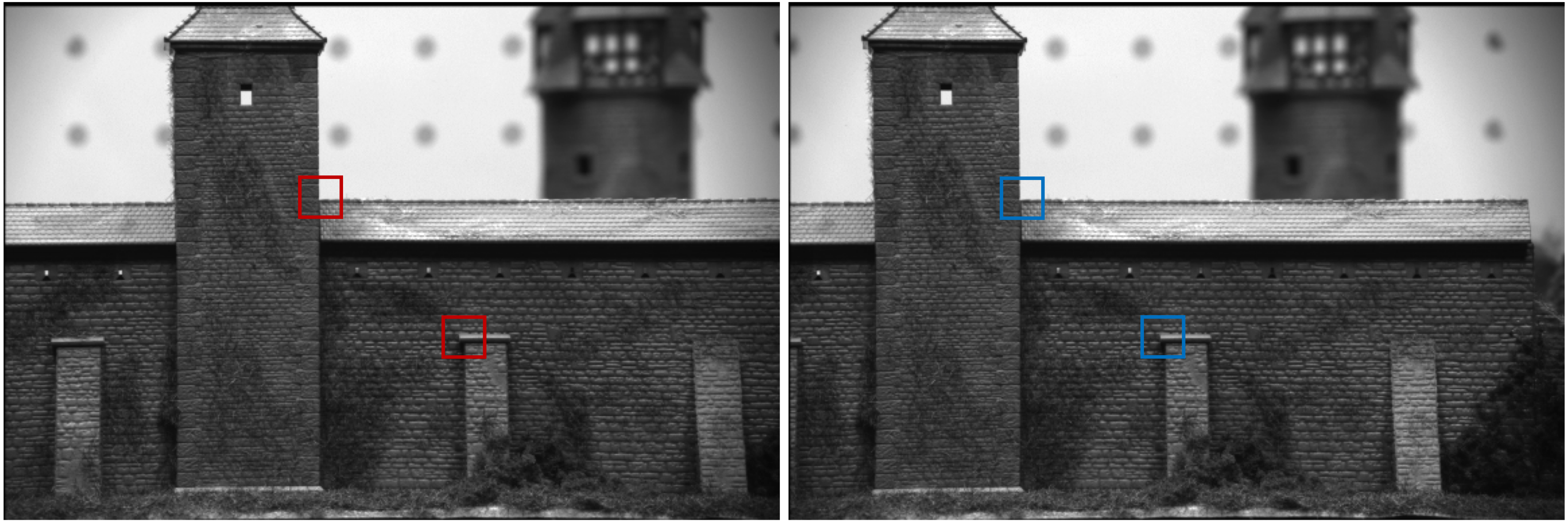
Patch Matching



Task: Find the patch in the right image which is most similar to the red patch in the left image

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Good Features to Match

- **Uniform regions**
  - Same intensity in both x and y directions
  - Bad. 2D degrees of freedom in matching.

- **Edges**
  - Same intensity along the edge
  - Bad. 1 degree of freedom in matching.

- **Corners**
  - Large variation in the neighborhood the point in all directions.
  - Best. No degree of freedom in matching. Just one match.

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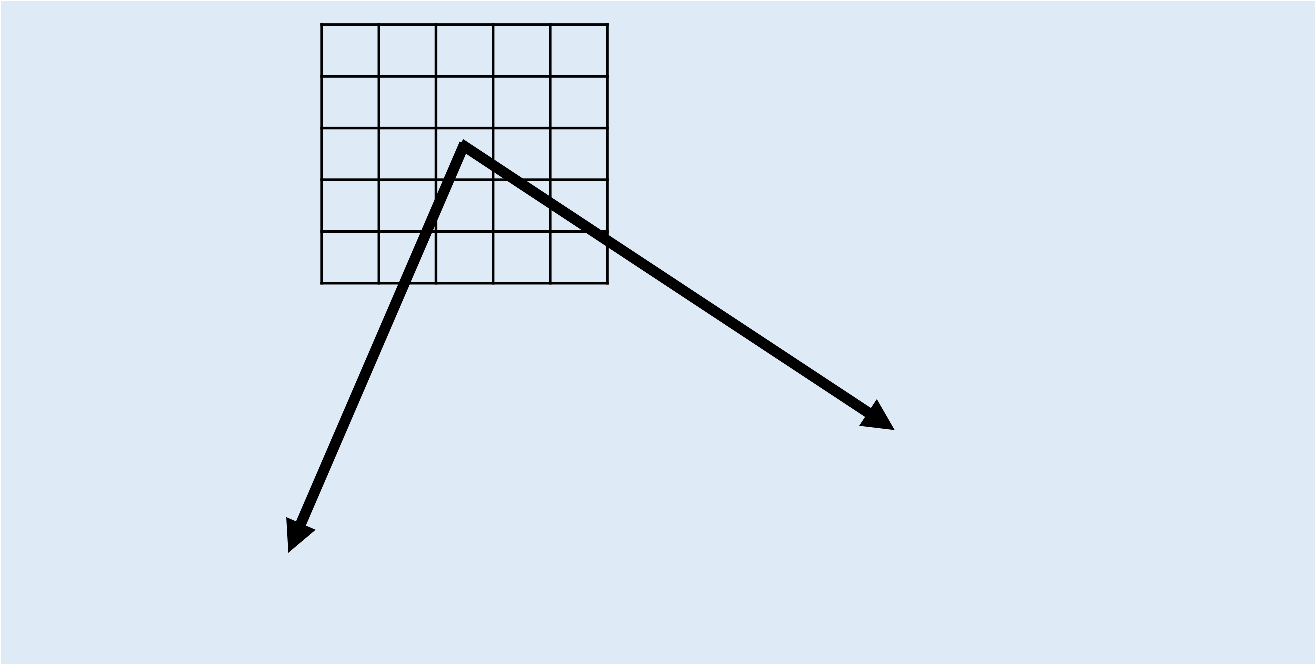
Corners as Feature Points

- We should easily recognize the point by looking at intensity values within a small window
- Shifting the window in any direction should yield a large change in appearance.



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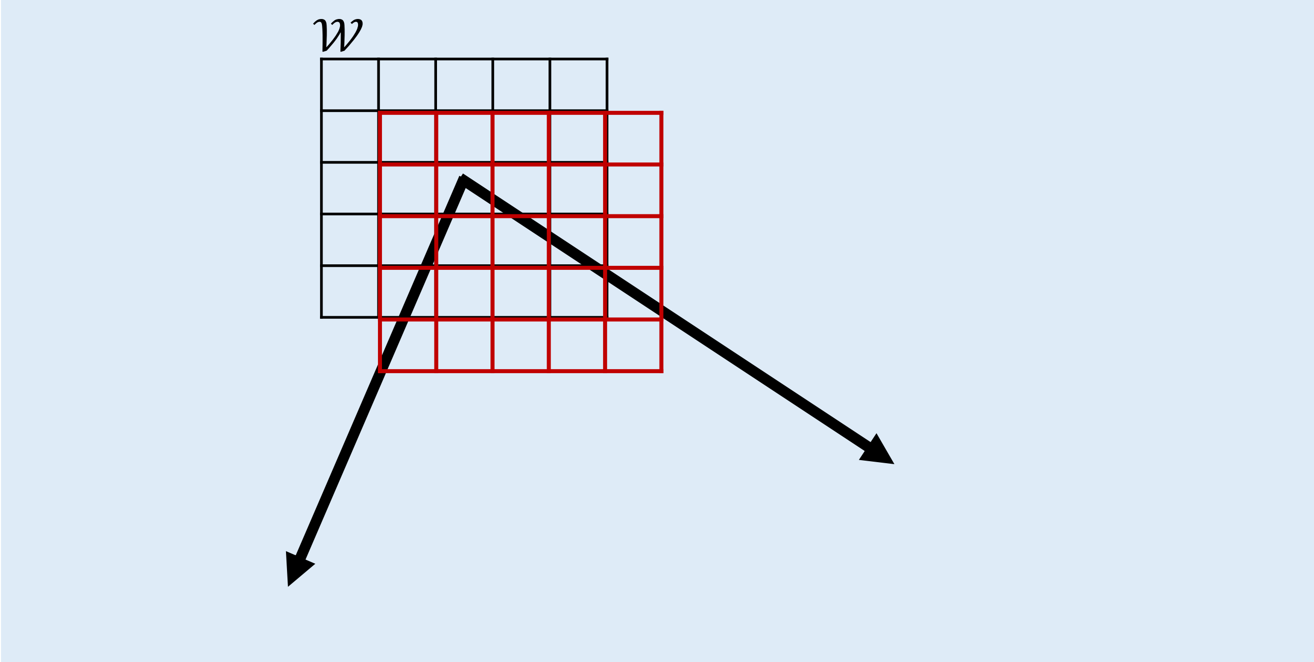
Patch Energy

- Change of intensity for the shift (u, v)

$$E(u,v) = \sum_{(x,y)\in\mathcal{W}} w(x,y)[I(x+u,y+v) - I(x,y)]^2$$

Windowing function
- Usually Gaussian
- We will ignore for simplicity



- For nearly constant patches, E will be near 0.
- For very distinctive patches, E will be larger.

We want patches where $E(u,v)$ is LARGE for all (u,v).

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Taylor Series for 2D Functions

$$f(x+u, y+v) = f(x,y) + uf_x(x,y) + vf_y(x,y)$$

First partial derivatives

$$+\frac{1}{2!}\left[u^2 f_{xx}(x,y) + uvf_{xy}(x,y) + v^2 f_y(x,y)\right]$$

Second partial derivatives

$+\ \ldots$ Higher Order Terms

First Order Approximation

$$\boldsymbol{f(x+u, y+v) \cong f(x,y) + uf_x(x,y) + vf_y(x,y)}$$

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Harris Corner Detector: Derivation

First Order Approximation

$$\sum_{(x,y)\in\mathcal{W}} [I(x+u,y+v) - I(x,y)]^2 \approx \sum_{(x,y)\in\mathcal{W}} \left[\boxed{I(x,y) + uI_x + vI_y} - I(x,y)\right]^2$$

$$= \sum_{(x,y)\in\mathcal{W}} \left[u^2 I_x^2 + 2uvI_xI_y + v^2 I_y^2\right]$$

Windowing function

$w(x,y)$

$$= \sum_{(x,y)\in\mathcal{W}} [u \quad v] \begin{bmatrix} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$= [u \quad v] \underbrace{\begin{bmatrix} \sum I_x^2 & \sum I_xI_y \\ \sum I_xI_y & \sum I_y^2 \end{bmatrix}}_{\boldsymbol{M}} \begin{bmatrix} u \\ v \end{bmatrix}$$

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Understanding Harris (Intuitively)

- Treat gradient vectors as a set of (dx, dy) points with a center of mass defined as being at $(0,0)$.
- Fit an ellipse to that set of points via scatter matrix
- Analyze ellipse parameters for varying cases…

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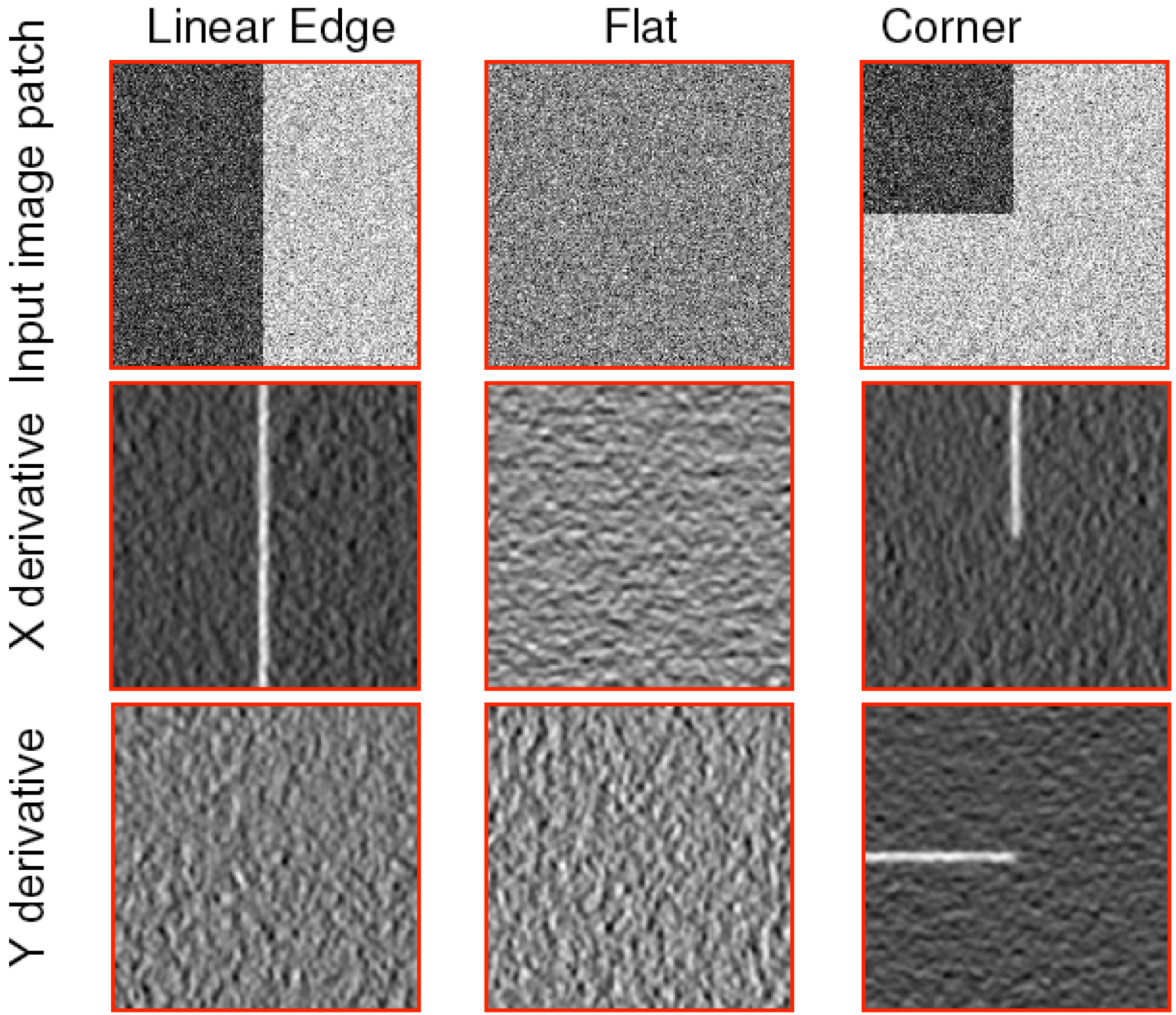
Example: Cases and 2D Derivatives



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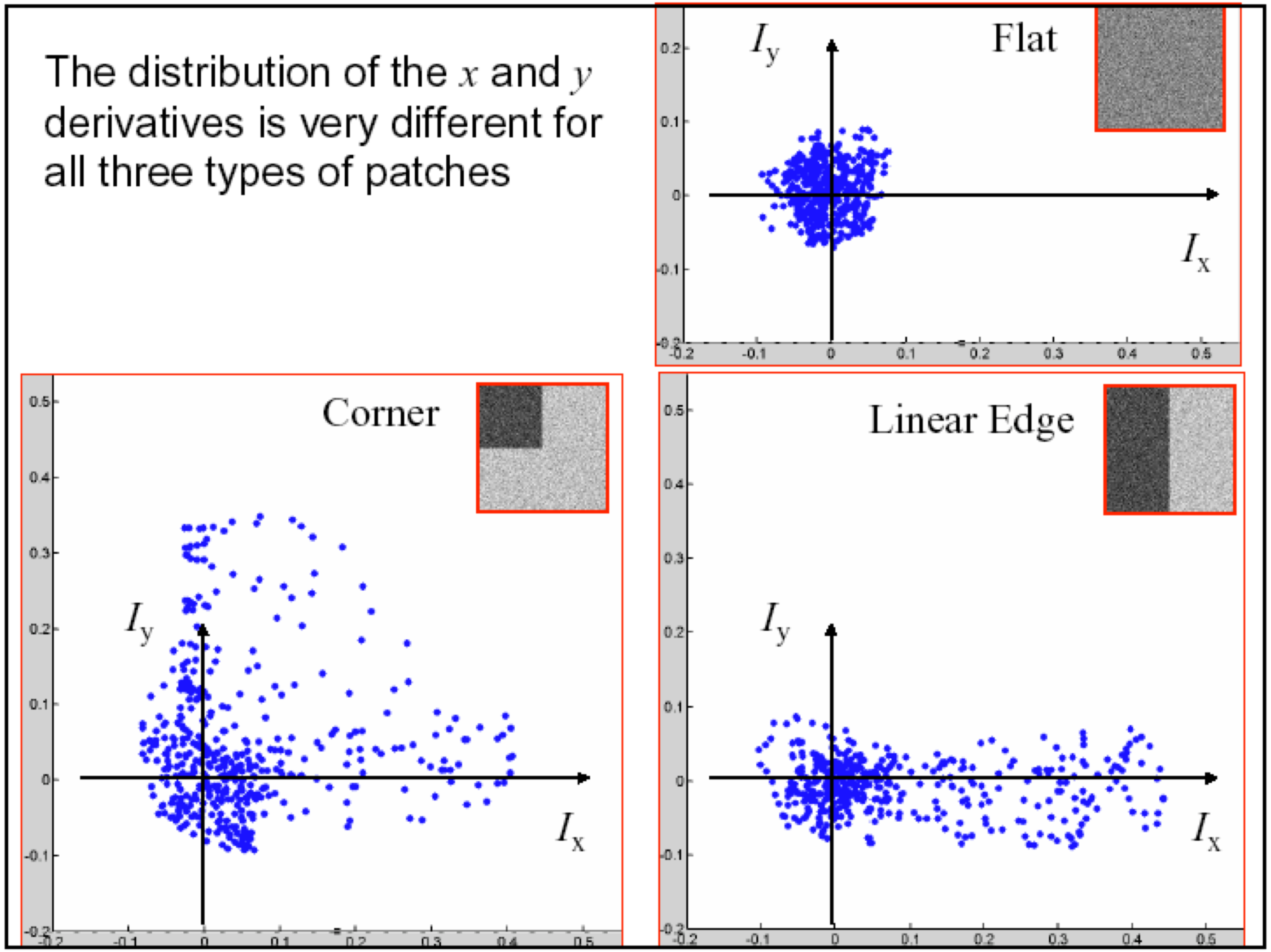
Plotting Derivatives as 2D Points



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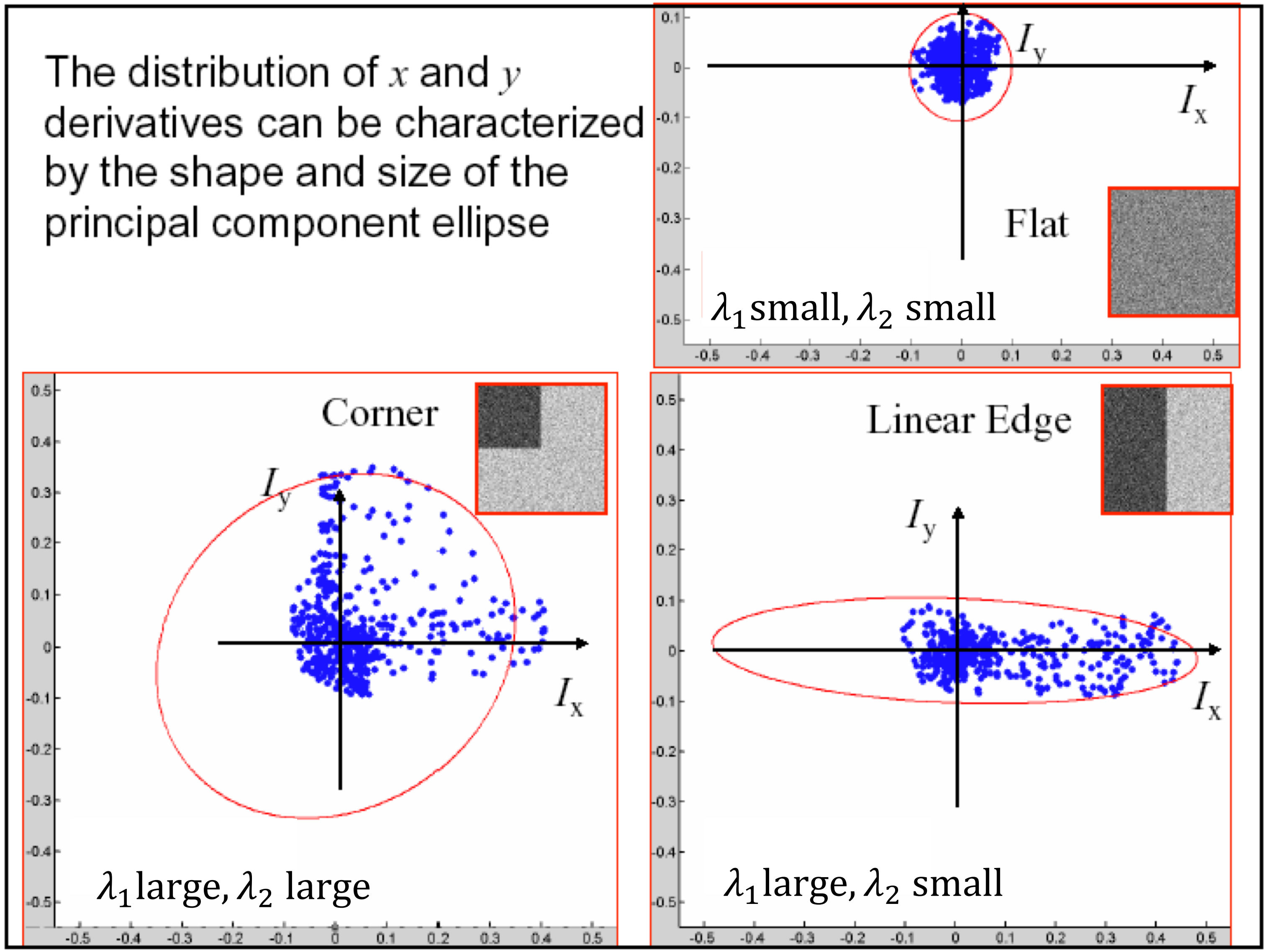
Fitting Ellipse to each Set of Points

- Recall that in PCA, the eigenvector with maximum eigenvalue represent the axis of the most variance.
- In terms of ellipse that we fit, the axis of the ellipse are the eigenvectors.
- Longer the axis larger the corresponding eigenvalue.



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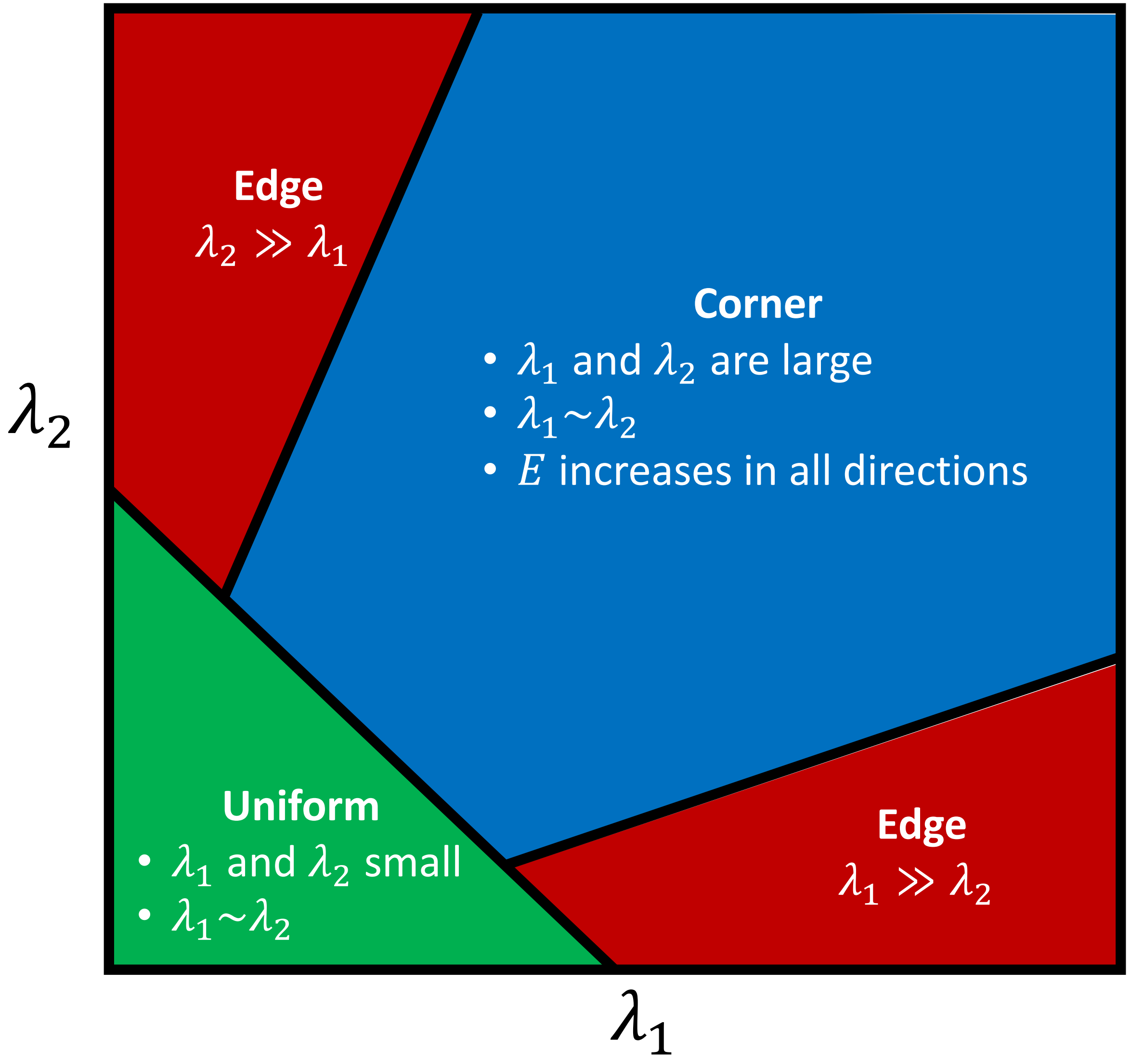
Classification of Points by Eigenvalues of M



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Corner Response Measure

- Measure of corner response:

$$R = \text{Det}(M) - k(\text{Trace}(M))^2$$

$$\text{Det}(\text{M}) = \lambda_1 \lambda_2$$

$$\text{Trace}(M) = \lambda_1 + \lambda_2$$

k is an empirically determined constant. Usually between $0.04 - 0.06$

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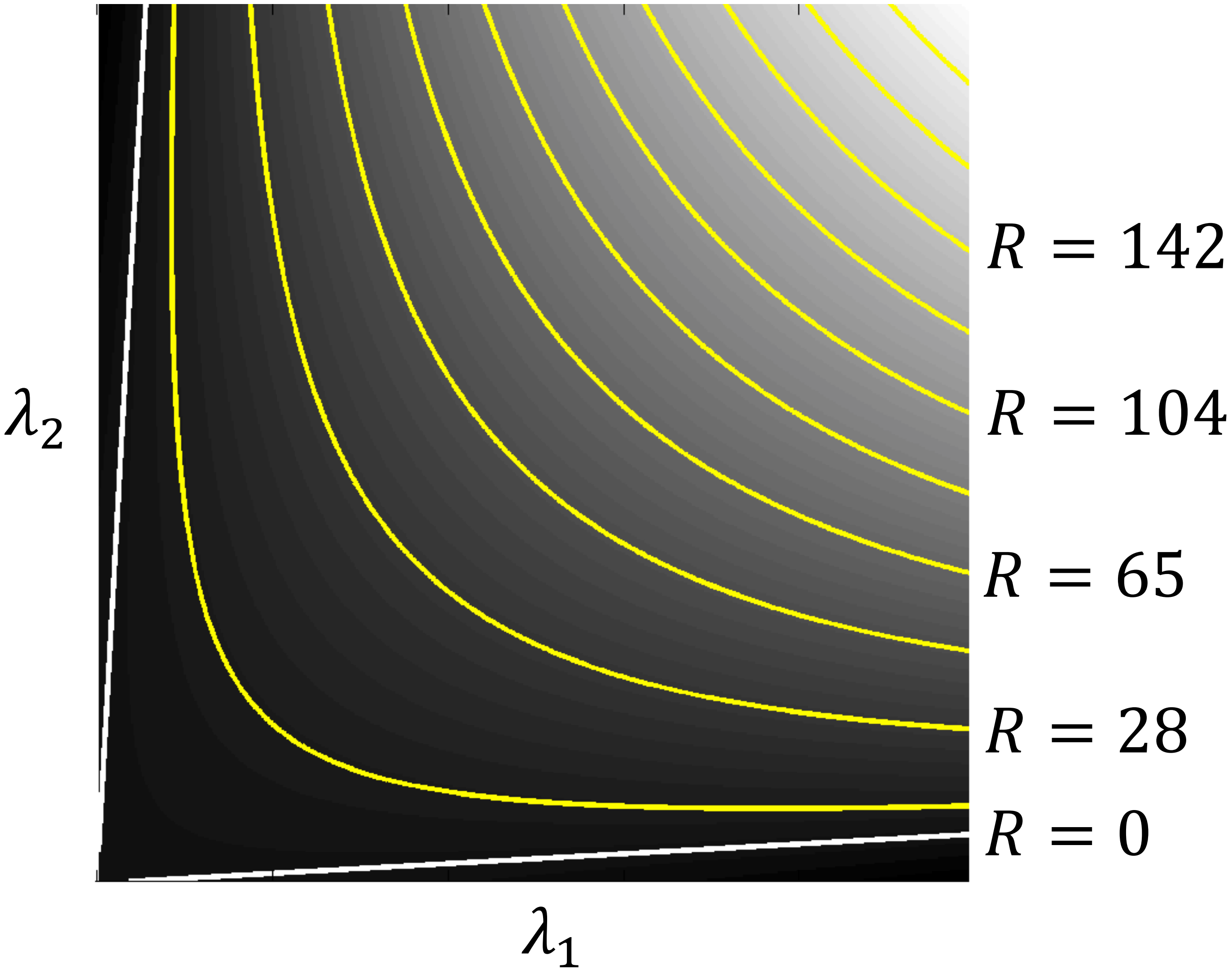
Corner Response Measure

- $R = \mathrm{Det}(M) - k(\mathrm{Trace}(M))^2$
- R depends only on the eigenvalues of M
- R is large for a corner, and negative with large magnitude for an edge.
- $|R|$ is small for uniform region.



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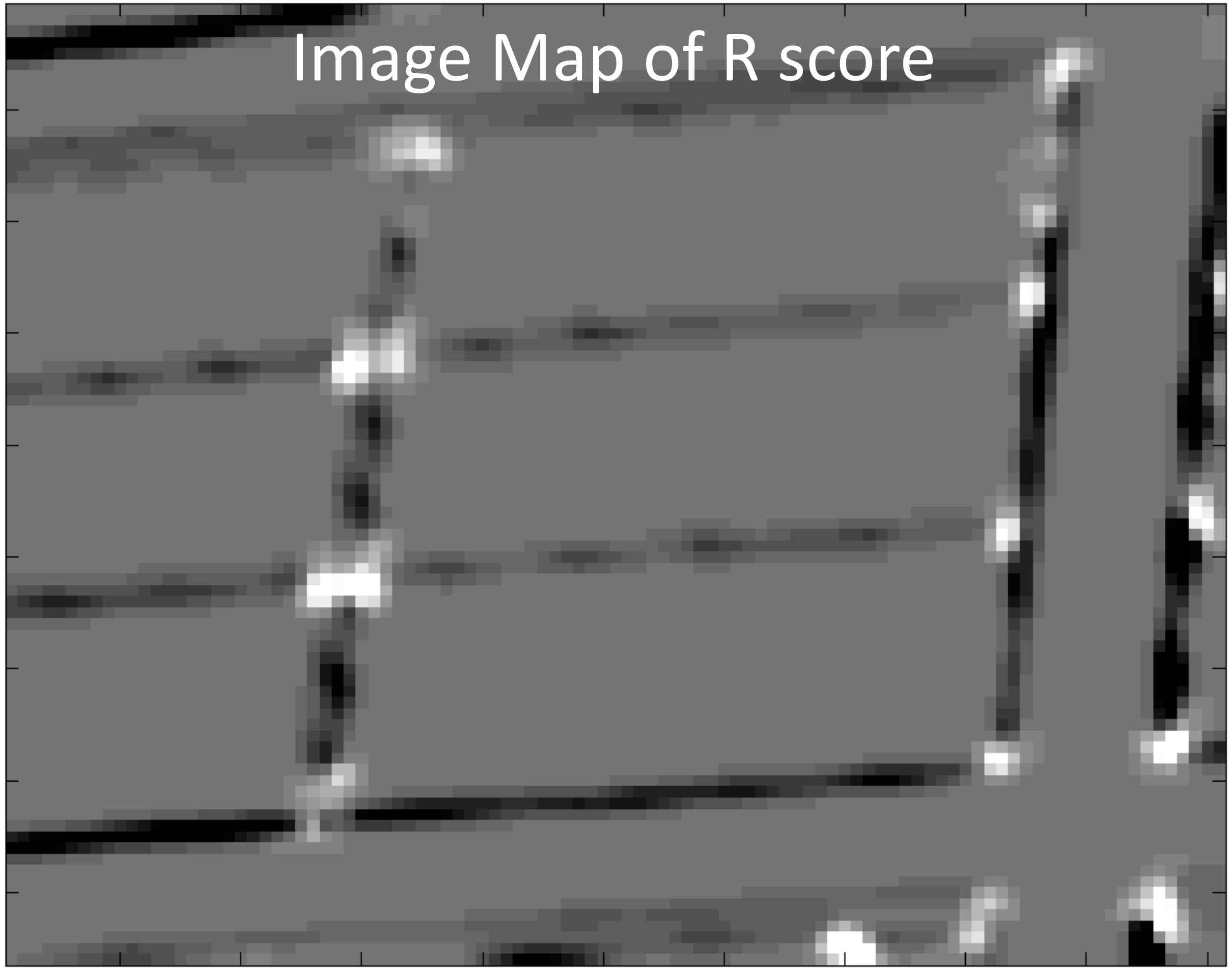
Example of Corner Response



I_x, I_y computed using Sobel. Windowing function $w = \text{Gaussian}\ (\sigma = 1)$

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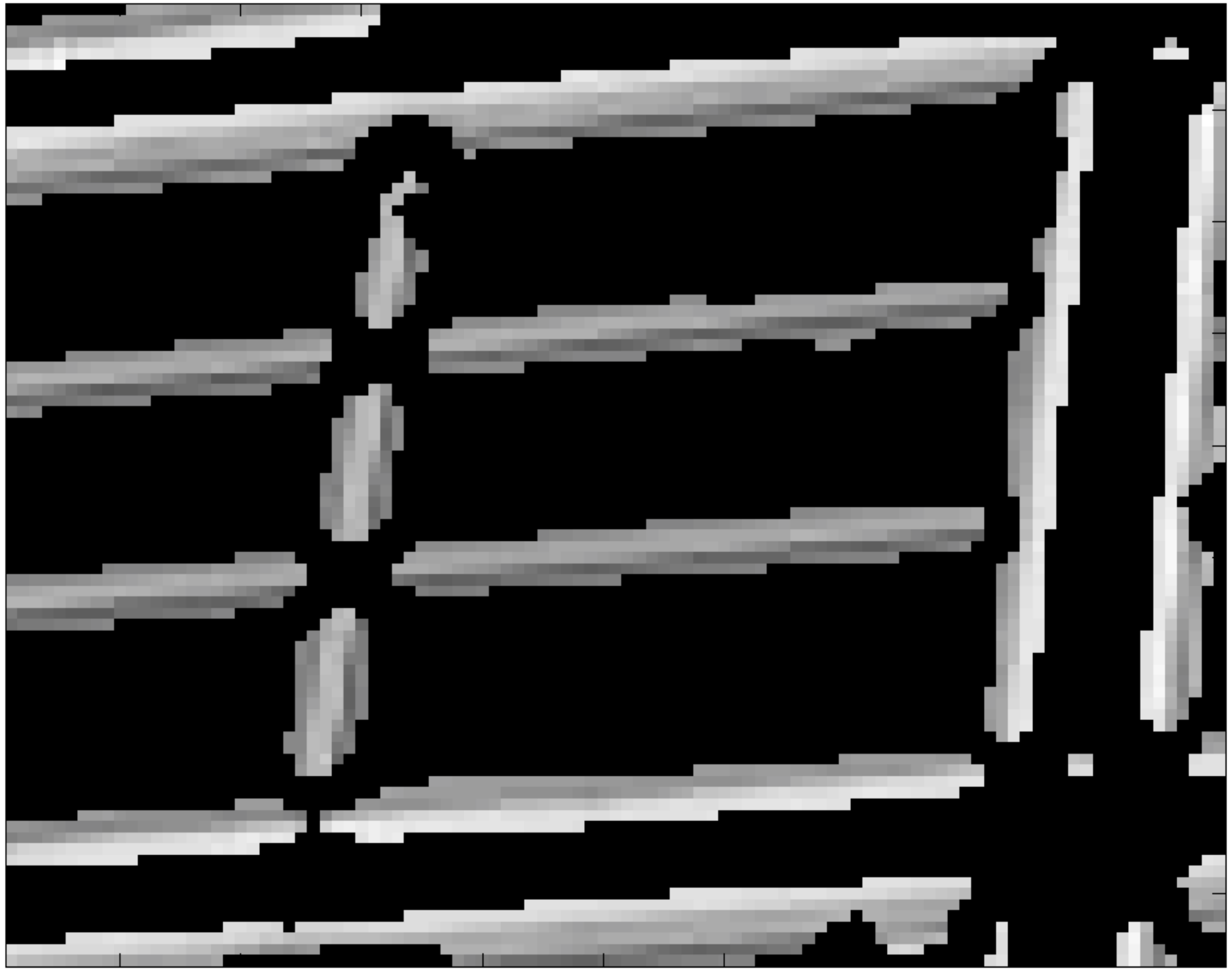
Example of Corner Response: Edges



Edges: $R < -10000$

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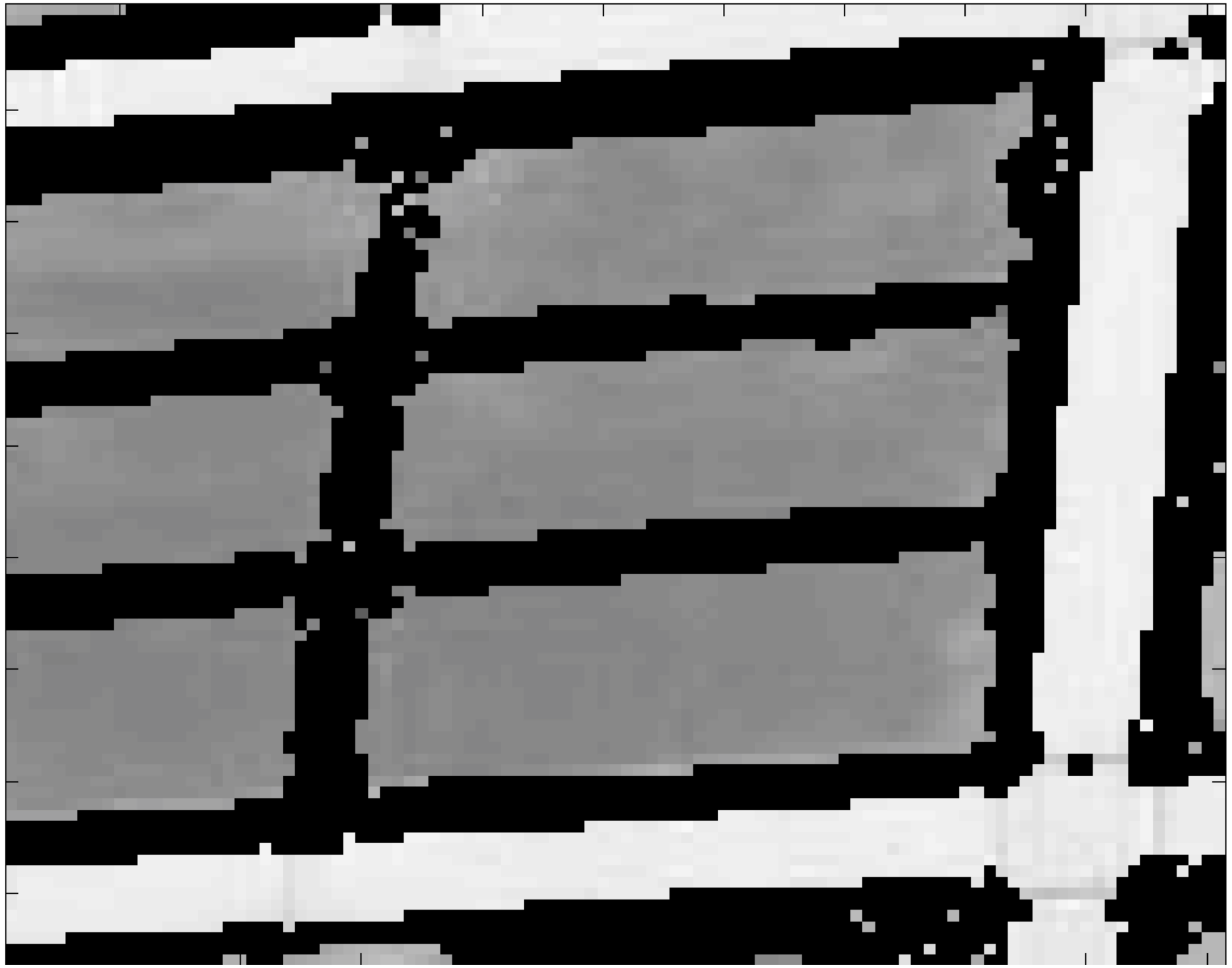
Example of Corner Response: Uniform Regions



Neither edges nor corners: $-10000 < R < 10000$

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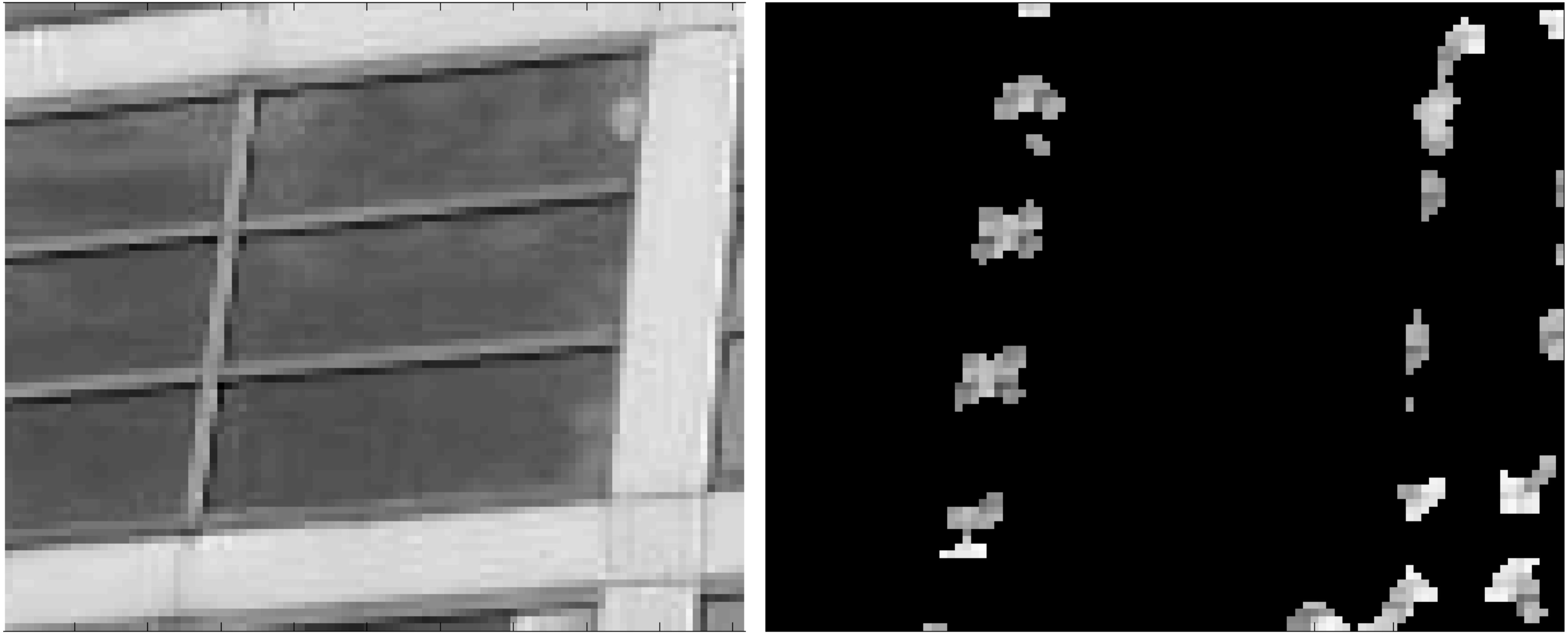
Example of Corner Response: Corners



Corners: $R > 10000$

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Harris Corner Detector: Full Algorithm

- Compute x and y derivatives of image using Sobel operator

$$I_x = G_\sigma^x * I \qquad I_y = G_\sigma^y * I$$

- Compute products of derivatives at every pixel:

$$I_{x^2} = I_x I_x \qquad I_{y^2} = I_y I_y \qquad I_{xy} = I_x I_y$$

- Compute the sums of products of derivatives with windowing function:

$$S_{x^2} = G_{\sigma'} * I_{x^2} \qquad S_{y^2} = G_{\sigma'} * I_{y^2} \qquad S_{xy} = G_{\sigma'} * I_{xy}$$

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Harris Corner Detector: Full Algorithm

- Define at each point (x, y) the matrix:

$$M(x,y) = \begin{bmatrix} S_{x^2}(x,y) & S_{xy}(x,y) \\ S_{xy}(x,y) & S_{y^2}(x,y) \end{bmatrix}$$

- Compute the response of detector at each pixel:

$$R = \mathrm{Det}(M) - k\big(\mathrm{Trace}(M)\big)^2$$

- Threshold on value of R. Compute non-maximal suppression.

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Harris Corner Detector: Invariances

- Translation Invariant?
- Rotation Invariant?
- Scale Invariant?
- Illumination Invariant?