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Multiple Linear Regression

Estimate the values of ω_1 and ω_2 by minimizing the loss function $L(\theta)$, where $\theta = [\omega_1, \omega_2]$. The loss function is defined as the mean squared error between the predicted and actual values of y:

$$\theta^* = \arg\min_\theta L(\theta)$$
$$L(\theta) = \frac{1}{N}\sum_{n=1}^{N}(f_\theta(x_n) - y_n)^2$$

where $f_\theta(x_n) = \omega_1 x_{1,n} + \omega_2 x_{2,n}$ and N is the number of data points.

Data Table:

x_1	x_2	y
-3.5	1.9	0.21
2.1	-3.3	0.01
-4.3	5.1	0.97
1.7	2.2	0.98
0.1	-0.1	0.45

We know, $w = [X^T X]^{-1} X^T Y$

Here $f_\theta(x)$ is $\rightarrow \omega_1 x_1 + \omega_2 x_2$, which means there is no intercept term. So,

$$X = \begin{bmatrix} -3.5 & 1.9 \\ 2.1 & -3.3 \\ -4.3 & 5.1 \\ 1.7 & 2.2 \\ 0.1 & -0.1 \end{bmatrix}, \quad Y = \begin{bmatrix} 0.21 \\ 0.01 \\ 0.97 \\ 0.98 \\ 0.45 \end{bmatrix}, \quad w = \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}$$

$$X^T X = \underbrace{\begin{bmatrix} -3.5 & 2.1 & -4.3 & 1.7 & 0.1 \\ 1.9 & -3.3 & 5.1 & 2.2 & -0.1 \end{bmatrix}}_{2\times 5} \underbrace{\begin{bmatrix} -3.5 & 1.9 \\ 2.1 & -3.3 \\ -4.3 & 5.1 \\ 1.7 & 2.2 \\ 0.1 & -0.1 \end{bmatrix}}_{5\times 2}$$

$$X^T X = \begin{bmatrix} 38.05 & -31.78 \\ -31.78 & 45.36 \end{bmatrix}$$

$$[X^T X]^{-1} = \frac{1}{715.98} \begin{bmatrix} 45.36 & 31.78 \\ 31.78 & 38.05 \end{bmatrix}$$

$$[X^T X]^{-1} = \begin{bmatrix} 0.06335 & 0.04438 \\ 0.04438 & 0.05314 \end{bmatrix}$$

$$[X^T X]^{-1} X^T = \begin{bmatrix} 0.06335 & 0.04438 \\ 0.04438 & 0.05314 \end{bmatrix} \begin{bmatrix} -3.5 & 2.1 & -4.3 & 1.7 & 0.1 \\ 1.9 & -3.3 & 5.1 & 2.2 & -0.1 \end{bmatrix}$$

$$[X^T X]^{-1} X^T = \begin{bmatrix} -0.137 & -0.013 & -0.045 & 0.205 & 0.0019 \\ -0.054 & -0.082 & 0.079 & 0.192 & -0.0009 \end{bmatrix}$$

$$[X^T X]^{-1} X^T Y =$$

$$\begin{bmatrix} -0.137 & -0.013 & -0.045 & 0.205 & 0.0019 \\ -0.054 & -0.082 & 0.079 & 0.192 & -0.0009 \end{bmatrix}_{2\times5} \begin{bmatrix} 0.21 \\ 0.01 \\ 0.97 \\ 0.98 \\ 0.45 \end{bmatrix}_{5\times1}$$

$$w = [X^T X]^{-1} X^T Y = \begin{bmatrix} 0.12871 \\ 0.25329 \end{bmatrix}$$

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 0.12871 \\ 0.25329 \end{bmatrix}$$

$$w_1 = 0.12871 \ , \ w_2 = 0.25329$$

Equation of best fit line $\rightarrow$ $0.12871 x_1 + 0.25329 x_2$