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Continuous wind speed models based on stochastic differential equations

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HIGHLIGHTS

- ▶ We propose two procedures to build wind speed models based on stochastic differential equations.
- ▶ The starting point of both procedures is a SDE defining an Ornstein–Uhlenbeck process.
- ▶ Proposed models show statistical properties similar to available wind speed measures.
- ▶ Proposed models can generate wind speed trajectories ranging from few minutes to several hours.
- ▶ Developed models can be coupled with wind generator models for power system dynamic studies.

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ABSTRACT

This paper proposes two general procedures to develop wind speed models based on stochastic differential equations. Models are intended to generate wind speed trajectories with statistical properties similar to those observed in the wind speed historical data available for a particular location. The developed models are parsimonious in the sense that they only use the information about the marginal distribution and the autocorrelation observed in the wind speed data. Since these models are continuous, they can be used to simulate wind speed trajectories at different time scales. However, their ability to reproduce the statistical properties of the wind speed is limited to a time frame of hours since diurnal and seasonal effects are not considered. The developed models can be embedded into dynamic wind turbine models to perform dynamic studies. Statistical properties of wind speed data from two real-world locations with significantly different characteristics are used to test the developed models.

1. Introduction

The power production of a wind power plant is affected by the statistical characteristics of the wind speed at its location. Due to the volatile and uncontrollable nature of this energy source, the power output of wind generators is a fluctuating signal. As a consequence, from the network perspective, wind power can be viewed as a source of stochastic perturbations coming from the generation side. These perturbations can affect the power quality and they must be taken into account in power system dynamic studies. The uncertainty in the power production of wind power plants has also economic implications since this kind of generation cannot be dispatched in a conventional way. To deal with these issues, careful studies have to be performed in which the appropriate representation of the wind variability represents a key modeling aspect. This paper is devoted to the development of mathematical models able to reproduce the wind speed behavior.

The wind speed at a specific location is characterized by its statistical properties. Through the analysis of recorded historical data, the marginal distribution of the wind speed can be estimated. Several probability distributions have been proposed to describe the wind speed behavior (e.g., [1–6]). From the studies reported in the literature, it can be concluded that the type of probability distribution depends on the particular location. For long time scales, the two-parameter Weibull distribution has shown a good fit to the observed wind speed empirical distribution in many locations around the world [2]. However, for wind speed fluctuations in time scales shorter than 10 min, turbulent effects take place, and the Weibull distribution is not considered a good fit [4,7].

Another characteristic observed from the wind speed data is that wind speed is an autocorrelated (two-point time-correlated) variable. The autocorrelation function of the wind speed is usually characterized by an exponential decay in the range of hours.

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Outside this time frame, non-stationary phenomena related to diurnal and seasonal effects can be observed [8–11].

Therefore, wind speed models have to reproduce the statistical properties discussed above in order to provide reliable results in wind power studies.

Wind speed models are of interests in dynamic studies and control of wind turbines (e.g., [12–16]), in generating capacity reliability evaluation (e.g., [17–19]), and in power system economics and operation (e.g., [20–22]). Techniques used in these areas include wind speed trajectories generation, wind speed forecasting, Monte Carlo simulation and wind speed scenarios. These techniques can accommodate different wind speed models. In this regard, reference [23] reports the state of the art of wind speed models used in power system dynamic analysis, whereas [24] provides a bibliographical survey on wind speed forecasting methods and models. Reference [25] uses Monte Carlo simulations to compare the performance of different wind speed models in the context of reliability evaluation of power systems. Finally, a general methodology to generate wind speed scenarios is proposed in [26].

Among the wind speed models used in different research fields, discrete models used in time-series analysis are the most common. These models are based on Box–Jenkins methods and include Auto-Regressive (AR), Moving Average (MA), Auto-Regressive Moving Average (ARMA), and Auto-Regressive Integrated Moving Average (ARIMA) models. The ability of these models to reproduce the statistical properties of the wind speed for a particular site depends on the wind speed data available and on the time frame of interest [24]. Other models include four-component composite model [27], models based on filters and wind power spectral density [12], and Markov chain models [28]. Recently, wind speed fluctuation models constructed from the solution of the stationary Fokker–Plank equation have been proposed in [29].

In this paper, two continuous wind speed models based on stochastic differential equations (SDEs) are developed. The starting point of the development of both models is a SDE defining an Ornstein–Uhlenbeck process. This process can be considered as the continuous-time equivalent of the discrete-time AR process of order one, AR (1), [30]. The developed models are intended to generate wind speed trajectories with similar statistical properties to those observed in the wind speed data available at a particular location. The developed models take into account the marginal distribution and the autocorrelation function of the wind speed. To illustrate the proposed procedures, the two-parameter Weibull distribution is used in the development of the models. However, it is important to note that the proposed methodology is not limited to the Weibull distribution and other probability distributions can be used.

Therefore, the developed models provide autocorrelated Weibull distributed wind speed trajectories. As they are continuous models, they can in principle be used to generate wind speed trajectories at any time scale. However, since the models strive to reproduce the observed exponential autocorrelation feature, their ability to reproduce the wind speed behavior is expected to be acceptable only in the time frame where the wind speed remains exponentially autocorrelated, typically for a few hours. Furthermore, the developed models can be used to perform short term dynamic studies. In these studies, the inclusion of autocorrelations into wind speed trajectories is critical.

For simulation purposes, each developed model can be used as an independent "block" that generates wind speed trajectories. Another possibility in the context of dynamic simulations is to merge the equations of the wind speed model within the differential equations of the wind generator.

The remainder of the paper is organized as follows. Section 2 briefly introduces SDEs and provides the main theoretical methods used for developing the wind speed models. Section 3 describes the developed models and their properties. Section 4 discusses the generation of wind speed trajectories and illustrates their statistical properties through numerical simulations. Finally, Section 5 provides relevant conclusions.

2. Brief outline on stochastic differential equations and stochastic calculus

Stochastic differential equations (SDEs) are widely used to model stochastic phenomena in several fields of science, engineering and finance (see, for example, [31,32]). This section defines SDEs and provides the relevant stochastic calculus background that will be used to build our wind models. The interested reader can find theoretical background on SDEs and stochastic calculus in [33–37].

2.1. Definition of stochastic differential equations

A one-dimensional stochastic differential equation has the general form

$$\begin{aligned} dy(t) &= a[y(t),t]dt + b[y(t),t]dW(t), \quad t \in [0,T], \\ y(0) &= y_0, \end{aligned} \tag{1}$$

where $a[y(t),t]$ and $b[y(t),t]$ are the drift and the diffusion terms of the SDE, respectively, and $W(t)$ represents a standard Wiener process. This kind of equation can be viewed as an ordinary differential equation where an additional term is included to model the stochastic dynamical behavior related to variable $y(t)$. The standard Wiener process $(W(t), t \in [0,+\infty))$ is a non-stationary diffusion process with the following characteristics [34]:

- $W(0) = 0$, with probability 1.
- The function $t \mapsto W(t)$ is almost surely continuous.
- For $0 \leqslant t_i < t_{i+1} \leqslant T$, the random variable defined by the increments $\Delta W_i = W(t_{i+1}) - W(t_i)$ is Gaussian distributed with mean zero and variance $h = t_{i+1} - t_i$, i.e., $\Delta W_i \sim \mathcal{N}(0,h)$.
- For $0 \leqslant t_i < t_{i+1} < t_{i+2} \leqslant T$, the non-overlapping increments $\Delta W_i = W(t_{i+1}) - W(t_i)$ and $\Delta W_{i+1} = W(t_{i+2}) - W(t_{i+1})$ are independent.

Therefore, a standard Wiener process describes a continuous Gaussian process whose sample paths (increments) are of unbounded variation [36]. Another characteristic of the Wiener process is that $W(t)$ is nowhere differentiable. Therefore, despite the fact that the differential formulation is widely used in the literature (and it will be used throughout this paper), in a strictly mathematical sense Eq. (1) is not fully correct. Actually, the only correct formulation for (1) is its equivalent integral form

$$y(t) = y(0) + \int_0^t a[y(s),s]ds + \int_0^t b[y(s),s]dW(s), \quad t \in [0,T], \tag{2}$$

where the first integral is an ordinary Riemann–Stieltjes integral and the second one is a stochastic integral. Due to the unbounded variation of the sample paths of the Wiener process, stochastic integrals cannot be interpreted as Riemann–Stieltjes integrals. In this regard, there are mainly two different interpretations of stochastic integrals: the Itô's and the Stratonovich's approaches. In order to exploit the advantages of the Itô's calculus, in this paper stochastic integrals will be interpreted in the Itô's sense.

In the general case, SDEs have to be solved numerically. Numerical methods for SDE can show two types of convergence: strong and weak. Strong convergence refers to the goodness of the approximation when the focus is on the process trajectories themselves. On the other hand, weak convergence refers to the goodness of the approximation of the moments of the solutions to the moments of the process. Ref. [38] provides a detailed description of the available methods for the numerical solution of SDEs.

2.2. Ornstein–Uhlenbeck process

The simplest SDE arises from (1) if $a[y(t),t] = 0$ and $b[y(t),t] = b =$ constant, that is,

$$\begin{aligned} dy(t) &= bdW(t), \quad t \in [0,T], \\ y(0) &= 0. \end{aligned} \tag{3}$$

With the assumptions above, the trajectories of $y(t)$ are similar to those of a standard Wiener process. Therefore, the main statistical properties (first and second order moments) of $y(t)$ are [37]:

$$\begin{aligned} &\mathrm{E}[y(t)] = 0, \quad \forall t \in [0,T], \\ &\mathrm{Var}[y(t)] = b^2 t, \quad \forall t \in [0,T], \\ &\mathrm{Aut}[y(t_i), y(t_j)] = \min(t_i, t_j), \quad \forall t_i, t_j \in [0,T], \end{aligned}$$

where $\mathrm{E}[\cdot]$ is the mean, $\mathrm{Var}[\cdot]$ is the variance, and $\mathrm{Aut}[\cdot]$ is the autocorrelation function. As a result, $y(t)$ is a non-stationary diffusion process distributed as $\mathcal{N}(0, b^2 t)$.

A stationary and two-point correlated diffusion process, the so-called Ornstein–Uhlenbeck process, can be defined by adding a drift term of the form $-\alpha y(t)$ to Eq. (3) or, equivalently, by setting $a[y(t),t] = -\alpha y(t)$ and $b[y(t),t] = b =$ constant in (1), leading to the following SDE:

$$dy(t) = -\alpha y(t)dt + bdW(t), \quad t \in [0,T]. \tag{4}$$

If the initial condition of (4) is

$$y(0) \sim \mathcal{N}(0, b^2/2\alpha), \tag{5}$$

the resulting process has the following statistical properties [37]:

$$\begin{aligned} &\mathrm{E}[y(t)] = 0, \quad \forall t \in [0,T], \\ &\mathrm{Var}[y(t)] = b^2/2\alpha, \quad \forall t \in [0,T], \\ &\mathrm{Aut}[y(t_i), y(t_j)] = e^{-\alpha|t_j - t_i|}, \quad \forall t_i, t_j \in [0,T]. \end{aligned}$$

Therefore, $y(t)$ is a stationary autocorrelated Gaussian diffusion process distributed as $\mathcal{N}(0, b^2/2\alpha)$.

In this paper, the SDE (4) is used as a starting point to developing two models defining an autocorrelated Weibull distributed process aimed to reproduce the wind speed behavior.

2.3. Change of variables in stochastic calculus: The Itô's formula

The Itô's formula is an adaptation of the deterministic chain rule to the stochastic calculus. This formula allows computing derivatives of functions involving variables defined by SDEs of the type (1). For a variable $x(t)$ expressed as an arbitrary function $g[\cdot]$ of the stochastic variable $y(t)$ defined by (1)

$$x(t) = g[y(t), t], \tag{6}$$

the stochastic dynamic behavior of $x(t)$ obeys the following SDE:

$$\begin{aligned} dx(t) &= c[x(t),t]dt + d[x(t),t]dW(t), \quad t \in [0,T], \\ x(0) &= g[y(0)], \end{aligned} \tag{7}$$

with

$$c[x(t),t] = \frac{\partial g}{\partial t}[y(t),t] + a[y(t),t]\frac{\partial g}{\partial y}[y(t),t] + \frac{1}{2}b^2[y(t),t]\frac{\partial^2 g}{\partial y^2} \times [y(t),t], \tag{8}$$

$$d[y(t),t] = b[y(t),t]\frac{\partial g}{\partial y}[y(t),t], \tag{9}$$

$$y(t) = g^{-1}[x(t),t], \tag{10}$$

where $a[y(t),t]$ and $b[y(t),t]$ are the drift and the diffusion terms of (1), respectively [38].

By an appropriate choice of the function $g[\cdot]$ and the process $y(t)$, the Itô's formula is applied in this paper to construct a SDE able to generate a stochastic process with statistical properties similar to those observed in wind speed measurements.

2.4. Memoryless transformations: Translation processes

Memoryless transformations are used to obtain non-Gaussian stochastic processes from Gaussian stochastic processes. For a Gaussian process $y(t)$, the transformation

$$x(t) = g[y(t)], \tag{11}$$

is said to be a memoryless transformation since the value of the new process $x(t)$ at an arbitrary instant t depends only on the value of $y(t)$ at t. As a result, process $x(t)$ is not Gaussian unless function $g[\cdot]$ is linear. Because $y(t)$ is a Gaussian process, the statistical properties of the process $x(t)$ only depend on the first and second order moments of $y(t)$ in a way which is controlled by the function $g[\cdot]$. Furthermore, if process $y(t)$ is stationary, then $x(t)$ is also stationary [39].

For the purpose of this paper, the memoryless transformations of interest are those related to translation processes. A translation process is the result of a memoryless transformation (11) where $g(\cdot)$ is given by

$$x(t) = g[y(t)] = F^{-1}[\Phi[y(t)]], \tag{12}$$

where $\Phi[\cdot]$ is the standard Gaussian cumulative distribution function and $F[\cdot]$ represents a continuous cumulative distribution function other than Gaussian. By using the transformation (12), we generate a stochastic process $x(t)$ whose distribution matches the distribution F [39,40].

The relationship between the autocorrelation functions of processes $x(t)$ and $y(t)$ as a result of transformation (12) is more complicated. In the general case, it is not always possible to find an autocorrelation function for the Gaussian process $y(t)$ that leads to a pre-specified autocorrelation function for $x(t)$. On the other hand, autocorrelation functions of $x(t)$ and $y(t)$ satisfy the following inequality [39,40]:

$$|\mathrm{Aut}[x(t)]| \leqslant |\mathrm{Aut}[y(t)]|. \tag{13}$$

In this paper, the relationship between $\mathrm{Aut}[x(t)]$ and $\mathrm{Aut}[y(t)]$ is analyzed numerically.

The translation process (12), where $F[\cdot]$ is the Weibull cumulative distribution function, has been used to model the wind speed in the context of time series, e.g., in [11,26]. This translation process is also applied to the development of the wind speed models discussed in the paper.

3. Autocorrelated Weibull distributed stochastic processes

In this section, two models that define autocorrelated Weibull distributed stochastic processes are described. Both models are based on the Ornstein–Uhlenbeck process (4) and on the application of a memoryless transformation. In the first model, the memoryless transformation is applied after the integration of (4). In the second model, the memoryless transformation is applied directly to (4), and a new SDE defining the desired stochastic process is obtained through the application of the Itô's formula (Section 2.3). Both methods provide a stochastic process with identical statistical properties.

3.1. Model I

As stated previously, the starting point for Model I is the SDE (4), that is,

$$\begin{aligned} dy(t) &= -\alpha y(t)dt + b dW(t), \quad t \in [0,T], \\ y(0) &\sim \mathcal{N}(0, b^2/2\alpha). \end{aligned} \tag{14}$$

By means of the numerical integration of (14) in a time interval $[0,T]$, trajectories $y(t)$ of an Ornstein–Uhlenbeck process are obtained at discrete times $t_i \in [0,T]$. As pointed out in Section 2.2, the Ornstein–Uhlenbeck process is an exponentially autocorrelated Gaussian process distributed as $\mathcal{N}(0, b^2/2\alpha)$. Therefore, to obtain an autocorrelated Weibull distributed process, a memoryless transformation is applied to the trajectories $y(t)$ as follows

$$x(t_i) = g[y(t_i)] = F_W^{-1}\left[\Phi\left[\frac{y(t_i)}{b/\sqrt{2\alpha}}\right]\right], \tag{15}$$

where $F_W[\cdot]$ represents the Weibull cumulative distribution function, i.e.,

$$F_W[u] = 1 - \exp\left[\left(\frac{u}{\lambda}\right)^k\right], \quad \forall u > 0, \tag{16}$$

with $\lambda > 0$ and $k > 0$, the scale and shape parameters of the Weibull distribution, respectively, and $\Phi[\cdot]$ is the Gaussian cumulative distribution function, i.e.,

$$\Phi\left[\frac{u - \mathrm{E}[u]}{\sqrt{\mathrm{Var}[u]}}\right] = \frac{1}{2}\left(1 + \mathrm{erf}\left[\frac{u - \mathrm{E}[u]}{\sqrt{2\mathrm{Var}[u]}}\right]\right), \quad \forall u \in \mathbb{R}. \tag{17}$$

Observe that, in (15), it has been taken into account that $y(t)$ is distributed as $\mathcal{N}(0, b^2/2\alpha)$. The resulting process $x(t)$ is an autocorrelated Weibull distributed stochastic process.

3.2. Model II

In this section a new SDE defining the desired process is constructed. As in Model I, the starting point is the SDE (14) defining the Ornstein–Uhlenbeck process. This SDE is transformed into a new SDE defining a Weibull distributed process by applying Itô's formula (Section 2.3) to the memoryless transformation

$$x(t) = g[y(t)] = F_W^{-1}\left[\Phi\left[\frac{y(t)}{b/\sqrt{2\alpha}}\right]\right], \tag{18}$$

where $F_W[\cdot]$ represents the Weibull cumulative distribution function (16), and $\Phi[\cdot]$ is the Gaussian cumulative distribution function (17).

From SDE (14), the application of Itô's formula to (18) leads to the following SDE:

$$dx(t) = c[x(t)]dt + d[x(t)]dW(t), \quad t \in [0,T], \tag{19}$$

where

$$c[x(t)] = -\alpha y(t)\frac{\partial g}{\partial y}[y(t)] + \frac{1}{2}b^2\frac{\partial^2 g}{\partial y^2}[y(t)], \tag{20}$$

$$d[y(t)] = b\frac{\partial g}{\partial y}[y(t)], \tag{21}$$

$$y(t) = (b/\sqrt{2\alpha})\Phi^{-1}[F_W[x(t)]], \tag{22}$$

and initial condition

$$x(0) = F_W^{-1}\left[\Phi\left[\frac{y(0)}{b/\sqrt{2\alpha}}\right]\right]. \tag{23}$$

The expression for the derivatives in (20) and (21) are as follows:

$$\begin{aligned} \frac{\partial g}{\partial y}[y(t)] = \frac{1}{\sqrt{\pi}}\frac{\lambda}{k}\frac{\sqrt{\alpha}}{b}&\frac{\left(-\ln\left[1-\Phi\left[\frac{y(t)}{b/\sqrt{2\alpha}}\right]\right]\right)^{\frac{1-k}{k}}}{1-\Phi\left[\frac{y(t)}{b/\sqrt{2\alpha}}\right]} \\ &\times \exp\left[-\frac{\alpha}{b^2}y^2(t)\right], \end{aligned} \tag{24}$$

$$\begin{aligned} \frac{\partial^2 g}{\partial y^2}[y(t)] = \frac{1}{\pi}\frac{\lambda}{k}\frac{\alpha}{b^2}\left(\frac{1-k}{k}\right)&\frac{\left(-\ln\left[1-\Phi\left[\frac{y(t)}{b/\sqrt{2\alpha}}\right]\right]\right)^{\frac{1-2k}{k}}}{\left(1-\Phi\left[\frac{y(t)}{b/\sqrt{2\alpha}}\right]\right)^2} \\ &\times \exp\left[-\frac{2\alpha}{b^2}y^2(t)\right] + \frac{1}{\pi}\frac{\lambda}{k}\frac{\alpha}{b^2} \\ &\times \frac{\left(-\ln\left[1-\Phi\left[\frac{y(t)}{b/\sqrt{2\alpha}}\right]\right]\right)^{\frac{1-k}{k}}}{\left(1-\Phi\left[\frac{y(t)}{b/\sqrt{2\alpha}}\right]\right)^2}\exp\left[-\frac{2\alpha}{b^2}y^2(t)\right] \\ &- \frac{1}{\sqrt{2\pi}}\frac{\lambda}{k}\left(\frac{\sqrt{2\alpha}}{b}\right)^3 y(t)\frac{\left(-\ln\left[1-\Phi\left[\frac{y(t)}{b/\sqrt{2\alpha}}\right]\right]\right)^{\frac{1-k}{k}}}{1-\Phi\left[\frac{y(t)}{b/\sqrt{2\alpha}}\right]} \\ &\times \exp\left[-\frac{\alpha}{b^2}y^2(t)\right]. \end{aligned} \tag{25}$$

The numerical integration of the SDE (19) provides an autocorrelated Weibull distributed stochastic process $x(t)$.

3.3. Statistical properties of the resulting process

As pointed out above, both Model I and Model II lead to stochastic processes $x(t)$ with the same statistical properties, as follows:

$$\mathrm{E}[x(t)] = \mu_W, \quad \forall t \in [0,T], \tag{26}$$

$$\mathrm{Var}[x(t)] = \sigma_W^2, \quad \forall t \in [0,T], \tag{27}$$

$$\mathrm{Aut}[x(t_i),\ x(t_j)] \approx e^{-\alpha|t_j - t_i|}, \quad \forall t_i, t_j \in [0,T], \tag{28}$$

where μ_W and σ_W^2 are the mean and the variance, respectively, of the Weibull distribution (16), that is,

$$\mu_W = \lambda\Gamma\left[1 + \frac{1}{k}\right], \tag{29}$$

$$\sigma_W^2 = \lambda^2\Gamma\left[1 + \frac{2}{k}\right] - \mu_W^2, \tag{30}$$

with $\Gamma[\cdot]$ the Gamma function. Eq. (28) states that the autocorrelation of the process $x(t)$ is well approximated by the autocorrelation of the Ornstein–Uhlenbeck process. This result has been determined empirically by analyzing a number of simulations. The goodness of this approximation depends on the parameters of the Weibull distribution, as it is discussed in Section 4.1.2. Therefore, the process $x(t)$ takes the moments of the Weibull distribution used in the memoryless transformation and shows a similar exponential autocorrelation to that of the Ornstein–Uhlenbeck process. These results are corroborated by the simulations discussed in Section 4.1.

It should be noted that the only parameters needed in Models I and II are λ, k, and α, which are the parameters that determine the statistical properties (26)–(28). Parameters λ and k are directly taken from the Weibull fit of the wind speed data, while α can be easily computed from Eq. (28) once the autocorrelation of the wind speed is known. Parameter b in Models I and II does not affect the statistical properties (26)–(28) of the process $x(t)$. Therefore, its value can be arbitrary chosen. For simplicity, in this paper parameter b is set to $b = \sqrt{2\alpha}$.

As we mentioned before, the procedure applied to develop the equations involved in the wind speed models is not constrained to the Weibull distribution. This feature is useful if the wind speed of a particular site is better described by distribution functions other than the Weibull one.

4. Generation of wind speed trajectories

The process of generating wind speed trajectories consists in either integrating the SDE (14) and subsequently applying transformation (15) (Model I), or in integrating directly the SDE (19) (Model II). In this section, both models are used for generating wind speed trajectories.

To solve (14) and (19) a numerical integration method is applied. In this paper, an implicit integration scheme belonging to family of implicit Milstein schemes is used, as follows:

$$x(t+h) = x(t) + \frac{h}{2}[a[x(t+h)] + a[x(t)]] + b[x(t)]\Delta W + \frac{1}{2}b[x(t)]\frac{\partial b}{\partial x}[x(t)]((\Delta W)^2 - h), \quad (31)$$

where $a[\cdot]$ and $b[\cdot]$ are the drift and diffusion terms, respectively, of the corresponding SDE, h is the integration time step and $\Delta W \sim \mathcal{N}(0,h)$ are random increments of the Wiener process. Other available numerical integration methods for SDEs can be found, for example, in [38].

As in the deterministic case, the convergence error (as well as the consistency and the numerical stability) of a numerical integration method for SDEs depends on the integration step length h. In the case of the implicit Milstein scheme (31) the convergence error is of order 1 ($O(h)$) in both the weak and the strong senses. Therefore, an appropriate integration step length h must be chosen to obtain a good approximation to the real trajectories and the statistical properties of $x(t)$ while maintaining a reasonable computational burden.

The marginal distribution and the autocorrelation of the wind speed are typically determined on the basis of hourly or averaged hourly measurement data. As a result, autocorrelation values are usually computed for time lags expressed in hours. Therefore, if the interest is in generating wind speed trajectories for dynamic studies, autocorrelation factors must be adapted to the time frame of seconds. In the proposed methods, this adaptation is directly done by properly scaling the corresponding hourly-based value of parameter α.

4.1. Simulations

To illustrate the performance of the developed models, the parameters of a Weibull fit and the 1-h autocorrelations of the wind speed data of two locations in Canada reported in [1,10] are used. The two sites are Cape St. James and Victoria Airport, and have been chosen because they show significantly different wind speed characteristics, as it can be observed in Table 1.

To test the statistical properties of the processes obtained with the developed models, 5000 wind speed trajectories are generated for a time frame of 24 h for each model. This time frame is chosen to illustrate the exponential decay of the autocorrelation of the processes obtained. For these simulations, a time scale of hours is considered and the integration step is set to $h = 5/60 = 0.0833$ h, i.e., 5 min. Parameters λ, k, and α in Eqs. (14) and (15) for Model I, and in the Eq. (19) for Model II, respectively, are set according to the data in Table 1. The α values are obtained from the autocorrelation function (28). They are $\alpha = 0.0954$ [1/h], for Cape St. James, and $\alpha = 0.2575$ [1/h], for Victoria Airport, respectively.

Table 1
Weibull parameters and 1-h autocorrelation of wind speed data [1,10].

Location	λ	k	Autocorrelation
Cape St. James	10.67	1.77	0.909
Victoria Airport	3.36	1.51	0.773

4.1.1. Marginal distribution

Fig. 1 shows the Weibull cumulative distribution functions and the cumulative probabilities of the processes generated by the developed models for the two locations. The upper part of this Figure shows the fit with the distribution function of wind speeds at Victoria Airport, whereas the lower part shows the fit with the distribution function of wind speeds at Cape St. James. These results correspond to the fit at hour 24, but similar results can be found at each point of the simulated time period ([0,24] h).

As it is observed in Fig. 1, the wind speed data generated by the developed models follow the corresponding Weibull distributions. This is also corroborated by Figs. 2 and 3, where the mean and the standard deviation of the wind speed trajectories generated by the developed models are compared with the expected theoretical mean (26) and standard deviation (square root of (27)) of the Weibull distribution, respectively. In both Figs. 2 and 3, the upper part corresponds to Cape St. James, while the lower part to Victoria Airport.

4.1.2. Autocorrelation function

Fig. 4 depicts the autocorrelation of the wind speeds generated by the developed models together with the autocorrelation of the Ornstein–Uhlenbeck process. As in the previous figures, the upper and lower curves correspond to the autocorrelation of the wind speeds generated with model parameters extracted from Cape St. James and Victoria Airport wind data, respectively. As it can be observed, the wind speed autocorrelation at Victoria Airport decays faster than the wind speed autocorrelation at Cape St. James, which is in concordance with the data in Table 1. Observe also that the autocorrelation of the wind speed data generated by the developed models approximates the exponential decay of the Ornstein–Uhlenbeck process.

As we pointed out before, the autocorrelation of the generated Weibull-distributed process is well approximated by the autocorrelation of the Ornstein–Uhlenbeck process. Here, we analyze numerically the parameter range in which this approximation holds. While empirical, this analysis is nevertheless important, since the parameter α in (14) for Model I, and in (19) for Model

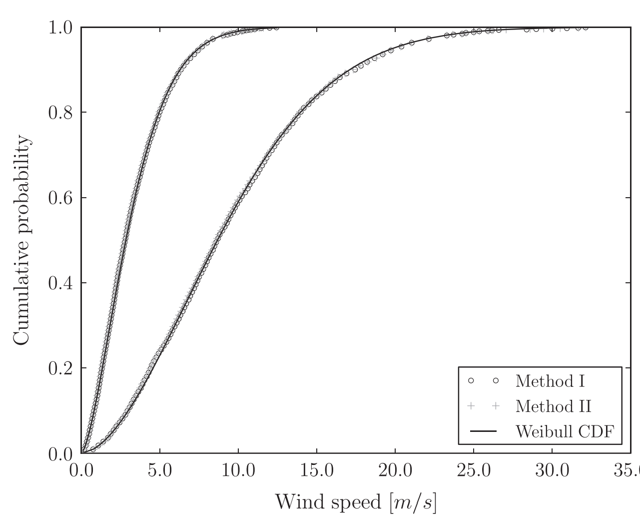

Fig. 1. Weibull cumulative distribution function vs. cumulative probabilities of the generated processes.

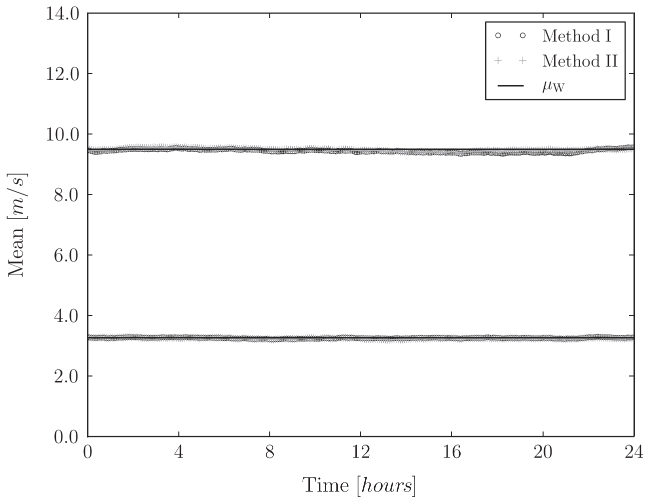

Fig. 2. Mean value of the Weibull distribution vs. mean value of the generated processes.

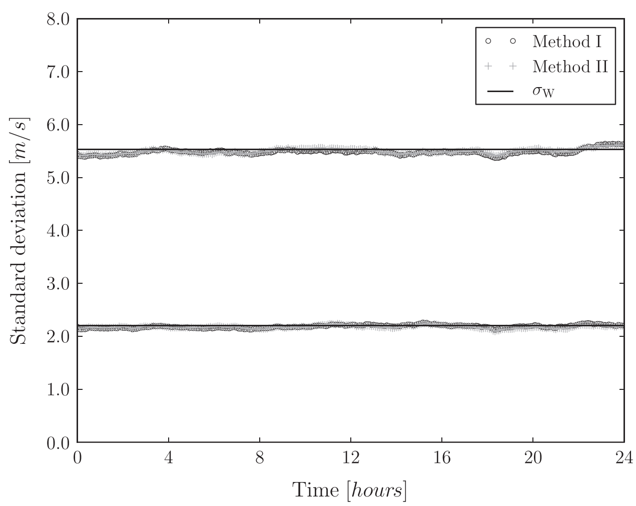

Fig. 3. Standard deviation of the Weibull distribution vs. standard deviation of the generated processes.

II, is computed assuming that the autocorrelation of the resulting processes is the exponential autocorrelation of the Ornstein–Uhlenbeck process. The deviation in the value of the autocorrelation of both processes is computed for different values of the shape parameter k of the Weibull distribution. In this study, the scale parameter of the Weibull fit of Cape St. James and of Victoria Airport is fixed, whereas the shape parameter k is progressively increased from 0.5 to 4.0 in steps of 0.1. The deviation between both autocorrelation values is evaluated in terms of the Root Mean Square Error (RMSE). The autocorrelation values are computed by averaging over 5000 trajectories simulated using Model I.

Fig. 5 depicts the results of this analysis. It can be observed that the deviation between autocorrelation values depends on the value of the shape parameter k of the Weibull distribution. The autocorrelation of the generated Weibull distributed process is better approximated by the autocorrelation of the Ornstein–Uhlenbeck process as the value of k increases. The values of parameter k of the Weibull fit for wind speeds at different locations reported in the literature (e.g., [1–3,6]) are usually in the range [1.3,2.5]. Therefore, approximating the autocorrelation of the Weibull process with the exponential autocorrelation of the Ornstein–Uhlenbeck process is acceptable. Finally, extensive simulations have also shown that the contribution of the scale parameter λ to the deviation of autocorrelation values is marginal in the usual range of values for the shape parameter k.

4.1.3. Trajectories for dynamic studies

The previous results have shown that Model I and Model II provide wind speed trajectories with identical statistical properties. An obvious question is: Which one is the most appropriate from a practical point of view? For applications where power system dynamics are not of interest, Model I is preferred due to its simplicity. On the other hand, dynamic analyses include device models based on a set of differential or differential–algebraic equations. In these cases, the equations of both models can be coupled with the device models. In this respect, Model I involves one SDE and one algebraic equation, whereas Model II involves just one SDE. Therefore, Model II is to be preferred in situations where it is of interest to maintain a pure differential structure in the model.

As stated in the introduction of this paper, the developed models are flexible enough to accommodate different time scales without major adjustments. To show this property, the hourly-based data available from Cape St. James and Victoria Airport were used

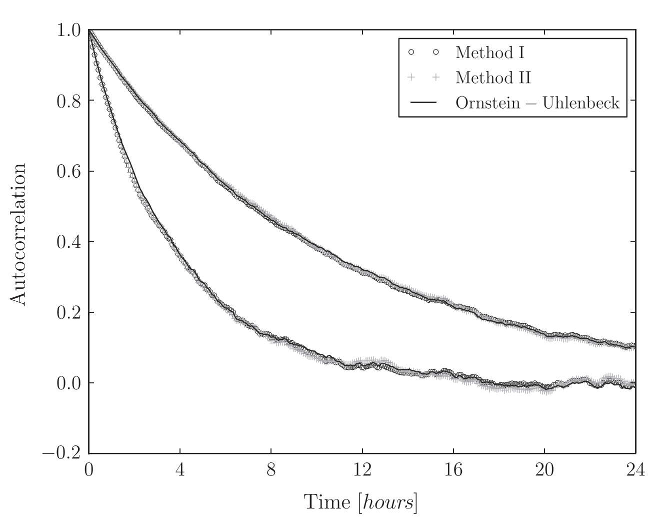

Fig. 4. Autocorrelation of the Ornstein–Uhlenbeck process vs. autocorrelation of the generated Weibull-distributed processes.

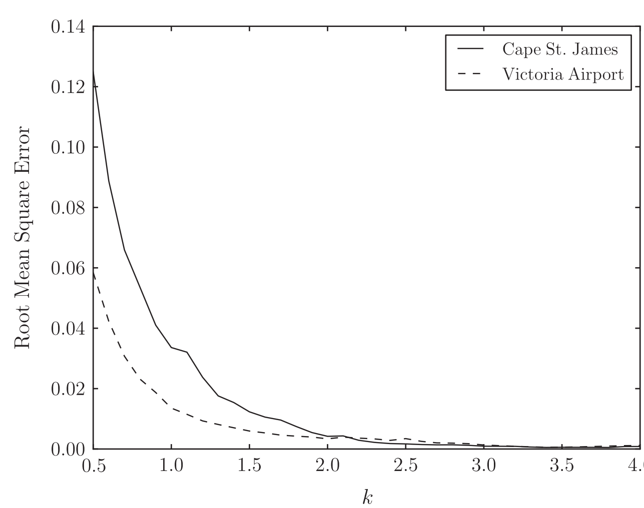

Fig. 5. RMSE between the autocorrelation of the Ornstein–Uhlenbeck process and the autocorrelation of the generated Weibull distributed process as a function of the shape parameter k of the Weibull distribution.

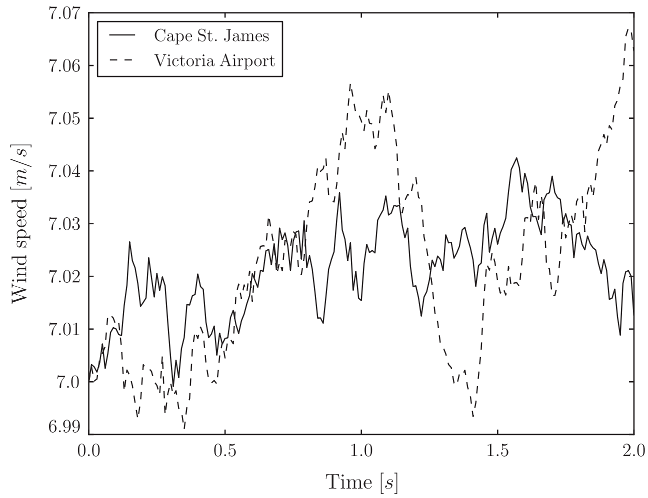

Fig. 6. Wind speed trajectories for dynamic analysis.

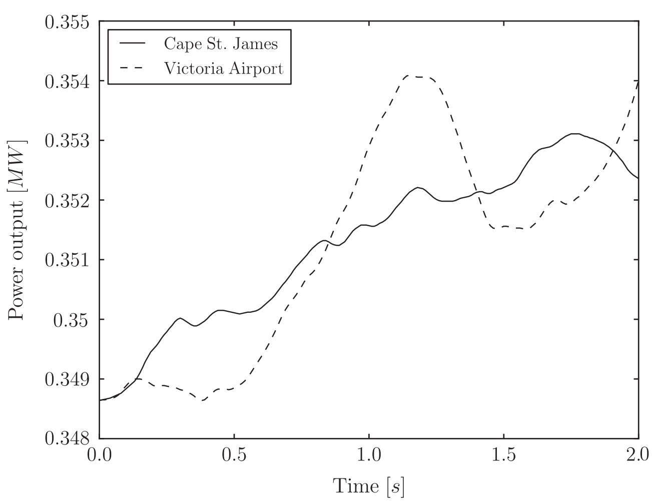

Fig. 7. Power output of a wind generator.

to extract wind model parameters for generating wind trajectories in the time frame of seconds. These trajectories are of interest in performing dynamic studies of power systems with inclusion of wind generators. The adaptation of the developed models consists in scaling the hourly-based autocorrelation data to the time frame of seconds. For that, the previously computed parameter α is divided by 3600 s/h (note that parameter $b = \sqrt{2\alpha}$ is also scaled accordingly). Therefore, the new values are $\alpha = 2.65 \times 10^{-5}$ [1/s] for Cape St. James, and $\alpha = 7.15 \times 10^{-5}$ [1/s] for Victoria Airport. The parameters of the Weibull distribution remain the same at both sites. Therefore, the models assume that the marginal distribution of the wind speed is independent of the time frame, as is the case for stationary processes.

Model II is coupled with a wind turbine model driving a Doubly-Fed Asynchronous Generator (DFIG). The simulations are carried out using Dome [41], a Python-based version of the Power System Analysis Toolbox (PSAT) [42]. The details of the dynamic model of the DFIG, as well as the technical data used, can be found in [43].

Fig. 6 depicts two wind speed trajectories generated for a time frame of 2 s with Model II. In the integration scheme, the step size is set to $h = 0.01$, i.e., 10 ms. The continuous black line represents a wind speed trajectory for Cape St. James, whereas the grey line represents a wind speed trajectory for Victoria Airport. Fig. 7 shows the power output of the wind generator for both trajectories. In order to compare the results, the initial value of the wind speed (7 m/s) is the same for both simulations. It can be observed that the variability of the wind speed, and therefore the variability of the power output, is relatively small over the time range of seconds. This fact partially justifies the use of constant wind speeds in large-disturbance dynamic studies. However, if the interest is in, for example, network voltage fluctuations or in small-perturbation dynamics, this variability cannot be ignored. It can also be observed that the variability of the wind speed at Victoria Airport is slightly larger than the variability of the wind speed at Cape St. James. This behavior is due to fact that the wind speed at Cape St. James is more correlated than the wind speed at Victoria Airport. This variability is also reflected in the wind generator power output, as it is observed in Fig. 7.

5. Conclusions

In this paper, two procedures to develop continuous wind speed models based on stochastic differential equations are proposed and tested. The models are constructed using parameters extracted from the statistical properties of real-world measurement data available for particular locations. According to the results of the simulations discussed in this paper, the two developed models are able to generate wind speed trajectories with similar marginal distributions and autocorrelation functions to those observed in wind speed data. The simulations also show that the developed models are flexible in the sense that they can be used for modeling wind speed trajectories for different time scales. However, since these are stationary stochastic models, diurnal and seasonal effects are not considered. Thus, the good performance of the models is limited to a time range of a few hours. In particular, the developed models are suitable to be merged with wind generator models for power system dynamic studies.

Future work will focus on the incorporation of non-stationary phenomena (e.g., diurnal cycle) to expand the time frame where these models are applicable. Another interesting task is to adapt the formulation of the developed models to take into account spatially cross-correlated phenomena.

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