

# Chapter 1

## Building Envelope and Thermal Balance

**Abstract** From the thermal balance point of view, in addition to air mass transfer due to infiltration and ventilation, building energy performance strictly depends on the characteristics of the envelope, in that it constitutes the boundary between the indoor and outdoor environments. Most of the currently available building performance assessment methods evaluate the heat exchange through the envelope by the means of a steady-state analysis, leading to the diffusion of strict regulations regarding the heat transmittance of the envelope elements. Although this approach is simple to use, it does not take into account the dynamic behaviour of the construction materials, which tend to store heat and release it after a certain length of time (Van Geem 1987). This phenomenon, usually referred to as thermal inertia or thermal mass, strongly affects the heat transfer process, influencing the building thermal energy need. This chapter summarises the theoretical basis of building thermal balance and heat transfer through the envelope. Furthermore, the different implications when considering the dynamic and the steady-state assessment methods are presented with the help of a practical example.

**Keywords** Building thermal balance • Heat transfer through building envelope • Dynamic and steady-state heat transfer • Building heat capacity • Thermal inertia • Climatic chamber tests

### 1.1 Building Thermal Balance

The building (or the considered thermal zone of a building) heating and cooling need calculation is mostly based on the air volume heat balance, which takes into account all the different heat flows affecting the indoor environments (Eq. 1.1). In order to simplify the calculation, the only sensible heat balance is considered, neglecting the thermal effects due to the water contained in the air (Cengel 2008).<sup>1</sup>

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<sup>1</sup>These effects are usually taken into account only when a complete air conditioning process is considered.

$$\sum \Phi_j = M \cdot c_{p,air} \frac{d\theta_{air}}{dt} \quad (1.1)$$

where

$\sum \Phi_j$  is the summation of the heat flow rates involving the air volume (W)

$M$  is the air volume mass (kg)

$c_{p,air}$  is the air specific heat capacity (J/(kg K))

$\theta_{air}$  is the air volume temperature (K)

$t$  is time (s)

The climatization system aims at maintaining a constant indoor air temperature, and so the second half of the balance usually equals zero. The summation of the sensible heat flows involving the air volume is defined in Eq. 1.2.

$$\sum \Phi_j = \Phi_{env} + \Phi_{ven} + \Phi_{int} + \Phi_{sol} + \Phi_{sys} \quad (1.2)$$

where

$\Phi_{env}$  is the heat flow rate from the envelope (W)

$\Phi_{ven}$  is the heat flow rate due to air mass exchange (W)

$\Phi_{int}$  is the heat gain due to internal heat sources (W)

$\Phi_{sol}$  is the solar heat gain through the transparent elements (W)

$\Phi_{sys}$  is the heat flow rate due to climatization system (W)

The balance is therefore solved using the heat flow rate due to the climatization system as dependent variable, as in Eq. 1.3.

$$\Phi_{sys} = \Phi_{env} + \Phi_{ven} + \Phi_{int} + \Phi_{sol} \quad (1.3)$$

The components of Eq. 1.3 have been traditionally analysed either in a detailed dynamic way or in a simplified way, which reduces the complexity of the underlying equations imposing steady-state boundary conditions. If the first approach guarantees a high accuracy level in reproducing the energy and mass flows occurring in the building, the latter one provides ease and flexibility in indicating the building performance.

### 1.1.1 Heat Flow from Envelope

The heat flow from the envelope is generally defined according to Eq. 1.4.

$$\Phi_{env} = \sum_i^{N_{el}} h_{s,i} A_i (\theta_{s,i} - \theta_z) \quad (1.4)$$

where

$h_s$  is the surface convective and radiative heat transfer coefficient (W/(m<sup>2</sup> K))

$A$  is the surface of the envelope element (m<sup>2</sup>)

$\theta_{s,i}$  is the internal surface temperature of the envelope element (°C)

$\theta_z$  is the indoor desired temperature (°C)

$N_{el}$  is the number of envelope elements

This parameter is essentially determined as the result of the external variability in terms of air temperature and solar radiation on the opaque elements of the envelope, which is determined by the general heat conduction equation (see Sect. 1.2).

### 1.1.2 Ventilation

The heat flow due to air mass exchange is defined according to Eq. 1.5.

$$\Phi_{vent} = \sum_i^{N_{flow}} \dot{m}_j c_{p,air} (\theta_{air,e} - \theta_z) \quad (1.5)$$

where

$\dot{m}$  is the air mass flow rate (kg/s)

$c_{p,air}$  is the air specific heat capacity (J/(kg K))

$\theta_{air,e}$  is the outdoor air temperature (°C)

$N_{flow}$  is the number of air flows

The calculation of this part of the heat balance is based on the same equation both in the steady-state and dynamic approaches, except for the time-step and therefore the application schedules.

The main parameter characterizing this heat flow is the air flow rate, also defined “discharge rate”, which can be either determined in detail according to opening size and presence of wind or in simplified ways according to standard air change rate values connected to the building use and provided by international and local regulations.

### 1.1.3 Internal Heat Sources

The heat production due to internal sources is a summation of the heat production of people, lighting and equipment present in the thermal zone. Similarly to the previously described discharge rate, also this component of the heat balance is based on the same calculation in the steady-state as well as in the dynamic approach, except for the time step and the application schedules.

The heat produced by these elements is usually summarized in overall load values, specific per floor area and provided by international and local regulations according to building use.

### 1.1.4 Solar Gain Through the Transparent Elements

The influence of the solar radiation entering the zone through the transparent elements on the indoor environment is defined according to Eq. 1.6 (Szokolay 2008).

$$Q_{sol} = A \cdot G \cdot SGF \quad (1.6)$$

where

$A$  is the glazed surface ( $m^2$ )

$G$  is the global (direct and diffuse) solar radiation incident on the glazed surface ( $W/m^2$ )

$SGF$  is the solar gain factor

The solar gain factor is a decimal fraction indicating what part of the incident radiation reaches the interior, depending on the characteristics of the glass and, if provided, of the shading elements.

Once arrived on the glazing surface, some part of the radiation is transmitted, some reflected and the remainder is absorbed within the body of the glass. The absorbed part will then heat up the glass, which will emit some of this heat to the outside, some of it to the inside, by re-radiation and convection. The  $SGF$  represents the sum of this inward re-emitted heat and the direct transmission.

This component of the heat balance is based on the same calculation both in steady-state and dynamic approaches, except for the considered time step.

## 1.2 Heat Transfer Through Building Elements

The heat transfer through the building elements is regulated by the law of heat conduction, the Fourier's law, which states that heat transfer is directly connected to the temperature distribution inside the same element. This law takes the form of Eq. 1.7.

$$\vec{q} = -\lambda \nabla T \quad (1.7)$$

where

$\vec{q}$  is the density of heat flow rate ( $W/m^2$ )

$\lambda$  is the thermal conductivity ( $W/(m \cdot K)$ )

$-\nabla T$  is the temperature gradient ( $K/m$ )

This component of the heat balance is calculated differently according to the steady-state and to the dynamic approach, as described in the following paragraphs.

### 1.2.1 Steady-State Analysis

Within the building heat balance, when dealing with the calculation of the maximum heat exchange due to reference boundary conditions (i.e. in case of the peak load needed to size the heating system), the effects of unsteady parameters such as the solar radiation and the internal heat loads can be safely neglected. The Fourier's law can be simplified considering flow crossing in steady-state conditions, without accounting for the dynamic phenomena and therefore neglecting the thermal inertia of the building elements (which depends on the density and specific heat of the materials), and in a single direction, which represents most of the heat exchanges through envelope elements (Eq. 1.8).

$$q = -\lambda \frac{dT}{dx} \quad (1.8)$$

where

$q$  is the heat flow rate density through the layer ( $\text{W/m}^2$ )

$\lambda$  is the conductivity of the layer material ( $\text{W/(m K)}$ )

$dT$  is the temperature difference of the layer surfaces ( $\text{K}$ )

$dx$  is the penetration depth of the heat flow ( $\text{m}$ )

With constant heat flow rate and penetration depth, which corresponds to the thickness of a homogeneous plane layer, thermal resistance  $R$  can be defined according to Eq. 1.9.

$$R = \frac{\Delta x}{\lambda} = \frac{s}{\lambda} \quad (1.9)$$

where

$R$  is the thermal resistance of the homogeneous plane wall ( $(\text{m}^2 \text{ K})/\text{W}$ )

$s$  is the plane wall thickness ( $\text{m}$ )

From this quantity it was possible to develop a model analogue to the one of electrical connections. This allows therefore calculating the overall resistance of a multilayer wall as the sum of the resistances of the  $n$  homogeneous layers that are part of it, taken in series, as in Eq. 1.10.

$$R_{wall} = \sum_{i=1}^n R_{layer,i} \quad (1.10)$$

Simplifying, then, in case of steady-state heat conduction an overall parameter characterizing the heat transfer through a multilayer wall can be used: the thermal conductance (Eq. 1.11).

$$C = \frac{1}{R_{wall}} \quad (1.11)$$

Finally, in order to take into account the surface heat transfer due to convection and radiation towards the surrounding air, the surface resistance value was also defined (Eq. 1.12). It depends partly on the surface characteristics and partly on the environmental conditions of the surrounding air.

$$R_s = \frac{1}{h} \quad (1.12)$$

where

$h$  is the surface heat transfer coefficient (W/(m<sup>2</sup> K))

Combining the conduction thermal characteristics with the superficial transfer coefficients it was possible to define an overall quantity describing the steady-state heat transfer characteristics of the complete element, which is the thermal transmittance, or U-value (W/(m<sup>2</sup> K)).

$$U = (R_{s,i} + R_{wall} + R_{s,e})^{-1} = \left( \frac{1}{h_e} + \sum_{i=1}^n \frac{s_i}{\lambda_i} + \frac{1}{h_i} \right)^{-1} \quad (1.13)$$

The calculation of thermal resistance and U-value for the envelope elements (Eq. 1.13) is described in norm EN ISO 6946 (2007), which also provides standard reference values for the surface resistances according to the element type.

Therefore, in case of heat exchange through the building envelope due to simple transmission in steady-state conditions Eq. 1.14 may be used when aiming at determining the winter peak load.

$$\Phi_{env,tr} = \sum_i^{N_{el}} U_i A_i (T_{air,e} - T_z) \quad (1.14)$$

where

$U$  is the thermal transmittance of the element (W/(m<sup>2</sup> K))

$A$  is the surface of the envelope element (m<sup>2</sup>)

$T_{air,e}$  is the external air temperature (°C)

$T_z$  is the internal desired temperature (°C)

This simplified approach was also extended to the calculation of the heat balance performed for longer time intervals (monthly or seasonal) in order to determine the building climatization energy need. In Eq. 1.14, however, only the heat flow caused by the temperature variation is taken into account, while the heat contribution due to the solar radiation arriving on the opaque envelope element is neglected. This approach is suitable for the winter peak load assessment, but in case of the energy need the solar component is very important and is usually added to the heat balance equation through Eq. 1.15.

$$\Phi_{env,sol,op} = \sum_j^{N_{el,op}} R_{se,j} U_j A_j (F_{sh,j} \alpha_j I_{sol} - h_{r,j} \Delta T_{er}) \quad (1.15)$$

where

$R_{s,e}$  is the thermal resistance of the external surface ((m<sup>2</sup> K)/W)

$U$  is the thermal transmittance of the element (W/(m<sup>2</sup> K))

$A$  is the surface of the element (m<sup>2</sup>)

$F_{sh}$  is the shading factor due to the surroundings

$\alpha$  is the surface absorption coefficient

$I_{sol}$  is the solar irradiance (W/m<sup>2</sup>)

$h_r$  is the radiative heat transfer coefficient of the external surface (W/(m<sup>2</sup> K))

$\Delta T_{er}$  is the average difference between air temperature and apparent sky temperature (K)

$N_{el,op}$  is the number of opaque envelope elements

Often Eqs. 1.14 and 1.15 are combined, grouping the  $UA$  parameter and adjusting the temperature difference parameter by substituting the outdoor air temperature with the sol-air one, which is defined according to Eq. 1.16 (ASHRAE 2001).

$$T_{sa,e} = T_{air,e} + \frac{\alpha I}{h_e} - \frac{\varepsilon \Delta R}{h_e} \quad (1.16)$$

where

$T_{sa,e}$  is the sol-air temperature (°C)

$T_{air,e}$  is the outdoor air temperature (°C)

$\alpha$  is the absorption coefficient of the external surface of the envelope element

$I$  is the global solar radiation on the envelope element (W/m<sup>2</sup>)

$\varepsilon$  is the emissivity of the external surface of the envelope element

- $\Delta R$  is the difference between the infrared radiation from the surroundings of the building (included the sky) and the one emitted by a black body at the outdoor air temperature ( $\text{W/m}^2$ )
- $h_e$  is the external surface heat transfer coefficient ( $\text{W}/(\text{m}^2 \text{ K})$ )

### 1.2.2 Transient Analysis

If the Fourier's equation is combined with the energy conservation principle for a minimal volume, it becomes the general heat conduction equation (Eq. 1.17).

$$\rho c \frac{\partial T}{\partial t} = \lambda \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] \quad (1.17)$$

where

- $\rho$  is the material density ( $\text{kg/m}^3$ )
- $c$  is the material specific heat ( $\text{J}/(\text{kg K})$ )
- $\partial T$  is the temperature variation ( $\text{K}$ )
- $\partial t$  is the time variation ( $\text{s}$ )
- $\lambda$  is the material conductivity ( $\text{W}/(\text{m K})$ )
- $x, y, z$  represent the space coordinates

As done for the steady-state analysis, also in the transient case the heat flow through building envelope elements can be approximated to a one-dimensional formulation and so the general heat conduction equation can be expressed as in Eq. 1.18.

$$\rho c \frac{\partial T}{\partial t} = \lambda \frac{\partial^2 T}{\partial x^2} \quad (1.18)$$

Equation 1.18 is a partial differential equation (PDE) and therefore it is not easily solvable outside specific boundary conditions (i.e. periodic analysis). The common way to face this equation is to adopt numerical methods that approximately solve it developing a function (or a discretization of this function) which simultaneously satisfies a certain relation between its derivatives calculated in specific space or time regions and some given boundary conditions at the extremes of the domain. Even if these methods simplify the calculation procedure, they require strong engineering knowledge to use the complicated application software.



### 1.2.2.1 Analysis Through Discretization

The main numerical methods that exactly solve PDEs are the finite difference, finite volumes and finite element analysis, which are based on an approximation of the reference domain in order to simplify the solution of the heat conduction equation.

The finite difference method is based on the reduction of the derivatives to a system of equations expressing the difference between the values of the function in discrete points.

The finite volume method is equally based on the values calculated in discrete points, but these points are found within a geometric mesh, whose vertices are placed inside infinitesimal volumes, and the heat flows are calculated at their boundaries (faces and edges). This technique is very common to solve fluid-dynamic problems.

Finally, the finite element method was developed in 1950s in order to face the complexity of some elasticity and structural problems, and is based on the simplification of the domain by its discretization in subdomains, which are geometric primitives (mostly triangles in case of two-dimensional domains and tetrahedra in case of three-dimensional domains) and are called “elements”. In this way it is possible to model objects with a very wide range of geometries, by the means of a mesh which appropriately discretizes the domain, a detailed description of all the relevant physical characteristics, and the description of the boundary conditions that activate the phenomenon.

As previously said, in case of the heat transfer through an envelope element a single direction flow is considered accurate enough, and so a wall can be easily modeled as its cross section, with all the different layers described by their thermal characteristics. The boundary conditions activating the heat exchange can be described directly as density of heat flow rate on the wall surfaces, or as a combination of air temperature coupled with a surface (radiative and convective) heat transfer coefficient, or even as a forced temperature for the wall surface.

### 1.2.2.2 Conduction Transfer Functions

The discretizing methods are complex to solve, and generally need the recourse to calculation means that have become available only recently. Therefore it was developed a simplified numerical method based on the “transfer functions” method, which represents systems by the means of an impulse-result scheme. In the field of heat transfer, this method took the name of conduction transfer functions or CTFs, and was developed in the 1960s by G.P. Mitalas and D.G. Stephenson from National Research Council of Canada (NRCC) and accepted by the American Society of Heating, Refrigerating, and Air-Conditioning Engineers (ASHRAE), which included it in its Handbook of Fundamentals. For this reason this technique is also known as “ASHRAE method”.

The transient nature of both inputs and outputs of the system is often faced using mathematical transforms. Among the most common ones regarding heat transfer

there are the Laplace transform for the analysis of continuous domains and the Z-transform, which was developed within the signal processing, for the analysis of data within a discrete domain.

Conduction transfer functions portray the whole dynamic behaviour of a building element and are a feature of the system depending solely on the thermo-physical characteristics of its single homogeneous layers. According to the boundary conditions at the element surface (i.e. the temperature oscillation) it is possible to determine the conduction heat flows through the boundary layers, characterized by unitary surface heat transfer coefficients (Eq. 1.19).

$$\begin{cases} \varphi_e(z) = \frac{D(z)}{B(z)} \cdot T_e(z) - \frac{1}{B(z)} \cdot T_i(z) \\ \varphi_i(z) = \frac{1}{B(z)} \cdot T_e(z) - \frac{A(z)}{B(z)} \cdot T_i(z) \end{cases} \quad (1.19)$$

where

- $\varphi(z)$  is the Z-transform of heat flow rate density
- $T(z)$  is the Z-transform of temperature
- $D(z)/B(z)$  is a conduction transfer function through the element
- $A(z)/B(z)$  is a conduction transfer function through the component
- $1/B(z)$  is a conduction transfer function through the component

Therefore, CTFs depend on:

- the thermo-physical characteristics of the layers materials (homogeneous or alike) in the element;
- the position of the layers within the element;
- the boundary conditions described by the surface heat transfer coefficients;
- the analysis time interval (usually 1 h).

Conduction transfer functions are generally determined according to how specific construction elements react to specific thermal stimuli (i.e. step, linear ramp, parabolic ramp).

The CTF method is the most common among the building dynamic simulation software since it has a relatively high computation speed (functions can be calculated once for each construction element at the beginning of the simulation) and it is generally accurate. Among the most famous tools for evaluating the transient energy balance of the buildings there are the DOE and TRNSYS (Klein et al. 2007) codes, based on the Z-transform according to the ASHRAE method, the BLAST one, based directly on the Laplace transform, and Energy Plus (Crawley et al. 2001), which implements a procedure called “state space method” which is based on several equations in matrix form.

### 1.2.2.3 Periodic Analysis

In case of specific problems, which are characterized by simple geometry, simple boundary conditions and material properties that are homogeneous and independent from temperature, PDEs also allow analytical solutions. In case of heat transfer, as alternative to both the complex numerical methods and the excessively simplified steady-state method, periodic analysis was therefore introduced, assuming that the envelope elements undertake a sinusoidal temperature variation.

A lot of physical phenomena can be approximated to sinusoidal trends, in particular when considering the thermal stimuli of buildings (i.e. the daily or yearly trends of climatic parameters): therefore some properties of periodic functions can reasonably be extended to portray the heat conduction through envelope elements in a simplified way. International standard EN ISO 13786 (2007), in fact, specifies the characteristics that describe the thermal behaviour of the envelope elements under periodic boundary conditions and suggests a procedure to calculate them. This norm assumes a sinusoidal temperature variation on one side of the element (usually the outside environment), while on the other side (usually the inside space) temperature is held constant. The effect of this stimulus is reflected on the heat flows through the element surfaces, which are one dimensional and have sinusoidal trends as well.

Within the standard, the following heat transfer matrix for a layer  $Z$  is defined (Eq. 1.20)

$$Z = \begin{pmatrix} \hat{\theta}_2 \\ \hat{q}_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \cdot \begin{pmatrix} \hat{\theta}_1 \\ \hat{q}_1 \end{pmatrix} \quad (1.20)$$

where

$\hat{q}$  is the complex amplitude of the heat flow rate density through the surface of the element ( $\text{W/m}^2$ )

$\hat{\theta}$  is the complex amplitude of the air temperature (K)

$Z_{ij}$  is an element of the transfer matrix, defined by Eq. 1.21

$$Z_{ij} = f(T, \lambda, \delta, \xi) \quad (1.21)$$

where

$T$  is the period of the variation (s)

$\lambda$  is the conductivity of the layer material ( $\text{W/(m K)}$ )

$\delta$  is the periodic penetration depth of the heat wave in the layer (m)

$\xi$  is the ratio of the thickness of the layer to the penetration depth

The transfer matrix for a multi-layer element results from the multiplication of the matrices characterizing the single layers.

This method can be adopted to solve any problem connecting the boundary conditions on one side of the element and the heat flow rate on the other side, consistently with the method assumptions.

Moreover, once the transfer matrix is defined it is possible to calculate some dynamic properties of the considered construction, representing the response of the element to a sinusoidal temperature variation on one of its sides.

Thermal admittance (Eq. 1.22), for instance, represents the complex amplitude of the heat flow rate density through the surface of the component divided by the complex amplitude of the temperature variation in the zone on the same side of the element ( $m$ ).

$$Y_{mm} = \frac{\hat{q}_m}{\hat{\theta}_m} \quad (1.22)$$

where

$Y_{mm}$  is the thermal admittance ( $\text{W}/(\text{m}^2 \text{ K})$ )

$\hat{q}_m$  is the complex amplitude of the heat flow rate density through the element surface adjacent to zone  $m$  ( $\text{W}/\text{m}^2$ )

$\hat{\theta}_m$  is the complex amplitude of the air temperature within zone  $m$  ( $\text{W}/\text{m}^2$ )

Periodic thermal transmittance (Eq. 1.23), differently, represents the complex amplitude of heat flow rate density through the element surface on the side with steady conditions, divided by the complex amplitude of the temperature variation on the other side.

$$Y_{mn} = \frac{\hat{q}_m}{\hat{\theta}_n} \quad (1.23)$$

where

$Y_{mn}$  is the periodic thermal transmittance ( $\text{W}/(\text{m}^2 \text{ K})$ )

$\hat{q}_m$  is the complex amplitude of the heat flow rate density through the element surface adjacent to zone  $m$  ( $\text{W}/\text{m}^2$ )

$\hat{\theta}_n$  is the complex amplitude of air temperature within zone  $n$  ( $\text{W}/\text{m}^2$ )

Thermal admittance and periodic thermal transmittance are therefore complex numbers as well, and the value usually taken into account for periodic analysis calculation is the modulus. In case of low thermal inertia both moduli tend to the element U-value.

From these fundamental characteristics it is also possible to determine the decrement factor (Eq. 1.24), which represents the dampening of the heat flow oscillation when crossing the element.

$$f = \frac{|Y_{mn}|}{U} \quad (1.24)$$

where

$Y_{mn}$  is the periodic thermal transmittance ( $\text{W}/(\text{m}^2 \text{ K})$ )

$U$  is the U-value ( $\text{W}/(\text{m}^2 \text{ K})$ )

As can the *time shift* (Eq. 1.25), which represents the delay of the heat flow in crossing the element:

$$\Delta t_f = \frac{T}{2\pi} \arg(Y_{mn}) \quad (1.25)$$

where

$T$  is the period of the variation (s)

Additionally, these phenomena can also be interpreted in terms of the ratio between the temperature oscillation on the outside and the temperature response on the inside surface facing a room at constant temperature (Ulgen 2002; Kaşka and Yumrutaş 2009), giving the Decrement Factor ( $DF$ ) and the Time Lag ( $TL$ , in h) definition described through Eqs. 1.26 and 1.27 respectively.

$$DF = \frac{T_{s,i,\max} - T_{s,i,\min}}{T_{sa,e,\max} - T_{sa,e,\min}} \quad (1.26)$$

and

$$TL = t_{T_{s,i,\max}} - t_{T_{sa,e,\max}} \quad (1.27)$$

where

$T_{s,i}$  inside surface temperature, in  $^{\circ}\text{C}$

$T_{sa,e}$  external sol-air temperature, in  $^{\circ}\text{C}$

$_{\min}$  local minimum

$_{\max}$  local maximum

$t$  time instant when the local maxima take place, in h

Among the properties described in the standard, the ones which are easier to understand and use are the time shift and the decrement factor. Moreover, the thermal admittance value is considered in standard EN ISO 13792 (2005), which suggests a simplified procedure to calculate the summer indoor temperature of a room without air-conditioning system (“admittance procedure”).

Another parameter introduced by the EN ISO 13786 (2007) standard is the areal heat capacity, which represents the effective heat capacity, relative to the layers directly interacting to the temperature variation on one of the side of the element

(Eq. 1.28). Therefore, for each of the building envelope elements there can be calculated both an internal areal heat capacity and an external one.

$$\kappa_m = \frac{T}{2\pi} |Y_{mm} - Y_{m}| \quad (1.28)$$

where

$\kappa_m$  is the areal heat capacity of the  $m$ -facing side of the element (J/(m<sup>2</sup> K))

$T$  is the period of the variation (s)

$Y_{mm}$  is the thermal admittance (W/(m<sup>2</sup> K))

$Y_m$  is the periodic thermal transmittance (W/(m<sup>2</sup> K))

Standard EN ISO 13790 (2008) refer to this last characteristic in the calculation of the utilization factors in order to take into account a dynamic parameter in the simplified steady-state building energy balance (see Chap. 3). In Standard EN ISO 13786 (2007) it is also possible to find some simplified methods to calculate the areal heat capacity without using the transfer matrix.

### 1.2.3 Steady-State Versus Transient Prediction

Within a larger study carried out in Politecnico di Milano (Ferrari and Zanotto 2013), it was possible to compare the actual behaviour of four selected walls, measured by the means of climatic chamber tests (ASTM C1363 2005; Brown and Stephenson 1993a, b; Burch et al. 1990; ISO 8890 1994), and the performance of the same walls assessed by the means of a finite element analysis tool.




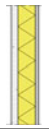
Since the main dynamic phenomenon that can be seen in walls is its thermal inertia, the selected constructions are characterized by the same U-value but by different aeric mass and layout (i.e. insulation position):

- a heavyweight masonry wall (“Heavy”), made by one layer of high density hollow bricks;
- a medium/heavyweight wall (“Mid-Ins”), made by outdoor face bricks and a hollow brick layer with insulation within;
- a medium/heavyweight wall (“Ext-Ins”) with hollow bricks and exterior insulation;
- a lightweight wall (“Light”) composed by an insulation sandwich panel.

The characteristics for each of these wall types are summarised in Table 1.1.

The walls were tested by the means of daily cycles of sol-air temperature (ASHRAE 2001), in order to represent a likely thermal impulse on a vertical wall facing South located in the city of Rome (considered as average representative of the Italian context). Two days were chosen from the Test Reference Year (TRY) climatic data of Rome: a very cold winter day (January 26th) and a very warm summer day (July 17th) without significant perturbation by occasional meteorological events.

**Table 1.1** Chosen wall types and related main thermal characteristics

	Heavy	Mid-ins	Ext-ins	Light
				
Thickness (m)	0.480	0.365	0.400	0.125
U-value (W/(m <sup>2</sup> K))	0.298	0.299	0.306	0.312
Aeric mass (kg/m <sup>2</sup> )	431	343	301	35
Heat capacity (kJ/(m <sup>2</sup> K))	368	291	259	28

For each sample, the test lasted at least 3 days, with a repetition of the daily cycle temperatures inside the climatic chamber, in order to stabilize the heat flows.

Regarding the finite element analysis, the wall samples have been modelled as 2 m long specimens (two-dimensional domains) and the analysis points have been placed in the middle, in order to reflect the sensors location for the climatic chamber tests.

The boundary conditions imposed to the domain are the surface temperatures registered during the tests, in order to neglect the variability connected to the heat transfer coefficient, which is difficult to estimate and strongly depends on the instantaneous environmental conditions. The simulation has been performed with the same boundary settings for a 30 days’ time in order to stabilize the model, in particular regarding the heavier samples.

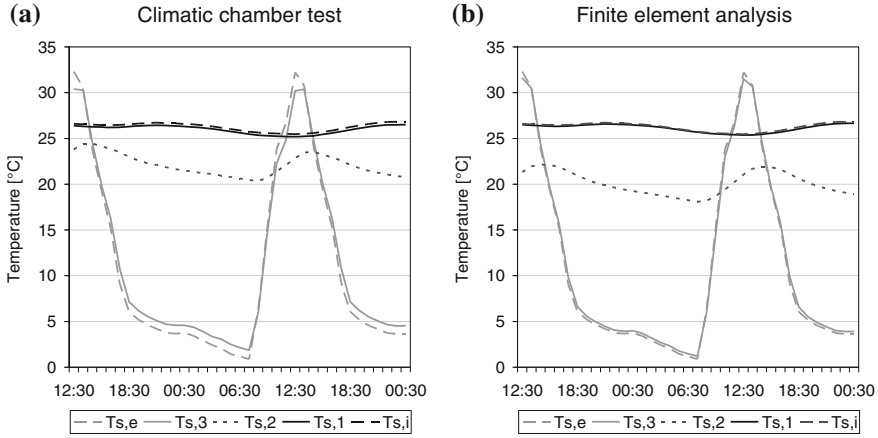
The temperatures predicted by the finite element method for the internal layers are very similar to the ones measured during the actual tests, both regarding the general trend and the absolute numeric values: the maximum differences are detected in the medium-weight solution with external insulation during the winter day, regarding the sensor placed between the hollow bricks and the insulation, but it is always less than 3 K, as can be seen in Fig. 1.1 ( $T_{s,2}$ ).

These small differences can be easily explained considering:

- the model approximation, which represents the hollow bricks as a single homogeneous layer with average equivalent density (consistently with the common practice in building simulation);
- since the tested samples were hand-crafted, the low precision of the actual construction praxis, in particular regarding plaster and mortar thicknesses;
- the moisture effects on the heat transfer, which were neglected in the finite element model aiming at calculation simplicity;
- three-dimensional heat transfer effects inside the real samples.

The finite element method shows, anyway, a very high accuracy in predicting the behaviour of the walls, both regarding the absolute values and the general trends.

As further analysis, the same model was used with different boundary conditions to simulate an indoor environment characterised by controlled temperature. The



**Fig. 1.1** Comparison of the hourly temperature values for the medium/heavyweight wall with external insulation during the winter day as detected by the thermocouples (a) and the ones calculated by the finite element simulations (b)

boundary conditions are based on the hourly sol-air temperature and the standard external heat transfer coefficient ( $25 \text{ W/(m}^2 \text{ K)}$ ), according to EN ISO 6946 (2007) on the outside surface, and due to constant temperature ( $20^\circ \text{C}$  for the winter season and  $26^\circ \text{C}$  for the summer season, according to EN 15251 (2007) and the standard internal heat transfer coefficient ( $7.7 \text{ W/(m}^2 \text{ K)}$ ), according to EN ISO 6946 (2007) on the inside surface.

In this case the simulation is extended to the whole heating season, which for Rome lasts from the November 1st to April 15th (D.P.R. 412 (1993)), and the whole cooling season, lasting from July 1st to the September 15th (selected as the whole period when the average daily outside sol-air temperature exceeds the comfort limit of  $26^\circ \text{C}$ ). Also for this analysis a stabilization time of 30 days has been respected.

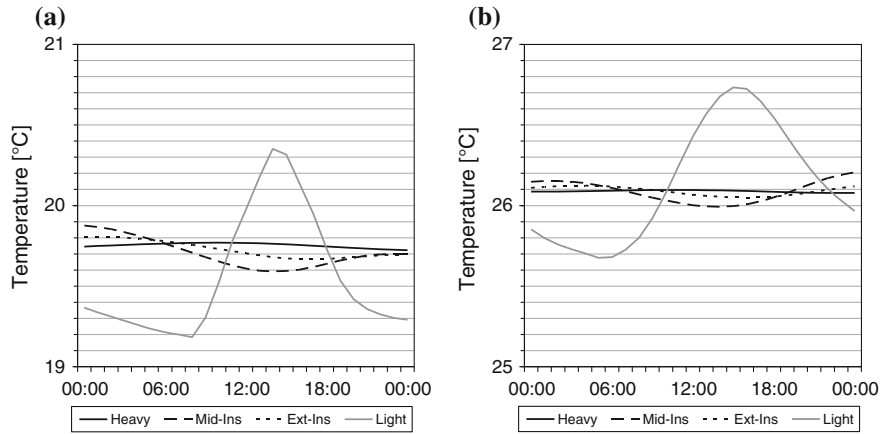
Figure 1.2 shows the hourly trends regarding the inside surface temperature for the different wall types during the reference winter (January 26th) and summer (July 17th) days.

As the inside air temperature is a constant value, a visible oscillation of the surface temperature is only detectable in case of the lightweight solution. This variability exists even if the air with constant temperature is considered directly in contact with the surface, attenuating the actual oscillation effect which would occur in a real room, where the air temperature is not evenly distributed.

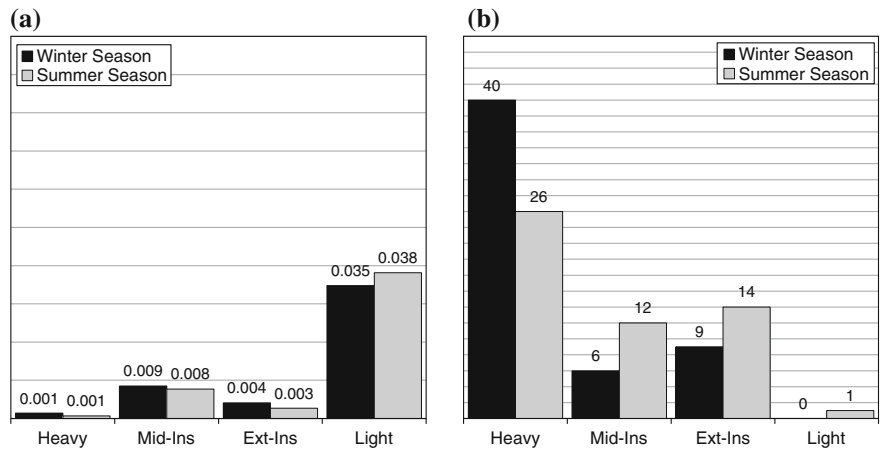
Acknowledging this behaviour, the attenuation and time delay phenomena, as described by Eqs. 1.25 and 1.26 for the periodic analysis, are calculated according to the finite element simulation results.

The outcoming values are shown in Fig. 1.3 and show the same general trend as the ones resulting from the experimental tests, but the differences between the lightweight construction and the others increase: regarding the time lag, the discrepancies among the various heavyweight samples can be seen, too. The summer





**Fig. 1.2** Trends of the inside surface temperature as calculated for the different wall types in the reference winter day (a) and in the reference summer day (b)



**Fig. 1.3** Decrement factor (*DF*) (a) and time lag (*TL*) (b) results for the different wall samples

season thermal delay values, in particular, are very similar to the ones resulting from the nominal periodic parameters. This can be explained considering that the summer solicitation is very close to the sinusoidal trend assumed in EN ISO 13786 (2007).

The finite element analysis allows to take into account not only the temperature results, but also the density of heat flow rate values crossing the analysis points, in  $\text{W/m}^2$ . In this work, the seasonal peak load has been estimated as the maximum values registered during the winter and summer seasons, as shown in Table 1.2.

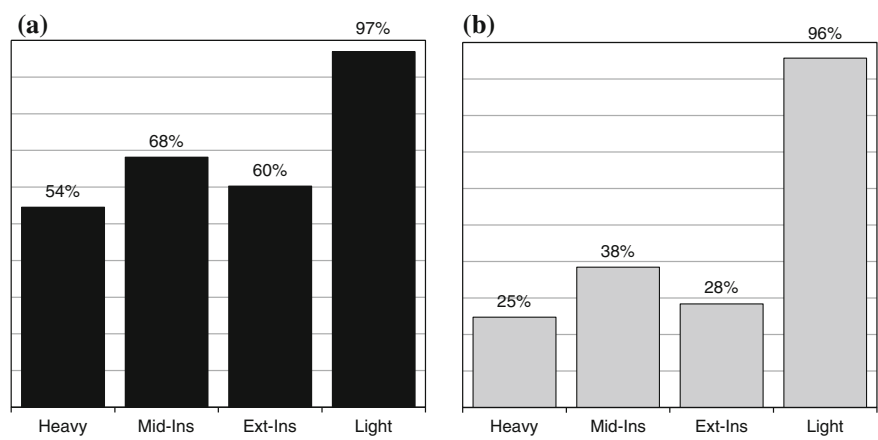
Differently, with the conventional simplified approach as reference, the peak load should be the same for all the walls, since it is calculated as the product of the nominal U-value and the seasonal maximum difference between the indoor air and the external sol-air temperatures.

The differences between the two evaluations are pointed out in Fig. 1.4 and show how the lightweight sample, with low thermal inertia, behaves closely to the steady-state estimation (with percentage near 100 %), while the other solutions strongly attenuate the effects of the sol-air temperature. It is useful to remind that in case of the heavyweight walls the peak load does not occur directly after the maximum solicitation, but only after a time lag corresponding to the thermal delay (the one shown in Fig. 1.3).

As overall results, the finite element simulation lead to various remarks in the field of energy performance implications, demonstrating the importance of correctly taking into account the dynamic behaviour of the building envelope.

**Table 1.2** Peak load values for the density of heat flow rate, in  $\text{W/m}^2$ , calculated by the means of the finite element analysis during the winter and summer season according to the different constructions

	Heavy	Mid-ins	Ext-ins	Light
Winter season	3.37	4.24	3.84	6.29
Summer season	1.69	2.64	1.99	6.86



**Fig. 1.4** Ratio (in percentage) between the peak loads calculated by the means of the finite element analysis and the ones estimated by the means of the steady-state approach in winter (a) and summer (b)

First of all, the attenuation of the heat wave oscillation represented by the temperature decrement factor can be interpreted as:

- lower peak load which would bring to a smaller size for the climatization system;
- smaller variability of the internal surface temperature, which means a more even use of the heating and cooling system (on/off operation) and more constant environmental conditions, influencing thermal comfort (radiant temperature).

The thermal delay, on the other hand, can bring to clear advantages on the indoor environmental conditions in the summer season, since the inside surface temperature (and therefore also the related heat exchange) has its local maxima during the night time, when most of the internal heat loads do not affect the zone volume and when the outdoor environmental conditions allow the use of attenuation and passive cooling strategies (i.e. night ventilation).

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