Chapter 17
Model Reduction of Geometrically Nonlinear Structures Via Physics-Informed Autoencoders

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Abstract High-fidelity finite element (FE) models are widely used to simulate the dynamic response of geometrically nonlinear structures. The high computational cost of running long-time-duration analyses; however, it has made nonlinear reduced-order models (ROMs) an attractive alternative. While there are a variety of reduced-order modeling techniques, in general, their shared goal is to project the nonlinear response of the system onto a smaller number of degrees of freedom (DOFs). For linear structures, modal-based projection methods are widely used for model reduction. This general procedure is akin to the dimensionality reduction that occurs when transforming data with autoencoders (AE), a machine learning technique used in unsupervised learning to extract representational features by restricting the capacity of the latent space. This idea is exploited to explore the use of AEs to perform dimensionality reduction of high-fidelity FE models and learn a latent space-based ROM. A physics-informed loss function is included to improve the network's performance and enhance interpretability. The approach is demonstrated by performing direct time integration of the equations of motion in latent space for a flat beam with geometric nonlinearity.

Keywords Nonlinear dynamics · Reduced order models · Geometric nonlinearity · Machine learning · Unsupervised learning

17.1 Introduction

The finite element (FE) method is a common numerical approach for computing the response of geometrically nonlinear mechanical systems that are represented by the following equation of motion:

$$M\ddot{x} + C\dot{x} + Kx + f_{\mathrm{nl}}(x) = f_{\mathrm{ext}} \tag{17.1}$$

where *M, C*, and *K* are the respective linear mass, damping, and stiffness of the model and the $\ddot{x}$, $\dot{x}$, $and\ x$ terms represent the respective acceleration, velocity, and displacement of the system. The nonlinear restoring force, f_{nl}, is a function of current displacements, and the external force of the system is f_{ext}. Numerical integration of the FE model's equation of motion is computationally expensive for large models and is often unfeasible when long-time-duration simulations are needed. An alternative to FE model integration is to use a reduced-order model (ROM) in its place. In general, reduced-order modeling techniques transform the original system into a lower-dimensional space, which can be integrated at a significantly lower cost. For linear structures, modal-based projection methods are commonly used as the basis set for the reduction. This procedure is similar to how autoencoders (AEs), a machine learning technique, perform dimensionality reduction on a given data set. The AE is a type of unsupervised learning technique where an input is reconstructed by passing it through an information bottleneck as shown in Fig. 17.1. The bottleneck is constructed by having layers of reduced dimensionality between an encoder and a decoder [1]. The effect of the information bottleneck is that the AE is forced to learn reduced latent representations of the original data. This idea is exploited to explore the use of AEs to perform dimensionality reduction of linear and geometrically nonlinear FE models and learn a latent space-based ROM.

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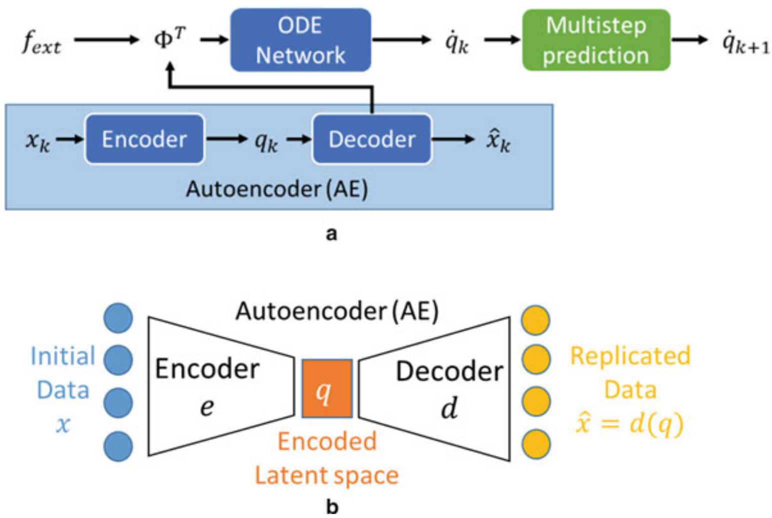

Fig. 17.1 (**a**) Schematic entire learning network architecture. (**b**) Schematic of autoencoder architecture

17.2 Network Architecture

In an AE's original form, it is trained by minimizing the mean squared error (MSE) of the reconstruction compared to the original data. In such case, the network learns reduced representations of the data by encoding the original data into latent features. In this work, the AE was enhanced to ensure that the latent representations of the data can be time-integrated and are useful in reconstructing the original data. To accomplish this objective, the "ODE Network," an additional multilayer perceptron (MLP), was used to compute derivatives of the latent space data. These derivatives were then used in a multistep integration scheme to predict the next time step. A diagram of the network architecture and flow of information is presented in Fig. 17.1 where the blue boxes represent different neural networks. The error between the predicted displacements and velocities at the next time step and the actual values was used as the additional loss term. The total loss included the reconstruction MSE and the one-step-ahead prediction error. This total loss term forces the network to learn a decoder that reconstructs the displacements in the physical domain and an encoder that generates a latent space that can be time-integrated to predict the next time step.

The ODE Network represents the following functional: $dq/dt = F(\boldsymbol{t}, \boldsymbol{q}, \boldsymbol{f}_{\text{ext}})$. In other words, the ODE Network finds the form of the nonlinear ordinary differential equation (ODE), which, in reality, is a function of the state variables, the state parameters, and the external source term. In this data-driven approach, the state parameters are assumed to be unknown. Instead, the network learns to represent these state parameters (i.e., mass, stiffness, and damping). Inspired by Maziar et al. [2], the one-step-ahead prediction is obtained by performing a multistep prediction that employs the Adams-Moulton coefficients:

$$\sum_{j=0}^{s} \alpha_j \boldsymbol{q}_{\boldsymbol{k+j}} = h \sum_{j=0}^{s} \beta_j F\left(\boldsymbol{t}_{k+j}, \boldsymbol{q}_{k+j}, \boldsymbol{f}_{\text{ext},k+j}\right) \tag{17.2}$$

where α and β are the Adams-Moulton coefficients and the number of multisteps, s, can be chosen by the user.

The transformation that the decoder performs can be interpreted as the nonlinear mode shapes. Mathematically, the transformation can be expressed as $\hat{\boldsymbol{x}} = \sigma^j \left(\mathbf{W}^j \sigma^{j-1} \left(\mathbf{W}^{j-1} \sigma^{j-2} \left(\ldots \sigma^1 \left(\mathbf{W}^1 \boldsymbol{q}\right)\right) \ldots\right)\right)$ where $\mathbf{W}^j$ is the weight matrix at layer j and σ^j is the activation function at layer j. A linearized representation of the modes is obtained by simply multiplying the decoder weight matrices $\Phi = \prod_j \mathbf{W}^j$. In the next section, an illustrative example is provided to demonstrate the methods.

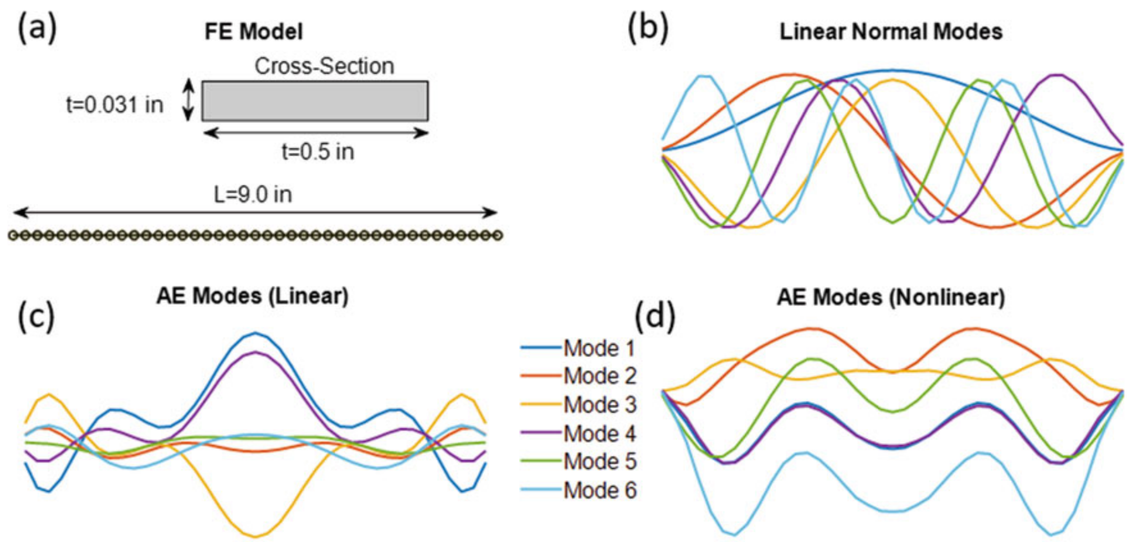

Fig. 17.2 Representation of the beam (**a**). Linear normal modes (**b**). Estimated shapes from AE for linear response (**c**). Estimated shapes from AE for nonlinear response (**d**)

17.3 Analysis

To demonstrate the approach, it was applied to a flat beam with geometric nonlinearities originating from axial stretching of the beam as it deforms, which has been used in previous studies [3]. The beam was modeled with 40 one-dimensional finite elements and solved using the open-source finite element research code (OSFERN) [4]. The beam was constrained to in-plane motion with the end nodes clamped, resulting in 117 free degrees of freedom (DOFs). A 110 dB Gaussian flat band (5–1000 Hz) random pressure load was applied transverse to the beam to excite its bending modes. Both linear and nonlinear responses were simulated and stored to use as training data for the network.

For both cases, the latent dimension was set to 6. For reference, the linear flat beam has four bending modes, including two symmetric modes, under 1000 Hz, so six latent dimensions should be enough to capture these modes. Both the encoder and the decoder had each only one nonlinear layer, followed by a linear output layer. The hyperbolic tangent function was used for the nonlinear activation. With this setup, the Φ matrix was extracted from the decoder, and the "modes" for the transverse DOFs are plotted in Fig. 17.2. As illustrated, the mode shapes are qualitatively different from the elastic mode shapes obtained from linear modal analysis and are not necessarily orthogonal to each other. The "mode shapes" extracted from the decoder with the nonlinear data were qualitatively different from the linear case and very different from the elastic mode shapes, as shown in Fig. 17.2.

The decoded displacements in the physical domain are shown in Fig. 17.3. To obtain these results, the original displacements (and velocities) were encoded and then time-integrated in the latent space using a Runge-Kutta routine with the transformed external force ($\Phi^t f_{\text{ext}}$) as input. The response in the latent space was then transformed to the physical space by passing it through the decoder. Again, there is an accumulation of error as time goes forward, but the predictions are close to the actual response. It should be noted that the predictions for the nonlinear problem were better than those for the linear case. This result is likely due to the nonlinear activation functions used to model the linear problem, where, in reality, there should not be any nonlinearities in the linear transformations.

17.4 Conclusion

A method for data-driven reduction of structural systems with autoencoders was demonstrated on a flat beam with geometric nonlinearities. The initial results are promising, suggesting that the reduced representations of the data can be used to perform time integration and obtain the structural response to new inputs or extend the response further in time. To address the challenge of error accumulation, future efforts will focus on incorporating physics constraints, such as ensuring that energy is conserved.

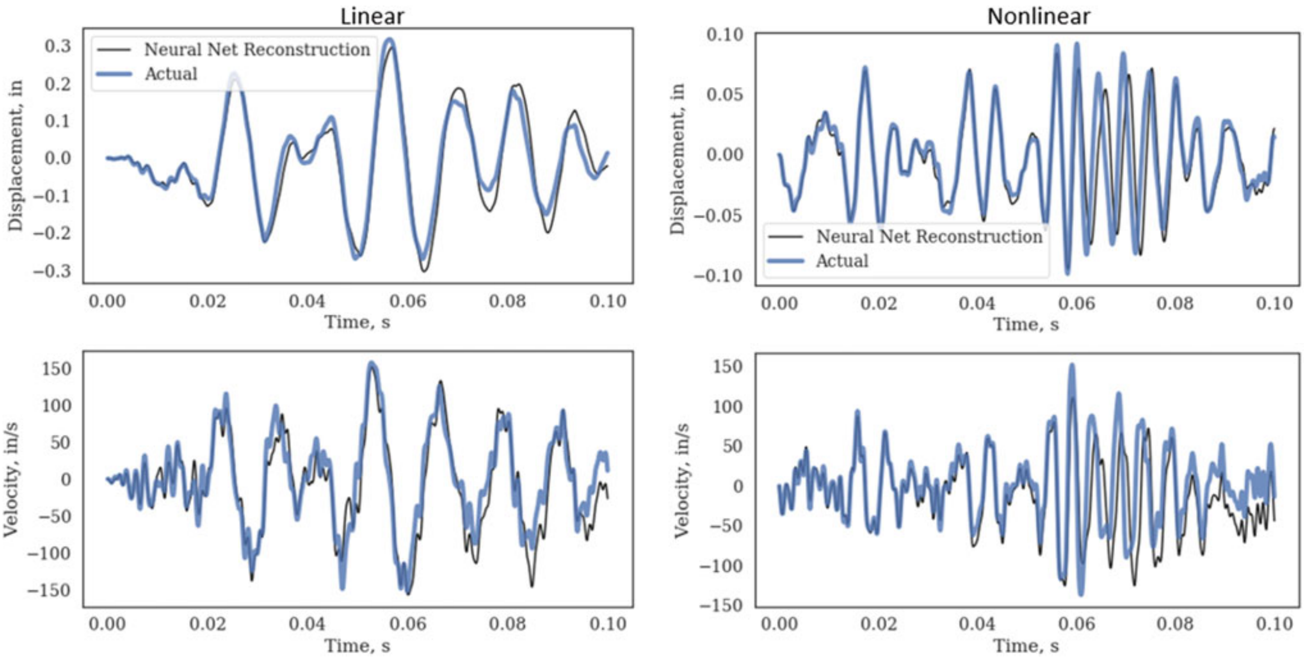

Fig. 17.3 Time-integrated displacements and velocities in the physical domain for linear (left) and nonlinear (right) beam

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