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Modelling uncertainties in electrical power systems with stochastic differential equations

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ABSTRACT

The presence of uncertainties in electrical power systems is a critical issue due to the effects that those perturbations can induce in the quality, safety and economy of the electrical supply. An accurate representation of these phenomena is needed in order to understand and predict possible damage scenarios, which allows the improvement of system operation. In this context, the method of stochastic differential equations provides a dynamic representation of power systems under continuous time uncertainties. Thus, this paper presents the application of stochastic differential equation in power system analysis and explain a systematic methodology to get a representative model of a perturbation, including estimation methods and a validation procedure. Three real examples are given, two of them related to non-conventional energy resources and the third related to energy consumption.

1. Introduction

The dynamics of an electrical power system (EPS) is continually affected by perturbations of random nature, ranging from system failures associated with external factors, such as accidents that damage the physical structure of the installations, the demand of electrical energy, the inclusion of non-conventional renewable energies (NCRE) (such as wind and tidal parks), etc.

The increasing inclusion of NCRE in energy supply systems implies a high presence of stochastic components, which did not occur in the past. Due to this, multiple methods of analysis on random dynamical scenarios have been developed to study their effect on the EPS [1–3] and thus have the correct instruments and information to perform operational and planning studies accordingly to the current state of the EPS.

The specialized literature of recent years has tried to model the uncertainties that non-conventional renewable energies can induce in the operation of power systems. Specifically, reference [4] shows certain classifications related to available analysis tools. The present work presents six analysis categories classified according to the following: Probabilistic approach [5], Possibilistic approach [6–8], Hybrid possibilistic?probabilistic approaches [9,10], Information gap decision theory [11], Robust optimization [12] and Interval analysis [13], where probabilistic density function (PDF); fuzzy logic concepts and error measurements are used to characterize the behavior of NCRE.

On the other hand, reference [14] offers a review of important research carried out in order to characterize the NCRE. In [15] it is presented several methodologies to determine the optimum size that solar and wind energy must have to coexist in a hybrid power system, understood as a system that operates with both conventional energies (thermal and hydraulic) and NCRE. In [16] it is shown the possibilistic-probabilistic approach to evaluate the impact of the NCRE on the distribution network. In [17] is it considered the impact of NCRE on the operations of power systems considering, as an analysis element, the low rotational inertia that they contribute to the system. The work reported in [18] mainly focuses on determining the economic impact that wind energy has on the operation of power systems. The reference [19] shows how the load frequency control (LFC) is used to evaluate the impact of several power generation sources, clearly including NCRE. Similarly, reference [20] presents a similar study to the one mentioned above, but considering the effects at a distribution level. The work presented in [21] displays the impact of wind energy on future electric networks. In [22] it is offered a review of the models which allow to carry out economic load dispatch in presence of high NCRE penetration. In [23] it is considered the presence of DC links in the operation of AC systems for the incorporation of NCRE on a great scale. On the other hand, [24] presents a review of how to design power systems that allow to include a high penetration of NCRE, mainly considering the

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evaluation of the economic impact on the operation.

Nowadays, it is of particular importance to analyze how the form in which we store power and the NCRE can help the operation of power systems efficiently. Accordingly, references [25,26] display mathematic models which allow to address the previously mentioned elements. Particularly, [27] shows how batteries allow to strengthen the solar energy penetration in power systems.

Other studies that require to model the NCRE, correspond to economic analyses associated to the economic dispatch [28–30], transmission expansion [31,32] and distribution development [33,34] where the probabilistic approach is used again as an analysis tool.

Considering all the important contributions achieved during recent years to characterize the NCRE in the operation of the power systems, there is still a pressing need for new models that allow to improve and enhance the dynamic studies, which consist in the resolution of non-linear algebraic differential equation systems. It is this context that provides the opportunity to present models based on stochastic differential equations, that have the potential to characterize the NCRE variability at every moment, so as to address the uncertainty as comprehensively as possible. Reference [35] proposes a model which allow to approach the transient stability problem by means of stochastic differential equations. Moreover, the work presented in [36] proposes to model wind energy by using a stochastic model. The present work proposes a generalization of the Ornstein-Uhlenbeck model to characterize different uncertainty situations that can affect in the operation of power systems.

A specific tool used to model the system in this context, is the application of stochastic differential equations (SDE), or also known as Ito calculus. The intuitive idea is to add a noise with some probability distribution into the equation of a dynamical system [37]. This allows a random dynamic representation of a EPS, which permits to perform real-time studies in the continuous effect of including on the system non-conventional renewable generation [38], load randomly changing, etc.

Among the most popular processes used in these applications, the Ornstein-Uhlenbeck process (OU) stands out. For example, in [39] this process is included as noise in the dynamical equation of a single machine-infinite bus (SMIB). It is also widely used and accepted for the representation of sources of wind generation, which allows a dynamic study of the impact that the inclusion of this type of energy sources produces. For example, [40] includes the wind power in the dynamic equations of a EPS, based on the OU process. With this model a method of long-term stability analysis is developed, whose effectiveness is validated and demonstrated by numerical examples. The OU process is also used in [41], where a hybrid stochastic model is presented, which integrates a wind generation model to represent the wind speed. It makes use of a deterministic scheme, that under certain conditions, approximates in a precise way the trajectories of the stochastic model and allows the analysis of stability. The need for stochastic models is highlighted in cases of high variability or operation near the stability limits.

In [42] the excitation of a wind turbine is modelled by means of the Brownian motion, a component that, when integrated into the dynamic equations of the system, generates a SDE, addressed by the Ito calculus. The general model of the generated system is used to study the transient stability in a test system (IEEE 39-bus). SDE are also applied in [43] for wind speed modelling, where a construction of the equation is proposed in which the probability distribution of the process associated with that equation can be manipulated. This allows to work with the experimental distributions of the wind behaviour found in the literature. The method is applied to a real case of study in New Zealand.

Another application of the SDE concerning wind generation can be found in [44], where three approaches to modelling wind speed based on SDE are discussed, with the statistical characteristics of the Weibull distribution and the exponential autocorrelation. These approaches are applied to the analysis of real data of a station in Ireland.

This method is also used at transmission level, as in the case of [45], where SDE are used to analyse the uncertainties affecting an electric transmission network with wind power generation and their impact on its reliability. The variables of consumption, ambient temperature, wind speed and wind generation are considered in the modelling. The result gives information about possible system states, in terms of power request and supply, that are critical for network stability.

The SDE also present a remarkable performance to analyse the stability, either of small signal, of voltage or transient. In [46], a single machine connected to an infinite bus is modelled with a SDE, adding noise with a Brownian motion. Then the equation of Fokker-Planck is used to model the probability density function of the stochastic SMIB model and describe its evolution over time. The probability density function can be used to determine the impact of random load perturbations on system stability by means of numerical analysis.

On the other hand, in [47] the small stability of a system is studied, when it is subject to sustained perturbations over time. These disturbances are modelled by the OU process, which directly affects the dynamics of the state variables. In addition, a stability indicator according to the random modelling scenario is presented. In [48] the Ito SDE theory is also used to investigate the stochastic small signal stability of power systems, but in this case the stochastic excitation of the mechanical power input of the asynchronous wind turbine is represented.

Regarding the transient stability, a model based on SDE is proposed in [49] to asses that analysis on a stochastic power systems. The model is tested with the nine-bus test system to demonstrate its effectiveness. Reference [50] also uses a similar method, but with the aim of optimizing the calculations that are demanded with Monte Carlo simulations. This method is applied in the four-generator and two-area system to verify its effectiveness.

The voltage stability is analysed in [3], where the effect of random loads in the classic three bus system model is studied, introducing a random component in the reactive power, which consists in a weighted Brownian motion. The addition of this disturbance transforms the system of ordinary differential equations into a SDE system. The effect of this random load on the voltage stability is illustrated by means of numerical resolutions of the SDE obtained. A similar work can be found in [51], where the consumption is considered as a stochastic process, that equation considers a deterministic function in addition to a Brownian motion weighted by a time-dependent function.

This paper conducts a general revision of the SDE method and shows its modelling effectiveness in three different application scenarios. It is organized as follows: Section 2 presents a general construction of a model based on SDE. It highlights the most important results of the Ito calculus, useful for this application. In Section 3 it is presented parametric estimation techniques for this type of equations and a validation method for the models. In Section 4 the application of the methodology is presented in three real examples, showing the results of estimation, validation and simulation for each case. Finally in Section 5 conclusions and future work are presented.

2. Modelling uncertainties with SDE

The general model that represents the dynamics of a EPS affected continuously over time by a perturbations is characterized by (1).

$$\dot{x} = f(x, y, \xi_t) \tag{1}$$

where x state variable, f function that characterizes the relationship between variables and ξ_t a perturbation of random character which can represent, in real applications: wind generation, wave power, electric consumption, etc.

As it was mentioned in the introduction, SDE are used to characterize the dynamic of a stochastic disturbance, considered as a random variable. In this case, ξ_t is considered as a stochastic process, which can be represented as a function g of a known stochastic process

η_t, which satisfies an equation of the form (2)

$$d\eta_t = \alpha(t, \eta_t)dt + \beta(t, \eta_t)dW_t \tag{2}$$

where α and β are functions known as drift and diffusion coefficient, respectively, and W_t Brownian motion.

Thus, the question is how to choose the function g and the process η_t in order to characterize the perturbation under study. Such task is directly related to the knowledge of the physical behaviour of the disturbance to be modelled and the statistical analysis of available real measurements. In the next section, it is explained with three real examples how to proceed in order to stablish a model.

Continuing with the theoretical framework of models based on SDE, this paper proposes to use the Ito formula to obtain the explicit SDE that satisfies the process ξ_t, which models a random perturbation sustained over time.

This formula is extracted from the theorem of the Ito formula [52], which states that given $g: [0, \infty] \times \Re \to \Re$ a function once continuously differentiable in t and twice continuously differentiable in x, where x represents the stochastic component and t the time, and given η_t a process with (2) as SDE. If $\xi_t = g(t, \eta_t)$, then ξ_t is a stochastic process that satisfies the SDE (3).

$$d\xi_t = \left(\frac{\partial g(t, \eta_t)}{\partial t} + \alpha(t, \eta_t)\frac{\partial g(t, \eta_t)}{\partial x} + \frac{1}{2}\beta^2(t, \xi_t)\frac{\partial^2 g(t, \xi_t)}{\partial x^2}\right)dt + \beta(t, \xi_t)\frac{\partial g(t, \xi_t)}{\partial x}dW_t \tag{3}$$

where α and β correspond to the functions of the Eq. (2) that satisfies η_t, the derivative $\frac{\partial g}{\partial t}$ refers to the derivatives with respect to the first coordinate of g, while the partial derivatives $\frac{\partial g}{\partial x}$ and $\frac{\partial^2 g}{\partial x^2}$ refer to the derivatives with respect to the coordinates with aleatory components of the function g.

With this Eq. (3) it is easy to obtain trajectories by means of simulations schemes. Those schemes are based on the Ito-Taylor expansion of the SDE. Depending on the number of terms of the expansion considered to perform an approximation, the schemes change their name, precision and complexity. For example, the linear approximation of the expansion is known as the Euler scheme, and it is the sufficient scheme to approximate linear SDE such as the OU process. For the expansion to the quadratic order the approach is known as the Milstein scheme. For more information see [37].

This work will use the Milstein scheme to simulate the resulting models. The approximation for a process η_t with SDE (2) is expressed in the Eq. (4).

$$\eta_{n+1} = \eta_n + \alpha\Delta + \beta\Delta W + \frac{1}{2}\beta\beta'((\Delta W)^2 - \Delta) \tag{4}$$

where $\Delta = t_{n+1} - t_n$ is the discretization of the time, $\Delta W = W_{t_{n+1}} - W_{t_n}$ corresponds to the Brownian increments, which is simulated by means of the relation $W_{t_{n+1}} - W_{t_n} = \sqrt{(t_{n+1} - t_n)}N_n$, where $N_n \sim N(0, 1)$ and, therefore β' is the derivative with respect to the stochastic coordinate.

Once a model is established, it is necessary to define the values of the parameters associated with the equation and validate the effectiveness of the proposed model.

This particular type of modelling procedure considers that there are real information of the disturbance to be modelled, so the parameters of the equation are estimated by statistical techniques and the validation of the model is based on the proximity between the real data and the data simulated by the equation.

3. Estimation and validation methods

In this section three parametric methods are presented in order to obtain estimators of the parameters involved in a model from real observations. In addition, as validation method for model based on SDE, the uniform residual technique will be presented.

3.1. Quasi maximum likelihood

To clarify ideas, we will consider an autonomous process defined by the differential Eq. (5).

$$d\eta_t = \alpha(\eta_t;\theta)dt + \beta(\eta_t;\theta)dW_t \quad , \eta_0 = x \tag{5}$$

where θ corresponds to the parameters present in the equation.

It is assumed that (5) has unique solution for all θ in some subset Θ of the real line and that the function β is positive. Besides, functions α and β are known and twice continuously differentiable with respect to their both arguments. The parameter θ is estimated from the discrete equidistant observations of η_t, denoted by $\eta_\Delta, \eta_{2\Delta}, \ldots, \eta_{n\Delta}$.

According to [53] the quasi maximum likelihood estimation function is:

$$G_n^*(\theta) = \sum_{i=1}^{n} \frac{\dot{F}(\eta_{(i-1)\Delta};\theta)}{\phi(\eta_{(i-1)\Delta};\theta)}(\eta_{i\Delta} - F(\eta_{(i-1)\Delta};\theta)) \tag{6}$$

where $F(x;\theta) = \mathbb{E}[\eta_\Delta|\eta_0 = x]$ corresponds to the conditional expectation, $\phi(\eta_{(i-1)\Delta};\theta) = \mathbb{E}[(\eta_{i\Delta} - F(\eta_{(i-1)\Delta};\theta))^2|\eta_{(i-1)\Delta}]$ conditional variance, and it is assumed that F is a differentiable function of θ.

Example 3.1. Ornstein-Uhlenbeck process

The OU process u_t satisfies the Eq. (7).

$$du_t = \omega u_t dt + \nu dW_t \tag{7}$$

where $\omega < 0$ and $\nu > 0$.

This process has as conditional expectation and variance the expressions $F(x;\theta) = u_0 e^{\omega t}$ y $\phi(u_{(i-1)\Delta};\theta) = \frac{\nu^2(e^{2\omega t} - 1)}{2\omega}$ respectively. With this it is possible to obtain an explicit formula for the estimation function $G_n^*(\theta)$, which corresponds to:

$$G_n^*(\eta) = \frac{1}{\nu^2}\sum_{i=1}^{n} u_{(i-1)\Delta}(u_{i\Delta} - u_{(i-1)\Delta}e^{\omega\Delta}) \tag{8}$$

Equating to zero, the estimator for ω is obtained:

$$\widetilde{\omega}_n = \frac{1}{\Delta}\ln\frac{\sum_{i=1}^{n} u_{(i-1)\Delta}u_{i\Delta}}{\sum_{i=1}^{n} u_{(i-1)\Delta}^2} \tag{9}$$

Provided that $\sum_{i=1}^{n} u_{(i-1)\Delta}u_{i\Delta} > 0$.

With this method the parameter ν is not possible to be estimated, because it vanishes when (8) is equated to zero.

3.2. Estimation by quadratic variation

In some cases the above method is not enough, since it does not provide information for all the parameters present in the equation. For these cases, the estimation by quadratic variation technique can be used. Consider a diffusion process η_t with SDE (2), the explicit form for the quadratic variation of this type of process is expressed in (10).

$$[\eta]_t = \int_0^t \beta^2(t, \eta_t)ds \tag{10}$$

The idea is to match the theoretical expression (10) with its discretization: $\sum_{k=1}^{n}(\eta_{t_k} - \eta_{t_{k-1}})^2$, where the aleatory variables η_{t_k}, with $k = 1, \ldots, n$ correspond to discrete observations of the process η.

Example 3.2. Ornstein-Uhlenbeck process

For this process the quadratic variation is:

$$[u]_t = \int_0^t \nu^2 ds = \nu^2 t \tag{11}$$

Equating with its discretization and clearing ν, the following estimator is obtained:

$$\tilde{\nu} = \sqrt{\frac{1}{t}\sum_{i=1}^{n}(u_{i\Delta} - u_{(i-1)\Delta})^2} \tag{12}$$

where t corresponds to the total time of observation.

3.3. Hybrid methods

In some cases the above estimators are very complicated, due to this, it is not possible to obtain explicit formulae of conditional expectation and variance, then it is necessary to implement numerical methods. In such cases it is convenient to use a stochastic process which can be written as a sum of other stochastic processes with known estimators. For example, as it will be seen in the next section, the electric consumption can be modelled as a sum of sines and cosines added to the OU process. For the estimation of the parameters associated to the sinusoidal functions it is much convenient to use methods of time series such as moments, least squares or maximum likelihood, and then, with the residuals obtained by the adjustment, to estimate the parameters associated to the noise represented by the OU process. Therefore, according to the structure of the stochastic process proposed it is possible to mix different estimation techniques based on observations of different statistical models.

3.4. Validation method

As validation method for models based on SDE, the uniform residuals technique [54] will be presented, which basically consists in determining if the real observations obtained, come from a process with the characteristics of the chosen model.

The systematic procedure begins with the calculation of the following random variables (13):

$$R_{t_i(\theta)} = \frac{\eta_{t_i} - F(x_{t_{i-1}};\theta)}{\sqrt{\phi(\eta_{(i-1)\Delta};\theta)}} \tag{13}$$

Those variables are known as standardized residuals. For the case of Gaussian processes, the standardized residuals must follow a standardized normal distribution, which implies that if they are compounded with the normal accumulated distribution function Φ, random variables with uniform distribution $U(0, 1)$ must be obtained, that is, variable (14)

$$\Psi_{t_i}(\theta) = \Phi(R_{t_i(\theta)}) \tag{14}$$

where $\Phi(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-t^2/2}dt$, must fulfil that $\Psi_{t_i}(\theta) \sim U(0, 1)$. For non Gaussian processes, the adequate distribution function must be used.

According to this method, if the proposed model represents the observations of the disturbance adequately, it must be obtained that the variables (14) follow a uniform distribution $U(0, 1)$, which can be checked by means of a quantile-quantile plot, which compares the quantiles of the random variable (14), with the theoretical quantiles of a variable with uniform distribution $U(0, 1)$.

4. Simulation results and discussion

Three real applications of models based on SDEs will be analysed. Two of them correspond to sources of NCRE of wind and wave, which introduce an important random component into the network, because the power generated by these sources depends on random factors such as speed and wind direction.

A third example corresponds to residential consumption, whose random component is directly related to the non-constancy in the use of energy by this consumption sector.

4.1. Wind energy

To analyse how wind generation really behaves, active power output measurements of a wind farm located in the north of Chile were used, which correspond to observations every second for three hours in kW.

The behaviour of wind generation in general is subject to several factors, among the most important is the wind speed and the installed capacity of the wind farm. In general, aspects such as the irregularity of the wind speed that translates into a generation without a defined tendency and seasonality are distinguished. The trajectories of the measurements of the wind generation are bounded in the interval $[0, P]$(where P corresponds to the total installed capacity of the park). In addition, the generation samples do not change discontinuously and the trajectories of the power generated vary around an average value. Finally, it is considered that the wind speed has normal logarithmic distribution, since it allows to work with the Brownian motion [55,56].

Since it is considered that the wind follows a log-normal distribution, it is natural to propose a process ξ_t corresponding to the composition of the exponential function $g(t, x) = e^{x+h}$ together with a Gaussian process as the OU process of the Eq. (7). The parameter h is included to ensure that the paths of the model move around the real average value of the observations. Thus, the model to be proposed is expressed in the Eq. (15).

$$\xi_t = e^{u_t+h} \tag{15}$$

According to the Ito formula (3), the process (15) satisfies the SDE (16).

$$d\xi_t = \left(\omega \ln\xi_t - \omega h + \frac{\nu^2}{2}\right)\xi_t dt + \nu\xi_t dW(t) \tag{16}$$

4.1.1. Estimation

In this example, it is possible to work directly with the O-U process, since $u_i = ln(\xi_i) - h$, where h is estimated by taking the expectation of $\ln\xi_t$, that is:

$$h = \frac{1}{n}\sum_{i=0}^{n}\ln(\xi_{i\Delta}) \tag{17}$$

Parameters ω and ν are computed from the estimators given in the Examples 3.1 and 3.2. To calculate this parameters, 80% of the total data are used, in order to use the remaining 20% to measure the performance of the model to predict future values. The results for the parameters are shown in Table 1.

4.1.2. Residual analysis

To validate this model it will be verified that the OU process adjusts to the data transformation $u_i = ln(\xi_i) - h$. The standardized residuals in this case are expressed in Eq. (18).

$$R_{t_i(\omega,\nu)} = \frac{u_{t_i} - u_{t_{i-1}}e^{\omega\Delta}}{\nu\sqrt{\frac{e^{2\omega\Delta}-1}{2\omega}}} \tag{18}$$

Fig. 1 shows the quantile-quantile plot of the values obtained from the composition of (18) with the function of normal distribution and the theoretical uniform distribution. This analysis is of particular relevance, as it allows to ensure that the adjustment made is accurate. The

Table 1
Estimated parameters of the wind power model.

Parameter	Estimated value
ω	−8.707802
ν	0.2309347
h	9.633223

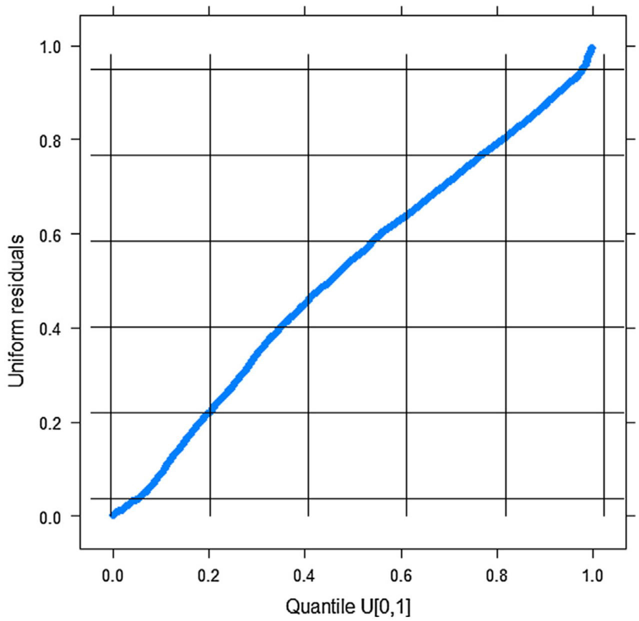

Fig. 1. Quantile quantile plot of the uniform residuals of the wind power model.

straighter the line obtained from the residuals, the better will be the approximation obtained [56].

Since the points follow approximately the identity line, it means that the observations obtained fit the type of model proposed and, therefore, there are no reasons to doubt of the adjustment.

4.1.3. Simulation

The discretization of the Eq. (16) according to the scheme (4), is expressed in Eq. (19).

$$\xi_{i+1} = Y\xi_i + (\omega\xi_i \ln(\xi_i) - \omega\xi_i h + \frac{1}{2}\xi_i \nu^2)\Delta + \nu\xi_i \Delta W + \frac{\nu^2\xi_i}{2}((\Delta W)^2 - \Delta) \quad (19)$$

In Fig. 2 a simulated trajectory of the process ξ_t with the previously estimated parameters is observed in black. As a comparison, in red the trajectory of the real data was plotted. The blue line marks the 80% of the data, that is, on its left the graph of the adjustment is observed, while on its right the graph of the projection of future data is observed.

According to Fig. 2, the simulated wind trajectories (black) follow the tendency of the real curve (red). Thus, with the remaining 20% of the samples, it is projected a simulated trajectory to estimate the value that it should have in function of the estimated proposal. The considered errors are presented below, which show that the results obtained are coherent with the residual analysis proposed.

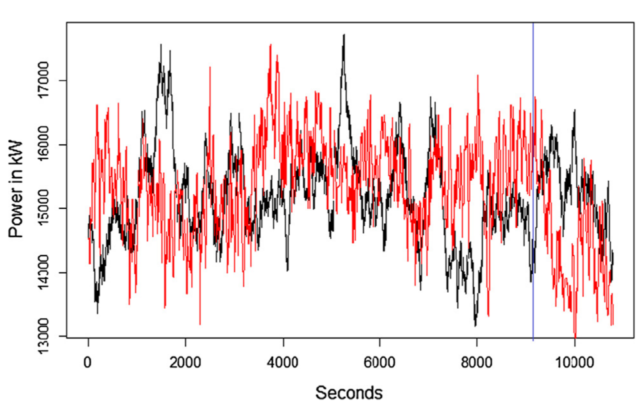

Fig. 2. Simulated trajectories of the wind power model.

Table 2
MPE of wind power model.

Error fit	Error forecast
0.194 %	4.690 %

4.1.4. Error measurements

Finally, in order to measure the precision of the model, the mean percentage error (MPE) of the adjustment was calculated, and also the projection errors, which use the remaining 20% of the data not considered for estimation, to compute the MPE value. The results are shown in Table 2.

4.2. Wave power

The power generation that can be extracted from the waves depends directly on two parameters: the significant wave high (SWH) and the wave period (WP), so the model proposed describes these two quantities separately. Then, with the series generated, it will be possible to obtain a representation of the power that is generated under these conditions.

In this case, there are measurements of SWH (in meters) and WP (in seconds) available from a buoy located on the coasts of Valparaiso, Chile, corresponding to observations every three hours during a month. This buoy is located on a depth of 145 m, which is considered in the literature as deep waters and in which case the expression for the power generation in these conditions is given by the Eq. (20)[57].

$$J = \frac{\rho g^2 H^2 T}{32\pi} \quad (20)$$

where H is the SWH, T is the WP, $\rho = 1025\,\mathrm{kg/m^3}$ is the mass density of sea water and $g = 9.8\mathrm{m/s^2}$ gravity acceleration.

Another way to obtain the power generation is to use the power matrix of a device that operates under the conditions of study. For this particular case the power matrix of the Floating oscillating water column [58] will be used, since this device meets with the operating conditions of the geographical location under study.

An important characteristic of the behaviour of the waves in deep waters, is that in these cases it is possible to apply the theory of linear waves, where it is feasible to demonstrate that the displacement of the surface of the wave is a Fourier series [59]. Then for the SWH the function $g_1(t, x) = a + \sum_{i=1}^{n}(\gamma_i \cos(2\pi t f_i) + \delta_i \sin(2\pi t f_i)) + x$ will be considered, where $x$will again represent the OU process. For the case of WP, also a sinusoidal function will be used, but composed by the exponential function, that is, $g_2(t, x) = e^{b+\sum_{i=1}^{n}\zeta_i sin(2\pi f_i t)+\kappa_i cos(2\pi f_i t)+x}$. Then, according to the Ito formula (3), the SDE that model SWH and WP are expressed in (21) and (22), respectively.

$$dH_t = (\sum_{i=1}^{n}((-\gamma_i 2\pi f_i - \eta\delta_i)sin(2\pi f_i t) + (\delta_i 2\pi f_i - \omega\gamma_i)cos(2\pi f_i t)) + \omega H_t - \omega a)dt + \nu dW_t \quad (21)$$

$$dT_t = T_t(\sum_{i=1}^{n} - \zeta_i 2\pi f_i sin(2\pi f_i t) + \kappa_i 2\pi f_i cos(2\pi f_i t) + \omega(\ln T_t - b - \sum_{i=1}^{n}\zeta_i cos(2\pi f_i t) + \kappa_i sin(2\pi f_i t)) + \frac{1}{2}\nu^2)dt + \nu T_t dW_t \quad (22)$$

where H_t is the SWH at the instant t, T_t is the WP at the instant t, f_i frequencies associated to seasonal changes, U_t O-U process and $a, \gamma, \delta, b, \zeta, \kappa$ parameter to be estimated.

4.2.1. Estimation

This type of model is an example of a hybrid representation and the method of estimation to be used is the least squares, applied to harmonic regression, together with the spectral analysis of the data series

Table 3
Significant frequencies of SWH and WP series.

SWH				WP	
Frequency	Value	Frequency	Value	Frequency	Value
f_1	0.004	f_8	0.056	f_1	0.02
f_2	0.008	f_9	0.012	f_2	0.024
f_3	0.02	f_{10}	0.085	f_3	0.028
f_4	0.04	f_{11}	0.035	f_4	0.036
f_5	0.044	f_{12}	0.08	f_5	0.148
f_6	0.016	f_{13}	0.095	f_6	0.04
f_7	0.028				

for the estimation of the significant frequencies f_i of both models (21) and (22).

The significant frequencies of both SWH and WP data samples are shown in Table 3, which were obtained from the periodogram (approximation of the spectral density).

With the information contained in Table 3, it is possible to estimate the coefficients a, γ_i and δ_i in the Eq. (21) and the coefficients b, ζ_i and κ_i in the Eq. (22), using the least square technique. The estimated values for each parameter of each model are summarized in Table 4.

To adjust the process u_t, the residuals left by the least square adjustment will be used as observations. According to Examples 3.1 and 3.2, the estimated values of the parameters ω and ν in both models are detailed in Table 5.

4.2.2. Validation procedure

As in the case of application to validate the wind power model, the uniform residuals of the adjustment of the OU process will be examined. Figs. 3 and 4 show the quantile-quantile plot of this analysis for each model.

It is observed in Figs. 3 and 4 that the points in the graphics follow approximately the identity line, so it can be affirmed that there is no reason to doubt of these adjustments. These figures show that the adjustments for SWH and WP are adequate since the trajectories obtained for the residual are uniform.

4.2.3. Simulated series of SWH, WP and power

Once the models have been validated, the projections of present and future values of SWH and WP must be done to apply the corresponding transformations to the series and obtain simulations of generated power.

The numerical scheme for Eqs. (21) and (22) according to (4) is presented in (23) and (24) respectively.

Table 5
Estimated parameters of O-U process for SWH and WP series.

SWH		WP	
Parameter	Value	Parameter	Value
ω	−22.0653	ω	−36.58495
ν	1.83798	ν	1.369082

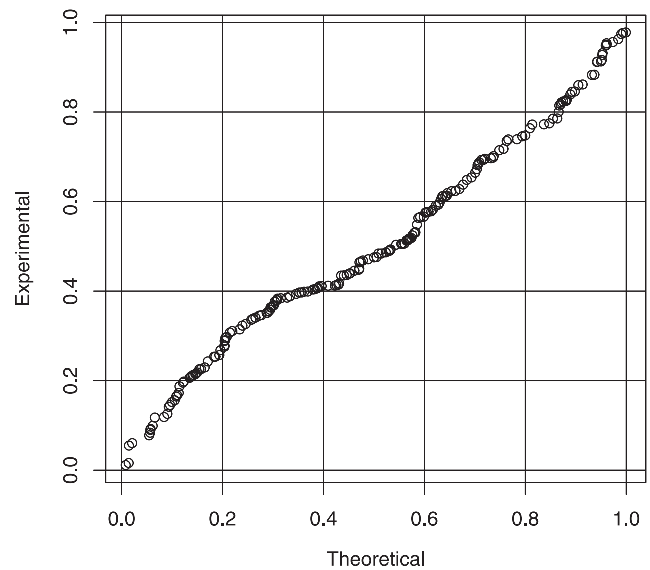

Fig. 3. Quantile-Quantile plot of uniform residuals for SWH.

$$H_{t_{j+1}} = H_{t_j} + \left(\sum_{i=1}^{n} ((-\gamma_i 2\pi f_i - \omega\delta_i) sin(2\pi f_i t_j) + (\delta_i 2\pi f_i - \omega\gamma_i) cos(2\pi f_i t_j)) + \omega H_{t_j} - \omega a \right)\Delta + \nu(\Delta W) \tag{23}$$

$$T_{t_{j+1}} = T_{t_j} + T_{t_j}\left(\sum_{i=1}^{n} -\zeta_i 2\pi f_i sin(2\pi f_i t_j) + \kappa_i 2\pi f_i cos(2\pi f_i t_j) + \eta(\ln T_{t_j} - \beta - \sum_{i=1}^{n} \zeta_i cos(2\pi f_i t_j) + \kappa_i sin(2\pi f_i t_j)) + \tfrac{1}{2}\nu \right)\Delta + \nu T_{t_j}(\Delta W) + \tfrac{\nu^2}{2} T_j((\Delta W)^2 - (\Delta)) \tag{24}$$

Both equations were simulated with a time interval ($\Delta = 0.12$ hours, which allows to generate more refined series of SWH and WP than the originals. A number of 500 simulations were performed of both equations, whose graphs can be observed in Figs. 5 and 6. By way of

Table 4
Estimated parameter of SWH and WP series.

SWH				WP	
Parameter	Value	Parameter	Value	Parameter	Value
a	2.16444961	δ_7	0.12739113	b	2.54443881
γ_1	−0.52041385	γ_8	0.05406861	ζ_1	−0.10642786
γ_2	0.41558115	δ_8	−0.11663150	κ_1	−0.03693054
γ_3	0.09459525	δ_9	−0.20366855	κ_2	0.05297084
δ_3	−0.25239546	γ_{10}	−0.13781228	κ_3	0.03609110
γ_4	−0.25232424	δ_{11}	−0.13920062	ζ_4	0.03558274
δ_4	−0.08251500	γ_{12}	0.10484563	κ_5	0.04967259
γ_5	0.24262507	δ_{12}	−0.07390363	ζ_6	−0.04178142
γ_6	0.14223323	γ_{13}	0.10370307	κ_6	−0.05086355
γ_7	0.09393246				

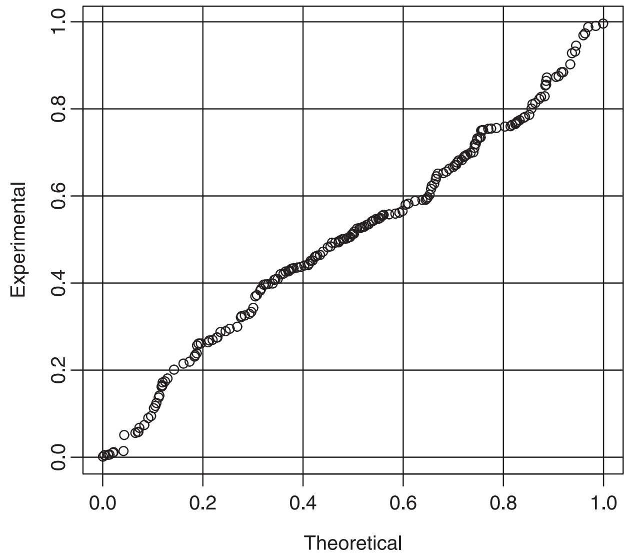

Fig. 4. Quantile-Quantile plot of uniform residuals for WP.

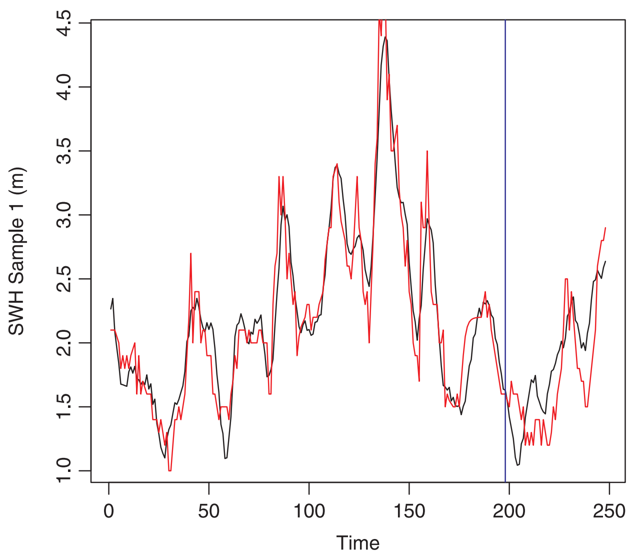

Fig. 5. Simulated trajectories of SWH.

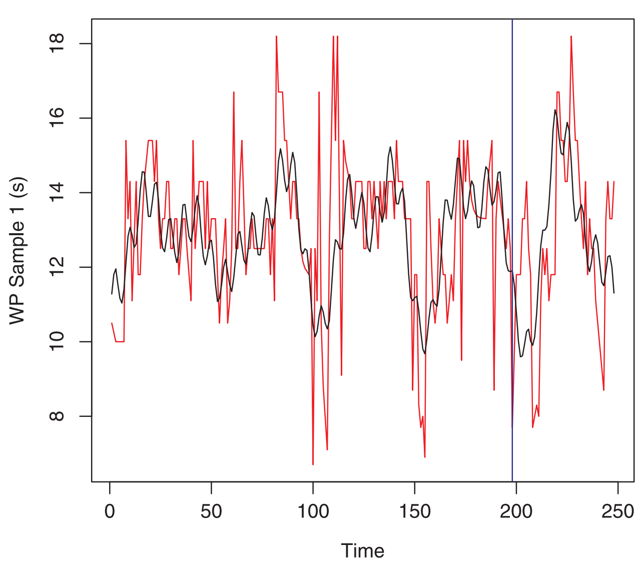

Fig. 6. Simulated trajectories of WP.

comparison, the average of the 500 simulated trajectories is plotted in black together with the graph of the real data trajectory in red. The blue line in the four graphs marks to the left side the 80% of the data with which the parametric adjustments were made and to the right side the 20% that was left out to be able to compare performance of the model at the moment of forecasting future values. This will allow to measure the prediction errors of the settings in the next stage.

Fig. 5 shows a good behaviour describing present and future values: it is possible to capture the variations of slope and the randomness present in these variations. Similar to the previous case, the parameters are estimated using 80% of the data and the projection is compared to the 20% of the remaining values. It can be clearly observed that the real trajectories (red) and the simulated ones (black) follow the same tendency.

In the case of WP, Fig. 6 shows that the variation of the model is much lower than that of the real data. However, the ability of the model to follow the trend of period variations stands out.

4.2.4. Physical Power

The interest variable of power generation will be obtained by means of two methods. The first corresponds to a transformation of the data generated in the previous section through the physical relationship for the deep-water wave-power level, expressed in the Eq. (20), where it takes as an input of H (SWH) and T (WP) the series generated by the proposed models.

The series for the power generated under the Eq. (20) is presented graphically in the Fig. 7.

For Fig. 7 a good adjustment of the present data is observed, following the slope changes correctly. For future projection, although the change in trend is well described, the random variation is greater in the real data than in the simulated average trajectory.

Graphically, it should be noted a decrease of the variance. This is because the simulated observed trajectory corresponds to an average of 500 simulations and not to a single trajectory.

4.2.5. Floating oscillating water column

The second method to obtain a series representing the power generation corresponds to a transformation of the data generated of SWH and WP by means of the power matrix of the floating oscillating water column device (OWC) [58], which is obtained in an experimental manner together with the Pierson-Moskowitz spectrum analysis. The selection of this device is based on the fact that the operating ranges of the machine match the data provided by the buoys. The power matrix

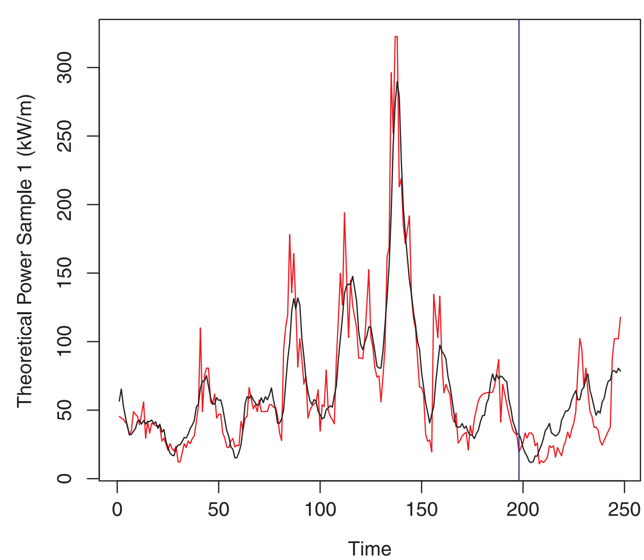

Fig. 7. Plot of theoretical power.

Table 6
Conversion matrix for the floating oscillating water column.

SWH	WP (seconds)												
(meters)	6	7	8	9	10	11	12	13	14	15	16	17	18
1.0	8	17	27	42	56	59	52	44	40	38	40	38	30
1.5	17	39	61	96	126	132	117	99	89	87	89	85	66
2.0	30	69	108	170	224	235	208	177	159	154	159	151	118
2.5	47	108	169	266	350	368	324	276	249	241	248	236	185
3	68	155	244	383	504	530	467	398	358	347	357	340	266
3.5	93	212	332	521	686	721	636	542	487	472	486	463	362
4	121	276	433	680	896	942	831	708	636	616	634	605	473
4.5	154	350	548	861	1130	1190	1050	896	805	780	803	765	599
5	190	432	677	1060	1400	1470	1300	1110	994	963	991	945	739
5.5		523	819	1290	1690	1780	1570	1340	1200	1170	1200	1140	894
6		622	975	1530	2020	2120	1870	1590	1430	1390	1430	1360	1860
6.5		730	1140	1800	2370	2490	2190	1870	1680	1630	1670	1600	1250
7		847	1330	2080	2750	2880	2540	2170	1950	1890	1940	1850	1450

for this device is presented in Table 6.

By means of a bilinear interpolation process, it was possible to obtain the intermediate values in the power matrix. The results of the transformation of the data series can be seen graphically in Fig. 8.

In Fig. 8 the black path corresponds to the power trajectory generated by the average of the 500 simulated SWH trajectories. In red, the power generated by the actual data is plotted and the blue vertical line represents the set point to the left, and the forecast to the right.

The graphical results are analogous to those observed in Fig. 7.

4.2.6. Error measurements

The models were subjected to a validation process and the errors of the results given by the models were measured by the mean percentage error (MPE). Table 7 summarizes the values obtained for the MPE of the SWH and WP models.

Table 7 shows the adjustment error, corresponding to the comparison of the 500 trajectories generated with the 80% of total data used for the parametric adjustment. On the other hand, the forecast error corresponds to the comparison of 20% of the total data with the respective part of the average of the 500 trajectories.

The adjustment errors of SWH and WP models of both data samples are acceptably low: less than 3% for the adjustment and less than 9% for the future forecast. However, these models use linear wave theory, and it is expected that the error of forecasting increases due to the non linearities associated to the physical measurements, but as the paper seeks for a robust model which can be useful in small signal analysis, the measurements errors are in an acceptable range.

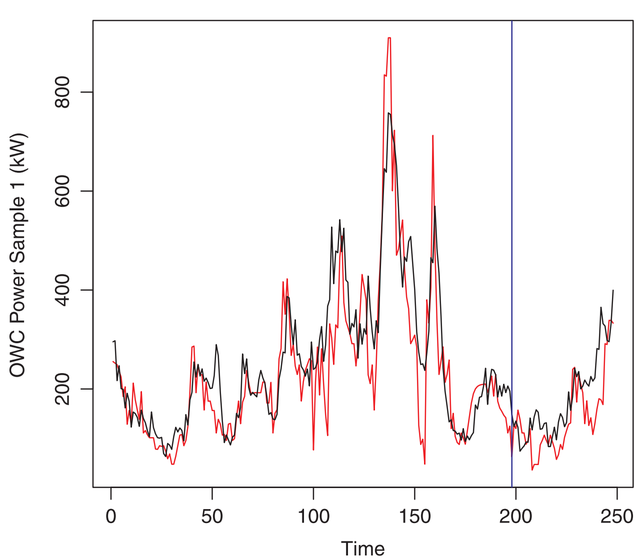

Fig. 8. Plot of OWC power.

Table 7
MPE values.

Model	MPE fit	MPE Forecast
SWH	−0.956104%	−8.851932%
WP	−1.078481%	−3.439984%

4.3. Residential consumption

To analyse the behaviour of residential consumption, real observations provided by a Chilean electric power distributor company were used. The data delivered by the company correspond to measures every 15 min during 120 days and 20 h of power in GW, from a feeder in Santiago de Chile.

In general, the behaviour of a load follows certain seasonal patterns, associated with changes in seasons of the year, day and night, weekends, etc. It is also subject to some growth due to population increase and urbanization [60]. A general characterization for this type of behaviour is expressed in the process of the Eq. (25)

$$D_t = a + bt + \sum_{i=1}^{n} (\gamma_i \cos(2\pi t f_i) + \delta_i \sin(2\pi t f_i)) + u_t \tag{25}$$

where D_t is the power demand, u_t OU process, f_i the frequencies associated with the seasonal changes and the rest of the coefficients are parameters to be estimated.

According to the Ito's formula (3), the process (25) satisfies (26).

$$dD_t = \left(b + \sum_{i=1}^{n} ((-2\pi f_i \gamma_i - \eta\delta_i)\sin(2\pi t f_i) + (2\pi f_i \delta_i - \omega\gamma_i)\cos(2\pi t f_i)) + \omega(D_t - a - bt)\right)dt + \nu dW_t \tag{26}$$

4.3.1. Estimation

The frequencies present in the Eq. (25) are found in the same manner as for the case of the wave energy. Table 8 shows the most relevant frequencies considered.

With this information it is possible to estimate a, b, γ and δ parameters of the Eq. (25) using the least square technique. The results obtained are shown in Table 9.

To estimate the parameters associated with the OU process, the residues obtained from the previously adjustment will be used. The estimation techniques and estimators are the same applied in the previous models. The results obtained are shown in Table 10.

Table 8
Frequencies associated to residential consumption.

Frequency	Value	Frequency	Value
f_1	0.00146	f_8	0.012
f_2	0.00154	f_9	0.01046
f_3	0.003	f_{10}	0.009
f_4	0.01055	f_{11}	0.006
f_5	0.03155	f_{12}	0.00454
f_6	0.02109	f_{13}	0.00163
f_7	0.021		

Table 9
Estimated parameters by leas squares of residential consumption.

Parameter	Value	Parameter	Value
α	3.664	γ_8	−2.868e-02
β	2.610e-05	δ_8	−1.478e-01
γ_1	2.956e-01	γ_9	2.123e-01
δ_1	5.207e-01	δ_9	6.856e-02
γ_2	−6.073e-01	γ_{10}	1.602e-01
δ_2	−5.442e-01	δ_{10}	5.722e-02
γ_3	−1.400e-01	γ_{11}	7.104e-02
δ_3	6.625e-01	δ_{11}	1.295e-01
γ_4	−5.464e-01	γ_{12}	−2.202e-01
δ_4	−5.166e-01	δ_{12}	1.127e-01
γ_5	1.769e-01	γ_{13}	−1.508e-01
δ_5	−3.201e-02	δ_{13}	−1.594e-01
γ_6	−2.135e-01	γ_7	7.866e-02
δ_6	1.249e-01	δ_7	−1.752e-01

Table 10
Estimated parameter of the OU process associated to residential consumption.

Parameter	Value
η_c	−5.578228
ν_c	2.164175

4.3.2. Validation procedure

Following the same procedure as in the residual analysis in the previous sections, the uniform residuals of the OU process associated to the residuals of the adjustment by harmonic regression will be studied. In Fig. 9 the obtained result is observed.

The quantile-quantile plot shows that the residuals of the harmonic regression is well adjusted by the chosen process.

4.3.3. Simulation

Discretizing 26 and using the scheme (4), it is possible to obtain (27).

$$D_{i+1} = D_i + (\beta + \sum_{i=1}^{n} ((-2\pi f_i \gamma_i - \eta\delta_i)\sin(2\pi t_i f_i) + (2\pi f_i \delta_i - \eta\gamma_i)\cos(2\pi t_i f_i)) + \omega(D_i - a - bt_i))(\Delta) + \nu(\Delta W) \tag{27}$$

In Fig. 10 it is observed a simulated trajectory in red and in black, in order to compare, the trajectory of the original data. The blue line shows the 80% of the data.

4.3.4. Error measurements

In order to provide a better comprehension of the results, the MPE will be measured, as all the previous cases. The results are presented in Table 11.

The values of MPE exhibit a good behaviour at the moment of describing past consumption, nevertheless, the projection of future series increases its errors. The presence of this inaccuracy may be due to the observation period. This period does not include the complete cycle, thus the model does not capture all the seasonal changes associated to energy consumption, it just considers the changes of a determined amount of month.

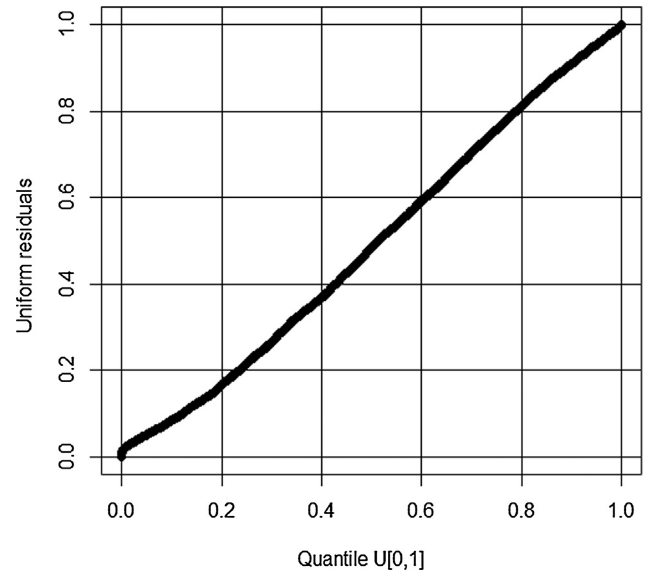

Fig. 9. Quantile quantile plot of uniform residuals of residential consumption model.

5. Conclusions and future work

The present work shows a methodology based on the Ito calculus, which considers random variables into the equation of a dynamical system. This allows a representation of random and self-sustained behaviour in time of a general perturbation affecting a EPS. When real data of some perturbation is available it is possible to use this methodology to construct parametric stochastic models. In the paper the modelling procedure is explained, including estimation methods, validation procedure and simulation schemes.

The technique is applied to three real Chilean cases. The first consist in a model of wind power, based on real data of the output power of a wind farm; the second one of a model of power energy, which uses observation of SWH and WP; and the third one of residential electric demand consumption, which is based on measures of real consumers.

The wind power is modelled as an exponential function of the Ornstein-Uhlenbeck process. On the other hand, the parameters of SWH and WP, and also the residential consumption are represented as a sum of sinusoidal functions and the Ornstein-Uhlenbeck process. The validation method shows the ability of the proposed methodology to represent and forecast data series of the exposed examples. Also the measured error of MPE is applied in the three cases in order to give a comprehensive indicator of the accuracy of the models.

The performance of the equation used to model some uncertainty depends directly on the ability of the expert to select an adequate function to explain the physical phenomenon and a stochastic process which captures the probabilistic characteristics of the aleatory component. However, the presented methodology allows the construction of a model, validated by comparing the probability characteristics of the theoretical model with those of the real data.

For future work, it is intended to generate the stochastic models proposed for multidimensional cases. It is a challenge the need to consider the interaction of the stochastic variables that represent the sustained in time random behavior introduced by the NCRE in the operation of power systems. Particularly, the approach of future work will be based on the use of stochastic differential equations to be

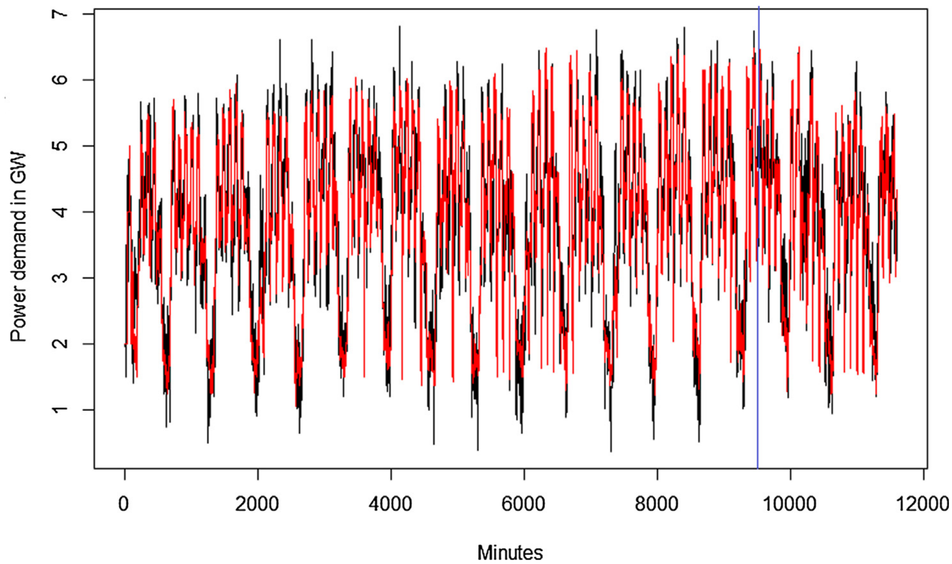

Fig. 10. Simulated residential consumption.

Table 11
Mean percentage errors of the fit and forecast.

MPE Fit	MPE Forecast
0.003519404 %	0.1215096 %

included in transient angular stability and steady state analysis.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.ijepes.2019.05.054.

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