Homework: Problem Solving

This document defines the **homework assignments** for the ["Algortihms" course @ Software University](). Please submit a single **`zip`** / **`rar`** / **`7z`** archive holding the solutions (source code) of all below described problems.

Problem 1. Shortest Path in Matrix

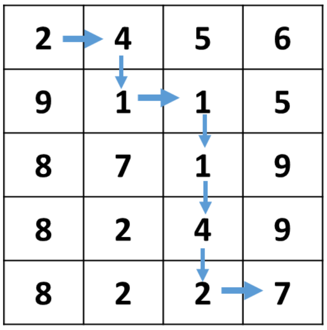
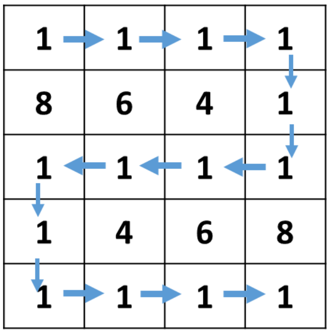
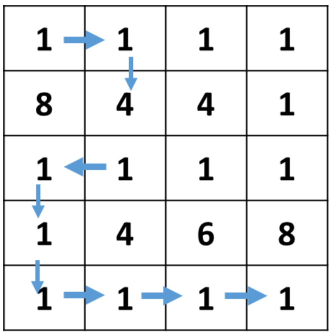
Write a program to find the **shortest path in a matrix of numbers** from the top-left corner to the bottom-right corner. The path consists of a sequence of cells, each sharing a common side with its next cell.

You will receive the number of **rows N** on the first line and the number of **columns M** on the second line. On each of the next N lines you'll receive the cells' values as a sequence of M **positive integers separated by a single space**.

Print the length of the path (sum of cell values) on the first line in format **`"Length: {length}"`**. On the second line, print the path in format **`"Path: {cell1} {cell2} …"`**. You can test your solution in the Judge system [here]().

Note: If multiple paths exist, print the one which moves through cells with lowest row and then column (traverse the matrix from top to bottom and from left to right).

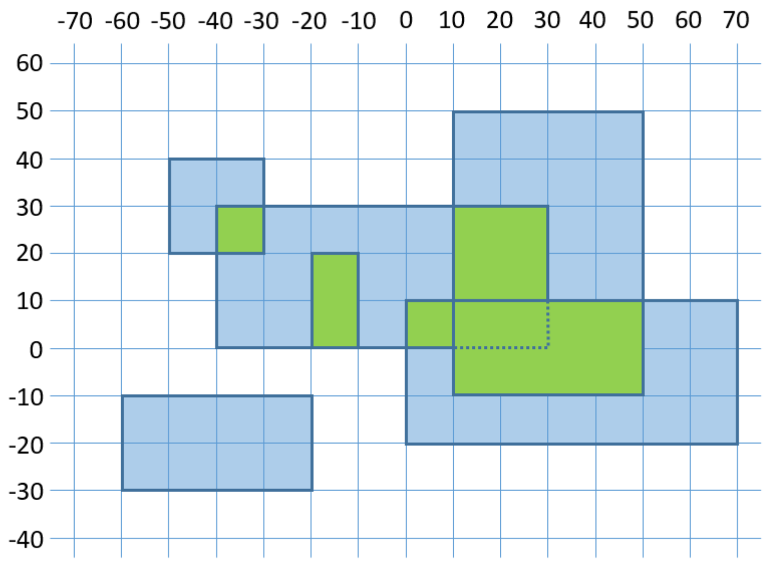
Examples:

Input	Output	Path (Visualized)
5 4 2 4 5 6 9 1 1 5 8 7 1 9 8 2 4 9 8 2 2 7	Length: 22 Path: 2 4 1 1 1 4 2 7	
5 4 1 1 1 1 8 6 4 1 1 1 1 1 1 4 6 8 1 1 1 1	Length: 14 Path: 1 1 1 1 1 1 1 1 1 1 1 1 1 1	
5 4 1 1 1 1 8 4 4 1 1 1 1 1 1 4 6 8 1 1 1 1	Length: 13 Path: 1 1 4 1 1 1 1 1 1 1	

Hint: Build a graph and use **Dijkstra's algorithm**.

Problem 2. Rectangle Intersection

You are given **N rectangles** in the plane. The rectangles are parallel to the coordinate axes and each is defined by its coordinates: **`{minX, maxX, minY, maxY}`**. Write a program to find the **total area** of all areas that belong to more than one of the initial rectangles. All coordinates are integers in the **range [-1000, 1000]**. Example:



We have 6 rectangles. Their intersection areas are shown in green. The intersection area is 1600.

On the first line you'll receive the **number of rectangles N**. On the next N lines, you'll receive the coordinates of each rectangle in format **`{minX} {maxX} {minY} {maxY}.`** On the only output line, print the total area belonging to more than one rectangle. You can test your solution in the Judge system here.

Examples:

Input	Output
6 -60 -20 -30 -10 -50 -30 20 40 -40 30 0 30 10 50 -10 50 0 70 -20 10 -20 -10 0 20	1600
3 40 80 -40 0 20 60 -20 30 50 100 -10 20	800
9 -851 88 546 659 990 999 608 998 815 835 -517 734 157 623 994 996 947 956 529 925 561 688 -241 434 -966 530 -825 273 396 780 -705 590 110 202 713 891	216777

Hints

- **Solution #1 (slow)**
  - Create a **matrix of size 2001 x 2001**.
  - **Paint** all rectangles in the matrix.
  - **Count** the painted cells.
- *** Solution #2 (faster)**
  - Extract all **X coordinates `x[]`** from all rectangles (**`minX`** and **`maxX`**) and **sort them** in increasingly.
  - For each two coordinates **`x[i]`** and **`x[i+1]`** find all rectangles **`rects[]`** that overlap with this interval, sorted by **`minY`**. To implement this efficiently, first pre-calculate the list of rectangles for each interval **`x[i]`** … **`x[i+1]`** by a single scan through the initial list of rectangles.
  - Extract all **Y coordinates `y[]`** from all rectangles **`rect[]`** (**`minY`** and **`maxY`**) and **sort them** in increasing order.
  - For each two coordinates **`y[i]`** and **`y[i+1]`** find how many rectangles overlap with this interval, **calculate the area** where **`rect_count ≥ 2`** and **sum it**. To implement this efficiently, first pre-calculate the number of overlapping rectangles for each interval **`y[i]`** … **`y[i+1]`** by a single scan through **`rect[]`**.
- *** **Solution #3 (fastest)**
  - Implement a solution based on **interval trees** as described in http://www.oi.edu.pl/static/attachment/20110713/boi-2001.pdf (see problem "**Mars Maps**")