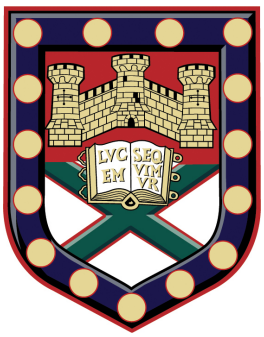

University of Exeter

College of Engineering, Mathematics and Physical Sciences

ECM3735 - Group 8

Finding the optimal strategy for the dice game 'Pig'

Authors

S. Bayliss
A. Dunford
E. Morison
J. Peet
L. Sutton

C. Crawford
R. Jones
C. Nash
A. Smith

Supervisor

Dr. B. Cooper

November 23, 2017

Abstract

Insert abstract here

Contents

1 Introduction

1.1 Aims and Objectives

josh

1.2 History of Pig

1.2.1 Basics of Pig

The dice game 'pig' is played by 2 people rolling a die. The number on the die signifies the points gained on that roll, which are collated in a sum total of points for the turn. These points can be banked to bring a players turn to an end. However, the points in this rolling total can be lost if the player rolls a 1, this will also automatically bring their turn to an end. The aim of the game is to be the first player to have a banked total of points greater than or equal to 100.

The probabilities of winning the game from any position can be simplified down into sets of linear equations. Suppose I have 2 strategies A and B. Consider that it is A's turn and we are in position (i, j, k) where i is A's banked points, j is B's banked points, and k is A's sum total of points so far on this turn. Let P_{ijk} denote the probability of A winning from the current position and Q_{ijk} denote probability of B winning from the equivalent position (where i then represents B's banked points etc.). Then,

$$p_{ijk} = \frac{1}{6}(1 - Q_{ij0}) + \frac{1}{6}\sum_{r=2}^{6} P_{ijk+r} \tag{1}$$

if A rolls and

$$p_{ijk} = 1 - Q_{jik} \tag{2}$$

if A banks their points.

1.2.2 Nellers Work

In *Dice Games Properly Explained*, Reiner Knizia takes the view of each roll of the die in 'pig' as a bet of not rolling a 1. He viewed the best strategy as "Whenever your accumulated points are less than 20, you should continue throwing, because the odds are in your favour."[Kni99]. However, Todd W Neller stated "*risking points is not the same as risking the probability of winning.*"[NP04], from this Neller proceeded to find a more optimal solution for 'pig' by taking the maximum of equation 1 and equation 2. This was not possible however as he ended up with an equation of the form $x = \max(A_1x + b_1, A_2x + b_2)$ for which there is no known general method. As a result Neller implemented value iteration to calculate an accurate estimate for the probabilities at each position (i, j, k) for both bank and roll. Figure 1 shows the resulting optimal strategy where you should roll if you are below the surface of the graph and bank if above.

1.3 Optimal Stratergy

chris C

1.4 Preliminary Findings

chris N

2 Methodology

2.1 Group Organisation

Mia

2.1.1 Meetings

2.1.2 Creation of Prject Plan

2.2 Piglet

Chris C

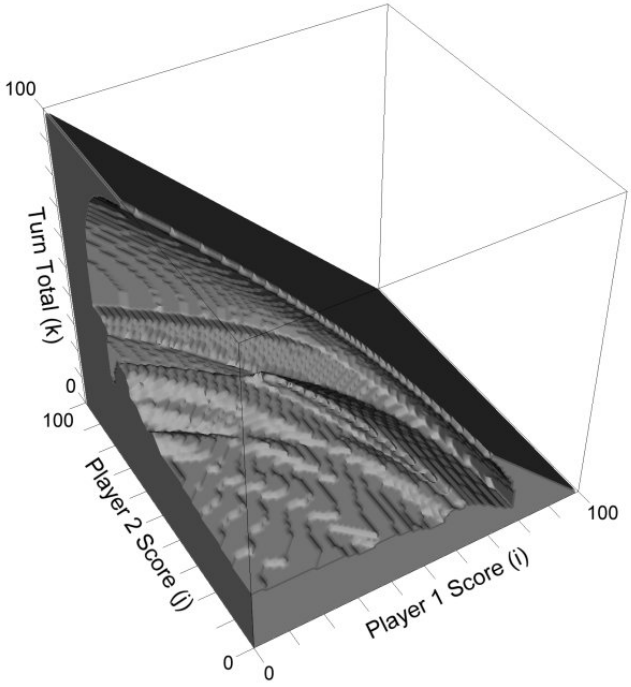

Figure 1: roll/hold boundary for the optimal Pig play policy (Neller)

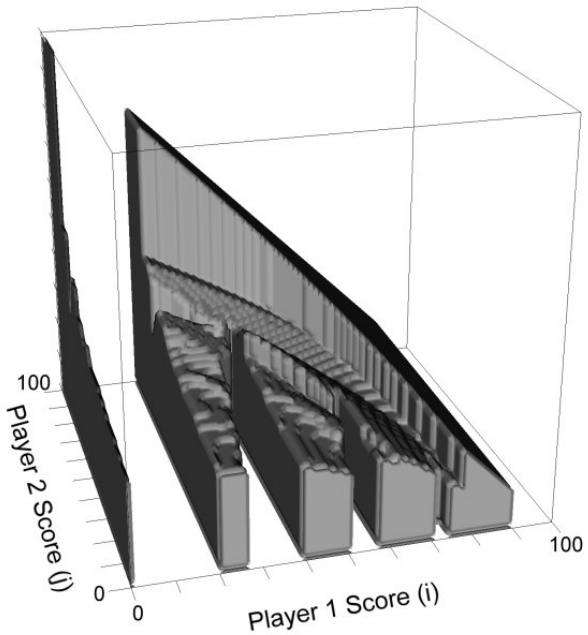

Figure 2: all of the states reachable by an optimal player (Neller)

2.2.1 Hand Written notes

2.2.2 Coding Pig

2.2.3 Piglet Findings

2.3 Pig

anthony and rhodri

2.3.1 Expansion from Piglet

I DID SOMETHING IOMPORTANT HERE

2.3.2 Debugging of the code

2.4 Behavioural Economics

Mia and Josh

2.5 Statistical Testing

eliot and sam

2.5.1 Fair test

2.5.2 Theory of Hypothosis testing

2.5.3 Running of tests

3 Findings

3.1 Pig

3.1.1 Did we solve Pig

YES.

3.2 Behavioural Economics

3.2.1 Do players stick to their risk preference

3.2.2 Players Interactions

3.3 Statistical Testing

3.3.1 Non-transitivity

3.3.2 Testing against our optimal

3.3.3 Testing against Nellers optimal

4 Conclusions

4.1 Overall findings

4.2 Determination of human affects on the optimal stratergy

4.3 Comparison to Nellers stratergy

References

[Kni99] Reiner Knizia. *Dice Games Properly Explained*. Elliot Right-Way Books, 1999.

[NP04] Todd W Neller and Clifton GM Presser. Optimal play of the dice game pig. *The UMAP Journal*, 25(1), 2004.

[NP10] Todd W Neller and Clifton GM Presser. Practical play of the dice game pig. *The UMAP Journal*, 31(1), 2010.