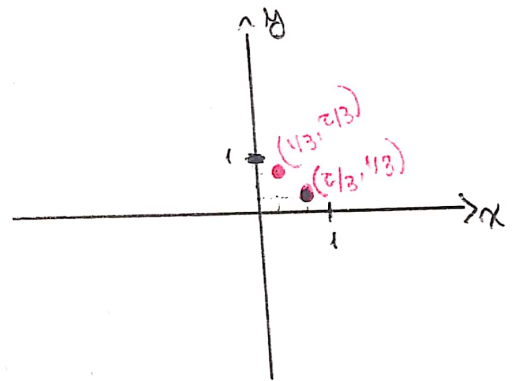2.6) a) $R = \{e_1, e_2\}$ (b. canónica de $\mathbb{R}^2$).   (e (9.2

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix} \quad , \quad T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$$

$\underbrace{}_{e_1}$ $\underbrace{}_{e_2}$

$$\rightarrow \boxed{Im R = \left\{ [\,^2\!/_3 \;\; ^1\!/_3\,]^T, \; [\,^1\!/_3 \;\; ^2\!/_3\,]^T \right\}} = \mathbb{R}_{um R}$$
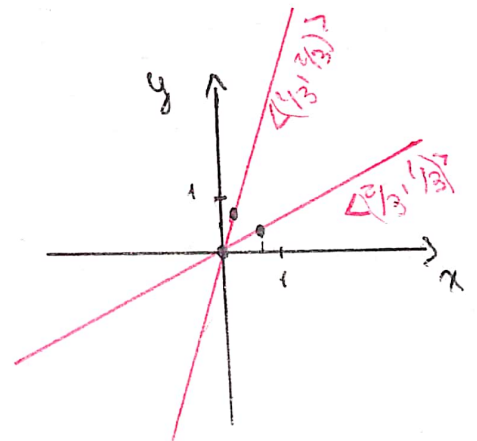
b) $R = \text{gen}\{e_1\} \cup \text{gen}\{e_2\}$ (ejes coordenados).

~~T(R)~~

$\longrightarrow \text{Im} R = \text{gen}\{T(e_1)\} \cup \text{gen}\{T(e_2)\}$

$\longrightarrow \boxed{\text{Im} R = \left\langle \begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix} \right\rangle \cup \left\langle \begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix} \right\rangle}$
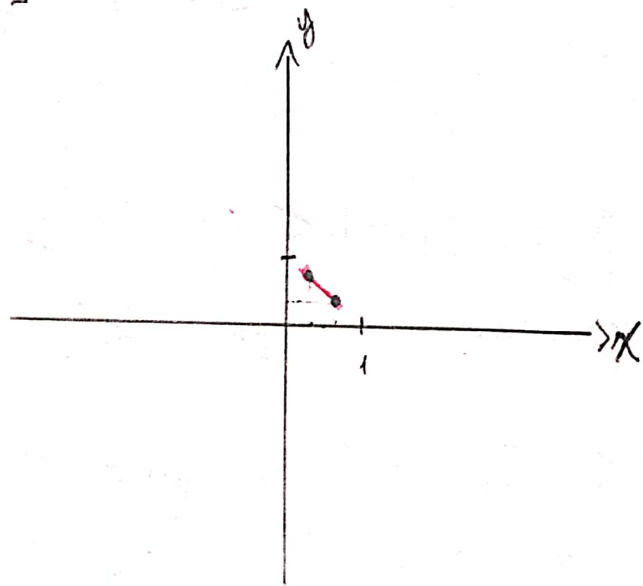
2.6) c) $R = [e_1, e_2]$ (segm. que une e_1 com e_2)

$\rightarrow R = t \cdot (e_2 - e_1) + e_1 = t \cdot (-1,1) + (1,0) = \underline{(-t+1, t)}$, $t \in [0,1]$

$$T\left(\begin{bmatrix} -t+1 \\ t \end{bmatrix}\right) = \begin{bmatrix} \frac{2}{3}(-t+1) + \frac{1}{3}t \\ \frac{1}{3}(-t+1) + \frac{2}{3}t \end{bmatrix} = \begin{bmatrix} -\frac{1}{3}t + \frac{2}{3} \\ \frac{1}{3}t + \frac{1}{3} \end{bmatrix} = t \cdot \begin{bmatrix} -\frac{1}{3} \\ \frac{1}{3} \end{bmatrix} + \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}, \ t \in [0,1]$$

$\rightarrow \boxed{\text{Im}\,R = \left\langle \begin{bmatrix} -\frac{1}{3} \\ \frac{1}{3} \end{bmatrix} \right\rangle + \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}}$, $t \in [0,1]$

$= t \cdot \begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \end{bmatrix}^T + \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \end{bmatrix}^T$

d) $R = \mathcal{P}_{e_1, e_2} = t_1 e_1 + t_2 e_2 = \text{gen}\{e_1, e_2\}$, $(t_1, t_2) \in [0,1] \times [0,1]$

$T\left(\underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{e_1}\right) = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$, $T\left(\underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{e_2}\right) = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$

$\rightarrow \boxed{\text{Im}\,R = \left\langle \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \end{bmatrix}^T, \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \end{bmatrix}^T \right\rangle = t_1 \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \end{bmatrix}^T + t_2 \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \end{bmatrix}^T}$, $(t_1, t_2) \in [0,1] \times [0,1]$