26/2/2015

1) $\vec{B}(0,0,1) = (-3\cdot 10^{-6}, -10^{-6}, 0)\,T$

a) $\vec{B}_{hilo\,1}(0,0,1) = \left(0, -\frac{\mu_0 \cdot 10A}{2\pi}, 0\right)T = (0, -2\cdot 10^{-6}, 0)\,T$

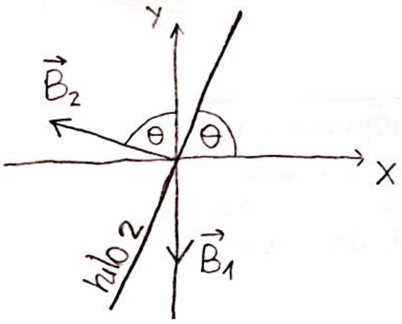
$\Rightarrow \vec{B}_{hilo\,2} = (-3\cdot 10^{-6}, -10^{-6}, 0) - \left(0, -\frac{\mu_0 \cdot 10A}{2\pi}, 0\right)$

$\vec{B}_{hilo\,2} = (-3\cdot 10^{-6}, 10^{-6}, 0)\,T$

$\Rightarrow B_{hilo\,2} = \frac{\mu_0 \cdot 10A}{2\pi\rho} = \sqrt{(-3\cdot 10^{-6})^2 + (10^{-6})^2} = 3{,}16\cdot 10^{-6}\,T$

$\Rightarrow \rho = 0{,}63$

$\Rightarrow$ el hilo 2 se ubica 0,63 por arriba o por debajo del hilo 1



$\theta = tg^{-1}\left(\frac{3\cdot 10^{-6}}{10^{-6}}\right) = 71{,}57°$

si se encuentra 0,63 por debajo la corriente irá para abajo y si se encuentra 0,63 por arriba irá para arriba la corriente

b) $q = 1\cdot 10^{-6}\,C$

$\vec{v} = 1000\,m/s\,\hat{j}$

$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B} = 0$

$\vec{E} = -(\vec{v}\times\vec{B})$

$\vec{E} = -(1000\,m/s\,\hat{j} \times (-3\cdot 10^{-6}, -10^{-6}, 0)\,T)$

$\vec{E} = -3\cdot 10^{-3}\,\frac{N}{C}\,\hat{k}$

2) $R_1 = 0{,}2\,m$

a) $R_2 = 2\,10^{-3}\,m$

$i_1 = 20A$

$\vec{v}_2 = 10\,m/s\,\hat{k}$

$$\vec{B}_{espira\,1} = \frac{\mu_0}{4\pi}\int \frac{i\,d\vec{l} \times (\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$$

$i = 20A$

$d\vec{l} = \rho\, d\varphi\, \hat{\varphi}$

$\vec{r} = (0,0,vt)$

$\vec{r}\,' = \rho\hat{\rho}$

$$\vec{B}_1 = \frac{\mu_0}{4\pi}\int_0^{2\pi} \frac{i\rho\, d\varphi\, \hat{\varphi} \times (-\rho\hat{\rho} + vt\,\hat{k})}{(\rho^2+(vt)^2)^{3/2}}$$

se considera solo el campo en $\hat{k}$ ya que R_2 es muy pequeño

$$= \frac{\mu_0}{4\pi}\int_0^{2\pi} \frac{i\rho^2 d\varphi\, \hat{k} + i\rho vt\, d\varphi\, \hat{\rho}}{(\rho^2+(vt)^2)^{3/2}}$$

$$= \frac{\mu_0 i\rho}{4\pi}\int_0^{2\pi} \frac{\rho\, d\varphi\, \hat{k} + vt(\cos\varphi\,\hat{i} + \operatorname{sen}\varphi\,\hat{j})\, d\varphi}{(\rho^2+(vt)^2)^{3/2}}$$

$$= \frac{\mu_0 i\rho}{4\pi(\rho^2+(vt)^2)^{3/2}}\left(vt\left(\cancel{(\operatorname{sen}\varphi\,\hat{i})_0^{2\pi}} + \cancel{(-\cos\varphi)_0^{2\pi}}\right) + \rho 2\pi\,\hat{k}\right)$$

$$= \frac{\mu_0 i\rho}{4\pi(\rho^2+(vt)^2)^{3/2}} \cdot \rho 2\pi\,\hat{k} = \frac{\mu_0 i\rho^2 \cancel{2\pi}\,\hat{k}}{\cancel{4\pi}_{\,2}(\rho^2+(vt)^2)^{3/2}} = \frac{\mu_0 . 20A . 0{,}2^2\,\hat{k}}{2(0{,}2^2+(10t)^2)^{3/2}}$$

$$= \frac{5{,}03 . 10^{-7}\,\hat{k}}{(0{,}04+100t^2)^{3/2}}\ T$$

$$\phi_{21} = \frac{5{,}03 . 10^{-7}}{(0{,}04+100t^2)^{3/2}} . \pi . (2 . 10^{-3}\,m)^2 = \frac{6{,}32 . 10^{-12}}{(0{,}04+100t^2)^{3/2}}\ Wb$$

$$\mathcal{E}_{ind} = -\frac{d\phi}{dt} = -\frac{d}{dt}\left(\frac{6{,}32.10^{-12}}{(0{,}04+100t^2)^{3/2}}\right)$$

$$= -6{,}32.10^{-12} \cdot \frac{d}{dt}\left((0{,}04+100t^2)^{-3/2}\right)$$

$$= -6{,}32.10^{-12} \cdot \left(-\frac{3}{2}\right)(0{,}04+100t^2)^{-5/2} \cdot 200t$$

$$= \frac{1{,}896.10^{-9}\,t}{(0{,}04+100t^2)^{5/2}} \text{ V}$$

La corriente inducida irá en sentido antihorario. Esto lo sabemos por el signo de la fem inducida (que luego calculamos la corriente) y además se puede deducir ya que al alejarse, el flujo sobre la espira pequeña disminuye y ésta creará una corriente, que a su vez crea un campo, que se opone a este cambio de flujo, es decir, ayudándo al campo de la espira grande a que no disminuya. Entonces como el campo inducido es hacia arriba, la corriente circulará en sentido antihorario

$\boxed{b)}$ $R = 1\,\Omega$

$$\frac{dU}{dt} = \frac{\mathcal{E}^2}{R} \Rightarrow \int_{U(0)}^{U(1)} dU = \int_0^1 \frac{\mathcal{E}^2}{R}\,dt \Rightarrow \Delta U = \int_0^1 \frac{3{,}59.10^{-18}\,t^2}{(0{,}04+100t^2)^5}\,dt$$

3) $L = 1.10^{-3} H$

$a = 0{,}2\, m, \quad A = 0{,}04\, m^2$

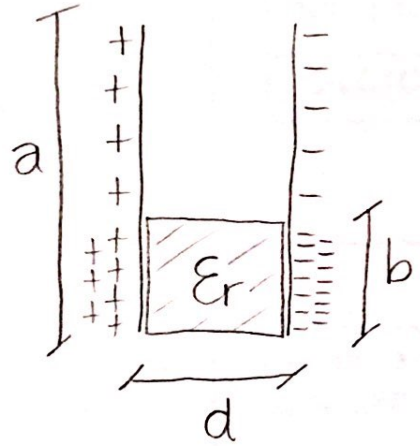
$d = 1.10^{-3} m$

$\varepsilon_r = 2$

$F_r = 218{,}5\, kHz = 218500\, Hz$

$$F_r = \frac{1}{2\pi\sqrt{LC}} \rightarrow 218500\, Hz = \frac{1}{2\pi\sqrt{1.10^{-3}.C}}$$

$$\Rightarrow C = 5{,}31.10^{-10}\, F$$



$$q = \sigma_1 a(a-b) + \sigma_2 ab$$

$$\vec{D} = \begin{cases} \sigma_1 \hat{i}, & 0<x<d,\ b<y<a \\ \sigma_2 \hat{i}, & 0<x<d,\ 0<y<b \end{cases}$$

$$\vec{E} = \begin{cases} \sigma_1/\varepsilon_0\, \hat{i}, & 0<x<d,\ b<y<a \\ \sigma_2/\varepsilon_0\varepsilon_r\, \hat{i}, & 0<x<d,\ 0<y<b \end{cases}$$

$\Rightarrow E_{1t} = E_{2t}$

$$\frac{\sigma_1}{\varepsilon_0} = \frac{\sigma_2}{\varepsilon_0 \varepsilon_r}$$

$$\Delta V_1 = \Delta V_2 = -\int_0^d \vec{E}\, d\vec{l} = -E.d$$

$$\Rightarrow -\frac{\sigma_1}{\varepsilon_0} d = \frac{-\sigma_2}{\varepsilon_0 \varepsilon_r} d$$

$$C = \frac{q}{|\Delta V|} = \frac{\sigma_1 a(a-b) + \sigma_2 ab}{\frac{\sigma_1}{\varepsilon_0} d = \frac{\sigma_2}{\varepsilon_0\varepsilon_r} d} = \frac{\sigma_1 a(a-b)}{\frac{\sigma_1}{\varepsilon_0} d} + \frac{\sigma_2 ab}{\frac{\sigma_2}{\varepsilon_0 \varepsilon_r} d}$$

$$C = \frac{\varepsilon_0 a(a-b)}{d} + \frac{\varepsilon_0 \varepsilon_r a b}{d}$$

$$\Rightarrow 5{,}31 \cdot 10^{-10} F = \frac{\varepsilon_0 \cdot 0{,}2(0{,}2-b)}{10^{-3}} + \frac{\varepsilon_0 2 \cdot 0{,}2 b}{10^{-3}}$$

$$\Rightarrow \boxed{b = 0{,}1\ m}$$

b) $R = |X_c| = \frac{1}{\omega C} = 1371{,}75\,\Omega$

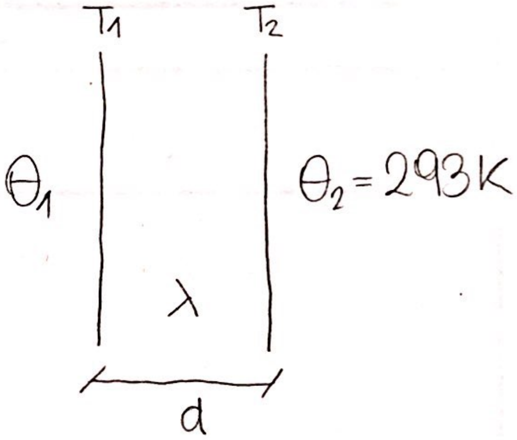
$$\boxed{\text{Factor de potencia} = \cos\varphi = \frac{R}{|Z|} = \frac{R}{\sqrt{R^2+(\omega L)^2}} = 0{,}71}$$

4) $\lambda = 1\ W/mK$

a) $d = 0{,}1\ m$

$A = 10\ m^2$

$|\vec{\nabla} T| = 100\ K/m$



$$\vec{\nabla} T = \frac{T_2 - T_1}{d} = \frac{293 - T_1}{0{,}1}$$

$$\frac{\dot{Q}}{S} = -K \vec{\nabla} T$$

$$100 = \frac{293 - T_1}{0{,}1} \qquad -100 = \frac{293 - T_1}{0{,}1}$$

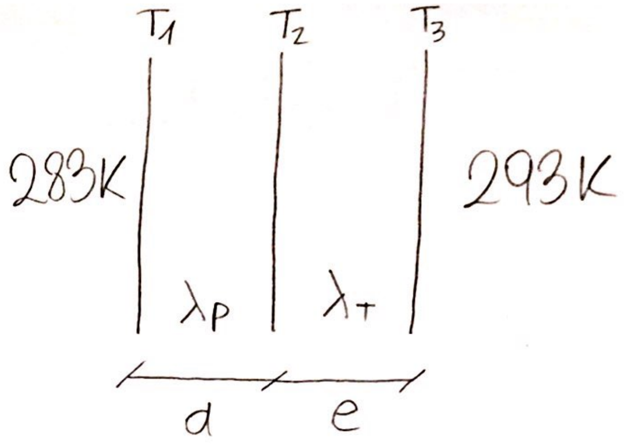
$$\boxed{T_1 = 283\ K} \qquad T_1 = 303\ K$$

$$\dot{Q} = \frac{\lambda (T_1 - T_2) A}{d}$$

$$\boxed{\dot{Q} = -1000\ W}$$

b) $\lambda_T = 0{,}03\ W/mK$

$\dot{Q} = -10\ W$



$$\dot{Q} = \frac{\lambda_P S (T_1 - T_2)}{d}$$

$$\dot{Q} = \frac{\lambda_T S (T_2 - T_3)}{e}$$

$$\Rightarrow \dot{Q}\left(\frac{d}{\lambda_P S} + \frac{e}{\lambda_T S}\right) = T_1 - T_3$$

$$-10W\left(\frac{0{,}1m}{1 W/mK \cdot 10m^2} + \frac{e}{0{,}03 W/mK \cdot 10m^2}\right) = 283K - 293K$$

$$\Rightarrow \boxed{e = 0{,}297\,m}$$

5) $L = 0{,}5m$ se agregan lentamente 20kg

$R = 0{,}05m$

$P = 100\,kPa = 1\,atm$

$T = 293K$

$P_1 = 1atm$ $\Rightarrow n = 0{,}16\,mol$

$T_1 = 293K$

$V_1 = \pi(0{,}05)^2 \cdot 0{,}5 = 3{,}93 \cdot 10^{-3} m^3$

$= 3{,}93\,l$

$P_2 = 1atm + 200N / \pi \cdot 0{,}05^2$

$\Rightarrow P_2 = 1atm + 0{,}251\,atm = 1{,}251\,atm$

$T_2 = 293K$ $V_2 = \frac{T_2 n R}{P_2} = 3{,}07\,l$

a) $W = \int \frac{nRT}{V} dV = \int_{V_1}^{V_2} \frac{nRT}{V} dV = nRT \ln\left(\frac{V_2}{V_1}\right)$

$= 0{,}16 . 0{,}082 . 293 . \ln\left(\frac{3{,}07}{3{,}93}\right) = -0{,}95 \text{ atm.l}$

$= \left[-96{,}27 \text{ J}\right]$

b) $W = -nC_v \Delta T$

$P_1 = 1 \text{ atm} \quad T_1 = 293 K \quad V_1 = 3{,}93 \text{ l} \quad n = 0{,}16 \text{ mol}$

$P_2 = 1{,}251 \text{ atm}$

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \rightarrow 1 \text{ atm} . (3{,}93 \text{l})^{7/5} = 1{,}251 \text{ atm} . V_2^{7/5}$

$V_2 = 3{,}35 \text{ l}$

$\Rightarrow T_2 = 319{,}42 K$

$\left[W = -0{,}16 . \frac{5}{2} 8{,}31 . (319{,}42 - 293) = -87{,}82 \text{ J}\right]$

$\boxed{4)}$ $C = 1 . 10^{-3} F$

$Q_0 = 0{,}1 C$

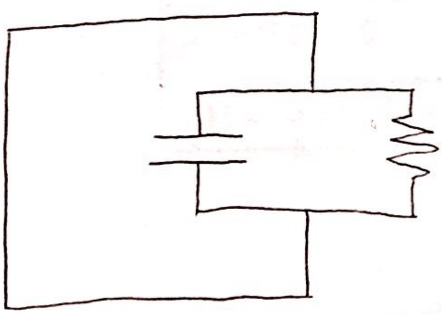
$R = 1000 \Omega$

$R . i(t) = -\frac{q(t)}{C}$

$i = -\frac{dq}{dt}$

$0 = \frac{q(t)}{C} + R \frac{dq(t)}{dt}$

$q(t) = q_h(t) + q_p(t)$



$q_p(t) \rightarrow 0 = \frac{q_p(t)}{C} \Rightarrow q_p(t) = 0$

$q_h(t) \rightarrow 0 = \frac{q_h(t)}{C} + R \frac{dq_h(t)}{dt}$

$$-\frac{q_h(t)}{C} = R\frac{dq_h(t)}{dt}$$

$$\int -\frac{dt}{RC} = \int \frac{dq_h(t)}{q_h(t)}$$

$$K - \frac{t}{RC} = \ln(q_h(t)) \rightarrow e^K \cdot e^{-t/RC} = q_h(t)$$

$$\Rightarrow q(t) = K e^{-t/RC}$$

como $q(0) = 0{,}1C \Rightarrow 0{,}1 = K e^0 \Rightarrow K = 0{,}1$

$$\Rightarrow q(t) = 0{,}1 e^{-t/RC}$$

$$q(0{,}1s) = 0{,}1 . e^{-0{,}1/(1000\Omega \cdot 10^{-3}F)}$$

$$\cancel{= 0{,}11C} \quad = 0{,}09C$$

5) $L_1 = 10^{-3} H$

$K = 0{,}5$

$M = 0{,}5\sqrt{L_1 L_2}$

$M = 10^{-3} H$

a) $U_1 = 2 \cdot 10^{-3} J$

$L_2 = 4 \cdot 10^{-3} H$

$U_2 = 18 \cdot 10^{-3} J$

$$U_1 = \frac{1}{2} i_1^2 L_1 \rightarrow 2 \cdot 10^{-3} = \frac{1}{2} 10^{-3} i_1^2 \rightarrow i_1 = 2A$$

$$U_2 = \frac{1}{2} i_2^2 L_2 \rightarrow 18 \cdot 10^{-3} = \frac{1}{2} \cdot 4 \cdot 10^{-3} i_2^2 \rightarrow i_2 = 3A$$

$$\boxed{U_T = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2 = 0{,}026 J \rightarrow MAX}$$

$$\boxed{U_T = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 = 0{,}02 J \rightarrow MIN}$$

b) $L_1 = 10^{-3} H$

$L_2 = 4 \cdot 10^{-3} H$

$M = 10^{-3} H$

$$L_{eq} = L_1 + L_2 + 2M$$

$$V(t) = V_L = L_{eq} \frac{di(t)}{dt}$$

$$12V e^{-t/10^{-3}s} = (10^{-3} + 4 \cdot 10^{-3} + 2 \cdot 10^{-3}) \frac{di(t)}{dt}$$

$$\Rightarrow 1714{,}29 e^{-t/10^{-3}} = \frac{di(t)}{dt} \Rightarrow \int 1714{,}29 e^{-t/10^{-3}} dt = \int di(t)$$

$$\Rightarrow -10^{-3} \cdot 1714{,}29 e^{-t/10^{-3}} + K = i(t)$$

$$i(t) = -1{,}71 e^{t/10^{-3}} + K$$

$$i(0) = 4A = -1{,}71 + K \rightarrow K = 5{,}71\,A$$

$$\Rightarrow \left[i(t) = -1{,}71\,A\,e^{t/10^{-3}s} + 5{,}71\,A \right]$$