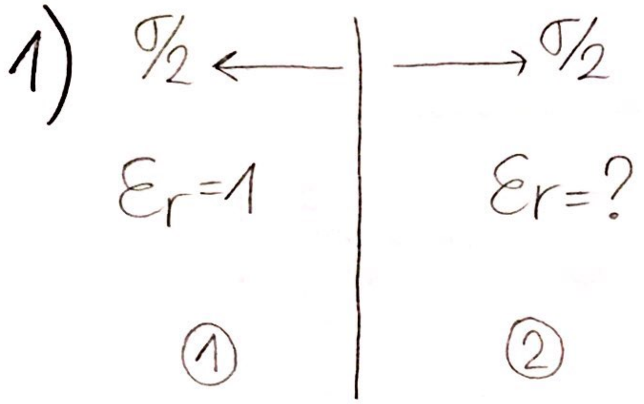
8/7/15

1) 

$$W_{-d\to d} = V_0$$

$$W_{-d\to 2d} = 0$$

$$W_{-d\to d} = -q\,\Delta V_{-d\to d} = -q\left(-\int_{-d}^{0}\vec{E}_1\,\vec{dl} - \int_{0}^{d}\vec{E}_2\,\vec{dl}\right)$$

$$= q\left(-E_{1n}\,d + E_{2n}\,d\right) = q\left(-\frac{D_{1n}}{\varepsilon_0}d + \frac{D_{2n}\,d}{\varepsilon_0\varepsilon_r}\right) = V_0$$

$$W_{-d\to 2d} = -q\,\Delta V_{-d\to 2d} = -q\left(-\int_{-d}^{0}\vec{E}_1\,\vec{dl} - \int_{0}^{2d}\vec{E}_2\,\vec{dl}\right)$$

$$= q\left(-E_{1n}\,d + E_{2n}\,2d\right) = q\left(-\frac{D_{1n}}{\varepsilon_0}d + \frac{D_{2n}}{\varepsilon_0\varepsilon_r}2d\right) = 0$$

$$D_{1n} = D_{2n} = \frac{\sigma}{2}$$

$$\Rightarrow -\frac{\sigma d}{2\varepsilon_0} + \frac{\sigma d}{2\varepsilon_0\varepsilon_r} = \frac{V_0}{q} \Rightarrow \frac{\sigma(-d2\varepsilon_0\varepsilon_r + d2\varepsilon_0)}{4\varepsilon_0^2\varepsilon_r} = \frac{V_0}{q}$$

$$\Rightarrow -\frac{\sigma d}{2\varepsilon_0} + \frac{\sigma d}{\varepsilon_0\varepsilon_r} = 0 \Rightarrow [\varepsilon_r = +2]\ ?$$

$$\boxed{\sigma = \frac{V_0 4\varepsilon_0^2\varepsilon_r}{q(-2d\varepsilon_0\varepsilon_r + 2d\varepsilon_0)}}$$

b) $V(X=0)=0$

$\underline{X<0}$

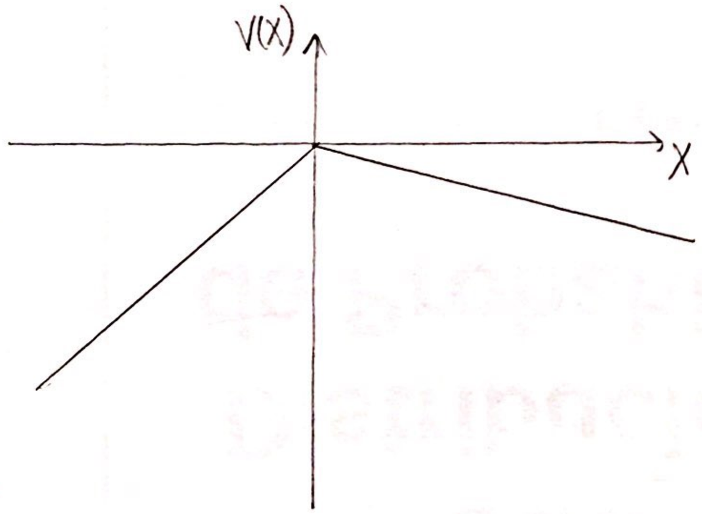
$$V(X) = -\int_0^X \vec{E}_1 \, d\vec{l} = E_{1n} X = \frac{D_{1n}}{\varepsilon_0} X = \frac{\sigma X}{2\varepsilon_0}$$

$$\Rightarrow V(X) = \frac{\sigma X}{2\varepsilon_0}$$

$\underline{X>0}$

$$V(X) = -\int_0^X \vec{E}_2 \, d\vec{l} = -E_{2n} X = \frac{-D_{2n}}{\varepsilon_0 \varepsilon_r} X = \frac{-\sigma X}{2\varepsilon_0 \varepsilon_r}$$

$$\Rightarrow V(X) = \frac{-\sigma X}{2\varepsilon_0 \varepsilon_r}$$



$$\vec{P} = \vec{D}_2 - \varepsilon_0 \vec{E}_2$$

$$\vec{P} = \frac{\sigma}{2} - \frac{\cancel{\varepsilon_0}\sigma}{2\cancel{\varepsilon_0}\varepsilon_r}$$

$$\vec{P} = \frac{\sigma}{2}\left(1 - \frac{1}{\varepsilon_r}\right)\hat{i}$$

$$\left[\sigma_p = \frac{\sigma}{2}\left(1-\frac{1}{\varepsilon_r}\right)\hat{i}\cdot(-\hat{i}) = -\frac{\sigma}{2}\left(1-\frac{1}{\varepsilon_r}\right)\right]$$

2) $\vec{B} = B_0 \operatorname{sen}(\omega t)\hat{k}$

a) $\vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} = -B_0 \cos(\omega t)\omega \hat{k}$

$= \boxed{-B_0 \omega \hat{k}}$

$\vec{E} = E_y(x)$

$\Rightarrow \vec{\nabla} \times \vec{E} = \begin{vmatrix} \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & E(x) & 0 \end{vmatrix} = \dfrac{\partial E(x)}{\partial x}\hat{k}$

$\Rightarrow \dfrac{\partial E(x)}{\partial x}\hat{k} = -B_0 \omega \hat{k} \Rightarrow \boxed{\vec{E}(x) = -B_0 \omega x \hat{k}}$

b) $\underline{\underline{x < d}}$

$\phi = \iint \vec{B}\, d\vec{S} = B_0 \operatorname{sen}(\omega t) . dx$

$\boxed{\mathcal{E}_{ind} = -\dfrac{d\phi}{dt} = -B_0 \omega \cos(\omega t) dx}$

$\underline{\underline{x \geqslant d}}$

$\phi = \iint \vec{B}\, d\vec{S} = B_0 \operatorname{sen}(\omega t) d^2$

$\boxed{\mathcal{E}_{ind} = -\dfrac{d\phi}{dt} = -B_0 \omega \cos(\omega t) d^2}$

3) $v(t) = V_0 \cos(\omega t)$

$V_0 = V_S = 5V$ (pico)

$V_L = 8V$ (pico)

$|V_L| = \omega L |i|$ $\qquad$ $|V_R| = R|i|$

$|V_S| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} |i|$ $\qquad$ $|V_C| = \frac{1}{\omega C} |i|$

$V_0 = \sqrt{V_R^2 + (V_L - V_C)^2}$ $\qquad$ $\begin{cases} V_S^2 = V_R^2 + V_C^2 \\ V_0^2 = V_R^2 + (V_L - V_C)^2 \end{cases}$

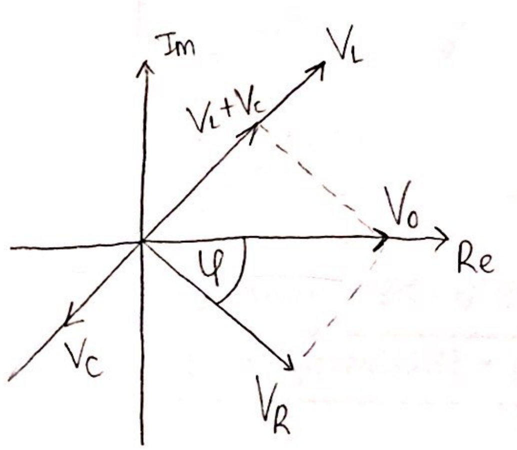
$\begin{cases} 5^2 = V_R^2 + V_C^2 \\ 5^2 = V_R^2 + (8 - V_C)^2 \end{cases}$ $\Rightarrow V_C^2 = (8 - V_C)^2$

$V_C^2 = 64 - 16V_C + V_C^2$

$16V_C = 64 \rightarrow \boxed{V_C = 4V}$

$\Rightarrow 5^2 = V_R^2 + 4^2$

$\Rightarrow V_R^2 = 9 \Rightarrow \boxed{V_R = 3V}$ $\qquad$ $\boxed{V_L > V_C}$



$tg^{-1}\left(\frac{4}{3}\right) = \boxed{53,1^\circ}$

5) $\vec{B}(\vec{r}) = \int \frac{\mu_0}{4\pi} \frac{i\,d\vec{l} \times (\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$

$d\vec{l} = dy'\hat{j} \qquad \vec{r} = (x,y,0) \quad \vec{r}\,' = (0,y',0)$

$(\vec{r}-\vec{r}\,') = (x,y,0)-(0,y',0) = (x,y-y',0)$

$|\vec{r}-\vec{r}\,'|^3 = \left(x^2+(y-y')^2\right)^{3/2}$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int_L^0 \frac{i\,dy'\hat{j} \times (x\hat{i}+(y-y')\hat{j})}{(x^2+(y-y')^2)^{3/2}} =$$

$$= \frac{\mu_0 i}{4\pi}\int_L^0 \frac{-x\,dy'\hat{k}}{(x^2+(y-y')^2)^{3/2}} = -\frac{\mu_0 x i\hat{k}}{4\pi}\int_L^0 \frac{dy'}{(x^2+(y-y')^2)^{3/2}}$$

$y-y' = u$

$-dy' = du$

$$\Rightarrow = \frac{\mu_0 x i\hat{k}}{4\pi}\int_L^0 \frac{du}{(x^2+u^2)^{3/2}} =$$

$$= \frac{\mu_0 x i\hat{k}}{4\pi}\left(\frac{u}{x^2\sqrt{u^2+x^2}}\right)_L^0 =$$

$$= \frac{\mu_0 x i\hat{k}}{4\pi}\left(-\frac{L}{x^2\sqrt{L^2+x^2}}\right)$$

$$L\to\infty \Rightarrow \vec{B} = \frac{\mu_0 x i\hat{k}}{4\pi}\left(\frac{-\cancel{L}}{x^2\sqrt{\cancel{L^2}+\cancel{x^2}}}\right) = -\frac{\mu_0 i\hat{k}}{4\pi x}$$

$$\Rightarrow \frac{\vec{B}_{\text{hilo infinito}}}{2}$$

$$\Rightarrow \vec{B}(d,0,0) = \frac{\mu_0 i}{4\pi d}(-\hat{k})$$

$$\Rightarrow \vec{B}(0,d,0) = \frac{\mu_0 i}{4\pi d}(-\hat{k})$$

b) $\vec{F} = q\vec{v} \times \vec{B}$

$$\vec{F} = q \cdot v_0 \hat{k} \times \left(\frac{\mu_0 i}{4\pi d}\hat{i} + \frac{\mu_0 i}{4\pi d}\hat{j}\right)$$

$$\boxed{\vec{F} = \frac{q v_0 \mu_0 i}{4\pi d}\hat{j} - \frac{q v_0 \mu_0 i}{4\pi d}\hat{i}}$$

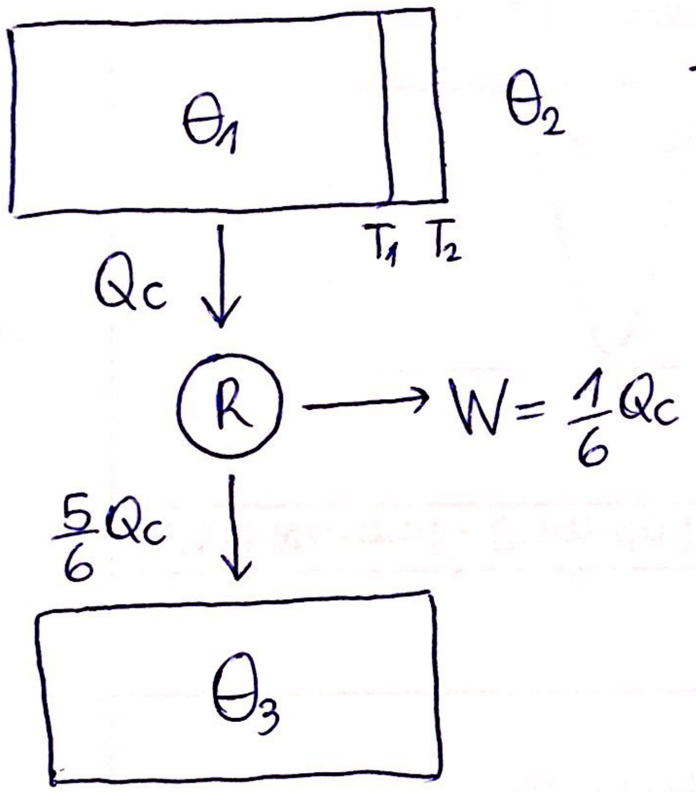
4) $d = 0{,}01\ m$

$S = 2{,}7\ m^2$

$\dot{Q} = 1000000\ W$

$h_a = 500\ kW/m^2K$

$\lambda_{cu} = 400\ W/mK$



a) sacar θ_1

b) max valor de fuente fria

$$\eta_C = 1 - \frac{\theta_3}{\theta_1} \quad \Big| \quad \eta_R = \frac{1/6\, Q_c}{Q_c} = \frac{1}{6}$$

$$\Rightarrow \frac{1}{6} = 1 - \frac{\theta_3}{\theta_1}$$

$\theta_1 \leftarrow$ lo saco de la parte a)

5) 1 mol

a) $C_p = \frac{5}{2}R$, $C_V = \frac{3}{2}R$

C1: ABCA → FRIGORÍFICA

C2: ADEA → MOTOR

CAE: ADIABATICA

BAD: ISOTERMA

ABCA:

AB: $Q = nRT\ln(V_B/V_A) = nRT\ln(1/2) < 0$

BC: $Q = nC_V\Delta T = nC_V(T_C - T_B) > 0$

CA: $Q = 0$

ADEA:

AD: $Q = nRT\ln(2) > 0$

DE: $Q = nC_V\Delta T = nC_V(T_E - T_D) < 0$

EA: $Q = 0$

b) $W_{ABCA} = nRT\ln(1/2) + 0 - nC_V(T_A - T_C)$

$W_{ADEA} = nRT\ln(2) + 0 - nC_V(T_A - T_E)$

$\varepsilon_1 = \frac{nC_V(T_C - T_B)}{nRT\ln(1/2) - nC_V(T_A - T_C)}$

$\eta_2 = \frac{nRT\ln(2) - nC_V(T_A - T_E)}{nRT\ln(2)}$