Data Analytics
Assignment -2
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Implementation

Assume the following orbit model and parameters:
1. The Sun is at the origin.
2. Mars's orbit is circular, with the center at a distance 1 unit from the Sun and at an angle c (degrees) from the Sun-Aries reference line.
3. Mars's orbit has a radius r (in units of the Sun-centre distance).
4. The equant is located at (e1, e2) in polar coordinates with the center taken to be the Sun, where e1 is the distance from the Sun and e2 is the angle in degrees with respect to the Sun-Aries reference line.
5. The 'equant 0' angle z (degrees) which is taken as the earliest opposition, is also taken as the reference time zero, with respect to the equant-Aries line (a line parallel to the Sun-Aries line since Aries is at infinity).
6. The angular velocity of Mars around the equant is **s** degrees per day.

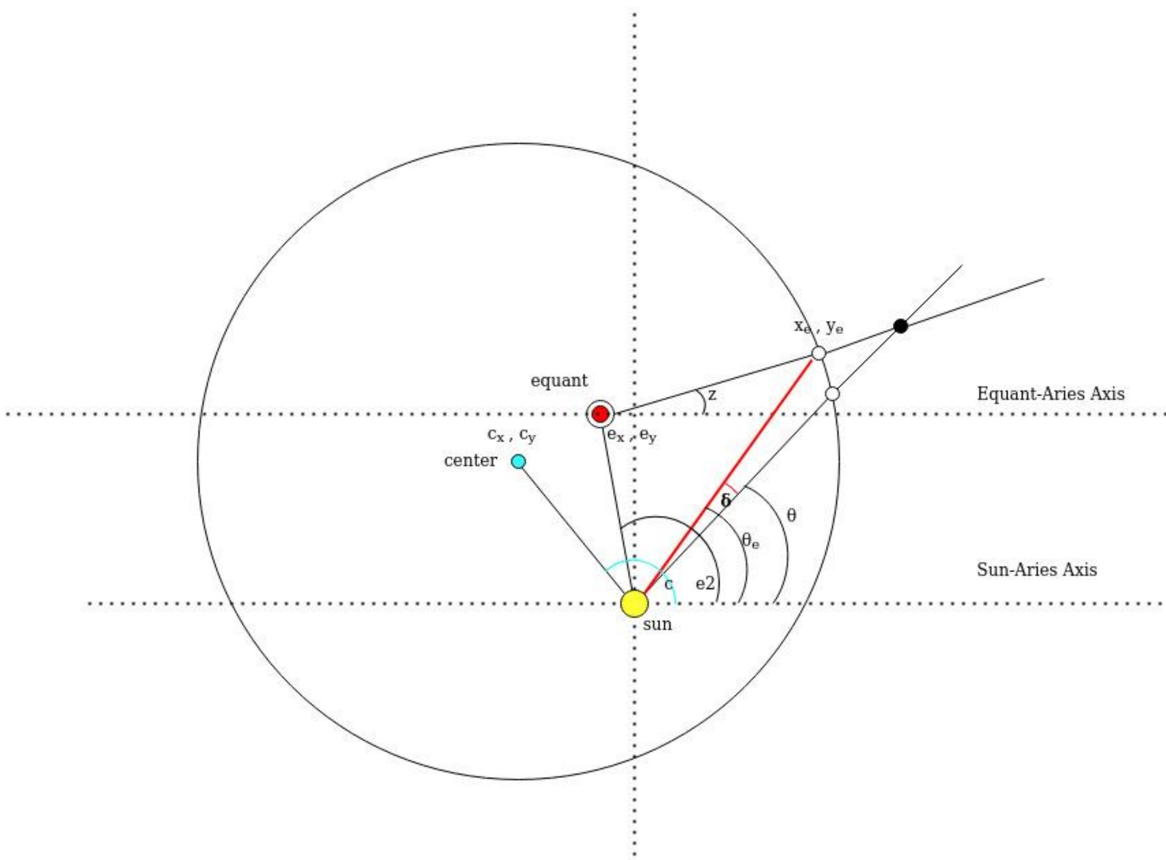

Fig . Variables Plot

Derivations

Let's assume that the Center of the circle is denoted by point (c_x, c_y). As the center of the circle is 1 unit away from the origin making an angle c from the Sun-Aries reference line, we get $c_x = \cos c$ and $c_y = \sin c$. Then, the equation of the circle is given

$$(x - c_x)^2 + (y - c_y)^2 = r^2 \qquad \text{......... (1)}$$

Similarly assume that the equant is denoted by point (e_x, e_y). Since the equant is e_1 distance from the sun and e_2 is the angle with respect to the Sun-Aries reference line, we get

$$e_x = e_1 \cos e_2 \quad \text{and,} \quad e_y = e_1 \sin e_2$$

Then, the equation of **equant 0** making angle **z (**ie. slope, $m = \tan z$ **)** wrt Equant-Aries line originating from Equant is given by

$$(y - e_y) = m\,(x - e_x) \qquad \text{......... (2)}$$

Let's find the point (x_e, y_e) which is the intersecting point of eq (1) and eq (2).
From eq (2) , we get

$$y = m\,(x - e_x) + e_y \qquad \text{.......... (3)}$$

Putting (3) in (1), we get

$$(x - c_x)^2 + (mx - me_x + e_y - c_y)^2 = r^2$$

Assuming $\beta = -me_x + e_y - c_y$ where $m = \tan z$, we get

$$\Rightarrow (x - c_x)^2 + (mx + \beta)^2 = r^2$$
$$\Rightarrow (1+m^2)\,x^2 - 2(c_x - m\beta)x + (\beta^2 + c_x^2 - r^2) = 0$$
$$\Rightarrow x^2 - [2(c_x - m\beta)/(1+m^2)]\,x + [(\beta^2 + c_x^2 - r^2)/(1+m^2)] = 0$$

Assuming $a = 1$, $b = -[2(c_x - m\beta)/(1+m^2)]$, $c = (\beta^2 + c_x^2 - r^2)/(1+m^2)$, using **quadratic formula** we can find x which is actually x_e. Then using (3), we can find y_e.
Once we get the point (x_e, y_e) point which is actually the predicted position of Mars. In order to find the oppositional discrepancy, we need to find the angle of the line joining origin and (x_e, y_e) wrt Sun-Aries reference line which we can easily find using the slope formula. We know that,

$$\text{Slope} = (y2 - y1) / (x2 - x1) = \tan \theta_e$$
$$\Rightarrow \tan \theta_e = (y_e - 0) / (x_e - 0)$$
$$\Rightarrow \theta_e = \tan^{-1}(y_e / x_e)$$

We can easily find the angle made by the observed position of Mars wrt Sun-Aries line from the longitude data (columns 5-8 in original data file) itself. Let's call this angle θ. Then, the oppositional discrepancy is given by

$$\delta = \theta_e - \theta \qquad \text{……… (4)}$$

The oppositional discrepancy is calculated for all 12 oppositional data points. In order to find the predicted position of Mars at all 12 oppositional positions, we make use of time data (columns 0-4 in the original data file) to calculate the time differences of each point from the starting point. Then, using the angular velocity of Mars around the equant (**s**), we calculate the angular distances traversed in those time differences. Since we assume the equant 0 makes 'z' angle wrt the Equant-Aries line, we will add z to all the angular distances.

Thus, we will get the predicted position of Mars at all 12 oppositional positions. Using (4), we can calculate all 12 oppositional discrepancies and the maximum discrepancies will be the maximum of absolute values of oppositional discrepancies.

Also, we have been asked to do a discretized search for r in the neighborhood of the average distance of the black dots from the center, we have computed the actual position of Mars on the longitudes using the below computations,

The equation of the longitude can be calculated using the θ and origin point, which is given by

$$y = x \tan \theta \qquad \text{……… (5)}$$

Now, we need to find the intersection of (2) and (3) in order to find the actual position of the Mars on actual opposition line. We get,

$$\Rightarrow x \tan \theta = m (x - e_x) + e_y$$

$$\Rightarrow x = (e_y - m e_x) / (\tan \theta - m) \qquad \text{………(6)}$$

Using (6) and (5) ,we will get y as well.

Now, using euclidean distance, we calculate the distance of the point (x, y) from the origin. We calculate this distance for each oppositional data point and take the average of all the distances to get the estimated r.

Please refer commented code for implementation details and understanding.

Observations

I have tried running the code for 2.5 hours straight but the code didn't stop so I selected the set of ranges for each parameter based on my intuition and was able to reduce the error to 22.4977 minutes. Among, all the ranges I have tried, the best result I have achieved is for

Fit parameters: r = 8.7000, s = 0.5240, c = 140.0000, e1 = 1.6000, e2 = 148.5000, z = 56.4800
The maximum angular error = 22.4977 (in minutes)

The next best result that I have achieved by running on different range is following :
Fit parameters: r = 8.2000, s = 0.5240175 , c = 118.8000 , e1 = 1.5000, e2 = 148.5000, z = 56.4700
The maximum angular error = 24.414 (in minutes)

Note that for second result, I have reported value of 's' up to 7 decimal digit. This is the exact value for which the code is giving 24.414 minutes as the maximum error and if we change it to 0.5240, the maximum error becomes 30.3891.

For reproducing the result, I have clipped the range of each parameter search so that the entire code can run in 10 minutes. The code will produce the best result which is giving the max error of 22.4977 minutes.