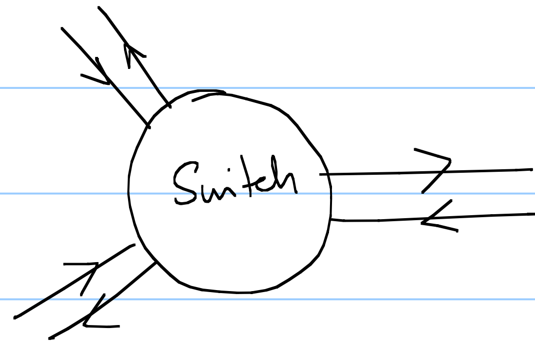
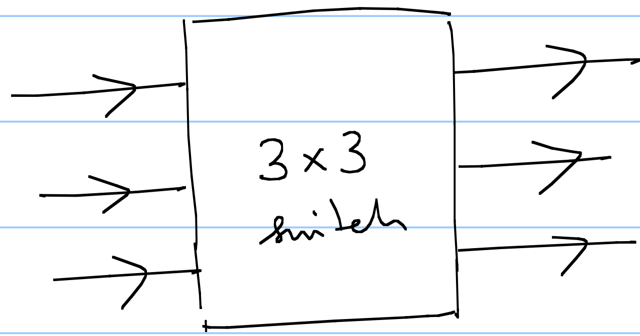
Note Title 1/20/2015

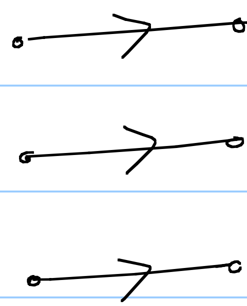
06. High-speed Switches



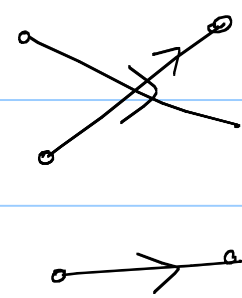
- Each link connected to a switch consists of an incoming wire and an outgoing wire
- These are interconnected using a switch fabric: the switch can connect each input to only output at each time and each output can be connected to only input at a time.
- N×N switch: N input ports and N output ports



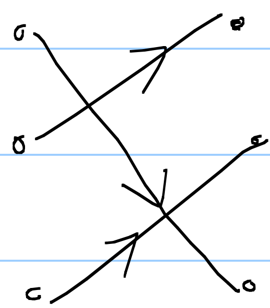
Possible input-output connections:



or



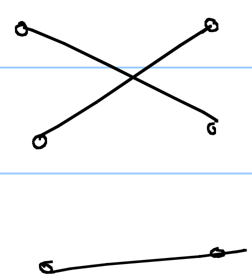
or



. . . . etc.

In general, in an $N \times N$ switch, there are $N!$ possible schedules (interconnections). Each schedule is a matching in a bipartite graph.

In matrix form, each schedule can be represented by a permutation matrix: exactly one 1 in each row and column



$$= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Capacity region or throughput region ($\mathcal{C}$)

Let $M_1, M_2, \ldots, M_{N!}$ be the set of permutation matrices for

an $N \times N$ switch. Let λ_{ij} be the arrival rate of packets from input i to output j.
The matrix λ is supportable only if

$$\lambda \leq \sum_i \alpha_i M_i$$

for some $\alpha_i \geq 0$, $\sum_i \alpha_i = 1$.

$\square$

<u>Question:</u> Can the queues be stabilized without knowing λ but only knowing that $\lambda(1+\varepsilon) \in \mathcal{C}$ for some $\varepsilon > 0$?
We will use Foster-Lyapunov theory to answer this question.

Assume $\boxed{a_{ij}(k) = \mathrm{Ber}(\lambda_{ij})}$

$$V = \sum_{i,j} q_{ij}^2$$

$$E\left(V(k+1) - V(k) \mid q(k) = q \right)$$

$$\leq E\left(\sum_{ij} (q_{ij} + a_{ij} - s_{ij})^2 - q_{ij}^2 \right)$$

$$\leq 2 \sum_{ij} q_{ij} (\lambda_{ij} - s_{ij}) + K$$

Choose $\{s_{ij}\} \in \underbrace{\{M_1, M_2, \ldots, M_{N!}\}}_{\triangleq \mathcal{M}}$

to solve

$$\max_{s \in \mathcal{M}} \sum_{i,j} q_{ij} s_{ij}$$

Useful insight: Recall

$\mathcal{C} = Co(\mathcal{M})$, where Co is the convex hull.

$$\max_{s \in \mathcal{M}} \sum_{i,j} q_{ij} s_{ij} = \max_{s \in \mathcal{C}} \sum_{i,j} q_{ij} s_{ij}$$

<u>Why?</u> if $s \in \mathcal{C}$, then

$$s = \sum_{\ell} \alpha_\ell M_\ell$$

$$\Rightarrow \quad s_{ij} = \sum_{\ell} \alpha_\ell M_\ell(i,j)$$

$$\sum_{i,j} q_{ij} s_{ij} = \sum_{i,j} q_{ij} \sum_{\ell} \alpha_\ell M_\ell(i,j)$$

$$= \sum_{\alpha} \alpha_\ell \sum_{i,j} q_{ij} M_\ell(i,j)$$

Thus, we will make $\alpha_\ell = 1$ for M_ℓ which maximizes

$$\sum_{i,j} q_{ij} M_\ell(i,j).$$

Back <u>to derivation</u>: since

$\lambda(1+\varepsilon) \in \mathcal{C}$

$$\sum_{i,j} \lambda_{ij}(1+\varepsilon) q_{ij} \le \sum_{i,j} s_{ij} q_{ij}$$

Thus, expected drift is

$$-\varepsilon \, E\left(\sum_{i,j} \lambda_{ij} q_{ij} \right) + K$$

which is negative for large q. By Foster-Lyapunov, the system is stable.

Scheduling algorithm (MaxWeight)

$$\max_{M_e \in M} \sum_{i,j} q_{ij} M_e(i,j)$$

Birkhoff-von Neumann theorem

Any $\{\lambda_{ij}\}$ s.t.

$$\left.\begin{aligned} &\sum_i \lambda_{ij} \le 1 \\ \text{and} \quad &\sum_j \lambda_{ij} \le 1 \end{aligned}\right\} \quad - (\mathrm{I})$$

can be written as

$$\Lambda = \sum_{\ell} \alpha_\ell M_\ell$$

for some

$$\alpha_\ell \geq 0 , \quad \sum_{\ell} \alpha_\ell = 1$$

$\square$

Thus, (I) is another characterization of the capacity region.