This is a part of SFQED-Loops script collection
developed for calculating loop processes in
Strong-Field Quantum Electrodynamics.
The scripts are available on https://github.com/ArsenyMironov/SFQED-Loops

If you use this script in your research, please, consider
citing our papers:
- A. A. Mironov, S. Meuren, and A. M. Fedotov, PRD 102, 053005 (2020),
  https://doi.org/10.1103/PhysRevD.102.053005
If you have any questions, please, don’t hesitate to contact me:
mironov.hep@gmail.com


```
NotebookEvaluate[NotebookOpen[NotebookDirectory[] <> "definitions.nb"]]
```

FeynCalc 9.2.0. For help, use the documentation center, check out the wiki or write to the mailing list.

See also the supplied examples. If you use FeynCalc in your research, please cite

- V. Shtabovenko, R. Mertig and F. Orellana,
  Comput. Phys. Commun., 207C, 432–444, 2016, arXiv:1601.01167
- R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun., 64, 345–359, 1991.

Electron propagator (proper time is dimensionless)

$$S^c(x_2, x_1) = \Lambda^{4-D} \int \frac{d^D p}{(2\pi)^D} E_p(x_2) \frac{i((\gamma p) + m)}{p^2 - m^2 + i0} \overline{E}_p(x_1)$$

$x = x_2 - x_1,$

$$X = \frac{1}{2}(x_1 + x_2),$$
$$\xi^2 = -\frac{e^2 a^2}{m^2},$$
$[\Lambda] = m$ - mass scale,
$$E_p(x_2) = \left[1 - \frac{e(\gamma k)(\gamma a)}{2(kp)}(kx_2)\right]$$
$$\text{Exp}\left[-i(p\,x_2) + i\frac{e(a\,p)}{2(k\,p)}(k\,x_2)^2 + i\frac{a^2 e^2}{6(k\,p)}(k\,x_2)^3\right];$$

$(\gamma p - m)S^c(p) = i$ - in E - p representation
$$S^c(p, q) = (2\pi)^4 \delta(p - q) S^c(p)$$

```
NewMomentum["p"]
NewCoordinate["x1"]
NewCoordinate["x2"]
NewCoordinate["x"]
NewCoordinate["X"]
```

$$\left\{p^\alpha, p^2, k\cdot p, \text{Fp}^\alpha, \text{FFp}^\alpha, \text{FDp}^\alpha, a\cdot p, 0, 0, 0, -a^2(k\cdot p), 0, 0, -\frac{m^6\chi\text{p}^2}{e^2}, -\frac{m^6\chi\text{p}^2}{e^2}, \frac{m^6\chi\text{p}^2}{e^2}, 0, 0, 0, 0, 0, 0\right\}$$

$$\left\{\text{x1}^\alpha, \text{x1}^2, k\cdot\text{x1}, a\cdot\text{x1}, \text{Fx1}^\alpha, \text{FFx1}^\alpha, \text{FDx1}^\alpha, k\cdot\text{x1}, 0, 0, 0, -a^2(k\cdot\text{x1}), 0, 0, -\frac{m^2\xi^2(k\cdot\text{x1})^2}{e^2}, -\frac{m^2\xi^2(k\cdot\text{x1})^2}{e^2}, \frac{m^2\xi^2(k\cdot\text{x1})^2}{e^2}, 0, 0, 0, 0, 0, 0\right\}$$

$$\left\{\text{x2}^\alpha, \text{x2}^2, k\cdot\text{x2}, a\cdot\text{x2}, \text{Fx2}^\alpha, \text{FFx2}^\alpha, \text{FDx2}^\alpha, k\cdot\text{x2}, 0, 0, 0, -a^2(k\cdot\text{x2}), 0, 0, -\frac{m^2\xi^2(k\cdot\text{x2})^2}{e^2}, -\frac{m^2\xi^2(k\cdot\text{x2})^2}{e^2}, \frac{m^2\xi^2(k\cdot\text{x2})^2}{e^2}, 0, 0, 0, 0, 0, 0\right\}$$

$$\left\{x^\alpha, x^2, k\cdot x, a\cdot x, \text{Fx}^\alpha, \text{FFx}^\alpha, \text{FDx}^\alpha, k\cdot x, 0, 0, 0, -a^2(k\cdot x), 0, 0, -\frac{m^2\xi^2(k\cdot x)^2}{e^2}, -\frac{m^2\xi^2(k\cdot x)^2}{e^2}, \frac{m^2\xi^2(k\cdot x)^2}{e^2}, 0, 0, 0, 0, 0, 0\right\}$$

$$\left\{X^\alpha, X^2, k\cdot X, a\cdot X, \text{FX}^\alpha, \text{FFX}^\alpha, \text{FDX}^\alpha, k\cdot X, 0, 0, 0, -a^2(k\cdot X), 0, 0, -\frac{m^2\xi^2(k\cdot X)^2}{e^2}, -\frac{m^2\xi^2(k\cdot X)^2}{e^2}, \frac{m^2\xi^2(k\cdot X)^2}{e^2}, 0, 0, 0, 0, 0, 0\right\}$$

```
Epx2 = Ep[x2, p]
EpBarx1 = EpC[x1, p]
```

$$\left\{1 - \frac{e(k\cdot\text{x2})(\gamma\cdot k).(\gamma\cdot a)}{2(k\cdot p)}, \frac{a^2 e^2(k\cdot\text{x2})^3}{6(k\cdot p)} + \frac{e(a\cdot p)(k\cdot\text{x2})^2}{2(k\cdot p)} - p\cdot\text{x2}\right\}$$

$$\left\{1 - \frac{e(k\cdot\text{x1})(\gamma\cdot a).(\gamma\cdot k)}{2(k\cdot p)}, -\frac{a^2 e^2(k\cdot\text{x1})^3}{6(k\cdot p)} - \frac{e(a\cdot p)(k\cdot\text{x1})^2}{2(k\cdot p)} + p\cdot\text{x1}\right\}$$

```
Matrix = Epx2[[1]].(GAD[α] Pair[Momentum[p, D], LorentzIndex[α, D]] + m).EpBarx1[[1]]
Coeff = ⅈ Λ^(4 - D) / (2 π)^D / (pv2 - m^2)
Phase = Epx2[[2]] + EpBarx1[[2]]
```

$$\left(1-\frac{e\,(k\cdot \mathrm{x2})\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)}\right).(m+\gamma^{\alpha}\,p^{\alpha}).\left(1-\frac{e\,(k\cdot \mathrm{x1})\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot p)}\right)$$

$$\frac{i\,(2\,\pi)^{-D}\,\Lambda^{4-D}}{p^2-m^2}$$

$$-\frac{a^2\,e^2\,(k\cdot \mathrm{x1})^3}{6\,(k\cdot p)}+\frac{a^2\,e^2\,(k\cdot \mathrm{x2})^3}{6\,(k\cdot p)}-\frac{e\,(a\cdot p)\,(k\cdot \mathrm{x1})^2}{2\,(k\cdot p)}+\frac{e\,(a\cdot p)\,(k\cdot \mathrm{x2})^2}{2\,(k\cdot p)}+p\cdot \mathrm{x1}-p\cdot \mathrm{x2}$$

```
Matrix1 = Contract[DiracSimplify[Matrix]]
Coeff1 = Coeff;
Phase1 = Phase;
```

$$-\frac{a^2\,e^2\,\gamma\cdot k\,(k\cdot \mathrm{x1})\,(k\cdot \mathrm{x2})}{2\,(k\cdot p)}-\frac{e\,m\,(k\cdot \mathrm{x1})\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot p)}-\frac{e\,m\,(k\cdot \mathrm{x2})\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)}-\frac{e\,(k\cdot \mathrm{x1})\,(\gamma\cdot p).(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot p)}-\frac{e\,(k\cdot \mathrm{x2})\,(\gamma\cdot k).(\gamma\cdot a).(\gamma\cdot p)}{2\,(k\cdot p)}+m+\gamma\cdot p$$

```
Matrix2 = Expand[ExpandScalarProduct[
    Matrix1 /. {Momentum[x1, D] → Momentum[X, D] - Momentum[x, D] / 2,
      Momentum[x2, D] → Momentum[X, D] + Momentum[x, D] / 2}]]
Coeff2 = Coeff1;
Phase2 = Expand[
   ExpandScalarProduct[Phase1 /. {Momentum[x1, D] → Momentum[X, D] - Momentum[x, D] / 2,
      Momentum[x2, D] → Momentum[X, D] + Momentum[x, D] / 2}]]
```

$$\frac{a^2\,e^2\,\gamma\cdot k\,(k\cdot x)^2}{8\,(k\cdot p)}-\frac{a^2\,e^2\,\gamma\cdot k\,(k\cdot X)^2}{2\,(k\cdot p)}+\frac{e\,m\,(k\cdot x)\,(\gamma\cdot a).(\gamma\cdot k)}{4\,(k\cdot p)}-\frac{e\,m\,(k\cdot x)\,(\gamma\cdot k).(\gamma\cdot a)}{4\,(k\cdot p)}-\frac{e\,m\,(k\cdot X)\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot p)}-\frac{e\,m\,(k\cdot X)\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)}-\frac{e\,(k\cdot x)\,(\gamma\cdot k).(\gamma\cdot a).(\gamma\cdot p)}{4\,(k\cdot p)}+\frac{e\,(k\cdot x)\,(\gamma\cdot p).(\gamma\cdot a).(\gamma\cdot k)}{4\,(k\cdot p)}-\frac{e\,(k\cdot X)\,(\gamma\cdot k).(\gamma\cdot a).(\gamma\cdot p)}{2\,(k\cdot p)}-\frac{e\,(k\cdot X)\,(\gamma\cdot p).(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot p)}+m+\gamma\cdot p$$

$$\frac{a^2\,e^2\,(k\cdot x)\,(k\cdot X)^2}{2\,(k\cdot p)}+\frac{a^2\,e^2\,(k\cdot x)^3}{24\,(k\cdot p)}+\frac{e\,(a\cdot p)\,(k\cdot x)\,(k\cdot X)}{k\cdot p}-p\cdot x$$

```
Matrix3 =
  (((Expand[Matrix2 /. TripleGamma]) /. EpsToF) /. FieldSubstitutions) /. akToF /.
   { DiracGamma[LorentzIndex[β, D], D].DiracGamma[5]
      Pair[LorentzIndex[β, D], Momentum[FDp, D]] → FVD[γγ5FD, α] pv[α]}
Coeff3 = Coeff2;
Phase3 = Phase2;
```

$$\frac{a^2 e^2 \gamma\cdot k(k\cdot x)^2}{8(k\cdot p)}-\frac{a^2 e^2 \gamma\cdot k(k\cdot X)^2}{2(k\cdot p)}-\frac{e(a\cdot p)\gamma\cdot k(k\cdot X)}{k\cdot p}+e\gamma\cdot a(k\cdot X)+\frac{i\, e\, m\, \sigma\mathrm{F}\,(k\cdot x)}{4(k\cdot p)}+\frac{i\, e\gamma\gamma 5\mathrm{FD}^{\alpha}\, p^{\alpha}\,(k\cdot x)}{2(k\cdot p)}+m+\gamma\cdot p$$

Expanding scalar products into components and changing variables

$p \to \{p_- = 1/2\,(p^0 - p^3),\ p_+ = p^0 + p^3,\ p_\perp\}$

$p_- = x_- / 2\,s$

$p_+ = (p^2 - p_\perp{}^2) / 2\,p_- = s\,(p^2 + p_\perp{}^2) / x_-$

Integration measure

$\int d^D p \ldots = \int \frac{ds}{2\,s}\, dp^2\, d^{D-2} p_\perp$

$x_- = kx / m$

$p_- = kx / 2\,m\,s$

$kp = m\,p_- = m\,xm / 2\,s = kx / 2\,s$

$ap = -at\,pt$

$\gamma p = \gamma_-\,p_+ + \gamma_+\,p_- - \gamma_\perp\,p_\perp = Gm\,\frac{s}{x_-}\,(p^2 + p_\perp{}^2) + Gp\,\frac{x_-}{2\,s} - Gt\,pt$

$\gamma k = \gamma_-\,k_+ = m\,Gm$

$kx = k_+\,x_- = m\,xm$

$(\gamma F^*)^\mu.\gamma^5 \to \{(\gamma F^*)_-.\gamma^5 = 0,\ (\gamma F^*)_+.\gamma^5,\ (\gamma F^*)_\perp.\gamma^5\} = \{0,\ \gamma\gamma 5FDp,\ \gamma\gamma 5FDt\}$

$(\gamma F^*)_\mu\,k^\mu = 0 \to (\gamma F^*)_- = 0$

$(\gamma F^*)_\mu\,a^\mu = 0 \to \gamma\gamma 5FDt\ at = 0$

$(\gamma F^*)^\mu.\gamma^5\,p_\mu = \gamma\gamma 5FDm * pp - \gamma\gamma 5FDt * pt$

```
D[{xm / 2 / s, s (p2 + pt2) / xm}, {{s, p2}}]
Abs[Det[%]]
```

$$\begin{pmatrix} -\frac{\mathrm{xm}}{2 s^2} & 0 \\ \frac{\mathrm{p2+pt2}}{\mathrm{xm}} & \frac{s}{\mathrm{xm}} \end{pmatrix}$$

$$\frac{1}{2\,|s|}$$

```
Matrix4 =
 Collect[Expand[Matrix3 /. {DiracGamma[Momentum[p, D], D] → Gp * pm + Gm * pp - Gt * pt,
        Pair[Momentum[a, D], Momentum[p, D]] → -at pt,
        FVD[γγ5FD, α_] pv[α_] → γγ5FDp * pm - γγ5FDt * pt,
        DiracGamma[Momentum[k, D], D] → m Gm} /. {kp → kx / 2 / s, pm → xm / 2 / s} /.
    {kx → m xm} /. {pp → s (p2 + pt^2) / xm}], {pt, p2}]
Coeff4 = Coeff3 / 2 / s
Phase4 = Collect[
  Expand[Phase3 /. {Pair[Momentum[p, D], Momentum[x, D]] → pp xm + pm xp - pt * xt,
        kp → m pm, Pair[Momentum[a, D], Momentum[p, D]] → -at pt } /. {kp → kx / 2 / s,
       pm → xm / 2 / s} /. {pp → s (p2 + pt^2) / xm} /. {xm → kx / m}], {pt, pp}]
```

$$-\frac{a^2\,e^2\,\mathrm{Gm}\,s\,(k\cdot X)^2}{\mathrm{xm}}+\frac{1}{4}\,a^2\,e^2\,\mathrm{Gm}\,m^2\,s\,\mathrm{xm}+e\,\gamma\cdot a\,(k\cdot X)+\mathrm{pt}\left(\frac{2\,\mathrm{at}\,e\,\mathrm{Gm}\,s\,(k\cdot X)}{\mathrm{xm}}-i\,\gamma\gamma5\mathrm{FDt}\,e\,s-\mathrm{Gt}\right)+$$
$$\frac{1}{2}\,i\,e\,m\,s\,\sigma\mathrm{F}+\frac{1}{2}\,i\,\gamma\gamma5\mathrm{FDp}\,e\,\mathrm{xm}+\frac{\mathrm{Gm}\,\mathrm{p2}\,s}{\mathrm{xm}}+\frac{\mathrm{Gm}\,\mathrm{pt}^2\,s}{\mathrm{xm}}+\frac{\mathrm{Gp}\,\mathrm{xm}}{2\,s}+m$$

$$\frac{i\,2^{-D-1}\,\pi^{-D}\,\Lambda^{4-D}}{s\left(p^2-m^2\right)}$$

$$\frac{1}{12}\,a^2\,e^2\,s\,(k\cdot x)^2+a^2\,e^2\,s\,(k\cdot X)^2+\mathrm{pt}\,(\mathrm{xt}-2\,\mathrm{at}\,e\,s\,(k\cdot X))-\frac{\mathrm{xp}\,(k\cdot x)}{2\,m\,s}-\mathrm{p2}\,s+\mathrm{pt}^2\,(-s)$$

Integration over

$\int d^{D-2} p_\perp \ldots$

$$I_0 = \int d^{D-2} p_\perp \, \mathrm{Exp}\left[-I\, A\, p_\perp^2 + I\, (J.p_\perp)\right] =$$

$$= \mathrm{Exp}\left[-I \frac{\pi}{2} \frac{D-2}{2}\right] \pi^{\frac{D-2}{2}} (\det A)^{-\frac{1}{2}} \mathrm{Exp}\left[I \frac{1}{4} J.A^{-1}.J\right]$$

$$I_1 = \int d^{D-2} p_\perp \, p_\perp \, \mathrm{Exp}\left[-I\, A\, p_\perp^2 + I\, (J.p_\perp)\right] =$$

$$= \frac{1}{2} A^{-1}.J\, I_0$$

$$I_2 = \int d^{D-2} p_\perp \, p_\perp \, p_\perp{}^2 \, \mathrm{Exp}\left[-I\, A\, p_\perp^2 + I\, (J.p_\perp)\right] =$$

$$= \left[-i \frac{1}{2} \mathrm{Tr}\, A^{-1} + \left(\frac{1}{2} A^{-1}.J\right)^2\right] I_0$$

where

$A = s,$

$J = x_\perp - 2\, e a_\perp\, s\, kX,$

$\det A = s^{D-2},$

$A^{-1} = 1/s$

We perform integrations

Integrations changes the coefficient (Coeff) and phase

Then recollect some scalar products

$a_\perp \wedge 2 = -a^2$

$a_\perp\, x_\perp = -(ax)$

$x_\perp^2 = 2\, x_-\, x_+ - x^2$

```
Amatr = -Coefficient[Phase4, pt^2]
J = Coefficient[Phase4, pt]
CI0 = Exp[-I Pi / 2 (D / 2 - 1)] Pi ^ (D / 2 - 1) / Amatr ^ (D / 2 - 1)
```

s

$\mathrm{xt} - 2\, \mathrm{at}\, e\, s\, (k \cdot X)$

$e^{-\frac{1}{2} i \pi \left(\frac{D}{2}-1\right)} \pi^{\frac{D}{2}-1} s^{1-\frac{D}{2}}$

```
Phase5 = Expand[Expand[((Phase4 /. {pt → 0}) + 1 / 4 J^2 / Amatr)] /.
    { at^2 → -av2, at xt → -ax, xt^2 → 2 xm xp - xv2} /. {xm → kx / m}]
Coeff5 = Coeff4 * CI0
Matrix5 = Collect[
  Expand[Expand[(Matrix4 /. {pt → 0}) + Coefficient[Matrix4, pt] * 1 / 2 / Amatr * J +
      Coefficient[Matrix4, pt^2] * (-I / 2 * (D - 2) / Amatr + (1 / 2 / Amatr * J) ^2)] /.
    {γγ5FDt at → 0, at^2 → -av2, at xt → -ax, xt^2 → 2 xm xp - xv2}],
  {p2, Gm, Gp, Gt, γγ5FDm, γγ5FDp, γγ5FDt}]
```

$$\frac{1}{12} a^2 e^2 s (k\cdot x)^2 + e (a\cdot x)(k\cdot X) - \text{p2}\, s - \frac{x^2}{4 s}$$

$$\frac{i\, 2^{-D-1} e^{-\frac{1}{2} i \pi \left(\frac{D}{2}-1\right)} \pi^{-\frac{D}{2}-1} \Lambda^{4-D} s^{-D/2}}{p^2 - m^2}$$

$$\text{Gm}\left(\frac{1}{4} a^2 e^2 m^2 s\, \text{xm} - \frac{i D}{2\, \text{xm}} - \frac{x^2}{4 s\, \text{xm}} + \frac{\text{xp}}{2 s} + \frac{i}{\text{xm}}\right) + e \gamma\cdot a (k\cdot X) + \text{Gt}\left(\text{at}\, e (k\cdot X) - \frac{\text{xt}}{2 s}\right) + \frac{1}{2} i e m s \sigma\text{F} + \frac{1}{2} i \gamma\gamma 5\text{FDp}\, e\, \text{xm} - \frac{1}{2} i \gamma\gamma 5\text{FDt}\, e\, \text{xt} + \frac{\text{Gm}\, \text{p2}\, s}{\text{xm}} + \frac{\text{Gp}\, \text{xm}}{2 s} + m$$

Next we substitute

$\gamma_\perp a_\perp = -\gamma a$

$\gamma_\perp x_\perp = \gamma_- x_+ - \gamma_+ x_- - \gamma x$

$\gamma_- x_- = \frac{\gamma k}{m} x_-$

$x_- = kx / m$

```
Matrix6 = Collect[Expand[Matrix5] /.
     {Gt at → -Contract[GAD[α] av[α]], Gt xt → Gm xp + Gp xm - Contract[GAD[α] xv[α]] ,
      γγ5FDt xt → γγ5FDp xm - DiracSlash[FDxv, Dimension → D].GA[5]} /.
    {Gm xm → DiracGamma[Momentum[k, D], D] / m * xm} /. {xm → kx / m},
  {p2, Gm, Gp, Gt, γγ5FDm, γγ5FDp, γγ5FDt}]
Coeff6 = Coeff5
Phase6 = Phase5
```

$$\frac{1}{2} i e (\gamma\cdot\text{FDxv}).\overline{\gamma}^5 + \frac{1}{4} a^2 e^2 s \gamma\cdot k (k\cdot x) + \text{Gm}\left(-\frac{i D m}{2 (k\cdot x)} - \frac{m x^2}{4 s (k\cdot x)} + \frac{i m}{k\cdot x}\right) + \frac{1}{2} i e m s \sigma\text{F} + \frac{\text{Gm}\, m\, \text{p2}\, s}{k\cdot x} + m + \frac{\gamma\cdot x}{2 s}$$

$$\frac{i\, 2^{-D-1} e^{-\frac{1}{2} i \pi \left(\frac{D}{2}-1\right)} \pi^{-\frac{D}{2}-1} \Lambda^{4-D} s^{-D/2}}{p^2 - m^2}$$

$$\frac{1}{12} a^2 e^2 s (k\cdot x)^2 + e (a\cdot x)(k\cdot X) - \text{p2}\, s - \frac{x^2}{4 s}$$

$\nu = p^2 - m^2$

$$\int \frac{d\nu}{\nu + i0} \exp[-i s \nu] = -2 \pi i\, \theta(s)$$

$$\int \mathrm{d}\nu \, \exp[-\mathrm{i} s \nu] = 2\pi\,\delta(s)$$

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}s}{s^{D/2}}\, s\, e^{\mathrm{i} g(s)}\, 2\pi\,\delta(s) = -2\pi\mathrm{i} \int_{-\infty}^{\infty} \frac{\mathrm{d}s}{s^{D/2}}\, e^{\mathrm{i} g(s)}\, \theta(s)\,[\mathrm{i}\,(D/2-1) + s\,g'(s)]$$

where

$g(s)$ = Phase6 with $p^2 = m^2$

Finally,

$Gm = \gamma_- = \gamma k / m$

```
Phase7 = Phase6 /. {p2 → m^2}
Coeff7 = Coeff6 * (pv2 - m^2) * (-2 π ⅈ)
g = Phase7;
Matrix7 = Expand[
    (Matrix6 /. {p2 → m^2}) + Coefficient[Matrix6, p2 s] * (ⅈ (D / 2 - 1) + s D[g, s])] /.
   {Gm → DiracGamma[Momentum[k, D], D] / m }
```

$$\frac{1}{12} a^2 e^2 s (k\cdot x)^2 + e(a\cdot x)(k\cdot X) + m^2(-s) - \frac{x^2}{4s}$$

$$2^{-D} e^{-\frac{1}{2} i \pi \left(\frac{D}{2}-1\right)} \pi^{-D/2} \Lambda^{4-D} s^{-D/2}$$

$$\frac{1}{2} i e (\gamma\cdot \mathrm{FDxv}).\bar{\gamma}^5 + \frac{1}{3} a^2 e^2 s \gamma\cdot k (k\cdot x) + \frac{1}{2} i e m s \sigma\mathrm{F} + m + \frac{\gamma\cdot x}{2s}$$

```
Matrix8 = Expand[Matrix7 / m /.
    {DiracGamma[Momentum[k, D], D] → Contract[GAD[α] FFxv[α]] / (-av2 kx) }]
Coeff8 = Coeff7 * m
Phase8 = Phase7 /. {av2 kx^2 → Fx^2}
```

$$\frac{i e (\gamma\cdot \mathrm{FDxv}).\bar{\gamma}^5}{2m} - \frac{e^2 s \gamma\cdot \mathrm{FFx}}{3m} + \frac{1}{2} i e s \sigma\mathrm{F} + \frac{\gamma\cdot x}{2ms} + 1$$

$$2^{-D} e^{-\frac{1}{2} i \pi \left(\frac{D}{2}-1\right)} \pi^{-D/2} m \Lambda^{4-D} s^{-D/2}$$

$$e(a\cdot x)(k\cdot X) + \frac{1}{12} e^2 \mathrm{Fx}^2 s - m^2 s - \frac{x^2}{4s}$$

```
Matrix9 = Matrix8 /. {s → s1 / m^2}
Coeff9 = Simplify[Coeff8 / m^2 /. {s → s1 / m^2}, Assumptions → m > 0]
Coeff9 /. {D → 4}
Phase9 = Phase8 /. {s → s1 / m^2}
```

$$\frac{i\,e\,(\gamma\cdot \mathrm{FDxv}).\bar{\gamma}^5}{2\,m}-\frac{e^2\,\mathrm{s1}\,\gamma\cdot \mathrm{FFx}}{3\,m^3}+\frac{i\,e\,\mathrm{s1}\,\sigma\mathrm{F}}{2\,m^2}+\frac{m\,\gamma\cdot x}{2\,\mathrm{s1}}+1$$

$$i\,2^{-D}\,e^{-\frac{1}{4}i\pi D}\,\pi^{-D/2}\,\Lambda^{4-D}\,m^{D-1}\,\mathrm{s1}^{-D/2}$$

$$-\frac{i\,m^3}{16\,\pi^2\,\mathrm{s1}^2}$$

$$e\,(a\cdot x)\,(k\cdot X)+\frac{e^2\,\mathrm{Fx}^2\,\mathrm{s1}}{12\,m^2}-\frac{m^2\,x^2}{4\,\mathrm{s1}}-\mathrm{s1}$$

```
Matrix91 =
  Expand[Matrix9 /. {DiracGamma[Momentum[FDxv, D], D].GA[5] → Contract[DiracGamma[
              LorentzIndex[μ, D], D].GA[5] FDxv[μ] /. {FDxv[μ_] → -LCD[μ, α1, α2, α3]
              xv[α1] av[α2] kv[α3]}, EpsContract → False]} /. antiTripleGamma /.
      {DiracGamma[Momentum[FFx, D], D] → -av2 kx DiracGamma[Momentum[k, D], D]} /.
    {σF → -2 ⅈ DiracGamma[Momentum[a, D], D].DiracGamma[Momentum[k, D], D]}]
```

$$\frac{a^2\,e^2\,\mathrm{s1}\,\gamma\cdot k\,(k\cdot x)}{3\,m^3}+\frac{e\,\mathrm{s1}\,(\gamma\cdot a).(\gamma\cdot k)}{m^2}-\frac{e\,(a\cdot x)\,\gamma\cdot k}{2\,m}+\frac{e\,\gamma\cdot a\,(k\cdot x)}{2\,m}+\frac{e\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{2\,m}+\frac{m\,\gamma\cdot x}{2\,\mathrm{s1}}+1$$

```
Expand[(Matrix91 / (m / 2 / s1) /. {s1 → s * m^2}) / 2 / s]
```

$$\frac{1}{3}a^2\,e^2\,s\,\gamma\cdot k\,(k\cdot x)+e\,m\,s\,(\gamma\cdot a).(\gamma\cdot k)-\frac{1}{2}e\,(a\cdot x)\,\gamma\cdot k+\frac{1}{2}e\,\gamma\cdot a\,(k\cdot x)+\frac{1}{2}e\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)+m+\frac{\gamma\cdot x}{2\,s}$$

Another representation of the γ - matrix preexponent term

```
MartixAnother =
  (2 m s1 * 1 / 2 / m / s1 GAD[α] (xv[α] - s1 e Fxv[α] + s1^2 / 3 e^2 FFxv[α]) + 2 m s1 * 1).
    (1 + e s1 DiracSlash[a, k, Dimension → D]) /. {s1 → s1 / m^2}
MartixAnother1 = DiracSimplify[Contract[DotSimplify[MartixAnother / (2 s1 / m) /.
      {Fxv[α_] → kv[α] ax - av[α] kx, FFxv[α_] → -av2 kx kv[α]}]]]
CoeffAnother = Coeff9 * m / 2 / s1
```

$$\left(\gamma^{\alpha}\left(\frac{e^2\,\mathrm{s1}^2\,\mathrm{FFx}^{\alpha}}{3\,m^4}-\frac{e\,\mathrm{s1}\,\mathrm{Fx}^{\alpha}}{m^2}+x^{\alpha}\right)+\frac{2\,\mathrm{s1}}{m}\right).\left(\frac{e\,\mathrm{s1}\,(\gamma\cdot a).(\gamma\cdot k)}{m^2}+1\right)$$

$$\frac{a^2\,e^2\,\mathrm{s1}\,\gamma\cdot k\,(k\cdot x)}{3\,m^3}+\frac{e\,\mathrm{s1}\,(\gamma\cdot a).(\gamma\cdot k)}{m^2}-\frac{e\,(a\cdot x)\,\gamma\cdot k}{2\,m}+\frac{e\,\gamma\cdot a\,(k\cdot x)}{2\,m}+\frac{e\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{2\,m}+\frac{m\,\gamma\cdot x}{2\,\mathrm{s1}}+1$$

$$i\,2^{-D-1}\,e^{-\frac{1}{4}i\pi D}\,\pi^{-D/2}\,\Lambda^{4-D}\,m^{D}\,\mathrm{s1}^{-\frac{D}{2}-1}$$

```
Matrix91 - Expand[MartixAnother1]
```

0

Final result for the electron propagator in a CCF

$$S^c(x_2, x_1) =$$

$$\Lambda^{4-D} \int \frac{d^D p}{(2\pi)^D} E_p(x_2) \frac{i((\gamma p) + m)}{p^2 - m^2 + i0} E_p^{bar}(x_1) = e^{-i\frac{\pi}{2}\left(\frac{D}{2}-1\right)} \frac{\Lambda^{4-D}}{2^D \pi^{D/2}} m^{D-1} e^{i\eta}$$

$$\int_0^\infty \frac{ds}{s^{D/2}} \left[1 + \frac{m(\gamma x)}{2s} - \frac{e^2 s(\gamma FFx)}{3m^3} + \frac{ies\left(\sigma^{\alpha\beta} F_{\alpha\beta}\right)}{2m^2} + \frac{i e F_{\alpha\beta}{}^* x^\beta \gamma^\alpha \gamma^5}{2m}\right]$$

$$e^{-is - i\frac{m^2 x^2}{4s} + i\frac{s}{12}\frac{e^2}{m^2}(Fx)^2} =$$

$$= e^{-i\frac{\pi}{2}\left(\frac{D}{2}-1\right)} \frac{\Lambda^{4-D}}{2^D \pi^{D/2}} m^{D-1} e^{i\eta}$$

$$\int_0^\infty \frac{ds}{s^{D/2}} \left[1 + \frac{m(\gamma x)}{2s} + \frac{e(\gamma a)(kx)}{2m} - \frac{e(\gamma k)(ax)}{2m} + \frac{e(\gamma x)(\gamma a)(\gamma k)}{2m} + \right.$$

$$\left. \frac{es(\gamma a)(\gamma k)}{m^2} + \frac{e^2 a^2 s(\gamma k)(kx)}{3m^3}\right] e^{-is - i\frac{m^2 x^2}{4s} + i\frac{s}{12}\frac{e^2}{m^2}(Fx)^2} =$$

$$= e^{-i\frac{\pi}{2}\left(\frac{D}{2}-1\right)} \frac{\Lambda^{4-D}}{2^{D+1} \pi^{D/2}} m^D e^{i\eta} \int_0^\infty \frac{ds}{s^{D/2+1}} \left[\frac{2s}{m} + \gamma^\alpha \left(g_{\alpha\beta} - \frac{es}{m^2} F_{\alpha\beta} + \frac{e^2 s^2}{3m^4} F_{\alpha\lambda} F^\lambda{}_\beta\right)\right.$$

$$\left. x^\beta\right] \left(1 + \frac{ies}{2} \sigma^{\alpha\beta} F_{\alpha\beta}\right) e^{-is - i\frac{m^2 x^2}{4s} + i\frac{s}{12}\frac{e^2}{m^2}(Fx)^2}$$

$\eta = e\,(ax)\,(k, (x_1 + x_2)/2)$,

$x = x_2 - x_1$,

$e > 0$,

$\sigma^{\alpha\beta} = \frac{i}{2}\left(\gamma^\alpha \gamma^\beta - \gamma^\beta \gamma^\alpha\right)$,

$\gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3$,

$$e^{-i\frac{\pi}{2}\left(\frac{D}{2}-1\right)} \frac{\Lambda^{4-D}}{2^D \pi^{D/2}} m^{D-1} \to \frac{(-i)\,m^3}{16\pi^2}, \quad D \to 4$$