

This is a part of SFQED-Loops script collection
developed for calculating loop processes in
Strong-Field Quantum Electrodynamics.

The scripts are available on <https://github.com/ArsenyMironov/SFQED-Loops>

If you use this script in your research, please, consider
citing our papers:

- A. A. Mironov, S. Meuren, and A. M. Fedotov, PRD 102, 053005 (2020),
<https://doi.org/10.1103/PhysRevD.102.053005>

- A. A. Mironov, A. M. Fedotov, arXiv:2109.00634 (2021)

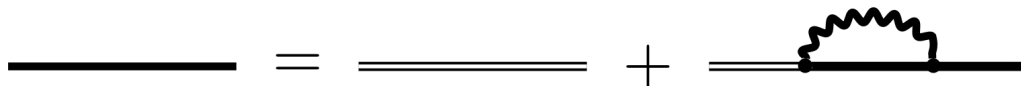
If you have any questions, please, don't hesitate to contact:
mironov.hep@gmail.com

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```
In[1]:= NotebookEvaluate[NotebookOpen[NotebookDirectory[] <> "definitions.nb"]]
```

FeynCalc 9.3.1 (stable version). For help, use the documentation center, check out the wiki or visit the forum.

To save your and our time, please check our FAQ for answers to some common FeynCalc questions.

See also the supplied examples. If you use FeynCalc in your research, please cite

- V. Shtabovenko, R. Mertig and F. Orellana, Comput.Phys.Commun. 256 (2020) 107478, arXiv:2001.04407.
- V. Shtabovenko, R. Mertig and F. Orellana, Comput.Phys.Commun. 207 (2016) 432–444, arXiv:1601.01167.
- R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun. 64 (1991) 345–359.

Electron propagator in E_p – representation satisfies the equation

$$S^c(p, q) = (2\pi)^4 \delta(p - q) S^c(p)$$

$$[\gamma p - m - M(p)] S^c(p) = i$$

Let

$$D(p) = \gamma p - m - M(p) = S + \gamma V + \sigma T + \gamma A \gamma^5$$

$$S = m s(p^2, \chi_p),$$

$$V^\mu = p^\mu v_1(p^2, \chi_p) + \frac{e^2 (F^2 p)^\mu}{m^4 \chi^2} v_2(p^2, \chi_p),$$

$$T_{\mu\nu} = \frac{e F_{\mu\nu}}{m \chi} t(p^2, \chi_p),$$

$$A^\mu = \frac{e (F^* p)^\mu}{m^2 \chi} a_s(p^2, \chi_p), \text{ where } s \text{ stands for 'scalar'};$$

then

$$S^c(p) = i D^{-1}(p) =$$

$$\begin{aligned} & \frac{i}{2} (S - \gamma V - \sigma T + \gamma A \gamma^5) \left[\frac{1}{D_+(p^2, \chi_p)} (1 + \gamma \epsilon^{(2)} \gamma^5) + \frac{1}{D_-(p^2, \chi_p)} (1 - \gamma \epsilon^{(2)} \gamma^5) \right] = \\ & = \frac{i}{2} \left[m s(p^2, \chi_p) - (\gamma p) v_1(p^2, \chi_p) - \frac{e^2 (\gamma F^2 p)}{m^4} v_2(p^2, \chi_p) - \right. \\ & \quad \left. \frac{e \sigma F}{m} t(p^2, \chi_p) + \frac{e (\gamma F^* p) \gamma^5}{m^2} a_s(p^2, \chi_p) \right] \times \\ & \quad \left[\frac{1}{D_+(p^2, \chi_p)} (1 + \gamma \epsilon^{(2)} \gamma^5) + \frac{1}{D_-(p^2, \chi_p)} (1 - \gamma \epsilon^{(2)} \gamma^5) \right], \end{aligned}$$

$$\epsilon^{(2)}_{\mu} = \frac{e (F^* p)^{\mu}}{m^3 \chi_p},$$

Electron propagator

$$S^c(x_2, x_1) = \Lambda^{4-D} \int \frac{d^D p}{(2\pi)^D} E_p(x_2) S^c(p) \bar{E}_p(x_1)$$

$$x = x_2 - x_1,$$

$$X = \frac{1}{2} (x_1 + x_2),$$

$$\xi^2 = -\frac{e^2 a^2}{m^2},$$

$$[\Lambda] = m \text{ - mass scale,}$$

$$E_p(x_2) = \left[1 - \frac{e(\gamma k)(\gamma a)}{2(kp)} (kx_2) \right] \text{Exp} \left[-i(p \cdot x_2) + i \frac{e(a \cdot p)}{2(k \cdot p)} (k \cdot x_2)^2 + i \frac{a^2 e^2}{6(k \cdot p)} (k \cdot x_2)^3 \right];$$

```
In[2]:= NewMomentum["p"]
NewCoordinate["x1"]
NewCoordinate["x2"]
NewCoordinate["x"]
NewCoordinate["X"]
```

$$\begin{aligned}
& \left\{ p^\alpha, p^2, k \cdot p, Fp^\alpha, FFp^\alpha, FDP^\alpha, a \cdot p, 0, 0, 0, -a^2 (k \cdot p), 0, 0, -\frac{m^6 \chi p^2}{e^2}, -\frac{m^6 \chi p^2}{e^2}, \frac{m^6 \chi p^2}{e^2}, 0, 0, 0, 0, 0, 0 \right\} \\
& \left\{ x1^\alpha, x1^2, k \cdot x1, a \cdot x1, Fx1^\alpha, FFx1^\alpha, FDX1^\alpha, k \cdot x1, 0, 0, 0, -a^2 (k \cdot x1), \right. \\
& \quad \left. 0, 0, -\frac{m^2 \xi^2 (k \cdot x1)^2}{e^2}, -\frac{m^2 \xi^2 (k \cdot x1)^2}{e^2}, \frac{m^2 \xi^2 (k \cdot x1)^2}{e^2}, 0, 0, 0, 0, 0, 0 \right\} \\
& \left\{ x2^\alpha, x2^2, k \cdot x2, a \cdot x2, Fx2^\alpha, FFx2^\alpha, FDX2^\alpha, k \cdot x2, 0, 0, 0, -a^2 (k \cdot x2), \right. \\
& \quad \left. 0, 0, -\frac{m^2 \xi^2 (k \cdot x2)^2}{e^2}, -\frac{m^2 \xi^2 (k \cdot x2)^2}{e^2}, \frac{m^2 \xi^2 (k \cdot x2)^2}{e^2}, 0, 0, 0, 0, 0, 0 \right\} \\
& \left\{ x^\alpha, x^2, k \cdot x, a \cdot x, Fx^\alpha, FFx^\alpha, FDX^\alpha, k \cdot x, 0, 0, 0, -a^2 (k \cdot x), \right. \\
& \quad \left. 0, 0, -\frac{m^2 \xi^2 (k \cdot x)^2}{e^2}, -\frac{m^2 \xi^2 (k \cdot x)^2}{e^2}, \frac{m^2 \xi^2 (k \cdot x)^2}{e^2}, 0, 0, 0, 0, 0, 0 \right\} \\
& \left\{ X^\alpha, X^2, k \cdot X, a \cdot X, FX^\alpha, FFX^\alpha, FDX^\alpha, k \cdot X, 0, 0, 0, -a^2 (k \cdot X), \right. \\
& \quad \left. 0, 0, -\frac{m^2 \xi^2 (k \cdot X)^2}{e^2}, -\frac{m^2 \xi^2 (k \cdot X)^2}{e^2}, \frac{m^2 \xi^2 (k \cdot X)^2}{e^2}, 0, 0, 0, 0, 0, 0 \right\}
\end{aligned}$$

$$\text{DdInv[sgn]} == \text{D}_{\text{sgn}}(p^2, \chi_p)$$

$$\epsilon == \epsilon^{(2)}$$

```

In[7]:= FToak = {Ft[α_, β_] → kv[α] × av[β] - av[α] × kv[β], DiracGamma[Momentum[Fp, D], D] →
  DiracGamma[Momentum[k, D], D] × Pair[Momentum[a, D], Momentum[p, D]] -
  DiracGamma[Momentum[a, D], D] × Pair[Momentum[k, D], Momentum[p, D]],
  DiracGamma[Momentum[FFp, D], D] → -av2 DiracGamma[Momentum[k, D], D] ×
  Pair[Momentum[k, D], Momentum[p, D]],
  FFpv[μ_] → -av2 kp kv[μ]}
FToEps =
{FDpv[μ_] → Contract[1/2 Eps[LorentzIndex[μ, D], LorentzIndex[v, D], LorentzIndex[α2, D],
  LorentzIndex[α3, D]] (kv[α2] × av[α3] - av[α2] × kv[α3]) pv[v]]}
Gamma5toTrippleGamma = {GAD[μ_].GA[5] × Eps[LorentzIndex[μ_, D],
  Momentum[a_, D], Momentum[b_, D], Momentum[c_, D]] →
  Contract[I Pair[LorentzIndex[α1, D], Momentum[a, D]] ×
  Pair[LorentzIndex[α2, D], Momentum[b, D]] × Pair[LorentzIndex[α3, D], Momentum[c, D]]
  (GAD[α1, α2, α3] - (MTD[α1, α2] × GAD[α3] + MTD[α2, α3] × GAD[α1] - MTD[α1, α3] × GAD[α2]))]}

Out[7]= {F(α_, β_) → a^β k^α - a^α k^β, γ · Fp → (a · p) γ · k - γ · a (k · p), γ · FFp → a^2 (-(γ · k) (k · p), FFp^{μ-} → a^2 (-(k^μ) (k · p))}

Out[8]= {FDp^{μ-} → -ε^{μ a k p}}

Out[9]= {γ^{μ-} . γ^5 ε^{μ- a- b- c-} → i (-(a · b) γ · c + (a · c) γ · b - γ · a (b · c) + (γ · a) . (γ · b) . (γ · c))}

```

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In[10]:= FxToak = {Ft[α_, β_] → kv[α] × av[β] - av[α] × kv[β], DiracGamma[Momentum[Fx, D], D] →
  DiracGamma[Momentum[k, D], D] × Pair[Momentum[a, D], Momentum[x, D]] -
  DiracGamma[Momentum[a, D], D] × Pair[Momentum[k, D], Momentum[x, D]],
  DiracGamma[Momentum[FFx, D], D] → -av2 DiracGamma[Momentum[k, D], D] ×
  Pair[Momentum[k, D], Momentum[x, D]],
  FFxv[μ_] → -av2 kx kv[μ], σF → -2 I DiracGamma[Momentum[a, D], D].
  DiracGamma[Momentum[k, D], D]}
FxToEps = {FDx[μ_] → Contract[1/2 Eps[LorentzIndex[μ, D], LorentzIndex[v, D],
  LorentzIndex[α2, D], LorentzIndex[α3, D]] (kv[α2] × av[α3] - av[α2] × kv[α3]) xv[v]],
  DiracGamma[Momentum[FDx, D], D].DiracGamma[5] → Contract[1/2
  Eps[LorentzIndex[μ, D], LorentzIndex[v, D], LorentzIndex[α2, D], LorentzIndex[α3, D]]
  (kv[α2] × av[α3] - av[α2] × kv[α3]) xv[v] × GAD[μ].DiracGamma[5]]}
Gamma5toTrippleGammax = {DiracGamma[LorentzIndex[μ_, D], D].DiracGamma[5]
  Eps[LorentzIndex[μ_, D], Momentum[a_, D], Momentum[b_, D], Momentum[c_, D]] →
  Contract[I Pair[LorentzIndex[α1, D], Momentum[a, D]] ×
  Pair[LorentzIndex[α2, D], Momentum[b, D]] × Pair[LorentzIndex[α3, D], Momentum[c, D]]
  (GAD[α1, α2, α3] - (MTD[α1, α2] × GAD[α3] + MTD[α2, α3] × GAD[α1] - MTD[α1, α3] × GAD[α2]))}

Out[10]= {F(α_-, β_-) → aβ kα - aα kβ, γ · Fx → (a · x) γ · k - γ · a (k · x),
  γ · FFx → a2 (-(γ · k) (k · x), FFxμ- → a2 (-kμ) (k · x), σF → -2 i (γ · a) · (γ · k)}

Out[11]= {FDx(μ_-) → -εμ a k x, (γ · FDx) · γ5 → -γμ · γ5 εμ a k x}

Out[12]= {γμ- · γ5 εμ- a- b- c- → i (-(a · b) γ · c + (a · c) γ · b - γ · a (b · c) + (γ · a) · (γ · b) · (γ · c))}

In[13]:= S = m sf
V[μ_] = pv[μ] v1 + e ^ 2 FFpv[μ] / m ^ 4 / χp ^ 2 v2
T[μ_, v_] = e Ft[μ, v] / m / χp tf
A[μ_] = e FDpv[μ] / m ^ 2 / χp af
ε[μ_] = e FDpv[μ] / m ^ 3 / χp

Out[13]= m sf

Out[14]= 
$$\frac{e^2 v_2 FFp^\mu}{m^4 \chi p^2} + v_1 p^\mu$$


Out[15]= 
$$\frac{e tf F(\mu, v)}{m \chi p}$$


Out[16]= 
$$\frac{af e FDp^\mu}{m^2 \chi p}$$


Out[17]= 
$$\frac{e FDp^\mu}{m^3 \chi p}$$


```

$$\text{In[18]:= Scp[sgn_] = (S - GAD[\alpha] \times V[\alpha] - I / 2 (GAD[\alpha, \beta] - GAD[\beta, \alpha]) T[\alpha, \beta] + A[\alpha] \times GAD[\alpha].GA[5]). \\ (1 + \text{sgn } \epsilon[\mu] \times GAD[\mu].GA[5]) /. \text{FTToEps} /. \text{FTToak} /. \text{Gamma5toTrippleGamma}$$

$$\text{Out[18]=} \left(-\frac{i \text{af } e((a \cdot p) \gamma \cdot k - \gamma \cdot a(k \cdot p) + (\gamma \cdot a)(\gamma \cdot k)(\gamma \cdot p))}{m^2 \chi p} - \gamma^\alpha \left(v1 p^\alpha - \frac{a^2 \epsilon^2 v2 k^\alpha (k \cdot p)}{m^4 \chi p^2} \right) - \right. \\ \left. \frac{i \text{etf } (\gamma^\alpha \cdot \gamma^\beta - \gamma^\beta \cdot \gamma^\alpha) (a^\beta k^\alpha - a^\alpha k^\beta)}{2 m \chi p} + m \text{sf} \right) \left(1 - \frac{i \text{esgn } ((a \cdot p) \gamma \cdot k - \gamma \cdot a(k \cdot p) + (\gamma \cdot a)(\gamma \cdot k)(\gamma \cdot p))}{m^3 \chi p} \right)$$

$$\text{In[19]:= Epx2 = Ep[x2, p]$$

$$\text{EpBarx1 = EpC[x1, p]}$$

$$\text{Out[19]=} \left\{ 1 - \frac{e(k \cdot x2) (\gamma \cdot k)(\gamma \cdot a)}{2 (k \cdot p)}, \frac{a^2 \epsilon^2 (k \cdot x2)^3}{6 (k \cdot p)} + \frac{e(a \cdot p) (k \cdot x2)^2}{2 (k \cdot p)} - p \cdot x2 \right\}$$

$$\text{Out[20]=} \left\{ 1 - \frac{e(k \cdot x1) (\gamma \cdot a)(\gamma \cdot k)}{2 (k \cdot p)}, -\frac{a^2 \epsilon^2 (k \cdot x1)^3}{6 (k \cdot p)} - \frac{e(a \cdot p) (k \cdot x1)^2}{2 (k \cdot p)} + p \cdot x1 \right\}$$

$$\text{In[21]:= Matrix = Epx2[[1]].Scp[\zeta].EpBarx1[[1]]$$

$$\text{Coeff} = i / 2 \wedge^{(4-D)} / (2 \pi)^D / \text{DdInv}[\zeta]$$

$$\text{Phase} = \text{Epx2}[[2]] + \text{EpBarx1}[[2]]$$

$$\text{Out[21]=} \left(1 - \frac{e(k \cdot x2) (\gamma \cdot k)(\gamma \cdot a)}{2 (k \cdot p)} \right) \left(-\frac{i \text{af } e((a \cdot p) \gamma \cdot k - \gamma \cdot a(k \cdot p) + (\gamma \cdot a)(\gamma \cdot k)(\gamma \cdot p))}{m^2 \chi p} - \right. \\ \left. \gamma^\alpha \left(v1 p^\alpha - \frac{a^2 \epsilon^2 v2 k^\alpha (k \cdot p)}{m^4 \chi p^2} \right) - \frac{i \text{etf } (\gamma^\alpha \cdot \gamma^\beta - \gamma^\beta \cdot \gamma^\alpha) (a^\beta k^\alpha - a^\alpha k^\beta)}{2 m \chi p} + m \text{sf} \right) \\ \left(1 - \frac{i \text{e}\zeta ((a \cdot p) \gamma \cdot k - \gamma \cdot a(k \cdot p) + (\gamma \cdot a)(\gamma \cdot k)(\gamma \cdot p))}{m^3 \chi p} \right) \left(1 - \frac{e(k \cdot x1) (\gamma \cdot a)(\gamma \cdot k)}{2 (k \cdot p)} \right)$$

$$\text{Out[22]=} \frac{i 2^{-D-1} \pi^{-D} \wedge^{4-D}}{\text{DdInv}(\zeta)}$$

$$\text{Out[23]=} -\frac{a^2 \epsilon^2 (k \cdot x1)^3}{6 (k \cdot p)} + \frac{a^2 \epsilon^2 (k \cdot x2)^3}{6 (k \cdot p)} - \frac{e(a \cdot p) (k \cdot x1)^2}{2 (k \cdot p)} + \frac{e(a \cdot p) (k \cdot x2)^2}{2 (k \cdot p)} + p \cdot x1 - p \cdot x2$$

In[24]:= **Matrix1 = Contract[DiracSimplify[Matrix]]**

Coeff1 = Coeff;

Phase1 = Phase;

$$\begin{aligned} \text{Out[24]} = & \frac{i v2 \zeta (\gamma \cdot k) (\gamma \cdot a) a^2 (k \cdot p)^2 e^3}{m^7 \chi p^3} + \frac{af \zeta (\gamma \cdot a) (\gamma \cdot k) a^2 (k \cdot p) (k \cdot x1) e^3}{2 m^5 \chi p^2} + \frac{af \zeta (\gamma \cdot k) (\gamma \cdot a) a^2 (k \cdot p) (k \cdot x2) e^3}{2 m^5 \chi p^2} - \\ & \frac{i v1 \zeta (\gamma \cdot a) (\gamma \cdot k) a^2 (k \cdot x1) (k \cdot x2) e^3}{2 m^3 \chi p} - \frac{af \zeta a^2 (k \cdot p)^2 e^2}{m^5 \chi p^2} + \frac{v2 \gamma \cdot k a^2 (k \cdot p) e^2}{m^4 \chi p^2} + \frac{2 tf \zeta \gamma \cdot k a^2 (k \cdot p) e^2}{m^4 \chi p^2} - \\ & \frac{af \zeta (\gamma \cdot a) (\gamma \cdot k) (a \cdot p) (k \cdot p) e^2}{m^5 \chi p^2} - \frac{af \zeta (\gamma \cdot k) (\gamma \cdot a) (a \cdot p) (k \cdot p) e^2}{m^5 \chi p^2} + \frac{i sf \zeta \gamma \cdot k a^2 (k \cdot x1) e^2}{2 m^2 \chi p} + \\ & \frac{i af \gamma \cdot k a^2 (k \cdot x1) e^2}{2 m^2 \chi p} - \frac{i v1 \zeta (\gamma \cdot p) (\gamma \cdot k) a^2 (k \cdot x1) e^2}{2 m^3 \chi p} - \frac{i sf \zeta \gamma \cdot k a^2 (k \cdot x2) e^2}{2 m^2 \chi p} - \frac{i af \gamma \cdot k a^2 (k \cdot x2) e^2}{2 m^2 \chi p} + \\ & \frac{i v1 \zeta (\gamma \cdot k) (\gamma \cdot p) a^2 (k \cdot x2) e^2}{2 m^3 \chi p} + \frac{i v1 \zeta (\gamma \cdot a) (\gamma \cdot k) (a \cdot p) (k \cdot x2) e^2}{m^3 \chi p} + \frac{i v1 \zeta (\gamma \cdot k) (\gamma \cdot a) (a \cdot p) (k \cdot x2) e^2}{m^3 \chi p} + \\ & \frac{v1 \gamma \cdot k a^2 (k \cdot x1) (k \cdot x2) e^2}{2 (k \cdot p)} + \frac{i tf (\gamma \cdot a) (\gamma \cdot k) e}{m \chi p} - \frac{i tf (\gamma \cdot k) (\gamma \cdot a) e}{m \chi p} - \frac{i sf \zeta (\gamma \cdot a) (\gamma \cdot k) (\gamma \cdot p) e}{m^2 \chi p} - \\ & \frac{i af (\gamma \cdot a) (\gamma \cdot k) (\gamma \cdot p) e}{m^2 \chi p} - \frac{i sf \zeta \gamma \cdot k (a \cdot p) e}{m^2 \chi p} - \frac{i af \gamma \cdot k (a \cdot p) e}{m^2 \chi p} + \frac{2 i v1 \zeta (\gamma \cdot k) (\gamma \cdot p) (a \cdot p) e}{m^3 \chi p} + \\ & \frac{i v1 \zeta (\gamma \cdot p) (\gamma \cdot k) (a \cdot p) e}{m^3 \chi p} + \frac{i sf \zeta \gamma \cdot a (k \cdot p) e}{m^2 \chi p} + \frac{i af \gamma \cdot a (k \cdot p) e}{m^2 \chi p} - \frac{2 i v1 \zeta (\gamma \cdot a) (\gamma \cdot p) (k \cdot p) e}{m^3 \chi p} - \\ & \frac{i v1 \zeta (\gamma \cdot p) (\gamma \cdot a) (k \cdot p) e}{m^3 \chi p} - \frac{m sf (\gamma \cdot a) (\gamma \cdot k) (k \cdot x1) e}{2 (k \cdot p)} + \frac{v1 (\gamma \cdot p) (\gamma \cdot a) (\gamma \cdot k) (k \cdot x1) e}{2 (k \cdot p)} - \\ & \frac{m sf (\gamma \cdot k) (\gamma \cdot a) (k \cdot x2) e}{2 (k \cdot p)} + \frac{v1 (\gamma \cdot k) (\gamma \cdot a) (\gamma \cdot p) (k \cdot x2) e}{2 (k \cdot p)} + \frac{i v1 \zeta (\gamma \cdot a) (\gamma \cdot k) p^2 e}{m^3 \chi p} + m sf - v1 \gamma \cdot p \end{aligned}$$

In[27]:= **(*a small test*)**

$$\begin{aligned} & ((\text{Matrix1} /. \{\text{sf} \rightarrow 1, v1 \rightarrow -1, v2 \rightarrow 0, t \rightarrow 0, af \rightarrow 0, \zeta \rightarrow 1\}) + \\ & (\text{Matrix1} /. \{\text{sf} \rightarrow 1, v1 \rightarrow -1, v2 \rightarrow 0, t \rightarrow 0, af \rightarrow 0, \zeta \rightarrow -1\})) / 2 \\ \text{Out[27]} = & \frac{1}{2} \left(-\frac{a^2 e^2 \gamma \cdot k (k \cdot x1) (k \cdot x2)}{k \cdot p} - \frac{e m (k \cdot x1) (\gamma \cdot a) (\gamma \cdot k)}{k \cdot p} - \frac{e m (k \cdot x2) (\gamma \cdot k) (\gamma \cdot a)}{k \cdot p} + \frac{2 i e tf (\gamma \cdot a) (\gamma \cdot k)}{m \chi p} - \right. \\ & \left. \frac{2 i e tf (\gamma \cdot k) (\gamma \cdot a)}{m \chi p} - \frac{e (k \cdot x1) (\gamma \cdot p) (\gamma \cdot a) (\gamma \cdot k)}{k \cdot p} - \frac{e (k \cdot x2) (\gamma \cdot k) (\gamma \cdot a) (\gamma \cdot p)}{k \cdot p} + 2 m + 2 \gamma \cdot p \right) \end{aligned}$$

In[28]:= **Matrix2 =**

**Expand[ExpandScalarProduct[Matrix1 /. {Momentum[x1, D] → Momentum[X, D] - Momentum[x, D] / 2,
Momentum[x2, D] → Momentum[X, D] + Momentum[x, D] / 2}]]**

Coeff2 = Coeff1;

Phase2 =

**Expand[ExpandScalarProduct[Phase1 /. {Momentum[x1, D] → Momentum[X, D] - Momentum[x, D] / 2,
Momentum[x2, D] → Momentum[X, D] + Momentum[x, D] / 2}]]**

$$\begin{aligned}
 \text{Out[28]} = & \frac{i v2 \zeta (\gamma \cdot k) (\gamma \cdot a) a^2 (k \cdot p)^2 e^3}{m^7 \chi p^3} + \frac{i v1 \zeta (\gamma \cdot a) (\gamma \cdot k) a^2 (k \cdot x)^2 e^3}{8 m^3 \chi p} - \frac{i v1 \zeta (\gamma \cdot a) (\gamma \cdot k) a^2 (k \cdot X)^2 e^3}{2 m^3 \chi p} - \\
 & \frac{af \zeta (\gamma \cdot a) (\gamma \cdot k) a^2 (k \cdot p) (k \cdot x) e^3}{4 m^5 \chi p^2} + \frac{af \zeta (\gamma \cdot k) (\gamma \cdot a) a^2 (k \cdot p) (k \cdot x) e^3}{4 m^5 \chi p^2} + \frac{af \zeta (\gamma \cdot a) (\gamma \cdot k) a^2 (k \cdot p) (k \cdot X) e^3}{2 m^5 \chi p^2} + \\
 & \frac{af \zeta (\gamma \cdot k) (\gamma \cdot a) a^2 (k \cdot p) (k \cdot X) e^3}{2 m^5 \chi p^2} - \frac{af \zeta a^2 (k \cdot p)^2 e^2}{m^5 \chi p^2} - \frac{v1 \gamma \cdot k a^2 (k \cdot x)^2 e^2}{8 (k \cdot p)} + \frac{v1 \gamma \cdot k a^2 (k \cdot X)^2 e^2}{2 (k \cdot p)} + \\
 & \frac{v2 \gamma \cdot k a^2 (k \cdot p) e^2}{m^4 \chi p^2} + \frac{2 tf \zeta \gamma \cdot k a^2 (k \cdot p) e^2}{m^4 \chi p^2} - \frac{af \zeta (\gamma \cdot a) (\gamma \cdot k) (a \cdot p) (k \cdot p) e^2}{m^5 \chi p^2} - \frac{af \zeta (\gamma \cdot k) (\gamma \cdot a) (a \cdot p) (k \cdot p) e^2}{m^5 \chi p^2} - \\
 & \frac{i sf \zeta \gamma \cdot k a^2 (k \cdot x) e^2}{2 m^2 \chi p} - \frac{i af \gamma \cdot k a^2 (k \cdot x) e^2}{2 m^2 \chi p} + \frac{i v1 \zeta (\gamma \cdot k) (\gamma \cdot p) a^2 (k \cdot x) e^2}{4 m^3 \chi p} + \frac{i v1 \zeta (\gamma \cdot p) (\gamma \cdot k) a^2 (k \cdot x) e^2}{4 m^3 \chi p} + \\
 & \frac{i v1 \zeta (\gamma \cdot a) (\gamma \cdot k) (a \cdot p) (k \cdot x) e^2}{2 m^3 \chi p} + \frac{i v1 \zeta (\gamma \cdot k) (\gamma \cdot a) (a \cdot p) (k \cdot x) e^2}{2 m^3 \chi p} + \frac{i v1 \zeta (\gamma \cdot k) (\gamma \cdot p) a^2 (k \cdot X) e^2}{2 m^3 \chi p} - \\
 & \frac{i v1 \zeta (\gamma \cdot p) (\gamma \cdot k) a^2 (k \cdot X) e^2}{2 m^3 \chi p} + \frac{i v1 \zeta (\gamma \cdot a) (\gamma \cdot k) (a \cdot p) (k \cdot X) e^2}{m^3 \chi p} + \frac{i v1 \zeta (\gamma \cdot k) (\gamma \cdot a) (a \cdot p) (k \cdot X) e^2}{m^3 \chi p} + \\
 & \frac{i tf (\gamma \cdot a) (\gamma \cdot k) e}{m \chi p} - \frac{i tf (\gamma \cdot k) (\gamma \cdot a) e}{m \chi p} - \frac{i sf \zeta (\gamma \cdot a) (\gamma \cdot k) (\gamma \cdot p) e}{m^2 \chi p} - \frac{i af (\gamma \cdot a) (\gamma \cdot k) (\gamma \cdot p) e}{m^2 \chi p} - \frac{i sf \zeta \gamma \cdot k (a \cdot p) e}{m^2 \chi p} - \\
 & \frac{i af \gamma \cdot k (a \cdot p) e}{m^2 \chi p} + \frac{2 i v1 \zeta (\gamma \cdot k) (\gamma \cdot p) (a \cdot p) e}{m^3 \chi p} + \frac{i v1 \zeta (\gamma \cdot p) (\gamma \cdot k) (a \cdot p) e}{m^3 \chi p} + \frac{i sf \zeta \gamma \cdot a (k \cdot p) e}{m^2 \chi p} + \frac{i af \gamma \cdot a (k \cdot p) e}{m^2 \chi p} - \\
 & \frac{2 i v1 \zeta (\gamma \cdot a) (\gamma \cdot p) (k \cdot p) e}{m^3 \chi p} - \frac{i v1 \zeta (\gamma \cdot p) (\gamma \cdot a) (k \cdot p) e}{m^3 \chi p} + \frac{m sf (\gamma \cdot a) (\gamma \cdot k) (k \cdot x) e}{4 (k \cdot p)} - \frac{m sf (\gamma \cdot k) (\gamma \cdot a) (k \cdot x) e}{4 (k \cdot p)} + \\
 & \frac{v1 (\gamma \cdot k) (\gamma \cdot a) (\gamma \cdot p) (k \cdot x) e}{4 (k \cdot p)} - \frac{v1 (\gamma \cdot p) (\gamma \cdot a) (\gamma \cdot k) (k \cdot x) e}{4 (k \cdot p)} - \frac{m sf (\gamma \cdot a) (\gamma \cdot k) (k \cdot X) e}{2 (k \cdot p)} - \frac{m sf (\gamma \cdot k) (\gamma \cdot a) (k \cdot X) e}{2 (k \cdot p)} + \\
 & \frac{v1 (\gamma \cdot k) (\gamma \cdot a) (\gamma \cdot p) (k \cdot X) e}{2 (k \cdot p)} + \frac{v1 (\gamma \cdot p) (\gamma \cdot a) (\gamma \cdot k) (k \cdot X) e}{2 (k \cdot p)} + \frac{i v1 \zeta (\gamma \cdot a) (\gamma \cdot k) p^2 e}{m^3 \chi p} + m sf - v1 \gamma \cdot p \\
 \text{Out[30]} = & \frac{a^2 e^2 (k \cdot x) (k \cdot X)^2}{2 (k \cdot p)} + \frac{a^2 e^2 (k \cdot x)^3}{24 (k \cdot p)} + \frac{e (a \cdot p) (k \cdot x) (k \cdot X)}{k \cdot p} - p \cdot x
 \end{aligned}$$

In[31]:= **Matrix3 =**

((Expand[Matrix2 /. TripleGamma] /. EpsToF) /. FieldSubstitutions) /. akToF

Coeff3 = Coeff2;

Phase3 = Phase2;

$$\begin{aligned}
 \text{Out[31]} = & \frac{v2 \zeta \sigma F a^2 (k \cdot p)^2 e^3}{2 m^7 \chi p^3} - \frac{v1 \zeta \sigma F a^2 (k \cdot x)^2 e^3}{16 m^3 \chi p} + \frac{v1 \zeta \sigma F a^2 (k \cdot X)^2 e^3}{4 m^3 \chi p} - \frac{i \text{af} \zeta \sigma F a^2 (k \cdot p) (k \cdot x) e^3}{4 m^5 \chi p^2} - \\
 & \frac{\text{af} \zeta a^2 (k \cdot p)^2 e^2}{m^5 \chi p^2} - \frac{v1 \gamma \cdot k a^2 (k \cdot x)^2 e^2}{8 (k \cdot p)} + \frac{v1 \gamma \cdot k a^2 (k \cdot X)^2 e^2}{2 (k \cdot p)} + \frac{v2 \gamma \cdot k a^2 (k \cdot p) e^2}{m^4 \chi p^2} + \\
 & \frac{2 \text{tf} \zeta \gamma \cdot k a^2 (k \cdot p) e^2}{m^4 \chi p^2} - \frac{i \text{sf} \zeta \gamma \cdot k a^2 (k \cdot x) e^2}{2 m^2 \chi p} - \frac{i \text{af} \gamma \cdot k a^2 (k \cdot x) e^2}{2 m^2 \chi p} + \frac{i v1 \zeta (\gamma \cdot k) (\gamma \cdot p) a^2 (k \cdot x) e^2}{4 m^3 \chi p} + \\
 & \frac{i v1 \zeta (\gamma \cdot p) (\gamma \cdot k) a^2 (k \cdot x) e^2}{4 m^3 \chi p} + \frac{i v1 \zeta (\gamma \cdot k) (\gamma \cdot p) a^2 (k \cdot X) e^2}{2 m^3 \chi p} - \frac{i v1 \zeta (\gamma \cdot p) (\gamma \cdot k) a^2 (k \cdot X) e^2}{2 m^3 \chi p} + \\
 & \frac{\text{sf} \zeta \gamma^\beta \cdot \bar{\gamma}^5 \text{FDp}^\beta e}{m^2 \chi p} + \frac{\text{af} \gamma^\beta \cdot \bar{\gamma}^5 \text{FDp}^\beta e}{m^2 \chi p} + \frac{2 i v1 \zeta (\gamma \cdot k) (\gamma \cdot p) (a \cdot p) e}{m^3 \chi p} + \frac{i v1 \zeta (\gamma \cdot p) (\gamma \cdot k) (a \cdot p) e}{m^3 \chi p} - \\
 & \frac{2 i v1 \zeta (\gamma \cdot a) (\gamma \cdot p) (k \cdot p) e}{m^3 \chi p} - \frac{i v1 \zeta (\gamma \cdot p) (\gamma \cdot a) (k \cdot p) e}{m^3 \chi p} + \frac{i m \text{sf} \sigma F (k \cdot x) e}{4 (k \cdot p)} - \frac{i v1 \gamma^\beta \cdot \bar{\gamma}^5 \text{FDp}^\beta (k \cdot x) e}{2 (k \cdot p)} - \\
 & v1 \gamma \cdot a (k \cdot X) e + \frac{v1 \gamma \cdot k (a \cdot p) (k \cdot X) e}{k \cdot p} - \frac{v1 \zeta \sigma F p^2 e}{2 m^3 \chi p} - \frac{\text{tf} \sigma F e}{m \chi p} + m \text{sf} - v1 \gamma \cdot p
 \end{aligned}$$

```

In[34]:= Matrix4 = Contract[
  DiracOrder[
    Matrix3 /. { DiracGamma[LorentzIndex[β, D], D].DiracGamma[5] ×
      Pair[LorentzIndex[β, D], Momentum[FDp, D]] → FVD[γγ5FD, α] × pv[α]
    } /. {
      DiracGamma[Momentum[a, D], D].DiracGamma[Momentum[p, D], D] → FVD[γαγ, α] × pv[α],
      DiracGamma[Momentum[k, D], D].DiracGamma[Momentum[p, D], D] → FVD[γkγ, α] × pv[α]
    }
  ]
  Coeff4 = Coeff3;
  Phase4 = Phase3;
Out[34]= - 
$$\frac{1}{16 m^7 \chi p^3 (k \cdot p)}$$


$$\begin{aligned} & (4 i a^2 \text{af} e^3 \zeta m^2 \sigma F \chi p (k \cdot x) (k \cdot p)^2 + 16 a^2 \text{af} e^2 \zeta m^2 \chi p (k \cdot p)^3 + a^2 e^3 \zeta m^4 \sigma F v1 \chi p^2 (k \cdot p) (k \cdot x)^2 - \\ & 4 a^2 e^3 \zeta m^4 \sigma F v1 \chi p^2 (k \cdot p) (k \cdot X)^2 - 8 a^2 e^3 \zeta \sigma F v2 (k \cdot p)^3 - 4 i e m^8 \text{sf} \sigma F \chi p^3 (k \cdot x) + \\ & 16 e m^6 \sigma F \text{tf} \chi p^2 (k \cdot p) + 8 e \zeta m^4 p^2 \sigma F v1 \chi p^2 (k \cdot p) - 16 m^8 \text{sf} \chi p^3 (k \cdot p)) + \frac{1}{8 m^4 \chi p^2 (k \cdot p)} \\ & e \gamma \cdot k (a^2 (-e) m^4 v1 \chi p^2 (k \cdot x)^2 + 4 a^2 e m^4 v1 \chi p^2 (k \cdot X)^2 + 16 a^2 e \zeta \text{tf} (k \cdot p)^2 + 8 a^2 e v2 (k \cdot p)^2 - \\ & 4 i a^2 \text{af} e m^2 \chi p (k \cdot p) (k \cdot x) - 4 i a^2 e \zeta m^2 \text{sf} \chi p (k \cdot p) (k \cdot x) + 8 m^4 v1 \chi p^2 (a \cdot p) (k \cdot X)) - \\ & \frac{i e \zeta v1 (k \cdot p) (2 (a \cdot p) - p \cdot \gamma a \gamma)}{m^3 \chi p} + \frac{i e \zeta v1 (2 (k \cdot p) - p \cdot \gamma k \gamma) (a^2 e (k \cdot x) - 2 a^2 e (k \cdot X) + 4 (a \cdot p))}{4 m^3 \chi p} + \\ & \frac{i e \zeta v1 (p \cdot \gamma k \gamma) (a^2 e (k \cdot x) + 2 a^2 e (k \cdot X) + 8 (a \cdot p))}{4 m^3 \chi p} - e v1 \gamma \cdot a (k \cdot X) + \\ & \frac{e (p \cdot \gamma \gamma 5 F D) (2 \text{af} (k \cdot p) - i m^2 v1 \chi p (k \cdot x) + 2 \zeta \text{sf} (k \cdot p))}{2 m^2 \chi p (k \cdot p)} - \\ & \frac{2 i e \zeta v1 (k \cdot p) (p \cdot \gamma a \gamma)}{m^3 \chi p} - v1 \gamma \cdot p \end{aligned}$$


```

Expanding scalar products into components and changing variables

$$p \rightarrow \{p_- = 1/2 (p^0 - p^3), p_+ = p^0 + p^3, p_\perp\}$$

$$p_- = x_- / 2 s$$

$$p_+ = (p^2 + p_\perp^2) / 2 p_- = s (p^2 + p_\perp^2) / x_-$$

Integration measure

$$\int d^D p \dots = \int \frac{ds}{2s} dp^2 d^{D-2} p_\perp$$

$$x_- = kx / m$$

$$p_- = kx / 2 m s$$

$$kp = m p_- = m xm / 2 s = kx / 2 s$$

$$ap = -at pt$$

$$\gamma p = \gamma_- p_+ + \gamma_+ p_- - \gamma_\perp p_\perp = Gm \frac{s}{x_-} (p^2 + p_\perp^2) + Gp \frac{x_-}{2s} - Gt pt$$

$$\gamma k = \gamma_- k_+ = m Gm$$

$$kx = k_+ x_- = m xm$$

$$(\gamma F^*)^\mu \cdot \gamma^5 \rightarrow \{(\gamma F^*)_- \cdot \gamma^5 = 0, (\gamma F^*)_+ \cdot \gamma^5, (\gamma F^*)_\perp \cdot \gamma^5\} = \{0, \gamma\gamma 5FDp, \gamma\gamma 5FDt\}$$

$$(\gamma F^*)_\mu k^\mu = 0 \rightarrow (\gamma F^*)_- = 0$$

$$(\gamma F^*)_\mu a^\mu = 0 \rightarrow \gamma\gamma 5FDt at = 0$$

$$(\gamma F^*)^\mu \cdot \gamma^5 p_\mu = \gamma\gamma 5FDm * pp$$

$$\gamma k \gamma_- = (\gamma k)^2 / m = 0$$

In[37]:= D[{xm / 2 / s, s (p2 + pt2) / xm}, {{s, p2}}]

Abs[Det[%]]

$$\text{Out[37]=} \begin{pmatrix} -\frac{xm}{2s^2} & 0 \\ \frac{p2+pt2}{xm} & -\frac{s}{xm} \end{pmatrix}$$

$$\text{Out[38]=} \frac{1}{2 |s|}$$

In[39]:= Matrix5 =

```
Collect[Expand[Matrix4 /. {DiracGamma[Momentum[p, D], D] → Gp * pm + Gm * pp - Gt[i] * pt[i],
  Pair[Momentum[a, D], Momentum[p, D]] → -at[i1] * pt[i1],
  Pair[Momentum[p, D], Momentum[γγ5FD, D]] → γγ5FDp * pm,
  Pair[Momentum[p, D], Momentum[γαγ, D]] → γαγp * pm + γαγm * pp - γαγt[i] * pt[i],
  Pair[Momentum[p, D], Momentum[γκγ, D]] → γκγp * pm - γκγt[i] * pt[i],
  (*av2→-at^2,*)
  DiracGamma[Momentum[k, D], D] → m Gm} /. {χp → ξ kp / m^2} /.
  {kp → kx / 2 / s, pm → xm / 2 / s} /. {kx → m xm} /. {pp → s (pv2 + pt[i2]^2) / xm}],
{v1, pt[i2] * pt[i], pt[i2], pt[i_], pv2, γγ5FDm, γγ5FDp, γγ5FDt[i],
  γκγm, γκγp, γκγt[i], γαγm, γαγp, γαγt[i]}]
```

Coefficient[Matrix5, γκγt[i]]

Coeff5 = Coeff4 / 2 / s

Phase5 =

```
Collect[Expand[Phase4 /. {Pair[Momentum[p, D], Momentum[x, D]] → pp xm + pm xp - pt * xt,
  kp → m pm, Pair[Momentum[a, D], Momentum[p, D]] → -at pt} /.
  {kp → kx / 2 / s, pm → xm / 2 / s} /. {pp → s (pv2 + pt^2) / xm} /. {xm → kx / m}], {pt, pp}]
```

$$\begin{aligned}
\text{Out[39]} = & v1 \left(\frac{a^2 e^3 \zeta s \sigma F (k \cdot X)^2}{2 m^2 \xi x m} + \frac{a^2 e^2 G m s (k \cdot X)^2}{x m} + \right. \\
& pt(i) \left(-\frac{2 i a^2 e^2 \zeta s \gamma k \gamma t(i) (k \cdot X)}{m^2 \xi x m} + \frac{2 i e \zeta s at(i1) \gamma k \gamma t(i) pt(i1)}{m^2 \xi x m} + \frac{i e \zeta \gamma a \gamma t(i)}{m \xi} + Gt(i) \right) - \\
& \frac{a^2 e^3 \zeta s \sigma F x m}{8 \xi} - \frac{1}{4} a^2 e^2 G m m^2 s x m + \frac{i a^2 \gamma k \gamma p e^2 \zeta (k \cdot X)}{m^2 \xi} - \frac{i a^2 e^2 \zeta (k \cdot X)}{m \xi} + \frac{i a^2 e^2 \zeta x m}{2 \xi} - \\
& e \gamma \cdot a(k \cdot X) + pt(i1) \left(-\frac{2 e G m s at(i1) (k \cdot X)}{x m} - \frac{i \gamma k \gamma p e \zeta at(i1)}{m^2 \xi} \right) + pt(i2)^2 \left(-\frac{G m s}{x m} - \frac{i \gamma a \gamma m e \zeta s}{m \xi x m} \right) + \\
& p^2 \left(-\frac{e \zeta s \sigma F}{m^2 \xi x m} - \frac{i \gamma a \gamma m e \zeta s}{m \xi x m} - \frac{G m s}{x m} \right) - \frac{i \gamma a \gamma p e \zeta x m}{2 m \xi s} - \frac{1}{2} i \gamma \gamma 5 F D p e x m - \frac{G p x m}{2 s} \Big) - \\
& \frac{i a^2 a f e^3 \zeta s \sigma F}{2 m \xi^2} - \frac{i a^2 a f e^2 G m m s}{\xi} - \frac{a^2 a f e^2 \zeta}{m \xi^2} + \frac{a^2 e^3 \zeta s \sigma F v2}{m^2 \xi^3 x m} - \\
& \frac{i a^2 e^2 \zeta G m m s s f}{\xi} + \\
& \frac{4 a^2 e^2 \zeta G m s t f}{\xi^2 x m} + \\
& \frac{2 a^2 e^2 G m s v2}{\xi^2 x m} + \gamma \gamma 5 F D p \left(\frac{a f e}{m \xi} + \frac{e \zeta s f}{m \xi} \right) + \\
& \frac{1}{2} i e m s s f \sigma F - \frac{2 e s \sigma F t f}{\xi x m} + m s f
\end{aligned}$$

$$\text{Out[40]=} \frac{2 i e \zeta s v l \text{ at}(i l) \text{ pt}(i) \text{ pt}(i l)}{m^2 \xi x m} - \frac{2 i a^2 e^2 \zeta s v l \text{ pt}(i) (k \cdot X)}{m^2 \xi x m}$$

$$\text{Out[41]=} \frac{i 2^{-D-2} \pi^{-D} \Lambda^{4-D}}{s \text{ DdInv}(\zeta)}$$

$$\text{Out[42]=} \frac{1}{12} a^2 e^2 s (k \cdot x)^2 + a^2 e^2 s (k \cdot X)^2 + \text{pt} (x t - 2 \text{ at } e s (k \cdot X)) - \frac{x p (k \cdot x)}{2 m s} - p^2 s + \text{pt}^2 (-s)$$

Integration over

$$\int d^{D-2} p_{\perp} \dots$$

$$\begin{aligned} I_0 &= \int d^{D-2} p_{\perp} \text{Exp}[-I A p_{\perp}^2 + I (J \cdot p_{\perp})] = \\ &= \text{Exp} \left[-I \frac{\pi}{2} \frac{D-2}{2} \right] \pi^{\frac{D-2}{2}} (\det A)^{-\frac{1}{2}} \text{Exp} \left[I \frac{1}{4} J \cdot A^{-1} \cdot J \right] \end{aligned}$$

$$\begin{aligned} I_{1 i} &= \int d^{D-2} p_{\perp} p_{\perp i} \text{Exp}[-I A p_{\perp}^2 + I (J \cdot p_{\perp})] = \\ &= \frac{1}{2} (A^{-1} \cdot J)_i I_0 \end{aligned}$$

$$\begin{aligned} I_2 &= \int d^{D-2} p_{\perp} p_{\perp}^2 \text{Exp}[-I A p_{\perp}^2 + I (J \cdot p_{\perp})] = \\ &= \left[-i \frac{1}{2} \text{Tr} A^{-1} + \left(\frac{1}{2} A^{-1} \cdot J \right)^2 \right] \end{aligned}$$

$$\begin{aligned} I_{2 i j} &= \int d^{D-2} p_{\perp} p_{\perp i} p_{\perp j} \text{Exp}[-I A p_{\perp}^2 + I (J \cdot p_{\perp})] = \\ &= \left[-i \frac{1}{2} A^{-1}_{ij} + \left(\frac{1}{2} A^{-1} \cdot J \right)_i \left(\frac{1}{2} A^{-1} \cdot J \right)_j \right] I_0 \end{aligned}$$

$$\begin{aligned} I_{3 i} &= -i \frac{\partial}{\partial J_i} I_2 = \int d^{D-2} p_{\perp} p_{\perp}^2 p_{\perp i} \text{Exp}[-I A p_{\perp}^2 + I (J \cdot p_{\perp})] = \\ &= \left\{ \left[-i \frac{1}{2} \text{Tr} A^{-1} + \left(\frac{1}{2} A^{-1} \cdot J \right)^2 \right] \frac{1}{2} (A^{-1} \cdot J)_i - i \frac{1}{2} (A^{-1 \top} A^{-1} \cdot J)_i \right\} I_0 = \\ &= \left(\frac{1}{2} A^{-1} \cdot J \right)_i I_2 - i (A^{-1 \top})_{ij} I_{1 j} \end{aligned}$$

where

$$A = s,$$

$$\begin{aligned} J &= x_{\perp} - 2 e a_{\perp} s k X, \\ \det A &= s^{D-2}, \\ A^{-1} &= 1/s \end{aligned}$$

We perform integrations

Integrations changes the coefficient (Coeff) and phase

Then recollect some scalar products

$$\begin{aligned} a_{\perp}^2 &= -a^2 \\ a_{\perp} x_{\perp} &= -(ax) \\ x_{\perp}^2 &= 2 x_{-} x_{+} - x^2 \end{aligned}$$

```
In[43]:= Clear[J]
Amatr = -Coefficient[Phase5, pt^2]
J[i_] = Coefficient[Phase5, pt] /. {at -> at[i], xt -> xt[i]}
CI0 = Exp[-I Pi / 2 (D / 2 - 1)] Pi^(D / 2 - 1) / Amatr^(D / 2 - 1)
```

Out[44]= s

Out[45]= $xt(i) - 2 e s at(i) (k \cdot X)$

Out[46]= $e^{-\frac{1}{2} i \pi \left(\frac{D}{2} - 1\right)} \pi^{\frac{D}{2} - 1} s^{1 - \frac{D}{2}}$

```

In[47]:= Phase6 = Expand[Expand[((Phase5 /. {pt → 0}) + 1 / 4 J[i]^2 / Amatr)] /.
  {at[i_]^2 → -av2, at[i] * xt[i] → -ax, xt[i]^2 → 2 xm xp - xv2} /. {xm → kx / m}]
Coeff6 = Coeff5 * CI0
Expand[
  Expand[Matrix5] /. {pt[i2_]^2 * pt[i1_] → ((-I / 2 * (D - 2) / Amatr + (1 / 2 / Amatr * J[i2])^2) *
    1 / 2 / Amatr * J[i1] - I * 1 / 2 / Amatr^2 * J[i1]))} /.
    {pt[i_] * pt[i1_] → (- I / 2 / Amatr δ[i, i1] + (1 / 2 / Amatr * J[i]) * (1 / 2 / Amatr * J[i1]))} /.
    {pt[i_]^2 → (-I / 2 * (D - 2) / Amatr + (1 / 2 / Amatr * J[i])^2)} /. {pt[i_] → (1 / 2 / Amatr * J[i])}
];
%-Coefficient[%, δ[i, i1]] * δ[i, i1] + (Coefficient[%, δ[i, i1]] /. {i1 → i});
Matrix6 =
Collect[% /. {γγ5FDt[i_] * at[i_] → 0} /. {at[i_]^2 → -av2} /. {at[i_] * xt[i_] → -ax} /.
  {xt[i_]^2 → 2 xm xp - xv2} /. {at[i_] ax kX xt[i_] → -ax ax kX},
  {v1, pt[i2] * pt[i], pt[i2], pt[i_], pv2, γγ5FDm, γγ5FDp, γγ5FDt[i],
    γkγm, γkγp, γkγt[i], γaym, γayp, γayt[i]}]
(*Matrix6=Collect[Expand[Expand[(Matrix5/.{pt[i_]→0})+
  Coefficient[Matrix5,pt]*1/2/Amatr*J+
  Coefficient[Matrix5,pt^2]*(-I/2*(D-2)/Amatr+( 1/2/Amatr*J)^2)+
  Coefficient[Matrix5,pt^3]*((-I/2*(D-2)/Amatr+( 1/2/Amatr*J)^2)*1/2/Amatr*J+
    1/2/Amatr^2*J)]/.{γγ5FDt at→ 0, at^4→ av2^2 ,
  at^3→ -av2 at, at^2→-av2,at xt→ -ax, xt^2→ 2xm xp -xv2}],
  {p2,Gm,Gp,Gt,γγ5FDm,γγ5FDp,γγ5FDt,γkγm,γkγp,γkγt,γaym,γayp,γayt}*)

```

$$\text{Out[47]} = \frac{1}{12} a^2 e^2 s (k \cdot x)^2 + e(a \cdot x) (k \cdot X) - p^2 s - \frac{x^2}{4 s}$$

$$\text{Out[48]} = \frac{i 2^{-D-2} e^{-\frac{1}{2} i \pi \left(\frac{D}{2}-1\right)} \pi^{-\frac{D}{2}-1} \Lambda^{4-D} s^{-D/2}}{\text{DdInv}(\zeta)}$$

$$\begin{aligned}
\text{Out[51]= } & \text{v1} \left(\gamma \gamma m \left(\frac{i a^2 e^3 \zeta s (k \cdot X)^2}{m \xi \text{ xm}} - \frac{i e^2 \zeta (a \cdot x) (k \cdot X)}{m \xi \text{ xm}} - \frac{D e \zeta}{2 m \xi \text{ xm}} - \frac{i e \zeta (2 \text{ xm xp} - x^2)}{4 m \xi s \text{ xm}} + \frac{e \zeta}{m \xi \text{ xm}} \right) + \right. \\
& \frac{a^2 e^3 \zeta s \sigma F (k \cdot X)^2}{2 m^2 \xi \text{ xm}} + \gamma k \gamma t(i) \left(\frac{i e^2 \zeta \text{ at}(i) (a \cdot x) (k \cdot X)}{m^2 \xi \text{ xm}} - \frac{i e \zeta \text{ xt}(i) (a \cdot x)}{2 m^2 \xi s \text{ xm}} + \frac{e \zeta \text{ at}(i)}{m^2 \xi \text{ xm}} \right) - \\
& \frac{a^2 e^3 \zeta s \sigma F \text{ xm}}{8 \xi} - \frac{1}{4} a^2 e^2 \text{ Gm } m^2 s \text{ xm} - \frac{i a^2 e^2 \zeta (k \cdot X)}{m \xi} + \frac{i a^2 e^2 \zeta \text{ xm}}{2 \xi} - \\
& e \gamma \cdot a (k \cdot X) + \frac{i \gamma k \gamma p e \zeta (a \cdot x)}{2 m^2 \xi s} + \gamma a \gamma t(i) \left(\frac{i e \zeta \text{ xt}(i)}{2 m \xi s} - \frac{i e^2 \zeta \text{ at}(i) (k \cdot X)}{m \xi} \right) - \\
& e \text{ at}(i) \text{ Gt}(i) (k \cdot X) + \frac{i D \text{ Gm}}{2 \text{ xm}} + p^2 \left(-\frac{e \zeta s \sigma F}{m^2 \xi \text{ xm}} - \frac{i \gamma a \gamma m e \zeta s}{m \xi \text{ xm}} - \frac{\text{Gm } s}{\text{xm}} \right) - \frac{i \gamma a \gamma p e \zeta \text{ xm}}{2 m \xi s} - \\
& \frac{1}{2} i \gamma \gamma 5 \text{ FDp } e \text{ xm} - \frac{\text{Gm} (2 \text{ xm xp} - x^2)}{4 s \text{ xm}} - \frac{i \text{ Gm}}{\text{xm}} - \frac{\text{Gp } \text{xm}}{2 s} + \frac{\text{Gt}(i) \text{ xt}(i)}{2 s} \Big) - \\
& \frac{i a^2 \text{ af } e^3 \zeta s \sigma F}{2 m \xi^2} - \frac{i a^2 \text{ af } e^2 \text{ Gm } m s}{\xi} - \frac{a^2 \text{ af } e^2 \zeta}{m \xi^2} + \frac{a^2 e^3 \zeta s \sigma F v2}{m^2 \xi^3 \text{ xm}} - \\
& \frac{i a^2 e^2 \zeta \text{ Gm } m s \text{ sf}}{\xi} + \\
& \frac{4 a^2 e^2 \zeta \text{ Gm } s \text{ tf}}{\xi^2 \text{ xm}} + \\
& \frac{2 a^2 e^2 \text{ Gm } s v2}{\xi^2 \text{ xm}} + \\
& \gamma \gamma 5 \text{ FDp} \left(\frac{\text{af } e}{m \xi} + \frac{e \zeta \text{ sf}}{m \xi} \right) + \\
& \frac{1}{2} i e m s \text{ sf } \sigma F - \frac{2 e s \sigma F \text{ tf}}{\xi \text{ xm}} + m \text{ sf}
\end{aligned}$$

Next we substitute

$$\gamma_{\perp} a_{\perp} = -\gamma a$$

$$\gamma_{\perp} x_{\perp} = \gamma_{-} x_{+} + \gamma_{+} x_{-} - \gamma x$$

$$\gamma_{-} x_{-} = \frac{\cancel{\gamma} k}{m} x_{-}$$

$$x_{-} = kx / m$$


```

In[52]:= Matrix7 = Collect[
  DiracSimplify[
    Expand[Matrix6] /. {Gt[i_] * at[i_] → -Contract[GAD[α] * av[α]],
      Gt[i_] * xt[i_] → Gm xp + Gp xm - Contract[GAD[α] * xv[α]] ,
      (*γγ5FDt[i_] xt[i_] → γγ5FDp xm - DiracSlash[FDx, Dimension → D].GA[5],*)
      γγ5FDp → DiracSlash[FDx, Dimension → D].GA[5] / xm,
      γkγt[i_] * xt[i_] → -Pair[Momentum[x, D], Momentum[γkγ, D]] + γkγp * xm,
      γkγt[i_] * at[i_] → -Pair[Momentum[a, D], Momentum[γkγ, D]],
      γayt[i_] * xt[i_] → -Pair[Momentum[x, D], Momentum[γay, D]] + γayp * xm +
      γaym * xp, γayt[i_] * at[i_] → -Pair[Momentum[a, D], Momentum[γay, D]]} /.
      {Gm → DiracGamma[Momentum[k, D], D] / m} /. {γkγm → 0} /.
      {γaym → DiracGamma[Momentum[a, D], D].DiracGamma[Momentum[k, D], D] / m}
    /. {xm → kx / m}
  ],
  {v1, p2, Gm, Gp, Gt, γγ5FDm, γγ5FDp, γγ5FDt, γkγm, γkγp, γkγt, γaym, γayp, γayt}

```

Coeff7 = Coeff6

Phase7 = Phase6

$$\begin{aligned}
 \text{Out[52]} = & -\frac{i a f s \zeta \sigma F a^2 e^3}{2 m \xi^2} + \frac{s v 2 \zeta \sigma F a^2 e^3}{m \xi^3 (k \cdot x)} - \frac{i a f s \gamma \cdot k a^2 e^2}{\xi} - \frac{i s f \zeta \gamma \cdot k a^2 e^2}{\xi} - \frac{a f \zeta a^2 e^2}{m \xi^2} + \frac{2 s v 2 \gamma \cdot k a^2 e^2}{\xi^2 (k \cdot x)} + \\
 & \frac{4 s t f \zeta \gamma \cdot k a^2 e^2}{\xi^2 (k \cdot x)} + \frac{1}{2} i m s f \sigma F e - \frac{2 m s t f \sigma F e}{\xi (k \cdot x)} + \frac{a f (\gamma \cdot F D x) \cdot \bar{\gamma}^5 e}{\xi (k \cdot x)} + \frac{s f \zeta (\gamma \cdot F D x) \cdot \bar{\gamma}^5 e}{\xi (k \cdot x)} + \\
 & m s f + v 1 \left(\frac{s \zeta \sigma F a^2 (k \cdot X)^2 e^3}{2 m \xi (k \cdot x)} + \frac{i s \zeta (\gamma \cdot a) (\gamma \cdot k) a^2 (k \cdot X)^2 e^3}{m \xi (k \cdot x)} - \frac{s \zeta \sigma F a^2 (k \cdot x) e^3}{8 m \xi} - \right. \\
 & \frac{1}{4} s \gamma \cdot k a^2 (k \cdot x) e^2 + \frac{i \zeta a^2 (k \cdot x) e^2}{2 m \xi} - \frac{i \zeta a^2 (k \cdot X) e^2}{m \xi} + \frac{i \zeta (a \cdot \gamma a \gamma) (k \cdot X) e^2}{m \xi} - \\
 & \frac{i \zeta (\gamma \cdot a) (\gamma \cdot k) (a \cdot x) (k \cdot X) e^2}{m \xi (k \cdot x)} - \frac{i \zeta (a \cdot x) (a \cdot \gamma k \gamma) (k \cdot X) e^2}{m \xi (k \cdot x)} - \frac{1}{2} i (\gamma \cdot F D x) \cdot \bar{\gamma}^5 e - \frac{s \zeta \sigma F p^2 e}{m \xi (k \cdot x)} - \\
 & \frac{i s \zeta (\gamma \cdot a) (\gamma \cdot k) p^2 e}{m \xi (k \cdot x)} + \frac{i \zeta (\gamma \cdot a) (\gamma \cdot k) x^2 e}{4 m s \xi (k \cdot x)} - \frac{i \zeta (x \cdot \gamma a \gamma) e}{2 m s \xi} + \frac{i \zeta (a \cdot x) (x \cdot \gamma k \gamma) e}{2 m s \xi (k \cdot x)} - \\
 & \left. \frac{D \zeta (\gamma \cdot a) (\gamma \cdot k) e}{2 m \xi (k \cdot x)} + \frac{\zeta (\gamma \cdot a) (\gamma \cdot k) e}{m \xi (k \cdot x)} - \frac{\zeta (a \cdot \gamma k \gamma) e}{m \xi (k \cdot x)} - \frac{\gamma \cdot x}{2 s} - \frac{s \gamma \cdot k p^2}{k \cdot x} + \frac{\gamma \cdot k x^2}{4 s (k \cdot x)} + \frac{i D \gamma \cdot k}{2 (k \cdot x)} - \frac{i \gamma \cdot k}{k \cdot x} \right) \\
 \text{Out[53]} = & \frac{i 2^{-D-2} e^{-\frac{1}{2} i \pi \left(\frac{D-1}{2} \right)} \pi^{-\frac{D}{2}-1} \Lambda^{4-D} s^{-D/2}}{\text{DdInv}(\zeta)}
 \end{aligned}$$

$$\text{Out[54]} = \frac{1}{12} a^2 e^2 s (k \cdot x)^2 + e (a \cdot x) (k \cdot X) - p^2 s - \frac{x^2}{4 s}$$

```

In[55]:= Matrix8 = Collect[
  Expand[
    DiracSimplify[
      Matrix7 /.
        {Pair[Momentum[x, D], Momentum[γ k γ, D]] →
          DiracGamma[Momentum[k, D], D].DiracGamma[Momentum[x, D], D],
        Pair[Momentum[a, D], Momentum[γ k γ, D]] → DiracGamma[Momentum[k, D], D].
          DiracGamma[Momentum[a, D], D],
        Pair[Momentum[x, D], Momentum[γ a γ, D]] → DiracGamma[Momentum[a, D], D].
          DiracGamma[Momentum[x, D], D],
        Pair[Momentum[a, D], Momentum[γ a γ, D]] → DiracGamma[Momentum[a, D], D].
          DiracGamma[Momentum[a, D], D]}
      ] /. FieldSubstitutions /. akToF /. {av2 → -ξ^2 m^2 / e^2}
    ],
  {sf, v1, v2, tf, af, ζ, σF}]
Coeff8 = Coeff7
Phase8 = Phase7 /. {av2 → -ξ^2 m^2 / e^2}
Out[55]= v1  $\left( -\frac{1}{2} i e (\gamma \cdot \text{FDx}) \bar{\gamma}^5 + \zeta \left( \frac{i e (a \cdot x) (\gamma \cdot k) (\gamma \cdot x)}{2 m \xi s (k \cdot x)} - \frac{i e (\gamma \cdot a) (\gamma \cdot x)}{2 m \xi s} + \sigma F \right. \right.$ 

$$\left( -\frac{i D e}{4 m \xi (k \cdot x)} - \frac{e p^2 s}{2 m \xi (k \cdot x)} + \frac{1}{8} e m \xi s (k \cdot x) - \frac{e x^2}{8 m \xi s (k \cdot x)} + \frac{i e}{m \xi (k \cdot x)} \right) - \frac{1}{2} i m \xi (k \cdot x) \Bigg) +$$


$$\frac{i D \gamma \cdot k}{2 (k \cdot x)} + \frac{1}{4} m^2 \xi^2 s \gamma \cdot k (k \cdot x) - \frac{p^2 s \gamma \cdot k}{k \cdot x} + \frac{x^2 \gamma \cdot k}{4 s (k \cdot x)} - \frac{i \gamma \cdot k}{k \cdot x} - \frac{\gamma \cdot x}{2 s} \Bigg) +$$


$$\text{af} \left( \frac{e (\gamma \cdot \text{FDx}) \bar{\gamma}^5}{\xi (k \cdot x)} + \zeta \left( m + \frac{1}{2} i e m s \sigma F \right) + i m^2 \xi s \gamma \cdot k \right) +$$


$$\text{sf} \left( \zeta \left( \frac{e (\gamma \cdot \text{FDx}) \bar{\gamma}^5}{\xi (k \cdot x)} + i m^2 \xi s \gamma \cdot k \right) + \frac{1}{2} i e m s \sigma F + m \right) +$$


$$\text{tf} \left( -\frac{2 e m s \sigma F}{\xi (k \cdot x)} - \frac{4 \zeta m^2 s \gamma \cdot k}{k \cdot x} \right) +$$


$$\text{v2} \left( -\frac{e \zeta m s \sigma F}{\xi (k \cdot x)} - \frac{2 m^2 s \gamma \cdot k}{k \cdot x} \right)$$


$$\frac{i 2^{-D-2} e^{-\frac{1}{2} i \pi \left( \frac{D}{2} - 1 \right)} \pi^{-\frac{D}{2} - 1} \Lambda^{4-D} s^{-D/2}}{\text{DdInv}(\zeta)}$$

Out[56]=
Out[57]=  $-\frac{1}{12} m^2 \xi^2 s (k \cdot x)^2 + e (a \cdot x) (k \cdot X) - p^2 s - \frac{x^2}{4 s}$ 

```

```
In[58]:= (*a small test*)Expand[(((Matrix8 /. {sf → 1, v1 → -1, v2 → 0, tf → 0, af → 0, ζ → 1}) +
  (Matrix8 /. {sf → 1, v1 → -1, v2 → 0, tf → 0, af → 0, ζ → -1})))/2]
```

$$\text{Out[58]} = \frac{1}{2} i e (\gamma \cdot \text{FDx}) \cdot \overline{\gamma}^5 - \frac{i D \gamma \cdot k}{2 (k \cdot x)} + \frac{1}{2} i e m s \sigma F - \frac{1}{4} m^2 \xi^2 s \gamma \cdot k (k \cdot x) + \frac{p^2 s \gamma \cdot k}{k \cdot x} - \frac{x^2 \gamma \cdot k}{4 s (k \cdot x)} + \frac{i \gamma \cdot k}{k \cdot x} + m + \frac{\gamma \cdot x}{2 s}$$

```
In[59]:= DiracSimplify[DiracGamma[Momentum[Fx, D], D].DiracGamma[Momentum[x, D], D] /. FxToak]
DiracSimplify[
  I DiracGamma[Momentum[FDx, D], D].DiracGamma[5].DiracGamma[Momentum[x, D], D] /. FxToEps /.
  FxToak /. Gamma5toTrippleGammax]
```

$$\text{Out[59]} = (a \cdot x) (\gamma \cdot k) \cdot (\gamma \cdot x) - (k \cdot x) (\gamma \cdot a) \cdot (\gamma \cdot x)$$

$$\text{Out[60]} = (a \cdot x) (\gamma \cdot k) \cdot (\gamma \cdot x) - (k \cdot x) (\gamma \cdot a) \cdot (\gamma \cdot x) + x^2 (\gamma \cdot a) \cdot (\gamma \cdot k)$$

```

In[61]:= Matrix9 = Collect[
  Expand[
    Matrix8 /. {ax DiracGamma[Momentum[k, D], D].DiracGamma[Momentum[x, D], D] →
      I DiracGamma[Momentum[FDx, D], D].DiracGamma[5].DiracGamma[Momentum[x, D], D] +
      kx DiracGamma[Momentum[a, D], D].DiracGamma[Momentum[x, D], D] -
      (xv2 DiracGamma[Momentum[a, D], D].DiracGamma[Momentum[k, D], D] /. akToF),
      DiracGamma[Momentum[k, D], D] → - DiracGamma[Momentum[FFx, D], D] / av2 / kx} /.
      {av2 → -ξ^2 m^2 / e^2}
  ],
  {sf, v1, v2, tf, af, ζ, e σ F / m / ξ / kx,
    σ F, e^2 DiracGamma[Momentum[FFx, D], D] / m^2 / ξ^2 / kx^2}]
Coeff9 = Coeff8
Phase9 = Phase8

```

$$\begin{aligned}
\text{Out[61]} = & v1 \left(\zeta \left(-\frac{e(\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5 \cdot (\gamma \cdot x)}{2 m \xi s (k \cdot x)} + \frac{e \sigma F \left(-\frac{iD}{4} - \frac{p^2 s}{2} + \frac{x^2}{8s} + i \right)}{m \xi (k \cdot x)} + \frac{1}{8} e m \xi s \sigma F (k \cdot x) - \frac{1}{2} i m \xi (k \cdot x) \right) - \right. \\
& \left. \frac{1}{2} i e(\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5 + \frac{e^2 \gamma \cdot \text{FFx} \left(\frac{iD}{2} - p^2 s + \frac{x^2}{4s} - i \right)}{m^2 \xi^2 (k \cdot x)^2} + \frac{1}{4} e^2 s \gamma \cdot \text{FFx} - \frac{\gamma \cdot x}{2s} \right) + \\
& \text{af} \left(\frac{e(\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5}{\xi (k \cdot x)} + \frac{i e^2 s \gamma \cdot \text{FFx}}{\xi (k \cdot x)} + \zeta \left(m + \frac{1}{2} i e m s \sigma F \right) \right) + \\
& \text{sf} \left(\zeta \left(\frac{e(\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5}{\xi (k \cdot x)} + \frac{i e^2 s \gamma \cdot \text{FFx}}{\xi (k \cdot x)} \right) + \frac{1}{2} i e m s \sigma F + m \right) + \\
& \text{tf} \left(-\frac{4 e^2 \zeta s \gamma \cdot \text{FFx}}{\xi^2 (k \cdot x)^2} - \frac{2 e m s \sigma F}{\xi (k \cdot x)} \right) + v2 \left(-\frac{2 e^2 s \gamma \cdot \text{FFx}}{\xi^2 (k \cdot x)^2} - \frac{e \zeta m s \sigma F}{\xi (k \cdot x)} \right) \\
\text{Out[62]} = & \frac{i 2^{-D-2} e^{-\frac{1}{2} i \pi \left(\frac{D-1}{2} \right)} \pi^{-\frac{D-1}{2}} \Lambda^{4-D} s^{-D/2}}{\text{DdInv}(\zeta)}
\end{aligned}$$

$$\text{Out[63]} = -\frac{1}{12} m^2 \xi^2 s (k \cdot x)^2 + e(a \cdot x) (k \cdot X) - p^2 s - \frac{x^2}{4s}$$

```

In[64]:= Collect[Expand[Simplify[D[Coeff8 s f[χ[s]] Exp[I Phase8], s] / Exp[I Phase8] / Coeff8]], f[χ[s]]]
pv2Sol = pv2 /. Solve[% == dfds, pv2][[1]]

```

$$\begin{aligned}
\text{Out[64]} = & s \chi'(s) f(\chi(s)) + f(\chi(s)) \left(-\frac{1}{12} i m^2 \xi^2 s (k \cdot x)^2 - \frac{D}{2} - i p^2 s + \frac{i x^2}{4s} + 1 \right) \\
\text{Out[65]} = & \frac{-m^2 \xi^2 s^2 f(\chi(s)) (k \cdot x)^2 + 6 i (D s f(\chi(s)) + 2 dfds s - 2 s^2 \chi'(s) f(\chi(s)) - 2 s f(\chi(s))) + 3 x^2 f(\chi(s))}{12 s^2 f(\chi(s))}
\end{aligned}$$

Let

$$f(\zeta, p^2, \chi(s)) = v_1(p^2, \chi(s)) D_{\zeta}^{-1}(p^2, \chi(s))$$

```
In[66]:= Matrix9pv2 = Coefficient[Matrix9, pv2] /. {v1 -> f[\zeta, pv2, \chi[s]]} /. {\chi[s] -> \xi kx / 2 / m ^ 2 / s}
CoeffNoD = Coeff9 * DdInv[\zeta]
PhaseNopv2 = Phase9 - Coefficient[Phase9, pv2] pv2
Collect[
  Expand[
    Simplify[-I D[Matrix9pv2 * CoeffNoD * Exp[I PhaseNopv2], s] / (CoeffNoD * Exp[I PhaseNopv2])
  ]
],
  {f[\zeta, pv2, \xi kx / 2 / m ^ 2 / s]}]
Matrix91 = Coefficient[%, f[\zeta, pv2, \xi kx / 2 / m ^ 2 / s]] v1
Matrix92 = %% - Matrix91 / v1 * f[\zeta, pv2, \xi kx / 2 / m ^ 2 / s]
```

$$\text{Out[66]=} -\frac{e^2 s \gamma \cdot \text{FFx} f\left(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s}\right)}{m^2 \xi^2 (k \cdot x)^2} - \frac{e \zeta s \sigma F f\left(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s}\right)}{2 m \xi (k \cdot x)}$$

$$\text{Out[67]=} i 2^{-D-2} e^{-\frac{1}{2} i \pi \left(\frac{D}{2}-1\right)} \pi^{-\frac{D}{2}-1} \Lambda^{4-D} s^{-D/2}$$

$$\text{Out[68]=} -\frac{1}{12} m^2 \xi^2 s (k \cdot x)^2 + e(a \cdot x) (k \cdot X) - \frac{x^2}{4 s}$$

$$\text{Out[69]=} f\left(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s}\right) \left(-\frac{i D e^2 \gamma \cdot \text{FFx}}{2 m^2 \xi^2 (k \cdot x)^2} - \frac{e^2 x^2 \gamma \cdot \text{FFx}}{4 m^2 \xi^2 s (k \cdot x)^2} + \frac{i e^2 \gamma \cdot \text{FFx}}{m^2 \xi^2 (k \cdot x)^2} - \right. \\ \left. \frac{i D e \zeta \sigma F}{4 m \xi (k \cdot x)} + \frac{1}{12} e^2 s \gamma \cdot \text{FFx} + \frac{1}{24} e \zeta m \xi s \sigma F(k \cdot x) - \frac{e \zeta \sigma F x^2}{8 m \xi s (k \cdot x)} + \frac{i e \zeta \sigma F}{2 m \xi (k \cdot x)} \right) \\ - \frac{i e^2 \gamma \cdot \text{FFx} f^{(0,0,1)}\left(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s}\right)}{2 m^4 \xi s (k \cdot x)} - \frac{i e \zeta \sigma F f^{(0,0,1)}\left(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s}\right)}{4 m^3 s}$$

$$\text{Out[70]=} v1 \left(-\frac{i D e^2 \gamma \cdot \text{FFx}}{2 m^2 \xi^2 (k \cdot x)^2} - \frac{e^2 x^2 \gamma \cdot \text{FFx}}{4 m^2 \xi^2 s (k \cdot x)^2} + \frac{i e^2 \gamma \cdot \text{FFx}}{m^2 \xi^2 (k \cdot x)^2} - \right. \\ \left. \frac{i D e \zeta \sigma F}{4 m \xi (k \cdot x)} + \frac{1}{12} e^2 s \gamma \cdot \text{FFx} + \frac{1}{24} e \zeta m \xi s \sigma F(k \cdot x) - \frac{e \zeta \sigma F x^2}{8 m \xi s (k \cdot x)} + \frac{i e \zeta \sigma F}{2 m \xi (k \cdot x)} \right)$$

$$\text{Out[71]=} -\frac{i e^2 \gamma \cdot \text{FFx} f^{(0,0,1)}\left(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s}\right)}{2 m^4 \xi s (k \cdot x)} - \frac{i e \zeta \sigma F f^{(0,0,1)}\left(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s}\right)}{4 m^3 s}$$

```

In[72]:= Matrix101 = Collect[
  (Expand[Matrix9] - Coefficient[Matrix9, pv2] pv2 + Matrix91) / m,
  {sf, v1, v2, tf, af, ζ, σF, DiracGamma[Momentum[FFx, D], D],
   DiracGamma[Momentum[x, D], D], DiracGamma[Momentum[FDx, D], D]}, Simplify]
Matrix102 = Expand[Matrix92 / m]
(*Matrix101=Collect[Matrix9/m,{sf,v1,v2,tf,af,ζ,σF, DiracGamma[Momentum[FFx,D],D],
  DiracGamma[Momentum[x,D],D],DiracGamma[Momentum[FDx,D],D]},Simplify]
  Matrix102=0*)
Coeff10 = Coeff9 * m
Phase10 = Phase9
Out[72]= af  $\left( \frac{e(\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5}{m \xi(k \cdot x)} + \frac{i e^2 s \gamma \cdot \text{FFx}}{m \xi(k \cdot x)} + \zeta \left( 1 + \frac{1}{2} i e s \sigma F \right) \right) +$ 
v1  $\left( \zeta \left( -\frac{e(\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5 \cdot (\gamma \cdot x)}{2 m^2 \xi s(k \cdot x)} + \sigma F \left( \frac{1}{6} e \xi s(k \cdot x) - \frac{i(D-3)e}{2 m^2 \xi(k \cdot x)} \right) - \frac{1}{2} i \xi(k \cdot x) \right) - \right.$ 
 $\left. \frac{i e(\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5}{2 m} + \frac{e^2 s \gamma \cdot \text{FFx}}{3 m} - \frac{\gamma \cdot x}{2 m s} \right) + \text{sf} \left( \zeta \left( \frac{e(\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5}{m \xi(k \cdot x)} + \frac{i e^2 s \gamma \cdot \text{FFx}}{m \xi(k \cdot x)} \right) + \frac{1}{2} i e s \sigma F + 1 \right) +$ 
tf  $\left( -\frac{4 e^2 \zeta s \gamma \cdot \text{FFx}}{m \xi^2(k \cdot x)^2} - \frac{2 e s \sigma F}{\xi(k \cdot x)} \right) + \text{v2} \left( -\frac{2 e^2 s \gamma \cdot \text{FFx}}{m \xi^2(k \cdot x)^2} - \frac{e \zeta s \sigma F}{\xi(k \cdot x)} \right)$ 
Out[73]=  $-\frac{i e^2 \gamma \cdot \text{FFx} f^{(0,0,1)}(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s})}{2 m^5 \xi s(k \cdot x)} - \frac{i e \zeta \sigma F f^{(0,0,1)}(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s})}{4 m^4 s}$ 
Out[74]=  $\frac{i 2^{-D-2} e^{-\frac{1}{2} i \pi \left( \frac{D}{2} - 1 \right)} \pi^{-\frac{D}{2} - 1} m \Lambda^{4-D} s^{-D/2}}{\text{DdInv}(\zeta)}$ 
Out[75]=  $-\frac{1}{12} m^2 \xi^2 s(k \cdot x)^2 + e(a \cdot x)(k \cdot X) - p^2 s - \frac{x^2}{4 s}$ 
In[76]:= Matrix101 // StandardForm;

```

```
In[77]:= Matrix10 = Collect[Expand[Matrix101 + Matrix102],
  {e^2 DiracGamma[Momentum[FFx, D], D], e DiracGamma[Momentum[FDx, D], D].DiracGamma[5],
  e DiracGamma[Momentum[FDx, D], D].DiracGamma[5].DiracGamma[Momentum[x, D], D],
  σF, DiracGamma[Momentum[x, D], D], sf, v1, v2, tf, af, ζ}]
```

$$\begin{aligned}
\text{Out[77]} = & e(\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5 \left(\frac{\text{af}}{m \xi(k \cdot x)} + \frac{\zeta \text{sf}}{m \xi(k \cdot x)} - \frac{i v1}{2 m} \right) - \frac{e \zeta v1 (\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5 (\gamma \cdot x)}{2 m^2 \xi s(k \cdot x)} + \\
& e^2 \gamma \cdot \text{FFx} \left(- \frac{i f^{0,0,1}(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s})}{2 m^5 \xi s(k \cdot x)} - \frac{4 \zeta s \text{tf}}{m \xi^2(k \cdot x)^2} - \frac{2 s v2}{m \xi^2(k \cdot x)^2} + \frac{i \text{af} s}{m \xi(k \cdot x)} + \frac{i \zeta s \text{sf}}{m \xi(k \cdot x)} + \frac{s v1}{3 m} \right) + \\
& \sigma F \left(- \frac{i e \zeta f^{0,0,1}(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s})}{4 m^4 s} + \frac{1}{2} i \text{af} e \zeta s + \zeta v1 \left(- \frac{i D e}{2 m^2 \xi(k \cdot x)} + \frac{3 i e}{2 m^2 \xi(k \cdot x)} + \frac{1}{6} e \xi s(k \cdot x) \right) - \right. \\
& \left. \frac{2 e s \text{tf}}{\xi(k \cdot x)} - \frac{e \zeta s v2}{\xi(k \cdot x)} + \frac{1}{2} i e s \text{sf} \right) + \text{af} \zeta - \frac{1}{2} i \zeta \xi v1(k \cdot x) - \frac{v1 \gamma \cdot x}{2 m s} + \text{sf}
\end{aligned}$$

```

In[78]:= Matrix10ReorderPart1 =
  Collect[Coefficient[Matrix10, e^2 DiracGamma[Momentum[FFx, D], D]] * m^1 ξ^2 (kx)^2, {sf,
    v1, v2, tf, af, ζ}, Simplify] e^2 DiracGamma[Momentum[FFx, D], D] / (m^1 ξ^2 (kx)^2) +
  Collect[Coefficient[Matrix10, e DiracGamma[Momentum[FDx, D], D].DiracGamma[5]] * m ξ (kx),
    {sf, v1, v2, tf, af, ζ}, Simplify] e
  DiracGamma[Momentum[FDx, D], D].DiracGamma[5] / (m ξ (kx)) +
  Collect[Coefficient[Matrix10, e σF] * 2 m^0 ξ (kx) / ζ, {sf, v1, v2, tf, af, ζ}, Simplify]
  e σF / (2 m^0 ξ (kx) / ζ) +
  Coefficient[Matrix10, e DiracGamma[Momentum[FDx, D], D].
    DiracGamma[5].DiracGamma[Momentum[x, D], D]] e
  DiracGamma[Momentum[FDx, D], D].DiracGamma[5].DiracGamma[Momentum[x, D], D] +
  Coefficient[Matrix10, DiracGamma[Momentum[x, D], D]] * DiracGamma[Momentum[x, D], D];
Matrix10Reorder =
  Matrix10ReorderPart1 + Expand[Matrix10 - Matrix10ReorderPart1] /. {1 / ζ → ζ}
Coeff10
Phase10

```

$$\begin{aligned}
\text{Out[79]} = & \frac{e(\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5 \left(\text{af} - \frac{1}{2} i \xi v1 (k \cdot x) + \zeta \text{sf} \right)}{m \xi (k \cdot x)} - \frac{e \zeta v1 (\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5 (\gamma \cdot x)}{2 m^2 \xi s (k \cdot x)} + \\
& \frac{e \zeta \sigma F \left(-\frac{i \xi (k \cdot x) f^{(0,0,1)}(\zeta, p^2, \frac{\xi (k \cdot x)}{2 m^2 s})}{2 m^4 s} + \frac{v1 (m^2 \xi^2 s (k \cdot x)^2 - 3 i D + 9 i)}{3 m^2} + i \text{af} \xi s (k \cdot x) + i \zeta \xi s \text{sf} (k \cdot x) - 4 \zeta s \text{tf} - 2 s v2 \right)}{2 \xi (k \cdot x)} + \\
& \frac{e^2 \gamma \cdot \text{FFx} \left(-\frac{i \xi (k \cdot x) f^{(0,0,1)}(\zeta, p^2, \frac{\xi (k \cdot x)}{2 m^2 s})}{2 m^4 s} + \frac{1}{3} \xi^2 s v1 (k \cdot x)^2 + i \text{af} \xi s (k \cdot x) + i \zeta \xi s \text{sf} (k \cdot x) - 4 \zeta s \text{tf} - 2 s v2 \right)}{m \xi^2 (k \cdot x)^2} + \\
& \text{af} \zeta - \frac{1}{2} i \zeta \xi v1 (k \cdot x) - \frac{v1 \gamma \cdot x}{2 m s} + \text{sf} \\
\text{Out[80]} = & \frac{i 2^{-D-2} e^{-\frac{1}{2} i \pi \left(\frac{D}{2} - 1 \right)} \pi^{-\frac{D}{2} - 1} m \Lambda^{4-D} s^{-D/2}}{\text{DdInv}(\zeta)}
\end{aligned}$$

$$\text{Out[81]} = -\frac{1}{12} m^2 \xi^2 s (k \cdot x)^2 + e(a \cdot x) (k \cdot X) - p^2 s - \frac{x^2}{4 s}$$

Matrix10 without removing p^2 with integration by parts

$$\begin{aligned}
\text{In[82]:= } & \text{sf} + \text{af} \, \zeta - \frac{\text{v1 DiracGamma[Momentum[x, D], D]}}{2 \, \text{m} \, \text{s}} - \\
& \left(\text{e v1} \, \zeta \, \text{DiracGamma[Momentum[FDx, D], D].DiracGamma[5].DiracGamma[Momentum[x, D], D]} \right) / \\
& \left(2 \, \text{m}^2 \, \text{s} \, \xi \, \text{Pair[Momentum[k, D], Momentum[x, D]]} \right) - \\
& \frac{1}{2} \, i \, \text{v1} \, \zeta \, \xi \, \text{Pair[Momentum[k, D], Momentum[x, D]]} + \\
& \left(\text{e DiracGamma[Momentum[FDx, D], D].DiracGamma[5]} \right. \\
& \left. \left(\text{af} + \text{sf} \, \zeta - \frac{1}{2} \, i \, \text{v1} \, \xi \, \text{Pair[Momentum[k, D], Momentum[x, D]]} \right) \right) / \\
& \left(\text{m} \, \xi \, \text{Pair[Momentum[k, D], Momentum[x, D]]} \right) + \\
& \left(\text{e} \, \zeta \, \sigma \text{F} \left(-2 \, \text{s} \, \text{v2} - 4 \, \text{s} \, \text{tf} \, \zeta + i \, \text{af} \, \text{s} \, \xi \, \text{Pair[Momentum[k, D], Momentum[x, D]]} + \right. \right. \\
& \quad i \, \text{s} \, \text{sf} \, \zeta \, \xi \, \text{Pair[Momentum[k, D], Momentum[x, D]]} + \\
& \quad \frac{1}{4 \, \text{m}^2 \, \text{s}} \, \text{v1} \left(8 \, i \, \text{s} - 2 \, i \, \text{D} \, \text{s} + \text{m}^2 \, \text{s}^2 \, \xi^2 \, \text{Pair[Momentum[k, D], Momentum[x, D]]}^2 - \right. \\
& \quad \left. \left. 4 \, \text{s}^2 \, \text{Pair[Momentum[p, D], Momentum[p, D]]} + \text{Pair[Momentum[x, D], Momentum[x, D]]} \right) \right) / \\
& \left(2 \, \xi \, \text{Pair[Momentum[k, D], Momentum[x, D]]} \right) + \left(\text{e}^2 \, \text{DiracGamma[Momentum[FFx, D], D]} \right. \\
& \left(-2 \, \text{s} \, \text{v2} - 4 \, \text{s} \, \text{tf} \, \zeta + i \, \text{af} \, \text{s} \, \xi \, \text{Pair[Momentum[k, D], Momentum[x, D]]} + \right. \\
& \quad i \, \text{s} \, \text{sf} \, \zeta \, \xi \, \text{Pair[Momentum[k, D], Momentum[x, D]]} + \\
& \quad \frac{1}{4 \, \text{m}^2 \, \text{s}} \, \text{v1} \left(-4 \, i \, \text{s} + 2 \, i \, \text{D} \, \text{s} + \text{m}^2 \, \text{s}^2 \, \xi^2 \, \text{Pair[Momentum[k, D], Momentum[x, D]]}^2 - \right. \\
& \quad \left. \left. 4 \, \text{s}^2 \, \text{Pair[Momentum[p, D], Momentum[p, D]]} + \text{Pair[Momentum[x, D], Momentum[x, D]]} \right) \right) / \\
& \left(\text{m} \, \xi^2 \, \text{Pair[Momentum[k, D], Momentum[x, D]]}^2 \right)
\end{aligned}$$

$$\begin{aligned}
\text{Out[82]} = & \frac{e(\gamma \cdot \text{FDx}).\bar{\gamma}^5 \left(\text{af} - \frac{1}{2} i \xi \text{v1} (k \cdot x) + \zeta \text{sf} \right)}{m \xi (k \cdot x)} - \frac{e \zeta \text{v1} (\gamma \cdot \text{FDx}).\bar{\gamma}^5.(\gamma \cdot x)}{2 m^2 \xi s (k \cdot x)} + \\
& \frac{e^2 \gamma \cdot \text{FFx} \left(\frac{\text{v1} (m^2 \xi^2 s^2 (k \cdot x)^2 + 2 i D s - 4 p^2 s^2 - 4 i s + x^2)}{4 m^2 s} + i \text{af} \xi s (k \cdot x) + i \zeta \xi s \text{sf} (k \cdot x) - 4 \zeta s \text{tf} - 2 s \text{v2} \right)}{m \xi^2 (k \cdot x)^2} + \\
& \frac{e \zeta \sigma \text{F} \left(\frac{\text{v1} (m^2 \xi^2 s^2 (k \cdot x)^2 - 2 i D s - 4 p^2 s^2 + 8 i s + x^2)}{4 m^2 s} + i \text{af} \xi s (k \cdot x) + i \zeta \xi s \text{sf} (k \cdot x) - 4 \zeta s \text{tf} - 2 s \text{v2} \right)}{2 \xi (k \cdot x)} + \\
& \text{af} \zeta - \frac{1}{2} i \zeta \xi \text{v1} (k \cdot x) - \frac{\text{v1} \gamma \cdot x}{2 m s} + \text{sf} \\
\text{In[83]} := & (\text{*a small test*}) \text{Expand}[\left((\text{Matrix101} /. \{\text{sf} \rightarrow 1, \text{v1} \rightarrow -1, \text{v2} \rightarrow 0, \text{tf} \rightarrow 0, \text{af} \rightarrow 0, \zeta \rightarrow 1\}) + \right. \\
& \left. (\text{Matrix101} /. \{\text{sf} \rightarrow 1, \text{v1} \rightarrow -1, \text{v2} \rightarrow 0, \text{tf} \rightarrow 0, \text{af} \rightarrow 0, \zeta \rightarrow -1\}) \right) / 2] \\
\text{Out[83]} = & \frac{i e(\gamma \cdot \text{FDx}).\bar{\gamma}^5}{2 m} - \frac{e^2 s \gamma \cdot \text{FFx}}{3 m} + \frac{1}{2} i e s \sigma \text{F} + \frac{\gamma \cdot x}{2 m s} + 1
\end{aligned}$$

```

In[84]:= Matrix111 = Collect[
  DiracOrder[DiracSimplify[Expand[Matrix101] /. FxToEps /. FxToak /.
    Gamma5toTrippleGammax /. {av2 → -ξ^2 m^2 / e^2}]],
  {sf, v1, v2, tf, af, ζ}, Simplify]
Matrix112 = Expand[Matrix102 /. FxToEps /. FxToak /. Gamma5toTrippleGammax]
Coeff10
Phase10

```

$$\begin{aligned}
\text{Out[84]} = & \text{v1} \left(\frac{s(\gamma \cdot k (2 m^2 \xi^2 s(k \cdot x) - 3 e(a \cdot x)) + 3 e \gamma \cdot a(k \cdot x) - 3 e(\gamma \cdot a) \cdot (\gamma \cdot k) \cdot (\gamma \cdot x)) - 3 \gamma \cdot x}{6 m s} - \right. \\
& \frac{1}{6 m^2 \xi s(k \cdot x)} i \zeta (e(\gamma \cdot a) \cdot (\gamma \cdot k) (2 m^2 \xi^2 s^2(k \cdot x)^2 - 6 i(D-3) s - 3 x^2) + \\
& \left. 3(k \cdot x)(e(\gamma \cdot a) \cdot (\gamma \cdot x) + m^2 \xi^2 s(k \cdot x)) - 3 e(a \cdot x)(\gamma \cdot k) \cdot (\gamma \cdot x)) \right) + \\
& \text{af} \left(\zeta (e s(\gamma \cdot a) \cdot (\gamma \cdot k) + 1) + \frac{i(\gamma \cdot k(m^2 \xi^2 s(k \cdot x) - e(a \cdot x)) + e \gamma \cdot a(k \cdot x) - e(\gamma \cdot a) \cdot (\gamma \cdot k) \cdot (\gamma \cdot x))}{m \xi(k \cdot x)} \right) + \\
& \text{sf} \left(\frac{i \zeta (\gamma \cdot k(m^2 \xi^2 s(k \cdot x) - e(a \cdot x)) + e \gamma \cdot a(k \cdot x) - e(\gamma \cdot a) \cdot (\gamma \cdot k) \cdot (\gamma \cdot x))}{m \xi(k \cdot x)} + e s(\gamma \cdot a) \cdot (\gamma \cdot k) + 1 \right) + \\
& \text{tf} \left(-\frac{4 \zeta m s \gamma \cdot k}{k \cdot x} + \frac{4 i e s(\gamma \cdot a) \cdot (\gamma \cdot k)}{\xi(k \cdot x)} \right) + \\
& \text{v2} \left(-\frac{2 m s \gamma \cdot k}{k \cdot x} + \frac{2 i e \zeta s(\gamma \cdot a) \cdot (\gamma \cdot k)}{\xi(k \cdot x)} \right) \\
\text{Out[85]} = & -\frac{e \zeta (\gamma \cdot a) \cdot (\gamma \cdot k) f^{(0,0,1)}\left(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s}\right)}{2 m^4 s} + \frac{i a^2 e^2 \gamma \cdot k f^{(0,0,1)}\left(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s}\right)}{2 m^5 \xi s} \\
\text{Out[86]} = & \frac{i 2^{-D-2} e^{-\frac{1}{2} i \pi \left(\frac{D}{2}-1\right)} \pi^{-\frac{D}{2}-1} m \Lambda^{4-D} s^{-D/2}}{\text{DdInv}(\zeta)} \\
\text{Out[87]} = & -\frac{1}{12} m^2 \xi^2 s(k \cdot x)^2 + e(a \cdot x)(k \cdot X) - p^2 s - \frac{x^2}{4 s}
\end{aligned}$$

```
In[88]:= Collect[Expand[Matrix111],
```

```
{
  DiracGamma[Momentum[x, D], D],
  DiracGamma[Momentum[k, D], D],
  DiracGamma[Momentum[a, D], D],
  DiracGamma[Momentum[a, D], D].DiracGamma[Momentum[k, D], D],
  DiracGamma[Momentum[a, D], D].DiracGamma[Momentum[x, D], D],
  DiracGamma[Momentum[k, D], D].DiracGamma[Momentum[x, D], D],
  DiracGamma[Momentum[a, D], D].
  DiracGamma[Momentum[k, D], D].DiracGamma[Momentum[x, D], D],
  sf, v1, v2, tf, af, ζ}]
```

$$\begin{aligned} \text{Out[88]} = & (\gamma \cdot a) \cdot (\gamma \cdot k) \left(\text{af} \, e \, \zeta \, s + \zeta \, v1 \left(-\frac{D e}{m^2 \xi(k \cdot x)} + \frac{i e x^2}{2 m^2 \xi s(k \cdot x)} + \frac{3 e}{m^2 \xi(k \cdot x)} - \frac{1}{3} i e \xi s(k \cdot x) \right) + \right. \\ & \left. \frac{4 i e s t f}{\xi(k \cdot x)} + \frac{2 i e \zeta s v2}{\xi(k \cdot x)} + e s s f \right) + \gamma \cdot k \left(\text{af} \left(i m \xi s - \frac{i e(a \cdot x)}{m \xi(k \cdot x)} \right) + \right. \\ & \left. \zeta s f \left(i m \xi s - \frac{i e(a \cdot x)}{m \xi(k \cdot x)} \right) + v1 \left(\frac{1}{3} m \xi^2 s(k \cdot x) - \frac{e(a \cdot x)}{2 m} \right) - \frac{4 \zeta m s t f}{k \cdot x} - \frac{2 m s v2}{k \cdot x} \right) + \\ & (\gamma \cdot a) \cdot (\gamma \cdot k) \cdot (\gamma \cdot x) \left(-\frac{i \text{af} e}{m \xi(k \cdot x)} - \frac{i e \zeta s f}{m \xi(k \cdot x)} - \frac{e v1}{2 m} \right) + \gamma \cdot a \left(\frac{i \text{af} e}{m \xi} + \frac{e v1(k \cdot x)}{2 m} + \frac{i e \zeta s f}{m \xi} \right) + \\ & \frac{i e \zeta v1(a \cdot x)(\gamma \cdot k) \cdot (\gamma \cdot x)}{2 m^2 \xi s(k \cdot x)} - \frac{i e \zeta v1(\gamma \cdot a) \cdot (\gamma \cdot x)}{2 m^2 \xi s} + \\ & \text{af} \, \zeta - \frac{1}{2} i \zeta \xi \zeta v1(k \cdot x) - \frac{v1 \gamma \cdot x}{2 m s} + s f \end{aligned}$$

Final result for the electron propagator in a CCF

$$\begin{aligned} S^C(x_2, x_1) = & \Lambda^{4-D} \int \frac{d^D p}{(2\pi)^D} E_p(x_2) \sum_{\zeta=\pm} \frac{i}{2} \left[m s(p^2, x_p) - (\gamma p) v_1(p^2, x_p) - \frac{e^2 (\gamma F^2 p)}{m^4} v_2(p^2, x_p) - \right. \\ & \left. \frac{e \sigma F}{m} t(p^2, x_p) + \frac{e (\gamma F^* p) \gamma^5}{m^2} a_s(p^2, x_p) \right] \frac{1 + \zeta \gamma \epsilon^{(2)} \gamma^5}{D_\zeta(p^2, x_p)} E_p^{\text{bar}}(x_1) = \\ & = e^{-i \frac{\pi}{2} \left(\frac{D}{2} - 2 \right)} \frac{m \Lambda^{4-D}}{2^{D+2} \pi^{D/2+1}} e^{i \eta} \sum_{\zeta=\pm} \int_0^\infty \frac{ds}{s^{D/2}} \left(\int_0^\infty \frac{dp^2}{D_\zeta(p^2, x_p(s))} e^{-i s p^2 - i \frac{x^2}{4s} + i \frac{x}{12} e^2 (F x)^2 x} \right. \\ & \left. \left\{ s(p^2, x_p(s)) \left[1 + \frac{1}{2} i s e \sigma F + \zeta \left(\frac{i e^2 s (\gamma F F x)}{m \xi(kx)} + \frac{e F_{\alpha\beta}^* x^\beta \gamma^\alpha \gamma^5}{m \xi(kx)} \right) \right] \right\} \right) \end{aligned}$$

$$\begin{aligned}
& + v_1(p^2, \chi_p(s)) \left[-\frac{(\gamma x)}{2 m s} + \frac{e^2 s (\gamma F F x)}{3 m} - \frac{i e F_{\alpha\beta}^* x^\beta \gamma^\alpha \gamma^5}{2 m} \right. \\
& \quad + \zeta \left(-\frac{1}{2} i \xi(kx) - \frac{e \sigma F}{2 m^2 \xi(kx)} \right. \\
& \quad \quad \left. \left(i(D-3) - \frac{1}{3} m^2 s \xi^2(kx)^2 \right) - \frac{e (\gamma F^* x) \gamma^5 (\gamma x)}{2 m^2 \xi s(kx)} \right] \\
& + v_2(p^2, \chi_p(s)) \left[-\frac{2 s e^2 (\gamma F F x)}{m \xi^2(kx)^2} - \zeta \frac{s e \sigma F}{\xi(kx)} \right] \\
& + t(p^2, \chi_p(s)) \left[-\frac{2 s e \sigma F}{\xi(kx)} - \zeta \frac{4 s e^2 (\gamma F F x)}{m \xi^2(kx)^2} \right] \\
& + a(p^2, \chi_p(s)) \left[\right. \\
& \quad \left. \frac{e F_{\alpha\beta}^* x^\beta \gamma^\alpha \gamma^5}{m \xi(kx)} + \frac{i s e^2 (\gamma F F x)}{m \xi(kx)} + \zeta \left(1 + \frac{1}{2} i s e \sigma F \right) \right] \Big\} \\
& + \int_0^\infty d p^2 \frac{\partial}{\partial \chi_p} \left(\frac{v_1(p^2, \chi_p)}{D_\zeta(p^2, \chi_p)} \right) e^{-i s p^2 - i \frac{x^2}{4 s} + i \frac{x^2}{12} e^2 (F x)^2} \left[-\frac{i e^2 (\gamma F F x)}{2 m^5 s \xi(kx)} - \zeta \frac{i e \sigma F}{4 m^4 s} \right] \\
& = e^{-i \frac{x^2}{2} \left(\frac{D}{2} - 2 \right)} \frac{m \Lambda^{4-D}}{2^{D+2} \pi^{D/2+1}} e^{i \eta} \sum_{\zeta=\pm} \int_0^\infty \frac{d s}{s^{D/2}} \int_0^\infty \frac{d p^2}{D_\zeta(p^2, \chi_p(s))} e^{-i s p^2 - i \frac{x^2}{4 s} + i \frac{x^2}{12} e^2 (F x)^2} \Big\{ \\
& \quad s(p) + \zeta \left[a(p) - \frac{i}{2} \xi(kx) v_1(p) \right] \\
& \quad - \frac{(\gamma x)}{2 m s} v_1(p) \\
& \quad + \frac{e^2 (\gamma F^2 x)}{m \xi^2(kx)^2} \left[i \zeta \xi s s(p) + \left(\frac{1}{4} \xi^2 s(kx)^2 - s \frac{p^2}{m^2} + \frac{x^2}{4 m^2 s} + \frac{i(D-2)}{2 m^2} \right) \right. \\
& \quad \quad \left. v_1(p) - 2 s v_2(p) - 4 \zeta s t(p) + i \xi s(kx) a(p) \right] \\
& \quad + \zeta \frac{e \sigma F}{\xi(kx)} \left[i \zeta \xi s s(p) + \left(\frac{1}{4} \xi^2 s(kx)^2 - s \frac{p^2}{m^2} + \frac{x^2}{4 m^2 s} - \frac{i(D-4)}{2 m^2} \right) v_1(p) - \right. \\
& \quad \quad \left. 2 s v_2(p) - 4 \zeta s t(p) + i \xi s(kx) a(p) \right] \\
& \quad + \frac{e (\gamma F^* x) \gamma^5}{m \xi(kx)} \left[\zeta s(p) - \frac{i}{2} \xi(kx) v_1(p) + a(p) \right] \\
& \quad \left. - \zeta \frac{e (\gamma F^* x) \gamma^5 (\gamma x)}{2 m^2 s \xi(kx)} v_1(p) \right\}
\end{aligned}$$

$$\begin{aligned}
&= e^{-i \frac{\pi}{2} \left(\frac{n}{2} - 2 \right)} \frac{m \Lambda^{4-D}}{2^{D+2} \pi^{D/2+1}} e^{i \eta} \sum_{\zeta=\pm} \int_0^\infty \frac{ds}{s^{D/2}} \int_0^\infty \frac{dp^2}{D_\zeta(p^2, \chi_p(s))} e^{-i s p^2 - i \frac{s^2}{4s} + i \frac{s}{12} e^2 (F_x)^2} \left\{ \right. \\
&\quad s(p) + \zeta \left[a(p) - \frac{i}{2} \xi(kx) v_1(p) \right] \\
&\quad - \frac{(\gamma x)}{2 m s} v_1(p) \\
&\quad + \frac{e^2 (\gamma F^2 x)}{m^3 \xi^2(kx)^2} m^2 \left[i \zeta \xi(kx) s s(p) + \frac{1}{3} s \xi^2(kx)^2 v_1(p) - 2 s v_2(p) - 4 \zeta s t \right. \\
&\quad \left. (p) + i \xi s(kx) a(p) - \frac{i \xi(kx)}{2 m^4 s} D_\zeta(p^2, \chi_p(s)) \frac{\partial}{\partial \chi_p} \left(\frac{v_1(p^2, \chi_p)}{D_\zeta(p^2, \chi_p)} \right) \right] \\
&\quad + \zeta \frac{e \sigma F}{m^2 \xi(kx)} m^2 \left[i \zeta \xi(kx) s s(p) + \left(\frac{1}{3} s \xi^2(kx)^2 - \frac{i(D-3)}{m^2} \right) v_1(p) - \right. \\
&\quad \left. 2 s v_2(p) - 4 \zeta s t(p) + i \xi s(kx) a(p) - \right. \\
&\quad \left. \frac{i \xi(kx)}{2 m^4 s} D_\zeta(p^2, \chi_p(s)) \frac{\partial}{\partial \chi_p} \left(\frac{v_1(p^2, \chi_p)}{D_\zeta(p^2, \chi_p)} \right) \right] \\
&\quad + \frac{e (\gamma F^* x) \gamma^5}{m \xi(kx)} \left[\zeta s(p) - \frac{i}{2} \xi(kx) v_1(p) + a(p) \right] \\
&\quad \left. - \zeta \frac{e (\gamma F^* x) \gamma^5 (\gamma x)}{2 m^2 s \xi(kx)} v_1(p) \right\} \\
&= e^{-i \frac{\pi}{2} \left(\frac{n}{2} - 1 \right)} \frac{\Lambda^{4-D}}{2^D \pi^{D/2}} m^{D-1} e^{i \eta} \\
&\quad \int_0^\infty \frac{ds}{s^{D/2}} \left[1 + \frac{m(\gamma x)}{2 s} + \frac{e(\gamma a)(kx)}{2 m} - \frac{e(\gamma k)(ax)}{2 m} + \frac{e(\gamma x)(\gamma a)(\gamma k)}{2 m} + \right. \\
&\quad \left. \frac{e s(\gamma a)(\gamma k)}{m^2} + \frac{e^2 a^2 s(\gamma k)(kx)}{3 m^3} \right] e^{-i s - i \frac{s^2}{4s} + i \frac{s}{12} \frac{e^2}{m^2} (F_x)^2} = \\
&= e^{-i \frac{\pi}{2} \left(\frac{n}{2} - 1 \right)} \frac{\Lambda^{4-D}}{2^{D+1} \pi^{D/2}} m^D e^{i \eta} \int_0^\infty \frac{ds}{s^{D/2+1}} \left[\frac{2 s}{m} + \gamma^\alpha \left(g_{\alpha\beta} - \frac{e s}{m^2} F_{\alpha\beta} + \frac{e^2 s^2}{3 m^4} F_{\alpha\lambda} F^\lambda_\beta \right) x^\beta \right] \\
&\quad \left(1 + \frac{i e s}{2} \sigma^{\alpha\beta} F_{\alpha\beta} \right) e^{-i s - i \frac{s^2}{4s} + i \frac{s}{12} \frac{e^2}{m^2} (F_x)^2} \\
&\eta = e(ax)(k, (x_1 + x_2)/2), \\
&x = x_2 - x_1, \\
&e > 0, \\
&\sigma^{\alpha\beta} = \frac{i}{2} (\gamma^\alpha \gamma^\beta - \gamma^\beta \gamma^\alpha),
\end{aligned}$$

$$\gamma^5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3,$$

$$e^{-i \frac{\pi}{2} \binom{p}{2} - 1} \frac{\Lambda^{4-D}}{2^D \pi^{D/2}} m^{D-1} \rightarrow \frac{(-i) m^3}{16 \pi^2}, \quad D \rightarrow 4$$