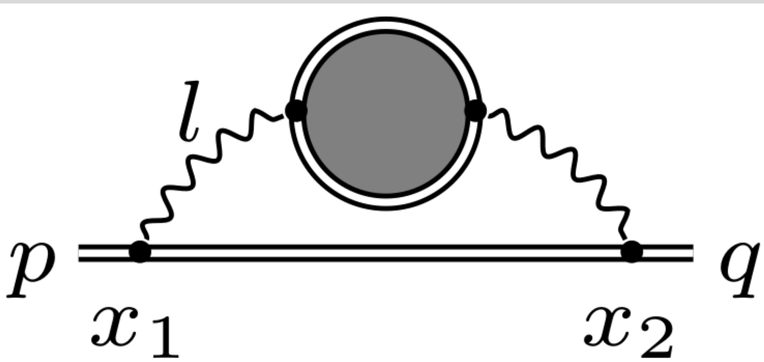
This is a part of SFQED-Loops script collection
developed for calculating loop processes in
Strong-Field Quantum Electrodynamics.
The scripts are available on https://github.com/ArsenyMironov/SFQED-Loops

If you use this script in your research, please, consider
citing our papers:
- A. A. Mironov, S. Meuren, and A. M. Fedotov, PRD 102, 053005 (2020),
  https://doi.org/10.1103/PhysRevD.102.053005
- A. A. Mironov, A. M. Fedotov, arXiv:2109.00634 (2021)

If you have any questions, please, don’t hesitate to contact:
mironov.hep@gmail.com




```
NotebookEvaluate[NotebookOpen[NotebookDirectory[] <> "definitions.nb"]]
```

FeynCalc 9.2.0. For help, use the documentation center, check out the wiki or write to the mailing list.

See also the supplied examples. If you use FeynCalc in your research, please cite

- V. Shtabovenko, R. Mertig and F. Orellana,
  Comput. Phys. Commun., 207C, 432–444, 2016, arXiv:1601.01167
- R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun., 64, 345–359, 1991.

Mass operator $(e > 0)$

$$-iM(q,p) =$$

$$(ie)^2 \Lambda^{2D-8} \int d^D x_1 \, d^D x_2 \, \overline{E}_q(x_2)\, \gamma^\mu S^c(x_2, x_1)\, \gamma^\nu E_p(x_1)\, D^c{}_{\mu\nu}(x_1 - x_2);$$

x_1^μ, x_2^μ – position of the left and right vertices of the diagram;
p^μ, q^μ – initial and final electron momenta;
S^c, $D^c{}_{\mu\nu}$ – electron and photon casual propagators;

The Ritus E_p – function

$$E_p(x_1) = \left[1 - \frac{e\,(\gamma k)\,(\gamma a)}{2\,(kp)}\,(kx_2)\right]$$

$$\text{Exp}\left[-i\,(p\,x_1) + i\,\frac{e\,(a\,p)}{2\,(k\,p)}\,(k\,x_1)^2 + i\,\frac{e^2\,a^2}{6\,(k\,p)}\,(k\,x_1)^3\right];$$

$$\overline{E}_q(x_2) = \gamma^0 E_q(x_2)\, \gamma^0;$$

Electron propagator in a CCF in D dimensions

$$S^c(x_2, x_1) = e^{i\eta} S^c_{\text{diag}}(x_2 - x_1) =$$

$$= e^{i\eta}\, e^{-i\frac{\pi}{2}\left(\frac{D}{2}-1\right)} \frac{\Lambda^{4-D}}{2^D \pi^{D/2}}\, m$$

$$\int_0^\infty \frac{ds}{s^{D/2}} \left[1 + \frac{(\gamma x)}{2\,m\,s} - \frac{e^2\,s\,(\gamma\, FFx)}{3\,m} + \frac{i}{2}\,e\,s\left(\sigma^{\alpha\beta} F_{\alpha\beta}\right) + \frac{i\,e\,F_{\alpha\beta}{}^*\, x^\beta \gamma^\alpha \gamma^5}{2\,m}\right]$$

$$e^{-is - i\frac{x^2}{4s} + i\frac{s}{12} e^2 (Fx)^2}$$

$$= e^{i\eta}\, e^{-i\frac{\pi}{2}\left(\frac{D}{2}-1\right)} \frac{\Lambda^{4-D}}{2^D \pi^{D/2}}\, m \int_0^\infty \frac{ds}{s^{D/2}} \left[1 + \frac{(\gamma x)}{2\,m\,s} + \frac{e\,(\gamma a)\,(kx)}{2\,m} - \frac{e\,(\gamma k)\,(ax)}{2\,m} + \right.$$

$$\left. \frac{e\,(\gamma x)\,(\gamma a)\,(\gamma k)}{2\,m} + e\,s\,(\gamma a)\,(\gamma k) + \frac{e^2\,a^2\,s\,(\gamma k)\,(kx)}{3\,m}\right] e^{-is - i\frac{x^2}{4s} + i\frac{s}{12} e^2 (Fx)^2}$$

$\eta = e\,(ax)\,(k,\,(x_1 + x_2)/2)$,
$x = x_2 - x_1$,
$e > 0$,
$[\Lambda] = m$ – mass scale,

$$\sigma^{\alpha\beta} = \frac{i}{2}\left(\gamma^\alpha \gamma^\beta - \gamma^\beta \gamma^\alpha\right),$$

$$\gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3,$$

$$F_{\alpha\beta} = k_\alpha a_\beta - k_\beta a_\alpha,$$

$$F_{\alpha\beta} F^\beta{}_\lambda = -a^2 k_\alpha k_\lambda$$

$$e^{-i\frac{\pi}{2}\left(\frac{D}{2}-1\right)} \frac{\Lambda^{4-D}}{2^D \pi^{D/2}} m^{D-1} \to \frac{(-i)\, m^3}{16\,\pi^2},\quad D \to 4$$

Exact photon propagator in momentum representation

$$D^c{}_{\mu\nu}(l) =$$
$$D_0\left(l^2\right) g_{\mu\nu} + D_1\left(l^2, \chi_l\right) \epsilon_\mu^{(1)}(l)\, \epsilon_\nu^{(1)}(l) + D_2\left(l^2, \chi_l\right) \epsilon_\mu^{(2)}(l)\, \epsilon_\nu^{(2)}(l);$$

l^μ – the photon 4 – momentum;

$$\chi_l = \frac{e}{m^3}\sqrt{-\left(F_{\mu\nu} l^\nu\right)^2};$$

$$\epsilon_\mu^{(1)}(l) = \frac{eF_{\mu\nu} l^\nu}{m^3 \chi_l};$$

$$\epsilon_\mu^{(2)}(l) = \frac{eF^*{}_{\mu\nu} l^\nu}{m^3 \chi_l};\qquad F^{*\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\delta\lambda} F_{\delta\lambda};$$

$$\left(\epsilon^{(i)}(l)\right)^2 = -1;$$

$$D_0\left(l^2\right) = \frac{-i}{l^2 + i0},\quad D_{1,2}\left(l^2, \chi_l\right) = \frac{i\Pi_{1,2}}{\left(l^2 + i0\right)\left(l^2 - \Pi_{1,2}\right)};$$

$\Pi_{1,2} = \Pi_{1,2}\left(l^2, \chi_l\right)$ – polarization operator eigenfunctions;

Exact photon propagator in coordinate representation

$$D^c{}_{\mu\nu}(x) = \frac{\Lambda^{4-D}}{(2\pi)^D}\int d^D l\, D^c{}_{\mu\nu}(l)\, e^{-ilx} =$$

$$= \mathrm{Exp}\left[-i\frac{\pi}{2}\left(\frac{D}{2}-2\right)\right] \frac{1}{2^D \pi^{D/2}} \frac{\Lambda^{4-D}}{m^{2-D}}$$
$$\int_0^\infty \frac{dt}{t^{D/2}} e^{-i\frac{m^2 x^2}{4t}} \left\{g_{\mu\nu} J_0(t) + \left(-2\, i\, t\frac{k_\mu k_\nu}{m^2 \phi^2} + \frac{e^2\, Fx_\mu\, Fx_\nu}{m^2 \xi^2 \phi^2}\right) J_1(t, \chi_l)\right.$$
$$\left. + \left(-2\, i\, t\frac{k_\mu k_\nu}{m^2 \phi^2} + \frac{e^2\, FDx_\mu\, FDx_\nu}{m^2 \xi^2 \phi^2}\right) J_2(t, \chi_l)\right\};$$

$$J_k(t, \chi_l) = -i\int_0^\infty dl^2\, D_k\left(l^2, \chi_l\right) e^{-il^2 t};$$

$$\chi_l = \xi\, kl \big/ m^2 = \xi\,\phi / 2\, m^2\, t;$$

Preliminaries

Let us define momenta and coordinate variables

$x = x_2 - x_1;$

$X = \frac{1}{2}(x_1 + x_2);$

$\phi = kx;$

$\Phi = kX;$

$\phi 1 = kx_1;$

$\phi 2 = kx_2;$

The functions NewMomentum and NewCoordinate are predefined in the file definitions.nb

They provide the corresponding 4 - vector along with all possible contractions with the field tensor $F_{\mu\nu}$ and 4 - vectors a_μ, k_μ (e.g. Fxv[μ] == $(Fx)_\mu = F_{\mu\nu} x^\nu$, see more details in the file definitions.nb)

```
NewMomentum["p"]
NewMomentum["q"]
NewMomentum["l"]
NewCoordinate["x1"]
NewCoordinate["x2"]
NewCoordinate["x"]
NewCoordinate["X"]
ScalarProduct[k, x] = ϕ;
ScalarProduct[k, X] = Φ;
ScalarProduct[k, x1] = ϕ1;
ScalarProduct[k, x2] = ϕ2;
ScalarProduct[Fx, Fx] = -ξ^2 ϕ^2 * m^2 / e^2;
ScalarProduct[FDx, FDx] = -ξ^2 ϕ^2 * m^2 / e^2;
ScalarProduct[x, FFx] = ξ^2 ϕ^2 * m^2 / e^2;
```

$$\left\{p^{\alpha},\ p^2,\ k\cdot p,\ \mathrm{Fp}^{\alpha},\ \mathrm{FFp}^{\alpha},\ \mathrm{FDp}^{\alpha},\ a\cdot p,\ 0,\ 0,\ 0,\ -a^2\,(k\cdot p),\ 0,\ 0,\ -\frac{m^6\,\chi\mathrm{p}^2}{e^2},\ -\frac{m^6\,\chi\mathrm{p}^2}{e^2},\ \frac{m^6\,\chi\mathrm{p}^2}{e^2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}$$

$$\left\{q^{\alpha},\ q^2,\ k\cdot q,\ \mathrm{Fq}^{\alpha},\ \mathrm{FFq}^{\alpha},\ \mathrm{FDq}^{\alpha},\ a\cdot q,\ 0,\ 0,\ 0,\ -a^2\,(k\cdot q),\ 0,\ 0,\ -\frac{m^6\,\chi\mathrm{q}^2}{e^2},\ -\frac{m^6\,\chi\mathrm{q}^2}{e^2},\ \frac{m^6\,\chi\mathrm{q}^2}{e^2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}$$

$$\left\{l^{\alpha},\ l^2,\ k\cdot l,\ \mathrm{Fl}^{\alpha},\ \mathrm{FFl}^{\alpha},\ \mathrm{FDl}^{\alpha},\ a\cdot l,\ 0,\ 0,\ 0,\ -a^2\,(k\cdot l),\ 0,\ 0,\ -\frac{m^6\,\chi\mathrm{l}^2}{e^2},\ -\frac{m^6\,\chi\mathrm{l}^2}{e^2},\ \frac{m^6\,\chi\mathrm{l}^2}{e^2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}$$

$$\left\{\mathrm{x1}^{\alpha},\ \mathrm{x1}^2,\ k\cdot\mathrm{x1},\ a\cdot\mathrm{x1},\ \mathrm{Fx1}^{\alpha},\ \mathrm{FFx1}^{\alpha},\ \mathrm{FDx1}^{\alpha},\ k\cdot\mathrm{x1},\ 0,\ 0,\ 0,\ -a^2\,(k\cdot\mathrm{x1}),\right.$$
$$\left.0,\ 0,\ -\frac{m^2\,\xi^2\,(k\cdot\mathrm{x1})^2}{e^2},\ -\frac{m^2\,\xi^2\,(k\cdot\mathrm{x1})^2}{e^2},\ \frac{m^2\,\xi^2\,(k\cdot\mathrm{x1})^2}{e^2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}$$

$$\left\{\mathrm{x2}^{\alpha},\ \mathrm{x2}^2,\ k\cdot\mathrm{x2},\ a\cdot\mathrm{x2},\ \mathrm{Fx2}^{\alpha},\ \mathrm{FFx2}^{\alpha},\ \mathrm{FDx2}^{\alpha},\ k\cdot\mathrm{x2},\ 0,\ 0,\ 0,\ -a^2\,(k\cdot\mathrm{x2}),\right.$$
$$\left.0,\ 0,\ -\frac{m^2\,\xi^2\,(k\cdot\mathrm{x2})^2}{e^2},\ -\frac{m^2\,\xi^2\,(k\cdot\mathrm{x2})^2}{e^2},\ \frac{m^2\,\xi^2\,(k\cdot\mathrm{x2})^2}{e^2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}$$

$$\left\{x^{\alpha},\ x^2,\ k\cdot x,\ a\cdot x,\ \mathrm{Fx}^{\alpha},\ \mathrm{FFx}^{\alpha},\ \mathrm{FDx}^{\alpha},\ k\cdot x,\ 0,\ 0,\ 0,\ -a^2\,(k\cdot x),\right.$$
$$\left.0,\ 0,\ -\frac{m^2\,\xi^2\,(k\cdot x)^2}{e^2},\ -\frac{m^2\,\xi^2\,(k\cdot x)^2}{e^2},\ \frac{m^2\,\xi^2\,(k\cdot x)^2}{e^2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}$$

$$\left\{X^{\alpha},\ X^2,\ k\cdot X,\ a\cdot X,\ \mathrm{FX}^{\alpha},\ \mathrm{FFX}^{\alpha},\ \mathrm{FDX}^{\alpha},\ k\cdot X,\ 0,\ 0,\ 0,\ -a^2\,(k\cdot X),\right.$$
$$\left.0,\ 0,\ -\frac{m^2\,\xi^2\,(k\cdot X)^2}{e^2},\ -\frac{m^2\,\xi^2\,(k\cdot X)^2}{e^2},\ \frac{m^2\,\xi^2\,(k\cdot X)^2}{e^2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}$$

$\mathrm{E_p}$ – functions

The functions Ep and EpC are predefined in the file definitions.nb

They provide a list with two data fields : the preexponent and the phase of the $\mathrm{E_p}$ – function (or the adjoint $\overline{\mathrm{E}}_\mathrm{p}$ – function)

```
Epx1 = Ep[x1, p]
EqBarx2 = EpC[x2, q]
```

$$\left\{1-\frac{e\,\phi1\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)},\ \frac{a^2\,e^2\,\phi1^3}{6\,(k\cdot p)}+\frac{e\,\phi1^2\,(a\cdot p)}{2\,(k\cdot p)}-p\cdot\mathrm{x1}\right\}$$

$$\left\{1-\frac{e\,\phi2\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot q)},\ -\frac{a^2\,e^2\,\phi2^3}{6\,(k\cdot q)}-\frac{e\,\phi2^2\,(a\cdot q)}{2\,(k\cdot q)}+q\cdot\mathrm{x2}\right\}$$

Propagators in coordinate representation and proper times

s – electron proper time of dimension m^{-2};

t – photon proper time of dimension m^{-2}

The functions DiracElectronPropagatorXRepr and

PhotonPropagatorExactXRepr are predefined in the file definitions.nb

They provide a list with three data fields : a γ -matrix (for S^c) or tensor (for D^c) preexponential, a scalar prefactor and the phase of the exponent.
It is implied that there is an integraion over the proper time from 0 to ∞

```
Sc = DiracElectronPropagatorXRepr[x, X, m^2 s];
Sc[[2]] = Simplify[Sc[[2]] * m^2, Assumptions → {m > 0}];
Sc
```

$$\left\{-\frac{e\,(a\cdot x)\,\gamma\cdot k}{2\,m}+\frac{e\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{2\,m}+e\,s\,(\gamma\cdot a).(\gamma\cdot k)+\frac{e\,\phi\,\gamma\cdot a}{2\,m}-\frac{1}{3}\,m\,\xi^2\,s\,\phi\,\gamma\cdot k+\frac{\gamma\cdot x}{2\,m\,s}+1,\right.$$
$$\left. i\,2^{-D}\,e^{-\frac{1}{4}\,i\,\pi\,D}\,\pi^{-D/2}\,m\,\Lambda^{4-D}\,s^{-D/2},\ e\,\Phi\,(a\cdot x)-\frac{1}{12}\,m^2\,\xi^2\,s\,\phi^2-m^2\,s-\frac{x^2}{4\,s}\right\}$$

```
Dc = PhotonPropagatorExactXRepr[x, m^2 t, μ, ν];
Dc[[2]] = Simplify[Dc[[2]] * m^2, Assumptions → {m > 0}];
Dc
```

$$\left\{\mathrm{J2}\!\left(t,\frac{\xi\,\phi}{2\,m^2\,t}\right)\left(\frac{1}{m^2\,\xi^2\,\phi^2}\,e^2\left(-k^\mu\,k^\nu\,(a\cdot x)^2+a^2\,\phi^2\,g^{\mu\nu}-a^2\,\phi\,k^\nu\,x^\mu-a^2\,\phi\,k^\mu\,x^\nu+\right.\right.\right.$$
$$\left.\left.\phi\,a^\nu\,k^\mu\,(a\cdot x)+\phi\,a^\mu\,k^\nu\,(a\cdot x)+a^2\,x^2\,k^\mu\,k^\nu+\phi^2\,(-a^\mu)\,a^\nu\right)-\frac{2\,i\,t\,k^\mu\,k^\nu}{\phi^2}\right)+$$
$$\mathrm{J1}\!\left(t,\frac{\xi\,\phi}{2\,m^2\,t}\right)\left(\frac{1}{m^2\,\xi^2\,\phi^2}\,e^2\left(k^\mu\,k^\nu\,(a\cdot x)^2-\phi\,a^\nu\,k^\mu\,(a\cdot x)-\phi\,a^\mu\,k^\nu\,(a\cdot x)+\phi^2\,a^\mu\,a^\nu\right)-\frac{2\,i\,t\,k^\mu\,k^\nu}{\phi^2}\right)+$$
$$g^{\mu\nu}\,\mathrm{J0}\!\left(t,\frac{\xi\,\phi}{2\,m^2\,t}\right),$$
$$-2^{-D-1}\,e^{-\frac{1}{4}\,i\,\pi\,D}\,\pi^{-\frac{D}{2}-1}\,\Lambda^{4-D}\,t^{-D/2},$$
$$\left.-\frac{x^2}{4\,t}\right\}$$

Calculation of M

The mass operator

M (q, p) = i $\int$ $d^D x_1\, d^D x_2$ ...;

We will write the integrand in the following form

Coeff * Tr[Matrix] * Exp[i Phase],

where
Coeff – is a dimensional coefficient in

front of the expression (also depends on s and t),
Matrix - the γ - martix factor,
Phase - the total phase of the exponential

We also introduce the notation

$$\text{Jk} == \text{Jk}\left[\text{t}, \frac{\xi\,\phi}{2\,\text{m}^2\,\text{t}}\right]$$

```
Coeff = I (I e)^2 Λ^(2 D-8) Sc[[2]] Dc[[2]]
Phase = EqBarx2[[2]] + Sc[[3]] + Dc[[3]] + Epx1[[2]]
Matrix = (EqBarx2[[1]].GAD[μ].Sc[[1]].GAD[ν].Epx1[[1]] Dc[[1]]) /.
  {J0[t, ξ ϕ/2/t/m^2] → J0, J1[t, ξ ϕ/2/t/m^2] → J1, J2[t, ξ ϕ/2/t/m^2] → J2}
```

$$-2^{-2D-1}\, e^{-\frac{1}{2} i \pi D}\, \pi^{-D-1}\, e^2\, m\, s^{-D/2}\, t^{-D/2}$$

$$\frac{a^2 e^2 \phi 1^3}{6\,(k\cdot p)} - \frac{a^2 e^2 \phi 2^3}{6\,(k\cdot q)} + \frac{e\,\phi 1^2\,(a\cdot p)}{2\,(k\cdot p)} - \frac{e\,\phi 2^2\,(a\cdot q)}{2\,(k\cdot q)} + e\,\Phi\,(a\cdot x) - \frac{1}{12} m^2 \xi^2 s\,\phi^2 - m^2 s - p\cdot \text{x1} + q\cdot \text{x2} - \frac{x^2}{4\,s} - \frac{x^2}{4\,t}$$

$$\Bigg(\text{J2}\left(\frac{1}{m^2\,\xi^2\,\phi^2} e^2 \left(-k^\mu k^\nu (a\cdot x)^2 + a^2 \phi^2 g^{\mu\nu} - a^2 \phi\, k^\nu x^\mu - a^2 \phi\, k^\mu x^\nu + \phi\, a^\nu k^\mu (a\cdot x) + \phi\, a^\mu k^\nu (a\cdot x) + a^2 x^2 k^\mu k^\nu + \phi^2 (-a^\mu)\, a^\nu\right) - \frac{2\,i\,t\,k^\mu k^\nu}{\phi^2}\right) +$$
$$\text{J1}\left(\frac{1}{m^2\,\xi^2\,\phi^2} e^2 \left(k^\mu k^\nu (a\cdot x)^2 - \phi\, a^\nu k^\mu (a\cdot x) - \phi\, a^\mu k^\nu (a\cdot x) + \phi^2 a^\mu a^\nu\right) - \frac{2\,i\,t\,k^\mu k^\nu}{\phi^2}\right) +$$
$$\text{J0}\, g^{\mu\nu}\Bigg)\left(1 - \frac{e\,\phi 2\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot q)}\right).\gamma^\mu.$$
$$\left(-\frac{e\,(a\cdot x)\,\gamma\cdot k}{2\,m} + \frac{e\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{2\,m} + e\,s\,(\gamma\cdot a).(\gamma\cdot k) + \frac{e\,\phi\,\gamma\cdot a}{2\,m} - \frac{1}{3} m\,\xi^2 s\,\phi\,\gamma\cdot k + \frac{\gamma\cdot x}{2\,m\,s} + 1\right).$$
$$\gamma^\nu.\left(1 - \frac{e\,\phi 1\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)}\right)$$

γ - matrix algebra

We perform simplifications of the γ -matrix factor with the aid of FeynCalc functions DotSimplify, DiracSimplify, and then Contract the Lorentz indices.

```
Coeff1 = Coeff;
Phase1 = Phase;
Contract[DotSimplify[Expand[Matrix]]];
Matrix1 = Collect[Contract[DiracSimplify[%]], {J0, J1, J2}]
```

$$\mathrm{J0}\left(-\frac{D\,\phi\,\phi1\,\gamma\cdot k\,a^2\,e^2}{4\,m\,(k\cdot p)}+\frac{3\,\phi\,\phi1\,\gamma\cdot k\,a^2\,e^2}{2\,m\,(k\cdot p)}+\frac{D\,\phi\,\phi2\,\gamma\cdot k\,a^2\,e^2}{4\,m\,(k\cdot q)}-\frac{3\,\phi\,\phi2\,\gamma\cdot k\,a^2\,e^2}{2\,m\,(k\cdot q)}+\frac{D\,\phi\,\phi1\,\phi2\,\gamma\cdot k\,a^2\,e^2}{4\,m\,s\,(k\cdot p)\,(k\cdot q)}-\right.$$
$$\frac{\phi\,\phi1\,\phi2\,\gamma\cdot k\,a^2\,e^2}{2\,m\,s\,(k\cdot p)\,(k\cdot q)}-\frac{D\,\phi\,\gamma\cdot a\,e}{2\,m}+\frac{\phi\,\gamma\cdot a\,e}{m}+D\,s\,(\gamma\cdot a).(\gamma\cdot k)\,e-4\,s\,(\gamma\cdot a).(\gamma\cdot k)\,e-\frac{(\gamma\cdot k).(\gamma\cdot a).(\gamma\cdot x)\,e}{m}-$$
$$\frac{D\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)\,e}{2\,m}+\frac{2\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)\,e}{m}+\frac{D\,\gamma\cdot k\,(a\cdot x)\,e}{2\,m}-\frac{\gamma\cdot k\,(a\cdot x)\,e}{m}-\frac{D\,\phi1\,(\gamma\cdot k).(\gamma\cdot a)\,e}{2\,(k\cdot p)}+$$
$$\frac{D\,\phi1\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,e}{4\,m\,s\,(k\cdot p)}-\frac{\phi1\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,e}{2\,m\,s\,(k\cdot p)}-\frac{D\,\phi2\,(\gamma\cdot a).(\gamma\cdot k)\,e}{2\,(k\cdot q)}+\frac{D\,\phi2\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)\,e}{4\,m\,s\,(k\cdot q)}-$$
$$\left.\frac{\phi2\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)\,e}{2\,m\,s\,(k\cdot q)}+D+\frac{1}{3}\,D\,m\,s\,\xi^2\,\phi\,\gamma\cdot k-\frac{2}{3}\,m\,s\,\xi^2\,\phi\,\gamma\cdot k-\frac{D\,\gamma\cdot x}{2\,m\,s}+\frac{\gamma\cdot x}{m\,s}\right)+$$
$$\mathrm{J1}\left(\frac{\phi\,\phi1\,\gamma\cdot k\,a^4\,e^4}{4\,m^3\,\xi^2\,(k\cdot p)}-\frac{\phi\,\phi2\,\gamma\cdot k\,a^4\,e^4}{4\,m^3\,\xi^2\,(k\cdot q)}+\frac{\phi\,\phi1\,\phi2\,\gamma\cdot k\,a^4\,e^4}{4\,m^3\,s\,\xi^2\,(k\cdot p)\,(k\cdot q)}+\frac{\phi\,\gamma\cdot a\,a^2\,e^3}{2\,m^3\,\xi^2}+\frac{s\,(\gamma\cdot k).(\gamma\cdot a)\,a^2\,e^3}{m^2\,\xi^2}-\right.$$
$$\frac{(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,a^2\,e^3}{2\,m^3\,\xi^2}-\frac{\gamma\cdot k\,a^2\,(a\cdot x)\,e^3}{2\,m^3\,\xi^2}-\frac{\phi1\,(\gamma\cdot k).(\gamma\cdot a)\,a^2\,e^3}{2\,m^2\,\xi^2\,(k\cdot p)}+\frac{\phi1\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,a^2\,e^3}{4\,m^3\,s\,\xi^2\,(k\cdot p)}-$$
$$\frac{\phi2\,(\gamma\cdot a).(\gamma\cdot k)\,a^2\,e^3}{2\,m^2\,\xi^2\,(k\cdot q)}+\frac{\phi2\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\,a^2\,e^3}{4\,m^3\,s\,\xi^2\,(k\cdot q)}-\frac{\phi2\,\gamma\cdot k\,a^2\,(a\cdot x)\,e^3}{2\,m^3\,s\,\xi^2\,(k\cdot q)}+\frac{\gamma\cdot k\,(a\cdot x)^2\,e^2}{m^3\,s\,\xi^2\,\phi}+$$
$$\frac{s\,\phi\,\gamma\cdot k\,a^2\,e^2}{3\,m}-\frac{\gamma\cdot x\,a^2\,e^2}{2\,m^3\,s\,\xi^2}+\frac{a^2\,e^2}{m^2\,\xi^2}+\frac{\gamma\cdot a\,(a\cdot x)\,e^2}{m^3\,s\,\xi^2}-\frac{(\gamma\cdot a).(\gamma\cdot k)\,(a\cdot x)\,e^2}{m^2\,\xi^2\,\phi}-$$
$$\left.\frac{(\gamma\cdot k).(\gamma\cdot a)\,(a\cdot x)\,e^2}{m^2\,\xi^2\,\phi}-\frac{(\gamma\cdot a).(\gamma\cdot x).(\gamma\cdot k)\,(a\cdot x)\,e^2}{2\,m^3\,s\,\xi^2\,\phi}-\frac{(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\,(a\cdot x)\,e^2}{2\,m^3\,s\,\xi^2\,\phi}-\frac{2\,i\,t\,\gamma\cdot k}{m\,s\,\phi}\right)+$$
$$\mathrm{J2}\left(-\frac{D\,\phi\,\phi1\,\gamma\cdot k\,a^4\,e^4}{4\,m^3\,\xi^2\,(k\cdot p)}+\frac{3\,\phi\,\phi1\,\gamma\cdot k\,a^4\,e^4}{4\,m^3\,\xi^2\,(k\cdot p)}+\frac{D\,\phi\,\phi2\,\gamma\cdot k\,a^4\,e^4}{4\,m^3\,\xi^2\,(k\cdot q)}-\frac{3\,\phi\,\phi2\,\gamma\cdot k\,a^4\,e^4}{4\,m^3\,\xi^2\,(k\cdot q)}+\frac{D\,\phi\,\phi1\,\phi2\,\gamma\cdot k\,a^4\,e^4}{4\,m^3\,s\,\xi^2\,(k\cdot p)\,(k\cdot q)}-\right.$$
$$\frac{3\,\phi\,\phi1\,\phi2\,\gamma\cdot k\,a^4\,e^4}{4\,m^3\,s\,\xi^2\,(k\cdot p)\,(k\cdot q)}-\frac{D\,\phi\,\gamma\cdot a\,a^2\,e^3}{2\,m^3\,\xi^2}+\frac{\phi\,\gamma\cdot a\,a^2\,e^3}{2\,m^3\,\xi^2}+\frac{D\,s\,(\gamma\cdot a).(\gamma\cdot k)\,a^2\,e^3}{m^2\,\xi^2}-$$
$$\frac{4\,s\,(\gamma\cdot a).(\gamma\cdot k)\,a^2\,e^3}{m^2\,\xi^2}-\frac{s\,(\gamma\cdot k).(\gamma\cdot a)\,a^2\,e^3}{m^2\,\xi^2}-\frac{(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)\,a^2\,e^3}{m^3\,\xi^2}-\frac{3\,(\gamma\cdot k).(\gamma\cdot a).(\gamma\cdot x)\,a^2\,e^3}{2\,m^3\,\xi^2}-$$
$$\frac{D\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)\,a^2\,e^3}{2\,m^3\,\xi^2}+\frac{3\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)\,a^2\,e^3}{2\,m^3\,\xi^2}+\frac{(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,a^2\,e^3}{2\,m^3\,\xi^2}+\frac{D\,\gamma\cdot k\,a^2\,(a\cdot x)\,e^3}{2\,m^3\,\xi^2}-$$
$$\frac{\gamma\cdot k\,a^2\,(a\cdot x)\,e^3}{2\,m^3\,\xi^2}-\frac{D\,\phi1\,(\gamma\cdot k).(\gamma\cdot a)\,a^2\,e^3}{2\,m^2\,\xi^2\,(k\cdot p)}+\frac{3\,\phi1\,(\gamma\cdot k).(\gamma\cdot a)\,a^2\,e^3}{2\,m^2\,\xi^2\,(k\cdot p)}+\frac{D\,\phi1\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,a^2\,e^3}{4\,m^3\,s\,\xi^2\,(k\cdot p)}-$$
$$\frac{3\,\phi1\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,a^2\,e^3}{4\,m^3\,s\,\xi^2\,(k\cdot p)}-\frac{D\,\phi2\,(\gamma\cdot a).(\gamma\cdot k)\,a^2\,e^3}{2\,m^2\,\xi^2\,(k\cdot q)}+\frac{3\,\phi2\,(\gamma\cdot a).(\gamma\cdot k)\,a^2\,e^3}{2\,m^2\,\xi^2\,(k\cdot q)}+$$
$$\frac{D\,\phi2\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)\,a^2\,e^3}{4\,m^3\,s\,\xi^2\,(k\cdot q)}-\frac{\phi2\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)\,a^2\,e^3}{2\,m^3\,s\,\xi^2\,(k\cdot q)}-\frac{\phi2\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\,a^2\,e^3}{4\,m^3\,s\,\xi^2\,(k\cdot q)}+$$
$$\frac{\phi2\,\gamma\cdot k\,a^2\,(a\cdot x)\,e^3}{2\,m^3\,s\,\xi^2\,(k\cdot q)}-\frac{\gamma\cdot k\,(a\cdot x)^2\,e^2}{m^3\,s\,\xi^2\,\phi}+\frac{D\,s\,\phi\,\gamma\cdot k\,a^2\,e^2}{3\,m}-\frac{s\,\phi\,\gamma\cdot k\,a^2\,e^2}{m}-\frac{D\,\gamma\cdot x\,a^2\,e^2}{2\,m^3\,s\,\xi^2}+\frac{3\,\gamma\cdot x\,a^2\,e^2}{2\,m^3\,s\,\xi^2}-$$
$$\frac{(\gamma\cdot k).(\gamma\cdot x)\,a^2\,e^2}{m^2\,\xi^2\,\phi}-\frac{(\gamma\cdot x).(\gamma\cdot k)\,a^2\,e^2}{m^2\,\xi^2\,\phi}+\frac{D\,a^2\,e^2}{m^2\,\xi^2}-\frac{a^2\,e^2}{m^2\,\xi^2}-\frac{\gamma\cdot a\,(a\cdot x)\,e^2}{m^3\,s\,\xi^2}+\frac{(\gamma\cdot a).(\gamma\cdot k)\,(a\cdot x)\,e^2}{m^2\,\xi^2\,\phi}+$$
$$\left.\frac{(\gamma\cdot k).(\gamma\cdot a)\,(a\cdot x)\,e^2}{m^2\,\xi^2\,\phi}+\frac{(\gamma\cdot a).(\gamma\cdot x).(\gamma\cdot k)\,(a\cdot x)\,e^2}{2\,m^3\,s\,\xi^2\,\phi}+\frac{(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\,(a\cdot x)\,e^2}{2\,m^3\,s\,\xi^2\,\phi}-\frac{2\,i\,t\,\gamma\cdot k}{m\,s\,\phi}\right)$$

Substitution of the variables

$$\mathrm{x1} = x - \frac{X}{2};$$

```
x2 = x + X/2;
ϕ1 = ϕ - Φ/2;
ϕ2 = ϕ + Φ/2;

ϕ = kx = m x₋;
Φ = kX = m X₋;

The integration measure :
    dᴰx₁ dᴰx₂ = dᴰx dᴰX
```

```
Coeff2 = Coeff1;
Phase2 = Expand[ExpandScalarProduct[
    Phase1 /. {ϕ2 → Φ + ϕ/2, ϕ1 → Φ - ϕ/2,  av2 → -m^2 ξ^2/e^2} /.
     {Momentum[x1, D] → Momentum[X, D] - Momentum[x, D]/2,
      Momentum[x2, D] → Momentum[X, D] + Momentum[x, D]/2}
   ]]
Matrix2 = Collect[
   Expand[
    Matrix1 /. {ϕ2 → Φ + ϕ/2, ϕ1 → Φ - ϕ/2, av2 → -m^2 ξ^2/e^2}
   ],
   {J0, J1, J2}]
```

$$\frac{e\Phi^2(a\cdot p)}{2(k\cdot p)}+\frac{e\phi^2(a\cdot p)}{8(k\cdot p)}-\frac{e\Phi\,\phi(a\cdot p)}{2(k\cdot p)}-\frac{e\Phi^2(a\cdot q)}{2(k\cdot q)}-\frac{e\phi^2(a\cdot q)}{8(k\cdot q)}-\frac{e\Phi\,\phi(a\cdot q)}{2(k\cdot q)}+ e\Phi(a\cdot x)-\frac{m^2\xi^2\Phi^3}{6(k\cdot p)}+\frac{m^2\xi^2\phi^3}{48(k\cdot p)}-\frac{m^2\xi^2\Phi\,\phi^2}{8(k\cdot p)}+\frac{m^2\xi^2\Phi^2\phi}{4(k\cdot p)}+\frac{m^2\xi^2\Phi^3}{6(k\cdot q)}+\frac{m^2\xi^2\phi^3}{48(k\cdot q)}+ \frac{m^2\xi^2\Phi\,\phi^2}{8(k\cdot q)}+\frac{m^2\xi^2\Phi^2\phi}{4(k\cdot q)}-\frac{1}{12}m^2\xi^2 s\phi^2-m^2 s+\frac{p\cdot x}{2}-p\cdot X+\frac{q\cdot x}{2}+q\cdot X-\frac{x^2}{4s}-\frac{x^2}{4t}$$

$$\mathrm{J1}\Bigg(-\frac{m\xi^2\gamma\cdot k\phi^3}{16s(k\cdot p)(k\cdot q)}-\frac{m\xi^2\gamma\cdot k\phi^2}{8(k\cdot p)}-\frac{m\xi^2\gamma\cdot k\phi^2}{8(k\cdot q)}-\frac{e\gamma\cdot a\phi}{2m}-\frac{1}{3}ms\xi^2\gamma\cdot k\phi+\frac{m\xi^2\Phi\gamma\cdot k\phi}{4(k\cdot p)}- \frac{e(\gamma\cdot k).(\gamma\cdot a)\phi}{4(k\cdot p)}+\frac{e(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\phi}{8ms(k\cdot p)}-\frac{m\xi^2\Phi\gamma\cdot k\phi}{4(k\cdot q)}+\frac{e(\gamma\cdot a).(\gamma\cdot k)\phi}{4(k\cdot q)}-\frac{e(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\phi}{8ms(k\cdot q)}+ \frac{e\gamma\cdot k(a\cdot x)\phi}{4ms(k\cdot q)}+\frac{m\xi^2\Phi^2\gamma\cdot k\phi}{4s(k\cdot p)(k\cdot q)}+\frac{\gamma\cdot x}{2ms}-es(\gamma\cdot k).(\gamma\cdot a)+\frac{e(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{2m}+ \frac{e^2\gamma\cdot a(a\cdot x)}{m^3 s\xi^2}+\frac{e\gamma\cdot k(a\cdot x)}{2m}+\frac{e\Phi(\gamma\cdot k).(\gamma\cdot a)}{2(k\cdot p)}-\frac{e\Phi(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{4ms(k\cdot p)}+\frac{e\Phi(\gamma\cdot a).(\gamma\cdot k)}{2(k\cdot q)}-$$

$$\frac{e\,\Phi\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)}{4\,m\,s\,(k\cdot q)}+\frac{e\,\Phi\,\gamma\cdot k\,(a\cdot x)}{2\,m\,s\,(k\cdot q)}-1+\frac{e^2\,\gamma\cdot k\,(a\cdot x)^2}{m^3\,s\,\xi^2\,\phi}-\frac{2\,i\,t\,\gamma\cdot k}{m\,s\,\phi}-\frac{e^2\,(\gamma\cdot a).(\gamma\cdot k)\,(a\cdot x)}{m^2\,\xi^2\,\phi}-$$

$$\frac{e^2\,(\gamma\cdot k).(\gamma\cdot a)\,(a\cdot x)}{m^2\,\xi^2\,\phi}-\frac{e^2\,(\gamma\cdot a).(\gamma\cdot x).(\gamma\cdot k)\,(a\cdot x)}{2\,m^3\,s\,\xi^2\,\phi}-\frac{e^2\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\,(a\cdot x)}{2\,m^3\,s\,\xi^2\,\phi}\Bigg)+$$

$$\mathrm{J0}\Bigg(\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)\,(k\cdot q)}-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^3}{8\,s\,(k\cdot p)\,(k\cdot q)}-\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{8\,(k\cdot p)}+\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}-\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{8\,(k\cdot q)}+\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot q)}-$$

$$\frac{D\,e\,\gamma\cdot a\,\phi}{2\,m}+\frac{e\,\gamma\cdot a\,\phi}{m}+\frac{1}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,\phi-\frac{2}{3}\,m\,s\,\xi^2\,\gamma\cdot k\,\phi+\frac{D\,m\,\xi^2\,\Phi\,\gamma\cdot k\,\phi}{4\,(k\cdot p)}-\frac{3\,m\,\xi^2\,\Phi\,\gamma\cdot k\,\phi}{2\,(k\cdot p)}+$$

$$\frac{D\,e\,(\gamma\cdot k).(\gamma\cdot a)\,\phi}{4\,(k\cdot p)}-\frac{D\,e\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,\phi}{8\,m\,s\,(k\cdot p)}+\frac{e\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,\phi}{4\,m\,s\,(k\cdot p)}-\frac{D\,m\,\xi^2\,\Phi\,\gamma\cdot k\,\phi}{4\,(k\cdot q)}+\frac{3\,m\,\xi^2\,\Phi\,\gamma\cdot k\,\phi}{2\,(k\cdot q)}-$$

$$\frac{D\,e\,(\gamma\cdot a).(\gamma\cdot k)\,\phi}{4\,(k\cdot q)}+\frac{D\,e\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)\,\phi}{8\,m\,s\,(k\cdot q)}-\frac{e\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)\,\phi}{4\,m\,s\,(k\cdot q)}-\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)\,(k\cdot q)}+\frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{2\,s\,(k\cdot p)\,(k\cdot q)}+$$

$$D-\frac{D\,\gamma\cdot x}{2\,m\,s}+\frac{\gamma\cdot x}{m\,s}+D\,e\,s\,(\gamma\cdot a).(\gamma\cdot k)-4\,e\,s\,(\gamma\cdot a).(\gamma\cdot k)-\frac{e\,(\gamma\cdot k).(\gamma\cdot a).(\gamma\cdot x)}{m}-\frac{D\,e\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{2\,m}+$$

$$\frac{2\,e\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{m}+\frac{D\,e\,\gamma\cdot k\,(a\cdot x)}{2\,m}-\frac{e\,\gamma\cdot k\,(a\cdot x)}{m}-\frac{D\,e\,\Phi\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)}+\frac{D\,e\,\Phi\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{4\,m\,s\,(k\cdot p)}-$$

$$\frac{e\,\Phi\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{2\,m\,s\,(k\cdot p)}-\frac{D\,e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot q)}+\frac{D\,e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)}{4\,m\,s\,(k\cdot q)}-\frac{e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)}{2\,m\,s\,(k\cdot q)}\Bigg)+$$

$$\mathrm{J2}\Bigg(-\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)\,(k\cdot q)}+\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)\,(k\cdot q)}+\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{8\,(k\cdot p)}-\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{8\,(k\cdot p)}+\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{8\,(k\cdot q)}-\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{8\,(k\cdot q)}+$$

$$\frac{D\,e\,\gamma\cdot a\,\phi}{2\,m}-\frac{e\,\gamma\cdot a\,\phi}{2\,m}-\frac{1}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,\phi+m\,s\,\xi^2\,\gamma\cdot k\,\phi-\frac{D\,m\,\xi^2\,\Phi\,\gamma\cdot k\,\phi}{4\,(k\cdot p)}+\frac{3\,m\,\xi^2\,\Phi\,\gamma\cdot k\,\phi}{4\,(k\cdot p)}-$$

$$\frac{D\,e\,(\gamma\cdot k).(\gamma\cdot a)\,\phi}{4\,(k\cdot p)}+\frac{3\,e\,(\gamma\cdot k).(\gamma\cdot a)\,\phi}{4\,(k\cdot p)}+\frac{D\,e\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,\phi}{8\,m\,s\,(k\cdot p)}-\frac{3\,e\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,\phi}{8\,m\,s\,(k\cdot p)}+\frac{D\,m\,\xi^2\,\Phi\,\gamma\cdot k\,\phi}{4\,(k\cdot q)}-$$

$$\frac{3\,m\,\xi^2\,\Phi\,\gamma\cdot k\,\phi}{4\,(k\cdot q)}+\frac{D\,e\,(\gamma\cdot a).(\gamma\cdot k)\,\phi}{4\,(k\cdot q)}-\frac{3\,e\,(\gamma\cdot a).(\gamma\cdot k)\,\phi}{4\,(k\cdot q)}-\frac{D\,e\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)\,\phi}{8\,m\,s\,(k\cdot q)}+\frac{e\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)\,\phi}{4\,m\,s\,(k\cdot q)}+$$

$$\frac{e\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\,\phi}{8\,m\,s\,(k\cdot q)}-\frac{e\,\gamma\cdot k\,(a\cdot x)\,\phi}{4\,m\,s\,(k\cdot q)}+\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)\,(k\cdot q)}-\frac{3\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)\,(k\cdot q)}-D+\frac{D\,\gamma\cdot x}{2\,m\,s}-\frac{3\,\gamma\cdot x}{2\,m\,s}-$$

$$D\,e\,s\,(\gamma\cdot a).(\gamma\cdot k)+4\,e\,s\,(\gamma\cdot a).(\gamma\cdot k)+e\,s\,(\gamma\cdot k).(\gamma\cdot a)+\frac{e\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)}{m}+\frac{3\,e\,(\gamma\cdot k).(\gamma\cdot a).(\gamma\cdot x)}{2\,m}+$$

$$\frac{D\,e\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{2\,m}-\frac{3\,e\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{2\,m}-\frac{e\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{2\,m}-\frac{e^2\,\gamma\cdot a\,(a\cdot x)}{m^3\,s\,\xi^2}-\frac{D\,e\,\gamma\cdot k\,(a\cdot x)}{2\,m}+$$

$$\frac{e\,\gamma\cdot k\,(a\cdot x)}{2\,m}+\frac{D\,e\,\Phi\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)}-\frac{3\,e\,\Phi\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)}-\frac{D\,e\,\Phi\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{4\,m\,s\,(k\cdot p)}+\frac{3\,e\,\Phi\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{4\,m\,s\,(k\cdot p)}+$$

$$\frac{D\,e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot q)}-\frac{3\,e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot q)}-\frac{D\,e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)}{4\,m\,s\,(k\cdot q)}+\frac{e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)}{2\,m\,s\,(k\cdot q)}+$$

$$\frac{e\,\Phi\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)}{4\,m\,s\,(k\cdot q)}-\frac{e\,\Phi\,\gamma\cdot k\,(a\cdot x)}{2\,m\,s\,(k\cdot q)}+1-\frac{e^2\,\gamma\cdot k\,(a\cdot x)^2}{m^3\,s\,\xi^2\,\phi}-\frac{2\,i\,t\,\gamma\cdot k}{m\,s\,\phi}+\frac{(\gamma\cdot k).(\gamma\cdot x)}{\phi}+\frac{(\gamma\cdot x).(\gamma\cdot k)}{\phi}+$$

$$\frac{e^2\,(\gamma\cdot a).(\gamma\cdot k)\,(a\cdot x)}{m^2\,\xi^2\,\phi}+\frac{e^2\,(\gamma\cdot k).(\gamma\cdot a)\,(a\cdot x)}{m^2\,\xi^2\,\phi}+\frac{e^2\,(\gamma\cdot a).(\gamma\cdot x).(\gamma\cdot k)\,(a\cdot x)}{2\,m^3\,s\,\xi^2\,\phi}+\frac{e^2\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\,(a\cdot x)}{2\,m^3\,s\,\xi^2\,\phi}\Bigg)$$

Integration over
$\int d^{D-2}X_\perp\, dX_+ \ldots$

Preexp does not depend on $X_\perp$ and X_+. Phase contains $X_\perp$ and X_+ in scalar products

$pX = p_- X_+ + p_+ X_- - p_\perp X_\perp$;

$qX = q_- X_+ + q_+ X_- - q_\perp X_\perp$;

so

$\int d^{D-2}X_\perp\, dX_+\, e^{-i\,(p-q).X} \ldots = (2\pi)^{D-1}\,\delta^{(D-2)}\,(p_\perp - q_\perp)\,\delta\,(p_- - q_-)$.

We will not write the δ - functions explicitly,
but we will assume that they are present;

Due to the conservation law $\delta^{(D-2)}\,(p_\perp - q_\perp)\,\delta\,(p_- - q_-)$

$kq \to kp$

$aq \to ap$

Notations

$pp = p_+$;

$qp = q_+$;

The remaining integrals : $\int dX_-\, d^D x \ldots$

```
Coeff3 = Coeff2 * (2 π) ^ (D - 1)
Phase3 = Phase2 /. {Pair[Momentum[p, D], Momentum[X, D]] → pp Xm,
      Pair[Momentum[q, D], Momentum[X, D]] → qp Xm} /.
    {Xm → Φ / m} /. {kq → kp, aq → ap}
Matrix3 = Matrix2 /. {kq → kp, aq → ap}
```

$$-\frac{2^{-D-2}\, e^{-\frac{1}{2} i \pi D}\, e^2\, m\, s^{-D/2}\, t^{-D/2}}{\pi^2}$$

$$-\frac{e\Phi\,\phi\,(a\cdot p)}{k\cdot p} + e\Phi\,(a\cdot x) + \frac{m^2\,\xi^2\,\phi^3}{24\,(k\cdot p)} + \frac{m^2\,\xi^2\,\Phi^2\,\phi}{2\,(k\cdot p)} - \frac{1}{12}\, m^2\,\xi^2\, s\,\phi^2 - m^2\, s - \frac{\mathrm{pp}\,\Phi}{m} + \frac{\mathrm{qp}\,\Phi}{m} + \frac{p\cdot x}{2} + \frac{q\cdot x}{2} - \frac{x^2}{4\,s} - \frac{x^2}{4\,t}$$

$$\mathrm{J0}\left(\frac{D\, m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\, s\,(k\cdot p)^2} - \frac{m\,\xi^2\,\gamma\cdot k\,\phi^3}{8\, s\,(k\cdot p)^2} - \frac{D\, m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)} + \frac{3\, m\,\xi^2\,\gamma\cdot k\,\phi^2}{2\,(k\cdot p)} - \frac{D\, e\,\gamma\cdot a\,\phi}{2\, m} + \frac{e\,\gamma\cdot a\,\phi}{m} + \frac{1}{3} D\, m\, s\,\xi^2\,\gamma\cdot k\,\phi - \right.$$
$$\frac{2}{3}\, m\, s\,\xi^2\,\gamma\cdot k\,\phi - \frac{D\, e\,(\gamma\cdot a).(\gamma\cdot k)\,\phi}{4\,(k\cdot p)} + \frac{D\, e\,(\gamma\cdot k).(\gamma\cdot a)\,\phi}{4\,(k\cdot p)} + \frac{D\, e\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)\,\phi}{8\, m\, s\,(k\cdot p)} - \frac{e\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)\,\phi}{4\, m\, s\,(k\cdot p)} -$$

$$\frac{D\,e\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,\phi}{8\,m\,s\,(k\cdot p)}+\frac{e\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,\phi}{4\,m\,s\,(k\cdot p)}-\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}+\frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{2\,s\,(k\cdot p)^2}+D-\frac{D\,\gamma\cdot x}{2\,m\,s}+$$
$$\frac{\gamma\cdot x}{m\,s}+D\,e\,s\,(\gamma\cdot a).(\gamma\cdot k)-4\,e\,s\,(\gamma\cdot a).(\gamma\cdot k)-\frac{e\,(\gamma\cdot k).(\gamma\cdot a).(\gamma\cdot x)}{m}-\frac{D\,e\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{2\,m}+$$
$$\frac{2\,e\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{m}+\frac{D\,e\,\gamma\cdot k\,(a\cdot x)}{2\,m}-\frac{e\,\gamma\cdot k\,(a\cdot x)}{m}-\frac{D\,e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot p)}-\frac{D\,e\,\Phi\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)}+$$
$$\left.\frac{D\,e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)}{4\,m\,s\,(k\cdot p)}-\frac{e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)}{2\,m\,s\,(k\cdot p)}+\frac{D\,e\,\Phi\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{4\,m\,s\,(k\cdot p)}-\frac{e\,\Phi\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{2\,m\,s\,(k\cdot p)}\right)+$$
$$\mathrm{J2}\left(-\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}+\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}+\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}-\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}+\frac{D\,e\,\gamma\cdot a\,\phi}{2\,m}-\frac{e\,\gamma\cdot a\,\phi}{2\,m}-\frac{1}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,\phi+\right.$$
$$m\,s\,\xi^2\,\gamma\cdot k\,\phi+\frac{D\,e\,(\gamma\cdot a).(\gamma\cdot k)\,\phi}{4\,(k\cdot p)}-\frac{3\,e\,(\gamma\cdot a).(\gamma\cdot k)\,\phi}{4\,(k\cdot p)}-\frac{D\,e\,(\gamma\cdot k).(\gamma\cdot a)\,\phi}{4\,(k\cdot p)}+\frac{3\,e\,(\gamma\cdot k).(\gamma\cdot a)\,\phi}{4\,(k\cdot p)}-$$
$$\frac{D\,e\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)\,\phi}{8\,m\,s\,(k\cdot p)}+\frac{e\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)\,\phi}{4\,m\,s\,(k\cdot p)}+\frac{e\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\,\phi}{8\,m\,s\,(k\cdot p)}+\frac{D\,e\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,\phi}{8\,m\,s\,(k\cdot p)}-$$
$$\frac{3\,e\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,\phi}{8\,m\,s\,(k\cdot p)}-\frac{e\,\gamma\cdot k\,(a\cdot x)\,\phi}{4\,m\,s\,(k\cdot p)}+\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}-\frac{3\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}-D+\frac{D\,\gamma\cdot x}{2\,m\,s}-\frac{3\,\gamma\cdot x}{2\,m\,s}-$$
$$D\,e\,s\,(\gamma\cdot a).(\gamma\cdot k)+4\,e\,s\,(\gamma\cdot a).(\gamma\cdot k)+e\,s\,(\gamma\cdot k).(\gamma\cdot a)+\frac{e\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)}{m}+\frac{3\,e\,(\gamma\cdot k).(\gamma\cdot a).(\gamma\cdot x)}{2\,m}+$$
$$\frac{D\,e\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{2\,m}-\frac{3\,e\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{2\,m}-\frac{e\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{2\,m}-\frac{e^2\,\gamma\cdot a\,(a\cdot x)}{m^3\,s\,\xi^2}-\frac{D\,e\,\gamma\cdot k\,(a\cdot x)}{2\,m}+$$
$$\frac{e\,\gamma\cdot k\,(a\cdot x)}{2\,m}+\frac{D\,e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot p)}-\frac{3\,e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot p)}+\frac{D\,e\,\Phi\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)}-\frac{3\,e\,\Phi\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)}-$$
$$\frac{D\,e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)}{4\,m\,s\,(k\cdot p)}+\frac{e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)}{2\,m\,s\,(k\cdot p)}+\frac{e\,\Phi\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)}{4\,m\,s\,(k\cdot p)}-\frac{D\,e\,\Phi\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{4\,m\,s\,(k\cdot p)}+$$
$$\frac{3\,e\,\Phi\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{4\,m\,s\,(k\cdot p)}-\frac{e\,\Phi\,\gamma\cdot k\,(a\cdot x)}{2\,m\,s\,(k\cdot p)}+1-\frac{e^2\,\gamma\cdot k\,(a\cdot x)^2}{m^3\,s\,\xi^2\,\phi}-\frac{2\,i\,t\,\gamma\cdot k}{m\,s\,\phi}+\frac{(\gamma\cdot k).(\gamma\cdot x)}{\phi}+\frac{(\gamma\cdot x).(\gamma\cdot k)}{\phi}+$$
$$\left.\frac{e^2\,(\gamma\cdot a).(\gamma\cdot k)\,(a\cdot x)}{m^2\,\xi^2\,\phi}+\frac{e^2\,(\gamma\cdot k).(\gamma\cdot a)\,(a\cdot x)}{m^2\,\xi^2\,\phi}+\frac{e^2\,(\gamma\cdot a).(\gamma\cdot x).(\gamma\cdot k)\,(a\cdot x)}{2\,m^3\,s\,\xi^2\,\phi}+\frac{e^2\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\,(a\cdot x)}{2\,m^3\,s\,\xi^2\,\phi}\right)+$$
$$\mathrm{J1}\left(-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}-\frac{e\,\gamma\cdot a\,\phi}{2\,m}-\frac{1}{3}\,m\,s\,\xi^2\,\gamma\cdot k\,\phi+\frac{e\,(\gamma\cdot a).(\gamma\cdot k)\,\phi}{4\,(k\cdot p)}-\frac{e\,(\gamma\cdot k).(\gamma\cdot a)\,\phi}{4\,(k\cdot p)}-\right.$$
$$\frac{e\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\,\phi}{8\,m\,s\,(k\cdot p)}+\frac{e\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,\phi}{8\,m\,s\,(k\cdot p)}+\frac{e\,\gamma\cdot k\,(a\cdot x)\,\phi}{4\,m\,s\,(k\cdot p)}+\frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}+\frac{\gamma\cdot x}{2\,m\,s}-$$
$$e\,s\,(\gamma\cdot k).(\gamma\cdot a)+\frac{e\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{2\,m}+\frac{e^2\,\gamma\cdot a\,(a\cdot x)}{m^3\,s\,\xi^2}+\frac{e\,\gamma\cdot k\,(a\cdot x)}{2\,m}+\frac{e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot p)}+\frac{e\,\Phi\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)}-$$
$$\frac{e\,\Phi\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)}{4\,m\,s\,(k\cdot p)}-\frac{e\,\Phi\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{4\,m\,s\,(k\cdot p)}+\frac{e\,\Phi\,\gamma\cdot k\,(a\cdot x)}{2\,m\,s\,(k\cdot p)}-1+\frac{e^2\,\gamma\cdot k\,(a\cdot x)^2}{m^3\,s\,\xi^2\,\phi}-\frac{2\,i\,t\,\gamma\cdot k}{m\,s\,\phi}-$$
$$\left.\frac{e^2\,(\gamma\cdot a).(\gamma\cdot k)\,(a\cdot x)}{m^2\,\xi^2\,\phi}-\frac{e^2\,(\gamma\cdot k).(\gamma\cdot a)\,(a\cdot x)}{m^2\,\xi^2\,\phi}-\frac{e^2\,(\gamma\cdot a).(\gamma\cdot x).(\gamma\cdot k)\,(a\cdot x)}{2\,m^3\,s\,\xi^2\,\phi}-\frac{e^2\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\,(a\cdot x)}{2\,m^3\,s\,\xi^2\,\phi}\right)$$

Reordering the γ - matrix terms in the preexponent

We employ the equality

$(\gamma\, a)\,(\gamma\, b)\,(\gamma\, c) \to$

$-i\,\gamma^{\beta}.\bar{\gamma}^{5}\,\epsilon^{\beta\,\mu\nu\delta}\,a_{\mu}\,b_{\nu}\,c_{\delta} + (a\,b)\,(\gamma\,c) - (a\,c)\,(\gamma\,b) + (\gamma\,a)\,(b\,c)$

to rewrite the terms with 3 gamma matrices

Then we recollect tensors $F_{\mu\nu}$,

$F^{*}{}_{\mu\nu}$ and $\left(F^{2}\right)_{\mu\nu}$ from the combinations of a_{μ}, k_{μ} and the antisymmetric tensor $\epsilon^{\alpha\beta\mu\nu}$

The scalar products (γa) and (γk) can be expressed as

$(\gamma a) = \frac{1}{\phi}[(\gamma k)\,(a x) - (\gamma\, F x)]$

$(\gamma k) = (\gamma k)\,(k x)/\phi = -a^{2}\,k_{\mu}\,k_{\nu}\,\gamma^{\mu}\,x^{\nu}\,\frac{1}{-a^{2}\,\phi} = \frac{e^{2}}{m^{2}\,\xi^{2}\,\phi}\left(\gamma F^{2}\,x\right)$

Then the result can be expressed as a linear combination of following γ - matrix structures :

1,

(γx),

$\left(\gamma F^{2}\,x\right)$,

$(\sigma F) = \sigma_{\mu\nu}\,F^{\mu\nu}$,

$\gamma^{\beta}\,\gamma^{5}\,(F^{*}\,x)_{\beta}$.

Also note that we treat $\bar{\gamma}^{5}$ as a 4 - dimensional object, assuming that the terms incorporating it are finite

```
Coeff4 = Coeff3;
Phase4 = Phase3;
Matrix4 = Collect[
  (((Expand[Matrix3 /. TripleGamma]) /. EpsToF) /. FieldSubstitutions) /. akToF /.
   {DiracGamma[Momentum[k, D], D].DiracGamma[Momentum[x, D], D] / ϕ →
     -DiracGamma[Momentum[x, D], D].DiracGamma[Momentum[k, D], D] / ϕ +
      2}, {J0, J1, J2}]
```

$$\text{J1}\left(-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}-\frac{1}{3}\,m\,s\,\xi^2\,\gamma\cdot k\,\phi+\frac{i\,e\,\sigma\text{F}\,\phi}{4\,(k\cdot p)}-\frac{e\,\Phi\,\gamma\cdot a\,\phi}{2\,m\,s\,(k\cdot p)}-\frac{i\,e\,\gamma^\beta.\overline{\gamma}^5\,\text{FDx}^\beta\,\phi}{4\,m\,s\,(k\cdot p)}+\frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}+\frac{1}{2}\,i\,e\,s\,\sigma\text{F}+\frac{\gamma\cdot x}{2\,m\,s}-\frac{i\,e\,\gamma^\beta.\overline{\gamma}^5\,\text{FDx}^\beta}{2\,m}+\frac{e\,\Phi\,\gamma\cdot k\,(a\cdot x)}{2\,m\,s\,(k\cdot p)}-1-\frac{2\,i\,t\,\gamma\cdot k}{m\,s\,\phi}\right)+$$

$$\text{J0}\left(\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^3}{8\,s\,(k\cdot p)^2}-\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}+\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{2\,(k\cdot p)}+\frac{1}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,\phi-\frac{2}{3}\,m\,s\,\xi^2\,\gamma\cdot k\,\phi-\frac{i\,D\,e\,\sigma\text{F}\,\phi}{4\,(k\cdot p)}+\frac{D\,e\,\Phi\,\gamma\cdot a\,\phi}{2\,m\,s\,(k\cdot p)}-\frac{e\,\Phi\,\gamma\cdot a\,\phi}{m\,s\,(k\cdot p)}+\frac{i\,D\,e\,\gamma^\beta.\overline{\gamma}^5\,\text{FDx}^\beta\,\phi}{4\,m\,s\,(k\cdot p)}-\frac{i\,e\,\gamma^\beta.\overline{\gamma}^5\,\text{FDx}^\beta\,\phi}{2\,m\,s\,(k\cdot p)}-\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}+\frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{2\,s\,(k\cdot p)^2}+D+\frac{1}{2}\,i\,D\,e\,s\,\sigma\text{F}-2\,i\,e\,s\,\sigma\text{F}-\frac{D\,\gamma\cdot x}{2\,m\,s}+\frac{\gamma\cdot x}{m\,s}-\frac{i\,D\,e\,\gamma^\beta.\overline{\gamma}^5\,\text{FDx}^\beta}{2\,m}+\frac{3\,i\,e\,\gamma^\beta.\overline{\gamma}^5\,\text{FDx}^\beta}{m}-\frac{D\,e\,\Phi\,\gamma\cdot k\,(a\cdot x)}{2\,m\,s\,(k\cdot p)}+\frac{e\,\Phi\,\gamma\cdot k\,(a\cdot x)}{m\,s\,(k\cdot p)}\right)+$$

$$\text{J2}\left(-\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}+\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}+\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}-\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}-\frac{1}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,\phi+m\,s\,\xi^2\,\gamma\cdot k\,\phi+\frac{i\,D\,e\,\sigma\text{F}\,\phi}{4\,(k\cdot p)}-\frac{3\,i\,e\,\sigma\text{F}\,\phi}{4\,(k\cdot p)}-\frac{D\,e\,\Phi\,\gamma\cdot a\,\phi}{2\,m\,s\,(k\cdot p)}+\frac{3\,e\,\Phi\,\gamma\cdot a\,\phi}{2\,m\,s\,(k\cdot p)}-\frac{i\,D\,e\,\gamma^\beta.\overline{\gamma}^5\,\text{FDx}^\beta\,\phi}{4\,m\,s\,(k\cdot p)}+\frac{3\,i\,e\,\gamma^\beta.\overline{\gamma}^5\,\text{FDx}^\beta\,\phi}{4\,m\,s\,(k\cdot p)}+\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}-\frac{3\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}-D-\frac{1}{2}\,i\,D\,e\,s\,\sigma\text{F}+\frac{3}{2}\,i\,e\,s\,\sigma\text{F}+\frac{D\,\gamma\cdot x}{2\,m\,s}-\frac{3\,\gamma\cdot x}{2\,m\,s}+\frac{i\,D\,e\,\gamma^\beta.\overline{\gamma}^5\,\text{FDx}^\beta}{2\,m}-\frac{3\,i\,e\,\gamma^\beta.\overline{\gamma}^5\,\text{FDx}^\beta}{2\,m}+\frac{D\,e\,\Phi\,\gamma\cdot k\,(a\cdot x)}{2\,m\,s\,(k\cdot p)}-\frac{3\,e\,\Phi\,\gamma\cdot k\,(a\cdot x)}{2\,m\,s\,(k\cdot p)}+3-\frac{2\,i\,t\,\gamma\cdot k}{m\,s\,\phi}\right)$$

Substituting the lightcone variables

We introduce the following notations

$(\text{xp}, \text{pp}, \text{qp}, \text{Gp}) == (x, p, q, \gamma)_+$,
$\text{xm} == x_- = \phi / m$,
$\text{pm} = p_- = \text{kp} / m$,
$(\text{xt}, \text{pt}, \text{Gt}, \text{at}) == (x, p, \gamma, a)_\perp$,
$\text{pp} = p_+$,

We introduce the following notations for $\gamma^5\ (F^*\,x)_\beta$
$\text{GFDp} == \left(\gamma^\beta\,\gamma^5\,F^*{}_{\beta\mu}\right)_+$,
$\text{GFDt} == \left(\gamma^\beta\,\gamma^5\,F^*{}_{\beta\mu}\right)_\perp$,
Note that $\left(\gamma^\beta\,\gamma^5\,F^*{}_{\beta\mu}\right)_- = 0$.

We also will use the conservation law
$\text{qm} = \text{pm}$,

qt = pt;

Scalar products will take the form

$x^2 = 2\, x_+\, x_- - x_\perp{}^2 = 2\,$xp xm $-$ xt^2,
$(px) = p_+\, x_- + p_-\, x_+ - p_\perp\, x_\perp =$ pp xm + pm xp $-$ pt xt,
$(ax) = -a_\perp\, x_\perp = -$at pt,
$\gamma p = \gamma_-\, p_+ + \gamma_+\, p_- - \gamma_\perp\, p_\perp =$ Gm $\frac{s}{x_-}\left(p^2 + p_\perp{}^2\right)$ + Gp $\frac{x_-}{2\,s}$ $-$ Gt pt,
where we used that $p_+ = \left(p^2 + p_\perp{}^2\right) / 2\, p_-$ and the definition of the proper time $s = x_- / 2\, p_-$;
$\gamma k = \gamma_-\, k_+ =$ m Gm,
$\gamma^\beta\, \gamma^5\, (F^*\, x)_\beta =$ GFDp xm $-$ GFDt xt.

The integration measure
$dx_- = d\phi / m$
$dX_- = d\Phi / m$
$\int dX_-\, d^D x \ldots = (m)^{-2} \int d\Phi\, d^{D-2} x_\perp\, d\phi\, dx_+ \ldots$

remaining integrals : $\int d\Phi\, d^{D-2} x_\perp\, d\phi\, dx_+$

```
Coeff5 = Coeff4 / (m) ^2
Phase5 = Collect[Phase4 /.
      {xv2 → 2 xm xp - xt ^ 2,
       ax → -at * xt,
       Pair[Momentum[p, D], Momentum[x, D]] → pp xm + pm xp - pt * xt,
       Pair[Momentum[q, D], Momentum[x, D]] → qp xm + qm xp - qt * xt,
       Pair[Momentum[a, D], Momentum[p, D]] → -at pt} /.
    {qm → pm, qt → pt} /. {xm → ϕ / m, pm → kp / m}, {xp, ϕ, xt}]
Matrix5 =
 Collect[Expand[Matrix4 /. {DiracGamma[Momentum[x, D], D] → Gp xm + Gm * xp - Gt * xt,
       DiracGamma[LorentzIndex[β, D], D].DiracGamma[5]
         Pair[LorentzIndex[β, D], Momentum[FDx, D]] → GFDp xm - GFDt * xt,
       Pair[Momentum[a, D], Momentum[x, D]] → -at * xt} /.
    {xm → ϕ / m}], {J0, J1, J2, xt, xp}]
```

$$-\frac{2^{-D-2}\, e^{-\frac{1}{2} i \pi D}\, e^2\, s^{-D/2}\, t^{-D/2}}{\pi^2\, m}$$

$$\phi\left(\frac{\text{at}\,e\,\text{pt}\,\Phi}{k\cdot p}+\frac{m^2\,\xi^2\,\Phi^2}{2\,(k\cdot p)}+\frac{\text{pp}}{2\,m}+\frac{\text{qp}}{2\,m}\right)+\text{xt}\,(-\text{at}\,e\,\Phi-\text{pt})+\frac{m^2\,\xi^2\,\phi^3}{24\,(k\cdot p)}+$$
$$\text{xp}\left(\frac{k\cdot p}{m}+\phi\left(-\frac{1}{2\,m\,s}-\frac{1}{2\,m\,t}\right)\right)-\frac{1}{12}\,m^2\,\xi^2\,s\,\phi^2-m^2\,s-\frac{\text{pp}\,\Phi}{m}+\frac{\text{qp}\,\Phi}{m}+\text{xt}^2\left(\frac{1}{4\,s}+\frac{1}{4\,t}\right)$$
$$\text{J1}\left(-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}-\frac{i\,e\,\text{GFDp}\,\phi^2}{4\,m^2\,s\,(k\cdot p)}-\frac{1}{3}\,m\,s\,\xi^2\,\gamma\cdot k\,\phi+\right.$$
$$\frac{\text{Gp}\,\phi}{2\,m^2\,s}+\frac{i\,e\,\sigma\text{F}\,\phi}{4\,(k\cdot p)}-\frac{e\,\Phi\,\gamma\cdot a\,\phi}{2\,m\,s\,(k\cdot p)}-\frac{i\,e\,\text{GFDp}\,\phi}{2\,m^2}+\frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}+\frac{\text{Gm}\,\text{xp}}{2\,m\,s}+$$
$$\left.\frac{1}{2}\,i\,e\,s\,\sigma\text{F}+\text{xt}\left(\frac{i\,e\,\text{GFDt}}{2\,m}+\frac{i\,e\,\phi\,\text{GFDt}}{4\,m\,s\,(k\cdot p)}-\frac{\text{Gt}}{2\,m\,s}-\frac{\text{at}\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\right)-1-\frac{2\,i\,t\,\gamma\cdot k}{m\,s\,\phi}\right)+$$
$$\text{J0}\left(\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^3}{8\,s\,(k\cdot p)^2}-\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}+\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{2\,(k\cdot p)}+\frac{i\,D\,e\,\text{GFDp}\,\phi^2}{4\,m^2\,s\,(k\cdot p)}-\right.$$
$$\frac{i\,e\,\text{GFDp}\,\phi^2}{2\,m^2\,s\,(k\cdot p)}+\frac{1}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,\phi-\frac{2}{3}\,m\,s\,\xi^2\,\gamma\cdot k\,\phi-\frac{D\,\text{Gp}\,\phi}{2\,m^2\,s}+\frac{\text{Gp}\,\phi}{m^2\,s}-\frac{i\,D\,e\,\sigma\text{F}\,\phi}{4\,(k\cdot p)}+$$
$$\frac{D\,e\,\Phi\,\gamma\cdot a\,\phi}{2\,m\,s\,(k\cdot p)}-\frac{e\,\Phi\,\gamma\cdot a\,\phi}{m\,s\,(k\cdot p)}-\frac{i\,D\,e\,\text{GFDp}\,\phi}{2\,m^2}+\frac{3\,i\,e\,\text{GFDp}\,\phi}{m^2}-\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}+$$
$$\frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{2\,s\,(k\cdot p)^2}+D+\left(\frac{\text{Gm}}{m\,s}-\frac{D\,\text{Gm}}{2\,m\,s}\right)\text{xp}+\frac{1}{2}\,i\,D\,e\,s\,\sigma\text{F}-2\,i\,e\,s\,\sigma\text{F}+$$
$$\left.\text{xt}\left(\frac{i\,D\,e\,\text{GFDt}}{2\,m}-\frac{3\,i\,e\,\text{GFDt}}{m}-\frac{i\,D\,e\,\phi\,\text{GFDt}}{4\,m\,s\,(k\cdot p)}+\frac{i\,e\,\phi\,\text{GFDt}}{2\,m\,s\,(k\cdot p)}+\frac{D\,\text{Gt}}{2\,m\,s}-\frac{\text{Gt}}{m\,s}-\frac{\text{at}\,e\,\Phi\,\gamma\cdot k}{m\,s\,(k\cdot p)}+\frac{\text{at}\,D\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\right)\right)+$$
$$\text{J2}\left(-\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}+\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}+\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}-\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}-\frac{i\,D\,e\,\text{GFDp}\,\phi^2}{4\,m^2\,s\,(k\cdot p)}+\frac{3\,i\,e\,\text{GFDp}\,\phi^2}{4\,m^2\,s\,(k\cdot p)}-\right.$$
$$\frac{1}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,\phi+m\,s\,\xi^2\,\gamma\cdot k\,\phi+\frac{D\,\text{Gp}\,\phi}{2\,m^2\,s}-\frac{3\,\text{Gp}\,\phi}{2\,m^2\,s}+\frac{i\,D\,e\,\sigma\text{F}\,\phi}{4\,(k\cdot p)}-\frac{3\,i\,e\,\sigma\text{F}\,\phi}{4\,(k\cdot p)}-\frac{D\,e\,\Phi\,\gamma\cdot a\,\phi}{2\,m\,s\,(k\cdot p)}+$$
$$\frac{3\,e\,\Phi\,\gamma\cdot a\,\phi}{2\,m\,s\,(k\cdot p)}+\frac{i\,D\,e\,\text{GFDp}\,\phi}{2\,m^2}-\frac{3\,i\,e\,\text{GFDp}\,\phi}{2\,m^2}+\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}-\frac{3\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}-D+$$
$$\left(\frac{D\,\text{Gm}}{2\,m\,s}-\frac{3\,\text{Gm}}{2\,m\,s}\right)\text{xp}-\frac{1}{2}\,i\,D\,e\,s\,\sigma\text{F}+\frac{3}{2}\,i\,e\,s\,\sigma\text{F}+\text{xt}\left(-\frac{i\,D\,e\,\text{GFDt}}{2\,m}+\frac{3\,i\,e\,\text{GFDt}}{2\,m}+\frac{i\,D\,e\,\phi\,\text{GFDt}}{4\,m\,s\,(k\cdot p)}-\right.$$
$$\left.\left.\frac{3\,i\,e\,\phi\,\text{GFDt}}{4\,m\,s\,(k\cdot p)}-\frac{D\,\text{Gt}}{2\,m\,s}+\frac{3\,\text{Gt}}{2\,m\,s}+\frac{3\,\text{at}\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}-\frac{\text{at}\,D\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\right)+3-\frac{2\,i\,t\,\gamma\cdot k}{m\,s\,\phi}\right)$$

Integration over
$\int dx_+$ and then $\int d\phi$ ...

The phase is linear in x_+,
the tems in the preexponent either do not depend or linear in x_+,
therefore we can perform the integration using the equality

$$\int dx_+ \begin{pmatrix} 1 \\ x_+ \end{pmatrix} \mathrm{Exp}[i\, x_+ P(\phi)] = 2\pi \begin{pmatrix} \delta(P(\phi)) \\ -i\,\delta'(P(\phi)) \end{pmatrix}$$

To use it, we separate the terms linear in xp == x_+.

Recall that Jk depends in ϕ, as
Jk == Jk$\left[t, \chi_l = \frac{\xi\,\phi}{2\,m^2\,t}\right]$,
In the next steps we will use the shorthand notation
Jk = Jk[ϕ].

```
Matrix52 = Collect[
  Coefficient[Expand[Matrix5], Gm xp] Gm xp /. {J0 → J0[ϕ], J1 → J1[ϕ], J2 → J2[ϕ]},
  {Gm xp, J0[ϕ], J1[ϕ], J2[ϕ]}]
Matrix51 = Collect[
  (Expand[Matrix5] /. {J0 → J0[ϕ], J1 → J1[ϕ], J2 → J2[ϕ]}) - Expand[Matrix52],
  {J0[ϕ], J1[ϕ], J2[ϕ], xt, xp}]
```

$$\mathrm{Gm\,xp}\left(\mathrm{J0}(\phi)\left(\frac{1}{m\,s}-\frac{D}{2\,m\,s}\right)+\mathrm{J2}(\phi)\left(\frac{D}{2\,m\,s}-\frac{3}{2\,m\,s}\right)+\frac{\mathrm{J1}(\phi)}{2\,m\,s}\right)$$

$$
\begin{aligned}
&\mathrm{J1}(\phi)\Bigg(-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}-\frac{i\,e\,\mathrm{GFDp}\,\phi^2}{4\,m^2\,s\,(k\cdot p)}-\\
&\quad\frac{1}{3}\,m\,s\,\xi^2\,\gamma\cdot k\,\phi+\frac{\mathrm{Gp}\,\phi}{2\,m^2\,s}+\frac{i\,e\,\sigma\mathrm{F}\,\phi}{4\,(k\cdot p)}-\frac{e\,\Phi\,\gamma\cdot a\,\phi}{2\,m\,s\,(k\cdot p)}-\frac{i\,e\,\mathrm{GFDp}\,\phi}{2\,m^2}+\frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}+\\
&\quad\frac{1}{2}\,i\,e\,s\,\sigma\mathrm{F}+\mathrm{xt}\left(\frac{i\,e\,\mathrm{GFDt}}{2\,m}+\frac{i\,e\,\phi\,\mathrm{GFDt}}{4\,m\,s\,(k\cdot p)}-\frac{\mathrm{Gt}}{2\,m\,s}-\frac{\mathrm{at}\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\right)-1-\frac{2\,i\,t\,\gamma\cdot k}{m\,s\,\phi}\Bigg)+\\
&\mathrm{J0}(\phi)\Bigg(\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^3}{8\,s\,(k\cdot p)^2}-\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}+\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{2\,(k\cdot p)}+\frac{i\,D\,e\,\mathrm{GFDp}\,\phi^2}{4\,m^2\,s\,(k\cdot p)}-\frac{i\,e\,\mathrm{GFDp}\,\phi^2}{2\,m^2\,s\,(k\cdot p)}+\\
&\quad\frac{1}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,\phi-\frac{2}{3}\,m\,s\,\xi^2\,\gamma\cdot k\,\phi-\frac{D\,\mathrm{Gp}\,\phi}{2\,m^2\,s}+\frac{\mathrm{Gp}\,\phi}{m^2\,s}-\frac{i\,D\,e\,\sigma\mathrm{F}\,\phi}{4\,(k\cdot p)}+\frac{D\,e\,\Phi\,\gamma\cdot a\,\phi}{2\,m\,s\,(k\cdot p)}-\frac{e\,\Phi\,\gamma\cdot a\,\phi}{m\,s\,(k\cdot p)}-\\
&\quad\frac{i\,D\,e\,\mathrm{GFDp}\,\phi}{2\,m^2}+\frac{3\,i\,e\,\mathrm{GFDp}\,\phi}{m^2}-\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}+\frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{2\,s\,(k\cdot p)^2}+D+\frac{1}{2}\,i\,D\,e\,s\,\sigma\mathrm{F}-2\,i\,e\,s\,\sigma\mathrm{F}+\\
&\quad\mathrm{xt}\left(\frac{i\,D\,e\,\mathrm{GFDt}}{2\,m}-\frac{3\,i\,e\,\mathrm{GFDt}}{m}-\frac{i\,D\,e\,\phi\,\mathrm{GFDt}}{4\,m\,s\,(k\cdot p)}+\frac{i\,e\,\phi\,\mathrm{GFDt}}{2\,m\,s\,(k\cdot p)}+\frac{D\,\mathrm{Gt}}{2\,m\,s}-\frac{\mathrm{Gt}}{m\,s}-\frac{\mathrm{at}\,e\,\Phi\,\gamma\cdot k}{m\,s\,(k\cdot p)}+\frac{\mathrm{at}\,D\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\right)\Bigg)+\\
&\mathrm{J2}(\phi)\Bigg(-\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}+\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}+\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}-\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}-\frac{i\,D\,e\,\mathrm{GFDp}\,\phi^2}{4\,m^2\,s\,(k\cdot p)}+\\
&\quad\frac{3\,i\,e\,\mathrm{GFDp}\,\phi^2}{4\,m^2\,s\,(k\cdot p)}-\frac{1}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,\phi+m\,s\,\xi^2\,\gamma\cdot k\,\phi+\frac{D\,\mathrm{Gp}\,\phi}{2\,m^2\,s}-\frac{3\,\mathrm{Gp}\,\phi}{2\,m^2\,s}+\frac{i\,D\,e\,\sigma\mathrm{F}\,\phi}{4\,(k\cdot p)}-\\
&\quad\frac{3\,i\,e\,\sigma\mathrm{F}\,\phi}{4\,(k\cdot p)}-\frac{D\,e\,\Phi\,\gamma\cdot a\,\phi}{2\,m\,s\,(k\cdot p)}+\frac{3\,e\,\Phi\,\gamma\cdot a\,\phi}{2\,m\,s\,(k\cdot p)}+\frac{i\,D\,e\,\mathrm{GFDp}\,\phi}{2\,m^2}-\frac{3\,i\,e\,\mathrm{GFDp}\,\phi}{2\,m^2}+\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}-\\
&\quad\frac{3\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}-D-\frac{1}{2}\,i\,D\,e\,s\,\sigma\mathrm{F}+\frac{3}{2}\,i\,e\,s\,\sigma\mathrm{F}+\mathrm{xt}\Bigg(-\frac{i\,D\,e\,\mathrm{GFDt}}{2\,m}+\frac{3\,i\,e\,\mathrm{GFDt}}{2\,m}+\frac{i\,D\,e\,\phi\,\mathrm{GFDt}}{4\,m\,s\,(k\cdot p)}-\\
&\qquad\frac{3\,i\,e\,\phi\,\mathrm{GFDt}}{4\,m\,s\,(k\cdot p)}-\frac{D\,\mathrm{Gt}}{2\,m\,s}+\frac{3\,\mathrm{Gt}}{2\,m\,s}+\frac{3\,\mathrm{at}\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}-\frac{\mathrm{at}\,D\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\Bigg)+3-\frac{2\,i\,t\,\gamma\cdot k}{m\,s\,\phi}\Bigg)
\end{aligned}
$$

```
Phase5xp =
  Collect[Simplify[Coefficient[Phase5, xp] /. {t → s ω / (s - ω)}], {kp}, Simplify]
ϕ0 = Collect[ϕ /. Solve[Phase5xp == 0, ϕ][[1]], {kp}, Simplify]
dP = Simplify[D[Phase5xp, ϕ]] /. {ϕ → ϕ0}
AbsdP = -dP
Phase5noxp = Collect[Phase5 /. {xp → 0} /. {t → s ω / (s - ω)}, {xp, ϕ, xt}, Simplify]
```

$$\frac{k\cdot p}{m}-\frac{\phi}{2\,m\,\omega}$$

$$2\,\omega\,(k\cdot p)$$

$$-\frac{1}{2\,m\,\omega}$$

$$\frac{1}{2\,m\,\omega}$$

$$\frac{1}{2}\,\phi\left(\frac{\Phi\left(2\,\mathrm{at}\,e\,\mathrm{pt}+m^2\,\xi^2\,\Phi\right)}{k\cdot p}+\frac{\mathrm{pp}+\mathrm{qp}}{m}\right)+\mathrm{xt}\,(-\mathrm{at}\,e\,\Phi-\mathrm{pt})+\frac{m^2\,\xi^2\,\phi^3}{24\,(k\cdot p)}-\frac{m^3\,s+\mathrm{pp}\,\Phi-\mathrm{qp}\,\Phi}{m}-\frac{1}{12}\,m^2\,\xi^2\,s\,\phi^2+\frac{\mathrm{xt}^2}{4\,\omega}$$

Now we can perform the integrations

$$\int dx_+ \begin{pmatrix} 1 \\ x_+ \end{pmatrix} \mathrm{Exp}[\mathbb{i}\, x_+ P(\phi)] =$$

$$2\pi \begin{pmatrix} \delta(P(\phi)) \\ -\mathbb{i}\delta'(P(\phi)) \end{pmatrix} = 2\pi \begin{pmatrix} \delta(\phi-\phi_0)\frac{1}{|P'(\phi_0)|} \\ -\mathbb{i}\delta(\phi-\phi_0)\frac{1}{|P'(\phi_0)|}\left(-\frac{d}{d\phi}\frac{1}{P'(\phi)}.\right) \end{pmatrix}$$

Here $\left(-\frac{d}{d\phi}\frac{1}{P'(\phi)}.\right)$ is an operator,

acting on a function in the place of ".". Then,

$$\int d\phi\, 2\pi \begin{pmatrix} \delta(\phi-\phi_0)\frac{1}{|P'(\phi_0)|} \\ -\mathbb{i}\,\delta(\phi-\phi_0)\frac{1}{|P'(\phi_0)|}\left(-\frac{d}{d\phi}\frac{1}{P'(\phi)}.\right) \end{pmatrix} f(\phi)\, e^{\mathbb{i}\, g(\phi)} =$$

$$\frac{2\pi}{|P'(\phi_0)|} e^{\mathbb{i}\, g(\phi_0)} \begin{pmatrix} f(\phi_0) \\ \mathbb{i}\frac{d}{d\phi}\left(f(\phi)\frac{1}{P'(\phi)}\right)|_{\phi_0} - f(\phi_0)\frac{g'(\phi_0)}{P'(\phi_0)} \end{pmatrix}$$

$P(\phi) = \frac{(k\,p)}{m} - \frac{\phi}{2\,m\,\omega}$,

$P'(\phi) = -\frac{1}{2\,m\,\omega}$,

$\omega^{-1} = s^{-1} + t^{-1}$, $\omega = s\,t/(s+t)$

Note that we will take the derivative of $Jk[\phi]$ in ϕ too;

We introduce the shorthand

$DJk == \frac{d}{d\phi} Jk[\phi] == \frac{d}{d\phi} J_k(t, \chi_l(\phi))$

The remaining integrals : $\int d\Phi\, d^{D-2} x_\perp$

```
Coeff6 = Coeff5 * 2 π / AbsdP
Phase6 =
 Collect[Expand[Simplify[Phase5 /. {xp → 0} /. {ϕ → ϕ0} /. {t →  s ω / (s - ω)}]], {xt, Φ}]
```

$$-\frac{2^{-D}\, e^{-\frac{1}{2} i \pi D}\, e^2\, \omega\, s^{-D/2}\, t^{-D/2}}{\pi}$$

$$\frac{1}{3} m^2 \xi^2 \omega^3 (k\cdot p)^2 - \frac{1}{3} m^2 \xi^2 s\,\omega^2 (k\cdot p)^2 + \Phi\left(2\,\mathrm{at}\, e\,\mathrm{pt}\,\omega - \frac{\mathrm{pp}}{m} + \frac{\mathrm{qp}}{m}\right) +$$
$$\mathrm{xt}\,(-\mathrm{at}\, e\,\Phi - \mathrm{pt}) + \frac{\mathrm{pp}\,\omega\,(k\cdot p)}{m} + \frac{\mathrm{qp}\,\omega\,(k\cdot p)}{m} + m^2 \xi^2 \Phi^2 \omega - m^2 s + \frac{\mathrm{xt}^2}{4\,\omega}$$

N**ote that Matrix52 produces terms that are proportinal to Jk and $\frac{dJk}{s\phi}$.**

We combine the ones that are proportional to Jk together in Matrix61, and leave the rest in Matrix62

We also again denote

$Jk == Jk\left[t, \frac{\xi \phi_0}{2 m^2 t}\right]$

```
Matrix61 =
 Collect[Expand[Matrix51 - (Matrix52 /. {xp → D[Phase5noxp, ϕ] / dP})] /. {J0[ϕ] → J0,
     J1[ϕ] → J1, J2[ϕ] → J2} /. {ϕ → ϕ0}, {J0, J1, J2, xt, Gm, Gp, GFDp, pp}]
Matrix62 = I D[1 / dP * Matrix52, ϕ] /. {xp → 1} /. {ϕ → ϕ0}
```

$$\mathrm{J1}\left(-\frac{m\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{2\,s}-m\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2+\frac{1}{2}\,i\,e\,\sigma\mathrm{F}\,\omega-\right.$$
$$\frac{e\,\Phi\,\gamma\cdot a\,\omega}{m\,s}-\frac{2}{3}\,m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega+\frac{\mathrm{Gp}\,(k\cdot p)\,\omega}{m^2\,s}+\frac{m\xi^2\,\Phi^2\,\gamma\cdot k\,\omega}{2\,s\,(k\cdot p)}+\frac{1}{2}\,i\,e\,s\,\sigma\mathrm{F}+$$
$$\mathrm{xt}\left(\frac{i\,e\,\omega\,\mathrm{GFDt}}{2\,m\,s}+\frac{i\,e\,\mathrm{GFDt}}{2\,m}-\frac{\mathrm{Gt}}{2\,m\,s}-\frac{\mathrm{at}\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\right)+\mathrm{GFDp}\left(-\frac{i\,e\,(k\cdot p)\,\omega^2}{m^2\,s}-\frac{i\,e\,(k\cdot p)\,\omega}{m^2}\right)+\mathrm{Gm}$$
$$\left.\left(\frac{m^2\,\xi^2\,(k\cdot p)\,\omega^3}{2\,s}-\frac{1}{3}\,m^2\,\xi^2\,(k\cdot p)\,\omega^2+\frac{\mathrm{pp}\,\omega}{2\,m\,s}+\frac{\mathrm{qp}\,\omega}{2\,m\,s}+\frac{m^2\,\xi^2\,\Phi^2\,\omega}{2\,s\,(k\cdot p)}+\frac{\mathrm{at}\,e\,\mathrm{pt}\,\Phi\,\omega}{s\,(k\cdot p)}\right)-1-\frac{i\,t\,\gamma\cdot k}{m\,s\,(k\cdot p)\,\omega}\right)+$$
$$\mathrm{J0}\left(\frac{D\,m\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{2\,s}-\frac{m\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{s}-D\,m\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2+6\,m\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2-\right.$$
$$\frac{1}{2}\,i\,D\,e\,\sigma\mathrm{F}\,\omega+\frac{D\,e\,\Phi\,\gamma\cdot a\,\omega}{m\,s}-\frac{2\,e\,\Phi\,\gamma\cdot a\,\omega}{m\,s}+\frac{2}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega-$$
$$\frac{4}{3}\,m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega-\frac{D\,m\xi^2\,\Phi^2\,\gamma\cdot k\,\omega}{2\,s\,(k\cdot p)}+\frac{m\xi^2\,\Phi^2\,\gamma\cdot k\,\omega}{s\,(k\cdot p)}+D+\frac{1}{2}\,i\,D\,e\,s\,\sigma\mathrm{F}-2\,i\,e\,s\,\sigma\mathrm{F}+$$
$$\mathrm{xt}\left(-\frac{i\,D\,e\,\omega\,\mathrm{GFDt}}{2\,m\,s}+\frac{i\,e\,\omega\,\mathrm{GFDt}}{m\,s}+\frac{i\,D\,e\,\mathrm{GFDt}}{2\,m}-\frac{3\,i\,e\,\mathrm{GFDt}}{m}+\frac{D\,\mathrm{Gt}}{2\,m\,s}-\frac{\mathrm{Gt}}{m\,s}-\frac{\mathrm{at}\,e\,\Phi\,\gamma\cdot k}{m\,s\,(k\cdot p)}+\frac{\mathrm{at}\,D\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\right)+$$
$$\mathrm{Gp}\left(\frac{2\,\omega\,(k\cdot p)}{m^2\,s}-\frac{D\,\omega\,(k\cdot p)}{m^2\,s}\right)+\mathrm{GFDp}\left(\frac{i\,D\,e\,(k\cdot p)\,\omega^2}{m^2\,s}-\frac{2\,i\,e\,(k\cdot p)\,\omega^2}{m^2\,s}-\frac{i\,D\,e\,(k\cdot p)\,\omega}{m^2}+\frac{6\,i\,e\,(k\cdot p)\,\omega}{m^2}\right)+$$
$$\mathrm{Gm}\left(-\frac{D\,m^2\,\xi^2\,(k\cdot p)\,\omega^3}{2\,s}+\frac{m^2\,\xi^2\,(k\cdot p)\,\omega^3}{s}+\frac{1}{3}\,D\,m^2\,\xi^2\,(k\cdot p)\,\omega^2-\frac{2}{3}\,m^2\,\xi^2\,(k\cdot p)\,\omega^2-\frac{D\,\mathrm{qp}\,\omega}{2\,m\,s}+\right.$$
$$\left.\left.\frac{\mathrm{qp}\,\omega}{m\,s}-\frac{D\,m^2\,\xi^2\,\Phi^2\,\omega}{2\,s\,(k\cdot p)}+\frac{m^2\,\xi^2\,\Phi^2\,\omega}{s\,(k\cdot p)}+\frac{2\,\mathrm{at}\,e\,\mathrm{pt}\,\Phi\,\omega}{s\,(k\cdot p)}-\frac{\mathrm{at}\,D\,e\,\mathrm{pt}\,\Phi\,\omega}{s\,(k\cdot p)}+\mathrm{pp}\left(\frac{\omega}{m\,s}-\frac{D\,\omega}{2\,m\,s}\right)\right)\right)+$$
$$\mathrm{J2}\left(-\frac{D\,m\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{2\,s}+\frac{3\,m\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{2\,s}+D\,m\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2-3\,m\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2+\right.$$
$$\frac{1}{2}\,i\,D\,e\,\sigma\mathrm{F}\,\omega-\frac{3}{2}\,i\,e\,\sigma\mathrm{F}\,\omega-\frac{D\,e\,\Phi\,\gamma\cdot a\,\omega}{m\,s}+\frac{3\,e\,\Phi\,\gamma\cdot a\,\omega}{m\,s}-\frac{2}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega+$$
$$2\,m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega+\frac{D\,m\xi^2\,\Phi^2\,\gamma\cdot k\,\omega}{2\,s\,(k\cdot p)}-\frac{3\,m\xi^2\,\Phi^2\,\gamma\cdot k\,\omega}{2\,s\,(k\cdot p)}-D-\frac{1}{2}\,i\,D\,e\,s\,\sigma\mathrm{F}+$$
$$\frac{3}{2}\,i\,e\,s\,\sigma\mathrm{F}+\mathrm{xt}\left(\frac{i\,D\,e\,\omega\,\mathrm{GFDt}}{2\,m\,s}-\frac{3\,i\,e\,\omega\,\mathrm{GFDt}}{2\,m\,s}-\frac{i\,D\,e\,\mathrm{GFDt}}{2\,m}+\frac{3\,i\,e\,\mathrm{GFDt}}{2\,m}-\frac{D\,\mathrm{Gt}}{2\,m\,s}+\right.$$
$$\left.\frac{3\,\mathrm{Gt}}{2\,m\,s}+\frac{3\,\mathrm{at}\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}-\frac{\mathrm{at}\,D\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\right)+\mathrm{Gp}\left(\frac{D\,\omega\,(k\cdot p)}{m^2\,s}-\frac{3\,\omega\,(k\cdot p)}{m^2\,s}\right)+$$
$$\mathrm{GFDp}\left(-\frac{i\,D\,e\,(k\cdot p)\,\omega^2}{m^2\,s}+\frac{3\,i\,e\,(k\cdot p)\,\omega^2}{m^2\,s}+\frac{i\,D\,e\,(k\cdot p)\,\omega}{m^2}-\frac{3\,i\,e\,(k\cdot p)\,\omega}{m^2}\right)+\mathrm{Gm}\left(\frac{D\,m^2\,\xi^2\,(k\cdot p)\,\omega^3}{2\,s}-\right.$$
$$\frac{3\,m^2\,\xi^2\,(k\cdot p)\,\omega^3}{2\,s}-\frac{1}{3}\,D\,m^2\,\xi^2\,(k\cdot p)\,\omega^2+m^2\,\xi^2\,(k\cdot p)\,\omega^2+\frac{D\,\mathrm{qp}\,\omega}{2\,m\,s}-\frac{3\,\mathrm{qp}\,\omega}{2\,m\,s}+\frac{D\,m^2\,\xi^2\,\Phi^2\,\omega}{2\,s\,(k\cdot p)}-$$
$$\left.\left.\frac{3\,m^2\,\xi^2\,\Phi^2\,\omega}{2\,s\,(k\cdot p)}-\frac{3\,\mathrm{at}\,e\,\mathrm{pt}\,\Phi\,\omega}{s\,(k\cdot p)}+\frac{\mathrm{at}\,D\,e\,\mathrm{pt}\,\Phi\,\omega}{s\,(k\cdot p)}+\mathrm{pp}\left(\frac{D\,\omega}{2\,m\,s}-\frac{3\,\omega}{2\,m\,s}\right)\right)+3-\frac{i\,t\,\gamma\cdot k}{m\,s\,(k\cdot p)\,\omega}\right)$$
$$-2\,i\,\mathrm{Gm}\,m\,\omega\left(\left(\frac{1}{m\,s}-\frac{D}{2\,m\,s}\right)\mathrm{J0}'(2\,\omega\,(k\cdot p))+\left(\frac{D}{2\,m\,s}-\frac{3}{2\,m\,s}\right)\mathrm{J2}'(2\,\omega\,(k\cdot p))+\frac{\mathrm{J1}'(2\,\omega\,(k\cdot p))}{2\,m\,s}\right)$$

Let us rewrite Matrix62

After the last integration

$Jk = Jk\left(t, \frac{\xi\,\phi_0}{2\,m^2\,t}\right);$

$Jk'\,(\phi_0) = \frac{\partial Jk\left(t, \frac{\xi\,\phi}{2\,m^2\,t}\right)}{\partial\phi}\,|_{\phi_0} == \frac{\partial}{\partial\chi_l} Jk\,(t, \chi_l\,(\phi_0)) * \frac{\xi}{2\,m^2\,t};$

$\chi_l\,(\phi_0) = \frac{\xi\,\phi_0}{2\,m^2\,t} = \frac{\xi\,\omega\,(k\,p)}{m^2\,t};$

In this step,

we use the initial assumption that J_0 does not depend on χ_l, therefore $\frac{\partial}{\partial\chi_l} J0 = 0.$

We denote

$dJkd\chi l == \frac{\partial}{\partial\chi_l} Jk\,(t, \chi_l\,(\phi_0)),\;\; k = 1, 2$

```
Coeff7 = Coeff6;
Phase7 = Phase6;
Matrix71 = Matrix61;
```

```
χlϕ = ξ ϕ / 2 / m^2 / t
χlϕ0 = χlϕ /. {ϕ → ϕ0}
dχldϕ = D[χlϕ, ϕ] /. {ϕ → ϕ0}
Matrix72d =
 Collect[Matrix62 /. { J0'[2 ω kp] →  dJ0dχl * dχldϕ, J1'[2 ω kp] →  dJ1dχl * dχldϕ,
     J2'[2 ω kp] →  dJ2dχl * dχldϕ} /. {dJ0dχl → 0}, {dJ0dχl, dJ1dχl, dJ2dχl}]
```

$$\frac{\xi\,\phi}{2\,m^2\,t}$$

$$\frac{\xi\,\omega\,(k\cdot p)}{m^2\,t}$$

$$\frac{\xi}{2\,m^2\,t}$$

$$-\frac{i\,\mathrm{dJ2d}\chi\mathrm{l}\,\mathrm{Gm}\,\xi\,\omega\left(\frac{D}{2\,m\,s}-\frac{3}{2\,m\,s}\right)}{m\,t}-\frac{i\,\mathrm{dJ1d}\chi\mathrm{l}\,\mathrm{Gm}\,\xi\,\omega}{2\,m^2\,s\,t}$$

```
Matrix7 = Collect[Matrix71 + Matrix72d,
  {J0, J1, J2, dJ0dχl, dJ1dχl, dJ2dχl, xt, Gm, Gp, GFDp, pp}]
```

$$
\begin{aligned}
&-\frac{i\,\mathrm{dJ2d\chi l}\,\mathrm{Gm}\left(\frac{D}{2\,m\,s}-\frac{3}{2\,m\,s}\right)\xi\,\omega}{m\,t}-\frac{i\,\mathrm{dJ1d\chi l}\,\mathrm{Gm}\,\xi\,\omega}{2\,m^2\,s\,t}+\\
&\mathrm{J1}\left(-\frac{m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{2\,s}-m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2+\frac{1}{2}\,i\,e\,\sigma\mathrm{F}\,\omega-\frac{e\,\Phi\,\gamma\cdot a\,\omega}{m\,s}-\right.\\
&\quad\frac{2}{3}\,m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega+\frac{\mathrm{Gp}\,(k\cdot p)\,\omega}{m^2\,s}+\frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\omega}{2\,s\,(k\cdot p)}+\frac{1}{2}\,i\,e\,s\,\sigma\mathrm{F}+\\
&\quad\mathrm{xt}\left(\frac{i\,e\,\omega\,\mathrm{GFDt}}{2\,m\,s}+\frac{i\,e\,\mathrm{GFDt}}{2\,m}-\frac{\mathrm{Gt}}{2\,m\,s}-\frac{\mathrm{at}\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\right)+\mathrm{GFDp}\left(-\frac{i\,e\,(k\cdot p)\,\omega^2}{m^2\,s}-\frac{i\,e\,(k\cdot p)\,\omega}{m^2}\right)+\mathrm{Gm}\\
&\quad\left.\left(\frac{m^2\,\xi^2\,(k\cdot p)\,\omega^3}{2\,s}-\frac{1}{3}\,m^2\,\xi^2\,(k\cdot p)\,\omega^2+\frac{\mathrm{pp}\,\omega}{2\,m\,s}+\frac{\mathrm{qp}\,\omega}{2\,m\,s}+\frac{m^2\,\xi^2\,\Phi^2\,\omega}{2\,s\,(k\cdot p)}+\frac{\mathrm{at}\,e\,\mathrm{pt}\,\Phi\,\omega}{s\,(k\cdot p)}\right)-1-\frac{i\,t\,\gamma\cdot k}{m\,s\,(k\cdot p)\,\omega}\right)+\\
&\mathrm{J0}\left(\frac{D\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{2\,s}-\frac{m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{s}-D\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2+6\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2-\right.\\
&\quad\frac{1}{2}\,i\,D\,e\,\sigma\mathrm{F}\,\omega+\frac{D\,e\,\Phi\,\gamma\cdot a\,\omega}{m\,s}-\frac{2\,e\,\Phi\,\gamma\cdot a\,\omega}{m\,s}+\frac{2}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega-\\
&\quad\frac{4}{3}\,m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega-\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\omega}{2\,s\,(k\cdot p)}+\frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\omega}{s\,(k\cdot p)}+D+\frac{1}{2}\,i\,D\,e\,s\,\sigma\mathrm{F}-2\,i\,e\,s\,\sigma\mathrm{F}+\\
&\quad\mathrm{xt}\left(-\frac{i\,D\,e\,\omega\,\mathrm{GFDt}}{2\,m\,s}+\frac{i\,e\,\omega\,\mathrm{GFDt}}{m\,s}+\frac{i\,D\,e\,\mathrm{GFDt}}{2\,m}-\frac{3\,i\,e\,\mathrm{GFDt}}{m}+\frac{D\,\mathrm{Gt}}{2\,m\,s}-\frac{\mathrm{Gt}}{m\,s}-\frac{\mathrm{at}\,e\,\Phi\,\gamma\cdot k}{m\,s\,(k\cdot p)}+\frac{\mathrm{at}\,D\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\right)+\\
&\quad\mathrm{Gp}\left(\frac{2\,\omega\,(k\cdot p)}{m^2\,s}-\frac{D\,\omega\,(k\cdot p)}{m^2\,s}\right)+\mathrm{GFDp}\left(\frac{i\,D\,e\,(k\cdot p)\,\omega^2}{m^2\,s}-\frac{2\,i\,e\,(k\cdot p)\,\omega^2}{m^2\,s}-\frac{i\,D\,e\,(k\cdot p)\,\omega}{m^2}+\frac{6\,i\,e\,(k\cdot p)\,\omega}{m^2}\right)+\\
&\quad\mathrm{Gm}\left(-\frac{D\,m^2\,\xi^2\,(k\cdot p)\,\omega^3}{2\,s}+\frac{m^2\,\xi^2\,(k\cdot p)\,\omega^3}{s}+\frac{1}{3}\,D\,m^2\,\xi^2\,(k\cdot p)\,\omega^2-\frac{2}{3}\,m^2\,\xi^2\,(k\cdot p)\,\omega^2-\frac{D\,\mathrm{qp}\,\omega}{2\,m\,s}+\right.\\
&\qquad\left.\left.\frac{\mathrm{qp}\,\omega}{m\,s}-\frac{D\,m^2\,\xi^2\,\Phi^2\,\omega}{2\,s\,(k\cdot p)}+\frac{m^2\,\xi^2\,\Phi^2\,\omega}{s\,(k\cdot p)}+\frac{2\,\mathrm{at}\,e\,\mathrm{pt}\,\Phi\,\omega}{s\,(k\cdot p)}-\frac{\mathrm{at}\,D\,e\,\mathrm{pt}\,\Phi\,\omega}{s\,(k\cdot p)}+\mathrm{pp}\left(\frac{\omega}{m\,s}-\frac{D\,\omega}{2\,m\,s}\right)\right)\right)+\\
&\mathrm{J2}\left(-\frac{D\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{2\,s}+\frac{3\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{2\,s}+D\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2-3\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2+\right.\\
&\quad\frac{1}{2}\,i\,D\,e\,\sigma\mathrm{F}\,\omega-\frac{3}{2}\,i\,e\,\sigma\mathrm{F}\,\omega-\frac{D\,e\,\Phi\,\gamma\cdot a\,\omega}{m\,s}+\frac{3\,e\,\Phi\,\gamma\cdot a\,\omega}{m\,s}-\frac{2}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega+\\
&\quad 2\,m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega+\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\omega}{2\,s\,(k\cdot p)}-\frac{3\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\omega}{2\,s\,(k\cdot p)}-D-\frac{1}{2}\,i\,D\,e\,s\,\sigma\mathrm{F}+\\
&\quad\frac{3}{2}\,i\,e\,s\,\sigma\mathrm{F}+\mathrm{xt}\left(\frac{i\,D\,e\,\omega\,\mathrm{GFDt}}{2\,m\,s}-\frac{3\,i\,e\,\omega\,\mathrm{GFDt}}{2\,m\,s}-\frac{i\,D\,e\,\mathrm{GFDt}}{2\,m}+\frac{3\,i\,e\,\mathrm{GFDt}}{2\,m}-\frac{D\,\mathrm{Gt}}{2\,m\,s}+\right.\\
&\qquad\left.\frac{3\,\mathrm{Gt}}{2\,m\,s}+\frac{3\,\mathrm{at}\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}-\frac{\mathrm{at}\,D\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\right)+\mathrm{Gp}\left(\frac{D\,\omega\,(k\cdot p)}{m^2\,s}-\frac{3\,\omega\,(k\cdot p)}{m^2\,s}\right)+\\
&\quad\mathrm{GFDp}\left(-\frac{i\,D\,e\,(k\cdot p)\,\omega^2}{m^2\,s}+\frac{3\,i\,e\,(k\cdot p)\,\omega^2}{m^2\,s}+\frac{i\,D\,e\,(k\cdot p)\,\omega}{m^2}-\frac{3\,i\,e\,(k\cdot p)\,\omega}{m^2}\right)+\mathrm{Gm}\left(\frac{D\,m^2\,\xi^2\,(k\cdot p)\,\omega^3}{2\,s}-\right.\\
&\qquad\frac{3\,m^2\,\xi^2\,(k\cdot p)\,\omega^3}{2\,s}-\frac{1}{3}\,D\,m^2\,\xi^2\,(k\cdot p)\,\omega^2+m^2\,\xi^2\,(k\cdot p)\,\omega^2+\frac{D\,\mathrm{qp}\,\omega}{2\,m\,s}-\frac{3\,\mathrm{qp}\,\omega}{2\,m\,s}+\frac{D\,m^2\,\xi^2\,\Phi^2\,\omega}{2\,s\,(k\cdot p)}-\\
&\qquad\left.\left.\frac{3\,m^2\,\xi^2\,\Phi^2\,\omega}{2\,s\,(k\cdot p)}-\frac{3\,\mathrm{at}\,e\,\mathrm{pt}\,\Phi\,\omega}{s\,(k\cdot p)}+\frac{\mathrm{at}\,D\,e\,\mathrm{pt}\,\Phi\,\omega}{s\,(k\cdot p)}+\mathrm{pp}\left(\frac{D\,\omega}{2\,m\,s}-\frac{3\,\omega}{2\,m\,s}\right)\right)+3-\frac{i\,t\,\gamma\cdot k}{m\,s\,(k\cdot p)\,\omega}\right)
\end{aligned}
$$

Integration over
$$\int d^{D-2} x_\perp \ldots :$$

The integral is gaussian

$$I_0 = \int d^{D-2} x_\perp \, \mathrm{Exp}\left[I\, A\, x_\perp^2 + I\, (J.x_\perp)\right] =$$

$$= \mathrm{Exp}\left[I \frac{\pi}{2} \frac{D-2}{2}\right] \pi^{\frac{D-2}{2}} (\det A)^{-\frac{1}{2}} \mathrm{Exp}\left[-I \frac{1}{4} J.A^{-1}.J\right]$$

The preexponent contains terms linear in $x_\perp$, so we will also need

$$I_1 = \int d^{D-2} x_\perp \, x_\perp \, \mathrm{Exp}\left[I\, A\, x_\perp^2 + I\, (J.x_\perp)\right] =$$

$$= -\frac{1}{2} \left(A^{-1}.J\right) I_0;$$

$$I_2 = \int d^{D-2} x_\perp \, x_\perp^2 \, \mathrm{Exp}\left[I\, A\, x_\perp^2 + I\, (J.x_\perp)\right] =$$

$$= \left[I \frac{1}{2} \mathrm{Tr}\, A^{-1} + \left(-\frac{1}{2} \left(A^{-1}.J\right)\right)^2\right] I_0$$

The remaining integrals : $\int d\Phi$

```
Amatr = Coefficient[Phase7, xt^2]
DetA = Amatr^(D - 2)
Jvec = Coefficient[Phase7, xt]
Jvec2 = Collect[Expand[Jvec^2], {Φ}]
CI0 = Exp[I Pi / 2 (D / 2 - 1)] Pi^(D / 2 - 1) (Amatr)^(-(D - 2) / 2)
PhaseI0 = -1 / 4 (1 / Amatr) Jvec2
xtQuadraticSubst = {xt^2 → (I * 1 / 2 / Amatr * (D - 2) + (-1 / 2 Amatr^(-1) Jvec)^2)}
xtLinearSubst = {xt → -1 / 2 Amatr^(-1) Jvec}
```

$$\frac{1}{4\,\omega}$$

$$4^{2-D} \left(\frac{1}{\omega}\right)^{D-2}$$

$$-\mathrm{at}\, e\, \Phi - \mathrm{pt}$$

$$\mathrm{at}^2\, e^2\, \Phi^2 + 2\, \mathrm{at}\, e\, \mathrm{pt}\, \Phi + \mathrm{pt}^2$$

$$2^{D-2}\, e^{\frac{1}{2} i \pi \left(\frac{D}{2}-1\right)}\, \pi^{\frac{D}{2}-1} \left(\frac{1}{\omega}\right)^{\frac{2-D}{2}}$$

$\omega\left(-\left(\mathrm{at}^2 e^2 \Phi^2 + 2\,\mathrm{at}\, e\, \mathrm{pt}\, \Phi + \mathrm{pt}^2\right)\right)$

$\left\{\mathrm{xt}^2 \to 4\,\omega^2 \left(-\mathrm{at}\, e\, \Phi - \mathrm{pt}\right)^2 + 2\, i\, (D-2)\, \omega\right\}$

$\{\mathrm{xt} \to -2\,\omega\,(-\mathrm{at}\, e\, \Phi - \mathrm{pt})\}$

We find that

$A = \frac{1}{4\,\omega}$,
$J = -e a_\perp \Phi - p_\perp$,
$\det A = \left(\frac{1}{4\,\omega}\right)^{D-2}$,
$A^{-1} = 4\,\omega$,

Also we use the following equalities for further simplifications :

$a^\mu \gamma^\beta \gamma^5 F^*{}_{\beta\mu} == -a_\perp \left(\gamma^\beta \gamma^5 F^*{}_{\beta\mu}\right)_\perp ==$ at GFDt $= 0$, as $a^\mu F^*{}_{\beta\mu} = 0$,
at Gt $== -(\gamma\, a)$,
at pt $== (ap)$,
$\mathrm{at}^2 == -a^2 = \xi^2 m^2 / e^2$

```
Coeff8 = Simplify[Coeff7 * CI0, Assumptions → {ω > 0, m > 0, t > 0, s > 0}]
Phase8 = Collect[(Phase7 /. {xt → 0}) + PhaseI0, Φ] /. {at^2 → ξ^2 m^2 / e^2}
Matrix8 = Collect[
  Expand[
    Matrix7 /. xtQuadraticSubst /. xtLinearSubst
   ] /. {at GFDt → 0, at Gt → -DiracGamma[Momentum[a, D], D],
    at^2 → ξ^2 m^2 / e^2, at pt → -Pair[Momentum[a, D], Momentum[p, D]]},
  {J0, J1, J2, dJ0dχl, dJ1dχl, dJ2dχl, Φ, Gm, Gp, GFDp, pp}]
```

$$\frac{1}{4}\, i\, e^{-\frac{1}{4} i \pi D}\, \pi^{\frac{D}{2}-2}\, e^2 \left(\frac{s\,t}{\omega}\right)^{-D/2}$$

$$\frac{1}{3} m^2 \xi^2 \omega^3 (k\cdot p)^2 - \frac{1}{3} m^2 \xi^2 s\, \omega^2 (k\cdot p)^2 + \frac{\mathrm{pp}\,\omega\,(k\cdot p)}{m} + \frac{\mathrm{qp}\,\omega\,(k\cdot p)}{m} - m^2 s + \Phi\left(\frac{\mathrm{qp}}{m} - \frac{\mathrm{pp}}{m}\right) - \mathrm{pt}^2\,\omega$$

$$-\frac{i\,\mathrm{dJ1d\chi l}\,\mathrm{Gm}\,\xi\,\omega}{2\,m^2\,s\,t} + \mathrm{dJ2d\chi l}\,\mathrm{Gm}\left(\frac{3\,i\,\xi\,\omega}{2\,m^2\,s\,t} - \frac{i\,D\,\xi\,\omega}{2\,m^2\,s\,t}\right) +$$
$$\mathrm{J1}\left(-\frac{m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{2\,s} - m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2 + \frac{i\,e\,\mathrm{GFDt}\,\mathrm{pt}\,\omega^2}{m\,s} + \frac{i\,e\,\mathrm{GFDt}\,\mathrm{pt}\,\omega}{m} + \frac{1}{2}\, i\, e\,\sigma\mathrm{F}\,\omega - \right.$$
$$\frac{2}{3} m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega + \frac{\mathrm{Gp}\,(k\cdot p)\,\omega}{m^2\,s} - \frac{\mathrm{Gt}\,\mathrm{pt}\,\omega}{m\,s} + \frac{1}{2}\, i\, e\, s\,\sigma\mathrm{F} + \Phi^2\left(\frac{\mathrm{Gm}\,m^2\,\xi^2\,\omega}{2\,s\,(k\cdot p)} - \frac{m\,\xi^2\,\omega\,\gamma\cdot k}{2\,s\,(k\cdot p)}\right) +$$
$$\Phi\left(\frac{e\,\omega\,\gamma\cdot k\,(a\cdot p)}{m\,s\,(k\cdot p)} - \frac{e\,\mathrm{Gm}\,\omega\,(a\cdot p)}{s\,(k\cdot p)}\right) + \mathrm{GFDp}\left(-\frac{i\,e\,(k\cdot p)\,\omega^2}{m^2\,s} - \frac{i\,e\,(k\cdot p)\,\omega}{m^2}\right) +$$

$$
\begin{aligned}
&\quad \mathrm{Gm}\left(\frac{m^2\,\xi^2\,(k\cdot p)\,\omega^3}{2\,s}-\frac{1}{3}\,m^2\,\xi^2\,(k\cdot p)\,\omega^2+\frac{\mathrm{pp}\,\omega}{2\,m\,s}+\frac{\mathrm{qp}\,\omega}{2\,m\,s}\right)-1-\frac{i\,t\,\gamma\cdot k}{m\,s\,(k\cdot p)\,\omega}\Bigg)+\\
&\mathrm{J0}\Bigg(\frac{D\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{2\,s}-\frac{m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{s}-D\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2+6\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2-\\
&\quad \frac{i\,D\,e\,\mathrm{GFDt}\,\mathrm{pt}\,\omega^2}{m\,s}+\frac{2\,i\,e\,\mathrm{GFDt}\,\mathrm{pt}\,\omega^2}{m\,s}+\frac{i\,D\,e\,\mathrm{GFDt}\,\mathrm{pt}\,\omega}{m}-\frac{6\,i\,e\,\mathrm{GFDt}\,\mathrm{pt}\,\omega}{m}-\frac{1}{2}\,i\,D\,e\,\sigma\mathrm{F}\,\omega+\\
&\quad \frac{2}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega-\frac{4}{3}\,m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega+\frac{D\,\mathrm{Gt}\,\mathrm{pt}\,\omega}{m\,s}-\frac{2\,\mathrm{Gt}\,\mathrm{pt}\,\omega}{m\,s}+D+\\
&\quad \frac{1}{2}\,i\,D\,e\,s\,\sigma\mathrm{F}-2\,i\,e\,s\,\sigma\mathrm{F}+\Phi^2\left(\frac{D\,m\,\omega\,\gamma\cdot k\,\xi^2}{2\,s\,(k\cdot p)}-\frac{m\,\omega\,\gamma\cdot k\,\xi^2}{s\,(k\cdot p)}+\mathrm{Gm}\left(\frac{m^2\,\xi^2\,\omega}{s\,(k\cdot p)}-\frac{D\,m^2\,\xi^2\,\omega}{2\,s\,(k\cdot p)}\right)\right)+\\
&\quad \Phi\left(-\frac{D\,e\,\omega\,\gamma\cdot k\,(a\cdot p)}{m\,s\,(k\cdot p)}+\frac{2\,e\,\omega\,\gamma\cdot k\,(a\cdot p)}{m\,s\,(k\cdot p)}+\mathrm{Gm}\left(\frac{D\,e\,\omega\,(a\cdot p)}{s\,(k\cdot p)}-\frac{2\,e\,\omega\,(a\cdot p)}{s\,(k\cdot p)}\right)\right)+\\
&\quad \mathrm{Gp}\left(\frac{2\,\omega\,(k\cdot p)}{m^2\,s}-\frac{D\,\omega\,(k\cdot p)}{m^2\,s}\right)+\mathrm{GFDp}\left(\frac{i\,D\,e\,(k\cdot p)\,\omega^2}{m^2\,s}-\frac{2\,i\,e\,(k\cdot p)\,\omega^2}{m^2\,s}-\frac{i\,D\,e\,(k\cdot p)\,\omega}{m^2}+\frac{6\,i\,e\,(k\cdot p)\,\omega}{m^2}\right)+\\
&\quad \mathrm{Gm}\Bigg(-\frac{D\,m^2\,\xi^2\,(k\cdot p)\,\omega^3}{2\,s}+\frac{m^2\,\xi^2\,(k\cdot p)\,\omega^3}{s}+\frac{1}{3}\,D\,m^2\,\xi^2\,(k\cdot p)\,\omega^2-\\
&\qquad \frac{2}{3}\,m^2\,\xi^2\,(k\cdot p)\,\omega^2-\frac{D\,\mathrm{qp}\,\omega}{2\,m\,s}+\frac{\mathrm{qp}\,\omega}{m\,s}+\mathrm{pp}\left(\frac{\omega}{m\,s}-\frac{D\,\omega}{2\,m\,s}\right)\Bigg)\Bigg)+\\
&\mathrm{J2}\Bigg(-\frac{D\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{2\,s}+\frac{3\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{2\,s}+D\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2-3\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2+\\
&\quad \frac{i\,D\,e\,\mathrm{GFDt}\,\mathrm{pt}\,\omega^2}{m\,s}-\frac{3\,i\,e\,\mathrm{GFDt}\,\mathrm{pt}\,\omega^2}{m\,s}-\frac{i\,D\,e\,\mathrm{GFDt}\,\mathrm{pt}\,\omega}{m}+\frac{3\,i\,e\,\mathrm{GFDt}\,\mathrm{pt}\,\omega}{m}+\frac{1}{2}\,i\,D\,e\,\sigma\mathrm{F}\,\omega-\\
&\quad \frac{3}{2}\,i\,e\,\sigma\mathrm{F}\,\omega-\frac{2}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega+2\,m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega-\frac{D\,\mathrm{Gt}\,\mathrm{pt}\,\omega}{m\,s}+\frac{3\,\mathrm{Gt}\,\mathrm{pt}\,\omega}{m\,s}-D-\\
&\quad \frac{1}{2}\,i\,D\,e\,s\,\sigma\mathrm{F}+\frac{3}{2}\,i\,e\,s\,\sigma\mathrm{F}+\Phi^2\left(-\frac{D\,m\,\omega\,\gamma\cdot k\,\xi^2}{2\,s\,(k\cdot p)}+\frac{3\,m\,\omega\,\gamma\cdot k\,\xi^2}{2\,s\,(k\cdot p)}+\mathrm{Gm}\left(\frac{D\,m^2\,\xi^2\,\omega}{2\,s\,(k\cdot p)}-\frac{3\,m^2\,\xi^2\,\omega}{2\,s\,(k\cdot p)}\right)\right)+\\
&\quad \Phi\left(\frac{D\,e\,\omega\,\gamma\cdot k\,(a\cdot p)}{m\,s\,(k\cdot p)}-\frac{3\,e\,\omega\,\gamma\cdot k\,(a\cdot p)}{m\,s\,(k\cdot p)}+\mathrm{Gm}\left(\frac{3\,e\,\omega\,(a\cdot p)}{s\,(k\cdot p)}-\frac{D\,e\,\omega\,(a\cdot p)}{s\,(k\cdot p)}\right)\right)+\mathrm{Gp}\\
&\quad\; \left(\frac{D\,\omega\,(k\cdot p)}{m^2\,s}-\frac{3\,\omega\,(k\cdot p)}{m^2\,s}\right)+\mathrm{GFDp}\left(-\frac{i\,D\,e\,(k\cdot p)\,\omega^2}{m^2\,s}+\frac{3\,i\,e\,(k\cdot p)\,\omega^2}{m^2\,s}+\frac{i\,D\,e\,(k\cdot p)\,\omega}{m^2}-\frac{3\,i\,e\,(k\cdot p)\,\omega}{m^2}\right)+\\
&\quad \mathrm{Gm}\Bigg(\frac{D\,m^2\,\xi^2\,(k\cdot p)\,\omega^3}{2\,s}-\frac{3\,m^2\,\xi^2\,(k\cdot p)\,\omega^3}{2\,s}-\frac{1}{3}\,D\,m^2\,\xi^2\,(k\cdot p)\,\omega^2+m^2\,\xi^2\,(k\cdot p)\,\omega^2+\\
&\qquad \frac{D\,\mathrm{qp}\,\omega}{2\,m\,s}-\frac{3\,\mathrm{qp}\,\omega}{2\,m\,s}+\mathrm{pp}\left(\frac{D\,\omega}{2\,m\,s}-\frac{3\,\omega}{2\,m\,s}\right)\Bigg)+3-\frac{i\,t\,\gamma\cdot k}{m\,s\,(k\cdot p)\,\omega}\Bigg)
\end{aligned}
$$

Next, we recollect all scalar products into covariant notations

After this step the dependence on Φ in the preexponent should vanish

GFDp pm $= \gamma^{\beta}\gamma^{5}\,(F^{*}p)_{\beta}$ + GFDt pt

Gp pm = − Gm pp + (γp) + GFDt pt

Gm = $(\gamma k)\,/\,m = (\gamma F^2 p)\,/\,m\left[-a^2\,(kp)\right]$

Note that after this step the preexponential does not depend on Φ

```
SubstitutionStep91 = {DiracGamma[Momentum[k, D], D] (kp) →
    DiracGamma[Momentum[FFp, D], D] e^2/m^2/ξ^2,
  GFDp → m/kp (GFDt pt + DiracGamma[LorentzIndex[β, D], D].DiracGamma[5]
       Pair[LorentzIndex[β, D], Momentum[FDp, D]]),
  Gp → m/kp (DiracGamma[Momentum[p, D], D] - Gm pp + Gt pt)}
SubstitutionStep92 = {Gm → DiracGamma[Momentum[k, D], D]/m}
SubstitutionStep93 = {DiracGamma[Momentum[k, D], D] →
    DiracGamma[Momentum[FFp, D], D] e^2/m^2/ξ^2/(kp)}
```

$$\left\{\gamma\cdot k\,(k\cdot p)\to\frac{e^2\,\gamma\cdot\mathrm{FFp}}{m^2\,\xi^2},\ \mathrm{GFDp}\to\frac{m\left(\mathrm{FDp}^{\beta}\,\gamma^{\beta}.\overline{\gamma}^{5}+\mathrm{GFDt\ pt}\right)}{k\cdot p},\ \mathrm{Gp}\to\frac{m\,(-\mathrm{Gm\ pp}+\mathrm{Gt\ pt}+\gamma\cdot p)}{k\cdot p}\right\}$$

$$\left\{\mathrm{Gm}\to\frac{\gamma\cdot k}{m}\right\}$$

$$\left\{\gamma\cdot k\to\frac{e^2\,\gamma\cdot\mathrm{FFp}}{m^2\,\xi^2\,(k\cdot p)}\right\}$$

```
Coeff9 = Coeff8
Phase9 = Phase8
Matrix9 =
 Collect[
  Expand[
   Simplify[
    Expand[
     Matrix8 /. SubstitutionStep91 /. SubstitutionStep92 /.
       SubstitutionStep93 /. {ω → s t / (s + t)}
    ]
   ]
  ],
  {J0, J1, J2, dJ0dχl, dJ1dχl, dJ2dχl, Φ, e^2 DiracGamma[Momentum[FFp, D], D],
   DiracGamma[LorentzIndex[β, D], D].DiracGamma[5]
    Pair[LorentzIndex[β, D], Momentum[FDp, D]],
   DiracGamma[Momentum[p, D], D], Pair[Momentum[a, D], Momentum[p, D]]}]
```

$$\frac{1}{4}\,i\,e^{-\frac{1}{4}\,i\,\pi\,D}\,\pi^{\frac{D}{2}-2}\,e^2\left(\frac{s\,t}{\omega}\right)^{-D/2}$$

$$\frac{1}{3}\,m^2\,\xi^2\,\omega^3\,(\,k\cdot p\,)^2-\frac{1}{3}\,m^2\,\xi^2\,s\,\omega^2\,(\,k\cdot p\,)^2+\frac{\mathrm{pp}\,\omega\,(k\cdot p)}{m}+\frac{\mathrm{qp}\,\omega\,(k\cdot p)}{m}-m^2\,s+\Phi\left(\frac{\mathrm{qp}}{m}-\frac{\mathrm{pp}}{m}\right)-\mathrm{pt}^2\,\omega$$

$$\text{dJ1d}\chi\text{l}\,\gamma\cdot\text{FFp}\left(-\frac{i\,s}{2\,m^5\,(s+t)^2\,\xi\,(k\cdot p)}-\frac{i\,t}{2\,m^5\,(s+t)^2\,\xi\,(k\cdot p)}\right)e^2+$$
$$\text{dJ2d}\chi\text{l}\,\gamma\cdot\text{FFp}\left(-\frac{i\,D\,s}{2\,m^5\,(s+t)^2\,\xi\,(k\cdot p)}+\frac{3\,i\,s}{2\,m^5\,(s+t)^2\,\xi\,(k\cdot p)}-\frac{i\,D\,t}{2\,m^5\,(s+t)^2\,\xi\,(k\cdot p)}+\frac{3\,i\,t}{2\,m^5\,(s+t)^2\,\xi\,(k\cdot p)}\right)e^2+$$
$$\text{J1}\left(\frac{i\,e\,\sigma\text{F}\,s^3}{2\,(s+t)^2}+\frac{3\,i\,e\,t\,\sigma\text{F}\,s^2}{2\,(s+t)^2}-\frac{s^2}{(s+t)^2}+\frac{i\,e\,t^2\,\sigma\text{F}\,s}{(s+t)^2}-\frac{2\,t\,s}{(s+t)^2}+\left(\frac{t^2}{m\,(s+t)^2}+\frac{s\,t}{m\,(s+t)^2}\right)\gamma\cdot p+\right.$$
$$\left(-\frac{i\,e\,t\,s^2}{m\,(s+t)^2}-\frac{2\,i\,e\,t^2\,s}{m\,(s+t)^2}\right)\gamma^\beta.\overline{\gamma}^5\,\text{FDp}^\beta+e^2\,\gamma\cdot\text{FFp}\left(-\frac{2\,t\,s^3}{3\,m\,(s+t)^2}-\frac{2\,t^2\,s^2}{m\,(s+t)^2}-\frac{\text{pp}\,t\,s}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}+\right.$$
$$\frac{\text{qp}\,t\,s}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}-\frac{i\,s}{m^3\,(s+t)^2\,\xi^2\,(k\cdot p)^2}-\frac{\text{pp}\,t^2}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}+\frac{\text{qp}\,t^2}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}-$$
$$\left.\left.\frac{3\,i\,t}{m^3\,(s+t)^2\,\xi^2\,(k\cdot p)^2}-\frac{3\,i\,t^2}{m^3\,(s+t)^2\,\xi^2\,(k\cdot p)^2\,s}-\frac{i\,t^3}{m^3\,(s+t)^2\,\xi^2\,(k\cdot p)^2\,s^2}\right)-\frac{t^2}{(s+t)^2}\right)+$$
$$\text{J0}\left(\frac{i\,D\,e\,\sigma\text{F}\,s^3}{2\,(s+t)^2}-\frac{2\,i\,e\,\sigma\text{F}\,s^3}{(s+t)^2}+\frac{i\,D\,e\,t\,\sigma\text{F}\,s^2}{2\,(s+t)^2}-\frac{4\,i\,e\,t\,\sigma\text{F}\,s^2}{(s+t)^2}+\frac{D\,s^2}{(s+t)^2}-\frac{2\,i\,e\,t^2\,\sigma\text{F}\,s}{(s+t)^2}+\right.$$
$$\frac{2\,D\,t\,s}{(s+t)^2}+\left(-\frac{D\,t^2}{m\,(s+t)^2}+\frac{2\,t^2}{m\,(s+t)^2}-\frac{D\,s\,t}{m\,(s+t)^2}+\frac{2\,s\,t}{m\,(s+t)^2}\right)\gamma\cdot p+$$
$$\left(-\frac{i\,D\,e\,t\,s^2}{m\,(s+t)^2}+\frac{6\,i\,e\,t\,s^2}{m\,(s+t)^2}+\frac{4\,i\,e\,t^2\,s}{m\,(s+t)^2}\right)\gamma^\beta.\overline{\gamma}^5\,\text{FDp}^\beta+$$
$$e^2\,\gamma\cdot\text{FFp}\left(\frac{2\,D\,t\,s^3}{3\,m\,(s+t)^2}-\frac{4\,t\,s^3}{3\,m\,(s+t)^2}+\frac{4\,t^2\,s^2}{m\,(s+t)^2}+\frac{D\,\text{pp}\,t\,s}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}-\frac{\text{pp}\,t\,s}{m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}-\right.$$
$$\frac{D\,\text{qp}\,t\,s}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}+\frac{\text{qp}\,t\,s}{m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}+\frac{D\,\text{pp}\,t^2}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}-$$
$$\left.\left.\frac{\text{pp}\,t^2}{m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}-\frac{D\,\text{qp}\,t^2}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}+\frac{\text{qp}\,t^2}{m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}\right)+\frac{D\,t^2}{(s+t)^2}\right)+$$
$$\text{J2}\left(-\frac{i\,D\,e\,\sigma\text{F}\,s^3}{2\,(s+t)^2}+\frac{3\,i\,e\,\sigma\text{F}\,s^3}{2\,(s+t)^2}-\frac{i\,D\,e\,t\,\sigma\text{F}\,s^2}{2\,(s+t)^2}+\frac{3\,i\,e\,t\,\sigma\text{F}\,s^2}{2\,(s+t)^2}-\frac{D\,s^2}{(s+t)^2}+\frac{3\,s^2}{(s+t)^2}-\frac{2\,D\,t\,s}{(s+t)^2}+\frac{6\,t\,s}{(s+t)^2}+\right.$$
$$\left(\frac{D\,t^2}{m\,(s+t)^2}-\frac{3\,t^2}{m\,(s+t)^2}+\frac{D\,s\,t}{m\,(s+t)^2}-\frac{3\,s\,t}{m\,(s+t)^2}\right)\gamma\cdot p+\left(\frac{i\,D\,e\,s^2\,t}{m\,(s+t)^2}-\frac{3\,i\,e\,s^2\,t}{m\,(s+t)^2}\right)\gamma^\beta.\overline{\gamma}^5\,\text{FDp}^\beta+$$
$$e^2\,\gamma\cdot\text{FFp}\left(-\frac{2\,D\,t\,s^3}{3\,m\,(s+t)^2}+\frac{2\,t\,s^3}{m\,(s+t)^2}-\frac{D\,\text{pp}\,t\,s}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}+\frac{3\,\text{pp}\,t\,s}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}+\right.$$
$$\frac{D\,\text{qp}\,t\,s}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}-\frac{3\,\text{qp}\,t\,s}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}-\frac{i\,s}{m^3\,(s+t)^2\,\xi^2\,(k\cdot p)^2}-\frac{D\,\text{pp}\,t^2}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}+$$
$$\frac{3\,\text{pp}\,t^2}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}+\frac{D\,\text{qp}\,t^2}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}-\frac{3\,\text{qp}\,t^2}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}-\frac{3\,i\,t}{m^3\,(s+t)^2\,\xi^2\,(k\cdot p)^2}-$$
$$\left.\left.\frac{3\,i\,t^2}{m^3\,(s+t)^2\,\xi^2\,(k\cdot p)^2\,s}-\frac{i\,t^3}{m^3\,(s+t)^2\,\xi^2\,(k\cdot p)^2\,s^2}\right)-\frac{D\,t^2}{(s+t)^2}+\frac{3\,t^2}{(s+t)^2}\right)$$

```
Coefficient[Matrix9, Φ]
Coefficient[Matrix9, Φ^2]
```

0

0

The integration over

$$\int_{-\infty}^{\infty} d\Phi \ldots$$

is now trivial,
as the phase in linear in Φ and the preexponent does not depend on Φ

$$\int d\Phi \, \mathrm{Exp}\left[i\, \Phi \, \frac{q_+ - p_+}{m}\right] = 2\,\pi\,\delta\left(\frac{q_+ - p_+}{m}\right) = 2\,\pi\, m\,\delta\,(q_+ - p_+)$$

After this step we have collected the full delta - function, so that
$M\,(q, p) = \Lambda^{D-4}\,(2\,\pi)^D\,\delta^{(D)}\,(q - p)\, M\,(p)$

In what follows we will consider $M\,(p)$

We substitute
$qp == q_+ \to pp = p_+,$
$pp = \frac{1}{2\,p_-}\left(p^2 + pt^2\right) = \frac{m}{2\,kp}\left(p^2 + pt^2\right)$

and also introduce
$\chi == \chi_p = \frac{\xi\,(kp)}{m^2}$

After this step all the feasible integrations are done.

We are left with two integrals over the proper times
$\int_0^\infty ds \int_0^\infty dt \ldots$
and the implicit integraion
$\int_{-\infty}^{\infty} dl^2$ in $J_k\,(t, \chi_l)$

```
Coeff10 = Coeff9 * 2 π * m / (2 π) ^D / Λ^ (D - 4)
Phase10 = Collect[
  Expand[
   Phase9 /. {qp → pp} /. {pp → (pv2 + pt^2) m / (2 kp)} /. {kp → m^2 χ / ξ}
  ],
  χ, Simplify]
Matrix10 = Collect[Expand[Matrix9 /. {qp → pp} /. {pp → (pv2 + pt^2) m / (2 kp)}] /.
    {DiracGamma[Momentum[k, D], D] →
      DiracGamma[Momentum[FFp, D], D] e^2 / m^2 / ξ^2 / (kp)} /. {kp → m^2 χ / ξ},
  {J0, J1, J2, dJ0dχl, dJ1dχl, dJ2dχl, e^2 DiracGamma[Momentum[FFp, D], D],
   e σF, DiracGamma[LorentzIndex[β, D], D].DiracGamma[5]
    Pair[LorentzIndex[β, D], Momentum[FDp, D]], DiracGamma[Momentum[p, D], D],
   Pair[Momentum[a, D], Momentum[p, D]], χ}, Simplify]
```

$$i\, 2^{-D-1}\, e^{-\frac{1}{4} i \pi D}\, \pi^{-\frac{D}{2}-1}\, e^2\, m\, \Lambda^{4-D} \left(\frac{s\,t}{\omega}\right)^{-D/2}$$

$$\frac{1}{3} m^6 \chi^2 \omega^2 (\omega - s) - m^2 s + p^2 \omega$$

$$\mathrm{J0}\left(-\frac{i\, e\, s\, t\, \mathrm{FDp}^{\beta}\, \gamma^{\beta}.\overline{\gamma}^{5}\, ((D-6)\, s - 4\, t)}{m\,(s+t)^2} + \frac{2\, e^2\, s^2\, t\, ((D-2)\, s + 6\, t)\, \gamma\cdot\mathrm{FFp}}{3\, m\,(s+t)^2} + \frac{i\, e\, s\, \sigma\mathrm{F}\, ((D-4)\, s - 4\, t)}{2\,(s+t)} - \frac{(D-2)\, t\, \gamma\cdot p}{m\,(s+t)} + D\right) +$$
$$\mathrm{J2}\left(\frac{i\,(D-3)\, e\, s^2\, t\, \mathrm{FDp}^{\beta}\, \gamma^{\beta}.\overline{\gamma}^{5}}{m\,(s+t)^2} + e^2\, \gamma\cdot\mathrm{FFp}\left(-\frac{2\,(D-3)\, s^3\, t}{3\, m\,(s+t)^2} - \frac{i\,(s+t)}{m^7\, s^2\, \chi^2}\right) - \frac{i\,(D-3)\, e\, s^2\, \sigma\mathrm{F}}{2\,(s+t)} + \frac{(D-3)\, t\, \gamma\cdot p}{m\,(s+t)} - D + 3\right) +$$
$$\mathrm{J1}\left(-\frac{i\, e\, s\, t\, \mathrm{FDp}^{\beta}\,(s+2\,t)\, \gamma^{\beta}.\overline{\gamma}^{5}}{m\,(s+t)^2} + e^2\, \gamma\cdot\mathrm{FFp}\left(-\frac{2\, s^2\, t\,(s+3\,t)}{3\, m\,(s+t)^2} - \frac{i\,(s+t)}{m^7\, s^2\, \chi^2}\right) + \frac{i\, e\, s\, \sigma\mathrm{F}\,(s+2\,t)}{2\,(s+t)} + \frac{t\, \gamma\cdot p}{m\, s + m\, t} - 1\right) -$$
$$\frac{i\,(D-3)\, \mathrm{dJ2d}\chi\mathrm{l}\, e^2\, \gamma\cdot\mathrm{FFp}}{2\, m^7\, \chi\,(s+t)} - \frac{i\, \mathrm{dJ1d}\chi\mathrm{l}\, e^2\, \gamma\cdot\mathrm{FFp}}{2\, m^7\, \chi\,(s+t)}$$

```
Coeff10 /. {D → 4}
Phase10 /. {D → 4}
Matrix10 /. {D → 4}
```

$$-\frac{i\, e^2\, m\, \omega^2}{32\, \pi^3\, s^2\, t^2}$$

$$\omega\, \overline{p}^2 + \frac{1}{3}\, m^6\, \chi^2\, \omega^2\, (\omega - s) - m^2\, s$$

$$-\frac{i\, \text{dJ1d}\chi\text{l}\, e^2\, \overline{\gamma}\cdot\overline{\text{FFp}}}{2\, m^7\, \chi\, (s+t)} - \frac{i\, \text{dJ2d}\chi\text{l}\, e^2\, \overline{\gamma}\cdot\overline{\text{FFp}}}{2\, m^7\, \chi\, (s+t)} +$$
$$\text{J0}\left(\frac{2\, e^2\, s^2\, t\, (2\, s + 6\, t)\, \overline{\gamma}\cdot\overline{\text{FFp}}}{3\, m\, (s+t)^2} - \frac{i\, e\, s\, t\, (-2\, s - 4\, t)\, \overline{\gamma}^{\beta}.\overline{\gamma}^{5}\, \overline{\text{FDp}}^{\beta}}{m\, (s+t)^2} - \frac{2\, t\, \overline{\gamma}\cdot\overline{p}}{m\, (s+t)} - \frac{2\, i\, e\, s\, \sigma\text{F}\, t}{s+t} + 4\right) +$$
$$\text{J1}\left(e^2\, \overline{\gamma}\cdot\overline{\text{FFp}}\left(-\frac{2\, s^2\, t\, (s + 3\, t)}{3\, m\, (s+t)^2} - \frac{i\, (s+t)}{m^7\, s^2\, \chi^2}\right) - \frac{i\, e\, s\, t\, (s + 2\, t)\, \overline{\gamma}^{\beta}.\overline{\gamma}^{5}\, \overline{\text{FDp}}^{\beta}}{m\, (s+t)^2} + \frac{t\, \overline{\gamma}\cdot\overline{p}}{m\, s + m\, t} + \frac{i\, e\, s\, \sigma\text{F}\, (s + 2\, t)}{2\, (s+t)} - 1\right) +$$
$$\text{J2}\left(e^2\, \overline{\gamma}\cdot\overline{\text{FFp}}\left(-\frac{2\, s^3\, t}{3\, m\, (s+t)^2} - \frac{i\, (s+t)}{m^7\, s^2\, \chi^2}\right) + \frac{i\, e\, s^2\, t\, \overline{\gamma}^{\beta}.\overline{\gamma}^{5}\, \overline{\text{FDp}}^{\beta}}{m\, (s+t)^2} + \frac{t\, \overline{\gamma}\cdot\overline{p}}{m\, (s+t)} - \frac{i\, e\, s^2\, \sigma\text{F}}{2\, (s+t)} - 1\right)$$

Let us change the variables :

$(s, t) \to (u, \sigma)$,

where

$$s = \frac{1}{m^2}\, \frac{1+u}{\chi^{2/3}\, u^{1/3}}\, \sigma$$
$$t = \frac{1}{m^2}\, \frac{1+u}{\chi^{2/3}\, u^{4/3}}\, \sigma$$

then

$$\omega = \frac{1}{m^2}\, \frac{1}{\chi^{2/3}\, u^{1/3}}\, \sigma,$$

$$\int_0^\infty dt \int_0^\infty ds \ldots = \int_0^\infty du \int_0^\infty d\sigma\, \left| J^{-1} \right| \ldots$$

$$\left| J^{-1} \right| = \frac{\sigma\, (u+1)^2}{m^4\, u^{8/3}\, \chi^{4/3}}$$

```
$Assumptions = {χ > 0, u > 0, σ > 0};
suσ = m^(-2) (1 + u) / u^(1/3) / χ^(2/3) σ
tuσ = m^(-2) (1 + u) / u^(4/3) / χ^(2/3) σ
Jac = Simplify[D[suσ, u] D[tuσ, σ] - D[suσ, σ] D[tuσ, u]]
Simplify[(1 / suσ + 1 / tuσ)^(-1)]
```

$$\frac{\sigma (u+1)}{m^2 \sqrt[3]{u}\ \chi^{2/3}}$$

$$\frac{\sigma (u+1)}{m^2 u^{4/3} \chi^{2/3}}$$

$$\frac{\sigma (u+1)^2}{m^4 u^{8/3} \chi^{4/3}}$$

$$\frac{\sigma}{m^2 \sqrt[3]{u \chi^2}}$$

```
Coeff11 = Simplify[Coeff10 * Jac /. {ω → s t / (s + t)} /. {s → suσ, t → tuσ},
   Assumptions → {χ > 0, u > 0, σ > 0}]
Phase11 = Collect[
   Expand[
    Simplify[
     (Phase10) /. {ω → s t / (s + t)} /. {s → suσ, t → tuσ},
     Assumptions → {χ > 0, u > 0, σ > 0}
    ]
   ], {pv2, σ},
   Simplify]
Matrix11 =
  Collect[
   Expand[
    Simplify[
     Matrix10 /. {kp → χ / ξ m^2} /. {ω → s t / (s + t)} /. {s → suσ, t → tuσ},
     Assumptions → {u > 0, σ > 0, χ > 0}]
   ],
   {J0, J1, J2, dJ0dχl, dJ1dχl, dJ2dχl, e^2 DiracGamma[Momentum[FFp, D], D],
    e σF, DiracGamma[LorentzIndex[β, D], D].DiracGamma[5]
     Pair[LorentzIndex[β, D], Momentum[FDp, D]],
    DiracGamma[Momentum[p, D], D], pv2, σ, χ}, Simplify]
```

$$\frac{i\, 2^{-D-1} e^{-\frac{1}{4} i \pi D} \pi^{-\frac{D}{2}-1} e^2 m \Lambda^{4-D} \left(\frac{\sigma (u+1)^2}{m^2 u^{4/3} \chi^{2/3}}\right)^{2-\frac{D}{2}}}{\sigma (u+1)^2}$$

$$\frac{p^2 \sigma}{m^2 \sqrt[3]{u \chi^2}} - \frac{\sigma^3}{3} - \frac{\sigma (u+1)}{\sqrt[3]{u \chi^2}}$$

$$
\begin{aligned}
&\mathrm{J0}\left(\sigma\,\mathrm{FDp}^{\beta}\,\gamma^{\beta}.\overline{\gamma}^{5}\left(\frac{4\,i\,e}{m^3\,\sqrt[3]{u}\,\,(u+1)^2\,\chi^{2/3}}-\frac{i\,e\,u^{2/3}\,(D\,u+D-6\,u-10)}{m^3\,(u+1)^2\,\chi^{2/3}}\right)+\frac{2\,e^2\,\sigma^2\,((D-2)\,u+6)\,\gamma\cdot\mathrm{FFp}}{3\,m^5\,u^{2/3}\,\chi^{4/3}}+\right.\\
&\qquad\left.e\,\sigma\,\sigma\mathrm{F}\left(\frac{i\,u^{2/3}\left(D\,(u+1)^2-4\left(u^2+3\,u+3\right)\right)}{2\,m^2\,(u+1)^2\,\chi^{2/3}}-\frac{2\,i}{m^2\,\sqrt[3]{u}\,\,(u+1)^2\,\chi^{2/3}}\right)+\frac{(2-D)\,\gamma\cdot p}{m\,u+m}+D\right)+\\
&\mathrm{J2}\left(\frac{i\,(D-3)\,e\,\sigma\,u^{2/3}\,\mathrm{FDp}^{\beta}\,\gamma^{\beta}.\overline{\gamma}^{5}}{m^3\,(u+1)\,\chi^{2/3}}+e^2\,\gamma\cdot\mathrm{FFp}\left(-\frac{2\,(D-3)\,\sigma^2\,\sqrt[3]{u}}{3\,m^5\,\chi^{4/3}}-\frac{i}{m^5\,\sigma\,u^{2/3}\,\chi^{4/3}}\right)-\right.\\
&\qquad\left.\frac{i\,(D-3)\,e\,\sigma\,\sigma\mathrm{F}\,u^{2/3}}{2\,m^2\,\chi^{2/3}}+\frac{(D-3)\,\gamma\cdot p}{m\,(u+1)}-D+3\right)+\\
&\mathrm{J1}\left(\sigma\,\mathrm{FDp}^{\beta}\,\gamma^{\beta}.\overline{\gamma}^{5}\left(-\frac{i\,e\,u^{2/3}\,(u+3)}{m^3\,(u+1)^2\,\chi^{2/3}}-\frac{2\,i\,e}{m^3\,\sqrt[3]{u}\,\,(u+1)^2\,\chi^{2/3}}\right)+e^2\,\gamma\cdot\mathrm{FFp}\left(-\frac{2\,\sigma^2\,(u+3)}{3\,m^5\,u^{2/3}\,\chi^{4/3}}-\frac{i}{m^5\,\sigma\,u^{2/3}\,\chi^{4/3}}\right)+\right.\\
&\qquad\left.e\,\sigma\,\sigma\mathrm{F}\left(\frac{i\,u^{2/3}\left(u^2+4\,u+5\right)}{2\,m^2\,(u+1)^2\,\chi^{2/3}}+\frac{i}{m^2\,\sqrt[3]{u}\,\,(u+1)^2\,\chi^{2/3}}\right)+\frac{\gamma\cdot p}{m\,u+m}-1\right)-\\
&\frac{i\,(D-3)\,\mathrm{dJ2d}\chi\mathrm{l}\,e^2\,u^{4/3}\,\gamma\cdot\mathrm{FFp}}{2\,m^5\,\sigma\,(u+1)^2\,\sqrt[3]{\chi}}-\frac{i\,\mathrm{dJ1d}\chi\mathrm{l}\,e^2\,u^{4/3}\,\gamma\cdot\mathrm{FFp}}{2\,m^5\,\sigma\,(u+1)^2\,\sqrt[3]{\chi}}
\end{aligned}
$$

In the next step we rewrite

dJkdχl == $\frac{\partial}{\partial \chi_l} J_k(t, \chi_l)$,
where $\chi_l = \chi_l(u) = \frac{u}{1+u}\chi$ and $t = t(u) = \frac{1}{m^2}\frac{1+u}{\chi^{2/3} u^{4/3}}\sigma$ as defined above

using integration by parts

First, we may use the equality
$\frac{\partial}{\partial \chi_l} J_k(t, \chi_l) = (\chi_l'(u))^{-1}\left[\frac{d}{du} J_k(t(u), \chi_l(u)) - \frac{\partial}{\partial t} J_k(t, \chi_l)\, t'(u)\right]$.

Then, we may integrate the first term by parts, i.e.
$$\int_0^\infty du\, f(u) \frac{d}{du} Jk(t, \chi_l) = -\int_0^\infty du\, Jk(t, \chi_l) \frac{d}{du} f(t),$$
where we assumed that $Jk(t, 0) = f(\infty) = 0$.

As for second term, we write
$$\frac{\partial}{\partial t} J_k(t, \chi_l) = \frac{\partial}{\partial t}(-i)\int_{-\infty}^{\infty} dl^2\, D_k(l^2, \chi_l)\, e^{-i l^2 t} =$$
$$(-i)^2 \int_{-\infty}^{\infty} dl^2\, l^2 D_k(l^2, \chi_l)\, e^{-i l^2 t} == -i\, m^2 \tilde{J}_k(t, \chi_l).$$

We denote
Jkt == $\tilde{J}_k$

```
χlu = Simplify[χlϕ0 /. {ω → s t / (s + t)} /. {s → suσ, t → tuσ} /. {kp → χ / ξ m^2}] /.
   {kp → χ / ξ m^2}
dχldu = Simplify[D[χlu, u]]
dtdu = Simplify[D[tuσ, u]]
Coefficient[Matrix11, dJ0dχl] * dJ0dχl +
 Coefficient[Matrix11, dJ1dχl] * dJ1dχl + Coefficient[Matrix11, dJ2dχl] * dJ2dχl
Matrix12t = % /. {dJ0dχl → 1 / dχldu (- (-I m^2 J0t) dtdu),
    dJ1dχl → 1 / dχldu (- (-I m^2 J1t) dtdu), dJ2dχl → 1 / dχldu (- (-I m^2 J2t) dtdu)}
Matrix12d = %% /. {dJ0dχl → 1 / dχldu dJ0du ,
    dJ1dχl → 1 / dχldu dJ1du , dJ2dχl → 1 / dχldu dJ2du }
```

$$\frac{u\chi}{u+1}$$

$$\frac{\chi}{(u+1)^2}$$

$$-\frac{\sigma\,(u+4)}{3\,m^2\,u^{7/3}\,\chi^{2/3}}$$

$$-\frac{i\,(D-3)\,\text{dJ2d}\chi\text{l}\,e^2\,u^{4/3}\,\gamma\cdot\text{FFp}}{2\,m^5\,\sigma\,(u+1)^2\,\sqrt[3]{\chi}}-\frac{i\,\text{dJ1d}\chi\text{l}\,e^2\,u^{4/3}\,\gamma\cdot\text{FFp}}{2\,m^5\,\sigma\,(u+1)^2\,\sqrt[3]{\chi}}$$

$$-\frac{(D-3)\,e^2\,\text{J2t}\,(u+4)\,\gamma\cdot\text{FFp}}{6\,m^5\,u\,\chi^2}-\frac{e^2\,\text{J1t}\,(u+4)\,\gamma\cdot\text{FFp}}{6\,m^5\,u\,\chi^2}$$

$$-\frac{i\,(D-3)\,\text{dJ2du}\,e^2\,u^{4/3}\,\gamma\cdot\text{FFp}}{2\,m^5\,\sigma\,\chi^{4/3}}-\frac{i\,\text{dJ1du}\,e^2\,u^{4/3}\,\gamma\cdot\text{FFp}}{2\,m^5\,\sigma\,\chi^{4/3}}$$

```
f = Coeff11 * Exp[I Phase11]
Matrix12dmod = Collect[
  Expand[Simplify[-D[Matrix12d * f, u] / f] /. {dJ0du → J0, dJ1du → J1, dJ2du → J2}],
  {J0, J1, J2}, Simplify]
```

$$\frac{1}{\sigma\,(u+1)^2}\,i\,2^{-D-1}\,\pi^{-\frac{D}{2}-1}\,e^2\,m\,\Lambda^{4-D}\left(\frac{\sigma\,(u+1)^2}{m^2\,u^{4/3}\,\chi^{2/3}}\right)^{2-\frac{D}{2}}\exp\left(i\left(\frac{p^2\,\sigma}{m^2\,\sqrt[3]{u\,\chi^2}}-\frac{\sigma^3}{3}-\frac{\sigma\,(u+1)}{\sqrt[3]{u\,\chi^2}}\right)-\frac{i\,\pi\,D}{4}\right)$$

$$\left(e^2\,\text{J1}\,\gamma\cdot\text{FFp}\left(p^2\,\sigma\,(u+1)+m^2\left(\sigma\left(2\,u^2+u-1\right)-i\,(D-2)\,(u-2)\,\sqrt[3]{u\,\chi^2}\right)\right)\right)\Big/\left(6\,m^7\,\sigma\,(u+1)\,\chi^2\right)+$$
$$\left((D-3)\,e^2\,\text{J2}\,\gamma\cdot\text{FFp}\left(p^2\,\sigma\,(u+1)+m^2\left(\sigma\left(2\,u^2+u-1\right)-i\,(D-2)\,(u-2)\,\sqrt[3]{u\,\chi^2}\right)\right)\right)\Big/\left(6\,m^7\,\sigma\,(u+1)\,\chi^2\right)$$

```
Coeff12 = Coeff11
Phase12 = Phase11
Matrix12 = Collect[
  (Matrix11 /. {dJ0dχl → 0, dJ1dχl → 0, dJ2dχl → 0}) + Matrix12t + Matrix12dmod,
  {J0, J1, J2, J0t, J1t, J2t, e^2 DiracGamma[Momentum[FFp, D], D],
   e σF, DiracGamma[LorentzIndex[β, D], D].DiracGamma[5]
    Pair[LorentzIndex[β, D], Momentum[FDp, D]],
   DiracGamma[Momentum[p, D], D], pv2, lv2, σ, χ}, Simplify]
```

$$\frac{i\, 2^{-D-1} e^{-\frac{1}{4} i \pi D} \pi^{-\frac{D}{2}-1} e^2 m \Lambda^{4-D} \left(\frac{\sigma (u+1)^2}{m^2 u^{4/3} \chi^{2/3}}\right)^{2-\frac{D}{2}}}{\sigma (u+1)^2}$$

$$\frac{p^2 \sigma}{m^2 \sqrt[3]{u \chi^2}} - \frac{\sigma^3}{3} - \frac{\sigma (u+1)}{\sqrt[3]{u \chi^2}}$$

$$\mathrm{J2}\left(\frac{i (D-3) e \sigma u^{2/3} \mathrm{FDp}^{\beta} \gamma^{\beta}.\overline{\gamma}^5}{m^3 (u+1) \chi^{2/3}} + e^2 \gamma\cdot\mathrm{FFp}\left(\frac{-\frac{i (D^2-5 D+6)(u-2) \sqrt[3]{u \chi^2}}{6 m^5 (u+1) \chi^2} - \frac{i}{m^5 u^{2/3} \chi^{4/3}}}{\sigma} + \frac{(D-3) p^2}{6 m^7 \chi^2} - \frac{2 (D-3) \sigma^2 \sqrt[3]{u}}{3 m^5 \chi^{4/3}} + \frac{(D-3)(2 u-1)}{6 m^5 \chi^2}\right) - \frac{i (D-3) e \sigma \sigma\mathrm{F} u^{2/3}}{2 m^2 \chi^{2/3}} + \frac{(D-3) \gamma\cdot p}{m (u+1)} - D + 3\right) +$$

$$\mathrm{J0}\left(-\frac{i e \sigma ((D-6) u-4) \mathrm{FDp}^{\beta} \gamma^{\beta}.\overline{\gamma}^5}{m^3 \sqrt[3]{u} (u+1) \chi^{2/3}} + \frac{2 e^2 \sigma^2 ((D-2) u+6) \gamma\cdot\mathrm{FFp}}{3 m^5 u^{2/3} \chi^{4/3}} + \frac{i e \sigma \sigma\mathrm{F} ((D-4) u-4)}{2 m^2 \sqrt[3]{u} \chi^{2/3}} + \frac{(2-D) \gamma\cdot p}{m u+m} + D\right) + \mathrm{J1}$$

$$\left(-\frac{i e \sigma (u+2) \mathrm{FDp}^{\beta} \gamma^{\beta}.\overline{\gamma}^5}{m^3 \sqrt[3]{u} (u+1) \chi^{2/3}} + e^2 \gamma\cdot\mathrm{FFp}\left(\frac{-\frac{i (D-2)(u-2) \sqrt[3]{u \chi^2}}{6 m^5 (u+1) \chi^2} - \frac{i}{m^5 u^{2/3} \chi^{4/3}}}{\sigma} + \frac{p^2}{6 m^7 \chi^2} - \frac{2 \sigma^2 (u+3)}{3 m^5 u^{2/3} \chi^{4/3}} + \frac{2 u-1}{6 m^5 \chi^2}\right) + \frac{i e \sigma \sigma\mathrm{F} (u+2)}{2 m^2 \sqrt[3]{u} \chi^{2/3}} + \frac{\gamma\cdot p}{m u+m} - 1\right) - \frac{(D-3) e^2 \mathrm{J2t} (u+4) \gamma\cdot\mathrm{FFp}}{6 m^5 u \chi^2} - \frac{e^2 \mathrm{J1t} (u+4) \gamma\cdot\mathrm{FFp}}{6 m^5 u \chi^2}$$

Let us rewrite the result using the following notation

$\text{MFFp} == \frac{e^2\,\gamma^\mu\,\text{FFp}_\mu}{m^5\,\chi^2}\left(\frac{\chi}{u}\right)^{2/3};$

$\text{MFDp} == \frac{e\gamma^\mu\,\gamma^5\,\text{FDp}_\mu}{m^3\,\chi}\left(\frac{\chi}{u}\right)^{1/3},\ \text{FDp}_\mu == (F^*\,p)_\mu;$

$\text{M}\sigma\text{F} == \frac{e\,\sigma^{\mu\nu}\,F_{\mu\nu}}{m^2\,\chi}\left(\frac{\chi}{u}\right)^{1/3};$

```
Coeff13 = Coeff12
Phase13 = Phase12
Matrix13 =
 Collect[
  Expand[
   Matrix12 /. {DiracGamma[Momentum[FFp, D], D] → MFFp / e^2 * m^5 χ^ (4 / 3) u^ (2 / 3),
     DiracGamma[LorentzIndex[β, D], D].DiracGamma[5]
       Pair[LorentzIndex[β, D], Momentum[FDp, D]] →
      MFDp / e * m^3 χ^ (2 / 3) u^ (1 / 3), σF → MσF / e * m^2 χ^ (2 / 3) u^ (1 / 3) }
  ],
  {J0, J1, J2, dJ0dχl, dJ1dχl, dJ2dχl, MFFp, MFDp, MσF,
   DiracGamma[Momentum[p, D], D], Pair[Momentum[a, D], Momentum[p, D]], σ}, Simplify]
```

$$\frac{i\,2^{-D-1}\,e^{-\frac{1}{4}i\pi D}\,\pi^{-\frac{D}{2}-1}\,e^2\,m\,\Lambda^{4-D}\left(\frac{\sigma\,(u+1)^2}{m^2\,u^{4/3}\,\chi^{2/3}}\right)^{2-\frac{D}{2}}}{\sigma\,(u+1)^2}$$

$$\frac{p^2\,\sigma}{m^2\,\sqrt[3]{u\,\chi^2}}-\frac{\sigma^3}{3}-\frac{\sigma\,(u+1)}{\sqrt[3]{u\,\chi^2}}$$

$$\begin{aligned}
&\text{J2}\left(\text{MFFp}\left(-\frac{i\left(\left(D^2-5D+6\right)u^2-2\left(D^2-5D+3\right)u+6\right)}{6\,\sigma\,(u+1)}+\frac{(D-3)\left(\frac{u}{\chi}\right)^{2/3}\left(m^2\,(2u-1)+p^2\right)}{6\,m^2}-\frac{2}{3}(D-3)\,\sigma^2\,u\right)+\right.\\
&\qquad\left.\frac{(D-3)\,\gamma\cdot p}{m\,(u+1)}+\frac{i\,(D-3)\,\text{MFDp}\,\sigma\,u}{u+1}-\frac{1}{2}\,i\,(D-3)\,\text{M}\sigma\text{F}\,\sigma\,u-D+3\right)+\\
&\text{J0}\left(\frac{(2-D)\,\gamma\cdot p}{m\,u+m}-\frac{i\,\text{MFDp}\,\sigma\,((D-6)\,u-4)}{u+1}+\text{MFFp}\,\sigma^2\left(\frac{2}{3}(D-2)\,u+4\right)+\frac{1}{2}\,i\,\text{M}\sigma\text{F}\,\sigma\,((D-4)\,u-4)+D\right)+\\
&\text{J1}\left(\text{MFFp}\left(-\frac{i\left((D-2)\,u^2-2\,(D-5)\,u+6\right)}{6\,\sigma\,(u+1)}+\frac{\left(\frac{u}{\chi}\right)^{2/3}\left(m^2\,(2u-1)+p^2\right)}{6\,m^2}-\frac{2}{3}\sigma^2\,(u+3)\right)+\right.\\
&\qquad\left.\frac{\gamma\cdot p}{m\,u+m}-\frac{i\,\text{MFDp}\,\sigma\,(u+2)}{u+1}+\frac{1}{2}\,i\,\text{M}\sigma\text{F}\,\sigma\,(u+2)-1\right)-\frac{\text{MFFp}\,(u+4)\,((D-3)\,\text{J2t}+\text{J1t})}{6\,\sqrt[3]{u\,\chi^2}}
\end{aligned}$$

In order to remove the terms in Matrix13 that are proportional to $(\mathrm{MFFp}\,\sigma^{-1})$,
we integrate over σ by parts.

Let us consider the integral
$\int_0^\infty d\sigma\, g(\sigma)\, \sigma^{3-D/2}\, \mathrm{Exp}[-i\,\sigma^3 - i z\,\sigma]$, $4 - D = \epsilon > 0$,
where $g(\sigma) = J_{1,2}(t(u, \sigma), \chi_l)$, and

where $g(\sigma)$ satisfies the condition :
$g(\sigma)\,\sigma^{1-D/2} \to 0,\ \ \sigma \to 0;\ \ g(\infty) = 0.$

Then, we may write

$$\int_0^\infty d\sigma\, g(\sigma)\, \sigma^{3-D/2}\, \mathrm{Exp}[-i\,\sigma^3 - i z\,\sigma] =$$
$$= i \int_0^\infty d\left(\mathrm{Exp}[-i\sigma^3]\right) g(\sigma)\, \sigma^{1-D/2}\, \mathrm{Exp}[-i\,\sigma^3 - i z\,\sigma] =$$
$$= -i \int_0^\infty d\sigma\, \sigma^{1-D/2}\, \mathrm{Exp}[-i\,\sigma^3 - i z\,\sigma] \times \left[g'(\sigma) + \left(-\frac{D/2-1}{\sigma} - i z\right) g(\sigma)\right],$$

therefore

$$\int_0^\infty d\sigma\, \sigma^{1-D/2}\, \mathrm{Exp}[-i\,\sigma^3 - i z\,\sigma]\, \frac{1}{\sigma}\, g(\sigma) =$$
$$= \int_0^\infty d\sigma\, \sigma^{1-D/2} \left[\mathrm{Exp}[-i\,\sigma^3 - i z\,\sigma] \times \frac{1}{D/2-1} \left[g'(\sigma) - i\left(\sigma^2 + z\right) g(\sigma)\right]\right].$$

For $g'(\sigma)$ we have

$$g'(\sigma) = \frac{d}{d\sigma} J_{1,2}(t(u, \sigma), \chi_l) =$$
$$\frac{d}{d\sigma}(-i) \int_{-\infty}^\infty dl^2\, D_k\left(l^2, \chi_l\right) \mathrm{Exp}\left[-i\left(\frac{u}{\chi}\right)^{2/3} \frac{1+u}{u^2} \frac{l^2}{m^2}\, \sigma\right] =$$
$$= (-i) \int_{-\infty}^\infty dl^2 \left(-i\left(\frac{u}{\chi}\right)^{2/3} \frac{1+u}{u^2} \frac{l^2}{m^2}\right) D_k\left(l^2, \chi_l\right) \mathrm{Exp}\left[-i t(u, \sigma)\, l^2\right]$$
$$= -i\left(\frac{u}{\chi}\right)^{2/3} \frac{1+u}{u^2}\, \tilde{J}_k.$$

At this step we substitute J_k and $\tilde{J}_k$ explicitly :

$$J_k = -i \int_{-\infty}^\infty dl^2\, D_k\left(l^2, \chi_l\right) \mathrm{Exp}\left[-i t(u, \sigma)\, l^2\right],$$
$$\tilde{J}_k = -i \int_{-\infty}^\infty dl^2\, \frac{l^2}{m^2}\, D_k\left(l^2, \chi_l\right) \mathrm{Exp}\left[-i t(u, \sigma)\, l^2\right]$$

and introduce notations
$D0 == D_0\left(l^2\right)$,
$Dk == D_k\left(l^2, \chi_l\right)$, $k = 1, 2$.

Outer integration : $\int_0^\infty ds \int_0^\infty dt \int_{-\infty}^\infty dl^2 \ldots$

The nonrenormalized diagonal part of the mass operator in D dimensions :

```
Coeff14 = Coeff13
Phase14 = Phase13
Matrix14 = Collect[Expand[Simplify[Matrix13 - Coefficient[Matrix13, 1 / σ] / σ +
      (Coefficient[Matrix13, 1 / σ] * I (-σ - z / σ) * σ / (D / 2 - 1) /.
        {z → (u / χ) ^ (2 / 3) (1 - (pv2 - m ^ 2) / m ^ 2 / u + (1 + u) / u ^ 2 lv2 / m ^ 2)})
     ]] /. {lv2 J1 → m ^ 2 J1t, lv2 J2 → m ^ 2 J2t} , {J0, J1, J2, J1t, J2t, MFFp,
   MFDp, MσF, DiracGamma[Momentum[p, D], D], pv2, u / χ, σ}, Simplify]
```

$$\frac{i\, 2^{-D-1} e^{-\frac{1}{4} i \pi D} \pi^{-\frac{D}{2}-1} e^2\, m\, \Lambda^{4-D} \left(\frac{\sigma (u+1)^2}{m^2 u^{4/3} \chi^{2/3}}\right)^{2-\frac{D}{2}}}{\sigma (u+1)^2}$$

$$\frac{p^2 \sigma}{m^2 \sqrt[3]{u \chi^2}} - \frac{\sigma^3}{3} - \frac{\sigma (u+1)}{\sqrt[3]{u \chi^2}}$$

$$\text{J2}\left(\text{MFFp}\left(\frac{p^2\left(\left(D^2-5D+6\right)u^2-\left(D^2-5D+2\right)u+4\right)}{2(D-2)\,m^2\sqrt[3]{u}\,(u+1)\,\chi^{2/3}}-\frac{\sigma^2\left(\left(D^2-5D+6\right)u^2+2u+2\right)}{(D-2)(u+1)}+\frac{\left(D^2-5D+2\right)u-4}{2(D-2)\sqrt[3]{u}\,\chi^{2/3}}\right)+\right.$$
$$\left.\frac{(D-3)\,\gamma\cdot p}{m(u+1)}+\frac{i(D-3)\,\text{MFDp}\,\sigma\, u}{u+1}-\frac{1}{2} i (D-3)\,\text{M}\sigma\text{F}\,\sigma\, u - D + 3\right) -$$
$$\frac{\text{J2t}\,\text{MFFp}\left(\left(D^2-5D+6\right)u^2+4u+4\right)}{2(D-2)\,u^{4/3}\chi^{2/3}}+$$
$$\text{J0}\left(\frac{(2-D)\,\gamma\cdot p}{m u + m}-\frac{i\,\text{MFDp}\,\sigma((D-6)u-4)}{u+1}+\text{MFFp}\,\sigma^2\left(\frac{2}{3}(D-2)u+4\right)+\frac{1}{2} i\,\text{M}\sigma\text{F}\,\sigma((D-4)u-4)+D\right)+$$
$$\text{J1}\left(\text{MFFp}\left(\frac{p^2\left((D-2)u^2-(D-6)u+4\right)}{2(D-2)\,m^2\sqrt[3]{u}\,(u+1)\,\chi^{2/3}}+\frac{\sigma^2\left(2\left(u^2+u+1\right)-D\left(u^2+2u+2\right)\right)}{(D-2)(u+1)}+\frac{(D-6)u-4}{2(D-2)\sqrt[3]{u}\,\chi^{2/3}}\right)+\right.$$
$$\left.\frac{\gamma\cdot p}{m u + m}-\frac{i\,\text{MFDp}\,\sigma(u+2)}{u+1}+\frac{1}{2} i\,\text{M}\sigma\text{F}\,\sigma(u+2)-1\right)-\frac{\text{J1t}\,\text{MFFp}\left((D-2)u^2+4u+4\right)}{2(D-2)\,u^{4/3}\chi^{2/3}}$$

```
(*Coeff14=Coeff13/(I)
   Phase14=Phase13-lv2 tuσ
     Expand[Matrix13]/.
   {J0→ D0,J1→ D1,J2→ D2,J0t→ lv2/m^2 D0,J1t→ lv2/m^2  D1,J2t→ lv2/m^2  D2};
Matrix14=Collect[Expand[Simplify[%-Coefficient[%,1/σ]/σ+
     (Coefficient[%,1/σ]*I(-σ-z/σ)*σ/(D/2-1)/.
       {z→(u/χ)^(2/3)(1-(pv2-m^2)/m^2/u+(1+u)/u^2 lv2/m^2)})]],
  {D0,D1,D2,MFFp,MFDp, MσF, DiracGamma[Momentum[p,D],D],pv2,lv2,χ,σ},
  Simplify]*)
```

```
"z = " (u / χ) ^ (2 / 3)
 Collect[Expand[-Coefficient[Phase14, σ] * (χ / u) ^ (2 / 3)], {pv2, lv2}, Simplify]
```

$$z = \left(\frac{u}{\chi}\right)^{2/3} \left(-\frac{p^2}{m^2\, u} + \frac{1}{u} + 1\right)$$

The nonrenormalized diagonal part of the mass operator in D = 4 :

```
Coeff14D4 = Coeff14 * σ /. {D → 4}
Phase14D4 = Phase14 /. {D → 4}
Matrix14D4 = Collect[
  Matrix14 / σ /. {D → 4},
  {J0, J1, J2, J1t, J2t, MFFp, MFDp, MσF, DiracGamma[Momentum[p, D], D] /. {D → 4},
   pv2 /. {D → 4}, lv2 /. {D → 4}, χ, σ}, Simplify]
```

$$-\frac{i\, e^2\, m}{32\, \pi^3\, (u+1)^2}$$

$$\frac{\sigma\, \overline{p}^2}{m^2 \sqrt[3]{u\, \chi^2}} - \frac{\sigma^3}{3} - \frac{\sigma\,(u+1)}{\sqrt[3]{u\, \chi^2}}$$

$$\mathrm{J0}\left(-\frac{2\, \overline{\gamma}\cdot\overline{p}}{\sigma\,(m\, u+m)} + \frac{2\, i\, \mathrm{MFDp}\,(u+2)}{u+1} + \frac{4}{3}\mathrm{MFFp}\, \sigma\,(u+3) - 2\, i\, \mathrm{M\sigma F} + \frac{4}{\sigma}\right) +$$
$$\mathrm{J1}\left(\mathrm{MFFp}\left(\frac{\left(u^2+u+2\right)\overline{p}^2}{2\, m^2\, \sigma \sqrt[3]{u}\,\, (u+1)\, \chi^{2/3}} - \frac{\sigma\left(u^2+3\, u+3\right)}{u+1} + \frac{-u-2}{2\, \sigma \sqrt[3]{u}\,\, \chi^{2/3}}\right) + \right.$$
$$\left. \frac{\overline{\gamma}\cdot\overline{p}}{\sigma\,(m\, u+m)} - \frac{i\, \mathrm{MFDp}\,(u+2)}{u+1} + \frac{1}{2}\, i\, \mathrm{M\sigma F}\,(u+2) - \frac{1}{\sigma}\right) +$$
$$\mathrm{J2}\left(\mathrm{MFFp}\left(\frac{\left(u^2+u+2\right)\overline{p}^2}{2\, m^2\, \sigma \sqrt[3]{u}\,\, (u+1)\, \chi^{2/3}} - \frac{\sigma\left(u^2+u+1\right)}{u+1} + \frac{-u-2}{2\, \sigma \sqrt[3]{u}\,\, \chi^{2/3}}\right) + \frac{\overline{\gamma}\cdot\overline{p}}{\sigma\,(m\, u+m)} + \right.$$
$$\left. \frac{i\, \mathrm{MFDp}\, u}{u+1} - \frac{1}{2}\, i\, \mathrm{M\sigma F}\, u - \frac{1}{\sigma}\right) - \frac{\mathrm{J1t}\, \mathrm{MFFp}\left(u^2+2\, u+2\right)}{2\, \sigma\, u^{4/3}\, \chi^{2/3}} - \frac{\mathrm{J2t}\, \mathrm{MFFp}\left(u^2+2\, u+2\right)}{2\, \sigma\, u^{4/3}\, \chi^{2/3}}$$

Renormalization

$M = M_0 + \delta M$

where we attribute the term incorporating J_0 to M_0 ,
and which coincides with the 1 – loop order unrenormalized mass operator,
and the rest to δM,
which is associated with the polarization loop insertions.

M_0 should be renormalized, δM is finite. Therefore,
we take the limit $D \to 4$ in δM.

We renormalize M_0 as follows
$M_0 (p, F) \to [M_0 (p, 0)]_{ren} + [M_0 (p, F) - M_0 (p, 0)]$

The second term gives a regular field dependent part. $[M_0 (p, 0)]_{ren}$ is the renormalized field – free mass operator.

In what follows, we write only the field – dependent part of M

Let us introduce the Ritus functions

$$f(z) = i\int_0^\infty d\sigma\, \mathrm{Exp}\left(-i\,\frac{\sigma^3}{3} - iz\sigma\right),$$

$$f'(z) = \int_0^\infty d\sigma\,\sigma\,\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3} - iz\sigma\right),$$

$$f_1(z) = \int_0^\infty \frac{d\sigma}{\sigma}\, e^{-iz\sigma}\left[\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3}\right) - 1\right]$$

$$z = \left(\frac{u}{\chi}\right)^{2/3}\left(1 - \frac{1}{u}\,\frac{p^2 - m^2}{m^2} + \frac{1+u}{u^2}\,\frac{l^2}{m^2}\right)$$

In effect,
in the limit $D \to 4$ the renormalization of M_0 is reduced to the substitutuion

$$\frac{1}{\sigma}\,\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3} - iz\sigma\right) \to \frac{1}{\sigma}\, e^{-iz\sigma}\left[\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3}\right) - 1\right]$$

or

$$\int_0^\infty \frac{d\sigma}{\sigma}\,\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3} - iz\sigma\right) \to f_1(z)$$

We also substitute

$$\int_0^\infty d\sigma\,\sigma\,\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3} - iz\sigma\right) \to f'(z)$$

$$\int_0^\infty d\sigma\,\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3} - iz\sigma\right) \to -i f(z)$$

and use

$J_0 = 2\pi i\,\theta(t)$

Matrix15M0 now contains the phase factor inside the Ritus f – functions.

```
pv2D4 = pv2 /. {D → 4};
lv2D4 = lv2 /. {D → 4};
zarg = Collect[Phase14D4 /. {σ^3 → 0} /. {σ → -1} /. {1/∛(u χ^2) → (u/χ)^(2/3)/u,
    1/(u^(4/3) χ^(2/3)) → (u/χ)^(2/3)/u^2}, {(u/χ)^(2/3)}, Simplify]
Clear[f];
Coeff15M0 = Coeff14D4 * 2 π I * 2 /. {e^2 → α 4 π}
Matrix15M0 =
  Collect[Expand[Coefficient[Matrix14D4, J0]/2] /. {1/σ → f1[z], σ → f'[z],
    MFDp → -I MFDp f[z], MσF → -I MσF f[z]}, {f[z], f'[z], f1[z]}]
```

$$\left(\frac{u}{\chi}\right)^{2/3}\left(-\frac{\overline{p}^2}{m^2\,u} + \frac{1}{u} + 1\right)$$

$$\frac{\alpha\, m}{2\,\pi\,(u+1)^2}$$

$$\mathrm{f1}(z)\left(2-\frac{\overline{\gamma}\cdot\overline{p}}{m\,u+m}\right)+\left(\frac{2\,\mathrm{MFFp}\,u}{3}+2\,\mathrm{MFFp}\right)f'(z)+f(z)\left(\frac{\mathrm{MFDp}\,u}{u+1}+\frac{2\,\mathrm{MFDp}}{u+1}-\mathrm{M\sigma F}\right)$$

The nontrivial part of δM

```
Coeff15δM = Coeff14D4 /. {e^2 → α 4 π}
Phase15 = Phase14D4
Matrix15δM = Matrix14D4 - Coefficient[Matrix14D4, J0] J0
```

$$-\frac{i\,\alpha\, m}{8\,\pi^2\,(u+1)^2}$$

$$\frac{\sigma\,\overline{p}^2}{m^2\,\sqrt[3]{u\,\chi^2}}-\frac{\sigma^3}{3}-\frac{\sigma\,(u+1)}{\sqrt[3]{u\,\chi^2}}$$

$$\mathrm{J1}\left(\mathrm{MFFp}\left(\frac{(u^2+u+2)\,\overline{p}^2}{2\,m^2\,\sigma\,\sqrt[3]{u}\,(u+1)\,\chi^{2/3}}-\frac{\sigma\,(u^2+3\,u+3)}{u+1}+\frac{-u-2}{2\,\sigma\,\sqrt[3]{u}\,\chi^{2/3}}\right)+\frac{\overline{\gamma}\cdot\overline{p}}{\sigma\,(m\,u+m)}-\frac{i\,\mathrm{MFDp}\,(u+2)}{u+1}+\frac{1}{2}\,i\,\mathrm{M\sigma F}\,(u+2)-\frac{1}{\sigma}\right)+$$
$$\mathrm{J2}\left(\mathrm{MFFp}\left(\frac{(u^2+u+2)\,\overline{p}^2}{2\,m^2\,\sigma\,\sqrt[3]{u}\,(u+1)\,\chi^{2/3}}-\frac{\sigma\,(u^2+u+1)}{u+1}+\frac{-u-2}{2\,\sigma\,\sqrt[3]{u}\,\chi^{2/3}}\right)+\frac{\overline{\gamma}\cdot\overline{p}}{\sigma\,(m\,u+m)}+\frac{i\,\mathrm{MFDp}\,u}{u+1}-\frac{1}{2}\,i\,\mathrm{M\sigma F}\,u-\frac{1}{\sigma}\right)-\frac{\mathrm{J1t}\,\mathrm{MFFp}\,(u^2+2\,u+2)}{2\,\sigma\,u^{4/3}\,\chi^{2/3}}-\frac{\mathrm{J2t}\,\mathrm{MFFp}\,(u^2+2\,u+2)}{2\,\sigma\,u^{4/3}\,\chi^{2/3}}$$

Let us rewrite the answer in the following form

$$M\,(p,\,F)=\sum_{n=0}^{2}\Big[m\,S_n\,(p^2,\,\chi)+(\gamma p)\,V_n^{(1)}\,(p^2,\,\chi)+\frac{(\gamma F^2\,p)}{m^4\,\chi^2}\,V_n^{(2)}\,(p^2,\,\chi)+\frac{(\sigma F)}{m\chi}\,T_n\,(p^2,\,\chi)+\frac{(\gamma F^*\,p)\,.\,\gamma^5}{m^2\,\chi}\,A_n\,(p^2,\,\chi)$$

$$\mathrm{MFFpV2}=\mathrm{MFFp}*m\,\left(\frac{\chi}{u}\right)^{-2/3}==\frac{e^2\,\gamma^\mu\,\mathrm{FFp}_\mu}{m^4\,\chi^2};$$
$$\mathrm{MFDpA}=\mathrm{MFDp}\,*m\,\left(\frac{\chi}{u}\right)^{-1/3}==\frac{e\gamma^\mu\,\gamma^5\,\mathrm{FDp}_\mu}{m^2\,\chi},\quad \mathrm{FDp}_\mu==(F^*\,p)_\mu;$$
$$\mathrm{M\sigma FT}=\mathrm{M\sigma F}\,*m\,\left(\frac{\chi}{u}\right)^{-1/3}==\frac{e\,\sigma^{\mu\nu}\,F_{\mu\nu}}{m\,\chi};$$

```
Coeff15M01 = Collect[Matrix15M0 * Coeff15M0 /. {MFFp → MFFpV2 / m (χ / u) ^ (2 / 3),
    MFDp → MFDpA / m (χ / u) ^ (1 / 3), MσF → MσFT / m (χ / u) ^ (1 / 3)},
  {D0, f[z], f'[z], f1[z], DiracGamma[Momentum[p]], MFDpA, MσFT}, Simplify]
Coeff15δM1 = Collect[Expand[Matrix15δM * Coeff15δM] /.
   {MFFp → MFFpV2 / m (χ / u) ^ (2 / 3),
    MFDp → MFDpA / m (χ / u) ^ (1 / 3), MσF → MσFT / m (χ / u) ^ (1 / 3)},
  {MFFpV2, J1, J1t, J2, J2t, DiracGamma[Momentum[p]], MFDpA, MσFT, σ}, Simplify]
```

$$\mathrm{f1}(z)\left(\frac{\alpha\, m}{\pi\,(u+1)^2}-\frac{\alpha\,\overline{\gamma}\cdot\overline{p}}{2\,\pi\,(u+1)^3}\right)+\frac{\alpha\,\mathrm{MFFpV2}\,(u+3)\left(\frac{\chi}{u}\right)^{2/3} f'(z)}{3\,\pi\,(u+1)^2}+f(z)\left(\frac{\alpha\,\mathrm{MFDpA}\,(u+2)\sqrt[3]{\frac{\chi}{u}}}{2\,\pi\,(u+1)^3}-\frac{\alpha\,\mathrm{M\sigma FT}\sqrt[3]{\frac{\chi}{u}}}{2\,\pi\,(u+1)^2}\right)$$

$$\mathrm{MFFpV2}\left(\mathrm{J1}\left(\frac{i\,\alpha\left(m^2\left(u^2+3\,u+2\right)-\left(u^2+u+2\right)\overline{p}^2\right)}{16\,\pi^2\,m^2\,\sigma\,u\,(u+1)^3}+\frac{i\,\alpha\,\sigma\left(u^2+3\,u+3\right)\left(\frac{\chi}{u}\right)^{2/3}}{8\,\pi^2\,(u+1)^3}\right)+\right.$$
$$\mathrm{J2}\left(\frac{i\,\alpha\left(m^2\left(u^2+3\,u+2\right)-\left(u^2+u+2\right)\overline{p}^2\right)}{16\,\pi^2\,m^2\,\sigma\,u\,(u+1)^3}+\frac{i\,\alpha\,\sigma\left(u^2+u+1\right)\left(\frac{\chi}{u}\right)^{2/3}}{8\,\pi^2\,(u+1)^3}\right)+$$
$$\left.\frac{i\,\alpha\,\mathrm{J1t}\left(u^2+2\,u+2\right)}{16\,\pi^2\,\sigma\,u^2\,(u+1)^2}+\frac{i\,\alpha\,\mathrm{J2t}\left(u^2+2\,u+2\right)}{16\,\pi^2\,\sigma\,u^2\,(u+1)^2}\right)+$$
$$\mathrm{J1}\left(-\frac{i\,\alpha\,\overline{\gamma}\cdot\overline{p}}{8\,\pi^2\,\sigma\,(u+1)^3}+\frac{i\,\alpha\,m}{8\,\pi^2\,\sigma\,(u+1)^2}-\frac{\alpha\,\mathrm{MFDpA}\,(u+2)\sqrt[3]{\frac{\chi}{u}}}{8\,\pi^2\,(u+1)^3}+\frac{\alpha\,\mathrm{M\sigma FT}\,(u+2)\sqrt[3]{\frac{\chi}{u}}}{16\,\pi^2\,(u+1)^2}\right)+$$
$$\mathrm{J2}\left(-\frac{i\,\alpha\,\overline{\gamma}\cdot\overline{p}}{8\,\pi^2\,\sigma\,(u+1)^3}+\frac{i\,\alpha\,m}{8\,\pi^2\,\sigma\,(u+1)^2}+\frac{\alpha\,\mathrm{MFDpA}\sqrt[3]{u^2\,\chi}}{8\,\pi^2\,(u+1)^3}-\frac{\alpha\,\mathrm{M\sigma FT}\sqrt[3]{u^2\,\chi}}{16\,\pi^2\,(u+1)^2}\right)$$

```
"S₀ = " TraditionalForm[
  Coeff15M01 / m /. {MFFpV2 → 0, DiracGamma[Momentum[p]] → 0, MFDpA → 0, MσFT → 0} ]
"V₀⁽¹⁾ = " TraditionalForm[ Coefficient[ Coeff15M01, DiracGamma[Momentum[p]]] ]
"V₀⁽²⁾ = " TraditionalForm[(- 8 π^2 / α (1 + u) ^2 / (-I) 2) ^ (-1) Collect[Expand[
     Coefficient[ Coeff15M01 * (- 8 π^2 / α (1 + u) ^2 / (-I) 2), MFFpV2]],
    {σ, J1, J1t, J2, J2t, (pv2 /. {D → 4})}, Simplify] ]
"T₀ = " TraditionalForm[(16 π^2 χ / α (χ / u) ^ (-1 / 3) (1 + u) ^2) ^ (-1) Collect[Expand[
     Coefficient[ Coeff15M01, MσFT] * (16 π^2 χ / α (χ / u) ^ (-1 / 3) (1 + u) ^2)],
    {J1, J1t, J2, J2t, σ, (pv2 /. {D → 4})}, Simplify]]
"A₀ = " TraditionalForm[(-8 π^2 χ / α (χ / u) ^ (-1 / 3) (1 + u) ^3) ^ (-1) Collect[
    Expand[Coefficient[ Coeff15M01, MFDpA] (-8 π^2 χ / α (χ / u) ^ (-1 / 3) (1 + u) ^3)],
    {σ, J1, J1t, J2, J2t, (pv2 /. {D → 4})}, Simplify]]
```

$$S_0 = \frac{\alpha\,\mathrm{f1}(z)}{\pi\,(u+1)^2}$$

$$V_0^{(1)} = \left(-\frac{\alpha\,\mathrm{f1}(z)}{2\,\pi\,(u+1)^3}\right)$$

$$V_0^{(2)} = \frac{(u+3)\,\alpha\left(\frac{\chi}{u}\right)^{2/3} f'(z)}{3\,\pi\,(u+1)^2}$$

$$T_0 = \left(-\frac{\alpha\sqrt[3]{\frac{\chi}{u}}\,f(z)}{2\,\pi\,(u+1)^2}\right)$$

$$A_0 = \frac{(u+2)\,\alpha\sqrt[3]{\frac{\chi}{u}}\,f(z)}{2\,\pi\,(u+1)^3}$$

```
"S1,2 = " TraditionalForm[
  Coeff15δM1 / m /. {MFFpV2 → 0, DiracGamma[Momentum[p]] → 0, MFDpA → 0, MσFT → 0} ]
"V(1)1,2 = " TraditionalForm[ Coefficient[ Coeff15δM1, DiracGamma[Momentum[p]]] ]
"V(2)1,2 = " TraditionalForm[ (- 8 π^2 / α (1 + u) ^2 / (-I) 2) ^ (-1) Collect[Expand[
     Coefficient[ Coeff15δM1 * (- 8 π^2 / α (1 + u) ^2 / (-I) 2), MFFpV2]],
    {σ, J1, J1t, J2, J2t, (pv2 /. {D → 4})}, Simplify] ]
"T1,2 = " TraditionalForm[ (16 π^2 / α (χ / u) ^ (-1 / 3) (1 + u) ^2) ^ (-1) Collect[Expand[
     Coefficient[ Coeff15δM1, MσFT] * (16 π^2 / α (χ / u) ^ (-1 / 3) (1 + u) ^2)],
    {J1, J1t, J2, J2t, σ, (pv2 /. {D → 4})}, Simplify]]
"A1,2 = " TraditionalForm[ (-8 π^2 / α (χ / u) ^ (-1 / 3) (1 + u) ^3) ^ (-1) Collect[
    Expand[Coefficient[ Coeff15δM1, MFDpA] (-8 π^2 / α (χ / u) ^ (-1 / 3) (1 + u) ^3)],
    {σ, J1, J1t, J2, J2t, (pv2 /. {D → 4})}, Simplify]]
```

$$S_{1,2} = \frac{\frac{i\,\text{J1}\,m\,\alpha}{8\,\pi^2\,(u+1)^2\,\sigma} + \frac{i\,\text{J2}\,m\,\alpha}{8\,\pi^2\,(u+1)^2\,\sigma}}{m}$$

$$V^{(1)}_{1,2} = \left(-\frac{i\,\text{J1}\,\alpha}{8\,\pi^2\,(u+1)^3\,\sigma} - \frac{i\,\text{J2}\,\alpha}{8\,\pi^2\,(u+1)^3\,\sigma}\right)$$

$$V^{(2)}_{1,2} = \frac{1}{16\,\pi^2\,(u+1)^2}\,i\,\alpha\left(\sigma\left(\frac{2\,\text{J2}\left(\frac{\chi}{u}\right)^{2/3}\left(u^2+u+1\right)}{u+1} + \frac{2\,\text{J1}\left(u^2+3\,u+3\right)\left(\frac{\chi}{u}\right)^{2/3}}{u+1}\right) + \frac{1}{\sigma}\left(\frac{\text{J1t}\left(u^2+2\,u+2\right)}{u^2} + \frac{\text{J2t}\left(u^2+2\,u+2\right)}{u^2} + \text{J1}\left(\frac{u+2}{u} - \frac{\left(u^2+u+2\right)\overline{p}^2}{m^2\,u\,(u+1)}\right) + \text{J2}\left(\frac{u+2}{u} - \frac{\left(u^2+u+2\right)\overline{p}^2}{m^2\,u\,(u+1)}\right)\right)\right)$$

$$T_{1,2} = \frac{(\text{J1}\,(u+2) - \text{J2}\,u)\,\alpha\,\sqrt[3]{\frac{\chi}{u}}}{16\,\pi^2\,(u+1)^2}$$

$$A_{1,2} = \left(-\frac{(\text{J1}\,(u+2) - \text{J2}\,u)\,\alpha\,\sqrt[3]{\frac{\chi}{u}}}{8\,\pi^2\,(u+1)^3}\right)$$

The elastic scattering amplitude $\mathcal{M} = \overline{u}_p\, M(p)\, u_p$

We assume that $p^2 = 0$ and calculate the matrix element

$\mathcal{M} = \overline{u}_p\, M\,(p)\, u_p$,

where u_p is a free Dirac bispinor and $\overline{u}_p = \gamma^0\, u_p^\dagger$.

To perform this calculation, we use the following relations
$\overline{u}_p\, u_p = 2\,m$,
$\overline{u}_p\,(\gamma p)\, u_p = 2\,m^2$,
$\overline{u}_p\, e^2\,\left(\gamma F^2\, p\right)\, u_p = 2\,m^6\,\chi^2$,
$\overline{u}_p\, e\,(\sigma_{\mu\nu}\, F^{\mu\nu})\, u_p = \frac{2}{m}\, e\,\left(\overline{u}_p\,\gamma^\beta\,\gamma^5\, u_p\right)\,(F^*\, p)_\beta = 4\, s^\beta\, e\,(F^*\, p)_\beta$,
where $s^\beta = \frac{1}{2\,m}\,\overline{u}_p\,\gamma^\beta\,\gamma^5\, u_p$ is the electron spin 4 - vector.

Recall
$\text{MFFp} == \frac{e^2\,\gamma^\mu\,\text{FFp}_\mu}{m^5\,\chi^2}\,\left(\frac{\chi}{u}\right)^{2/3}$,
$\text{MFDp} == \frac{e\gamma^\mu\,\gamma^5\,\text{FDp}_\mu}{m^3\,\chi}\,\left(\frac{\chi}{u}\right)^{1/3}$, $\text{FDp}_\mu == (F^*\, p)_\mu$,
$\text{M}\sigma\text{F} == \frac{e\,\sigma^{\mu\nu}\, F_{\mu\nu}}{m^2\,\chi}\,\left(\frac{\chi}{u}\right)^{1/3}$,

in effect, we should perform the following substitutions
$\overline{u}_p\,\text{MFFp}\, u_p = 2\,m\,\left(\frac{\chi}{u}\right)^{2/3}$,
$\overline{u}_p\,\text{MFDp}\, u_p = \frac{2\,e}{m^2\,\chi}\, s^\nu\,\text{FDp}_\nu\,\left(\frac{\chi}{u}\right)^{1/3}$,
$\overline{u}_p\,\text{M}\sigma\text{F}\, u_p = \frac{4\,e}{m^2\,\chi}\, s^\nu\,\text{FDp}_\nu\,\left(\frac{\chi}{u}\right)^{1/3}$.

```
Simplify[S + DiracGamma[Momentum[p]] / m V1 +
    MFFp V2 (χ / u) ^ (-2 / 3) + MσF T (χ / u) ^ (-1 / 3) + MFDp A (χ / u) ^ (-1 / 3) /.
   {DiracGamma[Momentum[p]] → 2 m^2, MFFp → 2 m χ^ (2 / 3) / u^ (2 / 3),
    MσF → e Contract[FVD[s, ν] FDpv[ν]] 4 / m^2 / χ * (χ / u) ^ (1 / 3),
    MFDp → e Contract[FVD[s, ν] FDpv[ν]] 2 / m^2 / χ * (χ / u) ^ (1 / 3), S → 2 m S}]
```

$$\frac{2\,e\,(A+2\,T)\,(\text{FDp}\cdot s)}{m^2\,\chi}+2\,m\,(S+\text{V1}+\text{V2})$$

```
Coeff16M0 = Coeff15M0 * 2 m
zargOS = zarg /. {(pv2 /. {D → 4}) → m^2}
TermI0 = Expand[Simplify[
    Matrix15M0 /. {DiracGamma[Momentum[p]] → 0, MFFp → 0, MσF → 0, MFDp → 0}]];
1/2/m Expand[Matrix15M0 - TermI0 + 2 m TermI0] /.
    {DiracGamma[Momentum[p]] → 2 m^2, MFFp → 2 m χ^(2/3)/u^(2/3),
     MσF → e Contract[FVD[s, ν] FDpv[ν]] 4/m^2/χ * (χ/u)^(1/3),
     MFDp → e Contract[FVD[s, ν] FDpv[ν]] 2/m^2/χ * (χ/u)^(1/3)} /. {pv2 →
     m^2} /. {D → 4};
Matrix16M0 = Collect[
  Expand[
    Simplify[% - Coefficient[%, e Pair[Momentum[s], Momentum[FDp]]]
       e Pair[Momentum[s], Momentum[FDp]]]]
   + Simplify[Coefficient[%, e Pair[Momentum[s], Momentum[FDp]]]
     e Pair[Momentum[s], Momentum[FDp]]]
  ,
  {f[z], f'[z], f1[z]}, Simplify]
```

$$\frac{\alpha\, m^2}{\pi\, (u+1)^2}$$

$$\left(\frac{u}{\chi}\right)^{2/3}$$

$$-\frac{e\, f(z) \left(\frac{u}{\chi}\right)^{2/3} \left(\overline{\text{FDp}}\cdot\overline{s}\right)}{m^3\, (u+1)} + \frac{2}{3}\, (u+3) \left(\frac{\chi}{u}\right)^{2/3} f'(z) + \frac{(2\, u+1)\, \text{f1}(z)}{u+1}$$

```
Coeff16δM = Coeff15δM * 2 m
Phase16 = Collect[Phase15 /. {pv2D4 → m^2}, σ, Simplify]
TermIδ = Expand[Simplify[
    Matrix15δM /. {DiracGamma[Momentum[p]] → 0, MFFp → 0, MσF → 0, MFDp → 0}]];
1/2/m Expand[Matrix15δM - TermIδ + 2 m TermIδ] /.
    {DiracGamma[Momentum[p]] → 2 m^2, MFFp → 2 m χ^(2/3)/u^(2/3),
     MσF → e Contract[FVD[s, ν] FDpv[ν]] 4/m^2/χ * (χ / u)^(1/3),
     MFDp → e Contract[FVD[s, ν] FDpv[ν]] 2/m^2/χ * (χ / u)^(1/3)} /.
   {pv2D4 → m^2} /. {D → 4};
Matrix16δM = Collect[
  Expand[
    Simplify[% - Coefficient[%, e Pair[Momentum[s], Momentum[FDp]]]
       e Pair[Momentum[s], Momentum[FDp]]]]
   + Simplify[Coefficient[%, e Pair[Momentum[s], Momentum[FDp]]]
     e Pair[Momentum[s], Momentum[FDp]]]
  ,
  {J1, J2, J1t, J2t, MFFp, MFDp, MσF, DiracGamma[Momentum[p]], χ, σ}, Simplify]
```

$$-\frac{i\,\alpha\,m^2}{4\,\pi^2\,(u+1)^2}$$

$$-\frac{\sigma^3}{3}-\sigma\left(\frac{u}{\chi}\right)^{2/3}$$

$$\mathrm{J1}\left(\frac{i\,e\,(u+2)\left(\frac{u}{\chi}\right)^{2/3}\left(\overline{\mathrm{FDp}}\cdot\overline{s}\right)}{m^3\,(u+1)}-\frac{1}{\sigma}+\frac{\sigma\left(-u^2-3\,u-3\right)\chi^{2/3}}{u^{2/3}\,(u+1)}\right)+$$
$$\mathrm{J2}\left(-\frac{i\,e\,u\left(\frac{u}{\chi}\right)^{2/3}\left(\overline{\mathrm{FDp}}\cdot\overline{s}\right)}{m^3\,(u+1)}-\frac{1}{\sigma}-\frac{\sigma\left(u^{4/3}+\frac{1}{u^{2/3}}+\sqrt[3]{u}\right)\chi^{2/3}}{u+1}\right)-\frac{\mathrm{J1t}\left(u^2+2\,u+2\right)}{2\,\sigma\,u^2}-\frac{\mathrm{J2t}\left(u^2+2\,u+2\right)}{2\,\sigma\,u^2}$$

Let us rewrite Matrix16M0

we perform an additional transformation of the D_0 term

Let us use the following integral equality (we will prove it below)

$$\int_0^\infty \frac{du}{(1+u)^2}\left[\frac{2\,(u-2)\,u}{3\,(u+1)}\left(\frac{\chi}{u}\right)^{2/3} f'\,(z_0)\;+\;-\frac{2\,u}{u+1}\,f_1\,(z_0)\right] = 0$$

We add this expression to Matrix16M0

After these transformation we arrive at the expression corresponding to Eq. (11) in [A.A.Mironov, S.Meuren, A.M.Fedotov PRD 102, 053 005 (2020)].

```
Matrix16M01 = Collect[Matrix16M0 + (2 (u - 2) u f'[z] / (3 (u + 1)) (χ / u) ^ (2 / 3) + f1[z] (- 2 u / (u + 1))),
  {f[z], f'[z], f1[z]}, Simplify]
```

$$-\frac{e\,f(z)\left(\frac{u}{\chi}\right)^{2/3}\left(\overline{\mathrm{FDp}}\cdot\vec{s}\right)}{m^3\,(u+1)}+\frac{2\left(2\,u^2+2\,u+3\right)\left(\frac{\chi}{u}\right)^{2/3} f'(z)}{3\,(u+1)}+\frac{\mathrm{f1}(z)}{u+1}$$

Proof of the integral equality

$$\mathbf{I} = \int_0^\infty \frac{du}{(1+u)^2}\left[\frac{2\,(u-2)\,u}{3\,(u+1)}\left(\frac{\chi}{u}\right)^{2/3} f'(z) + \left(\frac{l^2}{m^2} - \frac{2\,u}{u+1}\right) f_1(z)\right] = 0$$

Let us restore the explicit integral form of the Ritus f - functions and integrate the term with f' (z) by parts

$$f'(z) = \int_0^\infty d\sigma\,\sigma\,\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3} - i z\sigma\right) = i\int_0^\infty d\left[\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3}\right) - 1\right]\times\frac{1}{\sigma}\,e^{-iz\sigma} =$$
$$= -i\int_0^\infty d\sigma\times\left[\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3}\right) - 1\right]\times\left[-\frac{1}{\sigma^2} - \frac{iz}{\sigma}\right] e^{-iz\sigma},$$

where we used the fact that
$\left[\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3}\right) - 1\right]\times\frac{1}{\sigma}\,e^{-iz\sigma} = 0$ for $\sigma = 0$ and $\sigma = \infty$.

Note that after rewriting f' (z),
the factor $\left[\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3}\right) - 1\right]$ will be common in I,

$$I = \left[\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3}\right) - 1\right]\int_0^\infty du\; e^{-iz(u)\,\sigma}[\;...]$$

so we will only write out the expression in [...].
We also substitute

$$z(u) = \left(\frac{u}{\chi}\right)^{2/3}\left(\frac{(u+1)\,\vec{l}^{\,2}}{m^2\,u^2} + 1\right).$$

```
zarg1 = zargOS + lv2D4 tuσ / σ
Int = Collect[1 / (1 + u) ^2
      (2 (u - 2) u / 3 / (1 + u) (χ / u) ^ (2 / 3) f'[z] + (lv2D4 / m^2 - 2 u / (1 + u)) f1[z]) /.
     {f'[z] → -I (-1 / σ^2 - I z / σ), f1[z] → 1 / σ} /. {z → zarg1},
  {σ, lv2D4}, Simplify]
```

$$\frac{(u+1)\,\vec{l}^{\,2}}{m^2\,u^{4/3}\,\chi^{2/3}} + \left(\frac{u}{\chi}\right)^{2/3}$$

$$\frac{\frac{(u+4)\,\vec{l}^{\,2}}{3\,m^2\,u\,(u+1)^2} - \frac{2\,u}{3\,(u+1)^2}}{\sigma} + \frac{2\,i\,\sqrt[3]{u}\,(u-2)\,\chi^{2/3}}{3\,\sigma^2\,(u+1)^3}$$

In the next step we prove that in fact this expression is a total derivative

$$I = \left[\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3}\right) - 1\right] \int_0^\infty du\,[\ \ldots\ ] = \left[\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3}\right) - 1\right] \int_0^\infty du\,\frac{d}{du}\left[P\,(u)\,e^{-i z\,(u)\,\sigma}\right],$$

and $P\,(0) = P\,(\infty) = 0$, which makes the statement evident.

To find $P\,(u)$, we expand the derivative $\frac{d}{du}[\ \ldots\ ]$ and equal the coefficient of l^2 to the corresponding coefficent in the expression for Int.

We find that

$$P\,(u) = -\frac{i\,u^{4/3}\,\chi^{2/3}}{\sigma^2\,(u+1)^2}.$$

In the last two lines we check that the result is correct.

$$I = \int_0^\infty \frac{du}{(1+u)^2}\left[\frac{2\,(u-2)\,u}{3\,(u+1)}\left(\frac{\chi}{u}\right)^{2/3} f'\,(z) + \left(\frac{l^2}{m^2} - \frac{2\,u}{u+1}\right) f_1\,(z)\right] =$$

$$= -\frac{i}{\sigma^2}\left[\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3}\right) - 1\right] \int_0^\infty du\,\frac{d}{du}\left[\frac{u^2}{(u+1)^2}\left(\frac{\chi}{u}\right)^{2/3} e^{-i z \sigma}\right] = 0.$$

```
IntTest = Collect[
  Simplify[D[P[u] Exp[-I zarg1 σ], u] / Exp[-I zarg1 σ]], {λ, ν, P'[u], σ}, Simplify]
```

$$P'(u) - \frac{i\,\sigma\,P(u)\left(2\,m^2\,u^2 - (u+4)\,\vec{l}^{\,2}\right)}{3\,m^2\,u^{7/3}\,\chi^{2/3}}$$

```
Psol[u] =
 P[u] /. Solve[Coefficient[Int, lv2D4] == Coefficient[IntTest, lv2D4], P[u]][[1]]
Collect[(IntTest /. {P[u] → Psol[u], P'[u] → D[Psol[u], u]}), {lv2D4, σ}, Simplify]
Simplify[Int - % /. {∛(u χ^2) → χ^(2/3) u^(1/3)}]
```

$$-\frac{i\,u^{4/3}\,\chi^{2/3}}{\sigma^2\,(u+1)^2}$$

$$\frac{(u+4)\,\vec{l}^{\,2}}{3\,m^2\,\sigma\,u\,(u+1)^2} + \frac{2\,i\,\sqrt[3]{u}\,(u-2)\,\chi^{2/3}}{3\,\sigma^2\,(u+1)^3} - \frac{2\,u}{3\,\sigma\,(u+1)^2}$$

$$0$$