

This is a part of SFQED-Loops script collection
developed for calculating loop processes in
Strong-Field Quantum Electrodynamics.
The scripts are available on <https://github.com/ArsenyMironov/SFQED-Loops>

If you use this script in your research, please, consider
citing our papers:

- A. A. Mironov, S. Meuren, and A. M. Fedotov, PRD 102, 053005 (2020),
<https://doi.org/10.1103/PhysRevD.102.053005>

If you have any questions, please, don't hesitate to contact me:
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`NotebookEvaluate[NotebookOpen[NotebookDirectory[] <> "definitions.nb"]]`

FeynCalc 9.2.0. For help, use the documentation center, check out the wiki or write to the mailing list.

See also the supplied examples. If you use FeynCalc in your research, please cite

- V. Shtabovenko, R. Mertig and F. Orellana,
Comput. Phys. Commun., 207C, 432–444, 2016, arXiv:1601.01167
- R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun., 64, 345–359, 1991.

Electron propagator in E_p – representation satisfies the equation

$$S^c(p, q) = (2\pi)^4 \delta(p - q) S^c(p)$$

$$[\gamma p - m - M(p)] S^c(p) = i$$

Let

$$D(p) = \gamma p - m - M(p) = S + \gamma V + \sigma T + \gamma A \gamma^5$$

$$S = m s(p^2, \chi_p),$$

$$V^\mu = p^\mu v_1(p^2, \chi_p) + \frac{e^2 (F^2 p)^\mu}{m^4 \chi^2} v_2(p^2, \chi_p),$$

$$T_{\mu\nu} = \frac{e F_{\mu\nu}}{m \chi} t(p^2, \chi_p),$$

$$A^\mu = \frac{e (F^* p)^\mu}{m^2 \chi} a_s(p^2, \chi_p), \text{ where } s \text{ stands for 'scalar'};$$

then

$$\begin{aligned} S^c(p) &= iD^{-1}(p) = \frac{i}{2} (S - \gamma V - \sigma T + \gamma A \gamma^5) \left[\frac{1}{D_+(p^2, \chi_p)} (1 + \gamma \epsilon^{(2)} \gamma^5) + \frac{1}{D_-(p^2, \chi_p)} (1 - \gamma \epsilon^{(2)} \gamma^5) \right] = \\ &= \frac{i}{2} \left[m s(p^2, \chi_p) - (\gamma p) v_1(p^2, \chi_p) - \frac{e^2 (\gamma F^2 p)}{m^4} v_2(p^2, \chi_p) - \right. \\ &\quad \left. \frac{e \sigma F}{m} t(p^2, \chi_p) + \frac{e (\gamma F^* p) \gamma^5}{m^2} a_s(p^2, \chi_p) \right] \times \\ &\quad \left[\frac{1}{D_+(p^2, \chi_p)} (1 + \gamma \epsilon^{(2)} \gamma^5) + \frac{1}{D_-(p^2, \chi_p)} (1 - \gamma \epsilon^{(2)} \gamma^5) \right], \\ \epsilon^{(2)}_\mu &= \frac{e (F^* p)^\mu}{m^3 \chi_p}, \end{aligned}$$

Electron propagator

$$S^c(x_2, x_1) = \Lambda^{4-D} \int \frac{d^D p}{(2\pi)^D} E_p(x_2) S^c(p) \bar{E}_p(x_1)$$

$$x = x_2 - x_1,$$

$$X = \frac{1}{2} (x_1 + x_2),$$

$$\xi^2 = -\frac{e^2 a^2}{m^2},$$

$$[\Lambda] = m - \text{mass scale},$$

$$E_p(x_2) = \left[1 - \frac{e (\gamma k) (\gamma a)}{2 (kp)} (kx_2) \right]$$

$$\text{Exp} \left[-\mathbb{i} (p \cdot x_2) + \mathbb{i} \frac{e (a \cdot p)}{2 (k \cdot p)} (k \cdot x_2)^2 + \mathbb{i} \frac{a^2 e^2}{6 (k \cdot p)} (k \cdot x_2)^3 \right];$$

NewMomentum["p"]

NewCoordinate["x1"]

NewCoordinate["x2"]

NewCoordinate["x"]

NewCoordinate["X"]

$$\left\{ p^\alpha, p^2, k \cdot p, Fp^\alpha, FFp^\alpha, FDP^\alpha, a \cdot p, 0, 0, 0, -a^2 (k \cdot p), 0, 0, -\frac{m^6 \chi p^2}{e^2}, -\frac{m^6 \chi p^2}{e^2}, \frac{m^6 \chi p^2}{e^2}, 0, 0, 0, 0, 0, 0 \right\}$$

$$\left\{ x1^\alpha, x1^2, k \cdot x1, a \cdot x1, Fx1^\alpha, FFX1^\alpha, FDX1^\alpha, k \cdot x1, 0, 0, 0, -a^2 (k \cdot x1), \right. \\ \left. 0, 0, -\frac{m^2 \xi^2 (k \cdot x1)^2}{e^2}, -\frac{m^2 \xi^2 (k \cdot x1)^2}{e^2}, \frac{m^2 \xi^2 (k \cdot x1)^2}{e^2}, 0, 0, 0, 0, 0, 0 \right\}$$

$$\left\{ x2^\alpha, x2^2, k \cdot x2, a \cdot x2, Fx2^\alpha, FFX2^\alpha, FDX2^\alpha, k \cdot x2, 0, 0, 0, -a^2 (k \cdot x2), \right. \\ \left. 0, 0, -\frac{m^2 \xi^2 (k \cdot x2)^2}{e^2}, -\frac{m^2 \xi^2 (k \cdot x2)^2}{e^2}, \frac{m^2 \xi^2 (k \cdot x2)^2}{e^2}, 0, 0, 0, 0, 0, 0 \right\}$$

$$\left\{ x^\alpha, x^2, k \cdot x, a \cdot x, Fx^\alpha, FFX^\alpha, FDX^\alpha, k \cdot x, 0, 0, 0, -a^2 (k \cdot x), \right. \\ \left. 0, 0, -\frac{m^2 \xi^2 (k \cdot x)^2}{e^2}, -\frac{m^2 \xi^2 (k \cdot x)^2}{e^2}, \frac{m^2 \xi^2 (k \cdot x)^2}{e^2}, 0, 0, 0, 0, 0, 0 \right\}$$

$$\left\{ X^\alpha, X^2, k \cdot X, a \cdot X, FX^\alpha, FFX^\alpha, FDX^\alpha, k \cdot X, 0, 0, 0, -a^2 (k \cdot X), \right. \\ \left. 0, 0, -\frac{m^2 \xi^2 (k \cdot X)^2}{e^2}, -\frac{m^2 \xi^2 (k \cdot X)^2}{e^2}, \frac{m^2 \xi^2 (k \cdot X)^2}{e^2}, 0, 0, 0, 0, 0, 0 \right\}$$

$$\text{DdInv}[\text{sgn}] == \text{Dsgn}(p^2, \chi_p)$$

$$\epsilon == \epsilon^{(2)}$$

```

FToak = {Ft[α_, β_] → kv[α] av[β] - av[α] kv[β], DiracGamma[Momentum[Fp, D], D] →
  DiracGamma[Momentum[k, D], D] Pair[Momentum[a, D], Momentum[p, D]] -
  DiracGamma[Momentum[a, D], D] Pair[Momentum[k, D], Momentum[p, D]],
  DiracGamma[Momentum[FFp, D], D] → -av2 DiracGamma[Momentum[k, D], D]
  Pair[Momentum[k, D], Momentum[p, D]],
  FFpv[μ_] → -av2 kp kv[μ]}
FToEps =
  {FDpv[μ_] → Contract[1/2 Eps[LorentzIndex[μ, D], LorentzIndex[ν, D], LorentzIndex[
    α2, D], LorentzIndex[α3, D]] (kv[α2] av[α3] - av[α2] kv[α3]) pv[ν]]}
Gamma5toTrippleGamma = {GAD[μ_].GA[5] Eps[LorentzIndex[μ_, D],
  Momentum[a_, D], Momentum[b_, D], Momentum[c_, D]] →
  Contract[I Pair[LorentzIndex[α1, D], Momentum[a, D]] Pair[LorentzIndex[α2, D],
    Momentum[b, D]] Pair[LorentzIndex[α3, D], Momentum[c, D]] (GAD[α1, α2, α3] -
    (MTD[α1, α2] GAD[α3] + MTD[α2, α3] GAD[α1] - MTD[α1, α3] GAD[α2]))]}
{F(α_, β_) → aβ kα - aα kβ, γ·Fp → (a·p) γ·k - γ·a (k·p), γ·FFp → a2 (-(γ·k)) (k·p), FFpμ- → a2 (-kμ) (k·p)}
{FDpμ- → -eμ ā k p̄}
{γμ-·γ5 eμ- ā b̄ c̄ → -i (a·b) γ·c + i (a·c) γ·b - i γ·a (b·c) + i (γ·a).(γ·b).(γ·c)}

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FxToak = {Ft[α_, β_] → kv[α] av[β] - av[α] kv[β], DiracGamma[Momentum[Fx, D], D] →
  DiracGamma[Momentum[k, D], D] Pair[Momentum[a, D], Momentum[x, D]] -
  DiracGamma[Momentum[a, D], D] Pair[Momentum[k, D], Momentum[x, D]],
  DiracGamma[Momentum[FFx, D], D] → -av2 DiracGamma[Momentum[k, D], D]
  Pair[Momentum[k, D], Momentum[x, D]],
  FFXv[μ_] → -av2 kx kv[μ], σF →
  -2 I DiracGamma[Momentum[a, D], D].DiracGamma[Momentum[k, D], D] }
FxToEps = {FDx[μ_] → Contract[1/2 Eps[LorentzIndex[μ, D], LorentzIndex[ν, D],
  LorentzIndex[α2, D], LorentzIndex[α3, D]] (kv[α2] av[α3] - av[α2] kv[α3])
  xv[ν]], DiracGamma[Momentum[FDx, D], D].DiracGamma[5] →
  Contract[1/2 Eps[LorentzIndex[μ, D], LorentzIndex[ν, D],
  LorentzIndex[α2, D], LorentzIndex[α3, D]]
  (kv[α2] av[α3] - av[α2] kv[α3]) xv[ν] GAD[μ].DiracGamma[5]]}
Gamma5toTrippleGammax = {DiracGamma[LorentzIndex[μ_, D], D].DiracGamma[5]
  Eps[LorentzIndex[μ_, D], Momentum[a_, D], Momentum[b_, D], Momentum[c_, D]] →
  Contract[I Pair[LorentzIndex[α1, D], Momentum[a, D]] Pair[LorentzIndex[α2, D],
  Momentum[b, D]] Pair[LorentzIndex[α3, D], Momentum[c, D]] (GAD[α1, α2, α3] -
  (MTD[α1, α2] GAD[α3] + MTD[α2, α3] GAD[α1] - MTD[α1, α3] GAD[α2]))]}
{F(α_, β_) → αβ kα - αα kβ, γ·Fx → (a·x) γ·k - γ·a (k·x),
  γ·FFx → α2 (-γ·k) (k·x), FFXμ- → α2 (-kμ) (k·x), σF → -2 i (γ·a).(γ·k)}
{FDx(μ_) → -eμ ā k x̄, (γ·FDx).γ5 → -γμ.γ5 eμ ā k x̄}
{γμ-.γ5 eμ- ā b c̄ → -i (a·b) γ·c + i (a·c) γ·b - i γ·a (b·c) + i (γ·a).(γ·b).(γ·c)}

S = m sf
V[μ_] = pv[μ] v1 + e^2 FFpv[μ] / m^4 / χp^2 v2
T[μ_, ν_] = e Ft[μ, ν] / m / χp tf
A[μ_] = e FDpv[μ] / m^2 / χp af
ε[μ_] = e FDpv[μ] / m^3 / χp

m sf


$$\frac{e^2 v2 FFp^\mu}{m^4 \chi p^2} + v1 p^\mu$$


$$\frac{e tf F(\mu, \nu)}{m \chi p}$$


$$\frac{af e FDp^\mu}{m^2 \chi p}$$


$$\frac{e FDp^\mu}{m^3 \chi p}$$


```

$$\text{Scp}[\text{sgn}_-] = \left(\text{S} - \text{GAD}[\alpha] \text{V}[\alpha] - \text{I} / 2 \left(\text{GAD}[\alpha, \beta] - \text{GAD}[\beta, \alpha] \right) \text{T}[\alpha, \beta] + \text{A}[\alpha] \text{GAD}[\alpha] \cdot \text{GA}[5] \right) \cdot \\ (1 + \text{sgn} \epsilon[\mu] \text{GAD}[\mu] \cdot \text{GA}[5]) / . \text{FToEps} / . \text{FToak} / . \text{Gamma5toTrippleGamma}$$

$$\left(-\frac{\text{af } e(i(a \cdot p) \gamma \cdot k - i \gamma \cdot a(k \cdot p) + i(\gamma \cdot a)(\gamma \cdot k)(\gamma \cdot p))}{m^2 \chi p} - \gamma^\alpha \left(v1 p^\alpha - \frac{a^2 e^2 v2 k^\alpha (k \cdot p)}{m^4 \chi p^2} \right) - \right. \\ \left. \frac{i \text{etf}(\gamma^\alpha \cdot \gamma^\beta - \gamma^\beta \cdot \gamma^\alpha)(a^\beta k^\alpha - a^\alpha k^\beta)}{2 m \chi p} + m \text{sf} \right) \cdot \left(1 - \frac{e \text{sgn}(i(a \cdot p) \gamma \cdot k - i \gamma \cdot a(k \cdot p) + i(\gamma \cdot a)(\gamma \cdot k)(\gamma \cdot p))}{m^3 \chi p} \right)$$

$$\text{Epx2} = \text{Ep}[\text{x2}, p]$$

$$\text{EpBarx1} = \text{EpC}[\text{x1}, p]$$

$$\left\{ 1 - \frac{e(k \cdot x2)(\gamma \cdot k)(\gamma \cdot a)}{2(k \cdot p)}, \frac{a^2 e^2 (k \cdot x2)^3}{6(k \cdot p)} + \frac{e(a \cdot p)(k \cdot x2)^2}{2(k \cdot p)} - p \cdot x2 \right\}$$

$$\left\{ 1 - \frac{e(k \cdot x1)(\gamma \cdot a)(\gamma \cdot k)}{2(k \cdot p)}, -\frac{a^2 e^2 (k \cdot x1)^3}{6(k \cdot p)} - \frac{e(a \cdot p)(k \cdot x1)^2}{2(k \cdot p)} + p \cdot x1 \right\}$$

$$\text{Matrix} = \text{Epx2}[[1]] \cdot \text{Scp}[\xi] \cdot \text{EpBarx1}[[1]]$$

$$\text{Coeff} = i / 2 \Lambda^{(4-D)} / (2 \pi)^D / \text{DdInv}[\xi]$$

$$\text{Phase} = \text{Epx2}[[2]] + \text{EpBarx1}[[2]]$$

$$\left(1 - \frac{e(k \cdot x2)(\gamma \cdot k)(\gamma \cdot a)}{2(k \cdot p)} \right) \cdot \left(-\frac{\text{af } e(i(a \cdot p) \gamma \cdot k - i \gamma \cdot a(k \cdot p) + i(\gamma \cdot a)(\gamma \cdot k)(\gamma \cdot p))}{m^2 \chi p} - \right. \\ \left. \gamma^\alpha \left(v1 p^\alpha - \frac{a^2 e^2 v2 k^\alpha (k \cdot p)}{m^4 \chi p^2} \right) - \frac{i \text{etf}(\gamma^\alpha \cdot \gamma^\beta - \gamma^\beta \cdot \gamma^\alpha)(a^\beta k^\alpha - a^\alpha k^\beta)}{2 m \chi p} + m \text{sf} \right) \cdot \\ \left(1 - \frac{e \xi(i(a \cdot p) \gamma \cdot k - i \gamma \cdot a(k \cdot p) + i(\gamma \cdot a)(\gamma \cdot k)(\gamma \cdot p))}{m^3 \chi p} \right) \cdot \left(1 - \frac{e(k \cdot x1)(\gamma \cdot a)(\gamma \cdot k)}{2(k \cdot p)} \right)$$

$$i 2^{-D-1} \pi^{-D} \Lambda^{4-D}$$

$$\text{DdInv}(\zeta)$$

$$-\frac{a^2 e^2 (k \cdot x1)^3}{6(k \cdot p)} + \frac{a^2 e^2 (k \cdot x2)^3}{6(k \cdot p)} - \frac{e(a \cdot p)(k \cdot x1)^2}{2(k \cdot p)} + \frac{e(a \cdot p)(k \cdot x2)^2}{2(k \cdot p)} + p \cdot x1 - p \cdot x2$$

Matrix1 = Contract[DiracSimplify[Matrix]]

Coeff1 = Coeff;

Phase1 = Phase;

$$\begin{aligned}
& \frac{i v2 \zeta(\gamma \cdot k).(\gamma \cdot a) a^2 (k \cdot p)^2 e^3}{m^7 \chi p^3} + \frac{a f \zeta(\gamma \cdot a).(\gamma \cdot k) a^2 (k \cdot p) (k \cdot x1) e^3}{2 m^5 \chi p^2} + \frac{a f \zeta(\gamma \cdot k).(\gamma \cdot a) a^2 (k \cdot p) (k \cdot x2) e^3}{2 m^5 \chi p^2} - \\
& \frac{i v1 \zeta(\gamma \cdot a).(\gamma \cdot k) a^2 (k \cdot x1) (k \cdot x2) e^3}{2 m^3 \chi p} - \frac{a f \zeta a^2 (k \cdot p)^2 e^2}{m^5 \chi p^2} + \frac{v2 \gamma \cdot k a^2 (k \cdot p) e^2}{m^4 \chi p^2} + \frac{2 t f \zeta \gamma \cdot k a^2 (k \cdot p) e^2}{m^4 \chi p^2} - \\
& \frac{a f \zeta(\gamma \cdot a).(\gamma \cdot k) (a \cdot p) (k \cdot p) e^2}{m^5 \chi p^2} - \frac{a f \zeta(\gamma \cdot k).(\gamma \cdot a) (a \cdot p) (k \cdot p) e^2}{m^5 \chi p^2} + \frac{i s f \zeta \gamma \cdot k a^2 (k \cdot x1) e^2}{2 m^2 \chi p} + \\
& \frac{i a f \gamma \cdot k a^2 (k \cdot x1) e^2}{2 m^2 \chi p} - \frac{i v1 \zeta(\gamma \cdot p).(\gamma \cdot k) a^2 (k \cdot x1) e^2}{2 m^3 \chi p} - \frac{i s f \zeta \gamma \cdot k a^2 (k \cdot x2) e^2}{2 m^2 \chi p} - \frac{i a f \gamma \cdot k a^2 (k \cdot x2) e^2}{2 m^2 \chi p} + \\
& \frac{i v1 \zeta(\gamma \cdot k).(\gamma \cdot p) a^2 (k \cdot x2) e^2}{2 m^3 \chi p} + \frac{i v1 \zeta(\gamma \cdot a).(\gamma \cdot k) (a \cdot p) (k \cdot x2) e^2}{m^3 \chi p} + \frac{i v1 \zeta(\gamma \cdot k).(\gamma \cdot a) (a \cdot p) (k \cdot x2) e^2}{m^3 \chi p} + \\
& \frac{v1 \gamma \cdot k a^2 (k \cdot x1) (k \cdot x2) e^2}{2 (k \cdot p)} + \frac{i t f(\gamma \cdot a).(\gamma \cdot k) e}{m \chi p} - \frac{i t f(\gamma \cdot k).(\gamma \cdot a) e}{m \chi p} - \frac{i s f \zeta(\gamma \cdot a).(\gamma \cdot k).(\gamma \cdot p) e}{m^2 \chi p} - \\
& \frac{i a f(\gamma \cdot a).(\gamma \cdot k).(\gamma \cdot p) e}{m^2 \chi p} - \frac{i s f \zeta \gamma \cdot k (a \cdot p) e}{m^2 \chi p} - \frac{i a f \gamma \cdot k (a \cdot p) e}{m^2 \chi p} + \frac{2 i v1 \zeta(\gamma \cdot k).(\gamma \cdot p) (a \cdot p) e}{m^3 \chi p} + \\
& \frac{i v1 \zeta(\gamma \cdot p).(\gamma \cdot k) (a \cdot p) e}{m^3 \chi p} + \frac{i s f \zeta \gamma \cdot a (k \cdot p) e}{m^2 \chi p} + \frac{i a f \gamma \cdot a (k \cdot p) e}{m^2 \chi p} - \frac{2 i v1 \zeta(\gamma \cdot a).(\gamma \cdot p) (k \cdot p) e}{m^3 \chi p} - \\
& \frac{i v1 \zeta(\gamma \cdot p).(\gamma \cdot a) (k \cdot p) e}{m^3 \chi p} - \frac{m s f(\gamma \cdot a).(\gamma \cdot k) (k \cdot x1) e}{2 (k \cdot p)} + \frac{v1 (\gamma \cdot p).(\gamma \cdot a).(\gamma \cdot k) (k \cdot x1) e}{2 (k \cdot p)} - \\
& \frac{m s f(\gamma \cdot k).(\gamma \cdot a) (k \cdot x2) e}{2 (k \cdot p)} + \frac{v1 (\gamma \cdot k).(\gamma \cdot a).(\gamma \cdot p) (k \cdot x2) e}{2 (k \cdot p)} + \frac{i v1 \zeta(\gamma \cdot a).(\gamma \cdot k) p^2 e}{m^3 \chi p} + m s f - v1 \gamma \cdot p
\end{aligned}$$

(*a small test*)

$$\begin{aligned}
& \left(\left(\text{Matrix1} /. \{s f \rightarrow 1, v1 \rightarrow -1, v2 \rightarrow 0, t \rightarrow 0, a f \rightarrow 0, \xi \rightarrow 1\} \right) + \right. \\
& \quad \left. \left(\text{Matrix1} /. \{s f \rightarrow 1, v1 \rightarrow -1, v2 \rightarrow 0, t \rightarrow 0, a f \rightarrow 0, \xi \rightarrow -1\} \right) \right) / 2 \\
& \frac{1}{2} \left(-\frac{a^2 e^2 \gamma \cdot k (k \cdot x1) (k \cdot x2)}{k \cdot p} - \frac{e m (k \cdot x1) (\gamma \cdot a).(\gamma \cdot k)}{k \cdot p} - \frac{e m (k \cdot x2) (\gamma \cdot k).(\gamma \cdot a)}{k \cdot p} + \frac{2 i e t f(\gamma \cdot a).(\gamma \cdot k)}{m \chi p} - \right. \\
& \quad \left. \frac{2 i e t f(\gamma \cdot k).(\gamma \cdot a)}{m \chi p} - \frac{e (k \cdot x1) (\gamma \cdot p).(\gamma \cdot a).(\gamma \cdot k)}{k \cdot p} - \frac{e (k \cdot x2) (\gamma \cdot k).(\gamma \cdot a).(\gamma \cdot p)}{k \cdot p} + 2 m + 2 \gamma \cdot p \right)
\end{aligned}$$

$$\begin{aligned}
& \text{Matrix2} = \text{Expand}[\text{ExpandScalarProduct}[\\
& \quad \text{Matrix1} /. \{ \text{Momentum}[\mathbf{x1}, \mathbf{D}] \rightarrow \text{Momentum}[\mathbf{X}, \mathbf{D}] - \text{Momentum}[\mathbf{x}, \mathbf{D}] / 2, \\
& \quad \text{Momentum}[\mathbf{x2}, \mathbf{D}] \rightarrow \text{Momentum}[\mathbf{X}, \mathbf{D}] + \text{Momentum}[\mathbf{x}, \mathbf{D}] / 2 \}]] \\
& \text{Coeff2} = \text{Coeff1}; \\
& \text{Phase2} = \text{Expand}[\\
& \quad \text{ExpandScalarProduct}[\text{Phase1} /. \{ \text{Momentum}[\mathbf{x1}, \mathbf{D}] \rightarrow \text{Momentum}[\mathbf{X}, \mathbf{D}] - \text{Momentum}[\mathbf{x}, \mathbf{D}] / 2, \\
& \quad \text{Momentum}[\mathbf{x2}, \mathbf{D}] \rightarrow \text{Momentum}[\mathbf{X}, \mathbf{D}] + \text{Momentum}[\mathbf{x}, \mathbf{D}] / 2 \}]] \\
& \frac{i v2 \zeta(\gamma \cdot k)(\gamma \cdot a) a^2 (k \cdot p)^2 e^3}{m^7 \chi p^3} + \frac{i v1 \zeta(\gamma \cdot a)(\gamma \cdot k) a^2 (k \cdot x)^2 e^3}{8 m^3 \chi p} - \frac{i v1 \zeta(\gamma \cdot a)(\gamma \cdot k) a^2 (k \cdot X)^2 e^3}{2 m^3 \chi p} - \\
& \frac{a f \zeta(\gamma \cdot a)(\gamma \cdot k) a^2 (k \cdot p)(k \cdot x) e^3}{4 m^5 \chi p^2} + \frac{a f \zeta(\gamma \cdot k)(\gamma \cdot a) a^2 (k \cdot p)(k \cdot x) e^3}{4 m^5 \chi p^2} + \frac{a f \zeta(\gamma \cdot a)(\gamma \cdot k) a^2 (k \cdot p)(k \cdot X) e^3}{2 m^5 \chi p^2} + \\
& \frac{a f \zeta(\gamma \cdot k)(\gamma \cdot a) a^2 (k \cdot p)(k \cdot X) e^3}{2 m^5 \chi p^2} - \frac{a f \zeta a^2 (k \cdot p)^2 e^2}{m^5 \chi p^2} - \frac{v1 \gamma \cdot k a^2 (k \cdot x)^2 e^2}{8 (k \cdot p)} + \frac{v1 \gamma \cdot k a^2 (k \cdot X)^2 e^2}{2 (k \cdot p)} + \\
& \frac{v2 \gamma \cdot k a^2 (k \cdot p) e^2}{m^4 \chi p^2} + \frac{2 t f \zeta \gamma \cdot k a^2 (k \cdot p) e^2}{m^4 \chi p^2} - \frac{a f \zeta(\gamma \cdot a)(\gamma \cdot k)(a \cdot p)(k \cdot p) e^2}{m^5 \chi p^2} - \frac{a f \zeta(\gamma \cdot k)(\gamma \cdot a)(a \cdot p)(k \cdot p) e^2}{m^5 \chi p^2} - \\
& \frac{i s f \zeta \gamma \cdot k a^2 (k \cdot x) e^2}{2 m^2 \chi p} - \frac{i a f \gamma \cdot k a^2 (k \cdot x) e^2}{2 m^2 \chi p} + \frac{i v1 \zeta(\gamma \cdot k)(\gamma \cdot p) a^2 (k \cdot x) e^2}{4 m^3 \chi p} + \frac{i v1 \zeta(\gamma \cdot p)(\gamma \cdot k) a^2 (k \cdot x) e^2}{4 m^3 \chi p} + \\
& \frac{i v1 \zeta(\gamma \cdot a)(\gamma \cdot k)(a \cdot p)(k \cdot x) e^2}{2 m^3 \chi p} + \frac{i v1 \zeta(\gamma \cdot k)(\gamma \cdot a)(a \cdot p)(k \cdot x) e^2}{2 m^3 \chi p} + \frac{i v1 \zeta(\gamma \cdot k)(\gamma \cdot p) a^2 (k \cdot X) e^2}{2 m^3 \chi p} - \\
& \frac{i v1 \zeta(\gamma \cdot p)(\gamma \cdot k) a^2 (k \cdot X) e^2}{2 m^3 \chi p} + \frac{i v1 \zeta(\gamma \cdot a)(\gamma \cdot k)(a \cdot p)(k \cdot X) e^2}{m^3 \chi p} + \frac{i v1 \zeta(\gamma \cdot k)(\gamma \cdot a)(a \cdot p)(k \cdot X) e^2}{m^3 \chi p} + \\
& \frac{i t f(\gamma \cdot a)(\gamma \cdot k) e}{m \chi p} - \frac{i t f(\gamma \cdot k)(\gamma \cdot a) e}{m \chi p} - \frac{i s f \zeta(\gamma \cdot a)(\gamma \cdot k)(\gamma \cdot p) e}{m^2 \chi p} - \frac{i a f(\gamma \cdot a)(\gamma \cdot k)(\gamma \cdot p) e}{m^2 \chi p} - \\
& \frac{i s f \zeta \gamma \cdot k(a \cdot p) e}{m^2 \chi p} - \frac{i a f \gamma \cdot k(a \cdot p) e}{m^2 \chi p} + \frac{2 i v1 \zeta(\gamma \cdot k)(\gamma \cdot p)(a \cdot p) e}{m^3 \chi p} + \frac{i v1 \zeta(\gamma \cdot p)(\gamma \cdot k)(a \cdot p) e}{m^3 \chi p} + \\
& \frac{i s f \zeta \gamma \cdot a(k \cdot p) e}{m^2 \chi p} + \frac{i a f \gamma \cdot a(k \cdot p) e}{m^2 \chi p} - \frac{2 i v1 \zeta(\gamma \cdot a)(\gamma \cdot p)(k \cdot p) e}{m^3 \chi p} - \frac{i v1 \zeta(\gamma \cdot p)(\gamma \cdot a)(k \cdot p) e}{m^3 \chi p} + \\
& \frac{m s f(\gamma \cdot a)(\gamma \cdot k)(k \cdot x) e}{4 (k \cdot p)} - \frac{m s f(\gamma \cdot k)(\gamma \cdot a)(k \cdot x) e}{4 (k \cdot p)} + \frac{v1 (\gamma \cdot k)(\gamma \cdot a)(\gamma \cdot p)(k \cdot x) e}{4 (k \cdot p)} - \\
& \frac{v1 (\gamma \cdot p)(\gamma \cdot a)(\gamma \cdot k)(k \cdot x) e}{4 (k \cdot p)} - \frac{m s f(\gamma \cdot a)(\gamma \cdot k)(k \cdot X) e}{2 (k \cdot p)} - \frac{m s f(\gamma \cdot k)(\gamma \cdot a)(k \cdot X) e}{2 (k \cdot p)} + \\
& \frac{v1 (\gamma \cdot k)(\gamma \cdot a)(\gamma \cdot p)(k \cdot X) e}{2 (k \cdot p)} + \frac{v1 (\gamma \cdot p)(\gamma \cdot a)(\gamma \cdot k)(k \cdot X) e}{2 (k \cdot p)} + \frac{i v1 \zeta(\gamma \cdot a)(\gamma \cdot k) p^2 e}{m^3 \chi p} + m s f - v1 \gamma \cdot p \\
& \frac{a^2 e^2 (k \cdot x)(k \cdot X)^2}{2 (k \cdot p)} + \frac{a^2 e^2 (k \cdot x)^3}{24 (k \cdot p)} + \frac{e(a \cdot p)(k \cdot x)(k \cdot X)}{k \cdot p} - p \cdot x
\end{aligned}$$

Matrix3 =

((Expand[Matrix2 /. TripleGamma]) /. EpsToF) /. FieldSubstitutions) /. akToF

Coeff3 = Coeff2;

Phase3 = Phase2;

$$\begin{aligned}
& \frac{v2 \zeta \sigma F a^2 (k \cdot p)^2 e^3}{2 m^7 \chi p^3} - \frac{v1 \zeta \sigma F a^2 (k \cdot x)^2 e^3}{16 m^3 \chi p} + \frac{v1 \zeta \sigma F a^2 (k \cdot X)^2 e^3}{4 m^3 \chi p} - \frac{i \text{af} \zeta \sigma F a^2 (k \cdot p) (k \cdot x) e^3}{4 m^5 \chi p^2} - \\
& \frac{\text{af} \zeta a^2 (k \cdot p)^2 e^2}{m^5 \chi p^2} - \frac{v1 \gamma \cdot k a^2 (k \cdot x)^2 e^2}{8 (k \cdot p)} + \frac{v1 \gamma \cdot k a^2 (k \cdot X)^2 e^2}{2 (k \cdot p)} + \frac{v2 \gamma \cdot k a^2 (k \cdot p) e^2}{m^4 \chi p^2} + \\
& \frac{2 \text{tf} \zeta \gamma \cdot k a^2 (k \cdot p) e^2}{m^4 \chi p^2} - \frac{i \text{sf} \zeta \gamma \cdot k a^2 (k \cdot x) e^2}{2 m^2 \chi p} - \frac{i \text{af} \gamma \cdot k a^2 (k \cdot x) e^2}{2 m^2 \chi p} + \frac{i v1 \zeta (\gamma \cdot k) \cdot (\gamma \cdot p) a^2 (k \cdot x) e^2}{4 m^3 \chi p} + \\
& \frac{i v1 \zeta (\gamma \cdot p) \cdot (\gamma \cdot k) a^2 (k \cdot x) e^2}{4 m^3 \chi p} + \frac{i v1 \zeta (\gamma \cdot k) \cdot (\gamma \cdot p) a^2 (k \cdot X) e^2}{2 m^3 \chi p} - \frac{i v1 \zeta (\gamma \cdot p) \cdot (\gamma \cdot k) a^2 (k \cdot X) e^2}{2 m^3 \chi p} + \\
& \frac{\text{sf} \zeta \gamma^\beta \cdot \bar{\gamma}^5 \text{FDp}^\beta e}{m^2 \chi p} + \frac{\text{af} \gamma^\beta \cdot \bar{\gamma}^5 \text{FDp}^\beta e}{m^2 \chi p} + \frac{2 i v1 \zeta (\gamma \cdot k) \cdot (\gamma \cdot p) (a \cdot p) e}{m^3 \chi p} + \frac{i v1 \zeta (\gamma \cdot p) \cdot (\gamma \cdot k) (a \cdot p) e}{m^3 \chi p} - \\
& \frac{2 i v1 \zeta (\gamma \cdot a) \cdot (\gamma \cdot p) (k \cdot p) e}{m^3 \chi p} - \frac{i v1 \zeta (\gamma \cdot p) \cdot (\gamma \cdot a) (k \cdot p) e}{m^3 \chi p} + \frac{i m \text{sf} \sigma F (k \cdot x) e}{4 (k \cdot p)} - \frac{i v1 \gamma^\beta \cdot \bar{\gamma}^5 \text{FDp}^\beta (k \cdot x) e}{2 (k \cdot p)} - \\
& v1 \gamma \cdot a (k \cdot X) e + \frac{v1 \gamma \cdot k (a \cdot p) (k \cdot X) e}{k \cdot p} - \frac{v1 \zeta \sigma F p^2 e}{2 m^3 \chi p} - \frac{\text{tf} \sigma F e}{m \chi p} + m \text{sf} - v1 \gamma \cdot p
\end{aligned}$$

```

Matrix4 = Contract[
  DiracOrder[
    Matrix3 /. { DiracGamma[LorentzIndex[β, D], D].DiracGamma[5]
      Pair[LorentzIndex[β, D], Momentum[FDp, D]] → FVD[γγ5FD, α] pv[α] }
    ] /. {
      DiracGamma[Momentum[a, D], D].DiracGamma[Momentum[p, D], D] → FVD[γαγ, α] pv[α],
      DiracGamma[Momentum[k, D], D].DiracGamma[Momentum[p, D], D] → FVD[γkγ, α] pv[α] }
    ]

```

Coeff4 = Coeff3;

Phase4 = Phase3;

$$\begin{aligned}
& -\frac{a^2 \text{af} e^2 \zeta (k \cdot p)^2}{m^5 \chi p^2} + \frac{a^2 e^3 \zeta \sigma F v_2 (k \cdot p)^2}{2 m^7 \chi p^3} - \frac{a^2 e^3 \zeta \sigma F v_1 (k \cdot x)^2}{16 m^3 \chi p} + \frac{a^2 e^3 \zeta \sigma F v_1 (k \cdot X)^2}{4 m^3 \chi p} - \\
& \frac{a^2 e^2 v_1 \gamma \cdot k (k \cdot x)^2}{8 (k \cdot p)} + \frac{a^2 e^2 v_1 \gamma \cdot k (k \cdot X)^2}{2 (k \cdot p)} - \frac{i a^2 \text{af} e^3 \zeta \sigma F (k \cdot p) (k \cdot x)}{4 m^5 \chi p^2} - \frac{i a^2 \text{af} e^2 \gamma \cdot k (k \cdot x)}{2 m^2 \chi p} + \\
& \frac{2 a^2 e^2 \zeta \text{tf} \gamma \cdot k (k \cdot p)}{m^4 \chi p^2} + \frac{a^2 e^2 v_2 \gamma \cdot k (k \cdot p)}{m^4 \chi p^2} + \frac{i a^2 e^2 \zeta v_1 (k \cdot p) (k \cdot x)}{2 m^3 \chi p} + \frac{i a^2 e^2 \zeta v_1 (k \cdot X) (p \cdot \gamma k \gamma)}{m^3 \chi p} - \\
& \frac{i a^2 e^2 \zeta v_1 (k \cdot p) (k \cdot X)}{m^3 \chi p} - \frac{i a^2 e^2 \zeta \text{sf} \gamma \cdot k (k \cdot x)}{2 m^2 \chi p} + \frac{e v_1 (a \cdot p) \gamma \cdot k (k \cdot X)}{k \cdot p} - e v_1 \gamma \cdot a (k \cdot X) + \\
& \frac{i e \zeta v_1 (a \cdot p) (p \cdot \gamma k \gamma)}{m^3 \chi p} + \frac{\text{af} e (p \cdot \gamma \gamma 5 \text{FD})}{m^2 \chi p} - \frac{i e \zeta v_1 (k \cdot p) (p \cdot \gamma a \gamma)}{m^3 \chi p} + \frac{i e m \text{sf} \sigma F (k \cdot x)}{4 (k \cdot p)} - \\
& \frac{i e v_1 (k \cdot x) (p \cdot \gamma \gamma 5 \text{FD})}{2 (k \cdot p)} - \frac{e \zeta p^2 \sigma F v_1}{2 m^3 \chi p} + \frac{e \zeta \text{sf} (p \cdot \gamma \gamma 5 \text{FD})}{m^2 \chi p} - \frac{e \sigma F \text{tf}}{m \chi p} + m \text{sf} - v_1 \gamma \cdot p
\end{aligned}$$

Expanding scalar products into components and changing variables

$$p \rightarrow \{p_- = 1/2 (p^0 - p^3), p_+ = p^0 + p^3, p_\perp\}$$

$$p_- = x_- / 2 s$$

$$p_+ = (p^2 + p_\perp^2) / 2 p_- = s (p^2 + p_\perp^2) / x_-$$

Integration measure

$$\int d^D p \dots = \int \frac{ds}{2s} dp^2 d^{D-2} p_\perp$$

$$x_- = kx / m$$

$$p_- = kx / 2 m s$$

$$kp_- = m p_- = m xm / 2 s = kx / 2 s$$

$$ap = -at pt$$

$$\gamma p = \gamma_- p_+ + \gamma_+ p_- - \gamma_\perp p_\perp = Gm \frac{s}{x_-} (p^2 + p_\perp^2) + Gp \frac{x_-}{2s} - Gt pt$$

$$\gamma k = \gamma_- k_+ = m Gm$$

$$kx = k_+ x_- = m xm$$

$$(\gamma F^*)^\mu \cdot \gamma^5 \rightarrow \{(\gamma F^*)_- \cdot \gamma^5 = 0, (\gamma F^*)_+ \cdot \gamma^5, (\gamma F^*)_\perp \cdot \gamma^5\} = \{0, \gamma\gamma 5FDp, \gamma\gamma 5FDt\}$$

$$(\gamma F^*)_\mu k^\mu = 0 \rightarrow (\gamma F^*)_- = 0$$

$$(\gamma F^*)_\mu a^\mu = 0 \rightarrow \gamma\gamma 5FDt at = 0$$

$$(\gamma F^*)^\mu \cdot \gamma^5 p_\mu = \gamma\gamma 5FDm * pp$$

$$\gamma k \gamma_- = (\gamma k)^2 / m = 0$$

$$D[\{xm/2/s, s(p2+pt2)/xm\}, \{s, p2\}]$$

$$\text{Abs}[\text{Det}[\%]]$$

$$\begin{pmatrix} -\frac{xm}{2s^2} & 0 \\ \frac{p2+pt2}{xm} & \frac{s}{xm} \end{pmatrix}$$

$$\frac{1}{2|s|}$$

```

Matrix5 = Collect[
  Expand[Matrix4 /. {DiracGamma[Momentum[p, D], D] → Gp * pm + Gm * pp - Gt[i] * pt[i],
    Pair[Momentum[a, D], Momentum[p, D]] → -at[i1] pt[i1],
    Pair[Momentum[p, D], Momentum[γγ5FD, D]] → γγ5FDp * pm,
    Pair[Momentum[p, D], Momentum[γαγ, D]] →
      γαγp * pm + γαγm * pp - γαγt[i] * pt[i],
    Pair[Momentum[p, D], Momentum[γkγ, D]] → γkγp * pm - γkγt[i] * pt[i],
    (*av2→-at^2,*)
    DiracGamma[Momentum[k, D], D] → m Gm} /. {γkγm → 0} /. {χp → ξ kp / m^2} /.
    {kp → kx / 2 / s, pm → xm / 2 / s} /. {kx → m xm} /. {pp → s (pv2 + pt[i2]^2) / xm}],
  {v1, pt[i2] pt[i], pt[i2], pt[i_], pv2, γγ5FDm, γγ5FDp, γγ5FDt[i],
    γkγm, γkγp, γkγt[i], γαγm, γαγp, γαγt[i]}]
Coefficient[Matrix5, γkγt[i]]
Coeff5 = Coeff4 / 2 / s
Phase5 = Collect[
  Expand[Phase4 /. {Pair[Momentum[p, D], Momentum[x, D]] → pp xm + pm xp - pt * xt,
    kp → m pm, Pair[Momentum[a, D], Momentum[p, D]] → -at pt} /. {kp → kx / 2 / s,
    pm → xm / 2 / s} /. {pp → s (pv2 + pt^2) / xm} /. {xm → kx / m}], {pt, pp}]
v1 (
  (a^2 e^3 ζ s σF (k.X)^2) / (2 m^2 ξ xm) + (a^2 e^2 Gm s (k.X)^2) / xm +
  pt(i) (
    - (2 i a^2 e^2 ζ s γkγt(i) (k.X)) / (m^2 ξ xm) + (2 i e ζ s at(i1) γkγt(i) pt(i1)) / (m^2 ξ xm) + (i e ζ γαγt(i)) / (m ξ) + Gt(i)
  ) -
  (a^2 e^3 ζ s σF xm) / (8 ξ) - (1 / 4) a^2 e^2 Gm m^2 s xm + (i a^2 γkγp e^2 ζ (k.X)) / (m^2 ξ) - (i a^2 e^2 ζ (k.X)) / (m ξ) + (i a^2 e^2 ζ xm) / (2 ξ) -
  e γ . a (k.X) + pt(i1) (
    - (2 e Gm s at(i1) (k.X)) / xm - (i γkγp e ζ at(i1)) / (m^2 ξ)
  ) + pt(i2)^2 (
    - (Gm s) / xm - (i γαγm e ζ s) / (m ξ xm)
  ) +
  p^2 (
    - (e ζ s σF) / (m^2 ξ xm) - (i γαγm e ζ s) / (m ξ xm) - (Gm s) / xm
  ) - (i γαγp e ζ xm) / (2 m ξ s) - (1 / 2) i γγ5FDp e xm - (Gp xm) / (2 s) -
  (i a^2 af e^3 ζ s σF) / (2 m ξ^2) - (i a^2 af e^2 Gm m s) / ξ - (a^2 af e^2 ζ) / (m ξ^2) + (a^2 e^3 ζ s σF v2) / (m^2 ξ^3 xm) -
  (i a^2 e^2 ζ Gm m s sf) / ξ +
  (4 a^2 e^2 ζ Gm s tf) / ξ^2 xm +
  (2 a^2 e^2 Gm s v2) / ξ^2 xm + γγ5FDp (
    (af e) / (m ξ) + (e ζ sf) / (m ξ)
  ) +
  (1 / 2) i e m s sf σF - (2 e s σF tf) / ξ xm + m sf
  (2 i e ζ s v1 at(i1) pt(i) pt(i1)) / (m^2 ξ xm) - (2 i a^2 e^2 ζ s v1 pt(i) (k.X)) / (m^2 ξ xm)
)

```

$$\frac{i\,2^{-D-2}\,\pi^{-D}\,\Lambda^{4-D}}{s\,\text{DdInv}(\zeta)}\frac{1}{12}\,a^2\,e^2\,s\,(k\cdot x)^2+a^2\,e^2\,s\,(k\cdot X)^2+\text{pt}\,(\text{xt}-2\,\text{at}\,e\,s\,(k\cdot X))-\frac{\text{xp}\,(k\cdot x)}{2\,m\,s}-p^2\,s+\text{pt}^2\,(-s)$$

Integration over

$$\int d^{D-2} p_{\perp} \dots$$

$$\begin{aligned} I_0 &= \int d^{D-2} p_{\perp} \text{Exp} \left[-I A p_{\perp}^2 + I (\mathbf{J} \cdot \mathbf{p}_{\perp}) \right] = \\ &= \text{Exp} \left[-I \frac{\pi}{2} \frac{D-2}{2} \right] \pi^{\frac{D-2}{2}} (\det A)^{-\frac{1}{2}} \text{Exp} \left[I \frac{1}{4} \mathbf{J} \cdot \mathbf{A}^{-1} \cdot \mathbf{J} \right] \end{aligned}$$

$$\begin{aligned} I_{1i} &= \int d^{D-2} p_{\perp} p_{\perp i} \text{Exp} \left[-I A p_{\perp}^2 + I (\mathbf{J} \cdot \mathbf{p}_{\perp}) \right] = \\ &= \frac{1}{2} (A^{-1} \cdot \mathbf{J})_i I_0 \end{aligned}$$

$$\begin{aligned} I_2 &= \int d^{D-2} p_{\perp} p_{\perp}^2 \text{Exp} \left[-I A p_{\perp}^2 + I (\mathbf{J} \cdot \mathbf{p}_{\perp}) \right] = \\ &= \left[-i \frac{1}{2} \text{Tr} A^{-1} + \left(\frac{1}{2} A^{-1} \cdot \mathbf{J} \right)^2 \right] \end{aligned}$$

$$\begin{aligned} I_{2ij} &= \int d^{D-2} p_{\perp} p_{\perp i} p_{\perp j} \text{Exp} \left[-I A p_{\perp}^2 + I (\mathbf{J} \cdot \mathbf{p}_{\perp}) \right] = \\ &= \left[-i \frac{1}{2} A^{-1}_{ij} + \left(\frac{1}{2} A^{-1} \cdot \mathbf{J} \right)_i \left(\frac{1}{2} A^{-1} \cdot \mathbf{J} \right)_j \right] I_0 \end{aligned}$$

$$\begin{aligned} I_{3i} &= -i \frac{\partial}{\partial J_i} I_2 = \int d^{D-2} p_{\perp} p_{\perp}^2 p_{\perp i} \text{Exp} \left[-I A p_{\perp}^2 + I (\mathbf{J} \cdot \mathbf{p}_{\perp}) \right] = \\ &= \left\{ \left[-i \frac{1}{2} \text{Tr} A^{-1} + \left(\frac{1}{2} A^{-1} \cdot \mathbf{J} \right)^2 \right] \frac{1}{2} (A^{-1} \cdot \mathbf{J})_i - i \frac{1}{2} (A^{-1 T} A^{-1} \cdot \mathbf{J})_i \right\} I_0 = \\ &= \left(\frac{1}{2} A^{-1} \cdot \mathbf{J} \right)_i I_2 - i (A^{-1 T})_{ij} I_{1j} \end{aligned}$$

where

$$A = s,$$

$$\mathbf{J} = \mathbf{x}_{\perp} - 2 e a_{\perp} s \mathbf{k} X,$$

$$\det A = s^{D-2},$$

$$A^{-1} = 1 / s$$

We perform integrations

Integrations changes the coefficient (Coeff) and phase

Then recollect some scalar products

$$a_{\perp}^2 = -a^2$$

$$\mathbf{a}_{\perp} \cdot \mathbf{x}_{\perp} = -(\mathbf{a} \cdot \mathbf{x})$$

$$x_{\perp}^2 = 2 x_{-} x_{+} - x^2$$

```

Clear[J]
Amatr = -Coefficient[Phase5, pt^2]
J[i_] = Coefficient[Phase5, pt] /. {at → at[i], xt → xt[i]}
CI0 = Exp[-I Pi / 2 (D / 2 - 1)] Pi^ (D / 2 - 1) / Amatr^ (D / 2 - 1)

s

xt(i) - 2 e s at(i) (k · X)


$$e^{-\frac{1}{2} i \pi \left(\frac{D}{2}-1\right)} \pi^{\frac{D}{2}-1} s^{1-\frac{D}{2}}$$


Phase6 = Expand[Expand[(Phase5 /. {pt → 0}) + 1 / 4 J[i]^2 / Amatr]] /.
  { at[i]^2 → -av2, at[i] xt[i] → -ax, xt[i]^2 → 2 xm xp - xv2} /. {xm → kx / m}]
Coeff6 = Coeff5 * CI0
Expand[
  Expand[Matrix5] /.
    {pt[i2_]^2 pt[i1_] → ((-I / 2 * (D - 2) / Amatr + (1 / 2 / Amatr * J[i2])^2) *
      1 / 2 / Amatr * J[i1] - I * 1 / 2 / Amatr^2 * J[i1])} /. {pt[i_] pt[i1_] →
      (- I / 2 / Amatr δ[i, i1] + (1 / 2 / Amatr * J[i]) * (1 / 2 / Amatr * J[i1]))} /.
      {pt[i_]^2 → (-I / 2 * (D - 2) / Amatr + (1 / 2 / Amatr * J[i])^2)} /.
      {pt[i_] → (1 / 2 / Amatr * J[i])}
  ];
%- Coefficient[%, δ[i, i1]] δ[i, i1] + (Coefficient[%, δ[i, i1]] /. {i1 → i});
Matrix6 =
  Collect[% /. {γ5FDt[i_] at[i_] → 0} /. { at[i_] ^2 → -av2} /. {at[i_] xt[i_] → -ax} /.
    { xt[i_] ^2 → 2 xm xp - xv2} /. {at[i_] ax kX xt[i_] → -ax ax kX},
    {v1, pt[i2] pt[i], pt[i2], pt[i_], pv2, γ5FDm, γ5FDp,
      γ5FDt[i], γkγm, γkγp, γkγt[i], γaγm, γaγp, γaγt[i]}]
  (*Matrix6=Collect[Expand[Expand[(Matrix5/.{pt[i_]→0})+
    Coefficient[Matrix5,pt]*1/2/Amatr*J+
    Coefficient[Matrix5,pt^2]*(-I/2*(D-2)/Amatr+( 1/2/Amatr*J)^2)+
    Coefficient[Matrix5,pt^3]*((-I/2*(D-2)/Amatr+( 1/2/Amatr*J)^2)*1/2/Amatr*J+
      1/2/Amatr^2*J)]/.{γ5FDt at→ 0, at^4→ av2^2 ,
    at^3→ -av2 at, at^2→-av2,at xt→ -ax, xt^2→ 2xm xp -xv2}],
    {p2,Gm,Gp,Gt,γ5FDm,γ5FDp,γ5FDt,γkγm,γkγp,γkγt,γaγm,γaγp,γaγt}]**)


$$\frac{1}{12} a^2 e^2 s (k \cdot x)^2 + e(a \cdot x) (k \cdot X) - p^2 s - \frac{x^2}{4 s}$$



$$\frac{i 2^{-D-2} e^{-\frac{1}{2} i \pi \left(\frac{D}{2}-1\right)} \pi^{-\frac{D}{2}-1} \Lambda^{4-D} s^{-D/2}}{\text{DdInv}(\zeta)}$$


```

$$\begin{aligned}
& v_l \left(\gamma_a \gamma_m \left(\frac{i a^2 e^3 \zeta s (k \cdot X)^2}{m \xi x_m} - \frac{i e^2 \zeta (a \cdot x) (k \cdot X)}{m \xi x_m} - \frac{D e \zeta}{2 m \xi x_m} - \frac{i e \zeta (2 x_m x_p - x^2)}{4 m \xi s x_m} + \frac{e \zeta}{m \xi x_m} \right) + \right. \\
& \quad \frac{a^2 e^3 \zeta s \sigma F (k \cdot X)^2}{2 m^2 \xi x_m} + \gamma_k \gamma_t(i) \left(\frac{i e^2 \zeta \text{at}(i) (a \cdot x) (k \cdot X)}{m^2 \xi x_m} - \frac{i e \zeta \text{xt}(i) (a \cdot x)}{2 m^2 \xi s x_m} + \frac{e \zeta \text{at}(i)}{m^2 \xi x_m} \right) - \\
& \quad \frac{a^2 e^3 \zeta s \sigma F x_m}{8 \xi} - \frac{1}{4} a^2 e^2 G_m m^2 s x_m - \frac{i a^2 e^2 \zeta (k \cdot X)}{m \xi} + \frac{i a^2 e^2 \zeta x_m}{2 \xi} - \\
& \quad e \gamma \cdot a (k \cdot X) + \frac{i \gamma_k \gamma_p e \zeta (a \cdot x)}{2 m^2 \xi s} + \gamma_a \gamma_t(i) \left(\frac{i e \zeta \text{xt}(i)}{2 m \xi s} - \frac{i e^2 \zeta \text{at}(i) (k \cdot X)}{m \xi} \right) - \\
& \quad e \text{at}(i) G_t(i) (k \cdot X) + \frac{i D G_m}{2 x_m} + p^2 \left(-\frac{e \zeta s \sigma F}{m^2 \xi x_m} - \frac{i \gamma_a \gamma_m e \zeta s}{m \xi x_m} - \frac{G_m s}{x_m} \right) - \frac{i \gamma_a \gamma_p e \zeta x_m}{2 m \xi s} - \\
& \quad \frac{1}{2} i \gamma \gamma 5 F D p e x_m - \frac{G_m (2 x_m x_p - x^2)}{4 s x_m} - \frac{i G_m}{x_m} - \frac{G_p x_m}{2 s} + \frac{G_t(i) \text{xt}(i)}{2 s} \Big) - \\
& \quad \frac{i a^2 a f e^3 \zeta s \sigma F}{2 m \xi^2} - \frac{i a^2 a f e^2 G_m m s}{\xi} - \frac{a^2 a f e^2 \zeta}{m \xi^2} + \frac{a^2 e^3 \zeta s \sigma F v_2}{m^2 \xi^3 x_m} - \\
& \quad \frac{i a^2 e^2 \zeta G_m m s s f}{\xi} + \\
& \quad \frac{4 a^2 e^2 \zeta G_m s t f}{\xi^2 x_m} + \\
& \quad \frac{2 a^2 e^2 G_m s v_2}{\xi^2 x_m} + \\
& \quad \gamma \gamma 5 F D p \left(\frac{a f e}{m \xi} + \frac{e \zeta s f}{m \xi} \right) + \\
& \quad \frac{1}{2} i e m s s f \sigma F - \frac{2 e s \sigma F t f}{\xi x_m} + m s f
\end{aligned}$$

Next we substitute

$$\gamma_{\perp} \mathbf{a}_{\perp} = -\gamma \mathbf{a}$$

$$\gamma_{\perp} \mathbf{x}_{\perp} = \gamma_{-} \mathbf{x}_{+} + \gamma_{+} \mathbf{x}_{-} - \gamma \mathbf{x}$$

$$\gamma_{-} \mathbf{x}_{-} = \frac{\gamma \mathbf{k}}{m} \mathbf{x}_{-}$$

$$\mathbf{x}_{-} = \mathbf{k} \mathbf{x} / m$$


```

Matrix7 = Collect[
  DiracSimplify[
    Expand[Matrix6] /. {Gt[i_] at[i_] → -Contract[GAD[α] av[α]],
      Gt[i_] xt[i_] → Gm xp + Gp xm - Contract[GAD[α] xv[α]],
      (*γγ5FDt[i_] xt[i_] → γγ5FDp xm - DiracSlash[FDx, Dimension → D].GA[5], *)
      γγ5FDp → DiracSlash[FDx, Dimension → D].GA[5] / xm,
      γkγt[i_] xt[i_] → -Pair[Momentum[x, D], Momentum[γkγ, D]] + γkγp * xm,
      γkγt[i_] at[i_] → -Pair[Momentum[a, D], Momentum[γkγ, D]],
      γaγt[i_] xt[i_] → -Pair[Momentum[x, D], Momentum[γαγ, D]] + γaγp * xm +
      γaγm * xp, γaγt[i_] at[i_] → -Pair[Momentum[a, D], Momentum[γαγ, D]]} /.
      {Gm → DiracGamma[Momentum[k, D], D] / m} /. {γkγm → 0} /.
      {γαγm → DiracGamma[Momentum[a, D], D].DiracGamma[Momentum[k, D], D] / m}
    /. {xm → kx / m}
  ],
  {v1, p2, Gm, Gp, Gt, γγ5FDm, γγ5FDp, γγ5FDt, γkγm, γkγp, γkγt, γaγm, γaγp, γaγt}
]

```

Coeff7 = Coeff6

Phase7 = Phase6

$$\begin{aligned}
& -\frac{i \text{af} s \zeta \sigma F a^2 e^3}{2 m \xi^2} + \frac{s v 2 \zeta \sigma F a^2 e^3}{m \xi^3 (k \cdot x)} - \frac{i \text{af} s \gamma \cdot k a^2 e^2}{\xi} - \frac{i s s f \zeta \gamma \cdot k a^2 e^2}{\xi} - \frac{\text{af} \zeta a^2 e^2}{m \xi^2} + \frac{2 s v 2 \gamma \cdot k a^2 e^2}{\xi^2 (k \cdot x)} + \\
& \frac{4 s t f \zeta \gamma \cdot k a^2 e^2}{\xi^2 (k \cdot x)} + \frac{1}{2} i m s s f \sigma F e - \frac{2 m s t f \sigma F e}{\xi (k \cdot x)} + \frac{\text{af} (\gamma \cdot \text{FDx}).\bar{\gamma}^5 e}{\xi (k \cdot x)} + \frac{s f \zeta (\gamma \cdot \text{FDx}).\bar{\gamma}^5 e}{\xi (k \cdot x)} + m s f + \\
& v1 \left(\frac{s \zeta \sigma F a^2 (k \cdot X)^2 e^3}{2 m \xi (k \cdot x)} + \frac{i s \zeta (\gamma \cdot a).(\gamma \cdot k) a^2 (k \cdot X)^2 e^3}{m \xi (k \cdot x)} - \frac{s \zeta \sigma F a^2 (k \cdot x) e^3}{8 m \xi} - \frac{1}{4} s \gamma \cdot k a^2 (k \cdot x) e^2 + \right. \\
& \frac{i \zeta a^2 (k \cdot x) e^2}{2 m \xi} - \frac{i \zeta a^2 (k \cdot X) e^2}{m \xi} + \frac{i \zeta (a \cdot \gamma a \gamma) (k \cdot X) e^2}{m \xi} - \frac{i \zeta (\gamma \cdot a).(\gamma \cdot k) (a \cdot x) (k \cdot X) e^2}{m \xi (k \cdot x)} - \\
& \frac{i \zeta (a \cdot x) (a \cdot \gamma k \gamma) (k \cdot X) e^2}{m \xi (k \cdot x)} - \frac{1}{2} i (\gamma \cdot \text{FDx}).\bar{\gamma}^5 e - \frac{s \zeta \sigma F p^2 e}{m \xi (k \cdot x)} - \frac{i s \zeta (\gamma \cdot a).(\gamma \cdot k) p^2 e}{m \xi (k \cdot x)} + \\
& \frac{i \zeta (\gamma \cdot a).(\gamma \cdot k) x^2 e}{4 m s \xi (k \cdot x)} - \frac{i \zeta (x \cdot \gamma a \gamma) e}{2 m s \xi} + \frac{i \zeta (a \cdot x) (x \cdot \gamma k \gamma) e}{2 m s \xi (k \cdot x)} - \frac{D \zeta (\gamma \cdot a).(\gamma \cdot k) e}{2 m \xi (k \cdot x)} + \\
& \left. \frac{\zeta (\gamma \cdot a).(\gamma \cdot k) e}{m \xi (k \cdot x)} - \frac{\zeta (a \cdot \gamma k \gamma) e}{m \xi (k \cdot x)} - \frac{\gamma \cdot x}{2 s} - \frac{s \gamma \cdot k p^2}{k \cdot x} + \frac{\gamma \cdot k x^2}{4 s (k \cdot x)} + \frac{i D \gamma \cdot k}{2 (k \cdot x)} - \frac{i \gamma \cdot k}{k \cdot x} \right) \\
& \frac{i 2^{-D-2} e^{-\frac{1}{2} i \pi \left(\frac{D-1}{2} \right)} \pi^{-\frac{D}{2}-1} \Lambda^{4-D} s^{-D/2}}{\text{DdInv}(\zeta)} \\
& \frac{1}{12} a^2 e^2 s (k \cdot x)^2 + e (a \cdot x) (k \cdot X) - p^2 s - \frac{x^2}{4 s}
\end{aligned}$$

```

Matrix8 = Collect[
  Expand[
    DiracSimplify[
      Matrix7 /.
        {Pair[Momentum[x, D], Momentum[γkγ, D]] →
          DiracGamma[Momentum[k, D], D].DiracGamma[Momentum[x, D], D],
        Pair[Momentum[a, D], Momentum[γkγ, D]] →
          DiracGamma[Momentum[k, D], D].DiracGamma[Momentum[a, D], D],
        Pair[Momentum[x, D], Momentum[γαγ, D]] →
          DiracGamma[Momentum[a, D], D].DiracGamma[Momentum[x, D], D],
        Pair[Momentum[a, D], Momentum[γαγ, D]] →
          DiracGamma[Momentum[a, D], D].DiracGamma[Momentum[a, D], D]}
    ] /. FieldSubstitutions /. akToF /. {av2 → -ξ^2 m^2 / e^2}
  ],
  {sf, v1, v2, tf, af, ξ, σF}]
Coeff8 = Coeff7
Phase8 = Phase7 /. {av2 → -ξ^2 m^2 / e^2}
v1 ⎛ - $\frac{1}{2}$  i e(γ.FDx).γ̄5 + ζ ⎛  $\frac{i e(a \cdot x)(\gamma \cdot k)(\gamma \cdot x)}{2 m \xi s(k \cdot x)}$  -  $\frac{i e(\gamma \cdot a)(\gamma \cdot x)}{2 m \xi s}$  + σF
      ⎛ - $\frac{i D e}{4 m \xi(k \cdot x)}$  -  $\frac{e p^2 s}{2 m \xi(k \cdot x)}$  +  $\frac{1}{8} e m \xi s(k \cdot x)$  -  $\frac{e x^2}{8 m \xi s(k \cdot x)}$  +  $\frac{i e}{m \xi(k \cdot x)}$  ⎛ - $\frac{1}{2} i m \xi(k \cdot x)$  ⎛ +
       $\frac{i D \gamma \cdot k}{2(k \cdot x)}$  +  $\frac{1}{4} m^2 \xi^2 s \gamma \cdot k(k \cdot x)$  -  $\frac{p^2 s \gamma \cdot k}{k \cdot x}$  +  $\frac{x^2 \gamma \cdot k}{4 s(k \cdot x)}$  -  $\frac{i \gamma \cdot k}{k \cdot x}$  -  $\frac{\gamma \cdot x}{2 s}$  ⎛ +
      af ⎛  $\frac{e(\gamma \cdot \text{FDx}).\bar{\gamma}^5}{\xi(k \cdot x)}$  + ζ ⎛  $m + \frac{1}{2} i e m s \sigma F$  ⎛ + i m^2 ξ s γ · k ⎛ +
      sf ⎛ ζ ⎛  $\frac{e(\gamma \cdot \text{FDx}).\bar{\gamma}^5}{\xi(k \cdot x)}$  + i m^2 ξ s γ · k ⎛ +  $\frac{1}{2} i e m s \sigma F + m$  ⎛ +
      tf ⎛ - $\frac{2 e m s \sigma F}{\xi(k \cdot x)}$  -  $\frac{4 \zeta m^2 s \gamma \cdot k}{k \cdot x}$  ⎛ +
      v2 ⎛ - $\frac{e \zeta m s \sigma F}{\xi(k \cdot x)}$  -  $\frac{2 m^2 s \gamma \cdot k}{k \cdot x}$  ⎛
       $i 2^{-D-2} e^{-\frac{1}{2} i \pi \left(\frac{D-1}{2}\right)} \pi^{-\frac{D}{2}-1} \Lambda^{4-D} s^{-D/2}$ 
      DdInv(ξ)
      - $\frac{1}{12} m^2 \xi^2 s(k \cdot x)^2 + e(a \cdot x)(k \cdot X) - p^2 s - \frac{x^2}{4 s}$ 
      (* small test*)Expand[⎛ ⎛ Matrix8 /. {sf → 1, v1 → -1, v2 → 0, tf → 0, af → 0, ξ → 1} ⎛ +
      ⎛ Matrix8 /. {sf → 1, v1 → -1, v2 → 0, tf → 0, af → 0, ξ → -1} ⎛ ⎛ / 2 ⎛
       $\frac{1}{2} i e(\gamma \cdot \text{FDx}).\bar{\gamma}^5 - \frac{i D \gamma \cdot k}{2(k \cdot x)} + \frac{1}{2} i e m s \sigma F - \frac{1}{4} m^2 \xi^2 s \gamma \cdot k(k \cdot x) + \frac{p^2 s \gamma \cdot k}{k \cdot x} - \frac{x^2 \gamma \cdot k}{4 s(k \cdot x)} + \frac{i \gamma \cdot k}{k \cdot x} + m + \frac{\gamma \cdot x}{2 s}$ 

```

```

DiracSimplify[
  DiracGamma[Momentum[Fx, D], D].DiracGamma[Momentum[x, D], D] /. FxToak]
DiracSimplify[
  I DiracGamma[Momentum[FDx, D], D].DiracGamma[5].DiracGamma[Momentum[x, D], D] /.
    FxToEps /. FxToak /. Gamma5toTrippleGammax]
(a · x) (γ · k).(γ · x) - (k · x) (γ · a).(γ · x)
(a · x) (γ · k).(γ · x) - (k · x) (γ · a).(γ · x) + x2 (γ · a).(γ · k)

Matrix9 = Collect[
  Expand[
    Matrix8 /. {ax DiracGamma[Momentum[k, D], D].DiracGamma[Momentum[x, D], D] →
      I DiracGamma[Momentum[FDx, D], D].DiracGamma[5].DiracGamma[Momentum[x, D],
        D] + kx DiracGamma[Momentum[a, D], D].DiracGamma[Momentum[x, D], D] -
      ( xv2 DiracGamma[Momentum[a, D], D].DiracGamma[Momentum[k, D], D] /. akToF),
      DiracGamma[Momentum[k, D], D] → - DiracGamma[Momentum[FFx, D], D] / av2 / kx} /.
      {av2 → -ξ2 m2 / e2}
  ],
  {sf, v1, v2, tf, af, ξ, e σF / m / ξ / kx, σF,
    e2 DiracGamma[Momentum[FFx, D], D] / m2 / ξ2 / kx2}]
Coeff9 = Coeff8
Phase9 = Phase8
v1 ⎛ ζ ⎛ -  $\frac{e(\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5 \cdot (\gamma \cdot x)}{2 m \xi s (k \cdot x)} + \frac{e \sigma F \left( -\frac{i D}{4} - \frac{p^2 s}{2} + \frac{x^2}{8 s} + i \right)}{m \xi (k \cdot x)} + \frac{1}{8} e m \xi s \sigma F (k \cdot x) - \frac{1}{2} i m \xi (k \cdot x) \right) -$ 
 $\frac{1}{2} i e (\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5 + \frac{e^2 \gamma \cdot \text{FFx} \left( \frac{i D}{2} - p^2 s + \frac{x^2}{4 s} - i \right)}{m^2 \xi^2 (k \cdot x)^2} + \frac{1}{4} e^2 s \gamma \cdot \text{FFx} - \frac{\gamma \cdot x}{2 s} \right) +$ 
 $\text{af} \left( \frac{e(\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5}{\xi (k \cdot x)} + \frac{i e^2 s \gamma \cdot \text{FFx}}{\xi (k \cdot x)} + \zeta \left( m + \frac{1}{2} i e m s \sigma F \right) \right) +$ 
 $\text{sf} \left( \zeta \left( \frac{e(\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5}{\xi (k \cdot x)} + \frac{i e^2 s \gamma \cdot \text{FFx}}{\xi (k \cdot x)} \right) + \frac{1}{2} i e m s \sigma F + m \right) +$ 
 $\text{tf} \left( -\frac{4 e^2 \zeta s \gamma \cdot \text{FFx}}{\xi^2 (k \cdot x)^2} - \frac{2 e m s \sigma F}{\xi (k \cdot x)} \right) + v2 \left( -\frac{2 e^2 s \gamma \cdot \text{FFx}}{\xi^2 (k \cdot x)^2} - \frac{e \zeta m s \sigma F}{\xi (k \cdot x)} \right)$ 
 $\frac{i 2^{-D-2} e^{-\frac{1}{2} i \pi \left( \frac{D}{2} - 1 \right)} \pi^{-\frac{D}{2} - 1} \Lambda^{4-D} s^{-D/2}}{\text{DdInv}(\zeta)}$ 
 $-\frac{1}{12} m^2 \xi^2 s (k \cdot x)^2 + e(a \cdot x) (k \cdot X) - p^2 s - \frac{x^2}{4 s}$ 

```

```
Collect[Expand[Simplify[D[Coeff8 s f[χ[s]] Exp[I Phase8], s] / Exp[I Phase8] / Coeff8]],
  f[χ[s]]]
```

```
pv2Sol = pv2 /. Solve[% == dfds, pv2][[1]]
```

$$s \chi'(s) f'(\chi(s)) + f(\chi(s)) \left(-\frac{1}{12} i m^2 \xi^2 s (k \cdot x)^2 - \frac{D}{2} - i p^2 s + \frac{i x^2}{4 s} + 1 \right)$$

$$\frac{1}{12 s^2 f(\chi(s))} \left(-m^2 \xi^2 s^2 f(\chi(s)) (k \cdot x)^2 + 6 i (D s f(\chi(s)) + 2 dfds s - 2 s^2 \chi'(s) f'(\chi(s)) - 2 s f(\chi(s))) + 3 x^2 f(\chi(s)) \right)$$

Let

$$f(\xi, p^2, \chi(s)) = v_1(p^2, \chi(s)) D_\xi^{-1}(p^2, \chi(s))$$

Matrix9pv2 =

Coefficient[Matrix9, pv2] /. {v1 → f[ξ, pv2, x[s]]} /. {x[s] → ξ kx / 2 / m^2 / s}

CoeffNoD = Coeff9 * DdInv[ξ]

PhaseNopv2 = Phase9 - Coefficient[Phase9, pv2] pv2

Collect[

Expand[

Simplify[

- I D[Matrix9pv2 * CoeffNoD * Exp[I PhaseNopv2], s] / (CoeffNoD * Exp[I PhaseNopv2])

]

],

{f[ξ, pv2, ξ kx / 2 / m^2 / s]}]

Matrix91 = Coefficient[%, f[ξ, pv2, ξ kx / 2 / m^2 / s]] v1

Matrix92 = %% - Matrix91 / v1 * f[ξ, pv2, ξ kx / 2 / m^2 / s]

$$-\frac{e^2 s \gamma \cdot \text{FFx} f(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s})}{m^2 \xi^2 (k \cdot x)^2} - \frac{e \zeta s \sigma F f(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s})}{2 m \xi (k \cdot x)}$$

$$i 2^{-D-2} e^{-\frac{1}{2} i \pi (\frac{D}{2}-1)} \pi^{-\frac{D}{2}-1} \Lambda^{4-D} s^{-D/2}$$

$$-\frac{1}{12} m^2 \xi^2 s (k \cdot x)^2 + e(a \cdot x) (k \cdot X) - \frac{x^2}{4 s}$$

$$f\left(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s}\right) \left(-\frac{i D e^2 \gamma \cdot \text{FFx}}{2 m^2 \xi^2 (k \cdot x)^2} - \frac{e^2 x^2 \gamma \cdot \text{FFx}}{4 m^2 \xi^2 s (k \cdot x)^2} + \frac{i e^2 \gamma \cdot \text{FFx}}{m^2 \xi^2 (k \cdot x)^2} - \right. \\ \left. \frac{i D e \zeta \sigma F}{4 m \xi (k \cdot x)} + \frac{1}{12} e^2 s \gamma \cdot \text{FFx} + \frac{1}{24} e \zeta m \xi s \sigma F (k \cdot x) - \frac{e \zeta \sigma F x^2}{8 m \xi s (k \cdot x)} + \frac{i e \zeta \sigma F}{2 m \xi (k \cdot x)} \right) - \\ \frac{i e^2 \gamma \cdot \text{FFx} f^{(0,0,1)}(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s})}{2 m^4 \xi s (k \cdot x)} - \frac{i e \zeta \sigma F f^{(0,0,1)}(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s})}{4 m^3 s}$$

$$v1 \left(-\frac{i D e^2 \gamma \cdot \text{FFx}}{2 m^2 \xi^2 (k \cdot x)^2} - \frac{e^2 x^2 \gamma \cdot \text{FFx}}{4 m^2 \xi^2 s (k \cdot x)^2} + \frac{i e^2 \gamma \cdot \text{FFx}}{m^2 \xi^2 (k \cdot x)^2} - \right. \\ \left. \frac{i D e \zeta \sigma F}{4 m \xi (k \cdot x)} + \frac{1}{12} e^2 s \gamma \cdot \text{FFx} + \frac{1}{24} e \zeta m \xi s \sigma F (k \cdot x) - \frac{e \zeta \sigma F x^2}{8 m \xi s (k \cdot x)} + \frac{i e \zeta \sigma F}{2 m \xi (k \cdot x)} \right) \\ - \frac{i e^2 \gamma \cdot \text{FFx} f^{(0,0,1)}(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s})}{2 m^4 \xi s (k \cdot x)} - \frac{i e \zeta \sigma F f^{(0,0,1)}(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s})}{4 m^3 s}$$

```

Matrix101 = Collect[
  (Expand[Matrix9] - Coefficient[Matrix9, pv2] pv2 + Matrix91) / m,
  {sf, v1, v2, tf, af, g, sF, DiracGamma[Momentum[FFx, D], D],
   DiracGamma[Momentum[x, D], D], DiracGamma[Momentum[FDx, D], D]}, Simplify]
Matrix102 = Expand[Matrix92 / m]
(*Matrix101=Collect[Matrix9/m,{sf,v1,v2,tf,af,g,sF, DiracGamma[Momentum[FFx,D],D],
  DiracGamma[Momentum[x,D],D],DiracGamma[Momentum[FDx,D],D]},Simplify]
  Matrix102=0*)
Coeff10 = Coeff9 * m
Phase10 = Phase9
af  $\left( \frac{e(\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5}{m \xi(k \cdot x)} + \frac{i e^2 s \gamma \cdot \text{FFx}}{m \xi(k \cdot x)} + \zeta \left( 1 + \frac{1}{2} i e s \sigma F \right) \right) +$ 
v1  $\left( \zeta \left( -\frac{e(\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5 \cdot (\gamma \cdot x)}{2 m^2 \xi s(k \cdot x)} + \sigma F \left( \frac{1}{6} e \xi s(k \cdot x) - \frac{i(D-3)e}{2 m^2 \xi(k \cdot x)} \right) - \frac{1}{2} i \xi(k \cdot x) \right) - \right.$ 
 $\left. \frac{i e(\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5}{2 m} + \frac{e^2 s \gamma \cdot \text{FFx}}{3 m} - \frac{\gamma \cdot x}{2 m s} \right) + \text{sf} \left( \zeta \left( \frac{e(\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5}{m \xi(k \cdot x)} + \frac{i e^2 s \gamma \cdot \text{FFx}}{m \xi(k \cdot x)} \right) + \frac{1}{2} i e s \sigma F + 1 \right) +$ 
tf  $\left( -\frac{4 e^2 \zeta s \gamma \cdot \text{FFx}}{m \xi^2(k \cdot x)^2} - \frac{2 e s \sigma F}{\xi(k \cdot x)} \right) + \text{v2} \left( -\frac{2 e^2 s \gamma \cdot \text{FFx}}{m \xi^2(k \cdot x)^2} - \frac{e \zeta s \sigma F}{\xi(k \cdot x)} \right)$ 
 $-\frac{i e^2 \gamma \cdot \text{FFx} f^{(0,0,1)}(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s})}{2 m^5 \xi s(k \cdot x)} - \frac{i e \zeta \sigma F f^{(0,0,1)}(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s})}{4 m^4 s}$ 
 $\frac{i 2^{-D-2} e^{-\frac{1}{2} i \pi (\frac{D}{2}-1)} \pi^{-\frac{D}{2}-1} m \Lambda^{4-D} s^{-D/2}}{\text{DdInv}(\zeta)}$ 
 $-\frac{1}{12} m^2 \xi^2 s(k \cdot x)^2 + e(a \cdot x)(k \cdot X) - p^2 s - \frac{x^2}{4 s}$ 
Matrix101 // StandardForm;

```

Matrix10 =

```
Collect[Expand[Matrix101 + Matrix102], {e^2 DiracGamma[Momentum[FFx, D], D],
  e DiracGamma[Momentum[FDx, D], D].DiracGamma[5],
  e DiracGamma[Momentum[FDx, D], D].DiracGamma[5].DiracGamma[Momentum[x, D], D],
  σF, DiracGamma[Momentum[x, D], D], sf, v1, v2, tf, af, ξ}]
```

$$\begin{aligned}
 & e(\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5 \left(\frac{\text{af}}{m \xi(k \cdot x)} + \frac{\zeta \text{sf}}{m \xi(k \cdot x)} - \frac{i v1}{2 m} \right) - \frac{e \zeta v1 (\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5 \cdot (\gamma \cdot x)}{2 m^2 \xi s(k \cdot x)} + \\
 & e^2 \gamma \cdot \text{FFx} \left(-\frac{i f^{(0,0,1)}(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s})}{2 m^5 \xi s(k \cdot x)} - \frac{4 \zeta s \text{tf}}{m \xi^2(k \cdot x)^2} - \frac{2 s v2}{m \xi^2(k \cdot x)^2} + \frac{i \text{af} s}{m \xi(k \cdot x)} + \frac{i \zeta s \text{sf}}{m \xi(k \cdot x)} + \frac{s v1}{3 m} \right) + \\
 & \sigma F \left(-\frac{i e \zeta f^{(0,0,1)}(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s})}{4 m^4 s} + \frac{1}{2} i \text{af} e \zeta s + \zeta v1 \left(-\frac{i D e}{2 m^2 \xi(k \cdot x)} + \frac{3 i e}{2 m^2 \xi(k \cdot x)} + \frac{1}{6} e \xi s(k \cdot x) \right) - \right. \\
 & \left. \frac{2 e s \text{tf}}{\xi(k \cdot x)} - \frac{e \zeta s v2}{\xi(k \cdot x)} + \frac{1}{2} i e s \text{sf} \right) + \text{af} \zeta - \frac{1}{2} i \zeta \xi v1(k \cdot x) - \frac{v1 \gamma \cdot x}{2 m s} + \text{sf}
 \end{aligned}$$

Matrix10ReorderPart1 =

```
Collect[Coefficient[Matrix10, e^2 DiracGamma[Momentum[FFx, D], D]] *
  m^1 ξ^2 (kx)^2, {sf, v1, v2, tf, af, ξ}, Simplify]
  e^2 DiracGamma[Momentum[FFx, D], D] / (m^1 ξ^2 (kx)^2) +
Collect[Coefficient[Matrix10, e DiracGamma[Momentum[FDx, D], D].DiracGamma[5]] *
  m ξ (kx), {sf, v1, v2, tf, af, ξ}, Simplify] e
  DiracGamma[Momentum[FDx, D], D].DiracGamma[5] / (m ξ (kx)) +
Collect[Coefficient[Matrix10, e σF] * 2 m^0 ξ (kx) / ξ,
  {sf, v1, v2, tf, af, ξ}, Simplify] e σF / (2 m^0 ξ (kx) / ξ) +
Coefficient[Matrix10, e DiracGamma[Momentum[FDx, D], D].
  DiracGamma[5].DiracGamma[Momentum[x, D], D]] e
  DiracGamma[Momentum[FDx, D], D].DiracGamma[5].DiracGamma[Momentum[x, D], D] +
Coefficient[Matrix10, DiracGamma[Momentum[x, D], D]]
  DiracGamma[Momentum[x, D], D];
```

Matrix10Reorder = Matrix10ReorderPart1 + Expand[Matrix10 - Matrix10ReorderPart1] /. {1 / ξ → ξ}

Coeff10

Phase10

$$\begin{aligned}
& \frac{e(\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5 \left(\text{af} - \frac{1}{2} i \xi v1 (k \cdot x) + \zeta \text{sf} \right)}{m \xi (k \cdot x)} - \frac{e \zeta v1 (\gamma \cdot \text{FDx}) \cdot \bar{\gamma}^5 (\gamma \cdot x)}{2 m^2 \xi s (k \cdot x)} + \\
& \frac{1}{2 \xi (k \cdot x)} e \zeta \sigma F \left(- \frac{i \xi (k \cdot x) f^{(0,0,1)}(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s})}{2 m^4 s} + \frac{v1 (m^2 \xi^2 s (k \cdot x)^2 - 3 i D + 9 i)}{3 m^2} + \right. \\
& \quad \left. i \text{af} \xi s (k \cdot x) + i \zeta \xi s \text{sf} (k \cdot x) - 4 \zeta s \text{tf} - 2 s v2 \right) + \frac{1}{m \xi^2 (k \cdot x)^2} e^2 \gamma \cdot \text{FFx} \\
& \left(- \frac{i \xi (k \cdot x) f^{(0,0,1)}(\zeta, p^2, \frac{\xi(k \cdot x)}{2 m^2 s})}{2 m^4 s} + \frac{1}{3} \xi^2 s v1 (k \cdot x)^2 + i \text{af} \xi s (k \cdot x) + i \zeta \xi s \text{sf} (k \cdot x) - 4 \zeta s \text{tf} - 2 s v2 \right) + \\
& \text{af} \zeta - \frac{1}{2} i \zeta \xi v1 (k \cdot x) - \frac{v1 \gamma \cdot x}{2 m s} + \text{sf} \\
& \frac{i 2^{-D-2} e^{-\frac{1}{2} i \pi \left(\frac{D-1}{2} \right)} \pi^{-\frac{D}{2}-1} m \Lambda^{4-D} s^{-D/2}}{\text{DdInv}(\zeta)} \\
& - \frac{1}{12} m^2 \xi^2 s (k \cdot x)^2 + e(a \cdot x) (k \cdot X) - p^2 s - \frac{x^2}{4 s}
\end{aligned}$$

Matrix10 without removing p^2 with integration by parts

$$\begin{aligned}
& sf + af \xi - \frac{v1 \text{DiracGamma}[\text{Momentum}[x, D], D]}{2 m s} - (e v1 \xi \\
& \quad \text{DiracGamma}[\text{Momentum}[FDx, D], D] \cdot \text{DiracGamma}[5] \cdot \text{DiracGamma}[\text{Momentum}[x, D], D]) / \\
& (2 m^2 s \xi \text{Pair}[\text{Momentum}[k, D], \text{Momentum}[x, D]]) - \frac{1}{2} i v1 \xi \xi \\
& \quad \text{Pair}[\text{Momentum}[k, D], \text{Momentum}[x, D]] + \\
& (e \text{DiracGamma}[\text{Momentum}[FDx, D], D] \cdot \text{DiracGamma}[5] \\
& \quad (af + sf \xi - \frac{1}{2} i v1 \xi \text{Pair}[\text{Momentum}[k, D], \text{Momentum}[x, D]])) / \\
& (m \xi \text{Pair}[\text{Momentum}[k, D], \text{Momentum}[x, D]]) + \\
& (e \xi \sigma F (-2 s v2 - 4 s t f \xi + i a f s \xi \text{Pair}[\text{Momentum}[k, D], \text{Momentum}[x, D]] + \\
& \quad i s s f \xi \xi \text{Pair}[\text{Momentum}[k, D], \text{Momentum}[x, D]] + \frac{1}{4 m^2 s} v1 (8 i s - 2 i D s + \\
& \quad m^2 s^2 \xi^2 \text{Pair}[\text{Momentum}[k, D], \text{Momentum}[x, D]]^2 - 4 s^2 \text{Pair}[\text{Momentum}[p, D], \\
& \quad \text{Momentum}[p, D]] + \text{Pair}[\text{Momentum}[x, D], \text{Momentum}[x, D]])) / \\
& (2 \xi \text{Pair}[\text{Momentum}[k, D], \text{Momentum}[x, D]]) + (e^2 \text{DiracGamma}[\text{Momentum}[FFx, D], D] \\
& (-2 s v2 - 4 s t f \xi + i a f s \xi \text{Pair}[\text{Momentum}[k, D], \text{Momentum}[x, D]] + \\
& \quad i s s f \xi \xi \text{Pair}[\text{Momentum}[k, D], \text{Momentum}[x, D]] + \\
& \quad \frac{1}{4 m^2 s} v1 (-4 i s + 2 i D s + m^2 s^2 \xi^2 \text{Pair}[\text{Momentum}[k, D], \text{Momentum}[x, D]]^2 - \\
& \quad 4 s^2 \text{Pair}[\text{Momentum}[p, D], \text{Momentum}[p, D]] + \text{Pair}[\text{Momentum}[x, D], \\
& \quad \text{Momentum}[x, D]])) / (m \xi^2 \text{Pair}[\text{Momentum}[k, D], \text{Momentum}[x, D]]^2) \\
& \frac{e(\gamma \cdot FDx) \cdot \bar{\gamma}^5 (af - \frac{1}{2} i \xi v1 (k \cdot x) + \zeta sf)}{m \xi (k \cdot x)} - \frac{e \zeta v1 (\gamma \cdot FDx) \cdot \bar{\gamma}^5 (\gamma \cdot x)}{2 m^2 \xi s (k \cdot x)} + \\
& \frac{1}{m \xi^2 (k \cdot x)^2} e^2 \gamma \cdot FFx \left(\frac{v1 (m^2 \xi^2 s^2 (k \cdot x)^2 + 2 i D s - 4 p^2 s^2 - 4 i s + x^2)}{4 m^2 s} + \right. \\
& \quad \left. i a f \xi s (k \cdot x) + i \zeta \xi s sf (k \cdot x) - 4 \zeta s t f - 2 s v2 \right) + \frac{1}{2 \xi (k \cdot x)} \\
& e \zeta \sigma F \left(\frac{v1 (m^2 \xi^2 s^2 (k \cdot x)^2 - 2 i D s - 4 p^2 s^2 + 8 i s + x^2)}{4 m^2 s} + i a f \xi s (k \cdot x) + i \zeta \xi s sf (k \cdot x) - 4 \zeta s t f - 2 s v2 \right) + \\
& af \zeta - \frac{1}{2} i \zeta \xi v1 (k \cdot x) - \frac{v1 \gamma \cdot x}{2 m s} + sf
\end{aligned}$$

(*a small test*)

$$\begin{aligned}
& \text{Expand}[(\text{Matrix101} /. \{sf \rightarrow 1, v1 \rightarrow -1, v2 \rightarrow 0, t f \rightarrow 0, af \rightarrow 0, \xi \rightarrow 1\}) + \\
& \quad (\text{Matrix101} /. \{sf \rightarrow 1, v1 \rightarrow -1, v2 \rightarrow 0, t f \rightarrow 0, af \rightarrow 0, \xi \rightarrow -1\})] / 2] \\
& \frac{i e(\gamma \cdot FDx) \cdot \bar{\gamma}^5}{2 m} - \frac{e^2 s \gamma \cdot FFx}{3 m} + \frac{1}{2} i e s \sigma F + \frac{\gamma \cdot x}{2 m s} + 1
\end{aligned}$$

```

Matrix111 = Collect[
  DiracOrder[DiracSimplify[Expand[Matrix101] /. FxToEps /. FxToak /.
    Gamma5toTrippleGammax /. {av2 → -ξ^2 m^2 / e^2}]],
  {sf, v1, v2, tf, af, ξ}, Simplify]
Matrix112 = Expand[Matrix102 /. FxToEps /. FxToak /. Gamma5toTrippleGammax]
Coeff10
Phase10
v1 (1/(6 m s) (s (γ · k (2 m^2 ξ^2 s (k · x) - 3 e (a · x)) + 3 e γ · a (k · x) - 3 e (γ · a) · (γ · k) · (γ · x)) - 3 γ · x) -
  (i ξ (e (γ · a) · (γ · k) (2 m^2 ξ^2 s^2 (k · x)^2 - 6 i (D - 3) s - 3 x^2) +
    3 (k · x) (e (γ · a) · (γ · x) + m^2 ξ^2 s (k · x)) - 3 e (a · x) (γ · k) · (γ · x))) / (6 m^2 ξ s (k · x))) +
  af (ξ (e s (γ · a) · (γ · k) + 1) + 1/(m ξ (k · x)) i (γ · k (m^2 ξ^2 s (k · x) - e (a · x)) + e γ · a (k · x) - e (γ · a) · (γ · k) · (γ · x))) +
  sf (1/(m ξ (k · x)) i ξ (γ · k (m^2 ξ^2 s (k · x) - e (a · x)) + e γ · a (k · x) - e (γ · a) · (γ · k) · (γ · x)) + e s (γ · a) · (γ · k) + 1) +
  tf (-4 ξ m s γ · k / (k · x) + 4 i e s (γ · a) · (γ · k) / ξ (k · x)) +
  v2 (-2 m s γ · k / (k · x) + 2 i e ξ s (γ · a) · (γ · k) / ξ (k · x))
  - (e ξ (γ · a) · (γ · k) f^(0,0,1)(ξ, p^2, ξ(k · x)/(2 m^2 s)) / (2 m^4 s) + i a^2 e^2 γ · k f^(0,0,1)(ξ, p^2, ξ(k · x)/(2 m^2 s)) / (2 m^5 ξ s)
  + (i 2^{-D-2} e^{-1/2 i π (D/2 - 1)} π^{-D/2 - 1} m Λ^{4-D} s^{-D/2}) / DdInv(ξ)
  - 1/12 m^2 ξ^2 s (k · x)^2 + e (a · x) (k · X) - p^2 s - x^2 / (4 s)

```

Collect[Expand[Matrix111],

```
{
  DiracGamma[Momentum[x, D], D],
  DiracGamma[Momentum[k, D], D],
  DiracGamma[Momentum[a, D], D],
  DiracGamma[Momentum[a, D], D].DiracGamma[Momentum[k, D], D],
  DiracGamma[Momentum[a, D], D].DiracGamma[Momentum[x, D], D],
  DiracGamma[Momentum[k, D], D].DiracGamma[Momentum[x, D], D],
  DiracGamma[Momentum[a, D], D].
  DiracGamma[Momentum[k, D], D].DiracGamma[Momentum[x, D], D],
  sf, v1, v2, tf, af, ξ]}]
```

$(\gamma \cdot a)(\gamma \cdot k)$

$$\begin{aligned} & \left(af e \xi s + \zeta v1 \left(-\frac{D e}{m^2 \xi (k \cdot x)} + \frac{i e x^2}{2 m^2 \xi s (k \cdot x)} + \frac{3 e}{m^2 \xi (k \cdot x)} - \frac{1}{3} i e \xi s (k \cdot x) \right) + \frac{4 i e s t f}{\xi (k \cdot x)} + \frac{2 i e \zeta s v2}{\xi (k \cdot x)} + e s s f \right) + \\ & \gamma \cdot k \left(af \left(i m \xi s - \frac{i e (a \cdot x)}{m \xi (k \cdot x)} \right) + \zeta s f \left(i m \xi s - \frac{i e (a \cdot x)}{m \xi (k \cdot x)} \right) + v1 \left(\frac{1}{3} m \xi^2 s (k \cdot x) - \frac{e (a \cdot x)}{2 m} \right) - \right. \\ & \quad \left. \frac{4 \zeta m s t f}{k \cdot x} - \frac{2 m s v2}{k \cdot x} \right) + (\gamma \cdot a)(\gamma \cdot k)(\gamma \cdot x) \left(-\frac{i a f e}{m \xi (k \cdot x)} - \frac{i e \zeta s f}{m \xi (k \cdot x)} - \frac{e v1}{2 m} \right) + \\ & \gamma \cdot a \left(\frac{i a f e}{m \xi} + \frac{e v1 (k \cdot x)}{2 m} + \frac{i e \zeta s f}{m \xi} \right) + \frac{i e \zeta v1 (a \cdot x)(\gamma \cdot k)(\gamma \cdot x)}{2 m^2 \xi s (k \cdot x)} - \frac{i e \zeta v1 (\gamma \cdot a)(\gamma \cdot x)}{2 m^2 \xi s} + \\ & af \zeta - \frac{1}{2} i \zeta \xi v1 (k \cdot x) - \frac{v1 \gamma \cdot x}{2 m s} + s f \end{aligned}$$

Final result for the electron propagator in a CCF

$$\begin{aligned} S^c(x_2, x_1) &= \Lambda^{4-D} \int \frac{d^D p}{(2\pi)^D} E_p(x_2) \\ &= \sum_{\xi=\pm} \frac{i}{2} \left[m s(p^2, \chi_p) - (\gamma p) v_1(p^2, \chi_p) - \frac{e^2 (\gamma F^2 p)}{m^4} v_2(p^2, \chi_p) - \right. \\ & \quad \left. \frac{e \sigma F}{m} t(p^2, \chi_p) + \frac{e (\gamma F^* p) \gamma^5}{m^2} a_s(p^2, \chi_p) \right] \frac{1 + \xi \gamma \epsilon^{(2)} \gamma^5}{D_\xi(p^2, \chi_p)} E_p^{\text{bar}}(x_1) = \\ &= e^{-i \frac{\pi}{2} \left(\frac{D}{2} - 2 \right)} \frac{m \Lambda^{4-D}}{2^{D+2} \pi^{D/2+1}} e^{i \eta} \sum_{\xi=\pm} \int_0^\infty \frac{ds}{s^{D/2}} \left(\int_0^\infty \frac{dp^2}{D_\xi(p^2, \chi_p(s))} e^{-i s p^2 - i \frac{x^2}{4s} + i \frac{s}{12} e^2 (Fx)^2} \times \right. \\ & \quad \left. \{ s(p^2, \chi_p(s)) \left[1 + \frac{1}{2} i s e \sigma F + \xi \left(\frac{i e^2 s (\gamma F F x)}{m \xi(kx)} + \frac{e F_{\alpha\beta}^* x^\beta \gamma^\alpha \gamma^5}{m \xi(kx)} \right) \right] \right. \\ & \quad \left. + v_1(p^2, \chi_p(s)) \left[-\frac{(\gamma x)}{2 m s} + \frac{e^2 s (\gamma F F x)}{3 m} - \frac{i e F_{\alpha\beta}^* x^\beta \gamma^\alpha \gamma^5}{2 m} \right] \right\} \end{aligned}$$

$$\begin{aligned}
& + \zeta \left(-\frac{1}{2} \mathbb{i} \xi (kx) - \frac{e \sigma F}{2 m^2 \xi (kx)} \right. \\
& \quad \left. \left(\mathbb{i} (D-3) - \frac{1}{3} m^2 s \xi^2 (kx)^2 \right) - \frac{e (\gamma F^* x) \gamma^5 (\gamma x)}{2 m^2 \xi s (kx)} \right) \\
& + v_2 (p^2, \chi_p(s)) \left[-\frac{2 s e^2 (\gamma F F x)}{m \xi^2 (kx)^2} - \zeta \frac{s e \sigma F}{\xi (kx)} \right] \\
& + t (p^2, \chi_p(s)) \left[-\frac{2 s e \sigma F}{\xi (kx)} - \zeta \frac{4 s e^2 (\gamma F F x)}{m \xi^2 (kx)^2} \right] \\
& + a (p^2, \chi_p(s)) \left[\frac{e F_{\alpha\beta}^* x^\beta \gamma^\alpha \gamma^5}{m \xi (kx)} + \frac{\mathbb{i} s e^2 (\gamma F F x)}{m \xi (kx)} + \zeta \left(1 + \frac{1}{2} \mathbb{i} s e \sigma F \right) \right] \} \\
& + \int_0^\infty d p^2 \frac{\partial}{\partial \chi_p} \left(\frac{v_1 (p^2, \chi_p)}{D_\xi (p^2, \chi_p)} \right) e^{-i s p^2 - \mathbb{i} \frac{x^2}{4s} + \mathbb{i} \frac{s}{12} e^2 (F x)^2} \left[-\frac{\mathbb{i} e^2 (\gamma F F x)}{2 m^5 s \xi (kx)} - \zeta \frac{\mathbb{i} e \sigma F}{4 m^4 s} \right] \\
& = e^{-i \frac{\pi}{2} (\frac{D}{2}-2)} \frac{m \Lambda^{4-D}}{2^{D+2} \pi^{D/2+1}} e^{i \eta} \sum_{\xi=\pm} \int_0^\infty \frac{ds}{s^{D/2}} \int_0^\infty \frac{d p^2}{D_\xi (p^2, \chi_p(s))} e^{-i s p^2 - \mathbb{i} \frac{x^2}{4s} + \mathbb{i} \frac{s}{12} e^2 (F x)^2} \{ \\
& \quad s (p) + \zeta \left[a (p) - \frac{\mathbb{i}}{2} \xi (kx) v_1 (p) \right] \\
& \quad - \frac{(\gamma x)}{2 m s} v_1 (p) \\
& \quad + \frac{e^2 (\gamma F^2 x)}{m \xi^2 (kx)^2} \left[\mathbb{i} \zeta \xi s s (p) + \left(\frac{1}{4} \xi^2 s (kx)^2 - s \frac{p^2}{m^2} + \frac{x^2}{4 m^2 s} + \frac{\mathbb{i} (D-2)}{2 m^2} \right) \right. \\
& \quad \left. v_1 (p) - 2 s v_2 (p) - 4 \zeta s t (p) + \mathbb{i} \xi s (kx) a (p) \right] \\
& \quad + \zeta \frac{e \sigma F}{\xi (kx)} \left[\mathbb{i} \zeta \xi s s (p) + \left(\frac{1}{4} \xi^2 s (kx)^2 - s \frac{p^2}{m^2} + \frac{x^2}{4 m^2 s} - \frac{\mathbb{i} (D-4)}{2 m^2} \right) \right. \\
& \quad \left. v_1 (p) - 2 s v_2 (p) - 4 \zeta s t (p) + \mathbb{i} \xi s (kx) a (p) \right] \\
& \quad + \frac{e (\gamma F^* x) \gamma^5}{m \xi (kx)} \left[\zeta s (p) - \frac{\mathbb{i}}{2} \xi (kx) v_1 (p) + a (p) \right] \\
& \quad - \zeta \frac{e (\gamma F^* x) \gamma^5 (\gamma x)}{2 m^2 s \xi (kx)} v_1 (p) \} \\
& = e^{-i \frac{\pi}{2} (\frac{D}{2}-2)} \frac{m \Lambda^{4-D}}{2^{D+2} \pi^{D/2+1}} e^{i \eta} \sum_{\xi=\pm} \int_0^\infty \frac{ds}{s^{D/2}} \int_0^\infty \frac{d p^2}{D_\xi (p^2, \chi_p(s))} e^{-i s p^2 - \mathbb{i} \frac{x^2}{4s} + \mathbb{i} \frac{s}{12} e^2 (F x)^2} \{ \\
& \quad s (p) + \zeta \left[a (p) - \frac{\mathbb{i}}{2} \xi (kx) v_1 (p) \right] \\
& \quad - \frac{(\gamma x)}{2 m s} v_1 (p) \\
& \quad + \frac{e^2 (\gamma F^2 x)}{m^3 \xi^2 (kx)^2}
\end{aligned}$$

$$\begin{aligned}
& m^2 \left[i \xi \xi (kx) s s (p) + \frac{1}{3} s \xi^2 (kx)^2 v_1 (p) - 2 s v_2 (p) - 4 \xi s t (p) + \right. \\
& \quad \left. i \xi s (kx) a (p) - \frac{i \xi (kx)}{2 m^4 s} D_\xi (p^2, \chi_p (s)) \frac{\partial}{\partial \chi_p} \left(\frac{v_1 (p^2, \chi_p)}{D_\xi (p^2, \chi_p)} \right) \right] \\
& + \xi \frac{e \sigma F}{m^2 \xi (kx)} m^2 \left[i \xi \xi (kx) s s (p) + \left(\frac{1}{3} s \xi^2 (kx)^2 - \frac{i (D-3)}{m^2} \right) \right. \\
& \quad \left. v_1 (p) - 2 s v_2 (p) - 4 \xi s t (p) + i \xi s (kx) a (p) - \right. \\
& \quad \left. \frac{i \xi (kx)}{2 m^4 s} D_\xi (p^2, \chi_p (s)) \frac{\partial}{\partial \chi_p} \left(\frac{v_1 (p^2, \chi_p)}{D_\xi (p^2, \chi_p)} \right) \right] \\
& + \frac{e (\gamma F^* x) \gamma^5}{m \xi (kx)} \left[\xi s (p) - \frac{i}{2} \xi (kx) v_1 (p) + a (p) \right] \\
& - \xi \frac{e (\gamma F^* x) \gamma^5 (\gamma x)}{2 m^2 s \xi (kx)} v_1 (p) \} \\
& = e^{-i \frac{\pi}{2} (\frac{D}{2}-1)} \frac{\Lambda^{4-D}}{2^D \pi^{D/2}} m^{D-1} e^{i \eta} \\
& \quad \int_0^\infty \frac{ds}{s^{D/2}} \left[1 + \frac{m (\gamma x)}{2 s} + \frac{e (\gamma a) (kx)}{2 m} - \frac{e (\gamma k) (ax)}{2 m} + \frac{e (\gamma x) (\gamma a) (\gamma k)}{2 m} + \right. \\
& \quad \left. \frac{e s (\gamma a) (\gamma k)}{m^2} + \frac{e^2 a^2 s (\gamma k) (kx)}{3 m^3} \right] e^{-is - i \frac{m^2 x^2}{4 s} + i \frac{s}{12} \frac{e^2}{m^2} (Fx)^2} = \\
& = e^{-i \frac{\pi}{2} (\frac{D}{2}-1)} \frac{\Lambda^{4-D}}{2^{D+1} \pi^{D/2}} m^D e^{i \eta} \int_0^\infty \frac{ds}{s^{D/2+1}} \left[\frac{2 s}{m} + \gamma^\alpha \left(g_{\alpha\beta} - \frac{es}{m^2} F_{\alpha\beta} + \frac{e^2 s^2}{3 m^4} F_{\alpha\lambda} F^\lambda_{\beta} \right) x^\beta \right] \\
& \quad \left(1 + \frac{ies}{2} \sigma^{\alpha\beta} F_{\alpha\beta} \right) e^{-is - i \frac{m^2 x^2}{4 s} + i \frac{s}{12} \frac{e^2}{m^2} (Fx)^2} \\
& \eta = e (ax) (k, (x_1 + x_2) / 2), \\
& x = x_2 - x_1, \\
& e > 0, \\
& \sigma^{\alpha\beta} = \frac{i}{2} (\gamma^\alpha \gamma^\beta - \gamma^\beta \gamma^\alpha), \\
& \gamma^5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3, \\
& e^{-i \frac{\pi}{2} (\frac{D}{2}-1)} \frac{\Lambda^{4-D}}{2^D \pi^{D/2}} m^{D-1} \rightarrow \frac{(-i) m^3}{16 \pi^2}, D \rightarrow 4
\end{aligned}$$