```
Get["FeynCalc`"]
$PrePrint = TraditionalForm
```

FeynCalc is already loaded! To reload it, please restart the kernel.

$Aborted

TraditionalForm

```
kv[α_] = Pair[LorentzIndex[α, D], Momentum[k, D]];
av[α_] = Pair[LorentzIndex[α, D], Momentum[a, D]];
av2 = Contract[av[α] av[α]];
Ft[α_, β_] =
  TensorFunction[{F, "A"}, LorentzIndex[α, D], LorentzIndex[β, D], Dimension → D];
FFt[α_, β_] = TensorFunction[{FF, "S"}, LorentzIndex[α, D],
   LorentzIndex[β, D], Dimension → D];
FDt[α_, β_] = TensorFunction[{FD, "A"}, LorentzIndex[α, D],
   LorentzIndex[β, D], Dimension → D];
ScalarProduct[k, a] = 0;
ScalarProduct[k, k] = 0;
FieldSubstitutions = {Ft[α_, β_] Ft[β_, γ_] Ft[γ_, δ_] → 0,
   Ft[α_, β_] Ft[β_, ϵ_] Ft[δ_, ϵ_] → 0, Ft[α_, β_] Ft[α_, γ_] Ft[γ_, δ_] → 0,
   Ft[α_, β_] Ft[α_, ϵ_] Ft[δ_, ϵ_] → 0, Ft[α_, β_] Ft[α_, β_] → 0,
   FDt[α_, β_] FDt[β_, γ_] FDt[γ_, δ_] → 0, FDt[α_, β_] FDt[β_, ϵ_] FDt[δ_, ϵ_] → 0,
   FDt[α_, β_] FDt[α_, γ_] FDt[γ_, δ_] → 0,
   FDt[α_, β_] FDt[α_, ϵ_] FDt[δ_, ϵ_] → 0, FDt[α_, β_] FDt[α_, β_] → 0,
   FDt[α_, β_] FDt[β_, γ_] → Ft[α, β] Ft[β, γ],
   FDt[α_, β_] FDt[α_, γ_] → Ft[α, β] Ft[α, γ],
   Ft[α_, β_] Ft[β_, γ_] → FFt[α, γ],
   Ft[α_, β_] Ft[α_, γ_] → -FFt[β, γ],
   Ft[α_, γ_] Ft[β_, γ_] → -FFt[α, β],
   Ft[α_, β_] FDt[β_, γ_] → 0,
   Ft[α_, β_] FDt[α_, γ_] → 0,
   FDt[α_, β_] Ft[β_, γ_] → 0,
   FDt[α_, β_] Ft[α_, γ_] → 0,
   Contract[FFt[μ_, ν_] FFt[μ_, ν_]] → 0,
   Contract[FFt[α_, ν_] FFt[β_, ν_]] → 0,
   Contract[Ft[μ_, ν] kv[ν]] → 0,
   Contract[FDt[μ_, ν] kv[ν]] → 0,
   Contract[FFt[μ_, ν] kv[ν]] → 0,
   Contract[FFt[μ_, ν] av[ν]] → 0,
   Contract[Ft[μ_, ν] av[ν]] → av2 kv[μ],
   Contract[FFt[μ, ν] kv[μ] kv[ν]] → 0};

FTensor[α_, β_] = (kv[α] av[β] - kv[β] av[α])
FDTensor[μ_, ν_] = Contract[1/2 LC[μ, ν, α, β] FTensor[α, β] /. {D → 4}]
```

$a^\beta k^\alpha - a^\alpha k^\beta$

$-\epsilon^{\mu\nu\overline{a}\overline{k}}$

```
NewMomentum[p_] := Module[{},
  Print[Map[ToExpression,
    {p <> "v[α_]=Pair[LorentzIndex[α,D],Momentum[" <> p <> ",D]]",
     p <> "v2=Contract[" <> p <> "v[α]" <> p <> "v[α]]",
```

```
    "k" <> p <> " =Contract[kv[α]" <> p <> "v[α]]",
    "F" <> p <> "v[α_] = Pair[LorentzIndex[α, D], Momentum[F" <> p <> ", D]]",
    "FF" <> p <> "v[α_] = Pair[LorentzIndex[α, D], Momentum[FF" <> p <> ", D]]",
    "FD" <> p <> "v[α_] = Pair[LorentzIndex[α, D], Momentum[FD" <> p <> ", D]]",
    "a" <> p <> "=Contract[av[α]" <> p <> "v[α]]",
    "ScalarProduct[k, F" <> p <> "] = 0",
    "ScalarProduct[k, FF" <> p <> "] = 0",
    "ScalarProduct[a, FF" <> p <> "] = 0",
    "ScalarProduct[a, F" <> p <> "] = -av2 Contract[kv[μ] " <> p <> "v[μ]]",
    "ScalarProduct[k, FD" <> p <> "] = 0",
    "ScalarProduct[a, FD" <> p <> "] = 0",
    "ScalarProduct[F" <> p <> ", F" <> p <> "] = -χ" <> p <> "^2*m^6/e^2",
    "ScalarProduct[FD" <> p <> ", FD" <> p <> "] = -χ" <> p <> "^2*m^6/e^2",
    "ScalarProduct[" <> p <> ", FF" <> p <> "] = χ" <> p <> "^2*m^6/e^2",
    "ScalarProduct[" <> p <> ", FD" <> p <> "] = 0",
    "ScalarProduct[" <> p <> ", F" <> p <> "] = 0",
    "ScalarProduct[F" <> p <> ", FD" <> p <> "] = 0",
    "ScalarProduct[FF" <> p <> ", FF" <> p <> "] = 0",
    "ScalarProduct[F" <> p <> ", FF" <> p <> "] = 0",
    "ScalarProduct[FD" <> p <> ", FF" <> p <> "] = 0"
   }]];
 FieldSubstitutions = Join[FieldSubstitutions,
   Map[ToExpression,
    {"Contract[Ft[μ_, ν] " <> p <> "v[ν]] -> F" <> p <> "v[μ]",
     "Contract[FDt[μ_, ν] " <> p <> "v[ν]] -> FD" <> p <> "v[μ]",
     "Contract[FFt[μ_, ν] " <> p <> "v[ν]] -> FF" <> p <> "v[μ]",
     "Contract[Ft[μ_, ν] F" <> p <> "v[ν]] -> FF" <> p <> "v[μ]",
     "Contract[FDt[μ_, ν] FD" <> p <> "v[ν]] -> FF" <> p <> "v[μ]",
     "FF" <> p <> "v[μ_] FF" <> p <> "v[ν_] -> χ" <> p <> "^2*m^6/e^2 FFt[μ, ν]",
     "Contract[FFt[α, β] " <> p <> "v[α] " <> p <> "v[β]] -> χ" <> p <> "^2*m^6/e^2",
     "Contract[FFt[μ_, ν] FF" <> p <> "v[ν]] -> 0",
     "Contract[FFt[μ, ν] " <> p <> "v[μ] F" <> p <> "v[ν]] -> 0",
     "Contract[FFt[μ, ν] " <> p <> "v[μ] FF" <> p <> "v[ν]] -> 0",
     "Contract[FDt[μ_, ν] F" <> p <> "v[ν]] -> 0",
     "Contract[FDt[μ_, ν] FF" <> p <> "v[ν]] -> 0",
     "Contract[FFt[μ, ν] " <> p <> "v[μ] kv[ν]] -> 0",
     "Contract[Ft[μ_, ν] FD" <> p <> "v[ν]] -> 0",
     "Contract[FFt[μ_, ν] FD" <> p <> "v[ν]] -> 0",
     "Contract[FDt[μ_, ν] F" <> p <> "v[ν]] -> 0",
     "Contract[FDt[μ_, ν] FF" <> p <> "v[ν]] -> 0",
     "Contract[Ft[μ_, ν] FF" <> p <> "v[ν]] -> 0"}]
  ];
]
```

```
NewCoordinate[x_] := Module[{},
  Print[Map[ToExpression,
    {x <> "v[α_]=Pair[LorentzIndex[α,D],Momentum[" <> x <> ",D]]",
     x <> "v2 = Contract[" <> x <> "v[α] " <> x <> "v[α]]",
     "k" <> x <> " = Contract[kv[α] " <> x <> "v[α]]",
     "a" <> x <> " = Contract[av[α] " <> x <> "v[α]]",
     "F" <> x <> "v[α_] = Pair[LorentzIndex[α, D], Momentum[F" <> x <> ", D]]",
     "FF" <> x <> "v[α_] = Pair[LorentzIndex[α, D], Momentum[FF" <> x <> ", D]]",
     "FD" <> x <> "v[α_] = Pair[LorentzIndex[α, D], Momentum[FD" <> x <> ", D]]",
     "ScalarProduct[k, " <> x <> "] = k" <> x,
     "ScalarProduct[k, F" <> x <> "] = 0",
     "ScalarProduct[k, FF" <> x <> "] = 0",
     "ScalarProduct[a, FF" <> x <> "] = 0",
     "ScalarProduct[a, F" <>
      x <> "] = -av2 Contract[kv[μ] " <> x <> "v[μ]]",
     "ScalarProduct[k, FD" <> x <> "] = 0",
     "ScalarProduct[a, FD" <> x <> "] = 0",
     "ScalarProduct[F" <>
      x <> ", F" <> x <> "] = -ξ^2 k" <> x <> "^2*m^2/e^2",
     "ScalarProduct[FD" <> x <> ", FD" <> x <> "] = -ξ^2 k" <> x <> "^2*m^2/e^2",
     "ScalarProduct[" <> x <> ", FF" <> x <> "] = ξ^2 k" <> x <> "^2*m^2/e^2",
     "ScalarProduct[" <> x <> ", FD" <> x <> "] = 0",
     "ScalarProduct[" <> x <> ", F" <> x <> "] = 0",
     "ScalarProduct[F" <> x <> ", FD" <> x <> "] = 0",
     "ScalarProduct[FF" <> x <> ", FF" <> x <> "] = 0",
     "ScalarProduct[F" <> x <> ", FF" <> x <> "] = 0",
     "ScalarProduct[FD" <> x <> ", FF" <> x <> "] = 0"
    }]];
  FieldSubstitutions = Join[FieldSubstitutions,
    Map[ToExpression,
     {"Contract[Ft[μ_,ν] " <> x <> "v[ν]]→F" <> x <> "v[μ]",
      "Contract[FDt[μ_,ν] " <> x <> "v[ν]]→FD" <> x <> "v[μ]",
      "Contract[FFt[μ_,ν] " <> x <> "v[ν]]→FF" <> x <> "v[μ]",
      "Contract[Ft[μ_,ν] F" <> x <> "v[ν]]→FF" <> x <> "v[μ]",
      "Contract[FDt[μ_,ν] FD" <> x <> "v[ν]]→FF" <> x <> "v[μ]",
      "FF" <> x <> "v[μ_] FF" <> x <> "v[ν_]→ξ^2 k" <> x <> "^2*m^2/e^2 FFt[μ,ν]",
      "Contract[FFt[α,β] " <> x <> "v[α] " <> x <> "v[β]]→ξ^2k" <> x <> "^2*m^2/e^2",
      "Contract[FFt[μ_,ν] F" <> x <> "v[ν]]→0",
      "Contract[FFt[μ_,ν] FF" <> x <> "v[ν]]→0",
      "Contract[FFt[μ,ν] " <> x <> "v[μ] F" <> x <> "v[ν]]→0",
      "Contract[FFt[μ,ν] " <> x <> "v[μ] FF" <> x <> "v[ν]]→0",
      "Contract[FDt[μ_,ν] F" <> x <> "v[ν]]→0",
      "Contract[FDt[μ_,ν] FF" <> x <> "v[ν]]→0",
```

```
      "Contract[FFt[μ,ν] " <> x <> "v[μ] kv[ν]]→0",
      "Contract[Ft[μ_,ν] FD" <> x <> "v[ν]]→0",
      "Contract[FFt[μ_,ν] FD" <> x <> "v[ν]]→0",
      "Contract[FDt[μ_,ν] F" <> x <> "v[ν]]→0",
      "Contract[FDt[μ_,ν] FF" <> x <> "v[ν]]→0",
      "Contract[Ft[μ_,ν] FF" <> x <> "v[ν]]→0"}]];
 ]

(*Preparing symbolic substitutions for thriple γ-
 matrix combinations and recollecting F-tensors from vectors a[μ] and k[μ]*)
TripleGamma = {DiracGamma[Momentum[a_, D], D].
    DiracGamma[Momentum[b_, D], D].DiracGamma[Momentum[c_, D], D] →
   Contract[Pair[LorentzIndex[α1, D], Momentum[a, D]] Pair[LorentzIndex[α2, D],
      Momentum[b, D]] Pair[LorentzIndex[α3, D], Momentum[c, D]] (MTD[α1, α2] GAD[α3] +
       MTD[α2, α3] GAD[α1] - MTD[α1, α3] GAD[α2] - i LCD[β, α1, α2, α3] GAD[β].GA[5])]}
EpsToF = {Eps[LorentzIndex[α_, D], Momentum[a, D], Momentum[k, D], Momentum[V_, D],
    Dimension → D] → -Contract[FDt[α, β] Pair[LorentzIndex[β, D], Momentum[V, D]]],
  Eps[LorentzIndex[α_, D], Momentum[k, D], Momentum[a, D], Momentum[V_, D],
    Dimension → D] → Contract[FDt[α, β] Pair[LorentzIndex[β, D], Momentum[V, D]]],
  Eps[LorentzIndex[α_, D], Momentum[V_, D], Momentum[a, D], Momentum[k, D],
    Dimension → D] → - Contract[FDt[α, β] Pair[LorentzIndex[β, D], Momentum[V, D]]],
  Eps[LorentzIndex[α_, D], Momentum[V_, D], Momentum[k, D], Momentum[a, D],
    Dimension → D] → Contract[FDt[α, β] Pair[LorentzIndex[β, D], Momentum[V, D]]],
  Eps[LorentzIndex[α_, D], Momentum[a, D], Momentum[V_, D], Momentum[k, D],
    Dimension → D] → Contract[FDt[α, β] Pair[LorentzIndex[β, D], Momentum[V, D]]],
  Eps[LorentzIndex[α_, D], Momentum[k, D], Momentum[V_, D], Momentum[a, D],
    Dimension → D] → -Contract[FDt[α, β] Pair[LorentzIndex[β, D], Momentum[V, D]]]}
akToF = {DiracGamma[Momentum[k, D], D] Pair[Momentum[a, D], Momentum[x, D]] -
    DiracGamma[Momentum[a, D], D] Pair[Momentum[k, D], Momentum[x, D]] →
   DiracGamma[Momentum[Fx, D], D],
  -DiracGamma[Momentum[k, D], D] Pair[Momentum[a, D], Momentum[x, D]] +
    DiracGamma[Momentum[a, D], D] Pair[Momentum[k, D], Momentum[x, D]] →
   -DiracGamma[Momentum[Fx, D], D],
  DiracGamma[Momentum[a, D], D].DiracGamma[Momentum[k, D], D] → i / 2 σF,
  DiracGamma[Momentum[k, D], D].DiracGamma[Momentum[a, D], D] → -i / 2 σF}
```

$$\{(\gamma\cdot \text{a\_}).(\gamma\cdot \text{b\_}).(\gamma\cdot \text{c\_}) \to -i\,\gamma^{\beta}.\overline{\gamma}^{5}\,\epsilon^{\beta\,a\,b\,c} + (a\cdot b)\,\gamma\cdot c - (a\cdot c)\,\gamma\cdot b + \gamma\cdot a\,(b\cdot c)\}$$

$$\{\epsilon^{\alpha\_\,a\,k\,\text{V}\_} \to -\text{FD}(\alpha,\ V),\ \epsilon^{\alpha\_\,k\,a\,\text{V}\_} \to \text{FD}(\alpha,\ V),\ \epsilon^{\alpha\_\,\text{V}\_\,a\,k} \to -\text{FD}(\alpha,\ V),$$
$$\epsilon^{\alpha\_\,\text{V}\_\,k\,a} \to \text{FD}(\alpha,\ V),\ \epsilon^{\alpha\_\,a\,\text{V}\_\,k} \to \text{FD}(\alpha,\ V),\ \epsilon^{\alpha\_\,k\,\text{V}\_\,a} \to -\text{FD}(\alpha,\ V)\}$$

$$\left\{(a\cdot x)\,\gamma\cdot k - \gamma\cdot a\,(k\cdot x) \to \gamma\cdot \text{Fx},\ \gamma\cdot a\,(k\cdot x) - (a\cdot x)\,\gamma\cdot k \to -(\gamma\cdot \text{Fx}),\ (\gamma\cdot a).(\gamma\cdot k) \to \frac{i\,\sigma\text{F}}{2},\ (\gamma\cdot k).(\gamma\cdot a) \to -\frac{i\,\sigma\text{F}}{2}\right\}$$

```
antiTripleGamma =
 {DiracGamma[LorentzIndex[β_, D], D].DiracGamma[5] Eps[LorentzIndex[β_, D],
     Momentum[a_, D], Momentum[b_, D], Momentum[c_, D], Dimension → D] →
   -i Contract[Pair[LorentzIndex[α1, D], Momentum[a, D]]
      Pair[LorentzIndex[α2, D], Momentum[b, D]]
      Pair[LorentzIndex[α3, D], Momentum[c, D]] (MTD[α1, α2] GAD[α3] +
        MTD[α2, α3] GAD[α1] - MTD[α1, α3] GAD[α2] - DiracGamma[LorentzIndex[α1, D], D].
         DiracGamma[LorentzIndex[α2, D], D].DiracGamma[LorentzIndex[α3, D], D])]}
```

$$\left\{\gamma^{\beta_-}.\overline{\gamma}^5\,\epsilon^{\beta_-\,a_-\,b_-\,c_-} \to -i\left((a\cdot b)\,\gamma\cdot c-(a\cdot c)\,\gamma\cdot b+\gamma\cdot a\,(b\cdot c)-(\gamma\cdot a).(\gamma\cdot b).(\gamma\cdot c)\right)\right\}$$

```
Ep[z_, p_] := {
  1 - e / 2 / Pair[Momentum[k, D], Momentum[p, D]]
    Pair[Momentum[k, D], Momentum[z, D]] DiracSlash[k, a, Dimension → D],
  - Pair[Momentum[z, D], Momentum[p, D]] + e Pair[Momentum[a, D], Momentum[p, D]] / 2 /
    Pair[Momentum[k, D], Momentum[p, D]] (Pair[Momentum[k, D], Momentum[z, D]]) ^2 +
   e^2 av2 / 6 / Pair[Momentum[k, D], Momentum[p, D]]
    (Pair[Momentum[k, D], Momentum[z, D]]) ^3
 }
EpC[z_, p_] := {
  1 - e / 2 / Pair[Momentum[k, D], Momentum[p, D]]
    Pair[Momentum[k, D], Momentum[z, D]] DiracSlash[a, k, Dimension → D],
  - (- Pair[Momentum[z, D], Momentum[p, D]] +
      e Pair[Momentum[a, D], Momentum[p, D]] / 2 / Pair[Momentum[k, D], Momentum[p, D]]
       (Pair[Momentum[k, D], Momentum[z, D]]) ^2 +
      e^2 av2 / 6 / Pair[Momentum[k, D], Momentum[p, D]]
       (Pair[Momentum[k, D], Momentum[z, D]]) ^3)
 }
DiracElectronPropagatorXRepr[x_, X_, s_] := Module[{s1},
  {DiracSimplify[
     Contract[DotSimplify[(1 / 2 / m / s1 GAD[α] (Pair[Momentum[x, D], LorentzIndex[
                  α, D]] - s1 e (kv[α] Pair[Momentum[a, D], Momentum[x, D]] -
                   av[α] Pair[Momentum[k, D], Momentum[x, D]]) +
                s1^2 / 3 e^2 (- av2 Pair[Momentum[k, D], Momentum[x, D]] kv[α])) + 1).
          (1 + e s1 DiracSlash[a, k, Dimension → D]) /. {s1 → s / m^2}]]],
    Exp[-I Pi / 2 (D / 2 - 1)] Λ^ (4 - D) / 2^D / Pi^ (D / 2) * m^ (D - 1) / s^ (D / 2),
    -s - m^2 Pair[Momentum[x, D], Momentum[x, D]] / 4 / s -
     s / 12 e^2 / m^2 Contract[Pair[Momentum[x, D], LorentzIndex[α, D]]
        FTensor[α, β] FTensor[β, γ] Pair[Momentum[x, D], LorentzIndex[γ, D]]] +
     e Pair[Momentum[a, D], Momentum[x, D]] Pair[Momentum[k, D], Momentum[X, D]]
   } /. {av2 → -m^2 ξ^2 / e^2}
 ]
```

```
PhotonPropagator0[z_, t_, μ_, ν_] :=
 {MTD[μ, ν],
  -Exp[-I Pi / 2 (D / 2 - 1)] Λ^(4 - D) / 2^D / Pi^(D / 2) * m^(D - 2) / t^(D / 2),
  -m^2 Pair[Momentum[z, D], Momentum[z, D]] / 4 / t}

PhotonPropagatorExactXRepr[z_, t_, μ_, ν_] := Module[{TFzFzμν, TFDzFDzμν},
  TFzFzμν = Expand[Contract[
     Ft[μ, α] Pair[Momentum[z, D], LorentzIndex[α, D]] Ft[ν, β] Pair[Momentum[z, D],
       LorentzIndex[β, D]] /. {Ft[α_, β_] → kv[α] av[β] - kv[β] av[α]}
    ]];
  TFDzFDzμν = Expand[Contract[
     -Pair[Momentum[z, D], Momentum[z, D]] FFt[μ, ν] + FFt[μ, α] Pair[Momentum[z, D],
        LorentzIndex[α, D]] Pair[Momentum[z, D], LorentzIndex[ν, D]] +
      FFt[ν, α] Pair[Momentum[z, D], LorentzIndex[α, D]] Pair[Momentum[z, D],
        LorentzIndex[μ, D]] - TFzFzμν - Pair[Momentum[z, D], LorentzIndex[α, D]]
       Pair[Momentum[z, D], LorentzIndex[β, D]] FFt[α, β] MTD[μ, ν] /.
      {Ft[α_, β_] → kv[α] av[β] - kv[β] av[α], FFt[α_, β_] → -av2 kv[α] kv[β]}
      (*/.{av2→ -m^2 ξ^2/e^2}*)
    ]];
  {MTD[μ, ν] * J0[m^(-2) t, ξ Pair[Momentum[k, D], Momentum[z, D]] / 2 / t] +
    (-2 i t kv[μ] kv[ν] / m^2 / Pair[Momentum[z, D], Momentum[k, D]]^2 +
       e^2 TFzFzμν / m^2 / ξ^2 / Pair[Momentum[z, D], Momentum[k, D]]^2)
     J1[m^(-2) t, ξ Pair[Momentum[k, D], Momentum[z, D]] / 2 / t] +
    (-2 i t kv[μ] kv[ν] / m^2 / Pair[Momentum[z, D], Momentum[k, D]]^2 +
       e^2 TFDzFDzμν / m^2 / ξ^2 / Pair[Momentum[z, D], Momentum[k, D]]^2)
     J2[m^(-2) t, ξ Pair[Momentum[k, D], Momentum[z, D]] / 2 / t],
   Exp[-I Pi / 2 (D / 2 - 2)] Λ^(4 - D) / 2^D / Pi^(D / 2) * m^(D - 2) / t^(D / 2) / (2 Pi),
   -m^2 Pair[Momentum[z, D], Momentum[z, D]] / 4 / t}]
```

Uncomment to see examples

```
(*uncomment to see example*)
(*Contract[DiracGamma[LorentzIndex[μ,D],D].GA[5]FDxv[μ]/.
     {FDxv[μ_]→LCD[μ,α1,α2,α3]xv[α1]kv[α2]av[α3]}]
   %/.antiTripleGamma
   %/.TripleGamma*)

(*NewMomentum["p"]
 NewCoordinate["x"]
 FieldSubstitutions
 Expand[Contract[Ft[μ,α]xv[α]Ft[ν,β]xv[β]]/.FieldSubstitutions/.
   {Fxv[α_]→kv[α] ax-av[α] kx}]*)
```

```
(*uncomment to see examples*)
(*NewCoordinate["o"]
 NewCoordinate["O"]*)
(*Ep[x,p]
 EpC[y,q]*)
(*DiracElectronPropagatorXRepr[ov,Ov,s]
 DiracElectronPropagatorXRepr1[o,O,s]*)
(*PhotonPropagator0[x,t,λ,δ]*)

(*NewCoordinate["z"]
 PhotonPropagatorExact[z,t,μ,ν]*)
```