

This is a part of SFQED-Loops script collection
developed for calculating loop processes in
Strong-Field Quantum Electrodynamics.
The scripts are available on <https://github.com/ArsenyMironov/SFQED-Loops>

If you use this script in your research, please, consider
citing our papers:

- A. A. Mironov, S. Meuren, and A. M. Fedotov, PRD 102, 053005 (2020),
<https://doi.org/10.1103/PhysRevD.102.053005>
- A. A. Mironov, A. M. Fedotov, arXiv:2109.00634 (2021)

If you have any questions, please, don't hesitate to contact:
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$x_1 \equiv x_2$

`NotebookEvaluate[NotebookOpen[NotebookDirectory[] <> "definitions.nb"]]`

FeynCalc 9.2.0. For help, use the documentation center, check out the wiki or write to the mailing list.

See also the supplied examples. If you use FeynCalc in your research, please cite

- V. Shtabovenko, R. Mertig and F. Orellana,
Comput. Phys. Commun., 207C, 432–444, 2016, arXiv:1601.01167
- R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun., 64, 345–359, 1991.

Electron propagator (proper time is dimensionless)

$$S^c(x_2, x_1) = \Lambda^{4-D} \int \frac{d^D p}{(2\pi)^D} E_p(x_2) \frac{i(\not{\gamma} p) + m}{p^2 - m^2 + i0} \bar{E}_p(x_1)$$

$$x = x_2 - x_1,$$

$$X = \frac{1}{2} (x_1 + x_2),$$

$$\xi^2 = -\frac{e^2 a^2}{m^2},$$

$$[\Delta] = m - \text{mass scale},$$

$$E_p(x_2) = \left[1 - \frac{e(\gamma k)(\gamma a)}{2(kp)} (kx_2) \right]$$

$$\text{Exp} \left[-i(p \cdot x_2) + i \frac{e(a \cdot p)}{2(k \cdot p)} (k \cdot x_2)^2 + i \frac{a^2 e^2}{6(k \cdot p)} (k \cdot x_2)^3 \right];$$

$$(\gamma p - m) S^c(p) = i - \text{in E - p representation}$$

$$S^c(p, q) = (2\pi)^4 \delta(p - q) S^c(p)$$

NewMomentum["p"]

NewCoordinate["x1"]

NewCoordinate["x2"]

NewCoordinate["x"]

NewCoordinate["X"]

$$\left\{ p^\alpha, p^2, k \cdot p, Fp^\alpha, FFp^\alpha, FDP^\alpha, a \cdot p, 0, 0, 0, -a^2(k \cdot p), 0, 0, -\frac{m^6 \chi p^2}{e^2}, -\frac{m^6 \chi p^2}{e^2}, \frac{m^6 \chi p^2}{e^2}, 0, 0, 0, 0, 0, 0 \right\}$$

$$\left\{ x1^\alpha, x1^2, k \cdot x1, a \cdot x1, Fx1^\alpha, FFx1^\alpha, FDX1^\alpha, k \cdot x1, 0, 0, 0, -a^2(k \cdot x1), 0, 0, -\frac{m^2 \xi^2 (k \cdot x1)^2}{e^2}, -\frac{m^2 \xi^2 (k \cdot x1)^2}{e^2}, \frac{m^2 \xi^2 (k \cdot x1)^2}{e^2}, 0, 0, 0, 0, 0, 0 \right\}$$

$$\left\{ x2^\alpha, x2^2, k \cdot x2, a \cdot x2, Fx2^\alpha, FFx2^\alpha, FDX2^\alpha, k \cdot x2, 0, 0, 0, -a^2(k \cdot x2), 0, 0, -\frac{m^2 \xi^2 (k \cdot x2)^2}{e^2}, -\frac{m^2 \xi^2 (k \cdot x2)^2}{e^2}, \frac{m^2 \xi^2 (k \cdot x2)^2}{e^2}, 0, 0, 0, 0, 0, 0 \right\}$$

$$\left\{ x^\alpha, x^2, k \cdot x, a \cdot x, Fx^\alpha, FFx^\alpha, FDX^\alpha, k \cdot x, 0, 0, 0, -a^2(k \cdot x), 0, 0, -\frac{m^2 \xi^2 (k \cdot x)^2}{e^2}, -\frac{m^2 \xi^2 (k \cdot x)^2}{e^2}, \frac{m^2 \xi^2 (k \cdot x)^2}{e^2}, 0, 0, 0, 0, 0, 0 \right\}$$

$$\left\{ X^\alpha, X^2, k \cdot X, a \cdot X, FX^\alpha, FFX^\alpha, FDX^\alpha, k \cdot X, 0, 0, 0, -a^2(k \cdot X), 0, 0, -\frac{m^2 \xi^2 (k \cdot X)^2}{e^2}, -\frac{m^2 \xi^2 (k \cdot X)^2}{e^2}, \frac{m^2 \xi^2 (k \cdot X)^2}{e^2}, 0, 0, 0, 0, 0, 0 \right\}$$

$$\text{Epx2} = \text{Ep}[x2, p]$$

$$\text{EpBarx1} = \text{EpC}[x1, p]$$

$$\left\{ 1 - \frac{e(k \cdot x2)(\gamma \cdot k)(\gamma \cdot a)}{2(k \cdot p)}, \frac{a^2 e^2 (k \cdot x2)^3}{6(k \cdot p)} + \frac{e(a \cdot p)(k \cdot x2)^2}{2(k \cdot p)} - p \cdot x2 \right\}$$

$$\left\{ 1 - \frac{e(k \cdot x1)(\gamma \cdot a)(\gamma \cdot k)}{2(k \cdot p)}, -\frac{a^2 e^2 (k \cdot x1)^3}{6(k \cdot p)} - \frac{e(a \cdot p)(k \cdot x1)^2}{2(k \cdot p)} + p \cdot x1 \right\}$$

$$\text{Matrix} = \text{Epx2}[[1]] \cdot (\text{GAD}[\alpha] \text{Pair}[\text{Momentum}[p, D], \text{LorentzIndex}[\alpha, D]] + m) \cdot \text{EpBarx1}[[1]]$$

$$\text{Coeff} = i \wedge^4 (4 - D) / (2 \pi)^D / (p^2 - m^2)$$

$$\text{Phase} = \text{Epx2}[[2]] + \text{EpBarx1}[[2]]$$

$$\left(1 - \frac{e(k \cdot x2)(\gamma \cdot k)(\gamma \cdot a)}{2(k \cdot p)} \right) (m + \gamma^\alpha p^\alpha) \left(1 - \frac{e(k \cdot x1)(\gamma \cdot a)(\gamma \cdot k)}{2(k \cdot p)} \right)$$

$$\frac{i (2 \pi)^{-D} \wedge^{4-D}}{p^2 - m^2}$$

$$-\frac{a^2 e^2 (k \cdot x1)^3}{6(k \cdot p)} + \frac{a^2 e^2 (k \cdot x2)^3}{6(k \cdot p)} - \frac{e(a \cdot p)(k \cdot x1)^2}{2(k \cdot p)} + \frac{e(a \cdot p)(k \cdot x2)^2}{2(k \cdot p)} + p \cdot x1 - p \cdot x2$$

$$\text{Matrix1} = \text{Contract}[\text{DiracSimplify}[\text{Matrix}]]$$

$$\text{Coeff1} = \text{Coeff};$$

$$\text{Phase1} = \text{Phase};$$

$$-\frac{a^2 e^2 \gamma \cdot k (k \cdot x1)(k \cdot x2)}{2(k \cdot p)} - \frac{e m (k \cdot x1)(\gamma \cdot a)(\gamma \cdot k)}{2(k \cdot p)} - \frac{e m (k \cdot x2)(\gamma \cdot k)(\gamma \cdot a)}{2(k \cdot p)} -$$

$$\frac{e(k \cdot x1)(\gamma \cdot p)(\gamma \cdot a)(\gamma \cdot k)}{2(k \cdot p)} - \frac{e(k \cdot x2)(\gamma \cdot k)(\gamma \cdot a)(\gamma \cdot p)}{2(k \cdot p)} + m + \gamma \cdot p$$

$$\text{Matrix2} = \text{Expand}[\text{ExpandScalarProduct}[\text{Matrix1} /. \{ \text{Momentum}[x1, D] \rightarrow \text{Momentum}[X, D] - \text{Momentum}[x, D] / 2, \text{Momentum}[x2, D] \rightarrow \text{Momentum}[X, D] + \text{Momentum}[x, D] / 2 \}]]$$

$$\text{Coeff2} = \text{Coeff1};$$

$$\text{Phase2} = \text{Expand}[\text{ExpandScalarProduct}[\text{Phase1} /. \{ \text{Momentum}[x1, D] \rightarrow \text{Momentum}[X, D] - \text{Momentum}[x, D] / 2, \text{Momentum}[x2, D] \rightarrow \text{Momentum}[X, D] + \text{Momentum}[x, D] / 2 \}]]$$

$$\frac{a^2 e^2 \gamma \cdot k (k \cdot x)^2}{8(k \cdot p)} - \frac{a^2 e^2 \gamma \cdot k (k \cdot X)^2}{2(k \cdot p)} + \frac{e m (k \cdot x)(\gamma \cdot a)(\gamma \cdot k)}{4(k \cdot p)} - \frac{e m (k \cdot x)(\gamma \cdot k)(\gamma \cdot a)}{4(k \cdot p)} -$$

$$\frac{e m (k \cdot X)(\gamma \cdot a)(\gamma \cdot k)}{2(k \cdot p)} - \frac{e m (k \cdot X)(\gamma \cdot k)(\gamma \cdot a)}{2(k \cdot p)} - \frac{e(k \cdot x)(\gamma \cdot k)(\gamma \cdot a)(\gamma \cdot p)}{4(k \cdot p)} +$$

$$\frac{e(k \cdot x)(\gamma \cdot p)(\gamma \cdot a)(\gamma \cdot k)}{4(k \cdot p)} - \frac{e(k \cdot X)(\gamma \cdot k)(\gamma \cdot a)(\gamma \cdot p)}{2(k \cdot p)} - \frac{e(k \cdot X)(\gamma \cdot p)(\gamma \cdot a)(\gamma \cdot k)}{2(k \cdot p)} + m + \gamma \cdot p$$

$$\frac{a^2 e^2 (k \cdot x)(k \cdot X)^2}{2(k \cdot p)} + \frac{a^2 e^2 (k \cdot x)^3}{24(k \cdot p)} + \frac{e(a \cdot p)(k \cdot x)(k \cdot X)}{k \cdot p} - p \cdot x$$

```

Matrix3 =
  (( (Expand[Matrix2 /. TripleGamma]) /. EpsToF) /. FieldSubstitutions) /. akToF /.
  { DiracGamma[LorentzIndex[β, D], D].DiracGamma[5]
    Pair[LorentzIndex[β, D], Momentum[FDp, D]] → FVD[γγ5FD, α] pv[α] }

```

```
Coeff3 = Coeff2;
```

```
Phase3 = Phase2;
```

$$\begin{aligned}
 & \frac{a^2 e^2 \gamma \cdot k (k \cdot x)^2}{8 (k \cdot p)} - \frac{a^2 e^2 \gamma \cdot k (k \cdot X)^2}{2 (k \cdot p)} - \frac{e (a \cdot p) \gamma \cdot k (k \cdot X)}{k \cdot p} + \\
 & e \gamma \cdot a (k \cdot X) + \frac{i e m \sigma F (k \cdot x)}{4 (k \cdot p)} + \frac{i e \gamma \gamma 5 F D^\alpha p^\alpha (k \cdot x)}{2 (k \cdot p)} + m + \gamma \cdot p
 \end{aligned}$$

Expanding scalar products into components and changing variables

$$p \rightarrow \{p_- = 1/2 (p^0 - p^3), p_+ = p^0 + p^3, p_\perp\}$$

$$p_- = x_- / 2 s$$

$$p_+ = (p^2 - p_\perp^2) / 2 p_- = s (p^2 + p_\perp^2) / x_-$$

Integration measure

$$\int d^D p \dots = \int \frac{ds}{2s} dp^2 d^{D-2} p_\perp$$

$$x_- = kx / m$$

$$p_- = kx / 2 m s$$

$$kp = m p_- = m xm / 2 s = kx / 2 s$$

$$ap = -at pt$$

$$\gamma p = \gamma_- p_+ + \gamma_+ p_- - \gamma_\perp p_\perp = Gm \frac{s}{x_-} (p^2 + p_\perp^2) + Gp \frac{x_-}{2s} - Gt pt$$

$$\gamma k = \gamma_- k_+ = m Gm$$

$$kx = k_+ x_- = m xm$$

$$(\gamma F^*)^\mu \cdot \gamma^5 \rightarrow \{(\gamma F^*)_- \cdot \gamma^5 = 0, (\gamma F^*)_+ \cdot \gamma^5, (\gamma F^*)_\perp \cdot \gamma^5\} = \{0, \gamma \gamma 5 F D p, \gamma \gamma 5 F D t\}$$

$$(\gamma F^*)_\mu k^\mu = 0 \rightarrow (\gamma F^*)_- = 0$$

$$(\gamma F^*)_\mu a^\mu = 0 \rightarrow \gamma \gamma 5 F D t at = 0$$

$$(\gamma F^*)^\mu \cdot \gamma^5 p_\mu = \gamma \gamma 5 F D m * pp - \gamma \gamma 5 F D t * pt$$

$$D[\{xm/2/s, s(p^2 + pt^2)/xm\}, \{\{s, p^2\}\}]$$

$$\text{Abs}[\text{Det}[\%]]$$

$$\begin{pmatrix} -\frac{xm}{2s^2} & 0 \\ \frac{p^2 + pt^2}{xm} & \frac{s}{xm} \end{pmatrix}$$

$$\frac{1}{2|s|}$$

Matrix4 =

```
Collect[Expand[Matrix3 /. {DiracGamma[Momentum[p, D], D] → Gp * pm + Gm * pp - Gt * pt,
  Pair[Momentum[a, D], Momentum[p, D]] → -at pt,
  FVD[γγ5FD, α_] pv[α_] → γγ5FDp * pm - γγ5FDt * pt,
  DiracGamma[Momentum[k, D], D] → m Gm} /. {kp → kx / 2 / s, pm → xm / 2 / s} /.
  {kx → m xm} /. {pp → s (p2 + pt^2) / xm}], {pt, p2}]
```

Coeff4 = Coeff3 / 2 / s

Phase4 = Collect[

```
Expand[Phase3 /. {Pair[Momentum[p, D], Momentum[x, D]] → pp xm + pm xp - pt * xt,
  kp → m pm, Pair[Momentum[a, D], Momentum[p, D]] → -at pt} /. {kp → kx / 2 / s,
  pm → xm / 2 / s} /. {pp → s (p2 + pt^2) / xm} /. {xm → kx / m}], {pt, pp}]
```

$$-\frac{a^2 e^2 Gm s (k \cdot X)^2}{xm} + \frac{1}{4} a^2 e^2 Gm m^2 s xm + e \gamma \cdot a (k \cdot X) + pt \left(\frac{2 at e Gm s (k \cdot X)}{xm} - i \gamma \gamma 5 FDt e s - Gt \right) +$$

$$\frac{1}{2} i e m s \sigma F + \frac{1}{2} i \gamma \gamma 5 FDp e xm + \frac{Gm p2 s}{xm} + \frac{Gm pt^2 s}{xm} + \frac{Gp xm}{2 s} + m$$

$$\frac{i 2^{-D-1} \pi^{-D} \Lambda^{4-D}}{s(p^2 - m^2)}$$

$$\frac{1}{12} a^2 e^2 s (k \cdot x)^2 + a^2 e^2 s (k \cdot X)^2 + pt (xt - 2 at e s (k \cdot X)) - \frac{xp (k \cdot x)}{2 m s} - p2 s + pt^2 (-s)$$

Integration over

$$\int d^{D-2} p_{\perp} \dots$$

$$\begin{aligned} I_0 &= \int d^{D-2} p_{\perp} \text{Exp} \left[-I A p_{\perp}^2 + I (\mathbf{J} \cdot \mathbf{p}_{\perp}) \right] = \\ &= \text{Exp} \left[-I \frac{\pi}{2} \frac{D-2}{2} \right] \pi^{\frac{D-2}{2}} (\det A)^{-\frac{1}{2}} \text{Exp} \left[I \frac{1}{4} \mathbf{J} \cdot \mathbf{A}^{-1} \cdot \mathbf{J} \right] \end{aligned}$$

$$\begin{aligned} I_1 &= \int d^{D-2} p_{\perp} p_{\perp} \text{Exp} \left[-I A p_{\perp}^2 + I (\mathbf{J} \cdot \mathbf{p}_{\perp}) \right] = \\ &= \frac{1}{2} \mathbf{A}^{-1} \cdot \mathbf{J} I_0 \end{aligned}$$

$$\begin{aligned} I_2 &= \int d^{D-2} p_{\perp} p_{\perp}^2 \text{Exp} \left[-I A p_{\perp}^2 + I (\mathbf{J} \cdot \mathbf{p}_{\perp}) \right] = \\ &= \left[-i \frac{1}{2} \text{Tr} \mathbf{A}^{-1} + \left(\frac{1}{2} \mathbf{A}^{-1} \cdot \mathbf{J} \right)^2 \right] I_0 \end{aligned}$$

where

$$A = s,$$

$$\mathbf{J} = \mathbf{x}_{\perp} - 2 e a_{\perp} s k X,$$

$$\det A = s^{D-2},$$

$$A^{-1} = 1 / s$$

We perform integrations

Integrations changes the coefficient (Coeff) and phase

Then recollect some scalar products

$$a_{\perp}^2 = -a^2$$

$$a_{\perp} \cdot x_{\perp} = - (a x)$$

$$x_{\perp}^2 = 2 x_{\perp} \cdot x_{+} - x^2$$

$$\text{Amatr} = -\text{Coefficient}[\text{Phase4}, \text{pt}^2]$$

$$\mathbf{J} = \text{Coefficient}[\text{Phase4}, \text{pt}]$$

$$\text{CI0} = \text{Exp} \left[-I \pi / 2 (D/2 - 1) \right] \pi^{D/2 - 1} / \text{Amatr}^{D/2 - 1}$$

s

$$x t - 2 a t e s(k \cdot X)$$

$$e^{-\frac{1}{2} i \pi \left(\frac{D-1}{2} \right)} \pi^{\frac{D-1}{2}} s^{1-\frac{D}{2}}$$

```

Phase5 = Expand[Expand[(Phase4 /. {pt → 0}) + 1 / 4 J^2 / Amatr]] /.
  {at^2 → -av2, at xt → -ax, xt^2 → 2 xm xp - xv2} /. {xm → kx / m}
Coeff5 = Coeff4 * CI0
Matrix5 = Collect[
  Expand[Expand[(Matrix4 /. {pt → 0}) + Coefficient[Matrix4, pt] * 1 / 2 / Amatr * J +
    Coefficient[Matrix4, pt^2] * (-I / 2 * (D - 2) / Amatr + (1 / 2 / Amatr * J)^2)] /.
    {γγ5FDt at → 0, at^2 → -av2, at xt → -ax, xt^2 → 2 xm xp - xv2}],
  {p2, Gm, Gp, Gt, γγ5FDm, γγ5FDp, γγ5FDt}]

```

$$\frac{1}{12} a^2 e^2 s (k \cdot x)^2 + e(a \cdot x)(k \cdot X) - p^2 s - \frac{x^2}{4 s}$$

$$\frac{i 2^{-D-1} e^{-\frac{1}{2} i \pi \left(\frac{D-1}{2}\right)} \pi^{-\frac{D}{2}-1} \Lambda^{4-D} s^{-D/2}}{p^2 - m^2}$$

$$Gm \left(\frac{1}{4} a^2 e^2 m^2 s xm - \frac{i D}{2 xm} - \frac{x^2}{4 s xm} + \frac{xp}{2 s} + \frac{i}{xm} \right) + e \gamma \cdot a (k \cdot X) + Gt \left(at e(k \cdot X) - \frac{xt}{2 s} \right) +$$

$$\frac{1}{2} i e m s \sigma F + \frac{1}{2} i \gamma \gamma 5 FDp e xm - \frac{1}{2} i \gamma \gamma 5 FDt e xt + \frac{Gm p^2 s}{xm} + \frac{Gp xm}{2 s} + m$$

Next we substitute

$$\gamma_{\perp} a_{\perp} = -\gamma a$$

$$\gamma_{\perp} x_{\perp} = \gamma_{-} x_{+} - \gamma_{+} x_{-} - \gamma x$$

$$\gamma_{-} x_{-} = \frac{\gamma k}{m} x_{-}$$

$$x_{-} = kx / m$$

```

Matrix6 = Collect[Expand[Matrix5] /.
  {Gt at → -Contract[GAD[α] av[α]], Gt xt → Gm xp + Gp xm - Contract[GAD[α] xv[α]],
  γγ5FDt xt → γγ5FDp xm - DiracSlash[FDxv, Dimension → D].GA[5]} /.
  {Gm xm → DiracGamma[Momentum[k, D], D] / m * xm} /. {xm → kx / m},
  {p2, Gm, Gp, Gt, γγ5FDm, γγ5FDp, γγ5FDt}]

```

Coeff6 = Coeff5

Phase6 = Phase5

$$\frac{1}{2} i e (\gamma \cdot FDxv) \cdot \bar{\gamma}^5 + \frac{1}{4} a^2 e^2 s \gamma \cdot k (k \cdot x) +$$

$$Gm \left(-\frac{i D m}{2 (k \cdot x)} - \frac{m x^2}{4 s (k \cdot x)} + \frac{i m}{k \cdot x} \right) + \frac{1}{2} i e m s \sigma F + \frac{Gm m p^2 s}{k \cdot x} + m + \frac{\gamma \cdot x}{2 s}$$

$$\frac{i 2^{-D-1} e^{-\frac{1}{2} i \pi \left(\frac{D-1}{2}\right)} \pi^{-\frac{D}{2}-1} \Lambda^{4-D} s^{-D/2}}{p^2 - m^2}$$

$$\frac{1}{12} a^2 e^2 s (k \cdot x)^2 + e(a \cdot x)(k \cdot X) - p^2 s - \frac{x^2}{4 s}$$

$$v = p^2 - m^2$$

$$\int \frac{dv}{v + i0} \exp[-i s v] = -2 \pi i \Theta(s)$$

$$\int dV \exp[-i s V] = 2 \pi \delta(s)$$

$$\int_{-\infty}^{\infty} \frac{ds}{s^{D/2}} s e^{ig(s)} 2 \pi \delta(s) = -2 \pi i \int_{-\infty}^{\infty} \frac{ds}{s^{D/2}} e^{ig(s)} \Theta(s) [i(D/2 - 1) + s g'(s)]$$

where

$$g(s) = \text{Phase6 with } p^2 = m^2$$

Finally,

$$G_m = \gamma_- = \gamma k / m$$

$$\text{Phase7} = \text{Phase6} /. \{p^2 \rightarrow m^2\}$$

$$\text{Coeff7} = \text{Coeff6} * (p^2 - m^2) * (-2 \pi i)$$

$$g = \text{Phase7};$$

$$\text{Matrix7} = \text{Expand}[$$

$$(\text{Matrix6} /. \{p^2 \rightarrow m^2\}) + \text{Coefficient}[\text{Matrix6}, p^2 s] * (i(D/2 - 1) + s D[g, s])] /.$$

$$\{G_m \rightarrow \text{DiracGamma}[\text{Momentum}[k, D], D] / m\}$$

$$\frac{1}{12} a^2 e^2 s (k \cdot x)^2 + e(a \cdot x)(k \cdot X) + m^2(-s) - \frac{x^2}{4s}$$

$$2^{-D} e^{-\frac{1}{2} i \pi (\frac{D}{2} - 1)} \pi^{-D/2} \Lambda^{4-D} s^{-D/2}$$

$$\frac{1}{2} i e(\gamma \cdot F D x v) \cdot \bar{\gamma}^5 + \frac{1}{3} a^2 e^2 s \gamma \cdot k (k \cdot x) + \frac{1}{2} i e m s \sigma F + m + \frac{\gamma \cdot x}{2s}$$

$$\text{Matrix8} = \text{Expand}[\text{Matrix7} / m /.]$$

$$\{\text{DiracGamma}[\text{Momentum}[k, D], D] \rightarrow \text{Contract}[\text{GAD}[\alpha] \text{FFxv}[\alpha]] / (-av^2 kx)\}$$

$$\text{Coeff8} = \text{Coeff7} * m$$

$$\text{Phase8} = \text{Phase7} /. \{av^2 kx^2 \rightarrow Fx^2\}$$

$$\frac{i e(\gamma \cdot F D x v) \cdot \bar{\gamma}^5}{2m} - \frac{e^2 s \gamma \cdot F F x}{3m} + \frac{1}{2} i e s \sigma F + \frac{\gamma \cdot x}{2ms} + 1$$

$$2^{-D} e^{-\frac{1}{2} i \pi (\frac{D}{2} - 1)} \pi^{-D/2} m \Lambda^{4-D} s^{-D/2}$$

$$e(a \cdot x)(k \cdot X) + \frac{1}{12} e^2 F x^2 s - m^2 s - \frac{x^2}{4s}$$

Matrix9 = Matrix8 /. {s → s1/m^2}

Coeff9 = Simplify[Coeff8/m^2 /. {s → s1/m^2}, Assumptions → m > 0]

Coeff9 /. {D → 4}

Phase9 = Phase8 /. {s → s1/m^2}

$$\frac{i e(\gamma \cdot \text{FDxv}) \cdot \bar{\gamma}^5}{2 m} - \frac{e^2 s1 \gamma \cdot \text{FFx}}{3 m^3} + \frac{i e s1 \sigma F}{2 m^2} + \frac{m \gamma \cdot x}{2 s1} + 1$$

$$i 2^{-D} e^{-\frac{1}{4} i \pi D} \pi^{-D/2} \Lambda^{4-D} m^{D-1} s1^{-D/2}$$

$$-\frac{i m^3}{16 \pi^2 s1^2}$$

$$e(a \cdot x)(k \cdot X) + \frac{e^2 \text{Fx}^2 s1}{12 m^2} - \frac{m^2 x^2}{4 s1} - s1$$

Matrix91 =

Expand[Matrix9 /. {DiracGamma[Momentum[FDxv, D], D].GA[5] → Contract[DiracGamma[LorentzIndex[μ, D], D].GA[5] FDxv[μ] /. {FDxv[μ_] → -LCD[μ, α1, α2, α3] xv[α1] av[α2] kv[α3]}], EpsContract → False]} /. antiTripleGamma /. {DiracGamma[Momentum[FFx, D], D] → -av2 kx DiracGamma[Momentum[k, D], D]} /. {σF → -2 i DiracGamma[Momentum[a, D], D].DiracGamma[Momentum[k, D], D]}]

$$\frac{a^2 e^2 s1 \gamma \cdot k(k \cdot x)}{3 m^3} + \frac{e s1 (\gamma \cdot a)(\gamma \cdot k)}{m^2} - \frac{e(a \cdot x) \gamma \cdot k}{2 m} + \frac{e \gamma \cdot a(k \cdot x)}{2 m} + \frac{e(\gamma \cdot x)(\gamma \cdot a)(\gamma \cdot k)}{2 m} + \frac{m \gamma \cdot x}{2 s1} + 1$$

Expand[(Matrix91 / (m/2/s1) /. {s1 → s * m^2}) / 2 / s]

$$\frac{1}{3} a^2 e^2 s \gamma \cdot k(k \cdot x) + e m s (\gamma \cdot a)(\gamma \cdot k) - \frac{1}{2} e(a \cdot x) \gamma \cdot k + \frac{1}{2} e \gamma \cdot a(k \cdot x) + \frac{1}{2} e(\gamma \cdot x)(\gamma \cdot a)(\gamma \cdot k) + m + \frac{\gamma \cdot x}{2 s}$$

Another representation of the γ - matrix preexponent term

MartixAnother =

(2 m s1 * 1 / 2 / m / s1 GAD[α] (xv[α] - s1 e Fxv[α] + s1^2 / 3 e^2 FFxv[α]) + 2 m s1 * 1) . (1 + e s1 DiracSlash[a, k, Dimension → D]) /. {s1 → s1/m^2}

MartixAnother1 = DiracSimplify[Contract[DotSimplify[MartixAnother / (2 s1 / m) /. {Fxv[α_] → kv[α] ax - av[α] kx, FFxv[α_] → -av2 kx kv[α]}]]]

CoeffAnother = Coeff9 * m / 2 / s1

$$\left(\gamma^\alpha \left(\frac{e^2 s1^2 \text{FFx}^\alpha}{3 m^4} - \frac{e s1 \text{Fx}^\alpha}{m^2} + x^\alpha \right) + \frac{2 s1}{m} \right) \left(\frac{e s1 (\gamma \cdot a)(\gamma \cdot k)}{m^2} + 1 \right)$$

$$\frac{a^2 e^2 s1 \gamma \cdot k(k \cdot x)}{3 m^3} + \frac{e s1 (\gamma \cdot a)(\gamma \cdot k)}{m^2} - \frac{e(a \cdot x) \gamma \cdot k}{2 m} + \frac{e \gamma \cdot a(k \cdot x)}{2 m} + \frac{e(\gamma \cdot x)(\gamma \cdot a)(\gamma \cdot k)}{2 m} + \frac{m \gamma \cdot x}{2 s1} + 1$$

$$i 2^{-D-1} e^{-\frac{1}{4} i \pi D} \pi^{-D/2} \Lambda^{4-D} m^D s1^{-\frac{D}{2}-1}$$

Matrix91 - Expand[MartixAnother1]

0

Final result for the electron propagator in a CCF

$$\begin{aligned}
 S^c(x_2, x_1) &= \Lambda^{4-D} \int \frac{d^D p}{(2\pi)^D} E_p(x_2) \frac{i(\gamma p + m)}{p^2 - m^2 + i0} E_p^{\text{bar}}(x_1) = e^{-i\frac{\pi}{2}(\frac{D}{2}-1)} \frac{\Lambda^{4-D}}{2^D \pi^{D/2}} m^{D-1} e^{i\eta} \\
 &\int_0^\infty \frac{ds}{s^{D/2}} \left[1 + \frac{m(\gamma x)}{2s} - \frac{e^2 s(\gamma F F x)}{3m^3} + \frac{ies(\sigma^{\alpha\beta} F_{\alpha\beta})}{2m^2} + \frac{ie F_{\alpha\beta}^* x^\beta \gamma^\alpha \gamma^5}{2m} \right] \\
 &e^{-is - i\frac{m^2 x^2}{4s} + i\frac{s}{12} \frac{e^2}{m^2} (Fx)^2} = \\
 &= e^{-i\frac{\pi}{2}(\frac{D}{2}-1)} \frac{\Lambda^{4-D}}{2^D \pi^{D/2}} m^{D-1} e^{i\eta} \\
 &\int_0^\infty \frac{ds}{s^{D/2}} \left[1 + \frac{m(\gamma x)}{2s} + \frac{e(\gamma a)(kx)}{2m} - \frac{e(\gamma k)(ax)}{2m} + \frac{e(\gamma x)(\gamma a)(\gamma k)}{2m} + \right. \\
 &\quad \left. \frac{es(\gamma a)(\gamma k)}{m^2} + \frac{e^2 a^2 s(\gamma k)(kx)}{3m^3} \right] e^{-is - i\frac{m^2 x^2}{4s} + i\frac{s}{12} \frac{e^2}{m^2} (Fx)^2} = \\
 &= e^{-i\frac{\pi}{2}(\frac{D}{2}-1)} \frac{\Lambda^{4-D}}{2^{D+1} \pi^{D/2}} m^D e^{i\eta} \int_0^\infty \frac{ds}{s^{D/2+1}} \left[\frac{2s}{m} + \gamma^\alpha \left(g_{\alpha\beta} - \frac{es}{m^2} F_{\alpha\beta} + \frac{e^2 s^2}{3m^4} F_{\alpha\lambda} F^\lambda{}_\beta \right) \right. \\
 &\quad \left. x^\beta \right] \left(1 + \frac{ies}{2} \sigma^{\alpha\beta} F_{\alpha\beta} \right) e^{-is - i\frac{m^2 x^2}{4s} + i\frac{s}{12} \frac{e^2}{m^2} (Fx)^2}
 \end{aligned}$$

$\eta = e(ax)(k, (x_1 + x_2)/2),$
 $x = x_2 - x_1,$
 $e > 0,$
 $\sigma^{\alpha\beta} = \frac{i}{2} (\gamma^\alpha \gamma^\beta - \gamma^\beta \gamma^\alpha),$
 $\gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3,$

$$e^{-i\frac{\pi}{2}(\frac{D}{2}-1)} \frac{\Lambda^{4-D}}{2^D \pi^{D/2}} m^{D-1} \rightarrow \frac{(-i)m^3}{16\pi^2}, \quad D \rightarrow 4$$