This is a part of SFQED-Loops script collection
developed for calculating loop processes in
Strong-Field Quantum Electrodynamics.
The scripts are available on https://github.com/ArsenyMironov/SFQED-Loops

If you use this script in your research, please, consider
citing our papers:
- A. A. Mironov, S. Meuren, and A. M. Fedotov, PRD 102, 053005 (2020),
  https://doi.org/10.1103/PhysRevD.102.053005
- A. A. Mironov, A. M. Fedotov, arXiv:2109.00634 (2021)

If you have any questions, please, don’t hesitate to contact:
mironov.hep@gmail.com

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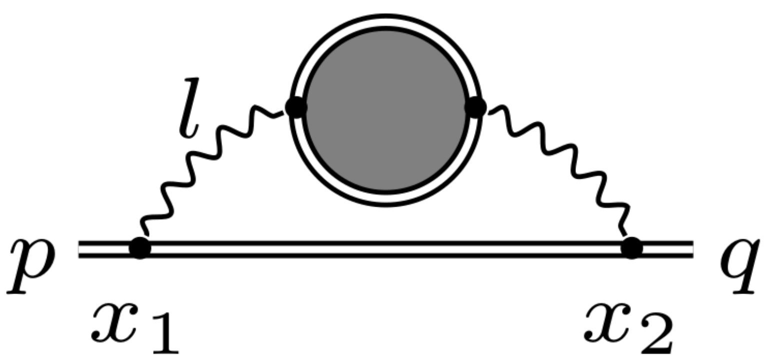


```
In[1]:= NotebookEvaluate[NotebookOpen[NotebookDirectory[] <> "definitions.nb"]]
```

FeynCalc 9.3.1 (stable version). For help, use the documentation center, check out the wiki or visit the forum.

To save your and our time, please check our FAQ for answers to some common FeynCalc questions.

See also the supplied examples. If you use FeynCalc in your research, please cite

- V. Shtabovenko, R. Mertig and F. Orellana, Comput.Phys.Commun. 256 (2020) 107478, arXiv:2001.04407.
- V. Shtabovenko, R. Mertig and F. Orellana, Comput.Phys.Commun. 207 (2016) 432–444, arXiv:1601.01167.
- R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun. 64 (1991) 345–359.

Mass operator $(e > 0)$

$$-i M(q, p) = (i e)^2 \Lambda^{2D-8} \int d^D x_1\, d^D x_2\, \overline{E}_q(x_2)\, \gamma^\mu\, S^c(x_2, x_1)\, \gamma^\nu\, E_p(x_1)\, D^c{}_{\mu\nu}(x_1 - x_2);$$

x_1^μ, x_2^μ – position of the left and right vertices of the diagram;
p^μ, q^μ – initial and final electron momenta;
S^c, $D^c{}_{\mu\nu}$ – electron and photon casual propagators;

The Ritus E_p – function

$$E_p(x_1) = \left[1 - \frac{e\,(\gamma k)\,(\gamma a)}{2\,(kp)}\,(k x_2)\right]$$

$$\text{Exp}\left[-i\,(p\, x_1) + i\,\frac{e\,(a\, p)}{2\,(k\, p)}\,(k\, x_1)^2 + i\,\frac{e^2\, a^2}{6\,(k\, p)}\,(k\, x_1)^3\right];$$

$$\overline{E}_q(x_2) = \gamma^0\, E_q(x_2)\, \gamma^0;$$

Electron propagator in a CCF in D dimensions

$$S^c(x_2, x_1) = e^{i\eta}\, S^c_{\text{diag}}(x_2 - x_1) =$$

$$= e^{i\eta}\, e^{-i\frac{\pi}{2}\left(\frac{D}{2}-1\right)}\, \frac{\Lambda^{4-D}}{2^D\, \pi^{D/2}}\, m$$

$$\int_0^\infty \frac{ds}{s^{D/2}} \left[1 + \frac{(\gamma x)}{2\, m\, s} - \frac{e^2\, s\, (\gamma\, F F x)}{3\, m} + \frac{i}{2}\, e\, s\, \left(\sigma^{\alpha\beta}\, F_{\alpha\beta}\right) + \frac{i\, e\, F_{\alpha\beta}{}^*\, x^\beta\, \gamma^\alpha\, \gamma^5}{2\, m}\right]$$

$$e^{-i s - i\frac{x^2}{4 s} + i\frac{s}{12} e^2 (F x)^2}$$

$$= e^{i\eta}\, e^{-i\frac{\pi}{2}\left(\frac{D}{2}-1\right)}\, \frac{\Lambda^{4-D}}{2^D\, \pi^{D/2}}\, m \int_0^\infty \frac{ds}{s^{D/2}} \left[1 + \frac{(\gamma x)}{2\, m\, s} + \frac{e\,(\gamma a)\,(k x)}{2\, m} - \frac{e\,(\gamma k)\,(a x)}{2\, m} + \right.$$

$$\frac{e\,(\gamma x)\,(\gamma a)\,(\gamma k)}{2\,m} + e\,s\,(\gamma a)\,(\gamma k) + \frac{e^2\,a^2\,s\,(\gamma k)\,(k x)}{3\,m}\Bigg]\,e^{-i s - i \frac{x^2}{4 s} + i \frac{s}{12} e^2 (F x)^2}$$

$\eta = e\,(a x)\,(k, (x_1 + x_2)/2)$,

$x = x_2 - x_1$,

$e > 0$,

$[\Lambda] = m$ – mass scale,

$\sigma^{\alpha\beta} = \frac{i}{2}\left(\gamma^\alpha\,\gamma^\beta - \gamma^\beta\,\gamma^\alpha\right)$,

$\gamma^5 = i\,\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3$,

$F_{\alpha\beta} = k_\alpha\,a_\beta - k_\beta\,a_\alpha$,

$F_{\alpha\beta}\,F^\beta{}_\lambda = -a^2\,k_\alpha\,k_\lambda$

$$e^{-i \frac{\pi}{2}\left(\frac{D}{2}-1\right)}\,\frac{\Lambda^{4-D}}{2^D\,\pi^{D/2}}\,m^{D-1} \to \frac{(-i)\,m^3}{16\,\pi^2},\quad D \to 4$$

Exact photon propagator in momentum representation

$D^c{}_{\mu\nu}(l) = D_0\left(l^2\right) g_{\mu\nu} + D_1\left(l^2, \chi_l\right) \epsilon_\mu^{(1)}(l)\,\epsilon_\nu^{(1)}(l) + D_2\left(l^2, \chi_l\right) \epsilon_\mu^{(2)}(l)\,\epsilon_\nu^{(2)}(l)$;

l^μ – the photon 4 – momentum;

$\chi_l = \frac{e}{m^3}\sqrt{-(F_{\mu\nu}\,l^\nu)^2}$;

$\epsilon_\mu^{(1)}(l) = \frac{e F_{\mu\nu}\,l^\nu}{m^3\,\chi_l}$;

$\epsilon_\mu^{(2)}(l) = \frac{e F^*{}_{\mu\nu}\,l^\nu}{m^3\,\chi_l}$; $\quad F^{*\mu\nu} = \frac{1}{2}\,\epsilon^{\mu\nu\delta\lambda}\,F_{\delta\lambda}$;

$\left(\epsilon^{(i)}(l)\right)^2 = -1$;

$D_0\left(l^2\right) = \frac{-i}{l^2 + i 0}$, $\quad D_{1,2}\left(l^2, \chi_l\right) = \frac{i\,\Pi_{1,2}}{(l^2 + i 0)\left(l^2 - \Pi_{1,2}\right)}$;

$\Pi_{1,2} = \Pi_{1,2}\left(l^2, \chi_l\right)$ – polarization operator eigenfunctions;

Exact photon propagator in coordinate representation

$$D^c{}_{\mu\nu}(x) = \frac{\Lambda^{4-D}}{(2\,\pi)^D}\int d^D l\, D^c{}_{\mu\nu}(l)\, e^{-i l x} =$$

$$= \mathrm{Exp}\left[-i\,\frac{\pi}{2}\left(\frac{D}{2} - 2\right)\right]\frac{1}{2^D\,\pi^{D/2}}\,\frac{\Lambda^{4-D}}{m^{2-D}}$$

$$\int_0^\infty \frac{dt}{t^{D/2}} e^{-i\frac{m^2 x^2}{4t}} \Big\{ g_{\mu\nu} J_0(t) + \left(-2 i t \frac{k_\mu k_\nu}{m^2 \phi^2} + \frac{e^2 Fx_\mu Fx_\nu}{m^2 \xi^2 \phi^2}\right) J_1(t, \chi_l)$$

$$+ \left(-2 i t \frac{k_\mu k_\nu}{m^2 \phi^2} + \frac{e^2 FDx_\mu FDx_\nu}{m^2 \xi^2 \phi^2}\right) J_2(t, \chi_l) \Big\};$$

$$J_k(t, \chi_l) = -i \int_0^\infty dl^2 D_k(l^2, \chi_l) e^{-i l^2 t};$$

$$\chi_l = \xi kl / m^2 = \xi \phi / 2 m^2 t;$$

Preliminaries

Let us define momenta and coordinate variables

$x = x_2 - x_1;$

$X = \frac{1}{2}(x_1 + x_2);$

$\phi = kx;$

$\Phi = kX;$

$\phi 1 = kx_1;$

$\phi 2 = kx_2;$

The functions NewMomentum and NewCoordinate
are predefined in the file definitions.nb

They provide the corresponding 4 − vector along with all possible
contractions with the field tensor $F_{\mu\nu}$ and 4 − vectors a_μ,
k_μ (e.g. $Fxv[\mu] == (Fx)_\mu = F_{\mu\nu} x^\nu$, see more details in the file definitions.nb)

In[2]:=
```
NewMomentum["p"]
NewMomentum["q"]
NewMomentum["l"]
NewCoordinate["x1"]
NewCoordinate["x2"]
NewCoordinate["x"]
NewCoordinate["X"]
ScalarProduct[k, x] = ϕ;
ScalarProduct[k, X] = Φ;
ScalarProduct[k, x1] = ϕ1;
ScalarProduct[k, x2] = ϕ2;
ScalarProduct[Fx, Fx] = -ξ^2 ϕ^2 * m^2 / e^2;
ScalarProduct[FDx, FDx] = -ξ^2 ϕ^2 * m^2 / e^2;
ScalarProduct[x, FFx] = ξ^2 ϕ^2 * m^2 / e^2;
```

$$\left\{p^{\alpha},\ p^2,\ k\cdot p,\ \mathrm{Fp}^{\alpha},\ \mathrm{FFp}^{\alpha},\ \mathrm{FDp}^{\alpha},\ a\cdot p,\ 0,\ 0,\ 0,\ -a^2\,(k\cdot p),\ 0,\ 0,\ -\frac{m^6\ \chi\mathrm{p}^2}{e^2},\ -\frac{m^6\ \chi\mathrm{p}^2}{e^2},\ \frac{m^6\ \chi\mathrm{p}^2}{e^2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}$$

$$\left\{q^{\alpha},\ q^2,\ k\cdot q,\ \mathrm{Fq}^{\alpha},\ \mathrm{FFq}^{\alpha},\ \mathrm{FDq}^{\alpha},\ a\cdot q,\ 0,\ 0,\ 0,\ -a^2\,(k\cdot q),\ 0,\ 0,\ -\frac{m^6\ \chi\mathrm{q}^2}{e^2},\ -\frac{m^6\ \chi\mathrm{q}^2}{e^2},\ \frac{m^6\ \chi\mathrm{q}^2}{e^2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}$$

$$\left\{l^{\alpha},\ l^2,\ k\cdot l,\ \mathrm{Fl}^{\alpha},\ \mathrm{FFl}^{\alpha},\ \mathrm{FDl}^{\alpha},\ a\cdot l,\ 0,\ 0,\ 0,\ -a^2\,(k\cdot l),\ 0,\ 0,\ -\frac{m^6\ \chi\mathrm{l}^2}{e^2},\ -\frac{m^6\ \chi\mathrm{l}^2}{e^2},\ \frac{m^6\ \chi\mathrm{l}^2}{e^2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}$$

$$\left\{\mathrm{x1}^{\alpha},\ \mathrm{x1}^2,\ k\cdot\mathrm{x1},\ a\cdot\mathrm{x1},\ \mathrm{Fx1}^{\alpha},\ \mathrm{FFx1}^{\alpha},\ \mathrm{FDx1}^{\alpha},\ k\cdot\mathrm{x1},\ 0,\ 0,\ 0,\ -a^2\,(k\cdot\mathrm{x1}),\right.$$
$$\left.0,\ 0,\ -\frac{m^2\ \xi^2\,(k\cdot\mathrm{x1})^2}{e^2},\ -\frac{m^2\ \xi^2\,(k\cdot\mathrm{x1})^2}{e^2},\ \frac{m^2\ \xi^2\,(k\cdot\mathrm{x1})^2}{e^2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}$$

$$\left\{\mathrm{x2}^{\alpha},\ \mathrm{x2}^2,\ k\cdot\mathrm{x2},\ a\cdot\mathrm{x2},\ \mathrm{Fx2}^{\alpha},\ \mathrm{FFx2}^{\alpha},\ \mathrm{FDx2}^{\alpha},\ k\cdot\mathrm{x2},\ 0,\ 0,\ 0,\ -a^2\,(k\cdot\mathrm{x2}),\right.$$
$$\left.0,\ 0,\ -\frac{m^2\ \xi^2\,(k\cdot\mathrm{x2})^2}{e^2},\ -\frac{m^2\ \xi^2\,(k\cdot\mathrm{x2})^2}{e^2},\ \frac{m^2\ \xi^2\,(k\cdot\mathrm{x2})^2}{e^2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}$$

$$\left\{x^{\alpha},\ x^2,\ k\cdot x,\ a\cdot x,\ \mathrm{Fx}^{\alpha},\ \mathrm{FFx}^{\alpha},\ \mathrm{FDx}^{\alpha},\ k\cdot x,\ 0,\ 0,\ 0,\ -a^2\,(k\cdot x),\right.$$
$$\left.0,\ 0,\ -\frac{m^2\ \xi^2\,(k\cdot x)^2}{e^2},\ -\frac{m^2\ \xi^2\,(k\cdot x)^2}{e^2},\ \frac{m^2\ \xi^2\,(k\cdot x)^2}{e^2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}$$

$$\left\{X^{\alpha},\ X^2,\ k\cdot X,\ a\cdot X,\ \mathrm{FX}^{\alpha},\ \mathrm{FFX}^{\alpha},\ \mathrm{FDX}^{\alpha},\ k\cdot X,\ 0,\ 0,\ 0,\ -a^2\,(k\cdot X),\right.$$
$$\left.0,\ 0,\ -\frac{m^2\ \xi^2\,(k\cdot X)^2}{e^2},\ -\frac{m^2\ \xi^2\,(k\cdot X)^2}{e^2},\ \frac{m^2\ \xi^2\,(k\cdot X)^2}{e^2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}$$

E_p – functions

The functions Ep and EpC are predefined in the file definitions.nb

They provide a list with two data fields : the preexponent and the phase of the E_p – function (or the adjoint $\overline{E_p}$ – function)

In[16]:=
```
Epx1 = Ep[x1, p]
EqBarx2 = EpC[x2, q]
```

Out[16]= $$\left\{1-\frac{e\,\phi1\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)},\ \frac{a^2\,e^2\,\phi1^3}{6\,(k\cdot p)}+\frac{e\,\phi1^2\,(a\cdot p)}{2\,(k\cdot p)}-p\cdot \mathrm{x1}\right\}$$

Out[17]= $$\left\{1-\frac{e\,\phi2\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot q)},\ -\frac{a^2\,e^2\,\phi2^3}{6\,(k\cdot q)}-\frac{e\,\phi2^2\,(a\cdot q)}{2\,(k\cdot q)}+q\cdot \mathrm{x2}\right\}$$

Propagators in coordinate representation and proper times

s – electron proper time of dimension m^{-2};

t – photon proper time of dimension m^{-2}

The functions DiracElectronPropagatorXRepr and PhotonPropagatorExactXRepr are predefined in the file definitions.nb

They provide a list with three data fields : a γ –matrix (for S^c) or tensor (for D^c) preexponential,
a scalar prefactor and the phase of the exponent.
It is implied that there is an integraion over the proper time from 0 to ∞

In[18]:=
```
Sc = DiracElectronPropagatorXRepr[x, X, m^2 s];
Sc[[2]] = Simplify[Sc[[2]] * m^2, Assumptions → {m > 0}];
Sc
```

Out[20]= $$\left\{-\frac{e\,(a\cdot x)\,\gamma\cdot k}{2\,m}+\frac{e\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{2\,m}+e\,s\,(\gamma\cdot a).(\gamma\cdot k)+\frac{e\,\phi\,\gamma\cdot a}{2\,m}-\frac{1}{3}\,m\,\xi^2\,s\,\phi\,\gamma\cdot k+\frac{\gamma\cdot x}{2\,m\,s}+1,\right.$$
$$\left. i\,2^{-D}\,e^{-\frac{1}{4}\,i\,\pi\,D}\,\pi^{-D/2}\,m\,\Lambda^{4-D}\,s^{-D/2},\ e\,\Phi\,(a\cdot x)-\frac{1}{12}\,m^2\,\xi^2\,s\,\phi^2-m^2\,s-\frac{x^2}{4\,s}\right\}$$

In[21]:=
```
Dc = PhotonPropagatorExactXRepr[x, m^2 t, μ, ν];
Dc[[2]] = Simplify[Dc[[2]] * m^2, Assumptions → {m > 0}];
Dc
```

Out[23]=
$$\left\{\mathrm{J2}\left(t, \frac{\xi\,\phi}{2\,m^2\,t}\right)\left(\frac{1}{m^2\,\xi^2\,\phi^2}e^2\left(-k^\mu\,k^\nu\,(a\cdot x)^2 + a^2\,\phi^2\,g^{\mu\,\nu} - a^2\,\phi\,k^\nu\,x^\mu - a^2\,\phi\,k^\mu\,x^\nu + \phi\,a^\nu\,k^\mu\,(a\cdot x) + \phi\,a^\mu\,k^\nu\,(a\cdot x) + a^2\,x^2\,k^\mu\,k^\nu + \phi^2\,(-a^\mu)\,a^\nu\right) - \frac{2\,i\,t\,k^\mu\,k^\nu}{\phi^2}\right) + \mathrm{J1}\left(t, \frac{\xi\,\phi}{2\,m^2\,t}\right)\left(\frac{e^2\left(k^\mu\,k^\nu\,(a\cdot x)^2 - \phi\,a^\nu\,k^\mu\,(a\cdot x) - \phi\,a^\mu\,k^\nu\,(a\cdot x) + \phi^2\,a^\mu\,a^\nu\right)}{m^2\,\xi^2\,\phi^2} - \frac{2\,i\,t\,k^\mu\,k^\nu}{\phi^2}\right) + g^{\mu\,\nu}\,\mathrm{J0}\left(t, \frac{\xi\,\phi}{2\,m^2\,t}\right), -2^{-D-1}\,e^{-\frac{1}{4}\,i\,\pi\,D}\,\pi^{-\frac{D}{2}-1}\,\Lambda^{4-D}\,t^{-D/2}, -\frac{x^2}{4\,t}\right\}$$

Calculation of M

The mass operator

$M(q, p) = i \int d^D x_1\, d^D x_2\ \ldots;$

We will write the integrand in the following form

Coeff * Tr[Matrix] * Exp[i Phase],

where

Coeff – is a dimensional coefficient

in front of the expression (also depends on s and t),

Matrix – the γ – martix factor,

Phase – the total phase of the exponential

We also introduce the notation

$$\mathrm{Jk} == \mathrm{Jk}\left[\mathrm{t}, \frac{\xi\,\phi}{2\,\mathrm{m}^2\,\mathrm{t}}\right]$$

In[24]:=
```
Coeff = I (I e)^2 Λ^(2 D-8) Sc[[2]] × Dc[[2]]
Phase = EqBarx2[[2]] + Sc[[3]] + Dc[[3]] + Epx1[[2]]
Matrix = (EqBarx2[[1]].GAD[μ].Sc[[1]].GAD[ν].Epx1[[1]] × Dc[[1]]) /.
  {J0[t, ξ ϕ / 2 / t / m^2] → J0, J1[t, ξ ϕ / 2 / t / m^2] → J1, J2[t, ξ ϕ / 2 / t / m^2] → J2}
```

Out[24]= $-2^{-2D-1}\, e^{-\frac{1}{2} i \pi D} \pi^{-D-1} e^2\, m\, s^{-D/2}\, t^{-D/2}$

Out[25]= $$\frac{a^2 e^2 \phi 1^3}{6(k\cdot p)} - \frac{a^2 e^2 \phi 2^3}{6(k\cdot q)} + \frac{e\,\phi 1^2 (a\cdot p)}{2(k\cdot p)} - \frac{e\,\phi 2^2 (a\cdot q)}{2(k\cdot q)} + e\,\Phi(a\cdot x) - \frac{1}{12} m^2 \xi^2 s\,\phi^2 - m^2 s - p\cdot \mathrm{x1} + q\cdot \mathrm{x2} - \frac{x^2}{4s} - \frac{x^2}{4t}$$

Out[26]= $$\Bigg(\mathrm{J2}\Bigg(\frac{1}{m^2 \xi^2 \phi^2} e^2 \big(-k^\mu k^\nu (a\cdot x)^2 + a^2 \phi^2 g^{\mu\nu} - a^2 \phi\, k^\nu x^\mu - a^2 \phi\, k^\mu x^\nu + \phi\, a^\nu k^\mu (a\cdot x) + \phi\, a^\mu k^\nu (a\cdot x) + a^2 x^2 k^\mu k^\nu + \phi^2 (-a^\mu) a^\nu\big) - \frac{2\, i\, t\, k^\mu k^\nu}{\phi^2}\Bigg) + \mathrm{J1}\Bigg(\frac{e^2 \big(k^\mu k^\nu (a\cdot x)^2 - \phi\, a^\nu k^\mu (a\cdot x) - \phi\, a^\mu k^\nu (a\cdot x) + \phi^2 a^\mu a^\nu\big)}{m^2 \xi^2 \phi^2} - \frac{2\, i\, t\, k^\mu k^\nu}{\phi^2}\Bigg) + \mathrm{J0}\, g^{\mu\nu}\Bigg)\Bigg(1 - \frac{e\,\phi 2\, (\gamma\cdot a).(\gamma\cdot k)}{2(k\cdot q)}\Bigg)\gamma^\mu.$$
$$\Bigg(-\frac{e(a\cdot x)\gamma\cdot k}{2m} + \frac{e(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{2m} + e\, s(\gamma\cdot a).(\gamma\cdot k) + \frac{e\,\phi\,\gamma\cdot a}{2m} - \frac{1}{3} m\,\xi^2 s\,\phi\,\gamma\cdot k + \frac{\gamma\cdot x}{2ms} + 1\Bigg)\gamma^\nu.\Bigg(1 - \frac{e\,\phi 1\,(\gamma\cdot k).(\gamma\cdot a)}{2(k\cdot p)}\Bigg)$$

γ − matrix algebra

We perform simplifications of the γ −matrix factor with the aid of FeynCalc functions DotSimplify, DiracSimplify, and then Contract the Lorentz indices.

In[27]:=
```
Coeff1 = Coeff;
Phase1 = Phase;
Contract[DotSimplify[Expand[Matrix]]];
Matrix1 = Collect[Contract[DiracSimplify[%]], {J0, J1, J2}]
```

Out[30]= $$\mathrm{J0}\Bigg(-\frac{D\,\phi\,\phi 1\,\gamma\cdot k\, a^2 e^2}{4m(k\cdot p)} + \frac{3\,\phi\,\phi 1\,\gamma\cdot k\, a^2 e^2}{2m(k\cdot p)} + \frac{D\,\phi\,\phi 2\,\gamma\cdot k\, a^2 e^2}{4m(k\cdot q)} - \frac{3\,\phi\,\phi 2\,\gamma\cdot k\, a^2 e^2}{2m(k\cdot q)} + \frac{D\,\phi\,\phi 1\,\phi 2\,\gamma\cdot k\, a^2 e^2}{4ms(k\cdot p)(k\cdot q)} - \frac{\phi\,\phi 1\,\phi 2\,\gamma\cdot k\, a^2 e^2}{2ms(k\cdot p)(k\cdot q)} - \frac{D\,\phi\,\gamma\cdot a\, e}{2m} + \frac{\phi\,\gamma\cdot a\, e}{m} + D\, s(\gamma\cdot a).(\gamma\cdot k)\, e - 4\, s(\gamma\cdot a).(\gamma\cdot k)\, e - \frac{(\gamma\cdot k).(\gamma\cdot a).(\gamma\cdot x)\, e}{m} - \frac{D(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)\, e}{2m} + \frac{2(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)\, e}{m} + \frac{D\,\gamma\cdot k(a\cdot x)\, e}{2m} - \frac{\gamma\cdot k(a\cdot x)\, e}{m} - \frac{D\,\phi 1(\gamma\cdot k).(\gamma\cdot a)\, e}{2(k\cdot p)} + $$

$$\frac{D\,\phi1\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,e}{4\,m\,s\,(k\cdot p)}-\frac{\phi1\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,e}{2\,m\,s\,(k\cdot p)}-\frac{D\,\phi2\,(\gamma\cdot a).(\gamma\cdot k)\,e}{2\,(k\cdot q)}+\frac{D\,\phi2\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)\,e}{4\,m\,s\,(k\cdot q)}-$$

$$\left.\frac{\phi2\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)\,e}{2\,m\,s\,(k\cdot q)}+D+\frac{1}{3}\,D\,m\,s\,\xi^2\,\phi\,\gamma\cdot k-\frac{2}{3}\,m\,s\,\xi^2\,\phi\,\gamma\cdot k-\frac{D\,\gamma\cdot x}{2\,m\,s}+\frac{\gamma\cdot x}{m\,s}\right)+$$

$$\text{J1}\left(\frac{\phi\,\phi1\,\gamma\cdot k\,a^4\,e^4}{4\,m^3\,\xi^2\,(k\cdot p)}-\frac{\phi\,\phi2\,\gamma\cdot k\,a^4\,e^4}{4\,m^3\,\xi^2\,(k\cdot q)}+\frac{\phi\,\phi1\,\phi2\,\gamma\cdot k\,a^4\,e^4}{4\,m^3\,s\,\xi^2\,(k\cdot p)\,(k\cdot q)}+\frac{\phi\,\gamma\cdot a\,a^2\,e^3}{2\,m^3\,\xi^2}+\frac{s\,(\gamma\cdot k).(\gamma\cdot a)\,a^2\,e^3}{m^2\,\xi^2}-\right.$$

$$\frac{(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,a^2\,e^3}{2\,m^3\,\xi^2}-\frac{\gamma\cdot k\,a^2\,(a\cdot x)\,e^3}{2\,m^3\,\xi^2}-\frac{\phi1\,(\gamma\cdot k).(\gamma\cdot a)\,a^2\,e^3}{2\,m^2\,\xi^2\,(k\cdot p)}+\frac{\phi1\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,a^2\,e^3}{4\,m^3\,s\,\xi^2\,(k\cdot p)}-$$

$$\frac{\phi2\,(\gamma\cdot a).(\gamma\cdot k)\,a^2\,e^3}{2\,m^2\,\xi^2\,(k\cdot q)}+\frac{\phi2\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\,a^2\,e^3}{4\,m^3\,s\,\xi^2\,(k\cdot q)}-\frac{\phi2\,\gamma\cdot k\,a^2\,(a\cdot x)\,e^3}{2\,m^3\,s\,\xi^2\,(k\cdot q)}+\frac{\gamma\cdot k\,(\,a\cdot x\,)^2\,e^2}{m^3\,s\,\xi^2\,\phi}+$$

$$\frac{s\,\phi\,\gamma\cdot k\,a^2\,e^2}{3\,m}-\frac{\gamma\cdot x\,a^2\,e^2}{2\,m^3\,s\,\xi^2}+\frac{a^2\,e^2}{m^2\,\xi^2}+\frac{\gamma\cdot a\,(a\cdot x)\,e^2}{m^3\,s\,\xi^2}-\frac{(\gamma\cdot a).(\gamma\cdot k)\,(a\cdot x)\,e^2}{m^2\,\xi^2\,\phi}-$$

$$\left.\frac{(\gamma\cdot k).(\gamma\cdot a)\,(a\cdot x)\,e^2}{m^2\,\xi^2\,\phi}-\frac{(\gamma\cdot a).(\gamma\cdot x).(\gamma\cdot k)\,(a\cdot x)\,e^2}{2\,m^3\,s\,\xi^2\,\phi}-\frac{(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\,(a\cdot x)\,e^2}{2\,m^3\,s\,\xi^2\,\phi}-\frac{2\,i\,t\,\gamma\cdot k}{m\,s\,\phi}\right)+$$

$$\text{J2}\left(-\frac{D\,\phi\,\phi1\,\gamma\cdot k\,a^4\,e^4}{4\,m^3\,\xi^2\,(k\cdot p)}+\frac{3\,\phi\,\phi1\,\gamma\cdot k\,a^4\,e^4}{4\,m^3\,\xi^2\,(k\cdot p)}+\frac{D\,\phi\,\phi2\,\gamma\cdot k\,a^4\,e^4}{4\,m^3\,\xi^2\,(k\cdot q)}-\frac{3\,\phi\,\phi2\,\gamma\cdot k\,a^4\,e^4}{4\,m^3\,\xi^2\,(k\cdot q)}+\frac{D\,\phi\,\phi1\,\phi2\,\gamma\cdot k\,a^4\,e^4}{4\,m^3\,s\,\xi^2\,(k\cdot p)\,(k\cdot q)}-\right.$$

$$\frac{3\,\phi\,\phi1\,\phi2\,\gamma\cdot k\,a^4\,e^4}{4\,m^3\,s\,\xi^2\,(k\cdot p)\,(k\cdot q)}-\frac{D\,\phi\,\gamma\cdot a\,a^2\,e^3}{2\,m^3\,\xi^2}+\frac{\phi\,\gamma\cdot a\,a^2\,e^3}{2\,m^3\,\xi^2}+\frac{D\,s\,(\gamma\cdot a).(\gamma\cdot k)\,a^2\,e^3}{m^2\,\xi^2}-$$

$$\frac{4\,s\,(\gamma\cdot a).(\gamma\cdot k)\,a^2\,e^3}{m^2\,\xi^2}-\frac{s\,(\gamma\cdot k).(\gamma\cdot a)\,a^2\,e^3}{m^2\,\xi^2}-\frac{(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)\,a^2\,e^3}{m^3\,\xi^2}-\frac{3\,(\gamma\cdot k).(\gamma\cdot a).(\gamma\cdot x)\,a^2\,e^3}{2\,m^3\,\xi^2}-$$

$$\frac{D\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)\,a^2\,e^3}{2\,m^3\,\xi^2}+\frac{3\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)\,a^2\,e^3}{2\,m^3\,\xi^2}+\frac{(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,a^2\,e^3}{2\,m^3\,\xi^2}+\frac{D\,\gamma\cdot k\,a^2\,(a\cdot x)\,e^3}{2\,m^3\,\xi^2}-$$

$$\frac{\gamma\cdot k\,a^2\,(a\cdot x)\,e^3}{2\,m^3\,\xi^2}-\frac{D\,\phi1\,(\gamma\cdot k).(\gamma\cdot a)\,a^2\,e^3}{2\,m^2\,\xi^2\,(k\cdot p)}+\frac{3\,\phi1\,(\gamma\cdot k).(\gamma\cdot a)\,a^2\,e^3}{2\,m^2\,\xi^2\,(k\cdot p)}+\frac{D\,\phi1\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,a^2\,e^3}{4\,m^3\,s\,\xi^2\,(k\cdot p)}-$$

$$\frac{3\,\phi1\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,a^2\,e^3}{4\,m^3\,s\,\xi^2\,(k\cdot p)}-\frac{D\,\phi2\,(\gamma\cdot a).(\gamma\cdot k)\,a^2\,e^3}{2\,m^2\,\xi^2\,(k\cdot q)}+\frac{3\,\phi2\,(\gamma\cdot a).(\gamma\cdot k)\,a^2\,e^3}{2\,m^2\,\xi^2\,(k\cdot q)}+$$

$$\frac{D\,\phi2\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)\,a^2\,e^3}{4\,m^3\,s\,\xi^2\,(k\cdot q)}-\frac{\phi2\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)\,a^2\,e^3}{2\,m^3\,s\,\xi^2\,(k\cdot q)}-\frac{\phi2\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\,a^2\,e^3}{4\,m^3\,s\,\xi^2\,(k\cdot q)}+$$

$$\frac{\phi2\,\gamma\cdot k\,a^2\,(a\cdot x)\,e^3}{2\,m^3\,s\,\xi^2\,(k\cdot q)}-\frac{\gamma\cdot k\,(\,a\cdot x\,)^2\,e^2}{m^3\,s\,\xi^2\,\phi}+\frac{D\,s\,\phi\,\gamma\cdot k\,a^2\,e^2}{3\,m}-\frac{s\,\phi\,\gamma\cdot k\,a^2\,e^2}{m}-\frac{D\,\gamma\cdot x\,a^2\,e^2}{2\,m^3\,s\,\xi^2}+\frac{3\,\gamma\cdot x\,a^2\,e^2}{2\,m^3\,s\,\xi^2}-$$

$$\frac{(\gamma\cdot k).(\gamma\cdot x)\,a^2\,e^2}{m^2\,\xi^2\,\phi}-\frac{(\gamma\cdot x).(\gamma\cdot k)\,a^2\,e^2}{m^2\,\xi^2\,\phi}+\frac{D\,a^2\,e^2}{m^2\,\xi^2}-\frac{a^2\,e^2}{m^2\,\xi^2}-\frac{\gamma\cdot a\,(a\cdot x)\,e^2}{m^3\,s\,\xi^2}+\frac{(\gamma\cdot a).(\gamma\cdot k)\,(a\cdot x)\,e^2}{m^2\,\xi^2\,\phi}+$$

$$\frac{(\gamma\cdot k).(\gamma\cdot a)\,(a\cdot x)\,e^2}{m^2\,\xi^2\,\phi} + \frac{(\gamma\cdot a).(\gamma\cdot x).(\gamma\cdot k)\,(a\cdot x)\,e^2}{2\,m^3\,s\,\xi^2\,\phi} + \frac{(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\,(a\cdot x)\,e^2}{2\,m^3\,s\,\xi^2\,\phi} - \frac{2\,i\,t\,\gamma\cdot k}{m\,s\,\phi}\Bigg)$$

Substitution of the variables

$x1 = x - \frac{X}{2};$

$x2 = x + \frac{X}{2};$

$\phi 1 = \phi - \frac{\Phi}{2};$

$\phi 2 = \phi + \frac{\Phi}{2};$

$\phi = kx = m\,x_-;$

$\Phi = kX = m\,X_-;$

The integration measure :

$d^D x_1\, d^D x_2 = d^D x\, d^D X$

In[31]:=
```
Coeff2 = Coeff1;
Phase2 = Expand[ExpandScalarProduct[
    Phase1 /. {ϕ2 → Φ + ϕ / 2, ϕ1 → Φ - ϕ / 2,  av2 → -m^2 ξ^2 / e^2} /.
     {Momentum[x1, D] → Momentum[X, D] - Momentum[x, D] / 2,
      Momentum[x2, D] → Momentum[X, D] + Momentum[x, D] / 2}
  ]]
Matrix2 = Collect[
   Expand[
    Matrix1 /. {ϕ2 → Φ + ϕ / 2, ϕ1 → Φ - ϕ / 2, av2 → -m^2 ξ^2 / e^2}
   ],
   {J0, J1, J2}]
```

Out[32]=

$$\frac{e\,\Phi^2\,(a\cdot p)}{2\,(k\cdot p)} + \frac{e\,\phi^2\,(a\cdot p)}{8\,(k\cdot p)} - \frac{e\,\Phi\,\phi\,(a\cdot p)}{2\,(k\cdot p)} - \frac{e\,\Phi^2\,(a\cdot q)}{2\,(k\cdot q)} - \frac{e\,\phi^2\,(a\cdot q)}{8\,(k\cdot q)} - \frac{e\,\Phi\,\phi\,(a\cdot q)}{2\,(k\cdot q)} + e\,\Phi\,(a\cdot x) - \frac{m^2\,\xi^2\,\Phi^3}{6\,(k\cdot p)} + \frac{m^2\,\xi^2\,\phi^3}{48\,(k\cdot p)} - \frac{m^2\,\xi^2\,\Phi\,\phi^2}{8\,(k\cdot p)} + \frac{m^2\,\xi^2\,\Phi^2\,\phi}{4\,(k\cdot p)} + \frac{m^2\,\xi^2\,\Phi^3}{6\,(k\cdot q)} + \frac{m^2\,\xi^2\,\phi^3}{48\,(k\cdot q)} + \frac{m^2\,\xi^2\,\Phi\,\phi^2}{8\,(k\cdot q)} + \frac{m^2\,\xi^2\,\Phi^2\,\phi}{4\,(k\cdot q)} - \frac{1}{12}\,m^2\,\xi^2\,s\,\phi^2 - m^2\,s + \frac{p\cdot x}{2} - p\cdot X + \frac{q\cdot x}{2} + q\cdot X - \frac{x^2}{4\,s} - \frac{x^2}{4\,t}$$

Out[33]=

$$
\begin{aligned}
&J1\left(-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)\,(k\cdot q)}-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^2}{8\,(k\cdot p)}-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^2}{8\,(k\cdot q)}-\frac{e\,\gamma\cdot a\,\phi}{2\,m}-\frac{1}{3}\,m\,s\,\xi^2\,\gamma\cdot k\,\phi+\frac{m\,\xi^2\,\Phi\,\gamma\cdot k\,\phi}{4\,(k\cdot p)}-\right.\\
&\frac{e\,(\gamma\cdot k).(\gamma\cdot a)\,\phi}{4\,(k\cdot p)}+\frac{e\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,\phi}{8\,m\,s\,(k\cdot p)}-\frac{m\,\xi^2\,\Phi\,\gamma\cdot k\,\phi}{4\,(k\cdot q)}+\frac{e\,(\gamma\cdot a).(\gamma\cdot k)\,\phi}{4\,(k\cdot q)}-\frac{e\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\,\phi}{8\,m\,s\,(k\cdot q)}+\\
&\frac{e\,\gamma\cdot k\,(a\cdot x)\,\phi}{4\,m\,s\,(k\cdot q)}+\frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)\,(k\cdot q)}+\frac{\gamma\cdot x}{2\,m\,s}-e\,s\,(\gamma\cdot k).(\gamma\cdot a)+\frac{e\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{2\,m}+\\
&\frac{e^2\,\gamma\cdot a\,(a\cdot x)}{m^3\,s\,\xi^2}+\frac{e\,\gamma\cdot k\,(a\cdot x)}{2\,m}+\frac{e\,\Phi\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)}-\frac{e\,\Phi\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{4\,m\,s\,(k\cdot p)}+\frac{e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot q)}-\\
&\frac{e\,\Phi\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)}{4\,m\,s\,(k\cdot q)}+\frac{e\,\Phi\,\gamma\cdot k\,(a\cdot x)}{2\,m\,s\,(k\cdot q)}-1+\frac{e^2\,\gamma\cdot k\,(a\cdot x)^2}{m^3\,s\,\xi^2\,\phi}-\frac{2\,i\,t\,\gamma\cdot k}{m\,s\,\phi}-\frac{e^2\,(\gamma\cdot a).(\gamma\cdot k)\,(a\cdot x)}{m^2\,\xi^2\,\phi}-\\
&\left.\frac{e^2\,(\gamma\cdot k).(\gamma\cdot a)\,(a\cdot x)}{m^2\,\xi^2\,\phi}-\frac{e^2\,(\gamma\cdot a).(\gamma\cdot x).(\gamma\cdot k)\,(a\cdot x)}{2\,m^3\,s\,\xi^2\,\phi}-\frac{e^2\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\,(a\cdot x)}{2\,m^3\,s\,\xi^2\,\phi}\right)+\\
&J0\left(\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)\,(k\cdot q)}-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^3}{8\,s\,(k\cdot p)\,(k\cdot q)}-\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{8\,(k\cdot p)}+\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}-\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{8\,(k\cdot q)}+\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot q)}-\right.\\
&\frac{D\,e\,\gamma\cdot a\,\phi}{2\,m}+\frac{e\,\gamma\cdot a\,\phi}{m}+\frac{1}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,\phi-\frac{2}{3}\,m\,s\,\xi^2\,\gamma\cdot k\,\phi+\frac{D\,m\,\xi^2\,\Phi\,\gamma\cdot k\,\phi}{4\,(k\cdot p)}-\frac{3\,m\,\xi^2\,\Phi\,\gamma\cdot k\,\phi}{2\,(k\cdot p)}+\\
&\frac{D\,e\,(\gamma\cdot k).(\gamma\cdot a)\,\phi}{4\,(k\cdot p)}-\frac{D\,e\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,\phi}{8\,m\,s\,(k\cdot p)}+\frac{e\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,\phi}{4\,m\,s\,(k\cdot p)}-\frac{D\,m\,\xi^2\,\Phi\,\gamma\cdot k\,\phi}{4\,(k\cdot q)}+\frac{3\,m\,\xi^2\,\Phi\,\gamma\cdot k\,\phi}{2\,(k\cdot q)}-\\
&\frac{D\,e\,(\gamma\cdot a).(\gamma\cdot k)\,\phi}{4\,(k\cdot q)}+\frac{D\,e\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)\,\phi}{8\,m\,s\,(k\cdot q)}-\frac{e\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)\,\phi}{4\,m\,s\,(k\cdot q)}-\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)\,(k\cdot q)}+\frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{2\,s\,(k\cdot p)\,(k\cdot q)}+\\
&D-\frac{D\,\gamma\cdot x}{2\,m\,s}+\frac{\gamma\cdot x}{m\,s}+D\,e\,s\,(\gamma\cdot a).(\gamma\cdot k)-4\,e\,s\,(\gamma\cdot a).(\gamma\cdot k)-\frac{e\,(\gamma\cdot k).(\gamma\cdot a).(\gamma\cdot x)}{m}-\frac{D\,e\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{2\,m}+\\
&\frac{2\,e\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{m}+\frac{D\,e\,\gamma\cdot k\,(a\cdot x)}{2\,m}-\frac{e\,\gamma\cdot k\,(a\cdot x)}{m}-\frac{D\,e\,\Phi\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)}+\frac{D\,e\,\Phi\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{4\,m\,s\,(k\cdot p)}-\\
&\left.\frac{e\,\Phi\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{2\,m\,s\,(k\cdot p)}-\frac{D\,e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot q)}+\frac{D\,e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)}{4\,m\,s\,(k\cdot q)}-\frac{e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)}{2\,m\,s\,(k\cdot q)}\right)+\\
&J2\left(-\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)\,(k\cdot q)}+\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)\,(k\cdot q)}+\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{8\,(k\cdot p)}-\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{8\,(k\cdot p)}+\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{8\,(k\cdot q)}-\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{8\,(k\cdot q)}+\right.\\
&\frac{D\,e\,\gamma\cdot a\,\phi}{2\,m}-\frac{e\,\gamma\cdot a\,\phi}{2\,m}-\frac{1}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,\phi+m\,s\,\xi^2\,\gamma\cdot k\,\phi-\frac{D\,m\,\xi^2\,\Phi\,\gamma\cdot k\,\phi}{4\,(k\cdot p)}+\frac{3\,m\,\xi^2\,\Phi\,\gamma\cdot k\,\phi}{4\,(k\cdot p)}-\\
&\frac{D\,e\,(\gamma\cdot k).(\gamma\cdot a)\,\phi}{4\,(k\cdot p)}+\frac{3\,e\,(\gamma\cdot k).(\gamma\cdot a)\,\phi}{4\,(k\cdot p)}+\frac{D\,e\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,\phi}{8\,m\,s\,(k\cdot p)}-\frac{3\,e\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,\phi}{8\,m\,s\,(k\cdot p)}+\frac{D\,m\,\xi^2\,\Phi\,\gamma\cdot k\,\phi}{4\,(k\cdot q)}-\\
&\frac{3\,m\,\xi^2\,\Phi\,\gamma\cdot k\,\phi}{4\,(k\cdot q)}+\frac{D\,e\,(\gamma\cdot a).(\gamma\cdot k)\,\phi}{4\,(k\cdot q)}-\frac{3\,e\,(\gamma\cdot a).(\gamma\cdot k)\,\phi}{4\,(k\cdot q)}-\frac{D\,e\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)\,\phi}{8\,m\,s\,(k\cdot q)}+\frac{e\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)\,\phi}{4\,m\,s\,(k\cdot q)}+
\end{aligned}
$$

$$\frac{e(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\,\phi}{8\,m\,s\,(k\cdot q)}-\frac{e\,\gamma\cdot k\,(a\cdot x)\,\phi}{4\,m\,s\,(k\cdot q)}+\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)\,(k\cdot q)}-\frac{3\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)\,(k\cdot q)}-D+\frac{D\,\gamma\cdot x}{2\,m\,s}-\frac{3\,\gamma\cdot x}{2\,m\,s}-$$
$$D\,e\,s\,(\gamma\cdot a).(\gamma\cdot k)+4\,e\,s\,(\gamma\cdot a).(\gamma\cdot k)+e\,s\,(\gamma\cdot k).(\gamma\cdot a)+\frac{e(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)}{m}+\frac{3\,e(\gamma\cdot k).(\gamma\cdot a).(\gamma\cdot x)}{2\,m}+$$
$$\frac{D\,e(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{2\,m}-\frac{3\,e(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{2\,m}-\frac{e(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{2\,m}-\frac{e^2\,\gamma\cdot a\,(a\cdot x)}{m^3\,s\,\xi^2}-\frac{D\,e\,\gamma\cdot k\,(a\cdot x)}{2\,m}+$$
$$\frac{e\,\gamma\cdot k\,(a\cdot x)}{2\,m}+\frac{D\,e\,\Phi\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)}-\frac{3\,e\,\Phi\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)}-\frac{D\,e\,\Phi\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{4\,m\,s\,(k\cdot p)}+\frac{3\,e\,\Phi\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{4\,m\,s\,(k\cdot p)}+$$
$$\frac{D\,e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot q)}-\frac{3\,e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot q)}-\frac{D\,e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)}{4\,m\,s\,(k\cdot q)}+\frac{e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)}{2\,m\,s\,(k\cdot q)}+$$
$$\frac{e\,\Phi\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)}{4\,m\,s\,(k\cdot q)}-\frac{e\,\Phi\,\gamma\cdot k\,(a\cdot x)}{2\,m\,s\,(k\cdot q)}+1-\frac{e^2\,\gamma\cdot k\,(\,a\cdot x)^2}{m^3\,s\,\xi^2\,\phi}-\frac{2\,i\,t\,\gamma\cdot k}{m\,s\,\phi}+\frac{(\gamma\cdot k).(\gamma\cdot x)}{\phi}+\frac{(\gamma\cdot x).(\gamma\cdot k)}{\phi}+$$
$$\left.\frac{e^2\,(\gamma\cdot a).(\gamma\cdot k)\,(a\cdot x)}{m^2\,\xi^2\,\phi}+\frac{e^2\,(\gamma\cdot k).(\gamma\cdot a)\,(a\cdot x)}{m^2\,\xi^2\,\phi}+\frac{e^2\,(\gamma\cdot a).(\gamma\cdot x).(\gamma\cdot k)\,(a\cdot x)}{2\,m^3\,s\,\xi^2\,\phi}+\frac{e^2\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\,(a\cdot x)}{2\,m^3\,s\,\xi^2\,\phi}\right)$$

Integration over
$\int d^{D-2}X_\perp\, dX_+ \ldots$

Preexp does not depend on $X_\perp$ and
X_+. Phase contains $X_\perp$ and X_+ in scalar products
$pX = p_- X_+ + p_+ X_- - p_\perp X_\perp$;
$qX = q_- X_+ + q_+ X_- - q_\perp X_\perp$;
so
$\int d^{D-2}X_\perp\, dX_+\, e^{-i\,(p-q).X} \ldots = (2\,\pi)^{D-1}\,\delta^{(D-2)}(p_\perp - q_\perp)\,\delta(p_- - q_-)$.

We will not write the δ − functions explicitly,
but we will assume that they are present;

Due to the conservation law $\delta^{(D-2)}(p_\perp - q_\perp)\,\delta(p_- - q_-)$
kq → kp
aq → ap

Notations
pp = p_+;
qp = q_+;

The remaining integrals : $\int dX_-\, d^D x \ldots$

In[34]:=
```
Coeff3 = Coeff2 * (2 π) ^ (D - 1)
Phase3 = Phase2 /. {Pair[Momentum[p, D], Momentum[X , D]] → pp Xm,
    Pair[Momentum[q, D], Momentum[X , D]] → qp Xm} /. {Xm → Φ / m} /. {kq → kp, aq → ap}
Matrix3 = Matrix2 /. {kq → kp, aq → ap}
```

Out[34]= $$-\frac{2^{-D-2}\, e^{-\frac{1}{2} i \pi D}\, e^2\, m\, s^{-D/2}\, t^{-D/2}}{\pi^2}$$

Out[35]= $$-\frac{e \Phi \phi (a\cdot p)}{k\cdot p} + e\Phi (a\cdot x) + \frac{m^2 \xi^2 \phi^3}{24 (k\cdot p)} + \frac{m^2 \xi^2 \Phi^2 \phi}{2 (k\cdot p)} - \frac{1}{12} m^2 \xi^2 s \phi^2 - m^2 s - \frac{\text{pp}\,\Phi}{m} + \frac{\text{qp}\,\Phi}{m} + \frac{p\cdot x}{2} + \frac{q\cdot x}{2} - \frac{x^2}{4 s} - \frac{x^2}{4 t}$$

Out[36]= $$\text{J0}\left(\frac{D m \xi^2 \gamma\cdot k \phi^3}{16 s (k\cdot p)^2} - \frac{m \xi^2 \gamma\cdot k \phi^3}{8 s (k\cdot p)^2} - \frac{D m \xi^2 \gamma\cdot k \phi^2}{4 (k\cdot p)} + \frac{3 m \xi^2 \gamma\cdot k \phi^2}{2 (k\cdot p)} - \frac{D e \gamma\cdot a \phi}{2 m} + \frac{e \gamma\cdot a \phi}{m} + \frac{1}{3} D m s \xi^2 \gamma\cdot k \phi - \frac{2}{3} m s \xi^2 \gamma\cdot k \phi - \frac{D e (\gamma\cdot a).(\gamma\cdot k) \phi}{4 (k\cdot p)} + \frac{D e (\gamma\cdot k).(\gamma\cdot a) \phi}{4 (k\cdot p)} + \frac{D e (\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x) \phi}{8 m s (k\cdot p)} - \frac{e (\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x) \phi}{4 m s (k\cdot p)} - \frac{D e (\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a) \phi}{8 m s (k\cdot p)} + \frac{e (\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a) \phi}{4 m s (k\cdot p)} - \frac{D m \xi^2 \Phi^2 \gamma\cdot k \phi}{4 s (k\cdot p)^2} + \frac{m \xi^2 \Phi^2 \gamma\cdot k \phi}{2 s (k\cdot p)^2} + D - \frac{D \gamma\cdot x}{2 m s} + \frac{\gamma\cdot x}{m s} + D e s (\gamma\cdot a).(\gamma\cdot k) - 4 e s (\gamma\cdot a).(\gamma\cdot k) - \frac{e (\gamma\cdot k).(\gamma\cdot a).(\gamma\cdot x)}{m} - \frac{D e (\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{2 m} + \frac{2 e (\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{m} + \frac{D e \gamma\cdot k (a\cdot x)}{2 m} - \frac{e \gamma\cdot k (a\cdot x)}{m} - \frac{D e \Phi (\gamma\cdot a).(\gamma\cdot k)}{2 (k\cdot p)} - \frac{D e \Phi (\gamma\cdot k).(\gamma\cdot a)}{2 (k\cdot p)} + \frac{D e \Phi (\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)}{4 m s (k\cdot p)} - \frac{e \Phi (\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)}{2 m s (k\cdot p)} + \frac{D e \Phi (\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{4 m s (k\cdot p)} - \frac{e \Phi (\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{2 m s (k\cdot p)}\right) +$$
$$\text{J2}\left(-\frac{D m \xi^2 \gamma\cdot k \phi^3}{16 s (k\cdot p)^2} + \frac{3 m \xi^2 \gamma\cdot k \phi^3}{16 s (k\cdot p)^2} + \frac{D m \xi^2 \gamma\cdot k \phi^2}{4 (k\cdot p)} - \frac{3 m \xi^2 \gamma\cdot k \phi^2}{4 (k\cdot p)} + \frac{D e \gamma\cdot a \phi}{2 m} - \frac{e \gamma\cdot a \phi}{2 m} - \frac{1}{3} D m s \xi^2 \gamma\cdot k \phi + m s \xi^2 \gamma\cdot k \phi + \frac{D e (\gamma\cdot a).(\gamma\cdot k) \phi}{4 (k\cdot p)} - \frac{3 e (\gamma\cdot a).(\gamma\cdot k) \phi}{4 (k\cdot p)} - \frac{D e (\gamma\cdot k).(\gamma\cdot a) \phi}{4 (k\cdot p)} + \frac{3 e (\gamma\cdot k).(\gamma\cdot a) \phi}{4 (k\cdot p)} - \frac{D e (\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x) \phi}{8 m s (k\cdot p)} + \frac{e (\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x) \phi}{4 m s (k\cdot p)} + \frac{e (\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a) \phi}{8 m s (k\cdot p)} + \frac{D e (\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a) \phi}{8 m s (k\cdot p)} - \frac{3 e (\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a) \phi}{8 m s (k\cdot p)} - \frac{e \gamma\cdot k (a\cdot x) \phi}{4 m s (k\cdot p)} + \frac{D m \xi^2 \Phi^2 \gamma\cdot k \phi}{4 s (k\cdot p)^2} - \frac{3 m \xi^2 \Phi^2 \gamma\cdot k \phi}{4 s (k\cdot p)^2} - D + \frac{D \gamma\cdot x}{2 m s} - \frac{3 \gamma\cdot x}{2 m s} - D e s (\gamma\cdot a).(\gamma\cdot k) + 4 e s (\gamma\cdot a).(\gamma\cdot k) + e s (\gamma\cdot k).(\gamma\cdot a) + \frac{e (\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)}{m} + \frac{3 e (\gamma\cdot k).(\gamma\cdot a).(\gamma\cdot x)}{2 m} + \frac{D e (\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{2 m} - \frac{3 e (\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{2 m} - \right.$$

$$\frac{e(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{2\,m}-\frac{e^2\,\gamma\cdot a\,(a\cdot x)}{m^3\,s\,\xi^2}-\frac{D\,e\,\gamma\cdot k\,(a\cdot x)}{2\,m}+\frac{e\,\gamma\cdot k\,(a\cdot x)}{2\,m}+\frac{D\,e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot p)}-$$

$$\frac{3\,e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot p)}+\frac{D\,e\,\Phi\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)}-\frac{3\,e\,\Phi\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)}-\frac{D\,e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)}{4\,m\,s\,(k\cdot p)}+$$

$$\frac{e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)}{2\,m\,s\,(k\cdot p)}+\frac{e\,\Phi\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)}{4\,m\,s\,(k\cdot p)}-\frac{D\,e\,\Phi\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{4\,m\,s\,(k\cdot p)}+\frac{3\,e\,\Phi\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{4\,m\,s\,(k\cdot p)}-$$

$$\frac{e\,\Phi\,\gamma\cdot k\,(a\cdot x)}{2\,m\,s\,(k\cdot p)}+1-\frac{e^2\,\gamma\cdot k\,(\,a\cdot x\,)^2}{m^3\,s\,\xi^2\,\phi}-\frac{2\,i\,t\,\gamma\cdot k}{m\,s\,\phi}+\frac{(\gamma\cdot k).(\gamma\cdot x)}{\phi}+\frac{(\gamma\cdot x).(\gamma\cdot k)}{\phi}+\frac{e^2\,(\gamma\cdot a).(\gamma\cdot k)\,(a\cdot x)}{m^2\,\xi^2\,\phi}+$$

$$\left.\frac{e^2\,(\gamma\cdot k).(\gamma\cdot a)\,(a\cdot x)}{m^2\,\xi^2\,\phi}+\frac{e^2\,(\gamma\cdot a).(\gamma\cdot x).(\gamma\cdot k)\,(a\cdot x)}{2\,m^3\,s\,\xi^2\,\phi}+\frac{e^2\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\,(a\cdot x)}{2\,m^3\,s\,\xi^2\,\phi}\right)+$$

$$\mathrm{J1}\left(-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(\,k\cdot p\,)^2}-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}-\frac{e\,\gamma\cdot a\,\phi}{2\,m}-\frac{1}{3}\,m\,s\,\xi^2\,\gamma\cdot k\,\phi+\frac{e\,(\gamma\cdot a).(\gamma\cdot k)\,\phi}{4\,(k\cdot p)}-\frac{e\,(\gamma\cdot k).(\gamma\cdot a)\,\phi}{4\,(k\cdot p)}-\right.$$

$$\frac{e\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\,\phi}{8\,m\,s\,(k\cdot p)}+\frac{e\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)\,\phi}{8\,m\,s\,(k\cdot p)}+\frac{e\,\gamma\cdot k\,(a\cdot x)\,\phi}{4\,m\,s\,(k\cdot p)}+\frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(\,k\cdot p\,)^2}+\frac{\gamma\cdot x}{2\,m\,s}-$$

$$e\,s\,(\gamma\cdot k).(\gamma\cdot a)+\frac{e\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{2\,m}+\frac{e^2\,\gamma\cdot a\,(a\cdot x)}{m^3\,s\,\xi^2}+\frac{e\,\gamma\cdot k\,(a\cdot x)}{2\,m}+\frac{e\,\Phi\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot p)}+\frac{e\,\Phi\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)}-$$

$$\frac{e\,\Phi\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)}{4\,m\,s\,(k\cdot p)}-\frac{e\,\Phi\,(\gamma\cdot x).(\gamma\cdot k).(\gamma\cdot a)}{4\,m\,s\,(k\cdot p)}+\frac{e\,\Phi\,\gamma\cdot k\,(a\cdot x)}{2\,m\,s\,(k\cdot p)}-1+\frac{e^2\,\gamma\cdot k\,(\,a\cdot x\,)^2}{m^3\,s\,\xi^2\,\phi}-\frac{2\,i\,t\,\gamma\cdot k}{m\,s\,\phi}-$$

$$\left.\frac{e^2\,(\gamma\cdot a).(\gamma\cdot k)\,(a\cdot x)}{m^2\,\xi^2\,\phi}-\frac{e^2\,(\gamma\cdot k).(\gamma\cdot a)\,(a\cdot x)}{m^2\,\xi^2\,\phi}-\frac{e^2\,(\gamma\cdot a).(\gamma\cdot x).(\gamma\cdot k)\,(a\cdot x)}{2\,m^3\,s\,\xi^2\,\phi}-\frac{e^2\,(\gamma\cdot k).(\gamma\cdot x).(\gamma\cdot a)\,(a\cdot x)}{2\,m^3\,s\,\xi^2\,\phi}\right)$$

Reordering the γ − matrix terms in the preexponent

We employ the equality

$(\gamma\ a)(\gamma\ b)(\gamma\ c) \to -i\ \gamma^{\beta}.\overline{\gamma}^{5}\ \epsilon^{\beta\,\mu\nu\delta}\ a_{\mu}\ b_{\nu}\ c_{\delta} + (a\ b)\left(\gamma\ c\right) - (a\ c)(\gamma\ b) + \left(\gamma\ a\right)(b\ c)$

to rewrite the terms with 3 gamma matrices

Then we recollect tensors $F_{\mu\nu}$,

$F^{*}_{\mu\nu}$ and $(F^{2})_{\mu\nu}$ from the combinations of a_{μ}, k_{μ} and the antisymmetric tensor $\epsilon^{\alpha\beta\mu\nu}$

The scalar products (γa) and (γk) can be expressed as

$(\gamma a) = \frac{1}{\phi}[(\gamma k)(ax) - (\gamma\ Fx)]$

$(\gamma k) = (\gamma k)(kx)/\phi = -a^{2}\ k_{\mu}\ k_{\nu}\ \gamma^{\mu}\ x^{\nu}\ \frac{1}{-a^{2}\ \phi} = \frac{e^{2}}{m^{2}\ \xi^{2}\ \phi}(\gamma F^{2}\ x)$

Then the result can be expressed as a linear

combination of following γ − matrix structures :

1,

(γx),

$(\gamma F^{2}\ x)$,

$(\sigma F) = \sigma_{\mu\nu}\ F^{\mu\nu}$,

$\gamma^{\beta}\ \gamma^{5}\ (F^{*}\ x)_{\beta}$.

Also note that we treat $\overline{\gamma}^{5}$ as a 4 − dimensional object,

assuming that the terms incorporating it are finite

In[37]:=
```
Coeff4 = Coeff3;
Phase4 = Phase3;
Matrix4 =
 Collect[(((Expand[Matrix3 /. TripleGamma]) /. EpsToF) /. FieldSubstitutions) /. akToF /.
   {DiracGamma[Momentum[k, D], D].DiracGamma[Momentum[x, D], D] / ϕ →
     -DiracGamma[Momentum[x, D], D].DiracGamma[Momentum[k, D], D] / ϕ + 2}, {J0, J1, J2}]
```

Out[39]=
$$J1\left(-\frac{m\,\xi^2\,\gamma\cdot k\phi^3}{16\,s\,(k\cdot p)^2}-\frac{m\,\xi^2\,\gamma\cdot k\phi^2}{4\,(k\cdot p)}-\frac{1}{3}\,m\,s\,\xi^2\,\gamma\cdot k\phi+\frac{i\,e\sigma\mathrm{F}\phi}{4\,(k\cdot p)}-\frac{e\Phi\,\gamma\cdot a\phi}{2\,m\,s\,(k\cdot p)}-\frac{i\,e\gamma^\beta.\overline{\gamma}^5\,\mathrm{FDx}^\beta\,\phi}{4\,m\,s\,(k\cdot p)}+\frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\phi}{4\,s\,(k\cdot p)^2}+\frac{1}{2}\,i\,e\,s\,\sigma\mathrm{F}+\frac{\gamma\cdot x}{2\,m\,s}-\frac{i\,e\gamma^\beta.\overline{\gamma}^5\,\mathrm{FDx}^\beta}{2\,m}+\frac{e\Phi\,\gamma\cdot k(a\cdot x)}{2\,m\,s\,(k\cdot p)}-1-\frac{2\,i\,t\gamma\cdot k}{m\,s\,\phi}\right)+$$
$$J0\left(\frac{D\,m\,\xi^2\,\gamma\cdot k\phi^3}{16\,s\,(k\cdot p)^2}-\frac{m\,\xi^2\,\gamma\cdot k\phi^3}{8\,s\,(k\cdot p)^2}-\frac{D\,m\,\xi^2\,\gamma\cdot k\phi^2}{4\,(k\cdot p)}+\frac{3\,m\,\xi^2\,\gamma\cdot k\phi^2}{2\,(k\cdot p)}+\frac{1}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\phi-\frac{2}{3}\,m\,s\,\xi^2\,\gamma\cdot k\phi-\frac{i\,D\,e\sigma\mathrm{F}\phi}{4\,(k\cdot p)}+\frac{D\,e\Phi\,\gamma\cdot a\phi}{2\,m\,s\,(k\cdot p)}-\frac{e\Phi\,\gamma\cdot a\phi}{m\,s\,(k\cdot p)}+\frac{i\,D\,e\gamma^\beta.\overline{\gamma}^5\,\mathrm{FDx}^\beta\,\phi}{4\,m\,s\,(k\cdot p)}-\frac{i\,e\gamma^\beta.\overline{\gamma}^5\,\mathrm{FDx}^\beta\,\phi}{2\,m\,s\,(k\cdot p)}-\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\phi}{4\,s\,(k\cdot p)^2}+\frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\phi}{2\,s\,(k\cdot p)^2}+D+\frac{1}{2}\,i\,D\,e\,s\,\sigma\mathrm{F}-2\,i\,e\,s\,\sigma\mathrm{F}-\frac{D\,\gamma\cdot x}{2\,m\,s}+\frac{\gamma\cdot x}{m\,s}-\frac{i\,D\,e\gamma^\beta.\overline{\gamma}^5\,\mathrm{FDx}^\beta}{2\,m}+\frac{3\,i\,e\gamma^\beta.\overline{\gamma}^5\,\mathrm{FDx}^\beta}{m}-\frac{D\,e\Phi\,\gamma\cdot k(a\cdot x)}{2\,m\,s\,(k\cdot p)}+\frac{e\Phi\,\gamma\cdot k(a\cdot x)}{m\,s\,(k\cdot p)}\right)+$$
$$J2\left(-\frac{D\,m\,\xi^2\,\gamma\cdot k\phi^3}{16\,s\,(k\cdot p)^2}+\frac{3\,m\,\xi^2\,\gamma\cdot k\phi^3}{16\,s\,(k\cdot p)^2}+\frac{D\,m\,\xi^2\,\gamma\cdot k\phi^2}{4\,(k\cdot p)}-\frac{3\,m\,\xi^2\,\gamma\cdot k\phi^2}{4\,(k\cdot p)}-\frac{1}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\phi+m\,s\,\xi^2\,\gamma\cdot k\phi+\frac{i\,D\,e\sigma\mathrm{F}\phi}{4\,(k\cdot p)}-\frac{3\,i\,e\sigma\mathrm{F}\phi}{4\,(k\cdot p)}-\frac{D\,e\Phi\,\gamma\cdot a\phi}{2\,m\,s\,(k\cdot p)}+\frac{3\,e\Phi\,\gamma\cdot a\phi}{2\,m\,s\,(k\cdot p)}-\frac{i\,D\,e\gamma^\beta.\overline{\gamma}^5\,\mathrm{FDx}^\beta\,\phi}{4\,m\,s\,(k\cdot p)}+\frac{3\,i\,e\gamma^\beta.\overline{\gamma}^5\,\mathrm{FDx}^\beta\,\phi}{4\,m\,s\,(k\cdot p)}+\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\phi}{4\,s\,(k\cdot p)^2}-\frac{3\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\phi}{4\,s\,(k\cdot p)^2}-D-\frac{1}{2}\,i\,D\,e\,s\,\sigma\mathrm{F}+\frac{3}{2}\,i\,e\,s\,\sigma\mathrm{F}+\frac{D\,\gamma\cdot x}{2\,m\,s}-\frac{3\,\gamma\cdot x}{2\,m\,s}+\frac{i\,D\,e\gamma^\beta.\overline{\gamma}^5\,\mathrm{FDx}^\beta}{2\,m}-\frac{3\,i\,e\gamma^\beta.\overline{\gamma}^5\,\mathrm{FDx}^\beta}{2\,m}+\frac{D\,e\Phi\,\gamma\cdot k(a\cdot x)}{2\,m\,s\,(k\cdot p)}-\frac{3\,e\Phi\,\gamma\cdot k(a\cdot x)}{2\,m\,s\,(k\cdot p)}+3-\frac{2\,i\,t\gamma\cdot k}{m\,s\,\phi}\right)$$

Substituting the lightcone variables

We introduce the following notations

(xp, pp, qp, Gp) == $(x, p, q, \gamma)_+$,
xm == $x_- = \phi / m$,
pm = p_- = kp / m,
(xt, pt, Gt, at) == $(x, p, \gamma, a)_\perp$,
pp = p_+,

We introduce the following notations for $\gamma^5\,(F^*\,x)_\beta$
GFDp == $(\gamma^\beta\,\gamma^5\,F^*{}_{\beta\mu})_+$,
GFDt == $(\gamma^\beta\,\gamma^5\,F^*{}_{\beta\mu})_\perp$,
Note that $(\gamma^\beta\,\gamma^5\,F^*{}_{\beta\mu})_- = 0$.

We also will use the conservation law

qm = pm,

qt = pt;

Scalar products will take the form

$x^2 = 2\, x_+\, x_- - x_\perp{}^2 = 2\, xp\, xm\ - xt^2$,

$(px) = p_+\, x_- + p_-\, x_+ - p_\perp\, x_\perp = pp\, xm + pm\, xp\ - pt\, xt$,

$(ax) = -a_\perp\, x_\perp = -at\, pt$,

$\gamma p = \gamma_-\, p_+ + \gamma_+\, p_- - \gamma_\perp\, p_\perp = Gm\, \frac{s}{x_-}\,(p^2 + p_\perp{}^2) + Gp\, \frac{x_-}{2\, s} - Gt\, pt$,

where we used that $p_+ = (p^2 + p_\perp{}^2) \big/ 2\, p_-$ and the definition of the proper time $s = x_- / 2\, p_-$;

$\gamma k = \gamma_-\, k_+ = m\, Gm$,

$\gamma^\beta\, \gamma^5\, (F^*\, x)_\beta = GFDp\, xm\ - GFDt\, xt$.

The integration measure

$d\!\!\!/\, x_- = d\phi / m$

$d\!\!\!/\, X_- = d\Phi / m$

$\int d\!\!\!/\, X_-\, d\!\!\!/^D x\ ... = (m)^{-2} \int d\!\!\!/\, \Phi\, d\!\!\!/^{D-2} x_\perp\, d\!\!\!/\, \phi\, d\!\!\!/\, x_+\ ...$

remaining integrals : $\int d\!\!\!/\, \Phi\, d\!\!\!/^{D-2} x_\perp\, d\!\!\!/\, \phi\, d\!\!\!/\, x_+$

In[40]:=
```
Coeff5 = Coeff4 / (m)^2
Phase5 = Collect[Phase4 /.
      {xv2 → 2 xm xp - xt^2,
       ax → -at * xt,
       Pair[Momentum[p, D], Momentum[x, D]] → pp xm + pm xp - pt * xt,
       Pair[Momentum[q, D], Momentum[x, D]] → qp xm + qm xp - qt * xt,
       Pair[Momentum[a, D], Momentum[p, D]] → -at pt} /.
    {qm → pm, qt → pt} /. {xm → ϕ / m, pm → kp / m}, {xp, ϕ, xt}]
Matrix5 = Collect[Expand[Matrix4 /. {DiracGamma[Momentum[x, D], D] → Gp xm + Gm * xp - Gt * xt,
       DiracGamma[LorentzIndex[β, D], D].DiracGamma[5] ×
         Pair[LorentzIndex[β, D], Momentum[FDx, D]] → GFDp xm - GFDt * xt,
       Pair[Momentum[a, D], Momentum[x, D]] → -at * xt} /. {xm → ϕ / m}], {J0, J1, J2, xt, xp}]
```

Out[40]= $$-\frac{2^{-D-2}\, e^{-\frac{1}{2} i \pi D}\, e^2\, s^{-D/2}\, t^{-D/2}}{\pi^2\, m}$$

Out[41]= $$\phi\left(\frac{\text{at}\ e\,\text{pt}\,\Phi}{k\cdot p}+\frac{m^2\,\xi^2\,\Phi^2}{2\,(k\cdot p)}+\frac{\text{pp}}{2\,m}+\frac{\text{qp}}{2\,m}\right)+\text{xt}\,(-\text{at}\ e\Phi-\text{pt})+\frac{m^2\,\xi^2\,\phi^3}{24\,(k\cdot p)}+$$
$$\text{xp}\left(\frac{k\cdot p}{m}+\phi\left(-\frac{1}{2\,m\,s}-\frac{1}{2\,m\,t}\right)\right)-\frac{1}{12}\,m^2\,\xi^2\,s\,\phi^2-m^2\,s-\frac{\text{pp}\,\Phi}{m}+\frac{\text{qp}\,\Phi}{m}+\text{xt}^2\left(\frac{1}{4\,s}+\frac{1}{4\,t}\right)$$

Out[42]= $$\text{J1}\left(-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}-\frac{i\,e\,\text{GFDp}\,\phi^2}{4\,m^2\,s\,(k\cdot p)}-\frac{1}{3}\,m\,s\,\xi^2\,\gamma\cdot k\,\phi+\right.$$
$$\frac{\text{Gp}\,\phi}{2\,m^2\,s}+\frac{i\,e\,\sigma\text{F}\phi}{4\,(k\cdot p)}-\frac{e\Phi\,\gamma\cdot a\,\phi}{2\,m\,s\,(k\cdot p)}-\frac{i\,e\,\text{GFDp}\,\phi}{2\,m^2}+\frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}+\frac{\text{Gm xp}}{2\,m\,s}+$$
$$\left.\frac{1}{2}\,i\,e\,s\,\sigma\text{F}+\text{xt}\left(\frac{i\,e\,\text{GFDt}}{2\,m}+\frac{i\,e\,\phi\,\text{GFDt}}{4\,m\,s\,(k\cdot p)}-\frac{\text{Gt}}{2\,m\,s}-\frac{\text{at}\ e\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\right)-1-\frac{2\,i\,t\,\gamma\cdot k}{m\,s\,\phi}\right)+$$
$$\text{J0}\left(\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^3}{8\,s\,(k\cdot p)^2}-\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}+\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{2\,(k\cdot p)}+\frac{i\,D\,e\,\text{GFDp}\,\phi^2}{4\,m^2\,s\,(k\cdot p)}-\right.$$
$$\frac{i\,e\,\text{GFDp}\,\phi^2}{2\,m^2\,s\,(k\cdot p)}+\frac{1}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,\phi-\frac{2}{3}\,m\,s\,\xi^2\,\gamma\cdot k\,\phi-\frac{D\,\text{Gp}\,\phi}{2\,m^2\,s}+\frac{\text{Gp}\,\phi}{m^2\,s}-\frac{i\,D\,e\,\sigma\text{F}\phi}{4\,(k\cdot p)}+$$
$$\frac{D\,e\Phi\,\gamma\cdot a\,\phi}{2\,m\,s\,(k\cdot p)}-\frac{e\Phi\,\gamma\cdot a\,\phi}{m\,s\,(k\cdot p)}-\frac{i\,D\,e\,\text{GFDp}\,\phi}{2\,m^2}+\frac{3\,i\,e\,\text{GFDp}\,\phi}{m^2}-\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}+$$
$$\frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{2\,s\,(k\cdot p)^2}+D+\left(\frac{\text{Gm}}{m\,s}-\frac{D\,\text{Gm}}{2\,m\,s}\right)\text{xp}+\frac{1}{2}\,i\,D\,e\,s\,\sigma\text{F}-2\,i\,e\,s\,\sigma\text{F}+$$
$$\left.\text{xt}\left(\frac{i\,D\,e\,\text{GFDt}}{2\,m}-\frac{3\,i\,e\,\text{GFDt}}{m}-\frac{i\,D\,e\,\phi\,\text{GFDt}}{4\,m\,s\,(k\cdot p)}+\frac{i\,e\,\phi\,\text{GFDt}}{2\,m\,s\,(k\cdot p)}+\frac{D\,\text{Gt}}{2\,m\,s}-\frac{\text{Gt}}{m\,s}-\frac{\text{at}\ e\Phi\,\gamma\cdot k}{m\,s\,(k\cdot p)}+\frac{\text{at}\ D\,e\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\right)\right)+$$
$$\text{J2}\left(-\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}+\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}+\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}-\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}-\frac{i\,D\,e\,\text{GFDp}\,\phi^2}{4\,m^2\,s\,(k\cdot p)}+\right.$$
$$\frac{3\,i\,e\,\text{GFDp}\,\phi^2}{4\,m^2\,s\,(k\cdot p)}-\frac{1}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,\phi+m\,s\,\xi^2\,\gamma\cdot k\,\phi+\frac{D\,\text{Gp}\,\phi}{2\,m^2\,s}-\frac{3\,\text{Gp}\,\phi}{2\,m^2\,s}+\frac{i\,D\,e\,\sigma\text{F}\phi}{4\,(k\cdot p)}-\frac{3\,i\,e\,\sigma\text{F}\phi}{4\,(k\cdot p)}-$$
$$\frac{D\,e\Phi\,\gamma\cdot a\,\phi}{2\,m\,s\,(k\cdot p)}+\frac{3\,e\Phi\,\gamma\cdot a\,\phi}{2\,m\,s\,(k\cdot p)}+\frac{i\,D\,e\,\text{GFDp}\,\phi}{2\,m^2}-\frac{3\,i\,e\,\text{GFDp}\,\phi}{2\,m^2}+\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}-\frac{3\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}-$$
$$D+\left(\frac{D\,\text{Gm}}{2\,m\,s}-\frac{3\,\text{Gm}}{2\,m\,s}\right)\text{xp}-\frac{1}{2}\,i\,D\,e\,s\,\sigma\text{F}+\frac{3}{2}\,i\,e\,s\,\sigma\text{F}+\text{xt}\left(-\frac{i\,D\,e\,\text{GFDt}}{2\,m}+\frac{3\,i\,e\,\text{GFDt}}{2\,m}+\right.$$
$$\left.\left.\frac{i\,D\,e\,\phi\,\text{GFDt}}{4\,m\,s\,(k\cdot p)}-\frac{3\,i\,e\,\phi\,\text{GFDt}}{4\,m\,s\,(k\cdot p)}-\frac{D\,\text{Gt}}{2\,m\,s}+\frac{3\,\text{Gt}}{2\,m\,s}+\frac{3\,\text{at}\ e\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}-\frac{\text{at}\ D\,e\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\right)+3-\frac{2\,i\,t\,\gamma\cdot k}{m\,s\,\phi}\right)$$

Integration over
$\int d\!\!\!/ x_+$ and then $\int d\!\!\!/ \phi$ ...

The phase is linear in x_+,
the tems in the preexponent either do not depend or linear in x_+,
therefore we can perform the integration using the equality

$$\int dx_+ \begin{pmatrix} 1 \\ x_+ \end{pmatrix} \mathrm{Exp}[i\, x_+\, P(\phi)] = 2\pi \begin{pmatrix} \delta(P(\phi)) \\ -i\, \delta'(P(\phi)) \end{pmatrix}$$

To use it, we separate the terms linear in xp == x_+.

Recall that Jk depends in ϕ, as
$Jk == Jk\left[t,\ \chi_l = \frac{\xi\,\phi}{2\,m^2\,t}\right]$,
In the next steps we will use the shorthand notation
$Jk = Jk[\phi]$.

```
In[43]:= Matrix52 = Collect[
           Coefficient[Expand[Matrix5], Gm xp] Gm xp /. {J0 → J0[ϕ], J1 → J1[ϕ], J2 → J2[ϕ]},
           {Gm xp, J0[ϕ], J1[ϕ], J2[ϕ]}]
         Matrix51 = Collect[
           (Expand[Matrix5] /. {J0 → J0[ϕ], J1 → J1[ϕ], J2 → J2[ϕ]}) - Expand[Matrix52],
           {J0[ϕ], J1[ϕ], J2[ϕ], xt, xp}]
```

Out[43]= $\mathrm{Gm\ xp}\left(\mathrm{J0}(\phi)\left(\frac{1}{m\,s} - \frac{D}{2\,m\,s}\right) + \mathrm{J2}(\phi)\left(\frac{D}{2\,m\,s} - \frac{3}{2\,m\,s}\right) + \frac{\mathrm{J1}(\phi)}{2\,m\,s}\right)$

Out[44]=

$$\mathrm{J1}(\phi)\left(-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}-\frac{i\,e\,\mathrm{GFDp}\,\phi^2}{4\,m^2\,s\,(k\cdot p)}-\frac{1}{3}\,m\,s\,\xi^2\,\gamma\cdot k\,\phi+\frac{\mathrm{Gp}\,\phi}{2\,m^2\,s}+\frac{i\,e\,\sigma\,\mathrm{F}\,\phi}{4\,(k\cdot p)}-\frac{e\,\Phi\,\gamma\cdot a\,\phi}{2\,m\,s\,(k\cdot p)}-\frac{i\,e\,\mathrm{GFDp}\,\phi}{2\,m^2}+\frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}+\frac{1}{2}\,i\,e\,s\,\sigma\,\mathrm{F}+\mathrm{xt}\left(\frac{i\,e\,\mathrm{GFDt}}{2\,m}+\frac{i\,e\,\phi\,\mathrm{GFDt}}{4\,m\,s\,(k\cdot p)}-\frac{\mathrm{Gt}}{2\,m\,s}-\frac{\mathrm{at}\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\right)-1-\frac{2\,i\,t\,\gamma\cdot k}{m\,s\,\phi}\right)+$$

$$\mathrm{J0}(\phi)\left(\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}-\frac{m\,\xi^2\,\gamma\cdot k\,\phi^3}{8\,s\,(k\cdot p)^2}-\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}+\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{2\,(k\cdot p)}+\frac{i\,D\,e\,\mathrm{GFDp}\,\phi^2}{4\,m^2\,s\,(k\cdot p)}-\frac{i\,e\,\mathrm{GFDp}\,\phi^2}{2\,m^2\,s\,(k\cdot p)}+\frac{1}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,\phi-\frac{2}{3}\,m\,s\,\xi^2\,\gamma\cdot k\,\phi-\frac{D\,\mathrm{Gp}\,\phi}{2\,m^2\,s}+\frac{\mathrm{Gp}\,\phi}{m^2\,s}-\frac{i\,D\,e\,\sigma\,\mathrm{F}\,\phi}{4\,(k\cdot p)}+\frac{D\,e\,\Phi\,\gamma\cdot a\,\phi}{2\,m\,s\,(k\cdot p)}-\frac{e\,\Phi\,\gamma\cdot a\,\phi}{m\,s\,(k\cdot p)}-\frac{i\,D\,e\,\mathrm{GFDp}\,\phi}{2\,m^2}+\frac{3\,i\,e\,\mathrm{GFDp}\,\phi}{m^2}-\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}+\frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{2\,s\,(k\cdot p)^2}+D+\frac{1}{2}\,i\,D\,e\,s\,\sigma\,\mathrm{F}-2\,i\,e\,s\,\sigma\,\mathrm{F}+\mathrm{xt}\left(\frac{i\,D\,e\,\mathrm{GFDt}}{2\,m}-\frac{3\,i\,e\,\mathrm{GFDt}}{m}-\frac{i\,D\,e\,\phi\,\mathrm{GFDt}}{4\,m\,s\,(k\cdot p)}+\frac{i\,e\,\phi\,\mathrm{GFDt}}{2\,m\,s\,(k\cdot p)}+\frac{D\,\mathrm{Gt}}{2\,m\,s}-\frac{\mathrm{Gt}}{m\,s}-\frac{\mathrm{at}\,e\,\Phi\,\gamma\cdot k}{m\,s\,(k\cdot p)}+\frac{\mathrm{at}\,D\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\right)\right)+$$

$$\mathrm{J2}(\phi)\left(-\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}+\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^3}{16\,s\,(k\cdot p)^2}+\frac{D\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}-\frac{3\,m\,\xi^2\,\gamma\cdot k\,\phi^2}{4\,(k\cdot p)}-\frac{i\,D\,e\,\mathrm{GFDp}\,\phi^2}{4\,m^2\,s\,(k\cdot p)}+\frac{3\,i\,e\,\mathrm{GFDp}\,\phi^2}{4\,m^2\,s\,(k\cdot p)}-\frac{1}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,\phi+m\,s\,\xi^2\,\gamma\cdot k\,\phi+\frac{D\,\mathrm{Gp}\,\phi}{2\,m^2\,s}-\frac{3\,\mathrm{Gp}\,\phi}{2\,m^2\,s}+\frac{i\,D\,e\,\sigma\,\mathrm{F}\,\phi}{4\,(k\cdot p)}-\frac{3\,i\,e\,\sigma\,\mathrm{F}\,\phi}{4\,(k\cdot p)}-\frac{D\,e\,\Phi\,\gamma\cdot a\,\phi}{2\,m\,s\,(k\cdot p)}+\frac{3\,e\,\Phi\,\gamma\cdot a\,\phi}{2\,m\,s\,(k\cdot p)}+\frac{i\,D\,e\,\mathrm{GFDp}\,\phi}{2\,m^2}-\frac{3\,i\,e\,\mathrm{GFDp}\,\phi}{2\,m^2}+\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}-\frac{3\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\phi}{4\,s\,(k\cdot p)^2}-D-\frac{1}{2}\,i\,D\,e\,s\,\sigma\,\mathrm{F}+\frac{3}{2}\,i\,e\,s\,\sigma\,\mathrm{F}+\mathrm{xt}\left(-\frac{i\,D\,e\,\mathrm{GFDt}}{2\,m}+\frac{3\,i\,e\,\mathrm{GFDt}}{2\,m}+\frac{i\,D\,e\,\phi\,\mathrm{GFDt}}{4\,m\,s\,(k\cdot p)}-\frac{3\,i\,e\,\phi\,\mathrm{GFDt}}{4\,m\,s\,(k\cdot p)}-\frac{D\,\mathrm{Gt}}{2\,m\,s}+\frac{3\,\mathrm{Gt}}{2\,m\,s}+\frac{3\,\mathrm{at}\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}-\frac{\mathrm{at}\,D\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\right)+3-\frac{2\,i\,t\,\gamma\cdot k}{m\,s\,\phi}\right)$$

In[45]:=
```
Phase5xp = Collect[Simplify[Coefficient[Phase5, xp] /. {t → s ω / (s - ω)}], {kp}, Simplify]
ϕ0 = Collect[ϕ /. Solve[Phase5xp == 0, ϕ][[1]], {kp}, Simplify]
dP = Simplify[D[Phase5xp, ϕ]] /. {ϕ → ϕ0}
AbsdP = -dP
Phase5noxp = Collect[Phase5 /. {xp → 0} /. {t → s ω / (s - ω)}, {xp, ϕ, xt}, Simplify]
```

Out[45]= $\frac{k\cdot p}{m}-\frac{\phi}{2\,m\,\omega}$

Out[46]= $2\,\omega\,(k\cdot p)$

Out[47]= $-\frac{1}{2\,m\,\omega}$

Out[48]= $\frac{1}{2\,m\,\omega}$

Out[49]= $$\frac{1}{2}\phi\left(\frac{\Phi\left(2\,\mathrm{at}\,e\,\mathrm{pt}+m^2\,\xi^2\,\Phi\right)}{k\cdot p}+\frac{\mathrm{pp}+\mathrm{qp}}{m}\right)+\mathrm{xt}\,(-\mathrm{at}\,e\,\Phi-\mathrm{pt})+\frac{m^2\,\xi^2\,\phi^3}{24\,(k\cdot p)}-\frac{m^3\,s+\mathrm{pp}\,\Phi-\mathrm{qp}\,\Phi}{m}-\frac{1}{12}m^2\,\xi^2\,s\,\phi^2+\frac{\mathrm{xt}^2}{4\,\omega}$$

Now we can perform the integrations

$$\int dx_+ \begin{pmatrix} 1 \\ x_+ \end{pmatrix} \mathrm{Exp}[i\, x_+ P(\phi)] =$$

$$2\pi \begin{pmatrix} \delta(P(\phi)) \\ -i\delta'(P(\phi)) \end{pmatrix} = 2\pi \begin{pmatrix} \delta(\phi-\phi_0)\frac{1}{|P'(\phi_0)|} \\ -i\,\delta(\phi-\phi_0)\frac{1}{|P'(\phi_0)|}\left(-\frac{d}{d\phi}\frac{1}{P'(\phi)}.\right) \end{pmatrix}$$

Here $\left(-\frac{d}{d\phi}\frac{1}{P'(\phi)}.\right)$ is an operator,

acting on a function in the place of ".". Then,

$$\int d\phi\, 2\pi \begin{pmatrix} \delta(\phi-\phi_0)\frac{1}{|P'(\phi_0)|} \\ -i\,\delta(\phi-\phi_0)\frac{1}{|P'(\phi_0)|}\left(-\frac{d}{d\phi}\frac{1}{P'(\phi)}.\right) \end{pmatrix} f(\phi)\, e^{i\, g(\phi)} =$$

$$\frac{2\pi}{|P'(\phi_0)|} e^{i\, g(\phi_0)} \begin{pmatrix} f(\phi_0) \\ i\frac{d}{d\phi}\left(f(\phi)\frac{1}{P'(\phi)}\right)\Big|_{\phi_0} - f(\phi_0)\frac{g'(\phi_0)}{P'(\phi_0)} \end{pmatrix}$$

$P(\phi) = \frac{(k\ p)}{m} - \frac{\phi}{2\,m\,\omega}$,

$P'(\phi) = -\frac{1}{2\,m\,\omega}$,

$\omega^{-1} = s^{-1} + t^{-1}$, $\omega = s\,t/(s+t)$

Note that we will take the derivative of Jk[ϕ] in ϕ too;

We introduce the shorthand

DJk == $\frac{d}{d\phi}$ Jk[ϕ] == $\frac{d}{d\phi} J_k(t, \chi_l(\phi))$

The remaining integrals : $\int d\Phi\, d^{D-2} x_\perp$

In[50]:=
```
Coeff6 = Coeff5 * 2 π / AbsdP
Phase6 =
 Collect[Expand[Simplify[Phase5 /. {xp → 0} /. {ϕ → ϕ0} /. {t → s ω / (s - ω)}]], {xt, Φ}]
```

Out[50]= $-\frac{2^{-D} e^{-\frac{1}{2} i \pi D} e^2\, \omega\, s^{-D/2}\, t^{-D/2}}{\pi}$

Out[51]=
$$\frac{1}{3} m^2 \xi^2 \omega^3 (k\cdot p)^2 - \frac{1}{3} m^2 \xi^2 s\,\omega^2 (k\cdot p)^2 + \Phi\left(2\,\text{at}\,e\,\text{pt}\,\omega - \frac{\text{pp}}{m} + \frac{\text{qp}}{m}\right) + \text{xt}\,(-\text{at}\,e\Phi - \text{pt}) + \frac{\text{pp}\,\omega\,(k\cdot p)}{m} + \frac{\text{qp}\,\omega\,(k\cdot p)}{m} + m^2 \xi^2 \Phi^2 \omega - m^2 s + \frac{\text{xt}^2}{4\,\omega}$$

Note that Matrix52 produces terms that are proportinal to Jk and $\frac{\text{dJk}}{\text{s}\phi}$.

We combine the ones that are proportional to Jk together in Matrix61, and leave the rest in Matrix62

We also again denote
Jk == Jk$\left[\text{t}, \frac{\xi\,\phi_0}{2\,\text{m}^2\,\text{t}}\right]$

In[52]:=
```
Matrix61 = Collect[Expand[Matrix51 - (Matrix52 /. {xp → D[Phase5noxp, ϕ] / dP})] /.
    {J0[ϕ] → J0, J1[ϕ] → J1, J2[ϕ] → J2} /. {ϕ → ϕ0}, {J0, J1, J2, xt, Gm, Gp, GFDp, pp}]
Matrix62 = I D[1 / dP * Matrix52, ϕ] /. {xp → 1} /. {ϕ → ϕ0}
```

Out[52]=
$$\text{J1}\left(-\frac{m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{2\,s} - m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2 + \frac{1}{2} i\,e\,\sigma\text{F}\omega - \frac{e\Phi\,\gamma\cdot a\,\omega}{m\,s} - \frac{2}{3} m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega + \frac{\text{Gp}\,(k\cdot p)\,\omega}{m^2\,s} + \frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\omega}{2\,s\,(k\cdot p)} + \frac{1}{2} i\,e\,s\,\sigma\text{F} + \text{xt}\left(\frac{i\,e\,\omega\,\text{GFDt}}{2\,m\,s} + \frac{i\,e\,\text{GFDt}}{2\,m} - \frac{\text{Gt}}{2\,m\,s} - \frac{\text{at}\,e\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\right) + \text{GFDp}\left(-\frac{i\,e\,(k\cdot p)\,\omega^2}{m^2\,s} - \frac{i\,e\,(k\cdot p)\,\omega}{m^2}\right) + \text{Gm}\left(\frac{m^2\,\xi^2\,(k\cdot p)\,\omega^3}{2\,s} - \frac{1}{3} m^2\,\xi^2\,(k\cdot p)\,\omega^2 + \frac{\text{pp}\,\omega}{2\,m\,s} + \frac{\text{qp}\,\omega}{2\,m\,s} + \frac{m^2\,\xi^2\,\Phi^2\,\omega}{2\,s\,(k\cdot p)} + \frac{\text{at}\,e\,\text{pt}\,\Phi\,\omega}{s\,(k\cdot p)}\right) - 1 - \frac{i\,t\,\gamma\cdot k}{m\,s\,(k\cdot p)\,\omega}\right) +$$
$$\text{J0}\left(\frac{D\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{2\,s} - \frac{m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{s} - D\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2 + 6\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2 - \frac{1}{2} i\,D\,e\,\sigma\text{F}\omega + \frac{D\,e\Phi\,\gamma\cdot a\,\omega}{m\,s} - \frac{2\,e\Phi\,\gamma\cdot a\,\omega}{m\,s} + \frac{2}{3} D\,m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega - \frac{4}{3} m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega - \frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\omega}{2\,s\,(k\cdot p)} + \frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\omega}{s\,(k\cdot p)} + D + \frac{1}{2} i\,D\,e\,s\,\sigma\text{F} - 2\,i\,e\,s\,\sigma\text{F} + \text{xt}\left(-\frac{i\,D\,e\,\omega\,\text{GFDt}}{2\,m\,s} + \frac{i\,e\,\omega\,\text{GFDt}}{m\,s} + \frac{i\,D\,e\,\text{GFDt}}{2\,m} - \frac{3\,i\,e\,\text{GFDt}}{m} + \frac{D\,\text{Gt}}{2\,m\,s} - \frac{\text{Gt}}{m\,s} - \frac{\text{at}\,e\Phi\,\gamma\cdot k}{m\,s\,(k\cdot p)} + \frac{\text{at}\,D\,e\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\right) + \text{Gp}\left(\frac{2\,\omega\,(k\cdot p)}{m^2\,s} - \frac{D\,\omega\,(k\cdot p)}{m^2\,s}\right) + \text{GFDp}\left(\frac{i\,D\,e\,(k\cdot p)\,\omega^2}{m^2\,s} - \frac{2\,i\,e\,(k\cdot p)\,\omega^2}{m^2\,s} - \frac{i\,D\,e\,(k\cdot p)\,\omega}{m^2} + \frac{6\,i\,e\,(k\cdot p)\,\omega}{m^2}\right) + \right.$$

$$\text{Gm}\left(-\frac{D\,m^2\,\xi^2\,(k\cdot p)\,\omega^3}{2\,s}+\frac{m^2\,\xi^2\,(k\cdot p)\,\omega^3}{s}+\frac{1}{3}\,D\,m^2\,\xi^2\,(k\cdot p)\,\omega^2-\frac{2}{3}\,m^2\,\xi^2\,(k\cdot p)\,\omega^2-\frac{D\,\text{qp}\,\omega}{2\,m\,s}+\right.$$
$$\left.\left.\frac{\text{qp}\,\omega}{m\,s}-\frac{D\,m^2\,\xi^2\,\Phi^2\,\omega}{2\,s\,(k\cdot p)}+\frac{m^2\,\xi^2\,\Phi^2\,\omega}{s\,(k\cdot p)}+\frac{2\,\text{at}\,e\,\text{pt}\,\Phi\,\omega}{s\,(k\cdot p)}-\frac{\text{at}\,D\,e\,\text{pt}\,\Phi\,\omega}{s\,(k\cdot p)}+\text{pp}\left(\frac{\omega}{m\,s}-\frac{D\,\omega}{2\,m\,s}\right)\right)\right)+$$
$$\text{J2}\left(-\frac{D\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{2\,s}+\frac{3\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{2\,s}+D\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2-3\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2+\right.$$
$$\frac{1}{2}\,i\,D\,e\,\sigma\,\text{F}\,\omega-\frac{3}{2}\,i\,e\,\sigma\,\text{F}\,\omega-\frac{D\,e\,\Phi\,\gamma\cdot a\,\omega}{m\,s}+\frac{3\,e\,\Phi\,\gamma\cdot a\,\omega}{m\,s}-\frac{2}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega+$$
$$2\,m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega+\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\omega}{2\,s\,(k\cdot p)}-\frac{3\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\omega}{2\,s\,(k\cdot p)}-D-\frac{1}{2}\,i\,D\,e\,s\,\sigma\,\text{F}+$$
$$\frac{3}{2}\,i\,e\,s\,\sigma\,\text{F}+\text{xt}\left(\frac{i\,D\,e\,\omega\,\text{GFDt}}{2\,m\,s}-\frac{3\,i\,e\,\omega\,\text{GFDt}}{2\,m\,s}-\frac{i\,D\,e\,\text{GFDt}}{2\,m}+\frac{3\,i\,e\,\text{GFDt}}{2\,m}-\right.$$
$$\left.\frac{D\,\text{Gt}}{2\,m\,s}+\frac{3\,\text{Gt}}{2\,m\,s}+\frac{3\,\text{at}\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}-\frac{\text{at}\,D\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\right)+\text{Gp}\left(\frac{D\,\omega\,(k\cdot p)}{m^2\,s}-\frac{3\,\omega\,(k\cdot p)}{m^2\,s}\right)+$$
$$\text{GFDp}\left(-\frac{i\,D\,e\,(k\cdot p)\,\omega^2}{m^2\,s}+\frac{3\,i\,e\,(k\cdot p)\,\omega^2}{m^2\,s}+\frac{i\,D\,e\,(k\cdot p)\,\omega}{m^2}-\frac{3\,i\,e\,(k\cdot p)\,\omega}{m^2}\right)+\text{Gm}\left(\frac{D\,m^2\,\xi^2\,(k\cdot p)\,\omega^3}{2\,s}-\right.$$
$$\frac{3\,m^2\,\xi^2\,(k\cdot p)\,\omega^3}{2\,s}-\frac{1}{3}\,D\,m^2\,\xi^2\,(k\cdot p)\,\omega^2+m^2\,\xi^2\,(k\cdot p)\,\omega^2+\frac{D\,\text{qp}\,\omega}{2\,m\,s}-\frac{3\,\text{qp}\,\omega}{2\,m\,s}+\frac{D\,m^2\,\xi^2\,\Phi^2\,\omega}{2\,s\,(k\cdot p)}-$$
$$\left.\left.\frac{3\,m^2\,\xi^2\,\Phi^2\,\omega}{2\,s\,(k\cdot p)}-\frac{3\,\text{at}\,e\,\text{pt}\,\Phi\,\omega}{s\,(k\cdot p)}+\frac{\text{at}\,D\,e\,\text{pt}\,\Phi\,\omega}{s\,(k\cdot p)}+\text{pp}\left(\frac{D\,\omega}{2\,m\,s}-\frac{3\,\omega}{2\,m\,s}\right)\right)+3-\frac{i\,t\,\gamma\cdot k}{m\,s\,(k\cdot p)\,\omega}\right)$$

Out[53]= $$-2\,i\,\text{Gm}\,m\,\omega\left(\left(\frac{1}{m\,s}-\frac{D}{2\,m\,s}\right)\text{J0}'(2\,\omega\,(k\cdot p))+\left(\frac{D}{2\,m\,s}-\frac{3}{2\,m\,s}\right)\text{J2}'(2\,\omega\,(k\cdot p))+\frac{\text{J1}'(2\,\omega\,(k\cdot p))}{2\,m\,s}\right)$$

Let us rewrite Matrix62

After the last integration

$Jk = Jk\left(t, \frac{\xi\,\phi_0}{2\,m^2\,t}\right);$

$Jk'(\phi_0) = \frac{\partial Jk\left(t, \frac{\xi\,\phi}{2\,m^2\,t}\right)}{\partial\phi}\Big|_{\phi_0} == \frac{\partial}{\partial\chi_l} Jk(t, \chi_l(\phi_0)) * \frac{\xi}{2\,m^2\,t};$

$\chi_l(\phi_0) = \frac{\xi\,\phi_0}{2\,m^2\,t} = \frac{\xi\,\omega\,(k\,p)}{m^2\,t};$

In this step,

we use the initial assumption that J_0 does not depend on χ_l, therefore $\frac{\partial}{\partial\chi_l} J0 = 0.$

We denote

$dJkd\chi l == \frac{\partial}{\partial\chi_l} Jk(t, \chi_l(\phi_0)),\ k = 1, 2$

In[54]:=
```
Coeff7 = Coeff6;
Phase7 = Phase6;
Matrix71 = Matrix61;
```

In[57]:=
```
χlϕ = ξ ϕ / 2 / m^2 / t
χlϕ0= χlϕ /. {ϕ → ϕ0}
dχldϕ = D[χlϕ, ϕ] /. {ϕ → ϕ0}
Matrix72d =
 Collect[Matrix62 /. { J0 '[2 ω kp] →  dJ0dχl * dχldϕ, J1 '[2 ω kp] →  dJ1dχl * dχldϕ,
     J2 '[2 ω kp] →  dJ2dχl * dχldϕ} /. {dJ0dχl → 0}, {dJ0dχl, dJ1dχl, dJ2dχl}]
```

Out[57]= $\frac{\xi\,\phi}{2\,m^2\,t}$

Out[58]= $\frac{\xi\,\omega\,(k\cdot p)}{m^2\,t}$

Out[59]= $\frac{\xi}{2\,m^2\,t}$

Out[60]= $-\frac{i\,\text{dJ2d}\chi\text{l}\,\text{Gm}\,\xi\,\omega\left(\frac{D}{2\,m\,s} - \frac{3}{2\,m\,s}\right)}{m\,t} - \frac{i\,\text{dJ1d}\chi\text{l}\,\text{Gm}\,\xi\,\omega}{2\,m^2\,s\,t}$

In[61]:=
```
Matrix7 =
  Collect[Matrix71 + Matrix72d, {J0, J1, J2, dJ0dχl, dJ1dχl, dJ2dχl, xt, Gm, Gp, GFDp, pp}]
```

Out[61]=

$$
\begin{aligned}
&-\frac{i\,\mathrm{dJ2d}\chi\mathrm{l}\,\mathrm{Gm}\left(\frac{D}{2\,m\,s}-\frac{3}{2\,m\,s}\right)\xi\,\omega}{m\,t}-\frac{i\,\mathrm{dJ1d}\chi\mathrm{l}\,\mathrm{Gm}\,\xi\,\omega}{2\,m^2\,s\,t}+\\
&\mathrm{J1}\Bigg(-\frac{m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{2\,s}-m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2+\frac{1}{2}\,i\,e\,\sigma\,\mathrm{F}\omega-\frac{e\,\Phi\,\gamma\cdot a\,\omega}{m\,s}-\\
&\quad\frac{2}{3}\,m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega+\frac{\mathrm{Gp}\,(k\cdot p)\,\omega}{m^2\,s}+\frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\omega}{2\,s\,(k\cdot p)}+\frac{1}{2}\,i\,e\,s\,\sigma\,\mathrm{F}+\\
&\quad\mathrm{xt}\left(\frac{i\,e\,\omega\,\mathrm{GFDt}}{2\,m\,s}+\frac{i\,e\,\mathrm{GFDt}}{2\,m}-\frac{\mathrm{Gt}}{2\,m\,s}-\frac{\mathrm{at}\;e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\right)+\mathrm{GFDp}\left(-\frac{i\,e\,(k\cdot p)\,\omega^2}{m^2\,s}-\frac{i\,e\,(k\cdot p)\,\omega}{m^2}\right)+\\
&\quad\mathrm{Gm}\left(\frac{m^2\,\xi^2\,(k\cdot p)\,\omega^3}{2\,s}-\frac{1}{3}\,m^2\,\xi^2\,(k\cdot p)\,\omega^2+\frac{\mathrm{pp}\;\omega}{2\,m\,s}+\frac{\mathrm{qp}\;\omega}{2\,m\,s}+\frac{m^2\,\xi^2\,\Phi^2\,\omega}{2\,s\,(k\cdot p)}+\frac{\mathrm{at}\;e\,\mathrm{pt}\;\Phi\,\omega}{s\,(k\cdot p)}\right)-\\
&\quad 1-\frac{i\,t\,\gamma\cdot k}{m\,s\,(k\cdot p)\,\omega}\Bigg)+\\
&\mathrm{J0}\Bigg(\frac{D\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{2\,s}-\frac{m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{s}-D\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2+6\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2-\\
&\quad\frac{1}{2}\,i\,D\,e\,\sigma\,\mathrm{F}\omega+\frac{D\,e\,\Phi\,\gamma\cdot a\,\omega}{m\,s}-\frac{2\,e\,\Phi\,\gamma\cdot a\,\omega}{m\,s}+\frac{2}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega-\\
&\quad\frac{4}{3}\,m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega-\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\omega}{2\,s\,(k\cdot p)}+\frac{m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\omega}{s\,(k\cdot p)}+D+\frac{1}{2}\,i\,D\,e\,s\,\sigma\,\mathrm{F}-2\,i\,e\,s\,\sigma\,\mathrm{F}+\\
&\quad\mathrm{xt}\left(-\frac{i\,D\,e\,\omega\,\mathrm{GFDt}}{2\,m\,s}+\frac{i\,e\,\omega\,\mathrm{GFDt}}{m\,s}+\frac{i\,D\,e\,\mathrm{GFDt}}{2\,m}-\frac{3\,i\,e\,\mathrm{GFDt}}{m}+\frac{D\,\mathrm{Gt}}{2\,m\,s}-\frac{\mathrm{Gt}}{m\,s}-\frac{\mathrm{at}\;e\,\Phi\,\gamma\cdot k}{m\,s\,(k\cdot p)}+\frac{\mathrm{at}\;D\,e\,\Phi\,\gamma\cdot k}{2\,m\,s\,(k\cdot p)}\right)+\\
&\quad\mathrm{Gp}\left(\frac{2\,\omega\,(k\cdot p)}{m^2\,s}-\frac{D\,\omega\,(k\cdot p)}{m^2\,s}\right)+\mathrm{GFDp}\left(\frac{i\,D\,e\,(k\cdot p)\,\omega^2}{m^2\,s}-\frac{2\,i\,e\,(k\cdot p)\,\omega^2}{m^2\,s}-\frac{i\,D\,e\,(k\cdot p)\,\omega}{m^2}+\frac{6\,i\,e\,(k\cdot p)\,\omega}{m^2}\right)+\\
&\quad\mathrm{Gm}\Bigg(-\frac{D\,m^2\,\xi^2\,(k\cdot p)\,\omega^3}{2\,s}+\frac{m^2\,\xi^2\,(k\cdot p)\,\omega^3}{s}+\frac{1}{3}\,D\,m^2\,\xi^2\,(k\cdot p)\,\omega^2-\frac{2}{3}\,m^2\,\xi^2\,(k\cdot p)\,\omega^2-\frac{D\,\mathrm{qp}\;\omega}{2\,m\,s}+\\
&\qquad\frac{\mathrm{qp}\;\omega}{m\,s}-\frac{D\,m^2\,\xi^2\,\Phi^2\,\omega}{2\,s\,(k\cdot p)}+\frac{m^2\,\xi^2\,\Phi^2\,\omega}{s\,(k\cdot p)}+\frac{2\,\mathrm{at}\;e\,\mathrm{pt}\;\Phi\,\omega}{s\,(k\cdot p)}-\frac{\mathrm{at}\;D\,e\,\mathrm{pt}\;\Phi\,\omega}{s\,(k\cdot p)}+\mathrm{pp}\left(\frac{\omega}{m\,s}-\frac{D\,\omega}{2\,m\,s}\right)\Bigg)\Bigg)+\\
&\mathrm{J2}\Bigg(-\frac{D\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{2\,s}+\frac{3\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^3}{2\,s}+D\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2-3\,m\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega^2+\\
&\quad\frac{1}{2}\,i\,D\,e\,\sigma\,\mathrm{F}\omega-\frac{3}{2}\,i\,e\,\sigma\,\mathrm{F}\omega-\frac{D\,e\,\Phi\,\gamma\cdot a\,\omega}{m\,s}+\frac{3\,e\,\Phi\,\gamma\cdot a\,\omega}{m\,s}-\frac{2}{3}\,D\,m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega+\\
&\quad 2\,m\,s\,\xi^2\,\gamma\cdot k\,(k\cdot p)\,\omega+\frac{D\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\omega}{2\,s\,(k\cdot p)}-\frac{3\,m\,\xi^2\,\Phi^2\,\gamma\cdot k\,\omega}{2\,s\,(k\cdot p)}-D-\frac{1}{2}\,i\,D\,e\,s\,\sigma\,\mathrm{F}+
\end{aligned}
$$

$$\frac{3}{2} i\, e\, s\, \sigma\, \mathrm{F} + \mathrm{xt}\left(\frac{i\, D\, e\, \omega\, \mathrm{GFDt}}{2\, m\, s} - \frac{3\, i\, e\, \omega\, \mathrm{GFDt}}{2\, m\, s} - \frac{i\, D\, e\, \mathrm{GFDt}}{2\, m} + \frac{3\, i\, e\, \mathrm{GFDt}}{2\, m} - \frac{D\, \mathrm{Gt}}{2\, m\, s} + \frac{3\, \mathrm{Gt}}{2\, m\, s} + \frac{3\, \mathrm{at}\, e\, \Phi\, \gamma\cdot k}{2\, m\, s\, (k\cdot p)} - \frac{\mathrm{at}\, D\, e\, \Phi\, \gamma\cdot k}{2\, m\, s\, (k\cdot p)}\right) + \mathrm{Gp}\left(\frac{D\, \omega\, (k\cdot p)}{m^2\, s} - \frac{3\, \omega\, (k\cdot p)}{m^2\, s}\right) +$$

$$\mathrm{GFDp}\left(-\frac{i\, D\, e\, (k\cdot p)\, \omega^2}{m^2\, s} + \frac{3\, i\, e\, (k\cdot p)\, \omega^2}{m^2\, s} + \frac{i\, D\, e\, (k\cdot p)\, \omega}{m^2} - \frac{3\, i\, e\, (k\cdot p)\, \omega}{m^2}\right) + \mathrm{Gm}\left(\frac{D\, m^2\, \xi^2\, (k\cdot p)\, \omega^3}{2\, s} - \frac{3\, m^2\, \xi^2\, (k\cdot p)\, \omega^3}{2\, s} - \frac{1}{3} D\, m^2\, \xi^2\, (k\cdot p)\, \omega^2 + m^2\, \xi^2\, (k\cdot p)\, \omega^2 + \frac{D\, \mathrm{qp}\, \omega}{2\, m\, s} - \frac{3\, \mathrm{qp}\, \omega}{2\, m\, s} + \frac{D\, m^2\, \xi^2\, \Phi^2\, \omega}{2\, s\, (k\cdot p)} - \frac{3\, m^2\, \xi^2\, \Phi^2\, \omega}{2\, s\, (k\cdot p)} - \frac{3\, \mathrm{at}\, e\, \mathrm{pt}\, \Phi\, \omega}{s\, (k\cdot p)} + \frac{\mathrm{at}\, D\, \mathrm{pt}\, \Phi\, \omega}{s\, (k\cdot p)} + \mathrm{pp}\left(\frac{D\, \omega}{2\, m\, s} - \frac{3\, \omega}{2\, m\, s}\right)\right) + 3 - \frac{i\, t\, \gamma\cdot k}{m\, s\, (k\cdot p)\, \omega}\Bigg)$$

Integration over

$$\int d^{D-2}\, x_\perp \ldots :$$

The integral is gaussian

$$I_0 = \int d^{D-2}\, x_\perp\, \mathrm{Exp}\left[I\, A\, x_\perp^2 + I\, (J.x_\perp)\right] =$$

$$= \mathrm{Exp}\left[I\, \frac{\pi}{2}\, \frac{D-2}{2}\right] \pi^{\frac{D-2}{2}} (\det A)^{-\frac{1}{2}}\, \mathrm{Exp}\left[-I\, \frac{1}{4}\, J.A^{-1}.J\right]$$

The preexponent contains terms linear in $x_\perp$, so we will also need

$$I_1 = \int d^{D-2}\, x_\perp\, x_\perp\, \mathrm{Exp}\left[I\, A\, x_\perp^2 + I\, (J.x_\perp)\right] =$$

$$= -\frac{1}{2} (A^{-1}.J)\, I_0;$$

$$I_2 = \int d^{D-2}\, x_\perp\, x_\perp^2\, \mathrm{Exp}\left[I\, A\, x_\perp^2 + I\, (J.x_\perp)\right] =$$

$$= \left[I\, \frac{1}{2}\, \mathrm{Tr}\, A^{-1} + \left(-\frac{1}{2} (A^{-1}.J)\right)^2\right] I_0$$

The remaining integrals : $\int d\Phi$

In[62]:=
```
Amatr = Coefficient[Phase7, xt^2]
DetA = Amatr^(D - 2)
Jvec = Coefficient[Phase7, xt]
Jvec2 = Collect[Expand[Jvec^2], {Φ}]
CI0 = Exp[I Pi / 2 (D / 2 - 1)] Pi^(D / 2 - 1) (Amatr)^(-(D - 2) / 2)
PhaseI0 = -1 / 4 (1 / Amatr) Jvec2
xtQuadraticSubst = {xt^2 → (I * 1 / 2 / Amatr * (D - 2) + ( -1 / 2 Amatr^(-1) Jvec)^2)}
xtLinearSubst = {xt → -1 / 2 Amatr^(-1) Jvec}
```

Out[62]= $\frac{1}{4\,\omega}$

Out[63]= $4^{2-D}\left(\frac{1}{\omega}\right)^{D-2}$

Out[64]= $-\mathrm{at}\; e\Phi - \mathrm{pt}$

Out[65]= $\mathrm{at}^2\; e^2\;\Phi^2 + 2\;\mathrm{at}\; e\,\mathrm{pt}\;\Phi + \mathrm{pt}^2$

Out[66]= $2^{D-2}\; e^{\frac{1}{2}\, i\,\pi\left(\frac{D}{2}-1\right)}\;\pi^{\frac{D}{2}-1}\left(\frac{1}{\omega}\right)^{\frac{2-D}{2}}$

Out[67]= $\omega\left(-\left(\mathrm{at}^2\; e^2\;\Phi^2 + 2\;\mathrm{at}\; e\,\mathrm{pt}\;\Phi + \mathrm{pt}^2\right)\right)$

Out[68]= $\left\{\mathrm{xt}^2 \to 4\,\omega^2\,(-\mathrm{at}\; e\Phi - \mathrm{pt})^2 + 2\, i\,(D-2)\,\omega\right\}$

Out[69]= $\{\mathrm{xt} \to -2\,\omega\,(-\mathrm{at}\; e\Phi - \mathrm{pt})\}$

We find that

$A = \frac{1}{4\,\omega}$,

$J = -e a_{\perp}\,\Phi - p_{\perp}$,

$\det A = \left(\frac{1}{4\,\omega}\right)^{D-2}$,

$A^{-1} = 4\,\omega$,

Also we use the following equalities for further simplifications :

$a^{\mu}\,\gamma^{\beta}\,\gamma^{5}\,F^{*}{}_{\beta\mu} == -a_{\perp}\,(\gamma^{\beta}\,\gamma^{5}\,F^{*}{}_{\beta\mu})_{\perp} ==$ at GFDt = 0, as $a^{\mu}\,F^{*}{}_{\beta\mu} = 0$,

at Gt == $-(\gamma\; a)$,

at pt == (ap),

$\mathrm{at}^2 == -a^2 = \xi^2\,m^2 / e^2$

```
In[70]:= Coeff8 = Simplify[Coeff7 * CI0, Assumptions → {ω > 0, m > 0, t > 0, s > 0}]
Phase8 = Collect[(Phase7 /. {xt → 0}) + PhaseI0, Φ] /. {at^2 → ξ^2 m^2 / e^2}
Matrix8 = Collect[
  Expand[
    Matrix7 /. xtQuadraticSubst /. xtLinearSubst
   ] /. {at GFDt → 0, at Gt → -DiracGamma[Momentum[a, D], D],
    at^2 → ξ^2 m^2 / e^2, at pt → -Pair[Momentum[a, D], Momentum[p, D]]},
  {J0, J1, J2, dJ0dχl, dJ1dχl, dJ2dχl, Φ, Gm, Gp, GFDp, pp}]
```

Out[70]= $\frac{1}{4} i e^{-\frac{1}{4} i \pi D} \pi^{\frac{D}{2}-2} e^2 \left(\frac{s t}{\omega}\right)^{-D/2}$

Out[71]= $\frac{1}{3} m^2 \xi^2 \omega^3 (k\cdot p)^2 - \frac{1}{3} m^2 \xi^2 s \omega^2 (k\cdot p)^2 + \frac{\text{pp}\,\omega (k\cdot p)}{m} + \frac{\text{qp}\,\omega (k\cdot p)}{m} - m^2 s + \Phi\left(\frac{\text{qp}}{m} - \frac{\text{pp}}{m}\right) - \text{pt}^2 \omega$

Out[72]= $-\frac{i\,\text{dJ1d}\chi\text{lGm}\,\xi\,\omega}{2 m^2 s t} + \text{dJ2d}\chi\text{lGm}\left(\frac{3 i \xi \omega}{2 m^2 s t} - \frac{i D \xi \omega}{2 m^2 s t}\right) +$

$\text{J1}\left(-\frac{m \xi^2 \gamma\cdot k (k\cdot p) \omega^3}{2 s} - m \xi^2 \gamma\cdot k (k\cdot p) \omega^2 + \frac{i e \text{GFDt pt}\,\omega^2}{m s} + \frac{i e \text{GFDt pt}\,\omega}{m} + \frac{1}{2} i e \sigma \text{F} \omega -\right.$

$\frac{2}{3} m s \xi^2 \gamma\cdot k (k\cdot p) \omega + \frac{\text{Gp}\,(k\cdot p)\,\omega}{m^2 s} - \frac{\text{Gt pt}\,\omega}{m s} + \frac{1}{2} i e s \sigma \text{F} + \Phi^2\left(\frac{\text{Gm}\, m^2 \xi^2 \omega}{2 s (k\cdot p)} - \frac{m \xi^2 \omega \gamma\cdot k}{2 s (k\cdot p)}\right) +$

$\Phi\left(\frac{e \omega \gamma\cdot k (a\cdot p)}{m s (k\cdot p)} - \frac{e \text{Gm}\, \omega (a\cdot p)}{s (k\cdot p)}\right) + \text{GFDp}\left(-\frac{i e (k\cdot p) \omega^2}{m^2 s} - \frac{i e (k\cdot p) \omega}{m^2}\right) +$

$\left.\text{Gm}\left(\frac{m^2 \xi^2 (k\cdot p) \omega^3}{2 s} - \frac{1}{3} m^2 \xi^2 (k\cdot p) \omega^2 + \frac{\text{pp}\,\omega}{2 m s} + \frac{\text{qp}\,\omega}{2 m s}\right) - 1 - \frac{i t \gamma\cdot k}{m s (k\cdot p) \omega}\right) +$

$\text{J0}\left(\frac{D m \xi^2 \gamma\cdot k (k\cdot p) \omega^3}{2 s} - \frac{m \xi^2 \gamma\cdot k (k\cdot p) \omega^3}{s} - D m \xi^2 \gamma\cdot k (k\cdot p) \omega^2 + 6 m \xi^2 \gamma\cdot k (k\cdot p) \omega^2 -\right.$

$\frac{i D e \text{GFDt pt}\,\omega^2}{m s} + \frac{2 i e \text{GFDt pt}\,\omega^2}{m s} + \frac{i D e \text{GFDt pt}\,\omega}{m} - \frac{6 i e \text{GFDt pt}\,\omega}{m} - \frac{1}{2} i D e \sigma \text{F} \omega +$

$\frac{2}{3} D m s \xi^2 \gamma\cdot k (k\cdot p) \omega - \frac{4}{3} m s \xi^2 \gamma\cdot k (k\cdot p) \omega + \frac{D \text{Gt pt}\,\omega}{m s} - \frac{2 \text{Gt pt}\,\omega}{m s} + D +$

$\frac{1}{2} i D e s \sigma \text{F} - 2 i e s \sigma \text{F} + \Phi^2\left(\frac{D m \omega \gamma\cdot k \xi^2}{2 s (k\cdot p)} - \frac{m \omega \gamma\cdot k \xi^2}{s (k\cdot p)} + \text{Gm}\left(\frac{m^2 \xi^2 \omega}{s (k\cdot p)} - \frac{D m^2 \xi^2 \omega}{2 s (k\cdot p)}\right)\right) +$

$\Phi\left(-\frac{D e \omega \gamma\cdot k (a\cdot p)}{m s (k\cdot p)} + \frac{2 e \omega \gamma\cdot k (a\cdot p)}{m s (k\cdot p)} + \text{Gm}\left(\frac{D e \omega (a\cdot p)}{s (k\cdot p)} - \frac{2 e \omega (a\cdot p)}{s (k\cdot p)}\right)\right) +$

$\text{Gp}\left(\frac{2 \omega (k\cdot p)}{m^2 s} - \frac{D \omega (k\cdot p)}{m^2 s}\right) + \text{GFDp}\left(\frac{i D e (k\cdot p) \omega^2}{m^2 s} - \frac{2 i e (k\cdot p) \omega^2}{m^2 s} - \frac{i D e (k\cdot p) \omega}{m^2} + \frac{6 i e (k\cdot p) \omega}{m^2}\right) +$

$\text{Gm}\left(-\frac{D m^2 \xi^2 (k\cdot p) \omega^3}{2 s} + \frac{m^2 \xi^2 (k\cdot p) \omega^3}{s} + \frac{1}{3} D m^2 \xi^2 (k\cdot p) \omega^2 -\right.$

$$\frac{2}{3} m^2\, \xi^2\, (k\cdot p)\, \omega^2 - \frac{D\,\text{qp}\,\omega}{2\, m\, s} + \frac{\text{qp}\,\omega}{m\, s} + \text{pp}\left(\frac{\omega}{m\, s} - \frac{D\,\omega}{2\, m\, s}\right)\Bigg)\Bigg) +$$

$$\text{J2}\Bigg(-\frac{D\, m\, \xi^2\, \gamma\cdot k\, (k\cdot p)\, \omega^3}{2\, s} + \frac{3\, m\, \xi^2\, \gamma\cdot k\, (k\cdot p)\, \omega^3}{2\, s} + D\, m\, \xi^2\, \gamma\cdot k\, (k\cdot p)\, \omega^2 - 3\, m\, \xi^2\, \gamma\cdot k\, (k\cdot p)\, \omega^2 +$$

$$\frac{i\, D\, e\,\text{GFDt pt}\,\omega^2}{m\, s} - \frac{3\, i\, e\,\text{GFDt pt}\,\omega^2}{m\, s} - \frac{i\, D\, e\,\text{GFDt pt}\,\omega}{m} + \frac{3\, i\, e\,\text{GFDt pt}\,\omega}{m} + \frac{1}{2} i\, D\, e\, \sigma\text{F}\omega -$$

$$\frac{3}{2} i\, e\, \sigma\text{F}\omega - \frac{2}{3} D\, m\, s\, \xi^2\, \gamma\cdot k\, (k\cdot p)\, \omega + 2\, m\, s\, \xi^2\, \gamma\cdot k\, (k\cdot p)\, \omega - \frac{D\,\text{Gt pt}\,\omega}{m\, s} + \frac{3\,\text{Gt pt}\,\omega}{m\, s} - D -$$

$$\frac{1}{2} i\, D\, e\, s\, \sigma\text{F} + \frac{3}{2} i\, e\, s\, \sigma\text{F} + \Phi^2\left(-\frac{D\, m\, \omega\, \gamma\cdot k\, \xi^2}{2\, s\, (k\cdot p)} + \frac{3\, m\, \omega\, \gamma\cdot k\, \xi^2}{2\, s\, (k\cdot p)} + \text{Gm}\left(\frac{D\, m^2\, \xi^2\, \omega}{2\, s\, (k\cdot p)} - \frac{3\, m^2\, \xi^2\, \omega}{2\, s\, (k\cdot p)}\right)\right) +$$

$$\Phi\left(\frac{D\, e\, \omega\, \gamma\cdot k\, (a\cdot p)}{m\, s\, (k\cdot p)} - \frac{3\, e\, \omega\, \gamma\cdot k\, (a\cdot p)}{m\, s\, (k\cdot p)} + \text{Gm}\left(\frac{3\, e\, \omega\, (a\cdot p)}{s\, (k\cdot p)} - \frac{D\, e\, \omega\, (a\cdot p)}{s\, (k\cdot p)}\right)\right) +$$

$$\text{Gp}\left(\frac{D\, \omega\, (k\cdot p)}{m^2\, s} - \frac{3\, \omega\, (k\cdot p)}{m^2\, s}\right) + \text{GFDp}\left(-\frac{i\, D\, e\, (k\cdot p)\, \omega^2}{m^2\, s} + \frac{3\, i\, e\, (k\cdot p)\, \omega^2}{m^2\, s} + \frac{i\, D\, e\, (k\cdot p)\, \omega}{m^2} - \frac{3\, i\, e\, (k\cdot p)\, \omega}{m^2}\right) +$$

$$\text{Gm}\Bigg(\frac{D\, m^2\, \xi^2\, (k\cdot p)\, \omega^3}{2\, s} - \frac{3\, m^2\, \xi^2\, (k\cdot p)\, \omega^3}{2\, s} - \frac{1}{3} D\, m^2\, \xi^2\, (k\cdot p)\, \omega^2 +$$

$$m^2\, \xi^2\, (k\cdot p)\, \omega^2 + \frac{D\,\text{qp}\,\omega}{2\, m\, s} - \frac{3\,\text{qp}\,\omega}{2\, m\, s} + \text{pp}\left(\frac{D\,\omega}{2\, m\, s} - \frac{3\,\omega}{2\, m\, s}\right)\Bigg) + 3 - \frac{i\, t\, \gamma\cdot k}{m\, s\, (k\cdot p)\, \omega}\Bigg)$$

Next, we recollect all scalar products into covariant notations

After this step the dependence on Φ in the preexponent should vanish

GFDp pm $= \gamma^{\beta}\, \gamma^{5}\, (F^{*}\, p)_{\beta}$ + GFDt pt

Gp pm = −Gm pp + (γp) + GFDt pt

Gm = (γk) / m = (γF^2 p) / m[$-a^2$ (kp)]

Note that after this step the preexponential does not depend on Φ

In[73]:=
```
SubstitutionStep91 =
 {DiracGamma[Momentum[k, D], D] (kp) → DiracGamma[Momentum[FFp, D], D] e^2 / m^2 / ξ^2,
  GFDp → m / kp (GFDt pt + DiracGamma[LorentzIndex[β, D], D].DiracGamma[5] ×
      Pair[LorentzIndex[β, D], Momentum[FDp, D]]),
  Gp → m / kp (DiracGamma[Momentum[p, D], D] - Gm pp + Gt pt)}
SubstitutionStep92 = {Gm → DiracGamma[Momentum[k, D], D] / m}
SubstitutionStep93 =
 {DiracGamma[Momentum[k, D], D] → DiracGamma[Momentum[FFp, D], D] e^2 / m^2 / ξ^2 / (kp)}
```

Out[73]= $\left\{\gamma\cdot k\,(k\cdot p)\to\frac{e^2\,\gamma\cdot\text{FFp}}{m^2\,\xi^2},\ \text{GFDp}\to\frac{m\left(\text{FDp}^{\beta}\,\gamma^{\beta}.\overline{\gamma}^{5}+\text{GFDt pt}\right)}{k\cdot p},\ \text{Gp}\to\frac{m\,(-\text{Gm pp}+\text{Gt pt}+\gamma\cdot p)}{k\cdot p}\right\}$

Out[74]= $\left\{\text{Gm}\to\frac{\gamma\cdot k}{m}\right\}$

Out[75]= $\left\{\gamma\cdot k\to\frac{e^2\,\gamma\cdot\text{FFp}}{m^2\,\xi^2\,(k\cdot p)}\right\}$

In[76]:=
```
Coeff9 = Coeff8
Phase9 = Phase8
Matrix9 =
 Collect[
  Expand[
   Simplify[
    Expand[
     Matrix8 /. SubstitutionStep91 /. SubstitutionStep92 /.
       SubstitutionStep93 /. {ω → s t / (s + t)}
    ]
   ]
  ],
  {J0, J1, J2, dJ0dχl, dJ1dχl, dJ2dχl, Φ,
   e^2 DiracGamma[Momentum[FFp, D], D], DiracGamma[LorentzIndex[β, D], D].DiracGamma[5] ×
    Pair[LorentzIndex[β, D], Momentum[FDp, D]],
   DiracGamma[Momentum[p, D], D], Pair[Momentum[a, D], Momentum[p, D]]}]
```

Out[76]= $\frac{1}{4}\,i\,e^{-\frac{1}{4}i\pi D}\,\pi^{\frac{D}{2}-2}\,e^2\left(\frac{s\,t}{\omega}\right)^{-D/2}$

Out[77]= $\frac{1}{3}\,m^2\,\xi^2\,\omega^3\,(k\cdot p)^2-\frac{1}{3}\,m^2\,\xi^2\,s\,\omega^2\,(k\cdot p)^2+\frac{\text{pp}\,\omega\,(k\cdot p)}{m}+\frac{\text{qp}\,\omega\,(k\cdot p)}{m}-m^2\,s+\Phi\left(\frac{\text{qp}}{m}-\frac{\text{pp}}{m}\right)-\text{pt}^2\,\omega$

Out[78]= $\text{dJ1d}\chi\text{l}\,\gamma\cdot\text{FFp}\left(-\frac{i\,s}{2\,m^5\,(s+t)^2\,\xi\,(k\cdot p)}-\frac{i\,t}{2\,m^5\,(s+t)^2\,\xi\,(k\cdot p)}\right)e^2+$

$\text{dJ2d}\chi\text{l}\,\gamma\cdot\text{FFp}\left(-\frac{i\,D\,s}{2\,m^5\,(s+t)^2\,\xi\,(k\cdot p)}+\frac{3\,i\,s}{2\,m^5\,(s+t)^2\,\xi\,(k\cdot p)}-\frac{i\,D\,t}{2\,m^5\,(s+t)^2\,\xi\,(k\cdot p)}+\frac{3\,i\,t}{2\,m^5\,(s+t)^2\,\xi\,(k\cdot p)}\right)e^2+$

$$\mathrm{J1}\left(\frac{i\,e\,\sigma\,\mathrm{F}\,s^3}{2\,(s+t)^2}+\frac{3\,i\,e\,t\,\sigma\,\mathrm{F}\,s^2}{2\,(s+t)^2}-\frac{s^2}{(s+t)^2}+\frac{i\,e\,t^2\,\sigma\,\mathrm{F}\,s}{(s+t)^2}-\frac{2\,t\,s}{(s+t)^2}+\left(\frac{t^2}{m\,(s+t)^2}+\frac{s\,t}{m\,(s+t)^2}\right)\gamma\cdot p+\right.$$
$$\left(-\frac{i\,e\,t\,s^2}{m\,(s+t)^2}-\frac{2\,i\,e\,t^2\,s}{m\,(s+t)^2}\right)\gamma^\beta.\overline{\gamma}^5\,\mathrm{FDp}^\beta+e^2\,\gamma\cdot\mathrm{FFp}\left(-\frac{2\,t\,s^3}{3\,m\,(s+t)^2}-\frac{2\,t^2\,s^2}{m\,(s+t)^2}-\frac{\mathrm{pp}\;t\,s}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}+\right.$$
$$\frac{\mathrm{qp}\;t\,s}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}-\frac{i\,s}{m^3\,(s+t)^2\,\xi^2\,(k\cdot p)^2}-\frac{\mathrm{pp}\;t^2}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}+\frac{\mathrm{qp}\;t^2}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}-$$
$$\left.\left.\frac{3\,i\,t}{m^3\,(s+t)^2\,\xi^2\,(k\cdot p)^2}-\frac{3\,i\,t^2}{m^3\,(s+t)^2\,\xi^2\,(k\cdot p)^2\,s}-\frac{i\,t^3}{m^3\,(s+t)^2\,\xi^2\,(k\cdot p)^2\,s^2}\right)-\frac{t^2}{(s+t)^2}\right)+$$
$$\mathrm{J0}\left(\frac{i\,D\,e\,\sigma\,\mathrm{F}\,s^3}{2\,(s+t)^2}-\frac{2\,i\,e\,\sigma\,\mathrm{F}\,s^3}{(s+t)^2}+\frac{i\,D\,e\,t\,\sigma\,\mathrm{F}\,s^2}{2\,(s+t)^2}-\frac{4\,i\,e\,t\,\sigma\,\mathrm{F}\,s^2}{(s+t)^2}+\frac{D\,s^2}{(s+t)^2}-\frac{2\,i\,e\,t^2\,\sigma\,\mathrm{F}\,s}{(s+t)^2}+\right.$$
$$\frac{2\,D\,t\,s}{(s+t)^2}+\left(-\frac{D\,t^2}{m\,(s+t)^2}+\frac{2\,t^2}{m\,(s+t)^2}-\frac{D\,s\,t}{m\,(s+t)^2}+\frac{2\,s\,t}{m\,(s+t)^2}\right)\gamma\cdot p+$$
$$\left(-\frac{i\,D\,e\,t\,s^2}{m\,(s+t)^2}+\frac{6\,i\,e\,t\,s^2}{m\,(s+t)^2}+\frac{4\,i\,e\,t^2\,s}{m\,(s+t)^2}\right)\gamma^\beta.\overline{\gamma}^5\,\mathrm{FDp}^\beta+$$
$$e^2\,\gamma\cdot\mathrm{FFp}\left(\frac{2\,D\,t\,s^3}{3\,m\,(s+t)^2}-\frac{4\,t\,s^3}{3\,m\,(s+t)^2}+\frac{4\,t^2\,s^2}{m\,(s+t)^2}+\frac{D\,\mathrm{pp}\;t\,s}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}-\frac{\mathrm{pp}\;t\,s}{m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}-\right.$$
$$\frac{D\,\mathrm{qp}\;t\,s}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}+\frac{\mathrm{qp}\;t\,s}{m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}+\frac{D\,\mathrm{pp}\;t^2}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}-$$
$$\left.\left.\frac{\mathrm{pp}\;t^2}{m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}-\frac{D\,\mathrm{qp}\;t^2}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}+\frac{\mathrm{qp}\;t^2}{m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}\right)+\frac{D\,t^2}{(s+t)^2}\right)+$$
$$\mathrm{J2}\left(-\frac{i\,D\,e\,\sigma\,\mathrm{F}\,s^3}{2\,(s+t)^2}+\frac{3\,i\,e\,\sigma\,\mathrm{F}\,s^3}{2\,(s+t)^2}-\frac{i\,D\,e\,t\,\sigma\,\mathrm{F}\,s^2}{2\,(s+t)^2}+\frac{3\,i\,e\,t\,\sigma\,\mathrm{F}\,s^2}{2\,(s+t)^2}-\frac{D\,s^2}{(s+t)^2}+\frac{3\,s^2}{(s+t)^2}-\frac{2\,D\,t\,s}{(s+t)^2}+\frac{6\,t\,s}{(s+t)^2}+\right.$$
$$\left(\frac{D\,t^2}{m\,(s+t)^2}-\frac{3\,t^2}{m\,(s+t)^2}+\frac{D\,s\,t}{m\,(s+t)^2}-\frac{3\,s\,t}{m\,(s+t)^2}\right)\gamma\cdot p+\left(\frac{i\,D\,e\,s^2\,t}{m\,(s+t)^2}-\frac{3\,i\,e\,s^2\,t}{m\,(s+t)^2}\right)\gamma^\beta.\overline{\gamma}^5\,\mathrm{FDp}^\beta+$$
$$e^2\,\gamma\cdot\mathrm{FFp}\left(-\frac{2\,D\,t\,s^3}{3\,m\,(s+t)^2}+\frac{2\,t\,s^3}{m\,(s+t)^2}-\frac{D\,\mathrm{pp}\;t\,s}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}+\frac{3\,\mathrm{pp}\;t\,s}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}+\right.$$
$$\frac{D\,\mathrm{qp}\;t\,s}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}-\frac{3\,\mathrm{qp}\;t\,s}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}-\frac{i\,s}{m^3\,(s+t)^2\,\xi^2\,(k\cdot p)^2}-\frac{D\,\mathrm{pp}\;t^2}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}+$$
$$\frac{3\,\mathrm{pp}\;t^2}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}+\frac{D\,\mathrm{qp}\;t^2}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}-\frac{3\,\mathrm{qp}\;t^2}{2\,m^4\,(s+t)^2\,\xi^2\,(k\cdot p)}-\frac{3\,i\,t}{m^3\,(s+t)^2\,\xi^2\,(k\cdot p)^2}-$$
$$\left.\left.\frac{3\,i\,t^2}{m^3\,(s+t)^2\,\xi^2\,(k\cdot p)^2\,s}-\frac{i\,t^3}{m^3\,(s+t)^2\,\xi^2\,(k\cdot p)^2\,s^2}\right)-\frac{D\,t^2}{(s+t)^2}+\frac{3\,t^2}{(s+t)^2}\right)$$

```
In[79]:= Coefficient[Matrix9, Φ]
         Coefficient[Matrix9, Φ^2]
Out[79]= 0
Out[80]= 0
```

The integration over

$$\int_{-\infty}^{\infty} d\Phi \ldots$$

is now trivial,
as the phase in linear in Φ and the preexponent does not depend on Φ

$$\int d\Phi\, \mathrm{Exp}\left[i\, \Phi\, \frac{q_+ - p_+}{m}\right] = 2\,\pi\,\delta\left(\frac{q_+ - p_+}{m}\right) = 2\,\pi\, m\,\delta\,(q_+ - p_+)$$

After this step we have collected the full delta − function, so that
$M\,(q,\ p) = \Lambda^{D-4}\,(2\,\pi)^D\,\delta^{(D)}\,(q - p)\,M\,(p)$

In what follows we will consider M (p)

We substitute
$qp == q_+ \to\ pp = p_+$,
$pp = \frac{1}{2\,p_-}\,(p^2 + pt^2) = \frac{m}{2\,kp}\,(p^2 + pt^2)$

and also introduce
$\chi == \chi_p = \frac{\xi\,(kp)}{m^2}$

After this step all the feasible integrations are done.

We are left with two integrals over the proper times
$\int_0^\infty ds \int_0^\infty dt \ldots$
and the implicit integraion
$\int_{-\infty}^\infty dl^2\quad$ in $J_k\,(t,\ \chi_l)$

In[81]:=
```
Coeff10 = Coeff9 * 2 π * m / (2 π)^D / Λ^(D - 4)
Phase10 = Collect[
   Expand[
    Phase9 /. {qp → pp} /. {pp → (pv2 + pt^2) m / (2 kp)} /. {kp → m^2 χ / ξ}
   ],
   χ, Simplify]
Matrix10 = Collect[Expand[Matrix9 /. {qp → pp} /. {pp → (pv2 + pt^2) m / (2 kp)}] /.
     {DiracGamma[Momentum[k, D], D] → DiracGamma[Momentum[FFp, D], D] e^2 / m^2 / ξ^2 / (kp)} /.
    {kp → m^2 χ / ξ}, {J0, J1, J2, dJ0dχl, dJ1dχl, dJ2dχl,
    e^2 DiracGamma[Momentum[FFp, D], D], e σF,
    DiracGamma[LorentzIndex[β, D], D].DiracGamma[5] ×
     Pair[LorentzIndex[β, D], Momentum[FDp, D]],
    DiracGamma[Momentum[p, D], D], Pair[Momentum[a, D], Momentum[p, D]], χ}, Simplify]
```

Out[81]= $i\, 2^{-D-1}\, e^{-\frac{1}{4} i \pi D}\, \pi^{-\frac{D}{2}-1}\, e^2\, m\, \Lambda^{4-D} \left(\frac{s\, t}{\omega}\right)^{-D/2}$

Out[82]= $\frac{1}{3} m^6\, \chi^2\, \omega^2\, (\omega - s) - m^2\, s + p^2\, \omega$

Out[83]= $$\mathrm{J0}\left(-\frac{i\, e\, s\, t\, \mathrm{FDp}^{\beta}\, \gamma^{\beta}.\overline{\gamma}^{5}\, ((D-6)\, s - 4\, t)}{m\, (s+t)^2} + \frac{2\, e^2\, s^2\, t\, ((D-2)\, s + 6\, t)\, \gamma\cdot\mathrm{FFp}}{3\, m\, (s+t)^2} + \frac{i\, e\, s\, \sigma\mathrm{F}\, ((D-4)\, s - 4\, t)}{2\, (s+t)} - \frac{(D-2)\, t\, \gamma\cdot p}{m\, (s+t)} + D\right) +$$
$$\mathrm{J2}\left(\frac{i\, (D-3)\, e\, s^2\, t\, \mathrm{FDp}^{\beta}\, \gamma^{\beta}.\overline{\gamma}^{5}}{m\, (s+t)^2} + e^2\, \gamma\cdot\mathrm{FFp}\left(-\frac{2\, (D-3)\, s^3\, t}{3\, m\, (s+t)^2} - \frac{i\, (s+t)}{m^7\, s^2\, \chi^2}\right) - \frac{i\, (D-3)\, e\, s^2\, \sigma\mathrm{F}}{2\, (s+t)} + \frac{(D-3)\, t\, \gamma\cdot p}{m\, (s+t)} - D + 3\right) +$$
$$\mathrm{J1}\left(-\frac{i\, e\, s\, t\, \mathrm{FDp}^{\beta}\, (s + 2\, t)\, \gamma^{\beta}.\overline{\gamma}^{5}}{m\, (s+t)^2} + e^2\, \gamma\cdot\mathrm{FFp}\left(-\frac{2\, s^2\, t\, (s + 3\, t)}{3\, m\, (s+t)^2} - \frac{i\, (s+t)}{m^7\, s^2\, \chi^2}\right) + \frac{i\, e\, s\, \sigma\mathrm{F}\, (s + 2\, t)}{2\, (s+t)} + \frac{t\, \gamma\cdot p}{m\, s + m\, t} - 1\right) -$$
$$\frac{i\, (D-3)\, \mathrm{dJ2d}\chi\mathrm{l}\, e^2\, \gamma\cdot\mathrm{FFp}}{2\, m^7\, \chi\, (s+t)} - \frac{i\, \mathrm{dJ1d}\chi\mathrm{l}\, e^2\, \gamma\cdot\mathrm{FFp}}{2\, m^7\, \chi\, (s+t)}$$

In[84]:=
```
Coeff10 /. {D → 4}
Phase10 /. {D → 4}
Matrix10 /. {D → 4}
```

Out[84]= $-\frac{i e^2 m \omega^2}{32 \pi^3 s^2 t^2}$

Out[85]= $\omega \overline{p}^2 + \frac{1}{3} m^6 \chi^2 \omega^2 (\omega - s) - m^2 s$

Out[86]=
$$-\frac{i\, \mathrm{dJ1d}\chi\mathrm{l}\, e^2\, \overline{\gamma}\cdot\overline{\mathrm{FFp}}}{2 m^7 \chi (s+t)} - \frac{i\, \mathrm{dJ2d}\chi\mathrm{l}\, e^2\, \overline{\gamma}\cdot\overline{\mathrm{FFp}}}{2 m^7 \chi (s+t)} +$$
$$\mathrm{J0}\left(\frac{2 e^2 s^2 t (2 s + 6 t)\, \overline{\gamma}\cdot\overline{\mathrm{FFp}}}{3 m (s+t)^2} - \frac{i e s t (-2 s - 4 t)\, \overline{\gamma}^{\beta}.\overline{\gamma}^{5}\, \overline{\mathrm{FDp}}^{\beta}}{m (s+t)^2} - \frac{2 t\, \overline{\gamma}\cdot\overline{p}}{m (s+t)} - \frac{2 i e s \sigma \mathrm{F} t}{s+t} + 4\right) +$$
$$\mathrm{J1}\left(e^2\, \overline{\gamma}\cdot\overline{\mathrm{FFp}} \left(-\frac{2 s^2 t (s + 3 t)}{3 m (s+t)^2} - \frac{i (s+t)}{m^7 s^2 \chi^2}\right) - \frac{i e s t (s + 2 t)\, \overline{\gamma}^{\beta}.\overline{\gamma}^{5}\, \overline{\mathrm{FDp}}^{\beta}}{m (s+t)^2} + \frac{t\, \overline{\gamma}\cdot\overline{p}}{m s + m t} + \frac{i e s \sigma \mathrm{F} (s + 2 t)}{2 (s+t)} - 1\right) +$$
$$\mathrm{J2}\left(e^2\, \overline{\gamma}\cdot\overline{\mathrm{FFp}} \left(-\frac{2 s^3 t}{3 m (s+t)^2} - \frac{i (s+t)}{m^7 s^2 \chi^2}\right) + \frac{i e s^2 t\, \overline{\gamma}^{\beta}.\overline{\gamma}^{5}\, \overline{\mathrm{FDp}}^{\beta}}{m (s+t)^2} + \frac{t\, \overline{\gamma}\cdot\overline{p}}{m (s+t)} - \frac{i e s^2 \sigma \mathrm{F}}{2 (s+t)} - 1\right)$$

Let us change the variables :

$(s, t) \to (u, \sigma)$,

where

$$s = \frac{1}{m^2} \frac{1+u}{\chi^{2/3} u^{1/3}} \sigma$$
$$t = \frac{1}{m^2} \frac{1+u}{\chi^{2/3} u^{4/3}} \sigma$$

then

$$\omega = \frac{1}{m^2} \frac{1}{\chi^{2/3} u^{1/3}} \sigma,$$

$$\int_0^\infty dt \int_0^\infty ds \ldots = \int_0^\infty du \int_0^\infty d\sigma \; \left| J^{-1} \right| \ldots$$

$$\left| J^{-1} \right| = \frac{\sigma (u+1)^2}{m^4 u^{8/3} \chi^{4/3}}$$

In[87]:=
```
$Assumptions = {χ > 0, u > 0, σ > 0};
suσ = m^(-2) (1 + u) / u^(1 / 3) / χ^(2 / 3) σ
tuσ = m^(-2) (1 + u) / u^(4 / 3) / χ^(2 / 3) σ
Jac = Simplify[D[suσ, u] × D[tuσ, σ] - D[suσ, σ] × D[tuσ, u]]
Simplify[(1 / suσ + 1 / tuσ)^(-1)]
```

Out[88]= $\frac{\sigma\,(u+1)}{m^2\,\sqrt[3]{u}\,\chi^{2/3}}$

Out[89]= $\frac{\sigma\,(u+1)}{m^2\,u^{4/3}\,\chi^{2/3}}$

Out[90]= $\frac{\sigma\,(u+1)^2}{m^4\,u^{8/3}\,\chi^{4/3}}$

Out[91]= $\frac{\sigma}{m^2\,\sqrt[3]{u\,\chi^2}}$

In[92]:=
```
Coeff11 = Simplify[Coeff10 * Jac /. {ω → s t / (s + t)} /. {s → suσ, t → tuσ},
  Assumptions → {χ > 0, u > 0, σ > 0}]
Phase11 = Collect[
  Expand[
   Simplify[
    (Phase10) /. {ω → s t / (s + t)} /. {s → suσ, t → tuσ}, Assumptions → {χ > 0, u > 0, σ > 0}
   ]
  ], {pv2, σ},
  Simplify]
Matrix11 =
 Collect[
  Expand[
   Simplify[
    Matrix10 /. {kp →  χ / ξ m^2} /. {ω → s t / (s + t)} /. {s → suσ, t → tuσ},
    Assumptions → {u > 0, σ > 0, χ > 0}]
  ],
  {J0, J1, J2, dJ0dχl, dJ1dχl, dJ2dχl, e^2 DiracGamma[Momentum[FFp, D], D], e σF,
   DiracGamma[LorentzIndex[β, D], D].DiracGamma[5] × Pair[LorentzIndex[β, D],
     Momentum[FDp, D]], DiracGamma[Momentum[p, D], D], pv2, σ, χ}, Simplify]
```

Out[92]= $\frac{i\,2^{-D-1}\,e^{-\frac{1}{4}i\pi D}\,\pi^{-\frac{D}{2}-1}\,e^2\,m\,\Lambda^{4-D}\left(\frac{\sigma\,(u+1)^2}{m^2\,u^{4/3}\,\chi^{2/3}}\right)^{2-\frac{D}{2}}}{\sigma\,(u+1)^2}$

Out[93]= $\frac{p^2\,\sigma}{m^2\,\sqrt[3]{u\,\chi^2}}-\frac{\sigma^3}{3}-\frac{\sigma\,(u+1)}{\sqrt[3]{u\,\chi^2}}$

Out[94]=
$$
\begin{aligned}
&J0\left(\sigma\, \mathrm{FDp}^{\beta}\, \gamma^{\beta}.\overline{\gamma}^{5}\left(\frac{4\, i\, e}{m^{3}\, \sqrt[3]{u}\, (u+1)^{2}\, \chi^{2/3}}-\frac{i\, e\, u^{2/3}\, (D\, u+D-6\, u-10)}{m^{3}\, (u+1)^{2}\, \chi^{2/3}}\right)+\frac{2\, e^{2}\, \sigma^{2}\, ((D-2)\, u+6)\, \gamma\cdot \mathrm{FFp}}{3\, m^{5}\, u^{2/3}\, \chi^{4/3}}+\right.\\
&\left. e\sigma\, \sigma \mathrm{F}\left(\frac{i\, u^{2/3}\left(D\,(u+1)^{2}-4\left(u^{2}+3\, u+3\right)\right)}{2\, m^{2}\, (u+1)^{2}\, \chi^{2/3}}-\frac{2\, i}{m^{2}\, \sqrt[3]{u}\, (u+1)^{2}\, \chi^{2/3}}\right)+\frac{(2-D)\, \gamma\cdot p}{m\, u+m}+D\right)+\\
&J2\left(\frac{i\,(D-3)\, e\sigma\, u^{2/3}\, \mathrm{FDp}^{\beta}\, \gamma^{\beta}.\overline{\gamma}^{5}}{m^{3}\, (u+1)\, \chi^{2/3}}+e^{2}\, \gamma\cdot \mathrm{FFp}\left(-\frac{2\,(D-3)\, \sigma^{2}\, \sqrt[3]{u}}{3\, m^{5}\, \chi^{4/3}}-\frac{i}{m^{5}\, \sigma\, u^{2/3}\, \chi^{4/3}}\right)-\right.\\
&\left.\frac{i\,(D-3)\, e\sigma\, \sigma \mathrm{F}\, u^{2/3}}{2\, m^{2}\, \chi^{2/3}}+\frac{(D-3)\, \gamma\cdot p}{m\,(u+1)}-D+3\right)+\\
&J1\left(\sigma\, \mathrm{FDp}^{\beta}\, \gamma^{\beta}.\overline{\gamma}^{5}\left(-\frac{i\, e\, u^{2/3}\, (u+3)}{m^{3}\, (u+1)^{2}\, \chi^{2/3}}-\frac{2\, i\, e}{m^{3}\, \sqrt[3]{u}\, (u+1)^{2}\, \chi^{2/3}}\right)+\right.\\
&e^{2}\, \gamma\cdot \mathrm{FFp}\left(-\frac{2\, \sigma^{2}\, (u+3)}{3\, m^{5}\, u^{2/3}\, \chi^{4/3}}-\frac{i}{m^{5}\, \sigma\, u^{2/3}\, \chi^{4/3}}\right)+\\
&\left. e\sigma\, \sigma \mathrm{F}\left(\frac{i\, u^{2/3}\left(u^{2}+4\, u+5\right)}{2\, m^{2}\, (u+1)^{2}\, \chi^{2/3}}+\frac{i}{m^{2}\, \sqrt[3]{u}\, (u+1)^{2}\, \chi^{2/3}}\right)+\frac{\gamma\cdot p}{m\, u+m}-1\right)-\\
&\frac{i\,(D-3)\, \mathrm{dJ2d}\chi \mathrm{l}\, e^{2}\, u^{4/3}\, \gamma\cdot \mathrm{FFp}}{2\, m^{5}\, \sigma\, (u+1)^{2}\, \sqrt[3]{\chi}}-\frac{i\, \mathrm{dJ1d}\chi \mathrm{l}\, e^{2}\, u^{4/3}\, \gamma\cdot \mathrm{FFp}}{2\, m^{5}\, \sigma\, (u+1)^{2}\, \sqrt[3]{\chi}}
\end{aligned}
$$

In the next step we rewrite

dJkdχl == $\frac{\partial}{\partial \chi_l} J_k(t, \chi_l)$,

where $\chi_l = \chi_l(u) = \frac{u}{1+u}\chi$ and $t = t(u) = \frac{1}{m^2}\frac{1+u}{\chi^{2/3}\,u^{4/3}}\sigma$ as defined above

using integration by parts

First, we may use the equality

$\frac{\partial}{\partial \chi_l} J_k(t, \chi_l) = (\chi_l'(u))^{-1}\left[\frac{d}{du} J_k(t(u), \chi_l(u)) - \frac{\partial}{\partial t} J_k(t, \chi_l)\, t'(u)\right].$

Then, we may integrate the first term by parts, i.e.

$$\int_0^\infty du\, f(u) \frac{d}{du} Jk(t, \chi_l) = -\int_0^\infty du\, Jk(t, \chi_l) \frac{d}{du} f(t),$$

where we assumed that Jk(t, 0) = f(∞) = 0.

As for second term, we write

$$\frac{\partial}{\partial t} J_k(t, \chi_l) = \frac{\partial}{\partial t}(-i) \int_{-\infty}^{\infty} dl^2\, D_k(l^2, \chi_l)\, e^{-i l^2 t} =$$

$$(-i)^2 \int_{-\infty}^{\infty} dl^2\; l^2\, D_k(l^2, \chi_l)\, e^{-i l^2 t} == -i\, m^2\, \tilde{J}_k(t, \chi_l).$$

We denote

Jkt == $\tilde{J}_k$

In[95]:=
```
χlu = Simplify[χlϕ0 /. {ω → s t / (s + t)} /. {s → suσ, t → tuσ} /. {kp → χ / ξ m ^ 2}] /.
   {kp → χ / ξ m ^ 2}
dχldu = Simplify[D[χlu, u]]
dtdu = Simplify[D[tuσ, u]]
Coefficient[Matrix11, dJ0dχl] * dJ0dχl +
  Coefficient[Matrix11, dJ1dχl] * dJ1dχl + Coefficient[Matrix11, dJ2dχl] * dJ2dχl
Matrix12t = % /. {dJ0dχl → 1 / dχldu (-(-I m ^ 2 J0t) dtdu),
    dJ1dχl → 1 / dχldu (-(-I m ^ 2 J1t) dtdu), dJ2dχl → 1 / dχldu (-(-I m ^ 2 J2t) dtdu)}
Matrix12d = %% /. {dJ0dχl → 1 / dχldu dJ0du , dJ1dχl → 1 / dχldu dJ1du ,
    dJ2dχl → 1 / dχldu dJ2du }
```

Out[95]= $\frac{u\,\chi}{u+1}$

Out[96]= $$\frac{\chi}{(u+1)^2}$$

Out[97]= $$-\frac{\sigma\,(u+4)}{3\,m^2\,u^{7/3}\,\chi^{2/3}}$$

Out[98]= $$-\frac{i\,(D-3)\,\text{dJ2d}\chi\text{l}\,e^2\,u^{4/3}\,\gamma\cdot\text{FFp}}{2\,m^5\,\sigma\,(u+1)^2\,\sqrt[3]{\chi}}-\frac{i\,\text{dJ1d}\chi\text{l}\,e^2\,u^{4/3}\,\gamma\cdot\text{FFp}}{2\,m^5\,\sigma\,(u+1)^2\,\sqrt[3]{\chi}}$$

Out[99]= $$-\frac{(D-3)\,e^2\,\text{J2t}\,(u+4)\,\gamma\cdot\text{FFp}}{6\,m^5\,u\,\chi^2}-\frac{e^2\,\text{J1t}\,(u+4)\,\gamma\cdot\text{FFp}}{6\,m^5\,u\,\chi^2}$$

Out[100]= $$-\frac{i\,(D-3)\,\text{dJ2du}\,e^2\,u^{4/3}\,\gamma\cdot\text{FFp}}{2\,m^5\,\sigma\,\chi^{4/3}}-\frac{i\,\text{dJ1du}\,e^2\,u^{4/3}\,\gamma\cdot\text{FFp}}{2\,m^5\,\sigma\,\chi^{4/3}}$$

In[101]:=
```
f = Coeff11 * Exp[I Phase11]
Matrix12dmod =
 Collect[Expand[Simplify[-D[Matrix12d * f, u] / f] /. {dJ0du → J0, dJ1du → J1, dJ2du → J2}],
  {J0, J1, J2}, Simplify]
```

Out[101]= $$\frac{i\,2^{-D-1}\,\pi^{-\frac{D}{2}-1}\,e^2\,m\,\Lambda^{4-D}\left(\frac{\sigma\,(u+1)^2}{m^2\,u^{4/3}\,\chi^{2/3}}\right)^{2-\frac{D}{2}}\exp\left(i\left(\frac{p^2\,\sigma}{m^2\,\sqrt[3]{u\,\chi^2}}-\frac{\sigma^3}{3}-\frac{\sigma\,(u+1)}{\sqrt[3]{u\,\chi^2}}\right)-\frac{i\,\pi\,D}{4}\right)}{\sigma\,(u+1)^2}$$

Out[102]= $$\frac{e^2\,\text{J1}\,\gamma\cdot\text{FFp}\left(p^2\,\sigma\,(u+1)+m^2\left(\sigma\left(2\,u^2+u-1\right)-i\,(D-2)\,(u-2)\,\sqrt[3]{u\,\chi^2}\right)\right)}{6\,m^7\,\sigma\,(u+1)\,\chi^2}+$$
$$\frac{(D-3)\,e^2\,\text{J2}\,\gamma\cdot\text{FFp}\left(p^2\,\sigma\,(u+1)+m^2\left(\sigma\left(2\,u^2+u-1\right)-i\,(D-2)\,(u-2)\,\sqrt[3]{u\,\chi^2}\right)\right)}{6\,m^7\,\sigma\,(u+1)\,\chi^2}$$

In[103]:=
```
Coeff12 = Coeff11
Phase12 = Phase11
Matrix12 = Collect[
  (Matrix11 /. {dJ0dχl → 0, dJ1dχl → 0, dJ2dχl → 0}) + Matrix12t + Matrix12dmod,
  {J0, J1, J2, J0t, J1t, J2t, e^2 DiracGamma[Momentum[FFp, D], D],
   e σF, DiracGamma[LorentzIndex[β, D], D].DiracGamma[5] ×
    Pair[LorentzIndex[β, D], Momentum[FDp, D]],
   DiracGamma[Momentum[p, D], D], pv2, lv2, σ, χ}, Simplify]
```

Out[103]= $$\frac{i\,2^{-D-1}\,e^{-\frac{1}{4}\,i\,\pi\,D}\,\pi^{-\frac{D}{2}-1}\,e^2\,m\,\Lambda^{4-D}\left(\frac{\sigma\,(u+1)^2}{m^2\,u^{4/3}\,\chi^{2/3}}\right)^{2-\frac{D}{2}}}{\sigma\,(u+1)^2}$$

Out[104]=

$$\frac{p^2\,\sigma}{m^2\,\sqrt[3]{u\,\chi^2}}-\frac{\sigma^3}{3}-\frac{\sigma\,(u+1)}{\sqrt[3]{u\,\chi^2}}$$

Out[105]=

$$\mathrm{J2}\left(\frac{i\,(D-3)\,e\sigma\,u^{2/3}\,\mathrm{FDp}^{\beta}\,\gamma^{\beta}.\overline{\gamma}^{5}}{m^3\,(u+1)\,\chi^{2/3}}+\right.$$

$$e^2\,\gamma\cdot\mathrm{FFp}\left(\frac{-\frac{i\,(D^2-5\,D+6)\,(u-2)\,\sqrt[3]{u\,\chi^2}}{6\,m^5\,(u+1)\,\chi^2}-\frac{i}{m^5\,u^{2/3}\,\chi^{4/3}}}{\sigma}+\frac{(D-3)\,p^2}{6\,m^7\,\chi^2}-\frac{2\,(D-3)\,\sigma^2\,\sqrt[3]{u}}{3\,m^5\,\chi^{4/3}}+\frac{(D-3)\,(2\,u-1)}{6\,m^5\,\chi^2}\right)-$$

$$\left.\frac{i\,(D-3)\,e\sigma\,\sigma\mathrm{F}\,u^{2/3}}{2\,m^2\,\chi^{2/3}}+\frac{(D-3)\,\gamma\cdot p}{m\,(u+1)}-D+3\right)+$$

$$\mathrm{J0}\left(-\frac{i\,e\sigma\,((D-6)\,u-4)\,\mathrm{FDp}^{\beta}\,\gamma^{\beta}.\overline{\gamma}^{5}}{m^3\,\sqrt[3]{u}\,(u+1)\,\chi^{2/3}}+\frac{2\,e^2\,\sigma^2\,((D-2)\,u+6)\,\gamma\cdot\mathrm{FFp}}{3\,m^5\,u^{2/3}\,\chi^{4/3}}+\right.$$

$$\left.\frac{i\,e\sigma\,\sigma\mathrm{F}((D-4)\,u-4)}{2\,m^2\,\sqrt[3]{u}\,\chi^{2/3}}+\frac{(2-D)\,\gamma\cdot p}{m\,u+m}+D\right)+\mathrm{J1}\left(-\frac{i\,e\sigma\,(u+2)\,\mathrm{FDp}^{\beta}\,\gamma^{\beta}.\overline{\gamma}^{5}}{m^3\,\sqrt[3]{u}\,(u+1)\,\chi^{2/3}}+\right.$$

$$e^2\,\gamma\cdot\mathrm{FFp}\left(\frac{-\frac{i\,(D-2)\,(u-2)\,\sqrt[3]{u\,\chi^2}}{6\,m^5\,(u+1)\,\chi^2}-\frac{i}{m^5\,u^{2/3}\,\chi^{4/3}}}{\sigma}+\frac{p^2}{6\,m^7\,\chi^2}-\frac{2\,\sigma^2\,(u+3)}{3\,m^5\,u^{2/3}\,\chi^{4/3}}+\frac{2\,u-1}{6\,m^5\,\chi^2}\right)+$$

$$\left.\frac{i\,e\sigma\,\sigma\mathrm{F}(u+2)}{2\,m^2\,\sqrt[3]{u}\,\chi^{2/3}}+\frac{\gamma\cdot p}{m\,u+m}-1\right)-\frac{(D-3)\,e^2\,\mathrm{J2t}\,(u+4)\,\gamma\cdot\mathrm{FFp}}{6\,m^5\,u\,\chi^2}-\frac{e^2\,\mathrm{J1t}\,(u+4)\,\gamma\cdot\mathrm{FFp}}{6\,m^5\,u\,\chi^2}$$

Let us rewrite the result using the following notation

$$\mathrm{MFFp} == \frac{e^2\,\gamma^{\mu}\,\mathrm{FFp}_{\mu}}{m^5\,\chi^2}\left(\frac{\chi}{u}\right)^{2/3};$$

$$\mathrm{MFDp} == \frac{e\gamma^{\mu}\,\gamma^5\,\mathrm{FDp}_{\mu}}{m^3\,\chi}\left(\frac{\chi}{u}\right)^{1/3},\quad \mathrm{FDp}_{\mu} == (F^*\,p)_{\mu};$$

$$\mathrm{M}\sigma\mathrm{F} == \frac{e\,\sigma^{\mu\nu}\,F_{\mu\nu}}{m^2\,\chi}\left(\frac{\chi}{u}\right)^{1/3};$$

In[106]:=
```
Coeff13 = Coeff12
Phase13 = Phase12
Matrix13 =
 Collect[
  Expand[
   Matrix12 /. {DiracGamma[Momentum[FFp, D], D] → MFFp / e^2 * m^5 χ^(4 / 3) u^(2 / 3),
     DiracGamma[LorentzIndex[β, D], D].DiracGamma[5] ×
       Pair[LorentzIndex[β, D], Momentum[FDp, D]] →
      MFDp / e * m^3 χ^(2 / 3) u^(1 / 3), σF → MσF / e * m^2 χ^(2 / 3) u^(1 / 3)}
  ],
  {J0, J1, J2, dJ0dχl, dJ1dχl, dJ2dχl, MFFp, MFDp, MσF,
   DiracGamma[Momentum[p, D], D], Pair[Momentum[a, D], Momentum[p, D]], σ}, Simplify]
```

Out[106]=
$$\frac{i\, 2^{-D-1} e^{-\frac{1}{4} i \pi D} \pi^{-\frac{D}{2}-1} e^2\, m \Lambda^{4-D} \left(\frac{\sigma\,(u+1)^2}{m^2\, u^{4/3}\, \chi^{2/3}}\right)^{2-\frac{D}{2}}}{\sigma\,(u+1)^2}$$

Out[107]=
$$\frac{p^2\,\sigma}{m^2 \sqrt[3]{u\,\chi^2}} - \frac{\sigma^3}{3} - \frac{\sigma\,(u+1)}{\sqrt[3]{u\,\chi^2}}$$

Out[108]=
$$\mathrm{J2}\left(\mathrm{MFFp}\left(-\frac{i\left(\left(D^2-5D+6\right)u^2-2\left(D^2-5D+3\right)u+6\right)}{6\,\sigma\,(u+1)}+\frac{(D-3)\left(\frac{u}{\chi}\right)^{2/3}\left(m^2\,(2u-1)+p^2\right)}{6\,m^2}-\frac{2}{3}(D-3)\,\sigma^2\,u\right)+\right.$$
$$\left.\frac{(D-3)\,\gamma\cdot p}{m\,(u+1)}+\frac{i\,(D-3)\,\mathrm{MFDp}\,\sigma\,u}{u+1}-\frac{1}{2}\,i\,(D-3)\,\mathrm{M\sigma F}\sigma\,u-D+3\right)+$$
$$\mathrm{J0}\left(\frac{(2-D)\,\gamma\cdot p}{m\,u+m}-\frac{i\,\mathrm{MFDp}\,\sigma\,((D-6)\,u-4)}{u+1}+\mathrm{MFFp}\,\sigma^2\left(\frac{2}{3}(D-2)\,u+4\right)+\frac{1}{2}\,i\,\mathrm{M\sigma F}\sigma\,((D-4)\,u-4)+D\right)+$$
$$\mathrm{J1}\left(\mathrm{MFFp}\left(-\frac{i\left((D-2)\,u^2-2\,(D-5)\,u+6\right)}{6\,\sigma\,(u+1)}+\frac{\left(\frac{u}{\chi}\right)^{2/3}\left(m^2\,(2u-1)+p^2\right)}{6\,m^2}-\frac{2}{3}\,\sigma^2\,(u+3)\right)+\right.$$
$$\left.\frac{\gamma\cdot p}{m\,u+m}-\frac{i\,\mathrm{MFDp}\,\sigma\,(u+2)}{u+1}+\frac{1}{2}\,i\,\mathrm{M\sigma F}\sigma\,(u+2)-1\right)-\frac{\mathrm{MFFp}\,(u+4)\,((D-3)\,\mathrm{J2t}+\mathrm{J1t})}{6\sqrt[3]{u\,\chi^2}}$$

In order to remove the terms in Matrix13 that are proportional to (MFFp σ^{-1}),
we integrate over σ by parts.

Let us consider the integral

$\int_0^\infty d\sigma\, g(\sigma)\, \sigma^{3-D/2}\, \mathrm{Exp}\left[-i\, \sigma^3 - i\, z\, \sigma\right],\ 4 - D = \epsilon > 0,$

where $g(\sigma) = J_{1,2}(t(u, \sigma), \chi_l)$, and

where $g(\sigma)$ satisfies the condition :

$g(\sigma)\, \sigma^{1-D/2} \to 0,\ \sigma \to 0;\ \ g(\infty) = 0.$

Then, we may write

$$\int_0^\infty d\sigma\, g(\sigma)\, \sigma^{3-D/2}\, \mathrm{Exp}\left[-i\, \sigma^3 - i\, z\, \sigma\right] =$$
$$= i \int_0^\infty d\left(\mathrm{Exp}\left[-i\sigma^3\right]\right) g(\sigma)\, \sigma^{1-D/2}\, \mathrm{Exp}\left[-i\, \sigma^3 - i\, z\, \sigma\right] =$$
$$= -i \int_0^\infty d\sigma\, \sigma^{1-D/2}\, \mathrm{Exp}\left[-i\, \sigma^3 - i\, z\, \sigma\right] \times \left[g'(\sigma) + \left(-\frac{D/2-1}{\sigma} - i z\right) g(\sigma)\right],$$

therefore

$$\int_0^\infty d\sigma\, \sigma^{1-D/2}\, \mathrm{Exp}\left[-i\, \sigma^3 - i\, z\, \sigma\right] \frac{1}{\sigma}\, g(\sigma) =$$
$$= \int_0^\infty d\sigma\, \sigma^{1-D/2} \left[\mathrm{Exp}\left[-i\, \sigma^3 - i\, z\, \sigma\right] \times \frac{1}{D/2-1} \left[g'(\sigma) - i\left(\sigma^2 + z\right) g(\sigma)\right]\right].$$

For $g'(\sigma)$ we have

$$g'(\sigma) =$$
$$\frac{d}{d\sigma} J_{1,2}(t(u, \sigma), \chi_l) = \frac{d}{d\sigma}(-i) \int_{-\infty}^\infty d\, l^2\, D_k(l^2, \chi_l)\, \mathrm{Exp}\left[-i \left(\frac{u}{\chi}\right)^{2/3} \frac{1+u}{u^2} \frac{l^2}{m^2}\, \sigma\right] =$$
$$= (-i) \int_{-\infty}^\infty d\, l^2 \left(-i \left(\frac{u}{\chi}\right)^{2/3} \frac{1+u}{u^2} \frac{l^2}{m^2}\right) D_k(l^2, \chi_l)\, \mathrm{Exp}\left[-i\, t(u, \sigma)\, l^2\right]$$
$$= -i \left(\frac{u}{\chi}\right)^{2/3} \frac{1+u}{u^2}\, \tilde{J}_k.$$

At this step we substitute J_k and $\tilde{J}_k$ explicitly :

$$J_k = -i \int_{-\infty}^\infty d\, l^2\, D_k(l^2, \chi_l)\, \mathrm{Exp}\left[-i\, t(u, \sigma)\, l^2\right],$$
$$\tilde{J}_k = -i \int_{-\infty}^\infty d\, l^2\, \frac{l^2}{m^2}\, D_k(l^2, \chi_l)\, \mathrm{Exp}\left[-i\, t(u, \sigma)\, l^2\right]$$

and introduce notations

$D0 == D_0(l^2),$

$Dk == D_k(l^2, \chi_l),\ k = 1, 2.$

Outer integration : $\int_0^\infty ds \int_0^\infty dt \int_{-\infty}^\infty d\, l^2 \ldots$

The nonrenormalized diagonal part of the mass operator in D dimensions :

In[109]:=
```
Coeff14 = Coeff13
Phase14 = Phase13
Matrix14 = Collect[Expand[Simplify[Matrix13 - Coefficient[Matrix13, 1 / σ] / σ +
      (Coefficient[Matrix13, 1 / σ] * I (-σ - z / σ) * σ / (D / 2 - 1) /.
        {z → (u / χ)^(2 / 3) (1 - (pv2 - m^2) / m^2 / u + (1 + u) / u^2 lv2 / m^2)})
    ]] /. {lv2 J1 → m^2 J1t, lv2 J2 → m^2 J2t} , {J0, J1, J2, J1t, J2t, MFFp,
  MFDp, MσF, DiracGamma[Momentum[p, D], D], pv2, u / χ, σ}, Simplify]
```

Out[109]=
$$\frac{i\, 2^{-D-1}\, e^{-\frac{1}{4} i \pi D} \pi^{-\frac{D}{2}-1} e^2\, m\, \Lambda^{4-D} \left(\frac{\sigma (u+1)^2}{m^2 u^{4/3} \chi^{2/3}}\right)^{2-\frac{D}{2}}}{\sigma (u+1)^2}$$

Out[110]=
$$\frac{p^2 \sigma}{m^2 \sqrt[3]{u \chi^2}} - \frac{\sigma^3}{3} - \frac{\sigma (u+1)}{\sqrt[3]{u \chi^2}}$$

Out[111]=
$$\text{J2}\left(\text{MFFp}\left(\frac{p^2 \left(\left(D^2-5D+6\right)u^2-\left(D^2-5D+2\right)u+4\right)}{2(D-2)m^2 \sqrt[3]{u}\,(u+1)\,\chi^{2/3}} - \frac{\sigma^2\left(\left(D^2-5D+6\right)u^2+2u+2\right)}{(D-2)(u+1)} + \frac{\left(D^2-5D+2\right)u-4}{2(D-2)\sqrt[3]{u}\,\chi^{2/3}}\right) + \frac{(D-3)\gamma\cdot p}{m(u+1)} + \frac{i(D-3)\,\text{MFDp}\,\sigma\, u}{u+1} - \frac{1}{2} i (D-3)\,\text{M}\sigma\text{F}\sigma\, u - D + 3\right) - \frac{\text{J2t MFFp}\left(\left(D^2-5D+6\right)u^2+4u+4\right)}{2(D-2)u^{4/3}\chi^{2/3}} + \text{J0}\left(\frac{(2-D)\gamma\cdot p}{m u + m} - \frac{i\,\text{MFDp}\,\sigma\,((D-6)u-4)}{u+1} + \text{MFFp}\,\sigma^2\left(\frac{2}{3}(D-2)u+4\right) + \frac{1}{2} i\,\text{M}\sigma\text{F}\sigma\,((D-4)u-4) + D\right) + \text{J1}\left(\text{MFFp}\left(\frac{p^2\left((D-2)u^2-(D-6)u+4\right)}{2(D-2)m^2\sqrt[3]{u}\,(u+1)\,\chi^{2/3}} + \frac{\sigma^2\left(2\left(u^2+u+1\right)-D\left(u^2+2u+2\right)\right)}{(D-2)(u+1)} + \frac{(D-6)u-4}{2(D-2)\sqrt[3]{u}\,\chi^{2/3}}\right) + \frac{\gamma\cdot p}{m u + m} - \frac{i\,\text{MFDp}\,\sigma\,(u+2)}{u+1} + \frac{1}{2} i\,\text{M}\sigma\text{F}\sigma\,(u+2) - 1\right) - \frac{\text{J1t MFFp}\left((D-2)u^2+4u+4\right)}{2(D-2)u^{4/3}\chi^{2/3}}$$

In[112]:=
```
(*Coeff14=Coeff13/(I)
   Phase14=Phase13-lv2 tuσ
     Expand[Matrix13]/.
   {J0→ D0,J1→ D1,J2→ D2,J0t→ lv2/m^2 D0,J1t→ lv2/m^2  D1,J2t→ lv2/m^2  D2};
Matrix14=Collect[Expand[Simplify[%-Coefficient[%,1/σ]/σ+(Coefficient[%,1/σ]*I
        (-σ-z/σ)*σ/(D/2-1)/.{z→(u/χ)^(2/3)(1-(pv2-m^2)/m^2/u+(1+u)/u^2 lv2/m^2)})]],
  {D0,D1,D2,MFFp,MFDp, MσF, DiracGamma[Momentum[p,D],D],pv2,lv2,χ,σ},Simplify]*)
```

In[113]:=
```
"z = " (u / χ)^(2 / 3)
 Collect[Expand[-Coefficient[Phase14, σ] * (χ / u)^(2 / 3)], {pv2, lv2}, Simplify]
```

Out[113]=
$$\text{z} = \left(\frac{u}{\chi}\right)^{2/3}\left(-\frac{p^2}{m^2 u} + \frac{1}{u} + 1\right)$$

The nonrenormalized diagonal part of the mass operator in D = 4 :

In[114]:=
```
Coeff14D4 = Coeff14 * σ /. {D → 4}
Phase14D4 = Phase14 /. {D → 4}
Matrix14D4 = Collect[
  Matrix14 / σ /. {D → 4},
  {J0, J1, J2, J1t, J2t, MFFp, MFDp, MσF,
   DiracGamma[Momentum[p, D], D] /. {D → 4}, pv2 /. {D → 4}, lv2 /. {D → 4}, χ, σ}, Simplify]
```

Out[114]=
$$-\frac{i\, e^2\, m}{32\, \pi^3\, (u+1)^2}$$

Out[115]=
$$\frac{\sigma\, \overline{p}^2}{m^2 \sqrt[3]{u\, \chi^2}} - \frac{\sigma^3}{3} - \frac{\sigma\, (u+1)}{\sqrt[3]{u\, \chi^2}}$$

Out[116]=
$$\mathrm{J0}\left(-\frac{2\, \overline{\gamma}\cdot\overline{p}}{\sigma\, (m\, u+m)} + \frac{2\, i\, \mathrm{MFDp}\, (u+2)}{u+1} + \frac{4}{3}\mathrm{MFFp}\, \sigma\, (u+3) - 2\, i\, \mathrm{M}\sigma\mathrm{F} + \frac{4}{\sigma}\right) +$$
$$\mathrm{J1}\left(\mathrm{MFFp}\left(\frac{\left(u^2+u+2\right)\overline{p}^2}{2\, m^2\, \sigma \sqrt[3]{u}\, (u+1)\, \chi^{2/3}} - \frac{\sigma\left(u^2+3\, u+3\right)}{u+1} + \frac{-u-2}{2\, \sigma \sqrt[3]{u}\, \chi^{2/3}}\right) + \right.$$
$$\left.\frac{\overline{\gamma}\cdot\overline{p}}{\sigma\, (m\, u+m)} - \frac{i\, \mathrm{MFDp}\, (u+2)}{u+1} + \frac{1}{2}\, i\, \mathrm{M}\sigma\mathrm{F}(u+2) - \frac{1}{\sigma}\right) +$$
$$\mathrm{J2}\left(\mathrm{MFFp}\left(\frac{\left(u^2+u+2\right)\overline{p}^2}{2\, m^2\, \sigma \sqrt[3]{u}\, (u+1)\, \chi^{2/3}} - \frac{\sigma\left(u^2+u+1\right)}{u+1} + \frac{-u-2}{2\, \sigma \sqrt[3]{u}\, \chi^{2/3}}\right) + \frac{\overline{\gamma}\cdot\overline{p}}{\sigma\, (m\, u+m)} + \right.$$
$$\left.\frac{i\, \mathrm{MFDp}\, u}{u+1} - \frac{1}{2}\, i\, \mathrm{M}\sigma\mathrm{F}\, u - \frac{1}{\sigma}\right) - \frac{\mathrm{J1t}\, \mathrm{MFFp}\left(u^2+2\, u+2\right)}{2\, \sigma\, u^{4/3}\, \chi^{2/3}} - \frac{\mathrm{J2t}\, \mathrm{MFFp}\left(u^2+2\, u+2\right)}{2\, \sigma\, u^{4/3}\, \chi^{2/3}}$$

Renormalization

$M = M_0 + \delta M$

where we attribute the term incorporating J_0 to M_0 ,
and which coincides with the 1 – loop order unrenormalized mass operator,
and the rest to δM,
which is associated with the polarization loop insertions.

M_0 should be renormalized, δM is finite. Therefore,
we take the limit $D \to 4$ in δM.

We renormalize M_0 as follows
$M_0(p, F) \to [M_0(p, 0)]_{ren} + [M_0(p, F) - M_0(p, 0)]$

The second term gives a regular field dependent
part. $[M_0(p, 0)]_{ren}$ is the renormalized field – free mass operator.

In what follows, we write only the field − dependent part of M

Let us introduce the Ritus functions

$f(z) = i \int_0^\infty d\sigma \, \mathrm{Exp}\left(-i \frac{\sigma^3}{3} - i z \sigma\right),$

$f'(z) = \int_0^\infty d\sigma \, \sigma \, \mathrm{Exp}\left(-i \frac{\sigma^3}{3} - i z \sigma\right),$

$f_1(z) = \int_0^\infty \frac{d\sigma}{\sigma} e^{-i z \sigma}\left[\mathrm{Exp}\left(-i \frac{\sigma^3}{3}\right) - 1\right]$

$z = \left(\frac{u}{\chi}\right)^{2/3} \left(1 - \frac{1}{u} \frac{p^2 - m^2}{m^2} + \frac{1+u}{u^2} \frac{l^2}{m^2}\right)$

In effect, in the limit D →
 4 the renormalization of M_0 is reduced to the substitutuion

$\frac{1}{\sigma} \mathrm{Exp}\left(-i \frac{\sigma^3}{3} - i z \sigma\right) \to \frac{1}{\sigma} e^{-i z \sigma}\left[\mathrm{Exp}\left(-i \frac{\sigma^3}{3}\right) - 1\right]$

or

$\int_0^\infty \frac{d\sigma}{\sigma} \mathrm{Exp}\left(-i \frac{\sigma^3}{3} - i z \sigma\right) \to f_1(z)$

We also substitute

$\int_0^\infty d\sigma \, \sigma \, \mathrm{Exp}\left(-i \frac{\sigma^3}{3} - i z \sigma\right) \to f'(z)$

$\int_0^\infty d\sigma \, \mathrm{Exp}\left(-i \frac{\sigma^3}{3} - i z \sigma\right) \to -i f(z)$

and use

$J_0 = 2 \pi i \theta(t)$

Matrix15M0 now contains the phase factor inside the Ritus f − functions.

In[117]:=
```
pv2D4 = pv2 /. {D → 4};
lv2D4 = lv2 /. {D → 4};

zarg = Collect[Phase14D4 /. {σ^3 → 0} /. {σ → -1} /. {1 / (u χ^2)^(1/3) → (u / χ)^(2 / 3) / u,
    1 / (u^(4/3) χ^(2/3)) → (u / χ)^(2 / 3) / u^2}, {(u / χ)^(2 / 3)}, Simplify]
Clear[f];
Coeff15M0 = Coeff14D4 * 2 π I * 2 /. {e^2 → α 4 π}
Matrix15M0 = Collect[Expand[Coefficient[Matrix14D4, J0] / 2] /.
   {1 / σ → f1[z], σ → f '[z], MFDp → -I MFDp f[z], MσF → -I MσF f[z]}, {f[z], f '[z], f1[z]}]
```

Out[119]= $$\left(\frac{u}{\chi}\right)^{2/3}\left(-\frac{\overline{p}^2}{m^2 u}+\frac{1}{u}+1\right)$$

Out[121]= $$\frac{\alpha m}{2\pi (u+1)^2}$$

Out[122]= $$\mathrm{f1}(z)\left(2-\frac{\overline{\gamma}\cdot\overline{p}}{m u+m}\right)+\left(\frac{2\,\mathrm{MFFp}\,u}{3}+2\,\mathrm{MFFp}\right)f'(z)+f(z)\left(\frac{\mathrm{MFDp}\,u}{u+1}+\frac{2\,\mathrm{MFDp}}{u+1}-\mathrm{M}\sigma\mathrm{F}\right)$$

The nontrivial part of δM

In[123]:=
```
Coeff15δM = Coeff14D4 /. {e^2 → α 4 π}
Phase15 = Phase14D4
Matrix15δM = Matrix14D4 - Coefficient[Matrix14D4, J0] J0
```

Out[123]= $$-\frac{i\alpha m}{8\pi^2 (u+1)^2}$$

Out[124]= $$\frac{\sigma \overline{p}^2}{m^2 \sqrt[3]{u\,\chi^2}}-\frac{\sigma^3}{3}-\frac{\sigma(u+1)}{\sqrt[3]{u\,\chi^2}}$$

Out[125]= $$\mathrm{J1}\left(\mathrm{MFFp}\left(\frac{(u^2+u+2)\overline{p}^2}{2m^2\sigma\sqrt[3]{u}\,(u+1)\chi^{2/3}}-\frac{\sigma(u^2+3u+3)}{u+1}+\frac{-u-2}{2\sigma\sqrt[3]{u}\,\chi^{2/3}}\right)+\frac{\overline{\gamma}\cdot\overline{p}}{\sigma(m u+m)}-\frac{i\,\mathrm{MFDp}\,(u+2)}{u+1}+\frac{1}{2}i\,\mathrm{M}\sigma\mathrm{F}(u+2)-\frac{1}{\sigma}\right)+$$
$$\mathrm{J2}\left(\mathrm{MFFp}\left(\frac{(u^2+u+2)\overline{p}^2}{2m^2\sigma\sqrt[3]{u}\,(u+1)\chi^{2/3}}-\frac{\sigma(u^2+u+1)}{u+1}+\frac{-u-2}{2\sigma\sqrt[3]{u}\,\chi^{2/3}}\right)+\frac{\overline{\gamma}\cdot\overline{p}}{\sigma(m u+m)}+\frac{i\,\mathrm{MFDp}\,u}{u+1}-\frac{1}{2}i\,\mathrm{M}\sigma\mathrm{F}\,u-\frac{1}{\sigma}\right)-\frac{\mathrm{J1t}\,\mathrm{MFFp}\,(u^2+2u+2)}{2\sigma u^{4/3}\chi^{2/3}}-\frac{\mathrm{J2t}\,\mathrm{MFFp}\,(u^2+2u+2)}{2\sigma u^{4/3}\chi^{2/3}}$$

Let us rewrite the answer in the following form

$$M(p,\ F) = \sum_{n=0}^{2} \Big[m\, S_n(p^2,\ \chi) + (\gamma p)\, V_n^{(1)}(p^2,\ \chi) + \frac{(\gamma F^2 p)}{m^4\, \chi^2} V_n^{(2)}(p^2,\ \chi) + \frac{(\sigma F)}{m\chi} T_n(p^2,\ \chi) + \frac{(\gamma F^* p)\cdot\gamma^5}{m^2\, \chi} A_n(p^2,\ \chi)$$

$$\mathrm{MFFpV2} = \mathrm{MFFp} * m \left(\frac{\chi}{u}\right)^{-2/3} == \frac{e^2\, \gamma^\mu\, \mathrm{FFp}_\mu}{m^4\, \chi^2};$$

$$\mathrm{MFDpA} = \mathrm{MFDp} * m \left(\frac{\chi}{u}\right)^{-1/3} == \frac{e \gamma^\mu\, \gamma^5\, \mathrm{FDp}_\mu}{m^2\, \chi},\quad \mathrm{FDp}_\mu == (F^*\, p)_\mu;$$

$$\mathrm{M\sigma FT} = \mathrm{M\sigma F} * m \left(\frac{\chi}{u}\right)^{-1/3} == \frac{e\, \sigma^{\mu\nu}\, F_{\mu\nu}}{m\, \chi};$$

In[126]:=
```
Coeff15M01 = Collect[Matrix15M0 * Coeff15M0 /. {MFFp → MFFpV2 / m (χ / u)^(2 / 3),
    MFDp → MFDpA / m (χ / u)^(1 / 3), MσF → MσFT / m (χ / u)^(1 / 3)},
  {D0, f[z], f '[z], f1[z], DiracGamma[Momentum[p]], MFDpA, MσFT}, Simplify]
Coeff15δM1 = Collect[Expand[Matrix15δM * Coeff15δM] /. {MFFp → MFFpV2 / m (χ / u)^(2 / 3),
    MFDp → MFDpA / m (χ / u)^(1 / 3), MσF → MσFT / m (χ / u)^(1 / 3)},
  {MFFpV2, J1, J1t, J2, J2t, DiracGamma[Momentum[p]], MFDpA, MσFT, σ}, Simplify]
```

Out[126]=

$$\mathrm{f1}(z)\left(\frac{\alpha\, m}{\pi\,(u+1)^2} - \frac{\alpha\, \overline{\gamma}\cdot\overline{p}}{2\,\pi\,(u+1)^3}\right) + \frac{\alpha\, \mathrm{MFFpV2}\,(u+3)\left(\frac{\chi}{u}\right)^{2/3} f'(z)}{3\,\pi\,(u+1)^2} + f(z)\left(\frac{\alpha\, \mathrm{MFDpA}\,(u+2)\sqrt[3]{\frac{\chi}{u}}}{2\,\pi\,(u+1)^3} - \frac{\alpha\, \mathrm{M\sigma FT}\sqrt[3]{\frac{\chi}{u}}}{2\,\pi\,(u+1)^2}\right)$$

Out[127]=
$$\text{MFFpV2}\left(\text{J1}\left(\frac{i\,\alpha\left(m^2\left(u^2+3\,u+2\right)-\left(u^2+u+2\right)\overline{p}^2\right)}{16\,\pi^2\,m^2\,\sigma\,u\,(u+1)^3}+\frac{i\,\alpha\,\sigma\left(u^2+3\,u+3\right)\left(\frac{\chi}{u}\right)^{2/3}}{8\,\pi^2\,(u+1)^3}\right)+\text{J2}\left(\frac{i\,\alpha\left(m^2\left(u^2+3\,u+2\right)-\left(u^2+u+2\right)\overline{p}^2\right)}{16\,\pi^2\,m^2\,\sigma\,u\,(u+1)^3}+\frac{i\,\alpha\,\sigma\left(u^2+u+1\right)\left(\frac{\chi}{u}\right)^{2/3}}{8\,\pi^2\,(u+1)^3}\right)+\frac{i\,\alpha\,\text{J1t}\left(u^2+2\,u+2\right)}{16\,\pi^2\,\sigma\,u^2\,(u+1)^2}+\frac{i\,\alpha\,\text{J2t}\left(u^2+2\,u+2\right)}{16\,\pi^2\,\sigma\,u^2\,(u+1)^2}\right)+\text{J1}\left(-\frac{i\,\alpha\,\overline{\gamma}\cdot\overline{p}}{8\,\pi^2\,\sigma\,(u+1)^3}+\frac{i\,\alpha\,m}{8\,\pi^2\,\sigma\,(u+1)^2}-\frac{\alpha\,\text{MFDpA}\,(u+2)\,\sqrt[3]{\frac{\chi}{u}}}{8\,\pi^2\,(u+1)^3}+\frac{\alpha\,\text{M}\sigma\text{FT}\,(u+2)\,\sqrt[3]{\frac{\chi}{u}}}{16\,\pi^2\,(u+1)^2}\right)+\text{J2}\left(-\frac{i\,\alpha\,\overline{\gamma}\cdot\overline{p}}{8\,\pi^2\,\sigma\,(u+1)^3}+\frac{i\,\alpha\,m}{8\,\pi^2\,\sigma\,(u+1)^2}+\frac{\alpha\,\text{MFDpA}\,\sqrt[3]{u^2\,\chi}}{8\,\pi^2\,(u+1)^3}-\frac{\alpha\,\text{M}\sigma\text{FT}\,\sqrt[3]{u^2\,\chi}}{16\,\pi^2\,(u+1)^2}\right)$$

In[128]:=
```
"S0 = " TraditionalForm[
  Coeff15M01 / m /. {MFFpV2 → 0, DiracGamma[Momentum[p]] → 0, MFDpA → 0, MσFT → 0}]
"V0(1) = " TraditionalForm[Coefficient[Coeff15M01, DiracGamma[Momentum[p]]]]
"V0(2) = " TraditionalForm[(- 8 π^2 / α (1 + u)^2 / (-I) 2)^(-1) Collect[Expand[
     Coefficient[Coeff15M01 * (- 8 π^2 / α (1 + u)^2 / (-I) 2), MFFpV2]],
   {σ, J1, J1t, J2, J2t, (pv2 /. {D → 4})}, Simplify]]
"T0 = " TraditionalForm[(16 π^2 χ / α (χ / u)^(-1 / 3) (1 + u)^2)^(-1) Collect[Expand[
     Coefficient[Coeff15M01, MσFT] * (16 π^2 χ / α (χ / u)^(-1 / 3) (1 + u)^2)],
   {J1, J1t, J2, J2t, σ, (pv2 /. {D → 4})}, Simplify]]
"A0 = " TraditionalForm[(-8 π^2 χ / α (χ / u)^(-1 / 3) (1 + u)^3)^(-1)
  Collect[Expand[Coefficient[Coeff15M01, MFDpA] (-8 π^2 χ / α (χ / u)^(-1 / 3) (1 + u)^3)],
   {σ, J1, J1t, J2, J2t, (pv2 /. {D → 4})}, Simplify]]
```

Out[128]= $S_0 = \frac{\alpha\, \mathrm{f1}(z)}{\pi\,(u+1)^2}$

Out[129]= $V_0^{(1)} = \left(-\frac{\alpha\, \mathrm{f1}(z)}{2\,\pi\,(u+1)^3}\right)$

Out[130]= $V_0^{(2)} = \frac{(u+3)\,\alpha\left(\frac{\chi}{u}\right)^{2/3} f'(z)}{3\,\pi\,(u+1)^2}$

Out[131]= $T_0 = \left(-\frac{\alpha \sqrt[3]{\frac{\chi}{u}}\; f(z)}{2\,\pi\,(u+1)^2}\right)$

Out[132]= $A_0 = \frac{(u+2)\,\alpha \sqrt[3]{\frac{\chi}{u}}\; f(z)}{2\,\pi\,(u+1)^3}$

```
In[133]:= "S1,2 = "TraditionalForm[
   Coeff15δM1/ m /. {MFFpV2 → 0, DiracGamma[Momentum[p]] → 0, MFDpA → 0, MσFT → 0}]
  "V(1)1,2 = "TraditionalForm[Coefficient[Coeff15δM1, DiracGamma[Momentum[p]]]]
  "V(2)1,2 = "TraditionalForm[(- 8 π ^2 / α (1 + u)^2 / (-I) 2)^(-1) Collect[Expand[
      Coefficient[Coeff15δM1* (- 8 π ^2 / α (1 + u)^2 / (-I) 2), MFFpV2]],
     {σ, J1, J1t, J2, J2t, (pv2 /. {D → 4})}, Simplify]]
  "T1,2 = "TraditionalForm[(16 π ^2 / α (χ / u)^(-1 / 3) (1 + u)^2)^(-1) Collect[Expand[
      Coefficient[Coeff15δM1, MσFT] * (16 π ^2 / α (χ / u)^(-1 / 3) (1 + u)^2)],
     {J1, J1t, J2, J2t, σ, (pv2 /. {D → 4})}, Simplify]]
  "A1,2 = "TraditionalForm[(-8 π ^2  / α (χ / u)^(-1 / 3) (1 + u)^3)^(-1)
    Collect[Expand[Coefficient[Coeff15δM1, MFDpA] (-8 π ^2  / α (χ / u)^(-1 / 3) (1 + u)^3)],
     {σ, J1, J1t, J2, J2t, (pv2 /. {D → 4})}, Simplify]]
```

Out[133]= $S_{1,2} = \dfrac{\dfrac{i\,\mathrm{J1}\,m\,\alpha}{8\,\pi^2\,(u+1)^2\,\sigma} + \dfrac{i\,\mathrm{J2}\,m\,\alpha}{8\,\pi^2\,(u+1)^2\,\sigma}}{m}$

Out[134]= $V^{(1)}_{1,2} = \left(-\dfrac{i\,\mathrm{J1}\,\alpha}{8\,\pi^2\,(u+1)^3\,\sigma} - \dfrac{i\,\mathrm{J2}\,\alpha}{8\,\pi^2\,(u+1)^3\,\sigma}\right)$

Out[135]= $V^{(2)}_{1,2} = \dfrac{i\,\alpha\left(\sigma\left(\dfrac{2\,\mathrm{J2}\left(\frac{\chi}{u}\right)^{2/3}\left(u^2+u+1\right)}{u+1} + \dfrac{2\,\mathrm{J1}\left(u^2+3\,u+3\right)\left(\frac{\chi}{u}\right)^{2/3}}{u+1}\right) + \dfrac{\frac{\mathrm{J1t}\left(u^2+2\,u+2\right)}{u^2} + \frac{\mathrm{J2t}\left(u^2+2\,u+2\right)}{u^2} + \mathrm{J1}\left(\frac{u+2}{u} - \frac{\left(u^2+u+2\right)\vec{p}^{\,2}}{m^2\,u\,(u+1)}\right) + \mathrm{J2}\left(\frac{u+2}{u} - \frac{\left(u^2+u+2\right)\vec{p}^{\,2}}{m^2\,u\,(u+1)}\right)}{\sigma}\right)}{16\,\pi^2\,(u+1)^2}$

Out[136]= $T_{1,2} = \dfrac{(\mathrm{J1}\,(u+2) - \mathrm{J2}\,u)\,\alpha\,\sqrt[3]{\frac{\chi}{u}}}{16\,\pi^2\,(u+1)^2}$

Out[137]= $A_{1,2} = \left(-\dfrac{(\mathrm{J1}\,(u+2) - \mathrm{J2}\,u)\,\alpha\,\sqrt[3]{\frac{\chi}{u}}}{8\,\pi^2\,(u+1)^3}\right)$

The elastic scattering amplitude $\mathcal{M} = \overline{u}_p\, M(p)\, u_p$

We assume that $p^2 = 0$ and calculate the matrix element

$\mathcal{M} = \overline{u}_p \, M(p) \, u_p$,

where u_p is a free Dirac bispinor and $\overline{u}_p = \gamma^0 u_p^\dagger$.

To perform this calculation, we use the following relations
$\overline{u}_p u_p = 2\,m$,
$\overline{u}_p (\gamma p) u_p = 2\,m^2$,
$\overline{u}_p \, e^2 (\gamma F^2 p) u_p = 2\,m^6 \chi^2$,
$\overline{u}_p \, e\,(\sigma_{\mu\nu} F^{\mu\nu}) u_p = \frac{2}{m} e \left(\overline{u}_p \gamma^\beta \gamma^5 u_p\right) (F^* p)_\beta = 4\,s^\beta e\,(F^* p)_\beta$,
where $s^\beta = \frac{1}{2\,m} \overline{u}_p \gamma^\beta \gamma^5 u_p$ is the electron spin 4 − vector.

Recall
$\text{MFFp} == \frac{e^2 \gamma^\mu \text{FFp}_\mu}{m^5 \chi^2} \left(\frac{\chi}{u}\right)^{2/3}$,
$\text{MFDp} == \frac{e \gamma^\mu \gamma^5 \text{FDp}_\mu}{m^3 \chi} \left(\frac{\chi}{u}\right)^{1/3}$, $\text{FDp}_\mu == (F^* p)_\mu$,
$\text{M}\sigma\text{F} == \frac{e\,\sigma^{\mu\nu} F_{\mu\nu}}{m^2 \chi} \left(\frac{\chi}{u}\right)^{1/3}$,

in effect, we should perform the following substitutions
$\overline{u}_p \, \text{MFFp} \, u_p = 2\,m \left(\frac{\chi}{u}\right)^{2/3}$,
$\overline{u}_p \, \text{MFDp} \, u_p = \frac{2\,e}{m^2 \chi} s^\nu \, \text{FDp}_\nu \left(\frac{\chi}{u}\right)^{1/3}$,
$\overline{u}_p \, \text{M}\sigma\text{F} \, u_p = \frac{4\,e}{m^2 \chi} s^\nu \, \text{FDp}_\nu \left(\frac{\chi}{u}\right)^{1/3}$.

In[138]:=
```
Simplify[S + DiracGamma[Momentum[p]] / m V1 + MFFp V2 (χ / u)^(-2 / 3) + MσF T (χ / u)^(-1 / 3) +
    MFDp A (χ / u)^(-1 / 3) /. {DiracGamma[Momentum[p]] → 2 m^2, MFFp → 2 m χ^(2 / 3) / u^(2 / 3),
    MσF → e Contract[FVD[s, ν] × FDpv[ν]] 4 / m^2 / χ * (χ / u)^(1 / 3),
    MFDp → e Contract[FVD[s, ν] × FDpv[ν]] 2 / m^2 / χ * (χ / u)^(1 / 3), S → 2 m S}]
```

Out[138]=
$$\frac{2\,e\,(A + 2\,T)\,(\text{FDp} \cdot s)}{m^2\,\chi} + 2\,m\,(S + \text{V1} + \text{V2})$$

In[139]:=
```
Coeff16M0 = Coeff15M0 * 2 m
zargOS = zarg /. {(pv2 /. {D → 4}) → m ^ 2}
TermI0 = Expand[
    Simplify[Matrix15M0 /. {DiracGamma[Momentum[p]] → 0, MFFp → 0, MσF → 0, MFDp → 0}]];
1 / 2 / m Expand[Matrix15M0 - TermI0 + 2 m TermI0] /.
     {DiracGamma[Momentum[p]] → 2 m ^ 2, MFFp → 2 m χ ^ (2 / 3) / u ^ (2 / 3),
      MσF → e Contract[FVD[s, ν] × FDpv[ν]] 4 / m ^ 2 / χ * (χ / u) ^ (1 / 3), MFDp →
       e Contract[FVD[s, ν] × FDpv[ν]] 2 / m ^ 2 / χ * (χ / u) ^ (1 / 3)} /. {pv2 → m ^ 2} /. {D → 4};
Matrix16M0 = Collect[
   Expand[
     Simplify[% -
       Coefficient[%, e Pair[Momentum[s], Momentum[FDp]]] e Pair[Momentum[s], Momentum[FDp]]]]
    + Simplify[Coefficient[%, e Pair[Momentum[s], Momentum[FDp]]]
      e Pair[Momentum[s], Momentum[FDp]]]
   ,
   {f[z], f '[z], f1[z]}, Simplify]
```

Out[139]= $\frac{\alpha\, m^2}{\pi\,(u+1)^2}$

Out[140]= $\left(\frac{u}{\chi}\right)^{2/3}$

Out[143]= $-\frac{e\, f(z)\left(\frac{u}{\chi}\right)^{2/3}\left(\overline{\mathrm{FDp}}\cdot\overline{s}\right)}{m^3\,(u+1)} + \frac{2}{3}\,(u+3)\left(\frac{\chi}{u}\right)^{2/3} f'(z) + \frac{(2\,u+1)\,\mathrm{f1}(z)}{u+1}$

In[144]:=
```
Coeff16δM = Coeff15δM * 2 m
Phase16 = Collect[Phase15 /. {pv2D4 → m ^ 2}, σ, Simplify]
TermIδ = Expand[
    Simplify[Matrix15δM /. {DiracGamma[Momentum[p]] → 0, MFFp → 0, MσF → 0, MFDp → 0}]];
1 / 2 / m Expand[Matrix15δM - TermIδ + 2 m TermIδ] /.
     {DiracGamma[Momentum[p]] → 2 m ^ 2, MFFp → 2 m χ ^ (2 / 3) / u ^ (2 / 3),
      MσF → e Contract[FVD[s, ν] × FDpv[ν]] 4 / m ^ 2 / χ * (χ / u) ^ (1 / 3), MFDp →
        e Contract[FVD[s, ν] × FDpv[ν]] 2 / m ^ 2 / χ * (χ / u) ^ (1 / 3)} /. {pv2D4 → m ^ 2} /. {D → 4};
Matrix16δM = Collect[
   Expand[
     Simplify[% -
        Coefficient[%, e Pair[Momentum[s], Momentum[FDp]]] e Pair[Momentum[s], Momentum[FDp]]]]
    + Simplify[Coefficient[%, e Pair[Momentum[s], Momentum[FDp]]]
      e Pair[Momentum[s], Momentum[FDp]]]
   ,
   {J1, J2, J1t, J2t, MFFp, MFDp, MσF, DiracGamma[Momentum[p]], χ, σ}, Simplify]
```

Out[144]= $$-\frac{i\,\alpha\,m^2}{4\,\pi^2\,(u+1)^2}$$

Out[145]= $$-\frac{\sigma^3}{3}-\sigma\left(\frac{u}{\chi}\right)^{2/3}$$

Out[148]= $$\mathrm{J1}\left(\frac{i\,e\,(u+2)\left(\frac{u}{\chi}\right)^{2/3}\left(\overline{\mathrm{FDp}}\cdot\overline{s}\right)}{m^3\,(u+1)}-\frac{1}{\sigma}+\frac{\sigma\left(-u^2-3\,u-3\right)\chi^{2/3}}{u^{2/3}\,(u+1)}\right)+$$
$$\mathrm{J2}\left(-\frac{i\,e\,u\left(\frac{u}{\chi}\right)^{2/3}\left(\overline{\mathrm{FDp}}\cdot\overline{s}\right)}{m^3\,(u+1)}-\frac{1}{\sigma}-\frac{\sigma\left(u^{4/3}+\frac{1}{u^{2/3}}+\sqrt[3]{u}\right)\chi^{2/3}}{u+1}\right)-\frac{\mathrm{J1t}\left(u^2+2\,u+2\right)}{2\,\sigma\,u^2}-\frac{\mathrm{J2t}\left(u^2+2\,u+2\right)}{2\,\sigma\,u^2}$$

Let us rewrite Matrix16M0

we perform an additional transformation of the D_0 term

Let us use the following integral equality (we will prove it below)

$$\int_0^\infty \frac{du}{(1+u)^2}\left[\frac{2(u-2)u}{3(u+1)}\left(\frac{\chi}{u}\right)^{2/3} f'(z_0) + -\frac{2u}{u+1} f_1(z_0)\right] = 0$$

We add this expression to Matrix16M0

After these transformation we arrive at the expression corresponding to Eq.(11) in [A.A.Mironov, S.Meuren, A.M.Fedotov PRD 102, 053 005 (2020)].

In[149]:=
```
Matrix16M01 = Collect[
  Matrix16M0 + (2 (u - 2) u f '[z] / (3 (u + 1)) (χ / u)^(2 / 3) + f1[z] (-2 u / (u + 1))), {f[z], f '[z], f1[z]}, Simplify]
```

Out[149]=
$$-\frac{e\, f(z)\left(\frac{u}{\chi}\right)^{2/3}\left(\overline{\text{FDp}}\cdot\overline{s}\right)}{m^3\,(u+1)}+\frac{2\left(2u^2+2u+3\right)\left(\frac{\chi}{u}\right)^{2/3} f'(z)}{3\,(u+1)}+\frac{\text{f1}(z)}{u+1}$$

Proof of the integral equality

$$\mathbf{I} = \int_0^\infty \frac{du}{(1+u)^2}\left[\frac{2(u-2)u}{3(u+1)}\left(\frac{\chi}{u}\right)^{2/3} f'(z) + \left(\frac{l^2}{m^2} - \frac{2u}{u+1}\right) f_1(z)\right] = 0$$

Let us restore the explicit integral form of the Ritus f – functions and integrate the term with f ' (z) by parts

$$f'(z) = \int_0^\infty d\sigma\, \sigma\, \mathrm{Exp}\left(-i\,\frac{\sigma^3}{3} - i z\sigma\right) = i\int_0^\infty d\left[\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3}\right) - 1\right] \times \frac{1}{\sigma}\, e^{-i z\sigma} =$$

$$= -i\int_0^\infty d\sigma \times \left[\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3}\right) - 1\right] \times \left[-\frac{1}{\sigma^2} - \frac{i z}{\sigma}\right] e^{-i z\sigma},$$

where we used the fact that

$\left[\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3}\right) - 1\right] \times \frac{1}{\sigma}\, e^{-i z\sigma} = 0$ for $\sigma = 0$ and $\sigma = \infty$.

Note that after rewriting f ' (z),

the factor $\left[\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3}\right) - 1\right]$ will be common in I,

$$I = \left[\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3}\right) - 1\right] \int_0^\infty du\, e^{-i z(u)\sigma}[\ldots]$$

so we will only write out the expression in [...].

We also substitute

$$z(u) = \left(\frac{u}{\chi}\right)^{2/3}\left(\frac{(u+1)\,\vec{l}^{\,2}}{m^2 u^2} + 1\right).$$

In[150]:=
```
zarg1 = zargOS + lv2D4 tuσ / σ
Int =
 Collect[1 / (1 + u) ^ 2 (2 (u − 2) u / 3 / (1 + u) (χ / u) ^ (2 / 3) f '[z] + (lv2D4 / m ^ 2 − 2 u / (1 + u)) f1[z]) /.
    {f '[z] → −I (−1 / σ ^ 2 − I z / σ), f1[z] → 1 / σ} /. {z → zarg1},
  {σ, lv2D4}, Simplify]
```

Out[150]=
$$\frac{(u+1)\,\vec{l}^{\,2}}{m^2\, u^{4/3}\, \chi^{2/3}} + \left(\frac{u}{\chi}\right)^{2/3}$$

Out[151]=
$$\frac{\frac{(u+4)\,\vec{l}^{\,2}}{3\, m^2\, u\, (u+1)^2} - \frac{2u}{3\,(u+1)^2}}{\sigma} + \frac{2\, i\, \sqrt[3]{u}\, (u-2)\, \chi^{2/3}}{3\, \sigma^2\, (u+1)^3}$$

In the next step we prove that in fact this expression is a total derivative

$$I = \left[\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3}\right) - 1\right] \int_0^\infty du\,[\,...\,] = \left[\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3}\right) - 1\right] \int_0^\infty du\,\frac{d}{du}\left[P(u)\,e^{-i z(u)\,\sigma}\right],$$

and $P(0) = P(\infty) = 0$, which makes the statement evident.

To find $P(u)$, we expand the derivative $\frac{d}{du}[\,...\,]$ and equal the coefficient of l^2 to the corresponding coeffcient in the expression for Int.

We find that

$$P(u) = -\frac{i\,u^{4/3}\,\chi^{2/3}}{\sigma^2\,(u+1)^2}.$$

In the last two lines we check that the result is correct.

$$I = \int_0^\infty \frac{du}{(1+u)^2}\left[\frac{2\,(u-2)\,u}{3\,(u+1)}\left(\frac{\chi}{u}\right)^{2/3} f'(z) + \left(\frac{l^2}{m^2} - \frac{2\,u}{u+1}\right) f_1(z)\right] =$$

$$= -\frac{i}{\sigma^2}\left[\mathrm{Exp}\left(-i\,\frac{\sigma^3}{3}\right) - 1\right] \int_0^\infty du\,\frac{d}{du}\left[\frac{u^2}{(u+1)^2}\left(\frac{\chi}{u}\right)^{2/3} e^{-i z \sigma}\right] = 0.$$

In[152]:=
```
IntTest =
 Collect[Simplify[D[P[u] Exp[-I zarg1 σ], u] / Exp[-I zarg1 σ]], {λ, ν, P'[u], σ}, Simplify]
```

Out[152]= $P'(u) - \frac{i\,\sigma\,P(u)\left(2\,m^2\,u^2 - (u+4)\,\vec{l}^{\,2}\right)}{3\,m^2\,u^{7/3}\,\chi^{2/3}}$

In[153]:=
```
Psol[u] = P[u] /. Solve[Coefficient[Int, lv2D4] == Coefficient[IntTest, lv2D4], P[u]][[1]]
Collect[(IntTest /. {P[u] → Psol[u], P'[u] → D[Psol[u], u]}), {lv2D4, σ}, Simplify]

Simplify[Int - % /. {∛u χ^2 → χ^(2/3) u^(1/3)}]
```

Out[153]= $-\frac{i\,u^{4/3}\,\chi^{2/3}}{\sigma^2\,(u+1)^2}$

Out[154]= $\frac{(u+4)\,\vec{l}^{\,2}}{3\,m^2\,\sigma\,u\,(u+1)^2} + \frac{2\,i\,\sqrt[3]{u}\,(u-2)\,\chi^{2/3}}{3\,\sigma^2\,(u+1)^3} - \frac{2\,u}{3\,\sigma\,(u+1)^2}$

Out[155]= 0