This is a part of SFQED-Loops script collection
developed for calculating loop processes in
Strong-Field Quantum Electrodynamics.
The scripts are available on https://github.com/ArsenyMironov/SFQED-Loops

If you use this script in your research, please, consider
citing our papers:
- A. A. Mironov, S. Meuren, and A. M. Fedotov, PRD 102, 053005 (2020),
  https://doi.org/10.1103/PhysRevD.102.053005
- A. A. Mironov, A. M. Fedotov, arXiv:2109.00634 (2021)

If you have any questions, please, don’t hesitate to contact:
mironov.hep@gmail.com



$$x_1 = x_2$$

In[1]:=
```
NotebookEvaluate[NotebookOpen[NotebookDirectory[] <> "definitions.nb"]]
```

**FeynCalc** 9.3.1 (stable version). For help, use the documentation center, check out the wiki or visit the forum.

To save your and our time, please check our FAQ for answers to some common FeynCalc questions.

See also the supplied examples. If you use FeynCalc in your research, please cite

- V. Shtabovenko, R. Mertig and F. Orellana, Comput.Phys.Commun. 256 (2020) 107478, arXiv:2001.04407.
- V. Shtabovenko, R. Mertig and F. Orellana, Comput.Phys.Commun. 207 (2016) 432–444, arXiv:1601.01167.
- R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun. 64 (1991) 345–359.

Electron propagator (proper time is dimensionless)

$$S^c(x_2, x_1) = \Lambda^{4-D} \int \frac{d^D p}{(2\pi)^D} E_p(x_2) \frac{i((\gamma p) + m)}{p^2 - m^2 + i0} \overline{E}_p(x_1)$$

$x = x_2 - x_1$,

$X = \frac{1}{2}(x_1 + x_2)$,

$\xi^2 = -\frac{e^2 a^2}{m^2}$,

$[\Lambda] = m$ – mass scale,

$$E_p(x_2) = \left[1 - \frac{e(\gamma k)(\gamma a)}{2(kp)}(k x_2)\right] \mathrm{Exp}\left[-i(p\, x_2) + i \frac{e(a\, p)}{2(k\, p)}(k\, x_2)^2 + i \frac{a^2 e^2}{6(k\, p)}(k\, x_2)^3\right];$$

$(\gamma p - m) S^c(p) = i$ – in E – p representation

$S^c(p, q) = (2\pi)^4 \delta(p - q) S^c(p)$

In[2]:=
```
NewMomentum["p"]
NewCoordinate["x1"]
NewCoordinate["x2"]
NewCoordinate["x"]
NewCoordinate["X"]
```

$$\left\{p^{\alpha},\ p^2,\ k\cdot p,\ \mathrm{Fp}^{\alpha},\ \mathrm{FFp}^{\alpha},\ \mathrm{FDp}^{\alpha},\ a\cdot p,\ 0,\ 0,\ 0,\ -a^2\,(k\cdot p),\ 0,\ 0,\ -\frac{m^6\ \chi\mathrm{p}^2}{e^2},\ -\frac{m^6\ \chi\mathrm{p}^2}{e^2},\ \frac{m^6\ \chi\mathrm{p}^2}{e^2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}$$

$$\left\{\mathrm{x1}^{\alpha},\ \mathrm{x1}^2,\ k\cdot\mathrm{x1},\ a\cdot\mathrm{x1},\ \mathrm{Fx1}^{\alpha},\ \mathrm{FFx1}^{\alpha},\ \mathrm{FDx1}^{\alpha},\ k\cdot\mathrm{x1},\ 0,\ 0,\ 0,\ -a^2\,(k\cdot\mathrm{x1}),\ 0,\ 0,\ -\frac{m^2\ \xi^2\,(\,k\cdot\mathrm{x1}\,)^2}{e^2},\ -\frac{m^2\ \xi^2\,(\,k\cdot\mathrm{x1}\,)^2}{e^2},\ \frac{m^2\ \xi^2\,(\,k\cdot\mathrm{x1}\,)^2}{e^2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}$$

$$\left\{\mathrm{x2}^{\alpha},\ \mathrm{x2}^2,\ k\cdot\mathrm{x2},\ a\cdot\mathrm{x2},\ \mathrm{Fx2}^{\alpha},\ \mathrm{FFx2}^{\alpha},\ \mathrm{FDx2}^{\alpha},\ k\cdot\mathrm{x2},\ 0,\ 0,\ 0,\ -a^2\,(k\cdot\mathrm{x2}),\ 0,\ 0,\ -\frac{m^2\ \xi^2\,(\,k\cdot\mathrm{x2}\,)^2}{e^2},\ -\frac{m^2\ \xi^2\,(\,k\cdot\mathrm{x2}\,)^2}{e^2},\ \frac{m^2\ \xi^2\,(\,k\cdot\mathrm{x2}\,)^2}{e^2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}$$

$$\left\{x^{\alpha},\ x^2,\ k\cdot x,\ a\cdot x,\ \mathrm{Fx}^{\alpha},\ \mathrm{FFx}^{\alpha},\ \mathrm{FDx}^{\alpha},\ k\cdot x,\ 0,\ 0,\ 0,\ -a^2\,(k\cdot x),\ 0,\ 0,\ -\frac{m^2\ \xi^2\,(\,k\cdot x\,)^2}{e^2},\ -\frac{m^2\ \xi^2\,(\,k\cdot x\,)^2}{e^2},\ \frac{m^2\ \xi^2\,(\,k\cdot x\,)^2}{e^2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}$$

$$\left\{X^{\alpha},\ X^2,\ k\cdot X,\ a\cdot X,\ \mathrm{FX}^{\alpha},\ \mathrm{FFX}^{\alpha},\ \mathrm{FDX}^{\alpha},\ k\cdot X,\ 0,\ 0,\ 0,\ -a^2\,(k\cdot X),\ 0,\ 0,\ -\frac{m^2\ \xi^2\,(\,k\cdot X)^2}{e^2},\ -\frac{m^2\ \xi^2\,(\,k\cdot X)^2}{e^2},\ \frac{m^2\ \xi^2\,(\,k\cdot X)^2}{e^2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}$$

In[7]:=
```
Epx2 = Ep[x2, p]
EpBarx1 = EpC[x1, p]
```

Out[7]= $$\left\{1-\frac{e\,(k\cdot\mathrm{x2})\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)},\ \frac{a^2\ e^2\,(\,k\cdot\mathrm{x2}\,)^3}{6\,(k\cdot p)}+\frac{e\,(a\cdot p)\,(\,k\cdot\mathrm{x2}\,)^2}{2\,(k\cdot p)}-p\cdot\mathrm{x2}\right\}$$

Out[8]= $$\left\{1-\frac{e\,(k\cdot\mathrm{x1})\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot p)},\ -\frac{a^2\ e^2\,(\,k\cdot\mathrm{x1}\,)^3}{6\,(k\cdot p)}-\frac{e\,(a\cdot p)\,(\,k\cdot\mathrm{x1}\,)^2}{2\,(k\cdot p)}+p\cdot\mathrm{x1}\right\}$$

In[9]:=
```
Matrix = Epx2[[1]].(GAD[α] × Pair[Momentum[p, D], LorentzIndex[α, D]] + m).EpBarx1[[1]]
Coeff = ⅈ Λ^(4 - D) / (2 π)^D / (pv2 - m^2)
Phase = Epx2[[2]] + EpBarx1[[2]]
```

Out[9]= $$\left(1-\frac{e\,(k\cdot\mathrm{x2})\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)}\right)(m+\gamma^{\alpha}\ p^{\alpha}).\left(1-\frac{e\,(k\cdot\mathrm{x1})\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot p)}\right)$$

Out[10]= $$\frac{i\,(2\,\pi)^{-D}\,\Lambda^{4-D}}{p^2-m^2}$$

Out[11]= $$-\frac{a^2\ e^2\,(\,k\cdot\mathrm{x1}\,)^3}{6\,(k\cdot p)}+\frac{a^2\ e^2\,(\,k\cdot\mathrm{x2}\,)^3}{6\,(k\cdot p)}-\frac{e\,(a\cdot p)\,(\,k\cdot\mathrm{x1}\,)^2}{2\,(k\cdot p)}+\frac{e\,(a\cdot p)\,(\,k\cdot\mathrm{x2}\,)^2}{2\,(k\cdot p)}+p\cdot\mathrm{x1}-p\cdot\mathrm{x2}$$

In[12]:=
```
Matrix1 = Contract[DiracSimplify[Matrix]]
Coeff1 = Coeff;
Phase1 = Phase;
```

Out[12]=
$$-\frac{a^2 e^2 \gamma\cdot k\,(k\cdot \mathrm{x1})\,(k\cdot \mathrm{x2})}{2\,(k\cdot p)}-\frac{e\,m\,(k\cdot \mathrm{x1})\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot p)}-\frac{e\,m\,(k\cdot \mathrm{x2})\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)}-\frac{e\,(k\cdot \mathrm{x1})\,(\gamma\cdot p).(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot p)}-\frac{e\,(k\cdot \mathrm{x2})\,(\gamma\cdot k).(\gamma\cdot a).(\gamma\cdot p)}{2\,(k\cdot p)}+m+\gamma\cdot p$$

In[15]:=
```
Matrix2 =
 Expand[ExpandScalarProduct[Matrix1 /. {Momentum[x1, D] → Momentum[X, D] - Momentum[x, D] / 2,
     Momentum[x2, D] → Momentum[X, D] + Momentum[x, D] / 2}]]
Coeff2 = Coeff1;
Phase2 =
 Expand[ExpandScalarProduct[Phase1 /. {Momentum[x1, D] → Momentum[X, D] - Momentum[x, D] / 2,
     Momentum[x2, D] → Momentum[X, D] + Momentum[x, D] / 2}]]
```

Out[15]=
$$\frac{a^2 e^2 \gamma\cdot k\,(k\cdot x)^2}{8\,(k\cdot p)}-\frac{a^2 e^2 \gamma\cdot k\,(k\cdot X)^2}{2\,(k\cdot p)}+\frac{e\,m\,(k\cdot x)\,(\gamma\cdot a).(\gamma\cdot k)}{4\,(k\cdot p)}-\frac{e\,m\,(k\cdot x)\,(\gamma\cdot k).(\gamma\cdot a)}{4\,(k\cdot p)}-\frac{e\,m\,(k\cdot X)\,(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot p)}-\frac{e\,m\,(k\cdot X)\,(\gamma\cdot k).(\gamma\cdot a)}{2\,(k\cdot p)}-\frac{e\,(k\cdot x)\,(\gamma\cdot k).(\gamma\cdot a).(\gamma\cdot p)}{4\,(k\cdot p)}+\frac{e\,(k\cdot x)\,(\gamma\cdot p).(\gamma\cdot a).(\gamma\cdot k)}{4\,(k\cdot p)}-\frac{e\,(k\cdot X)\,(\gamma\cdot k).(\gamma\cdot a).(\gamma\cdot p)}{2\,(k\cdot p)}-\frac{e\,(k\cdot X)\,(\gamma\cdot p).(\gamma\cdot a).(\gamma\cdot k)}{2\,(k\cdot p)}+m+\gamma\cdot p$$

Out[17]=
$$\frac{a^2 e^2 (k\cdot x)\,(k\cdot X)^2}{2\,(k\cdot p)}+\frac{a^2 e^2 (k\cdot x)^3}{24\,(k\cdot p)}+\frac{e\,(a\cdot p)\,(k\cdot x)\,(k\cdot X)}{k\cdot p}-p\cdot x$$

In[18]:=
```
Matrix3 = (((Expand[Matrix2 /. TripleGamma]) /. EpsToF) /. FieldSubstitutions) /. akToF /.
  { DiracGamma[LorentzIndex[β, D], D].DiracGamma[5] ×
     Pair[LorentzIndex[β, D], Momentum[FDp, D]] → FVD[γγ5FD, α] × pv[α]}
Coeff3 = Coeff2;
Phase3 = Phase2;
```

Out[18]=
$$\frac{a^2 e^2 \gamma\cdot k\,(k\cdot x)^2}{8\,(k\cdot p)}-\frac{a^2 e^2 \gamma\cdot k\,(k\cdot X)^2}{2\,(k\cdot p)}-\frac{e\,(a\cdot p)\,\gamma\cdot k\,(k\cdot X)}{k\cdot p}+e\,\gamma\cdot a\,(k\cdot X)+\frac{i\,e\,m\,\sigma\mathrm{F}\,(k\cdot x)}{4\,(k\cdot p)}+\frac{i\,e\,\gamma\gamma5\mathrm{FD}^{\alpha}\,p^{\alpha}\,(k\cdot x)}{2\,(k\cdot p)}+m+\gamma\cdot p$$

Expanding scalar products into components and changing variables

$p \to \{p_- = 1/2\,(p^0 - p^3),\ p_+ = p^0 + p^3,\ p_\perp\}$

$p_- = x_- / 2\,s$

$p_+ = (p^2 - p_\perp{}^2) \big/ 2\,p_- = s\,(p^2 + p_\perp{}^2) \big/ x_-$

Integration measure

$\int d\!\!{}^{-}{}^D p\ \ldots = \int \frac{d\!\!{}^{-} s}{2\,s}\, d\!\!{}^{-} p^2\, d\!\!{}^{-}{}^{D-2} p_\perp$

$x_- = kx / m$

$p_- = kx / 2\,m\,s$

$kp = m\,p_- = m\,xm / 2\,s = kx / 2\,s$

$ap = -at\,pt$

$\gamma p = \gamma_-\,p_+ + \gamma_+\,p_- - \gamma_\perp\,p_\perp = Gm\,\frac{s}{x_-}\,(p^2 + p_\perp{}^2) + Gp\,\frac{x_-}{2\,s} - Gt\,pt$

$\gamma k = \gamma_-\,k_+ = m\,Gm$

$kx = k_+\,x_- = m\,xm$

$(\gamma F^*)^\mu . \gamma^5 \to \{(\gamma F^*)_- . \gamma^5 = 0,\ (\gamma F^*)_+ . \gamma^5,\ (\gamma F^*)_\perp . \gamma^5\} = \{0,\ \gamma\gamma 5FDp,\ \gamma\gamma 5FDt\}$

$(\gamma F^*)_\mu\,k^\mu = 0\ \to\ (\gamma F^*)_- = 0$

$(\gamma F^*)_\mu\,a^\mu = 0 \to\ \gamma\gamma 5FDt\,at = 0$

$(\gamma F^*)^\mu . \gamma^5\,p_\mu = \gamma\gamma 5FDm * pp - \gamma\gamma 5FDt * pt$

In[21]:=
```
D[{xm / 2 / s, s (p2 + pt2) / xm}, {{s, p2}}]
Abs[Det[%]]
```

Out[21]=
$$\begin{pmatrix} -\frac{xm}{2\,s^2} & 0 \\ \frac{p2+pt2}{xm} & \frac{s}{xm} \end{pmatrix}$$

Out[22]=
$$\frac{1}{2\,|s|}$$

In[23]:=
```
Matrix4 =
  Collect[Expand[Matrix3 /. {DiracGamma[Momentum[p, D], D] → Gp * pm + Gm * pp - Gt * pt, Pair[
          Momentum[a, D], Momentum[p, D]] → -at pt, FVD[γγ5FD, α_] × pv[α_] →
         γγ5FDp * pm - γγ5FDt * pt, DiracGamma[Momentum[k, D], D] → m Gm} /.
       {kp → kx / 2 / s, pm → xm / 2 / s} /. {kx → m xm} /. {pp → s (p2 + pt ^ 2) / xm}], {pt, p2}]
Coeff4 = Coeff3 / 2 / s
Phase4 =
  Collect[Expand[Phase3 /. {Pair[Momentum[p, D], Momentum[x, D]] → pp xm + pm xp - pt * xt,
        kp → m pm, Pair[Momentum[a, D], Momentum[p, D]] → -at pt} /.
       {kp → kx / 2 / s, pm → xm / 2 / s} /. {pp → s (p2 + pt ^ 2) / xm} /. {xm → kx / m}], {pt, pp}]
```

Out[23]=
$$-\frac{a^2 e^2 \,\mathrm{Gm}\, s\,(k\cdot X)^2}{\mathrm{xm}} + \frac{1}{4} a^2 e^2 \,\mathrm{Gm}\, m^2 s\,\mathrm{xm} + e\gamma\cdot a\,(k\cdot X) + \mathrm{pt}\left(\frac{2\,\mathrm{at}\, e\,\mathrm{Gm}\, s\,(k\cdot X)}{\mathrm{xm}} - i\,\gamma\gamma 5\mathrm{FDt}\, e\, s - \mathrm{Gt}\right) + \frac{1}{2} i\, e\, m\, s\, \sigma\mathrm{F} + \frac{1}{2} i\,\gamma\gamma 5\mathrm{FDp}\, e\,\mathrm{xm} + \frac{\mathrm{Gm}\,\mathrm{p2}\, s}{\mathrm{xm}} + \frac{\mathrm{Gm}\,\mathrm{pt}^2 s}{\mathrm{xm}} + \frac{\mathrm{Gp}\,\mathrm{xm}}{2 s} + m$$

Out[24]=
$$\frac{i\, 2^{-D-1} \pi^{-D} \Lambda^{4-D}}{s\left(p^2 - m^2\right)}$$

Out[25]=
$$\frac{1}{12} a^2 e^2 s\,(k\cdot x)^2 + a^2 e^2 s\,(k\cdot X)^2 + \mathrm{pt}\,(\mathrm{xt} - 2\,\mathrm{at}\, e\, s\,(k\cdot X)) - \frac{\mathrm{xp}\,(k\cdot x)}{2\, m\, s} - \mathrm{p2}\, s + \mathrm{pt}^2\,(-s)$$

**Integration over**
$\int d\!\!{}^{-}{}^{D-2} p_\perp \ldots$

$$I_0 = \int d^{D-2} p_\perp \operatorname{Exp}\left[-I\, A\, p_\perp^2 + I\,(J.p_\perp)\right] =$$

$$= \operatorname{Exp}\left[-I \frac{\pi}{2} \frac{D-2}{2}\right] \pi^{\frac{D-2}{2}} (\det A)^{-\frac{1}{2}} \operatorname{Exp}\left[I \frac{1}{4} J.A^{-1}.J\right]$$

$$I_1 = \int d^{D-2} p_\perp\, p_\perp \operatorname{Exp}\left[-I\, A\, p_\perp^2 + I\,(J.p_\perp)\right] =$$

$$= \frac{1}{2} A^{-1}.J\, I_0$$

$$I_2 = \int d^{D-2} p_\perp\, p_\perp{}^2 \operatorname{Exp}\left[-I\, A\, p_\perp^2 + I\,(J.p_\perp)\right] =$$

$$= \left[-i \frac{1}{2} \operatorname{Tr} A^{-1} + \left(\frac{1}{2} A^{-1}.J\right)^2\right] I_0$$

where
$A = s,$
$J = x_\perp - 2\, ea_\perp\, s\, kX,$
$\det A = s^{D-2},$
$A^{-1} = 1/s$

We perform integrations
Integrations changes the coefficient (Coeff) and phase
Then recollect some scalar products
$a_\perp{}^\wedge 2 = -a^2$
$a_\perp\, x_\perp = -(ax)$
$x_\perp^2 = 2\, x_-\, x_+ - x^2$

In[26]:=
```
Amatr = -Coefficient[Phase4, pt^2]
J = Coefficient[Phase4, pt]
CI0 = Exp[-I Pi / 2 (D / 2 - 1)] Pi ^(D / 2 - 1) / Amatr ^(D / 2 - 1)
```

Out[26]= $s$

Out[27]= $\text{xt} - 2\,\text{at}\, e\, s\, (k \cdot X)$

Out[28]= $e^{-\frac{1}{2} i \pi \left(\frac{D}{2}-1\right)} \pi^{\frac{D}{2}-1} s^{1-\frac{D}{2}}$

In[29]:=
```
Phase5 = Expand[Expand[((Phase4 /. {pt → 0}) + 1 / 4 J^2 / Amatr)] /.
    { at^2 → -av2, at xt → -ax, xt^2 → 2 xm xp - xv2} /. {xm → kx / m}]
Coeff5 = Coeff4 * CI0
Matrix5 =
 Collect[Expand[Expand[(Matrix4 /. {pt → 0}) + Coefficient[Matrix4, pt] * 1 / 2 / Amatr * J +
      Coefficient[Matrix4, pt^2] * (-I / 2 * (D - 2) / Amatr + ( 1 / 2 / Amatr * J)^2)] /.
    {γγ5FDtat → 0, at^2 → -av2, at xt → -ax, xt^2 → 2 xm xp - xv2}],
  {p2, Gm, Gp, Gt, γγ5FDm, γγ5FDp, γγ5FDt}]
```

Out[29]= $\frac{1}{12} a^2 e^2 s (k\cdot x)^2 + e (a\cdot x)(k\cdot X) - \text{p2}\, s - \frac{x^2}{4 s}$

Out[30]= $\frac{i\, 2^{-D-1} e^{-\frac{1}{2} i \pi \left(\frac{D}{2}-1\right)} \pi^{-\frac{D}{2}-1} \Lambda^{4-D} s^{-D/2}}{p^2 - m^2}$

Out[31]= $\text{Gm}\left(\frac{1}{4} a^2 e^2 m^2 s\, \text{xm} - \frac{i D}{2\, \text{xm}} - \frac{x^2}{4 s\, \text{xm}} + \frac{\text{xp}}{2 s} + \frac{i}{\text{xm}}\right) + e \gamma\cdot a (k\cdot X) + \text{Gt}\left(\text{at}\; e (k\cdot X) - \frac{\text{xt}}{2 s}\right) + \frac{1}{2} i\, e\, m\, s\, \sigma \text{F} + \frac{1}{2} i\, \gamma\gamma 5\text{FDp}\; e \text{xm} - \frac{1}{2} i\, \gamma\gamma 5\text{FDt}\; e \text{xt} + \frac{\text{Gm p2}\, s}{\text{xm}} + \frac{\text{Gp xm}}{2 s} + m$

Next we substitute

$\gamma_\perp a_\perp = -\gamma a$

$\gamma_\perp x_\perp = \gamma_- x_+ - \gamma_+ x_- - \gamma x$

$\gamma_- x_- = \frac{\cancel{\gamma k}}{m} x_-$

$x_- = kx / m$

In[32]:=
```
Matrix6 = Collect[Expand[Matrix5] /.
      {Gt at → -Contract[GAD[α] × av[α]], Gt xt → Gm xp + Gp xm - Contract[GAD[α] × xv[α]] ,
       γγ5FDt xt  → γγ5FDp xm - DiracSlash[FDxv , Dimension → D].GA[5]} /.
     {Gm xm →  DiracGamma[Momentum[k , D], D] / m * xm} /. {xm →  kx / m},
   {p2, Gm , Gp, Gt, γγ5FDm, γγ5FDp, γγ5FDt}]
Coeff6 = Coeff5
Phase6 = Phase5
```

Out[32]=
$$\frac{1}{2} i\, e (\gamma \cdot \mathrm{FDxv}).\bar{\gamma}^5 + \frac{1}{4} a^2 e^2 s\, \gamma \cdot k (k \cdot x) + \mathrm{Gm} \left(-\frac{i D m}{2 (k \cdot x)} - \frac{m x^2}{4 s (k \cdot x)} + \frac{i m}{k \cdot x}\right) + \frac{1}{2} i\, e\, m\, s\, \sigma \mathrm{F} + \frac{\mathrm{Gm}\ m\, \mathrm{p2}\, s}{k \cdot x} + m + \frac{\gamma \cdot x}{2 s}$$

Out[33]=
$$\frac{i\, 2^{-D-1} e^{-\frac{1}{2} i \pi \left(\frac{D}{2}-1\right)} \pi^{-\frac{D}{2}-1} \Lambda^{4-D} s^{-D/2}}{p^2 - m^2}$$

Out[34]=
$$\frac{1}{12} a^2 e^2 s (k \cdot x)^2 + e (a \cdot x)(k \cdot X) - \mathrm{p2}\, s - \frac{x^2}{4 s}$$

$v = p^2 - m^2$

$$\int \frac{dv}{v + i0} \exp[-i s v] = -2 \pi i\, \theta(s)$$

$$\int dv \exp[-i s v] = 2 \pi \delta(s)$$

$$\int_{-\infty}^{\infty} \frac{ds}{s^{D/2}} s\, e^{i g(s)}\, 2 \pi \delta(s) = -2 \pi i \int_{-\infty}^{\infty} \frac{ds}{s^{D/2}} e^{i g(s)} \theta(s) [i (D/2 - 1) + s\, g'(s)]$$

where

$g(s) =$ Phase6 with $p^2 = m^2$

Finally,

$\mathrm{Gm} = \gamma_- = \gamma k / m$

In[35]:=
```
Phase7 = Phase6 /. {p2 → m^2}
Coeff7 = Coeff6 * (pv2 - m^2) * (-2 π ⅈ)
g = Phase7;
Matrix7 =
  Expand[(Matrix6 /. {p2 → m^2}) + Coefficient[Matrix6, p2 s] * (ⅈ (D / 2 - 1) + s D[g, s])] /.
   {Gm → DiracGamma[Momentum[k, D], D] / m}
```

Out[35]= $\frac{1}{12} a^2 e^2 s (k\cdot x)^2 + e (a\cdot x)(k\cdot X) + m^2 (-s) - \frac{x^2}{4 s}$

Out[36]= $2^{-D} e^{-\frac{1}{2} i \pi \left(\frac{D}{2}-1\right)} \pi^{-D/2} \Lambda^{4-D} s^{-D/2}$

Out[38]= $\frac{1}{2} i\, e (\gamma\cdot \text{FDxv}).\overline{\gamma}^5 + \frac{1}{3} a^2 e^2 s \gamma\cdot k (k\cdot x) + \frac{1}{2} i\, e\, m\, s\, \sigma \text{F} + m + \frac{\gamma\cdot x}{2 s}$

In[39]:=
```
Matrix8 =
  Expand[Matrix7 / m /. {DiracGamma[Momentum[k, D], D] → Contract[GAD[α] × FFxv[α]] / (-av2 kx)}]
Coeff8 = Coeff7 * m
Phase8 = Phase7 /. {av2 kx^2 → Fx^2}
```

Out[39]= $\frac{i\, e (\gamma\cdot \text{FDxv}).\overline{\gamma}^5}{2 m} - \frac{e^2 s \gamma\cdot \text{FFx}}{3 m} + \frac{1}{2} i\, e\, s\, \sigma \text{F} + \frac{\gamma\cdot x}{2 m s} + 1$

Out[40]= $2^{-D} e^{-\frac{1}{2} i \pi \left(\frac{D}{2}-1\right)} \pi^{-D/2} m \Lambda^{4-D} s^{-D/2}$

Out[41]= $e (a\cdot x)(k\cdot X) + \frac{1}{12} e^2 \text{Fx}^2 s - m^2 s - \frac{x^2}{4 s}$

In[42]:=
```
Matrix9 = Matrix8 /. {s → s1 / m^2}
Coeff9 = Simplify[Coeff8 / m^2 /. {s → s1 / m^2}, Assumptions → m > 0]
Coeff9 /. {D → 4}
Phase9 = Phase8 /. {s → s1 / m^2}
```

Out[42]= $\frac{i\, e (\gamma\cdot \text{FDxv}).\overline{\gamma}^5}{2 m} - \frac{e^2 \text{s1} \gamma\cdot \text{FFx}}{3 m^3} + \frac{i\, e\, \text{s1}\, \sigma \text{F}}{2 m^2} + \frac{m \gamma\cdot x}{2 \text{s1}} + 1$

Out[43]= $i\, 2^{-D} e^{-\frac{1}{4} i \pi D} \pi^{-D/2} \Lambda^{4-D} m^{D-1} \text{s1}^{-D/2}$

Out[44]= $-\frac{i\, m^3}{16 \pi^2 \text{s1}^2}$

Out[45]= $e (a\cdot x)(k\cdot X) + \frac{e^2 \text{Fx}^2 \text{s1}}{12 m^2} - \frac{m^2 x^2}{4 \text{s1}} - \text{s1}$

In[46]:=
```
Matrix91 = Expand[Matrix9 /. {DiracGamma[Momentum[FDxv, D], D].GA[5] →
      Contract[DiracGamma[LorentzIndex[μ, D], D].GA[5] × FDxv[μ] /.
        {FDxv[μ_] → -LCD[μ, α1, α2, α3] × xv[α1] × av[α2] × kv[α3]}, EpsContract → False]} /.
    antiTripleGamma /. {DiracGamma[Momentum[FFx, D], D] →
     -av2 kx DiracGamma[Momentum[k, D], D]} /.
  {σF → -2 ⅈ DiracGamma[Momentum[a, D], D].DiracGamma[Momentum[k, D], D]}]
```

Out[46]= $\frac{a^2 e^2 \text{s1}\, \gamma\cdot k\,(k\cdot x)}{3\, m^3}+\frac{e\, \text{s1}\,(\gamma\cdot a).(\gamma\cdot k)}{m^2}+\frac{e\,(a\cdot x)\,\gamma\cdot k}{2\, m}-\frac{e\,\gamma\cdot a\,(k\cdot x)}{2\, m}+\frac{e\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)}{2\, m}+\frac{m\,\gamma\cdot x}{2\, \text{s1}}+1$

In[47]:=
```
Expand[(Matrix91 / (m / 2 / s1) /. {s1 → s * m ^ 2}) / 2 / s]
```

Out[47]= $\frac{1}{3} a^2 e^2 s\, \gamma\cdot k\,(k\cdot x)+e\, m\, s\,(\gamma\cdot a).(\gamma\cdot k)+\frac{1}{2} e\,(a\cdot x)\,\gamma\cdot k-\frac{1}{2} e\,\gamma\cdot a\,(k\cdot x)+\frac{1}{2} e\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)+m+\frac{\gamma\cdot x}{2\, s}$

Another representation of the $\gamma$ − matrix preexponent term

In[48]:=
```
MartixAnother = (2 m s1 * 1 / 2 / m / s1 GAD[α] (xv[α] - s1 e Fxv[α] + s1 ^ 2 / 3 e ^ 2 FFxv[α]) + 2 m s1 * 1).
   (1 + e s1 DiracSlash[a, k, Dimension → D]) /. {s1 → s1 / m ^ 2}
MartixAnother1 = DiracSimplify[Contract[DotSimplify[
    MartixAnother / (2 s1 / m) /. {Fxv[α_] → kv[α] ax - av[α] kx, FFxv[α_] → -av2 kx kv[α]}]]]
CoeffAnother = Coeff9 * m / 2 / s1
```

Out[48]= $\left(\gamma^{\alpha}\left(\frac{e^2\, \text{s1}^2\, \text{FFx}^{\alpha}}{3\, m^4}-\frac{e\, \text{s1}\, \text{Fx}^{\alpha}}{m^2}+x^{\alpha}\right)+\frac{2\, \text{s1}}{m}\right)\left(\frac{e\, \text{s1}\,(\gamma\cdot a).(\gamma\cdot k)}{m^2}+1\right)$

Out[49]= $\frac{a^2 e^2 \text{s1}\, \gamma\cdot k\,(k\cdot x)}{3\, m^3}+\frac{e\, \text{s1}\,(\gamma\cdot a).(\gamma\cdot k)}{m^2}-\frac{e\,(a\cdot x)\,\gamma\cdot k}{2\, m}+\frac{e\,\gamma\cdot a\,(k\cdot x)}{2\, m}+\frac{e\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{2\, m}+\frac{m\,\gamma\cdot x}{2\, \text{s1}}+1$

Out[50]= $i\, 2^{-D-1}\, e^{-\frac{1}{4} i \pi D}\, \pi^{-D/2}\, \Lambda^{4-D}\, m^D\, \text{s1}^{-\frac{D}{2}-1}$

In[51]:=
```
Matrix91 - Expand[MartixAnother1]
```

Out[51]= $\frac{e\,(a\cdot x)\,\gamma\cdot k}{m}-\frac{e\,\gamma\cdot a\,(k\cdot x)}{m}+\frac{e\,(\gamma\cdot a).(\gamma\cdot k).(\gamma\cdot x)}{2\, m}-\frac{e\,(\gamma\cdot x).(\gamma\cdot a).(\gamma\cdot k)}{2\, m}$

# Final result for the electron propagator in a CCF

$$S^c\,(x_2,\, x_1) = \Lambda^{4-D} \int \frac{d^D p}{(2\,\pi)^D}\, E_p\,(x_2)\, \frac{i\,((\gamma p)+m)}{p^2 - m^2 + i0}\, E_p^{bar}\,(x_1) =$$

$$e^{-i\,\frac{\pi}{2}\left(\frac{D}{2}-1\right)}\, \frac{\Lambda^{4-D}}{2^D\,\pi^{D/2}}\, m^{D-1}\, e^{i\,\eta} \int_0^{\infty} \frac{ds}{s^{D/2}} \left[1 + \frac{m\,(\gamma x)}{2\,s} - \frac{e^2\,s\,(\gamma\, FFx)}{3\,m^3} + \frac{i\,e\,s\,(\sigma^{\alpha\beta}\,F_{\alpha\beta})}{2\,m^2} + \frac{i\,e\,F_{\alpha\beta}{}^{*}\,x^{\beta}\,\gamma^{\alpha}\,\gamma^{5}}{2\,m}\right] e^{-i\,s - i\,\frac{m^2\,x^2}{4\,s} + i\,\frac{s}{12}\,\frac{e^2}{m^2}\,(Fx)^2} =$$

$$= e^{-i\frac{\pi}{2}\left(\frac{D}{2}-1\right)} \frac{\Lambda^{4-D}}{2^D \pi^{D/2}} m^{D-1} e^{i\eta} \int_0^\infty \frac{ds}{s^{D/2}} \left[1 + \frac{m(\gamma x)}{2s} + \frac{e(\gamma a)(kx)}{2m} - \frac{e(\gamma k)(ax)}{2m} + \frac{e(\gamma x)(\gamma a)(\gamma k)}{2m} + \frac{e\,s(\gamma a)(\gamma k)}{m^2} + \frac{e^2 a^2 s(\gamma k)(kx)}{3m^3}\right] e^{-is - i\frac{m^2 x^2}{4s} + i\frac{s}{12}\frac{e^2}{m^2}(Fx)^2} =$$

$$= e^{-i\frac{\pi}{2}\left(\frac{D}{2}-1\right)} \frac{\Lambda^{4-D}}{2^{D+1} \pi^{D/2}} m^{D} e^{i\eta} \int_0^\infty \frac{ds}{s^{D/2+1}} \left[\frac{2s}{m} + \gamma^\alpha \left(g_{\alpha\beta} - \frac{es}{m^2} F_{\alpha\beta} + \frac{e^2 s^2}{3m^4} F_{\alpha\lambda} F^{\lambda}{}_{\beta}\right) x^\beta\right] \left(1 + \frac{ies}{2} \sigma^{\alpha\beta} F_{\alpha\beta}\right) e^{-is - i\frac{m^2 x^2}{4s} + i\frac{s}{12}\frac{e^2}{m^2}(Fx)^2}$$

$\eta = e(ax)(k, (x_1 + x_2)/2)$,

$x = x_2 - x_1$,

$e > 0$,

$\sigma^{\alpha\beta} = \frac{i}{2}\left(\gamma^\alpha \gamma^\beta - \gamma^\beta \gamma^\alpha\right)$,

$\gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3$,

$$e^{-i\frac{\pi}{2}\left(\frac{D}{2}-1\right)} \frac{\Lambda^{4-D}}{2^D \pi^{D/2}} m^{D-1} \to \frac{(-i)\,m^3}{16\pi^2}, \quad D \to 4$$