```
> restart;
    with(geometry) :; with(plots) :; with(LinearAlgebra) :; with(MTM) :;
> #f := x -> sin(20·Pi·x);
  f := x -> sqrt(x);
  #f := x -> e^(1/(x + 0.4)^2);
  #f := x -> sin(50·x);
```

$$f := x \mapsto \sqrt{x} \tag{1}$$

```
> xs := [seq(i/10, i = 0..10)];
```

$$xs := \left[0, \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{4}{5}, \frac{9}{10}, 1\right] \tag{2}$$

```
> ys := map(f, xs);
```

$$ys := \left[0, \frac{\sqrt{10}}{10}, \frac{\sqrt{5}}{5}, \frac{\sqrt{30}}{10}, \frac{\sqrt{10}}{5}, \frac{\sqrt{2}}{2}, \frac{\sqrt{15}}{5}, \frac{\sqrt{70}}{10}, \frac{2\sqrt{5}}{5}, \frac{3\sqrt{10}}{10}, 1\right] \tag{3}$$

```
> nintervals := 10;
  D2 := diff(diff(f(x), x), x);
   phi1 := eval(D2, x = 0);
  phi2 := eval(D2, x = 1);
```

$$nintervals := 10$$

$$D2 := -\frac{1}{4x^{3/2}}$$

Error, numeric exception: division by zero

$$\phi 2 := -\frac{1}{4} \tag{4}$$

```
> b := Vector(nintervals + 1, fill = 0) :;
> b[1] := 0 :;
  for i from 2 to nintervals + 1 do
    b[i] := (ys[i + 1] - ys[i]) / (1/10) - (ys[i] - ys[i - 1]) / (1/10) :;
  end do:;
  b[nintervals + 1] := 0 :;
  b;
```

Error, invalid subscript selector

$$\begin{bmatrix} 1 & 0 & \cdots \\ 2 & 2\sqrt{5} - 2\sqrt{10} & \cdots \\ 3 & \sqrt{30} - 4\sqrt{5} + \sqrt{10} & \cdots \\ 4 & 2\sqrt{10} - 2\sqrt{30} + 2\sqrt{5} & \cdots \\ 5 & 5\sqrt{2} - 4\sqrt{10} + \sqrt{30} & \cdots \\ 6 & 2\sqrt{15} - 10\sqrt{2} + 2\sqrt{10} & \cdots \\ 7 & \sqrt{70} - 4\sqrt{15} + 5\sqrt{2} & \cdots \\ 8 & 4\sqrt{5} - 2\sqrt{70} + 2\sqrt{15} & \cdots \\ 9 & 3\sqrt{10} - 8\sqrt{5} + \sqrt{70} & \cdots \\ 10 & 10 - 6\sqrt{10} + 4\sqrt{5} & \cdots \\ & \vdots & \end{bmatrix} \quad \textbf{(5)}$$

11 element Vector[column]

```
>
> A := Matrix(nintervals + 1, nintervals + 1) : ;
> for i from 2 to nintervals do
    A[i, i] := 4/10 :;
    if i < nintervals + 1 then
      A[i, i + 1] := 1/10 :;
```

$A[i, i-1] := \frac{1}{10}$:;

end if;

end do:;

$A[1, 1] := 1$:;

$A[nintervals + 1, nintervals + 1] := 1$:;

$A := \frac{1}{6} A$;

(6)

$$A := \begin{bmatrix} \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ \frac{1}{60} & \frac{1}{15} & \frac{1}{60} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & \frac{1}{60} & \frac{1}{15} & \frac{1}{60} & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & \frac{1}{60} & \frac{1}{15} & \frac{1}{60} & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \frac{1}{60} & \frac{1}{15} & \frac{1}{60} & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & \frac{1}{60} & \frac{1}{15} & \frac{1}{60} & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{60} & \frac{1}{15} & \frac{1}{60} & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{60} & \frac{1}{15} & \frac{1}{60} & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{60} & \frac{1}{15} & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{60} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{bmatrix} \quad \text{(6)}$$

11 × 11 Matrix

```
> M := LinearSolve(A, b) :;
> a := [seq(ys[i], i = 1 ..nintervals + 1)];
```

$$a := \left[0, \frac{\sqrt{10}}{10}, \frac{\sqrt{5}}{5}, \frac{\sqrt{30}}{10}, \frac{\sqrt{10}}{5}, \frac{\sqrt{2}}{2}, \frac{\sqrt{15}}{5}, \frac{\sqrt{70}}{10}, \frac{2\sqrt{5}}{5}, \frac{3\sqrt{10}}{10}, 1\right] \quad \text{(7)}$$

```
> c := [seq(M[i], i = 1 ..nintervals + 1) ] :;
```

> $d := \left[seq\left(\frac{(c[i]-c[i-1])}{\frac{1}{10}}, i=2..nintervals+1 \right) \right]:;$

> $b := \left[seq\left(\frac{(ys[i]-ys[i-1])}{\frac{1}{10}} + \left(\frac{1}{60} \right) \cdot (2\cdot c[i]+c[i-1]), i=2..nintervals+1 \right) \right]:;$

>

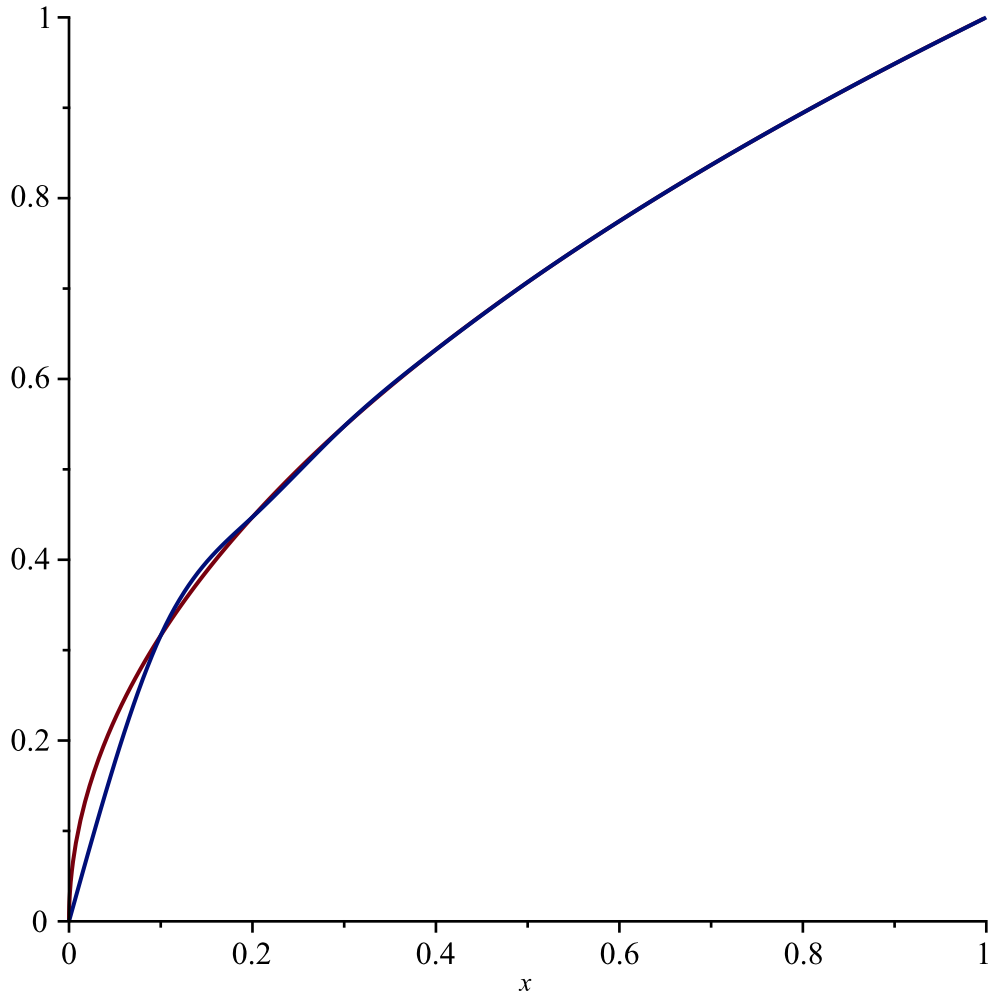
> $fii := (i,x) \rightarrow a[i]+b[i-1]\cdot(x-xs[i]) + \frac{c[i]}{2}\cdot(x-xs[i])^2 + \frac{d[i-1]}{6}\cdot(x-xs[i])^3 :;$

> $i := x \rightarrow \textbf{if } x=1 \textbf{ then } 11 \textbf{ else } \mathrm{floor}(10\cdot x)+2 \textbf{ end if}:;$

> $fl := x \rightarrow fii(i(x),x);$

$$fl := x \mapsto fii(i(x),x) \qquad \textbf{(8)}$$

> $plot([f(x),fl(x)],x=0..1);$



> $XXS := \left[seq\left(\frac{i}{100}, i=0..100 \right) \right]:;$

$errorm := (f,approx) \rightarrow evalf(\max(map(x \rightarrow (\mathrm{abs}(f(x)-approx(x))), XXS))):;$

```
errorm(f1, f);
```

0.06901604940 **(9)**

```
>
```