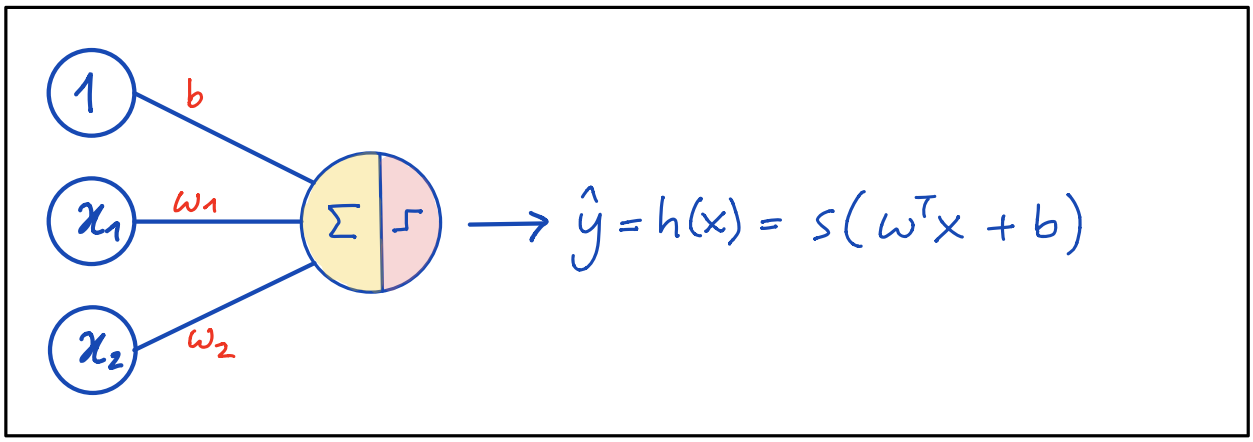
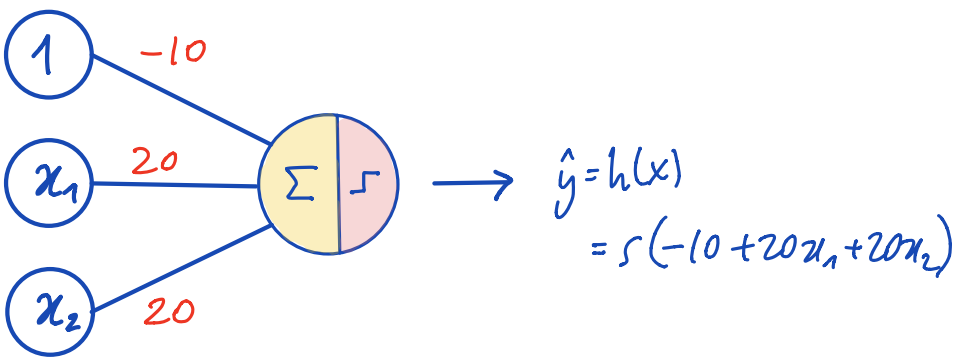
- **Multi Layer Perzeptron (MLP) = Tiefes Neuronales Netz**
  - Das Perzeptron = Einschichtiges Neuronales Netzwerk
    (auch: Linear Threshold Unit (LTU))

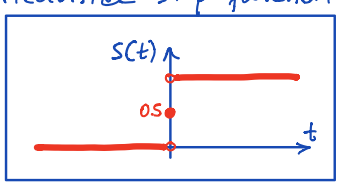


  - Grundbaustein von mehrschichtigen Netzwerken.
  - Kann nur lineare Beziehungen lernen
    (z.B. einfache Logik Funktionen lernen)

BSP.



$s(t) := H(t)$
Heaviside step function

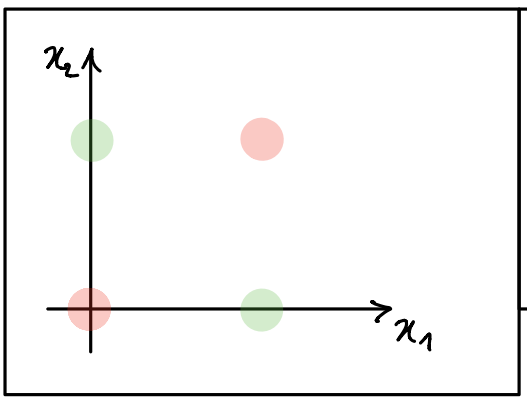


x_1	x_2	y
1	1	
0	1	
1	0	
0	0	

- ○ x_1 AND x_2
- ○ (NOT x_1) OR (NOT x_2)
- ○ x_1 OR x_2
- ○ (NOT x_1) AND (NOT x_2)

- Das Aneinanderreihen mehrerer Perzeptronen kann diese Einschränkung aufheben

BSP. $y = x_1 \text{ XOR } x_2 = (x_1 \text{ OR } x_2) \text{ AND } \neg(x_1 \text{ AND } x_2)$



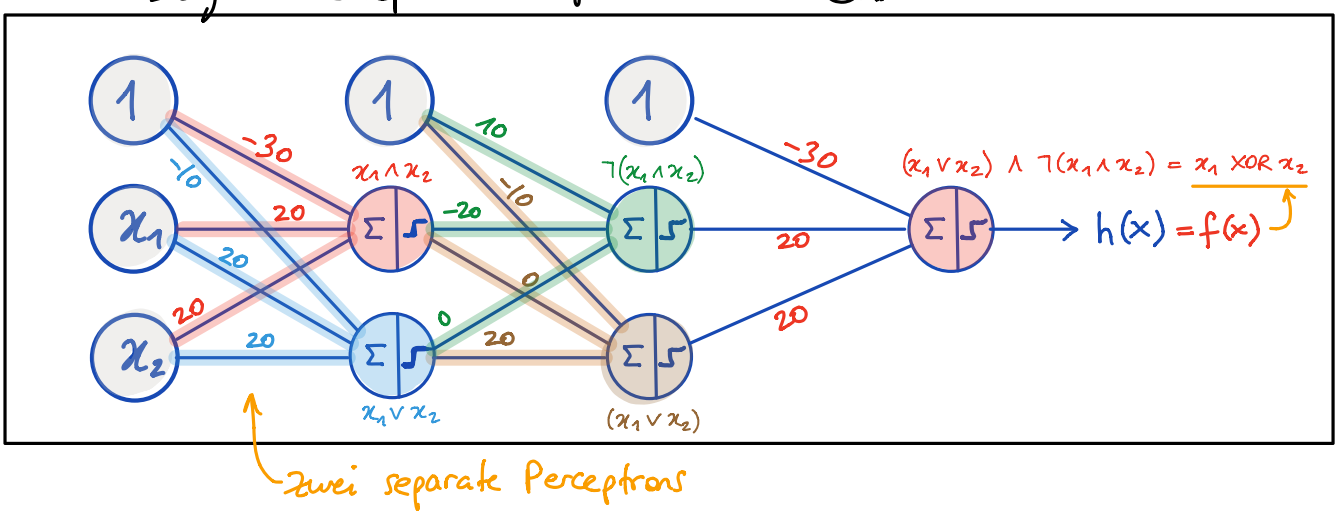
x_1	x_2	y
1	1	0
1	0	1
0	1	1
0	0	0

- $\chi = \{0,1\}$ $y = \{0,1\}$
- $(x_1 \text{ XOR } x_2)$ Gate

↑ Zielfunktion $f(x)$

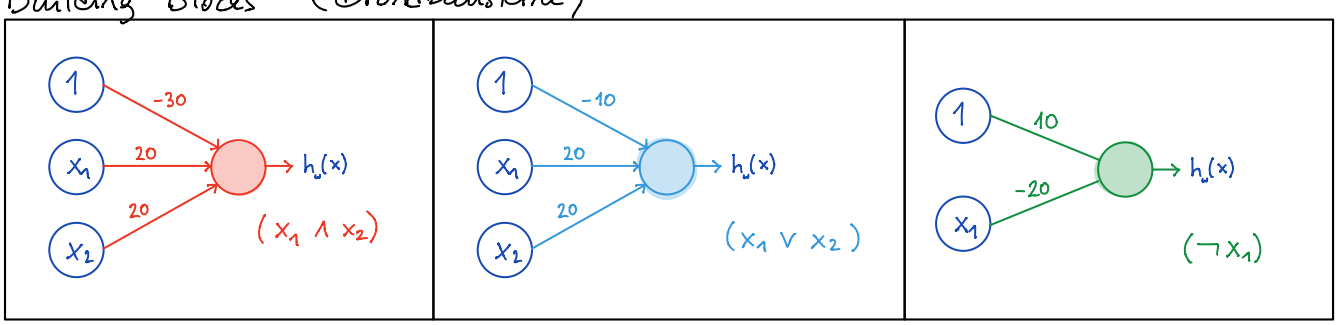
nicht linear separierbar!

Multi Layer Perzeptron für XOR Gate

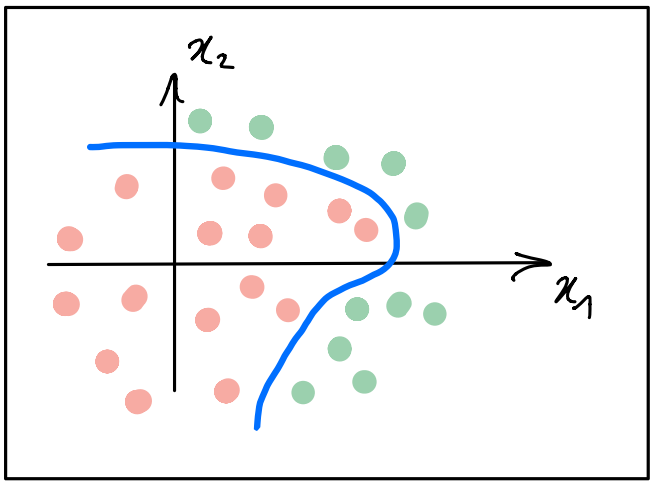


Zwei separate Perceptrons

Building Blocks (Grundbausteine)



- Warum Neuronale Netzwerke?



- Nichtlineare Grenzen mit Logistischer Regression durch das Einführen neuer Merkmale möglich:

$x_1^2, x_2^2, x_1 \cdot x_2, \dots$

$x_1^3, x_2^3, x_1^2 x_2, x_1 x_2^2, \dots$

- Warum brauchen wir tiefe Netzwerke?

BSP. $x_1, x_2, \dots, x_{100}$ (ursprüngliche Merkmale)

$x_1x_2, x_1x_3, \dots, x_1x_{100}, x_2x_3, x_2x_4, \dots, x_2x_{100}, \dots$ (quadr. Terme)

$x_1x_2x_3, x_1^2x_3, x_1x_{15}x_{17}, \dots$

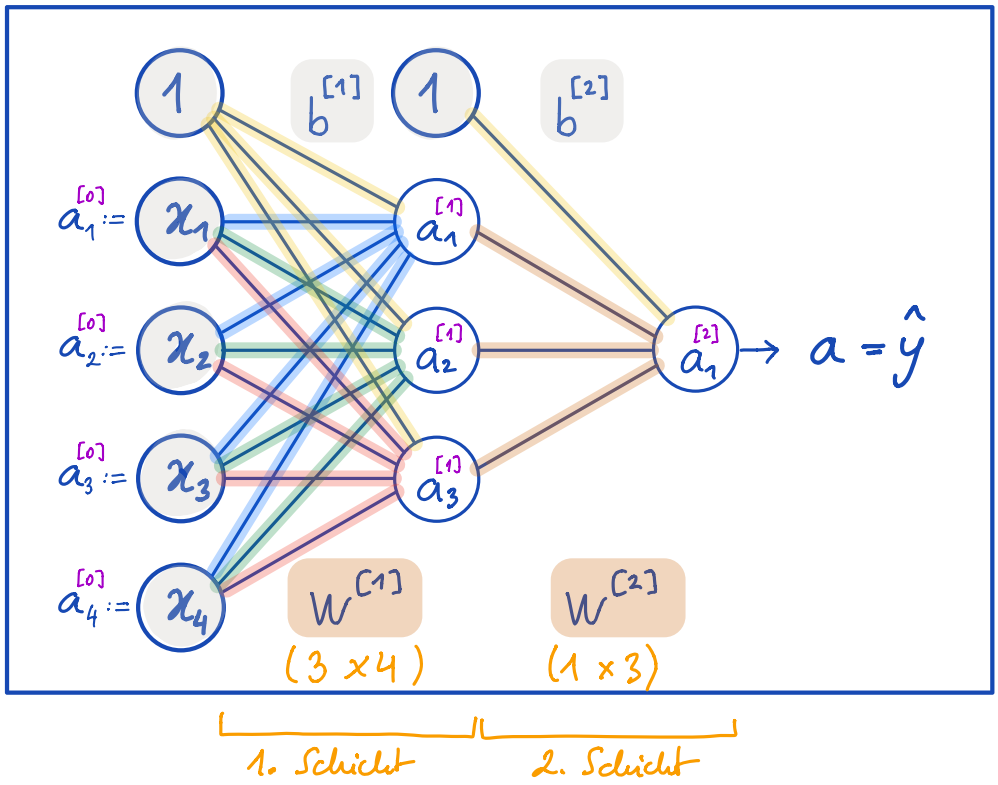
$\longrightarrow \approx 180000$

BSP. 50×50 px Graustufenbilder $\longrightarrow$ $n = 2500$

Graustufenwerte 0–255

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_{2500} \end{bmatrix} \in \mathbb{R}^{2500} \longrightarrow \text{mit quadratischen Termen ...?}$$

• Fully Connected Feed Forward Deep Neural Network



$a^{[1]}$: Aktivierungen der 1. Schicht (Vektor!)

$a^{[0]} := x$ (Eingabe)

$b^{[1]} = \begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \end{bmatrix}$: Bias Terme der 1. Schicht.

- Die "Aktivierungen" der ersten Schicht:

$$a_1^{[1]} = \text{relu}\left(w_{11}^{[1]} \cdot x_1 + w_{12}^{[1]} \cdot x_2 + w_{13}^{[1]} \cdot x_3 + w_{14}^{[1]} \cdot x_4 + b_1^{[1]} \right)$$ (ehem. w_{10})

$$a_2^{[1]} = \text{relu}\left(w_{21}^{[1]} \cdot x_1 + w_{22}^{[1]} \cdot x_2 + w_{23}^{[1]} \cdot x_3 + w_{24}^{[1]} \cdot x_4 + b_2^{[1]} \right)$$ (w_{20})

$$a_3^{[1]} = \text{relu}\left(w_{31}^{[1]} \cdot x_1 + w_{32}^{[1]} \cdot x_2 + w_{33}^{[1]} \cdot x_3 + w_{34}^{[1]} \cdot x_4 + b_3^{[1]} \right)$$ (w_{30})

- Matrix Notation für Vektor $a^{[1]}$:

$$\begin{bmatrix} a_1^{[1]} \\ a_2^{[1]} \\ a_3^{[1]} \end{bmatrix} = \text{relu}\left(\begin{bmatrix} w_{11}^{[1]} & w_{12}^{[1]} & w_{13}^{[1]} & w_{14}^{[1]} \\ w_{21}^{[1]} & w_{22}^{[1]} & w_{23}^{[1]} & w_{24}^{[1]} \\ w_{31}^{[1]} & w_{32}^{[1]} & w_{33}^{[1]} & w_{34}^{[1]} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \end{bmatrix} \right)$$

$$a^{[1]} = \text{relu}\left(W^{[1]} \cdot x + b^{[1]} \right)$$

$$z^{[1]} = W^{[1]} \cdot a^{[0]} + b^{[1]}$$

$(3\times1) \quad (3\times4) \cdot (4\times1) + (3\times1)$

Dimensionen?

$$a^{[1]} = \text{relu}\left(z^{[1]}\right)$$

$(3\times1) \qquad (3\times1)$

$$\sigma\begin{bmatrix}\cdot\\\cdot\\\cdot\end{bmatrix} = \begin{bmatrix}\sigma(\cdot)\\\sigma(\cdot)\\\sigma(\cdot)\end{bmatrix}$$

- Matrix Notation für Vektor $a^{[2]} = \hat{y}$:

$$a_1^{[2]} = \sigma\left(w_{11}^{[2]} \cdot a_1^{[1]} + w_{12}^{[2]} \cdot a_2^{[1]} + w_{13}^{[2]} \cdot a_3^{[1]} + b_1^{[2]}\right)$$

$$z^{[2]} = \underset{(1\times3)}{W^{[2]}} \cdot \underset{(3\times1)}{a^{[1]}} + b^{[2]} = \begin{bmatrix}w_{11}^{[2]} & w_{12}^{[2]} & w_{13}^{[2]}\end{bmatrix} \cdot \begin{bmatrix}a_1^{[1]}\\a_2^{[1]}\\a_3^{[1]}\end{bmatrix} + \begin{bmatrix}b_1^{[2]}\end{bmatrix}$$

$$a^{[2]} = \sigma\left(z^{[2]}\right)$$

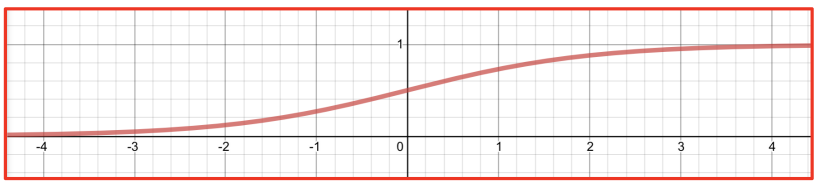
(1×1)

- Forward Propagation (Forwärtslauf)
- $a^{[0]} := x = \text{Eingabe}$
- $z^{[1]} = W^{[1]} \cdot x + b^{[1]}$

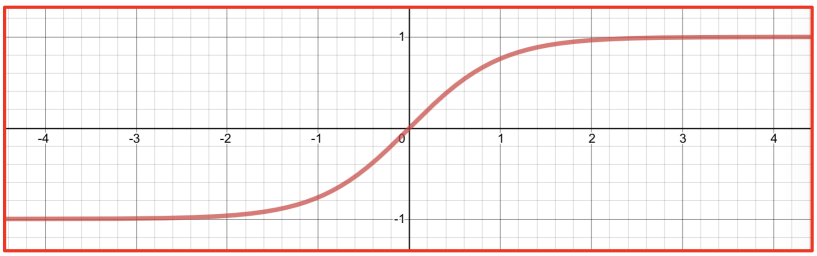
  $a^{[1]} = \text{ReLU}\left(z^{[1]}\right)$
- $z^{[2]} = W^{[2]} \cdot a^{[1]} + b^{[2]}$

  $a^{[2]} = \sigma\left(z^{[2]}\right) = \hat{y}$

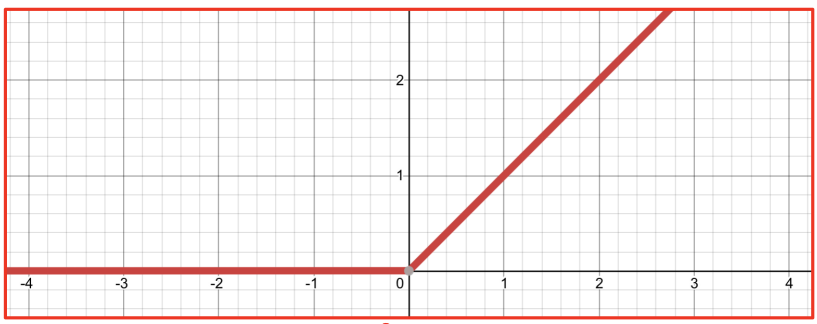
Andere Aktivierungsfunktionen



- Sigmoid(x) = $\sigma(x)$



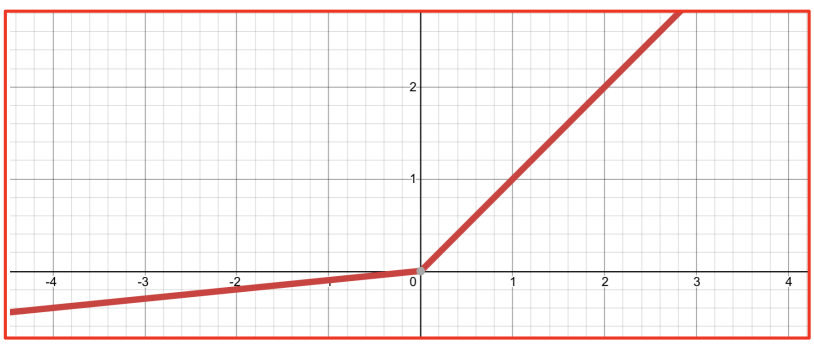
- tanh(x)



- ReLU = max(0, x)

$$= \begin{cases} 0 & , x \leq 0 \\ x & , x > 0 \end{cases}$$

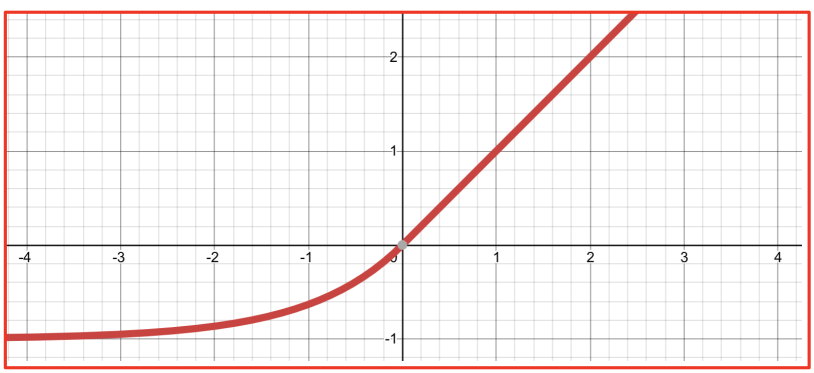
Good default choice (for hidden layers)



- Leaky ReLU

$$= \max(0.1x, x)$$

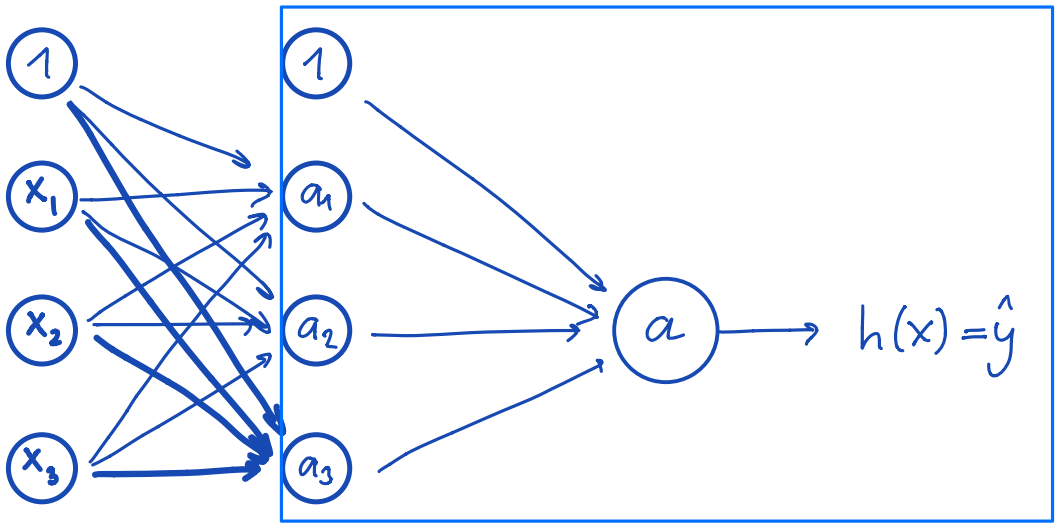
$$= \begin{cases} 0.1x & , x < 0 \\ x & , x \geq 0 \end{cases}$$



- ELU

$$= \begin{cases} \alpha(e^x - 1) & , x < 0 \\ x & , x \geq 0 \end{cases}$$

- Intuition : Das Lernen von Merkmalen für Logistische Regression

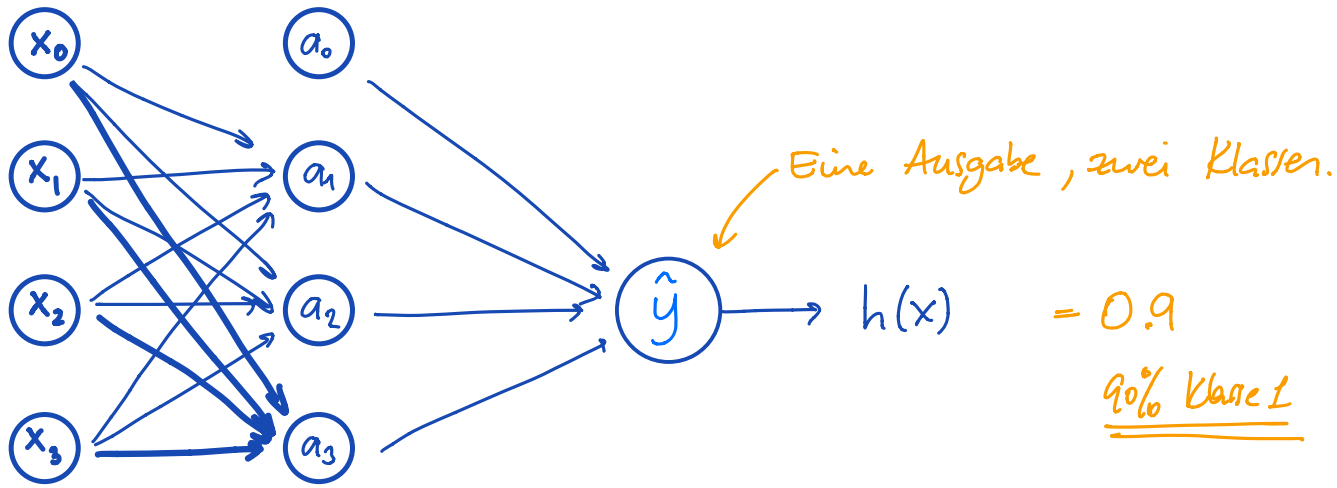


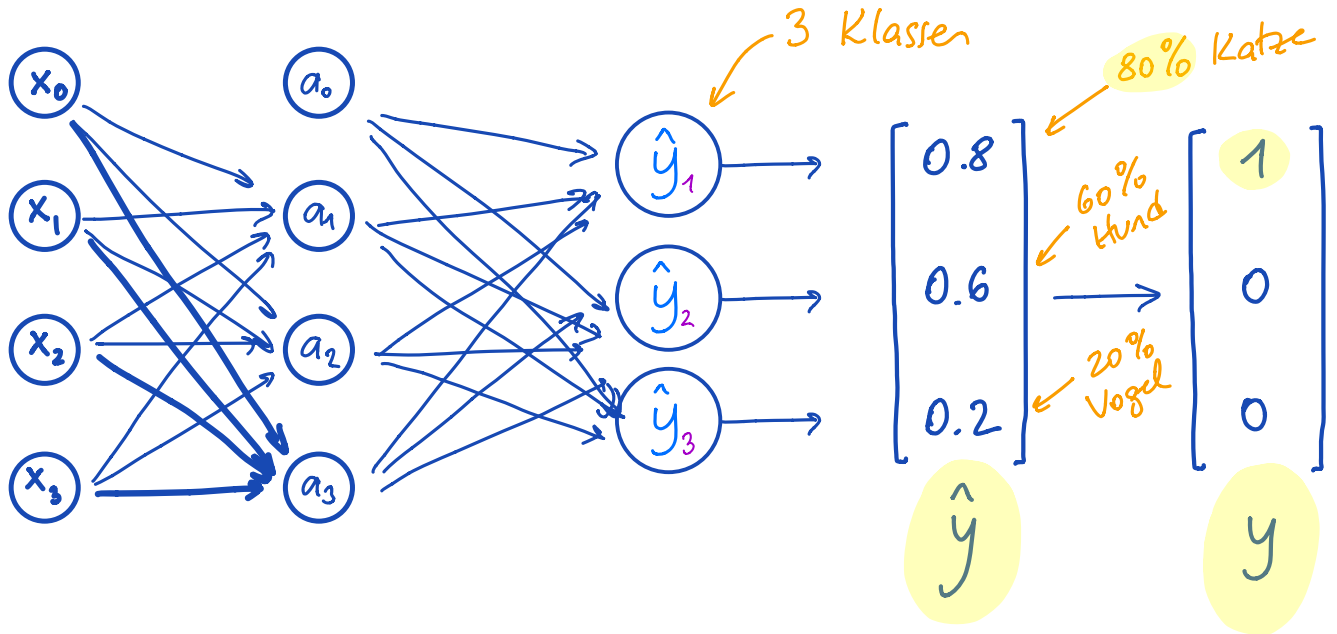
Logistische Repression
mit Merkmalen a_1, a_2, a_3

$$\hat{y} = \sigma\left(w_{11} a_1 + w_{12} a_2 + w_{13} a_3 + b_1 \right)$$

→ Das Netzwerk lernt eigene Merkmale ($a^{[1]}$) für logistische Regression

- **Mehrklassige Klassifizierung** (one-vs-all)





- Kostenfunktion (L Schichten, m Beispiele, K Klassen)

$$J(w) = \frac{-1}{m}\left[\sum_{i=1}^{m}\left[\sum_{k=1}^{K} y_k^{(i)} \cdot \log\left(\underbrace{h(x^{(i)})_k}_{\hat{y}_k^{(i)}}\right) + \left(1-y_k^{(i)}\right)\cdot\log\left(1-h(x^{(i)})_k\right)\right]\right]$$

$$+ \frac{\lambda}{2m}\sum_{l=1}^{L-1}\sum_{i=1}^{s_l}\sum_{j=1}^{s_{l+1}}\left(w_{ji}^{(l)}\right)^2$$

All weights in all layers