Stochastic simulation of a simple epidemic

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SIR model

$$S \xrightarrow{\beta SI} I \xrightarrow{\gamma I} R$$

N: number of individuals in the population

S: number of Susceptible individuals
I: number of Infective individuals
R: number of Removed (recovered/dead) individuals

Where $\beta > 0$, corresponds to the infection transmision rate and $\gamma > 0$ the recovery rate

$$N = S(t) + I(t) + R(t)$$

$$\begin{aligned} \frac{\delta S}{\delta t} &= -\beta I \frac{S}{N} \\ \frac{\delta I}{\delta t} &= \beta I \frac{S}{N} - \gamma I \\ \frac{\delta R}{\delta t} &= \gamma I \end{aligned}$$

$$\xrightarrow{\text{Stochastic}}$$

Continuos variable = Discrete variable

Process rate= Process probabilities

Gillespie algorithm

for *i=1 to 100* **do**
  while *Number of infection>0* ***and*** $t(j) \leq T$ **do**
    Generate uniform random number u_1 and u_2
    Define total transition probability P and probability of infection P_i
    Determine time step to next transition $T_x = -\frac{ln(u_1)}{P}$
    if $u_2 \leq P_i$ **then**
      $I \leftarrow I + 1$
      $S \leftarrow S - 1$
    else
      $I \leftarrow I - 1$
      $S \leftarrow S + 1$
    end
    j=j+1
  end
end
Calculate the mean of $I(t)$ for every inteval t.

$$a_1 = \beta S I$$
$$a_2 = \gamma I$$
$$P = \beta S I + \gamma I$$
$$P_1 = \frac{\beta S I}{\beta S I + \gamma I}$$
$$P_2 = \frac{\gamma I}{\beta S I + \gamma I}$$

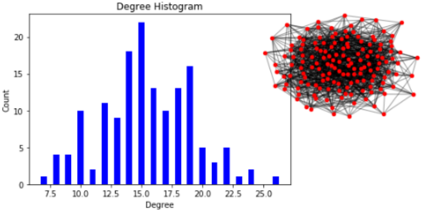
$$\beta = k\, T_r$$
$$\lambda(I) = \beta(I/N)$$

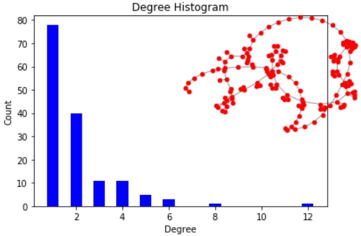
If I = 0, an absorbing state has been reached and the process stops.

Networks

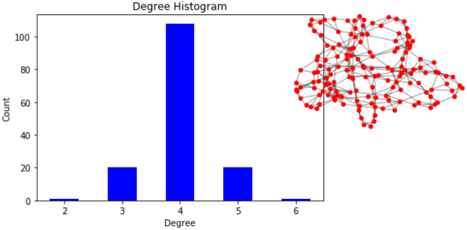
Random network $P(k) = \binom{n-1}{k} p^k (1-p)^{n-1-k}$

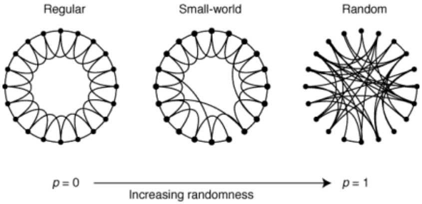
PL network $P(k) \sim k^{-\gamma}$





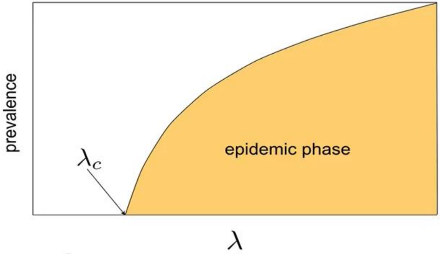
SW network





Epidemic threshold in maximally random graphs.

$$\lambda_c = \frac{\langle k \rangle}{\langle k^2 \rangle - \langle k \rangle}$$

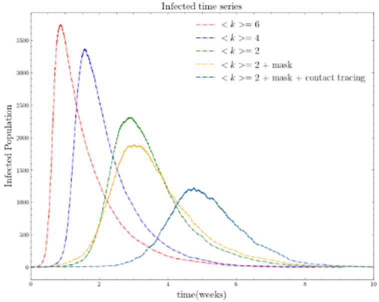


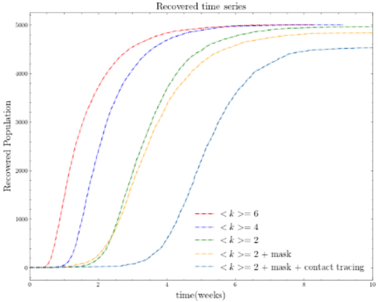
In scale free networks we have

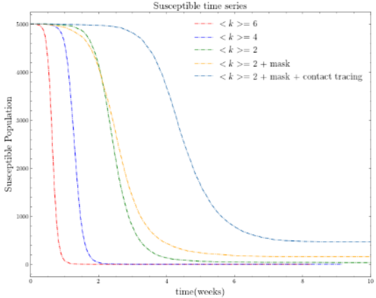
$$\langle k^2 \rangle \to \infty \qquad \lambda_c \to 0$$

$$2 < \gamma \leq 3$$

Results and discussion





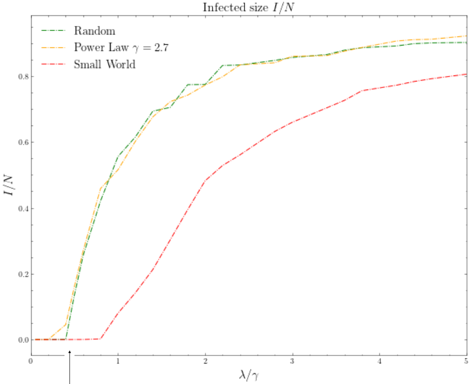


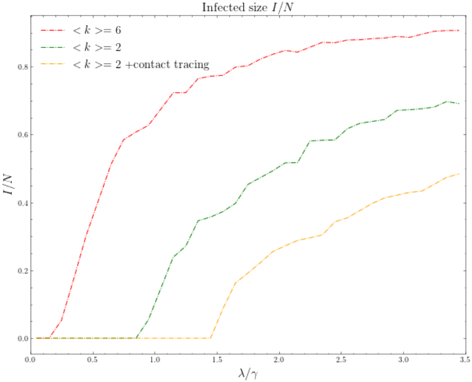
Decreasing expected degree = social distancing

Decreasing infection rate = mask

Detect infected person with probability p and remove = Contact tracing

Results and discussion





$\lambda_c = 0.19$ (Analytical)

$\lambda_c = 0.15$ (Numerical)

Conclusions

- Network models present great flexibility compared to the continuous model.
- Social distancing and contact tracing are useful measures to control infection.
- The simulation is very sensitive to initial conditions and infection parameters.