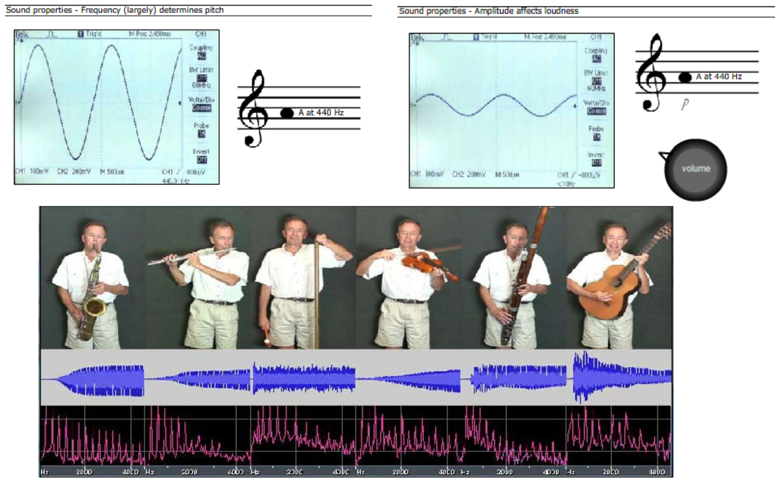
Music signals

U3

- Pitch, loudness, timbre (*tono, volumen, timbre*)

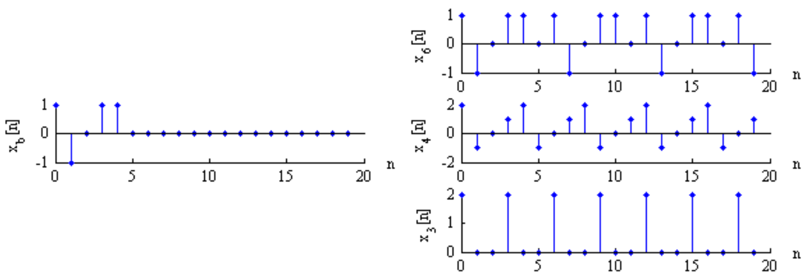


Periodic signals

U3

- $x[n] = x[n + P]$ (discrete-time signal of period P)
- Any periodic signal can be rewritten as the convolution of basic signal $x_b[n]$ with a train of deltas

$$x[n] = \sum_{i=-\infty}^{\infty} x_b[n - iP] = x_b[n] * \sum_{i=-\infty}^{\infty} \delta[n - iP]$$



DTFT of a train of deltas

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The sum of the harmonic frequencies of F_0=1/P:

$$\sum_{k=0}^{P-1} e^{j2\pi\frac{k}{P}n} = \begin{cases} P, & \text{if} \quad n = \dot{P} \\ \dfrac{1-e^{j2\pi n}}{1-e^{j2\pi\frac{1}{P}n}} = 0, & \text{if} \quad n \neq \dot{P} \end{cases}$$

This is a train of Kronecker deltas with period P and amplitude P

$$t[n] = \sum_{i=-\infty}^{\infty} \delta[n-iP] = \frac{1}{P}\sum_{k=0}^{P-1} e^{j2\pi\frac{k}{P}n}$$

$$T(F) = \frac{1}{P}\sum_{i=-\infty}^{\infty}\sum_{k=0}^{P-1} \delta\left(F-\frac{k}{P}-i\right)$$

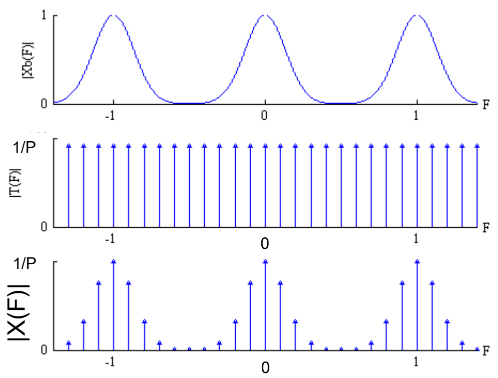
⟵ Dirac deltas

DTFT of periodic signals

U3

$$x[n] = x_b[n] * \sum_{i=-\infty}^{\infty} \delta[n-iP] \leftrightarrow \quad X(F) = X_b(F)\cdot T(F)$$

$$= \tfrac{1}{P}\textstyle\sum_{i=-\infty}^{\infty}\sum_{k=0}^{P-1} X_b\left(\frac{k}{P}\right)\delta\left(F-\frac{k}{P}-i\right)$$



Dirac deltas at the harmonic frequencies of $F_0 = \frac{1}{P}$

Windowing of signals

U3

- In real life, we can only work with a portion of a infinite signal: windowing

$$x_v[n] = x[n] \cdot w[n] \leftrightarrow \quad X_v(F) = X(F) \otimes W(F)$$

$$= W(F) * \sum_{k=0}^{P-1} \frac{1}{P} X_b\left(\frac{k}{P}\right) \delta\left(F - \frac{k}{P}\right)$$

$$= \sum_{k=0}^{P-1} \frac{1}{P} X_b\left(\frac{k}{P}\right) W\left(F - \frac{k}{P}\right)$$

Examples of windows (w[n])

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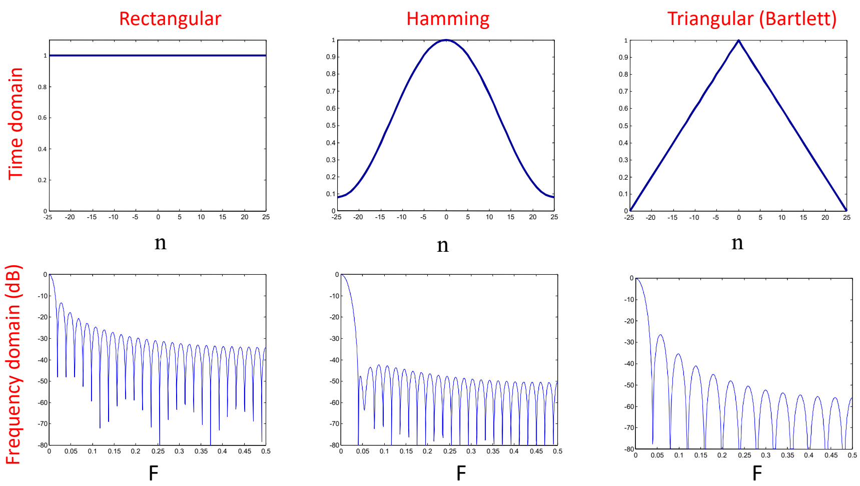

Tradeoff between width of the main lobe and level of secondary lobes

Features of windows

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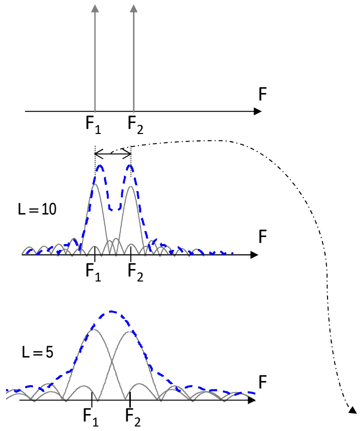
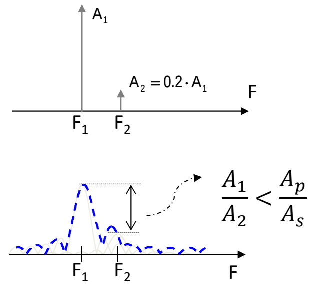
- Frequency resolution of the window: ability to detect spectral components of similar amplitude which are close in frequency



- Amplitude sensitivity of the window: ability to detect spectral components with very different amplitudes



$$\frac{A_1}{A_2} < \frac{A_p}{A_s}$$

$$|F_1 - F_2| > \Delta F$$

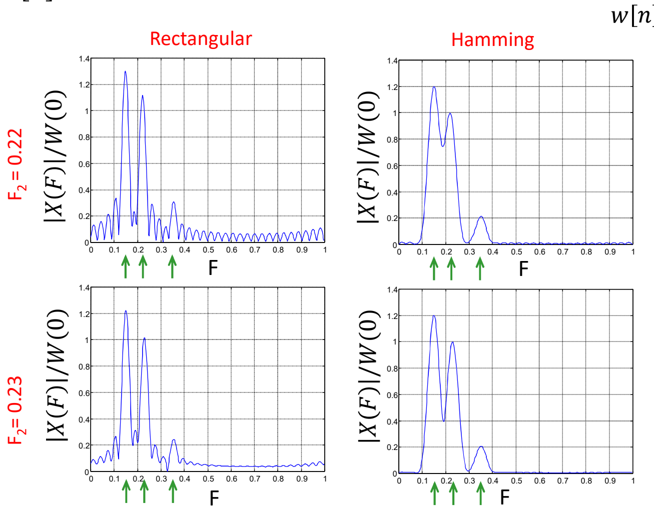
Comparison of windows

U3

$$s[n] = 1.2\, e^{j2\pi \cdot 0.15n} + 1\, e^{j2\pi F_2 n} + 0.2\, e^{j2\pi \cdot 0.35n}$$

$x[n] = s[n] \cdot w[n]$

$w[n]$: window of 32 samples



Fourier transforms of $x[n]$

Summary of windows

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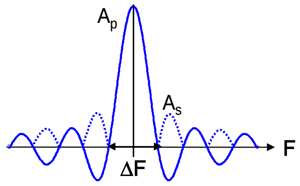
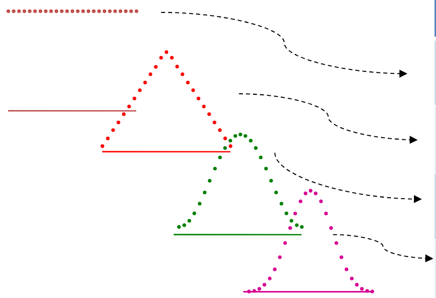
- Rectangular window
  - Good resolution if L>>1
  - Constant sensitivity (~13 dB) (and independent of L).



- Trade-off resolution/sensitivity:



Window	ΔF	α_{ps} dB
Rectangular	2/L	13
Barlett (triangular)	4/L	26
Hamming	4/L	41
Blackman	6/L	57