Probability distributions

Continuous Distributions

Beta(α, β)

$f(x) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 \le x \le 1, \quad \alpha, \beta > 0$

$E(X) = \frac{\alpha}{\alpha+\beta}, \quad V(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

$M(t) = 1 + \sum_{k=1}^{\infty} \left(\prod_{r=0}^{k-1} \frac{\alpha+r}{\alpha+\beta+r}\right) \frac{t^k}{k!}$

Cauchy(θ, σ)

$f(x) = \frac{1}{\pi\sigma} \frac{1}{1+(\frac{x-\theta}{\sigma})^2}, \quad -\infty < x < \infty, \quad -\infty < \theta < \infty, \quad \sigma > 0.$

$E(X)$ **n.a.** , $V(X)$ **n.a.**

$M(t)$ **n.a.**

$\chi^2(n)$

$f(x) = \frac{1}{\Gamma(n/2)2^{n/2}} x^{n/2-1} e^{-x/2}, \quad 0 \le x < \infty, \quad n = 1, 2, \ldots$

$E(X) = n, \quad V(X) = 2 \cdot n$

$M(t) = \left(\frac{1}{1-2t}\right)^{n/2}$

Double exponential(μ, σ)

$f(x) = \frac{1}{2\sigma} e^{-|x-\mu|/\sigma} \quad -\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \sigma > 0$

$E(X) = \mu, \quad V(X) = 2\sigma^2$

$M(t) = \frac{e^{\mu t}}{1-(\sigma t)^2}$

Exponential(λ)

$f(x) = \lambda e^{-\lambda x}, \quad x > 0, \quad \lambda > 0$

$E(X) = \frac{1}{\lambda}, \quad V(X) = \frac{1}{\lambda^2}$

$M(t) = \frac{1}{1-t/\lambda}$

$\varphi(t) = \frac{\lambda}{\lambda - it}$

Exponential (alternative parametrization)

$f(x) = \frac{1}{\beta} e^{-x/\beta}, \quad x \geq 0, \quad \beta > 0$

$E(X) = \beta, \quad V(X) = \beta^2$

$M(t) = \frac{1}{1-\beta t}, \quad t < \frac{1}{\beta}$

F(ν_1, ν_2)

$f(x) = \frac{\Gamma(\frac{\nu_1+\nu_2}{2})}{\Gamma(\frac{\nu_1}{2})\Gamma(\frac{\nu_2}{2})} \cdot \left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2} \cdot \frac{x^{(\nu_1-2)/2}}{(1+\frac{\nu_1}{\nu_2}x)^{(\nu_1+\nu_2)/2}}, \quad x \geq 0, \quad \nu_1, \nu_2 = 1, 2, \ldots$

$E(X) = \frac{\nu_2}{\nu_2 - 2}, \quad \nu_2 > 2, \quad V(X) = 2 \cdot \left(\frac{\nu_2}{\nu_2-2}\right)^2 \cdot \frac{\nu_1+\nu_2-2}{\nu_1(\nu_2-4)}, \quad \nu_2 > 4$

$M(t)$ **n.a.**

Gamma(α, β)

$f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta} \quad x > 0, \quad \alpha, \beta > 0$

$E(X) = \alpha\beta \quad V(X) = \alpha\beta^2$

$M(t) = \left(\frac{1}{1-\beta t}\right)^\alpha$

Gamma (alternative parametrization)

$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$

$E(X) = \frac{\alpha}{\beta} \quad V(X) = \frac{\alpha}{\beta^2}$

$M(t) = \left(\frac{1}{1-t/\beta}\right)^\alpha$

Inverse Gamma(α, β)

$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{-\alpha-1} e^{-\beta/x} \quad x > 0, \quad \alpha, \beta > 0$

$E(X) = \frac{\beta}{\alpha-1} \quad V(X) = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)}$

Logistic(μ, β)

$f(x) = \frac{1}{\beta} \frac{e^{-(x-\mu)/\beta}}{(1+e^{-(x-\mu)/\beta})^2} \quad -\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \beta > 0$

$E(X) = \mu, \quad V(X) = \frac{\pi^2 \beta^2}{3}$

$M(t) = e^{\mu t}\Gamma(1-\beta t)\Gamma(1+\beta t), \quad |t| < \frac{1}{\beta}$

Lognormal(μ, σ^2)

$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \frac{e^{-(\log x-\mu)^2/(2\sigma^2)}}{x}, \quad 0 \le x < \infty, \quad -\infty < \mu < \infty, \quad \sigma > 0$

$E(X) = e^{\mu + \frac{\sigma^2}{2}}, \quad V(X) = e^{2(\mu+\sigma^2)} - e^{2\mu+\sigma^2}$

$M(t) =$ **n.a.**

Normal(μ, σ^2)

$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \sigma > 0$

$E(X) = \mu, \quad V(X) = \sigma^2$

$M(t) = e^{\mu t + \sigma^2 t^2/2}$

$\varphi(t) = e^{it\mu - \frac{1}{2}\sigma^2 t^2}$

Pareto(α, β)

$f(x) = \frac{\beta \alpha^\beta}{x^{\beta+1}}, \quad \alpha < x < \infty, \quad \alpha > 0, \quad \beta > 0$

$E(X) = \frac{\beta\alpha}{\beta-1}, \quad \beta > 1. \quad V(X) = \frac{\beta\alpha^2}{(\beta-1)^2(\beta-2)}, \quad \beta > 2$

$M(t)$ **n.a**

t(ν)

$f(x) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})} \frac{1}{\sqrt{\nu\pi}} \frac{1}{(1+\frac{x^2}{\nu})^{(\nu+1)/2}} \quad -\infty < x < \infty, \quad \nu = 1, \ldots$

$E(X) = 0, \quad \nu > 1, \quad V(X) = \frac{\nu}{\nu-2}, \quad \nu > 2$

$M(t)$ **n.a.**

Uniform(a, b)

$f(x) = \frac{1}{b-a}, \quad a \leq x \leq b$

$E(X) = \frac{b+a}{2}, \quad V(X) = \frac{(b-a)^2}{12}$

$M(t) = \frac{e^{bt} - e^{at}}{(b-a)t}$

Weibull(γ, β)

$f(x) = \frac{\gamma}{\beta} x^{\gamma-1} e^{-x^{\gamma}/\beta}, \quad 0 < x < \infty, \quad \gamma > 0, \quad \beta > 0$

$E(X) = \beta^{1/\gamma} \Gamma(1 + \frac{1}{\gamma}), \quad V(X) = \beta^{2/\gamma} \left(\Gamma(1 + \frac{2}{\gamma}) - \Gamma^2(1 + \frac{1}{\gamma}) \right)$

Discrete Distributions

Bernoulli(p)

$P(X = x) = p^x(1-p)^{1-x}, \quad x = 0, 1; \quad 0 \leq p \leq 1$

$E(X) = p, \quad V(X) = p(1-p)$

$M(t) = (1-p) + pe^t$

Binomial(n, p)

$P(X = x) = \binom{n}{x} p^x(1-p)^{n-x}; \quad x = 0, 1, \ldots, n; \quad 0 \leq p \leq 1$

$E(X) = np, \quad V(X) = np(1-p)$

$M(t) = (pe^t + (1-p))^n$

$\varphi(t) = \left(1 - p + pe^{it}\right)^n$

Discrete Uniform

$P(X = x \mid N) = \frac{1}{N} \quad x = 1, \ldots, N$

$E(X) = \frac{N+1}{2}, \quad V(X) = \frac{(N+1)(N-1)}{12}$

$M(t) = \frac{1}{N} \sum_{i=1}^{N} e^{it}$

Geometric(p)

$P(X = x) = p(1-p)^{x-1}, \quad 0 \leq p \leq 1, \quad x = 1, \ldots \quad 0 \leq p \leq 1$

$E(X) = \frac{1}{p}, \quad V(X) = \frac{1-p}{p^2}$

$M(t) = \frac{pe^t}{1-(1-p)e^t}$

$\varphi(t) = \frac{pe^{it}}{1-(1-p)e^{it}}$

Geometric(p) (alternative formulation)

$P(X = x) = p(1-p)^{x}, \quad 0 \leq p \leq 1, \quad x = 0, \ldots \quad 0 \leq p \leq 1$

$E(X) = \frac{1-p}{p}, \quad V(X) = \frac{1-p}{p^2}$

$M(t) = \frac{p}{1-(1-p)e^t}$

$\varphi(t) = \frac{p}{1-(1-p)e^{it}}$

Hypergeometric

$P(X = x) = \frac{\binom{M}{x}\binom{N-M}{K-x}}{\binom{N}{K}}$

$E(X) = \frac{KM}{N}, \quad V(X) = \frac{KM}{N}\frac{(N-M)(N-K))}{N(N-1)}$

Multinomial

$P(\mathbf{X} = \mathbf{x}) = \frac{n!}{x_1!\cdots x_k!}p_1^{x_1}\cdots p_k^{x_k} \quad x_i = 0, 1, \ldots \quad x_i + \cdots x_k = n$

$E(X_i) = np_i, \quad V(X_i) = np_i(1-p_i), \quad Cov(X_i, X_j) = -np_ip_j$

Negative Binomial

$P(X = x) = \binom{r+x-1}{x}p^r(1-p)^x; \quad x = 0, 1, \ldots; \quad 0 \leq p \leq 1$

$E(X) = \frac{r(1-p)}{p}, \quad V(X) = \frac{r(1-p)}{p^2}$

$M(t) = \left(\frac{p}{1-(1-p)e^t}\right)^r$

Poisson

$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad x = 0, 1, \ldots \quad 0 \leq \lambda < \infty$

$E(X) = \lambda, \quad V(X) = \lambda$

$M(t) = e^{\lambda(e^t-1)}$

$\varphi(t) = e^{\lambda(e^{it}-1)}$

References

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