CAMBIO DE VARIABLES

Curso 2019-2020

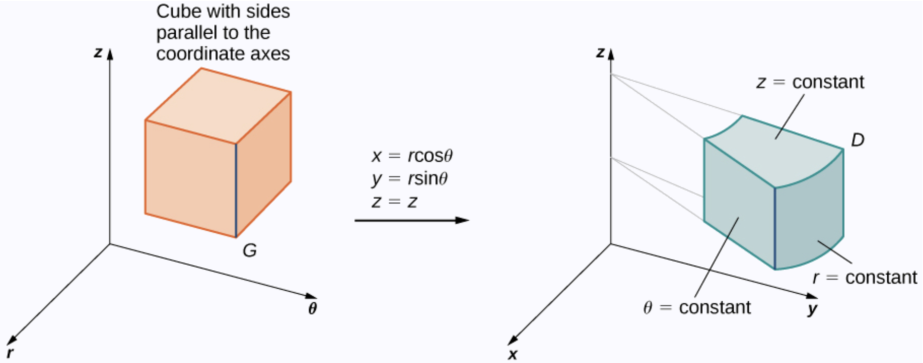

Imagen: wikipedia

Cambio de variable en $\mathbb{R}$

1. $I, J \subset \mathbb{R}$ intervalos abiertos
2. $\varphi\colon I \longrightarrow J$ biyección de clase $\mathcal{C}^1(I) \Longrightarrow \varphi^{-1} \in \mathcal{C}^1(J)$
3. $[a,b] \subset J \Longrightarrow \varphi^{-1}([a,b])$ es un intervalo cerrado $[\alpha,\beta]$.

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Para cada función continua $f\colon [a,b] \longrightarrow \mathbb{R}$ se satisface que

$$\int_a^b f(x)\,dx = \left[\begin{array}{l} x = \varphi(s) \\ dx = \varphi'(s)\,ds \end{array}\right]$$

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- Considerar $G(s) = \displaystyle\int_{\varphi^{-1}(a)}^{s} f\big(\varphi(u)\big)\,\varphi'(u)\,du$ y $F(y) = \displaystyle\int_a^y f(u)\,du$

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- Considerar $G(s) = \displaystyle\int_{\varphi^{-1}(a)}^{s} f\big(\varphi(u)\big)\,\varphi'(u)\,du$ y $F(y) = \displaystyle\int_a^y f(u)\,du$
- $G'(s) = f\big(\varphi(s)\big)\,\varphi'(s) = F\big(\varphi(s)\big)' \Longrightarrow G(s) = F\big(\varphi(s)\big) + k$

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- Como $\varphi \circ \varphi^{-1} = Id \Longrightarrow \varphi'(x) \neq 0$, para cada $x \in I$.

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- ¿Por qué aparece φ' en la fórmula del cambio?
- Si $f = 1 \Longrightarrow b - a = \displaystyle\int_\alpha^\beta |\varphi'(s)|\,ds$

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Cambio de variable

1. $U, V \subset \mathbb{R}^n$ abiertos
2. $F\colon U \longrightarrow V$ difeomorfismo de clase $\mathcal{C}^1(U) \Longrightarrow \det DF \neq 0$
3. $D \subset U$ elemental tal que $F(D) \subset V$ es elemental.

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En particular, considerando f, la función constantemente igual a 1,

$$\boxed{\mathsf{v}\big(F(D)\big) = \int_D |\mathrm{det}(DF)|}$$

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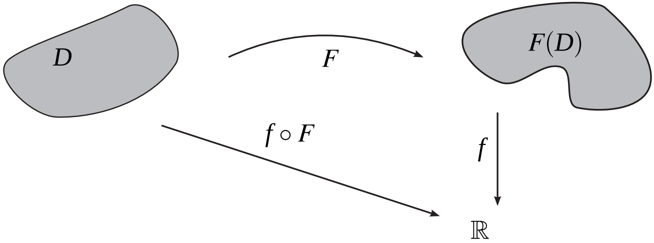

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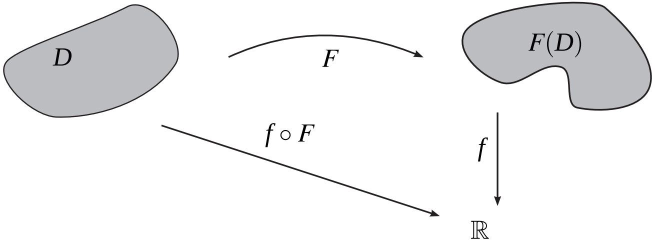

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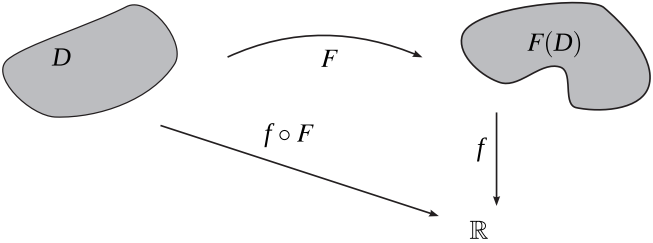

- $$\int_{F(D)} f = \int_D (f \circ F)|\det DF| \Longrightarrow \mathsf{v}\big(F(D)\big) = \int_D |\det DF|$$

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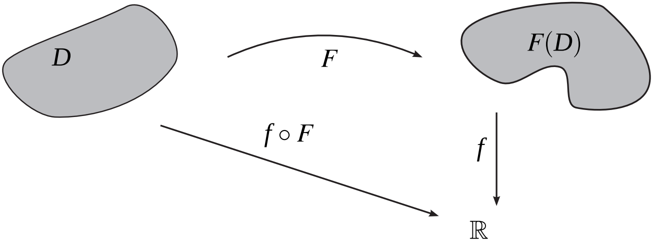

- Si $D^* = F(D)$, $\displaystyle\int_{D^*} f = \int_{F^{-1}(D^*)} (f \circ F)|\det DF|$

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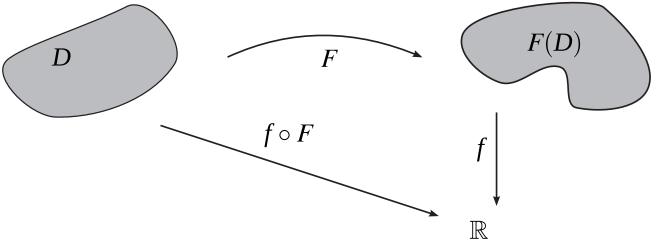


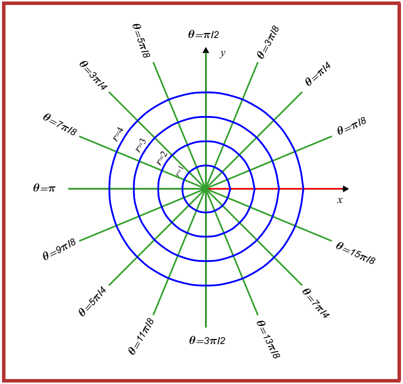
- Si $D^* = F(D)$, $\mathsf{v}\big(D^*\big) = \displaystyle\int_{F^{-1}(D^*)} |\det DF|$

Coordenadas polares

- $\varphi\colon (0,+\infty)\times(\theta_0,\theta_0+2\pi)\longrightarrow\mathbb{R}^2$, dada por

$$\varphi(r,\theta)=\big(r\cos(\theta),r\,\mathrm{sen}(\theta)\big)$$

- $\mathrm{Img}(\varphi)=\mathbb{R}^2\setminus\{(x,y): y=x\tan(\theta_0),\ x\geq 0\},\quad \mathsf{J}_\varphi=r$



Coordenadas polares

- Calcular $\displaystyle\int_A xy^2$, donde

$$A = \left\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1,\ x > 0,\ y > 0\right\}.$$

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- Calcular $\displaystyle\int_B x^2 + y^2$, donde

$$B = \Big\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 2ax\Big\}.$$

Coordenadas polares desplazadas

- $\varphi\colon (0,+\infty)\times(\theta_0,\theta_0+2\pi)\longrightarrow\mathbb{R}^2$, dada por
$$\varphi(r,\theta)=\big(a+r\cos(\theta),b+r\mathrm{sen}(\theta)\big)$$
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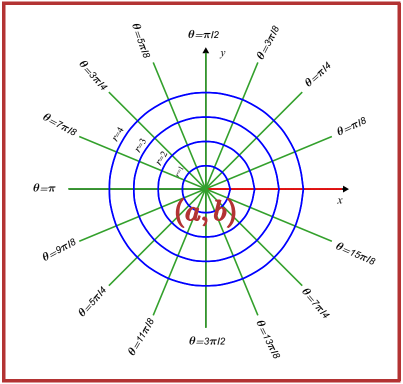

Coordenadas polares desplazadas

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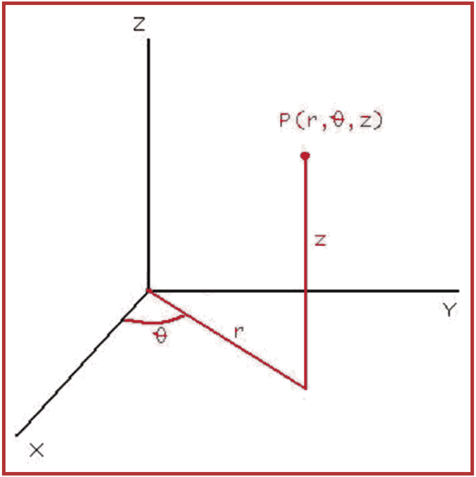
Coordenadas polares desplazadas

- Calcular $\int_B x^2 + y^2$, donde

$$B = \left\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 2ax + 2by\right\}.$$

Coordenadas cilíndricas

- $\Phi\colon (0,+\infty)\times(\theta_0,\theta_0+2\pi)\times\mathbb{R}\longrightarrow\mathbb{R}^3$, dada por
$$\Phi(r,\theta,z)=\big(r\cos(\theta),r\mathrm{sen}(\theta),z\big)$$



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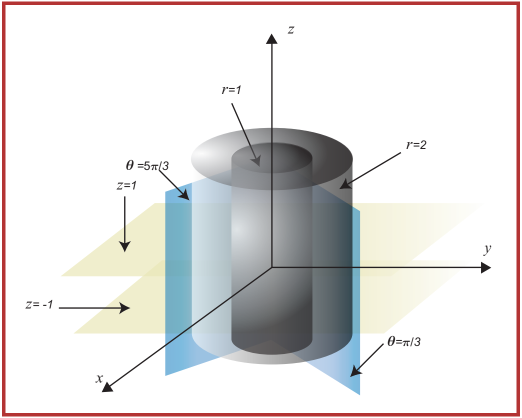


Coordenadas cilíndricas

- Calcular $\displaystyle\int_A 7yz$, donde

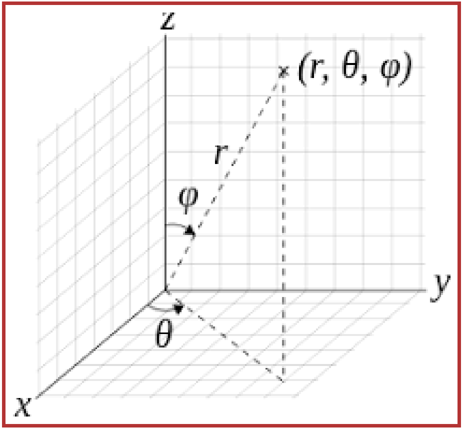
$$A = \Big\{(x,y,z) \in \mathbb{R}^3 : \; y > 0,\; 0 < z < a, \\ x^2 + y^2 < b^2\Big\}$$

y $a, b > 0$

Coordenadas esféricas

- $\Psi\colon (0,+\infty)\times(0,2\pi)\times(0,\pi)\longrightarrow \mathbb{R}^3$, dada por

$$\Psi(r,\theta,\varphi)=\big(r\cos(\theta)\mathrm{sen}(\varphi), r\mathrm{sen}(\theta)\mathrm{sen}(\varphi), r\cos(\varphi)\big)$$



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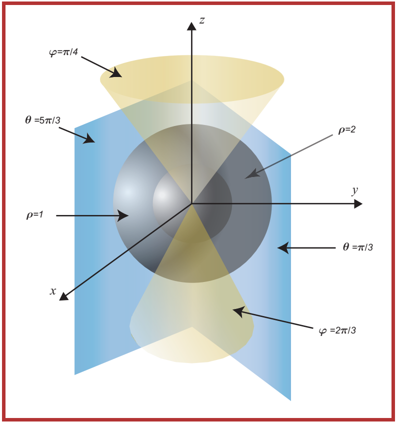

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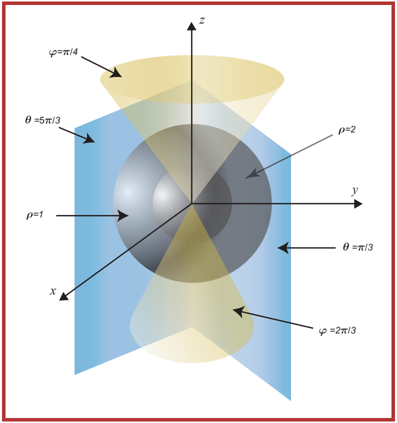

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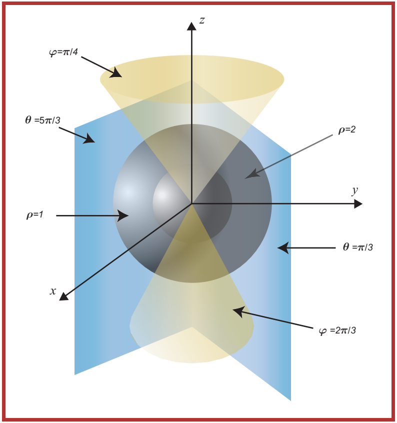
- $\Psi\colon (0,+\infty)\times(0,\pi)\times(0,2\pi)\longrightarrow \mathbb{R}^3$, dada por
  $$\Psi(r,\varphi_1,\varphi_2)=\big(r\operatorname{sen}(\varphi_1)\cos(\varphi_2), r\operatorname{sen}(\varphi_1)\operatorname{sen}(\varphi_2), r\cos(\varphi_1)\big)$$
- $\operatorname{Img}(\Psi)=\mathbb{R}^3\setminus\{(x,0,z): x\geq 0\},\quad \mathsf{J}_\Psi = r^2\operatorname{sen}(\varphi_1)$



Coordenadas esféricas

- Calcular $\displaystyle\int_A xyz$, donde

$$A = \Big\{(x,y,z) \in \mathbb{R}^3 : \; x^2 + y^2 + z^2 < 1, \\ x > 0,\; y > 0,\; z > 0\Big\}.$$

Coordenadas esféricas n-dimensionales

- $\Psi\colon (0,+\infty)\times(0,\pi)\times(0,2\pi)\longrightarrow\mathbb{R}^3$, dada por

$$\Psi(r,\varphi_2,\varphi_1)=\big(r\mathrm{sen}(\varphi_1)\cos(\varphi_2), r\mathrm{sen}(\varphi_1)\mathrm{sen}(\varphi_2), r\cos(\varphi_1)\big)$$

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- Si $n\geq 3$, $\Psi\colon (0,+\infty)\times(0,\pi)^{n-2}\times(0,2\pi)\longrightarrow\mathbb{R}^n$, dada por

$$\boxed{\begin{aligned}
x_1 &= r\cos(\varphi_1)\\
x_2 &= r\mathrm{sen}(\varphi_1)\cos(\varphi_2)\\
x_3 &= r\mathrm{sen}(\varphi_1)\mathrm{sen}(\varphi_2)\cos(\varphi_3)\\
x_4 &= r\mathrm{sen}(\varphi_1)\mathrm{sen}(\varphi_2)\mathrm{sen}(\varphi_3)\cos(\varphi_4)\\
&\vdots\\
x_{n-1} &= r\mathrm{sen}(\varphi_1)\mathrm{sen}(\varphi_2)\cdots\mathrm{sen}(\varphi_{n-2})\cos(\varphi_{n-1})\\
x_n &= r\mathrm{sen}(\varphi_1)\mathrm{sen}(\varphi_2)\cdots\mathrm{sen}(\varphi_{n-2})\mathrm{sen}(\varphi_{n-1})
\end{aligned}}$$

Coordenadas esféricas n-dimensionales

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$$\begin{aligned}
x_1 &= r\mathrm{sen}(\varphi_1)\cos(\varphi_2)\\
x_2 &= r\mathrm{sen}(\varphi_1)\mathrm{sen}(\varphi_2)\cos(\varphi_3)\\
x_3 &= r\mathrm{sen}(\varphi_1)\mathrm{sen}(\varphi_2)\mathrm{sen}(\varphi_3)\cos(\varphi_4)\\
&\vdots\\
x_{n-2} &= r\mathrm{sen}(\varphi_1)\mathrm{sen}(\varphi_2)\cdots\mathrm{sen}(\varphi_{n-2})\cos(\varphi_{n-1})\\
x_{n-1} &= r\mathrm{sen}(\varphi_1)\mathrm{sen}(\varphi_2)\cdots\mathrm{sen}(\varphi_{n-2})\mathrm{sen}(\varphi_{n-1})\\
x_n &= r\cos(\varphi_1)
\end{aligned}$$

Coordenadas esféricas n-dimensionales

- $\Psi\colon (0,+\infty)\times(0,\pi)\times(0,2\pi)\longrightarrow\mathbb{R}^3$, dada por

$$\Psi(r,\varphi_2,\varphi_1)=\big(r\mathrm{sen}(\varphi_1)\cos(\varphi_2), r\mathrm{sen}(\varphi_1)\mathrm{sen}(\varphi_2), r\cos(\varphi_1)\big)$$

- Si $n=3$, $\Psi\colon (0,+\infty)\times(0,\pi)^{n-2}\times(0,2\pi)\longrightarrow\mathbb{R}^n$, dada por

$$\boxed{\begin{aligned}
x_1 &= r\mathrm{sen}(\varphi_1)\cos(\varphi_2)\\
x_2 &= r\mathrm{sen}(\varphi_1)\mathrm{sen}(\varphi_2)\cos(\varphi_3)\\
x_3 &= r\mathrm{sen}(\varphi_1)\mathrm{sen}(\varphi_2)\mathrm{sen}(\varphi_3)\cos(\varphi_4)\\
&\vdots\\
x_{n-2} &= r\mathrm{sen}(\varphi_1)\mathrm{sen}(\varphi_2)\cdots\mathrm{sen}(\varphi_{n-2})\cos(\varphi_{n-1})\\
x_{n-1} &= r\mathrm{sen}(\varphi_1)\mathrm{sen}(\varphi_2)\cdots\mathrm{sen}(\varphi_{n-2})\mathrm{sen}(\varphi_{n-1})\\
x_n &= r\cos(\varphi_1)
\end{aligned}}$$

Coordenadas esféricas n-dimensionales

- Si $n \geq 3$, $\Psi\colon (0,+\infty) \times (0,\pi)^{n-2} \times (0,2\pi) \longrightarrow \mathbb{R}^n$, dada por

$$\boxed{\begin{aligned}
x_1 &= r\operatorname{sen}(\varphi_1)\cos(\varphi_2)\\
x_2 &= r\operatorname{sen}(\varphi_1)\operatorname{sen}(\varphi_2)\cos(\varphi_3)\\
x_3 &= r\operatorname{sen}(\varphi_1)\operatorname{sen}(\varphi_2)\operatorname{sen}(\varphi_3)\cos(\varphi_4)\\
&\vdots\\
x_{n-2} &= r\operatorname{sen}(\varphi_1)\operatorname{sen}(\varphi_2)\cdots\operatorname{sen}(\varphi_{n-2})\cos(\varphi_{n-1})\\
x_{n-1} &= r\operatorname{sen}(\varphi_1)\operatorname{sen}(\varphi_2)\cdots\operatorname{sen}(\varphi_{n-2})\operatorname{sen}(\varphi_{n-1})\\
x_n &= r\cos(\varphi_1)
\end{aligned}}$$

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x_3 &= r\mathrm{sen}(\varphi_1)\mathrm{sen}(\varphi_2)\mathrm{sen}(\varphi_3)\cos(\varphi_4)\\
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x_{n-1} &= r\mathrm{sen}(\varphi_1)\mathrm{sen}(\varphi_2)\cdots\mathrm{sen}(\varphi_{n-2})\mathrm{sen}(\varphi_{n-1})\\
x_n &= r\cos(\varphi_1)
\end{aligned}}$$

- $\Psi\big((0,+\infty) \times (0,\pi)^{n-2} \times (0,2\pi)\big) = A_n$, donde

$$A_n = \mathbb{R}^n \setminus \Big\{(x_1,\dots,x_n) \in \mathbb{R}^n : x_{n-2} \geq 0,\ x_{n-1} = 0\Big\}$$

Coordenadas esféricas n-dimensionales

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$$\boxed{\begin{aligned} x_1 &= r\mathrm{sen}(\varphi_1)\cos(\varphi_2) \\ x_2 &= r\mathrm{sen}(\varphi_1)\mathrm{sen}(\varphi_2)\cos(\varphi_3) \\ x_3 &= r\mathrm{sen}(\varphi_1)\mathrm{sen}(\varphi_2)\mathrm{sen}(\varphi_3)\cos(\varphi_4) \\ &\vdots \\ x_{n-2} &= r\mathrm{sen}(\varphi_1)\mathrm{sen}(\varphi_2)\cdots\mathrm{sen}(\varphi_{n-2})\cos(\varphi_{n-1}) \\ x_{n-1} &= r\mathrm{sen}(\varphi_1)\mathrm{sen}(\varphi_2)\cdots\mathrm{sen}(\varphi_{n-2})\mathrm{sen}(\varphi_{n-1}) \\ x_n &= r\cos(\varphi_1) \end{aligned}}$$

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$$A_n = \mathbb{R}^n \setminus \Big\{(x_1,\dots,x_n) \in \mathbb{R}^n : x_{n-2} \geq 0,\ \ x_{n-1} = 0\Big\}$$

- $\mathsf{J}_\Psi = (-1)^{n-1}\, r^{n-1} \prod_{j=1}^{n-2} \mathrm{sen}^{n-1-j}(\varphi_j)$

Volumen de las bolas n-dimensionales

- Si $n \geq 3$, $\Psi\colon (0,+\infty)\times(0,\pi)^{n-2}\times(0,2\pi)\longrightarrow \mathbb{R}^n$, dada por

$$\boxed{\begin{array}{c} x_k = r\cos(\varphi_{k+1})\displaystyle\prod_{i=1}^{k}\mathrm{sen}(\varphi_i),\;\; k=1,\dots,n-2; \\ x_{n-1} = r\displaystyle\prod_{i=1}^{n-1}\mathrm{sen}(\varphi_i), \qquad\qquad x_n = r\cos(\varphi_1). \end{array}}$$

- $B_n(a) = \big\{(x_1\dots,x_n)\in\mathbb{R}^n : x_1^2+\cdots+x_n^2\leq a^2\big\}$, $a>0$.
- $\Omega_n(a) = \mathsf{v}\big(B_n(a)\big)$
- $\mathsf{J}_\Psi = (-1)^{n-1}\,r^{n-1}\displaystyle\prod_{j=1}^{n-2}\mathrm{sen}^{n-1-j}(\varphi_j)$

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$$\boxed{\begin{aligned} x_k &= r\cos(\varphi_{k+1}) \prod_{i=1}^{k} \operatorname{sen}(\varphi_i), \;\; k = 1,\dots,n-2; \\ x_{n-1} &= r \prod_{i=1}^{n-1} \operatorname{sen}(\varphi_i), \qquad\qquad x_n = r\cos(\varphi_1). \end{aligned}}$$

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- $B_n(a) = T\big([0,a] \times [0,\pi]^{n-2} \times [0,2\pi]\big)$

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- $\Omega_n(a)=\mathsf{v}\big(B_n(a)\big)$
- $\mathsf{J}_\Psi=(-1)^{n-1}\,r^{n-1}\prod_{j=1}^{n-2}\operatorname{sen}^{n-1-j}(\varphi_j)$
- $B_n(a)=T\big([0,a]\times[0,\pi]^{n-2}\times[0,2\pi]\big)$
- $$\boxed{\Omega_n(a)=\int_{B_n(a)} d\boldsymbol{V}=\int_{[0,a]\times[0,\pi]^{n-2}\times[0,2\pi]}\big|J_\Psi\big|\,dr\,d\varphi_1\cdots d\varphi_{n-1}}$$

Volumen de las bolas n-dimensionales

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- $\Omega_n(a) = \mathsf{v}\big(B_n(a)\big)$
- $\mathsf{J}_\Psi = (-1)^{n-1}\, r^{n-1} \displaystyle\prod_{j=1}^{n-2} \operatorname{sen}^{n-1-j}(\varphi_j)$
- $B_n(a) = T\big([0,a] \times [0,\pi]^{n-2} \times [0,2\pi]\big)$
- $$\boxed{\Omega_n(a) = \frac{2\pi}{n}\, a^n \prod_{j=1}^{n-2} \int_0^\pi \operatorname{sen}^{n-1-j}(\varphi_j)\, d\varphi_j}$$

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- $$\int_0^{\pi} \operatorname{sen}^k(\varphi) d\varphi = B\Big(\frac{k+1}{2}, \frac{1}{2}\Big) = \frac{\Gamma\Big(\frac{k+1}{2}\Big)\Gamma\Big(\frac{1}{2}\Big)}{\Gamma\Big(\frac{k+2}{2}\Big)}$$

Volumen de las bolas n-dimensionales

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- $$\int_0^\pi \operatorname{sen}^k(\varphi) d\varphi = B\Big(\frac{k+1}{2}, \frac{1}{2}\Big) = \frac{\Gamma\Big(\dfrac{k+1}{2}\Big)\Gamma\Big(\dfrac{1}{2}\Big)}{\Gamma\Big(\dfrac{k+2}{2}\Big)} = \sqrt{\pi}\, \frac{\Gamma\Big(\dfrac{k+1}{2}\Big)}{\Gamma\Big(\dfrac{k+2}{2}\Big)}$$

Volumen de las bolas n-dimensionales

- $B_n(a) = \big\{(x_1 \dots, x_n) \in \mathbb{R}^n : x_1^2 + \cdots + x_n^2 \leq a^2\big\}$, $a > 0$.
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- $$\int_0^{\pi} \operatorname{sen}^k(\varphi) d\varphi = B\Big(\frac{k+1}{2}, \frac{1}{2}\Big) = \frac{\Gamma\Big(\frac{k+1}{2}\Big)\Gamma\Big(\frac{1}{2}\Big)}{\Gamma\Big(\frac{k+2}{2}\Big)} = \sqrt{\pi}\,\frac{\Gamma\Big(\frac{k+1}{2}\Big)}{\Gamma\Big(\frac{k+2}{2}\Big)}$$
- $$\boxed{\Omega_n(a) = \frac{2\pi}{n}\, a^n (\sqrt{\pi})^{n-2} \prod_{j=1}^{n-2} \frac{\Gamma\Big(\frac{n-j}{2}\Big)}{\Gamma\Big(\frac{n+1-j}{2}\Big)}}$$

Volumen de las bolas n-dimensionales

- $B_n(a) = \big\{(x_1 \dots, x_n) \in \mathbb{R}^n : x_1^2 + \cdots + x_n^2 \le a^2\big\}$, $a > 0$.
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- $$\boxed{\Omega_n(a) = \frac{2\pi}{n}\, a^n \prod_{j=1}^{n-2} \int_0^\pi \operatorname{sen}^{n-1-j}(\varphi_j)\, d\varphi_j \Longrightarrow \Omega_n(a) = a^n \Omega_n(1)}$$
- $$\int_0^\pi \operatorname{sen}^k(\varphi) d\varphi = B\Big(\frac{k+1}{2}, \frac{1}{2}\Big) = \frac{\Gamma\Big(\frac{k+1}{2}\Big)\Gamma\Big(\frac{1}{2}\Big)}{\Gamma\Big(\frac{k+2}{2}\Big)} = \sqrt{\pi}\, \frac{\Gamma\Big(\frac{k+1}{2}\Big)}{\Gamma\Big(\frac{k+2}{2}\Big)}$$
- $$\boxed{\Omega_n(a) = \frac{2\pi}{n}\, a^n (\sqrt{\pi})^{n-2} \prod_{j=1}^{n-2} \frac{\Gamma\Big(\frac{n-j}{2}\Big)}{\Gamma\Big(\frac{n+1-j}{2}\Big)} = \frac{2}{n}\, a^n \pi^{\frac{n}{2}} \frac{\Gamma(1)}{\Gamma\Big(\frac{n}{2}\Big)}}$$

Volumen de las bolas n-dimensionales

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- $$\boxed{\Omega_n(a) = \frac{2\pi}{n}\, a^n \prod_{j=1}^{n-2} \int_0^{\pi} \operatorname{sen}^{n-1-j}(\varphi_j)\, d\varphi_j \Longrightarrow \Omega_n(a) = a^n \Omega_n(1)}$$
- $$\boxed{\Omega_n(a) = \frac{2\pi}{n}\, a^n (\sqrt{\pi})^{n-2} \prod_{j=1}^{n-2} \frac{\Gamma\left(\dfrac{n-j}{2}\right)}{\Gamma\left(\dfrac{n+1-j}{2}\right)} = \frac{a^n \pi^{\frac{n}{2}}}{\frac{n}{2}\Gamma\left(\dfrac{n}{2}\right)}}$$

Volumen de las bolas n-dimensionales

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- $$\boxed{\Omega_n(a) = \frac{a^n \pi^{\frac{n}{2}}}{\Gamma\left(\frac{n}{2} + 1\right)}}$$

Volumen de las bolas n-dimensionales

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- $$\boxed{\Omega_n(a) = \frac{a^n \pi^{\frac{n}{2}}}{\Gamma\Big(\dfrac{n}{2} + 1\Big)} = a^n \begin{cases} \dfrac{\pi^m}{m!}, & \text{si } n = 2m, \\[2ex] \dfrac{2^n}{n!}\, \pi^m m!, & \text{si } n = 2m + 1. \end{cases}}$$

Volumen de las bolas n-dimensionales

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- $$\boxed{\Omega_n(a) = \frac{2\pi}{n}\, a^n \prod_{j=1}^{n-2} \int_0^\pi \operatorname{sen}^{n-1-j}(\varphi_j)\, d\varphi_j \Longrightarrow \Omega_n(a) = a^n \Omega_n(1)}$$
- $$\boxed{\Omega_n(a) = \frac{a^n \pi^{\frac{n}{2}}}{\Gamma\left(\frac{n}{2}+1\right)} = a^n \begin{cases} \dfrac{\pi^m}{m!}, & \text{si } n = 2m, \\[2ex] \dfrac{2^n}{n!}\, \pi^m m!, & \text{si } n = 2m+1. \end{cases}}$$
- $\Omega_1(a) = 2a$, $\Omega_2(a) = \pi a^2$, $\Omega_3(a) = \dfrac{4}{3}\pi a^3$, $\Omega_4(a) = \dfrac{\pi^2}{2}\, a^4$.