CAI: Cerca i Anàlisi de la Informació
Grau en Ciència i Enginyeria de Dades, UPC

Implementation

September 30, 2021

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Contents

Query answering

A bad algorithm:

input query q;
for every document d in database
    check if d matches q;
    if so, add its docid to list L;
output list L (perhaps sorted in some way);

Query answering

A bad algorithm:

input query q;
for every document d in database
    check if d matches q;
    if so, add its docid to list L;
output list L (perhaps sorted in some way);

Time should be largely independent of database size.
(Unavoidably) proportional to answer size.

Central Data Structure: Inverted file

A vocabulary or lexicon or dictionary, usually kept in main memory, maintains all the indexed terms (*set*, *map*...)

- Collection: document $\rightarrow$ words contained in the document

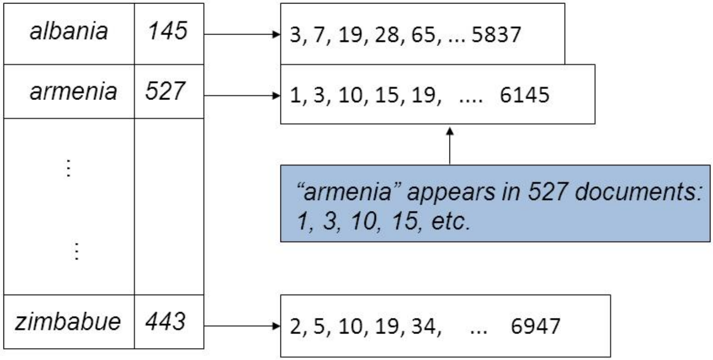
Central Data Structure: Inverted file

A vocabulary or lexicon or dictionary, usually kept in main memory, maintains all the indexed terms (*set*, *map*. . .)

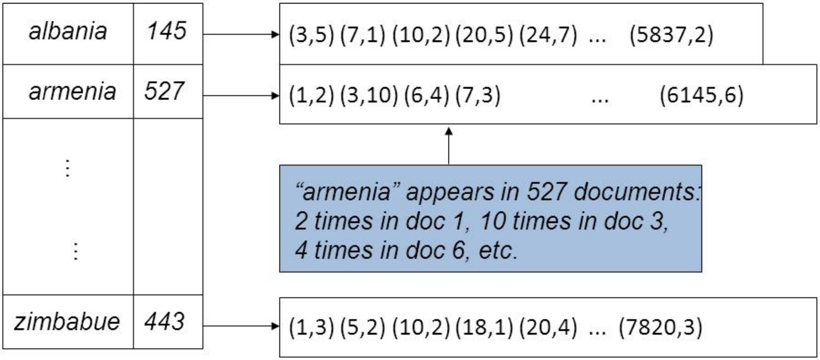
- Collection: document $\rightarrow$ words contained in the document
- Inverted file: word $\rightarrow$ documents that contain the word

Built at preprocessing time, not at query time: can afford to spend some time in its construction.

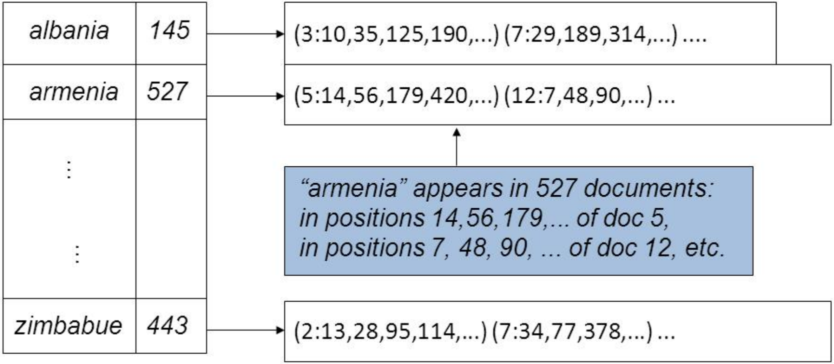
The inverted file: Variant 1



The inverted file: Variant 2



The inverted file: Variant 3



Postings

The inverted file is made of incidence/posting lists

We assign a *document identifier*, docid to each document.
The dictionary may fit in RAM for medium-size applications.

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We assign a *document identifier*, docid to each document.
The dictionary may fit in RAM for medium-size applications.

- For each indexed term, a posting list: list of docid's (plus maybe other info) where the term appears.
- Posting lists stored in disk for largish collections.
- Almost always sorted by *docid*.
- often compressed: minimize info to bring from disk!

Implementation of the Boolean Model

Simplest: Traverse posting lists

Conjunctive query: a AND b

- get the posting lists of a and b from inverted file
- ... and intersect them
- if sorted: can do a merge-like intersection;
- time: order of the sum of the lengths of posting lists.

Exercise. Similar algorithms for OR and BUTNOT.

Implementation of the Boolean Model

```
def intersect(L1,L2):
    i = j = 0
    Lres = []
    while i < len(L1) and j < len(L2):
        if L1[i] < L2[j]:
            ++i
        else if L1[i] > L2[j]
            ++j
        else  # L1[i] == L2[j]
            Lres.append(L1[i])
            ++i
            ++j
    return Lres
```

Query Optimization

Query Optimizer $\rightarrow$ evaluation plan for each query:

- Rewriting the query using laws of Boolean algebra
- Choosing other algorithms for intersection and union
- Using more data structures (computed offline)

Query Rewriting

What is the most efficient way to compute a AND b AND c?

- (a AND b) AND c?
- (b AND c) AND a?
- (a AND c) AND b?

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The following are equivalent. Which is cheapest?

- (a AND b) OR (a AND c)?
- a AND (b OR c)?

The cost of an execution plan depends on the sizes of the lists and the sizes of intermediate lists.

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Worst cases:

- $|L1 \cap L2| \leq \min(|L1|, |L2|)$
- $|L1 \cup L2| \leq |L1| + |L2| - |L1 \cap L2| \leq |L1| + |L2|$

Query Rewriting

a AND b AND $c \rightarrow$ (a AND b) AND c

Assume: $|L_a| = 1,000$, $|L_b| = 2,000$, $|L_c| = 300$.

Minimum comparisons if using sequential scanning = $1,000 + 2,000 + 300 = 3,300$.

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Minimum comparisons if using sequential scanning = $1,000 + 2,000 + 300 = 3,300$.

Instruction	Comparisons	Result $\leq$
1. $L_{a \cap b} =$ intersect(L_a, L_b)	1,000 + 2,000 = 3,000	1,000
2. $L_{res} =$ intersect$(L_{a \cap b}, L_c)$	1,000 + 300 = 1,300	300
Total comparisons	3,000 + 1,300 = **4,300**	–

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1. $L_{a \cap c} =$ intersect(L_a, L_c)	1,000 + 300 = 1,300	300
2. $L_{res} =$ intersect$(L_{a \cap c}, L_b)$	300 + 2,000 = 2,300	300
Total comparisons	1,300 + 2,300 = **3,600** < 4,300	–

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Total comparisons	1,300 + 2,300 = **3,600** < 4,300	–

Heuristic for AND-only queries: Intersect from shortest to longest.

Query Rewriting

a AND (b OR c)

Assume: $|L_a| = 300$, $|L_b| = 4,000$, $|L_c| = 5,000$.

Minimum comparisons if using sequential scanning = $300 + 4,000 + 5,000 = 9,300$.

Query Rewriting

a AND (b OR c)

Assume: $|L_a| = 300$, $|L_b| = 4,000$, $|L_c| = 5,000$.

Minimum comparisons if using sequential scanning = $300 + 4,000 + 5,000 = 9,300$.

Instruction	Comparisons	Result $\leq$
1. $L_{b \cup c}$ = union(L_b, L_c)	4,000+5,000 = 9,000	9,000
2. L_{res} = intersect$(L_a, L_{b \cup c})$	9,000 + 300 = 9,300	300
Total comparisons	9,000 + 9,300 = **18,300**	–

Query Rewriting

a AND (b OR c) $\rightarrow$ (a AND b) OR (a AND c)

Assume: $|L_a| = 300$, $|L_b| = 4,000$, $|L_c| = 5,000$.

Minimum comparisons if using sequential scanning = $300 + 4,000 + 5,000 = 9,300$.

Query Rewriting

a AND (b OR c) $\rightarrow$ (a AND b) OR (a AND c)

Assume: $|L_a| = 300$, $|L_b| = 4,000$, $|L_c| = 5,000$.

Minimum comparisons if using sequential scanning = $300 + 4,000 + 5,000 = 9,300$.

Instruction	Comparisons	Result $\leq$
1. $L_{a \cap b} =$ intersect(L_a, L_b)	300+4,000 = 4,300	300
2. $L_{a \cap c} =$ intersect(L_a, L_c)	300+5,000 = 5,300	300
3. $L_{res} =$ union$(L_{a \cap b}, L_{a \cap c})$	300 + 300 = 600	600
Total comparisons	4,300 + 5,300 + 600 = **9,900** < 18,300	–

Query Rewriting

The combinatorics may get complicated . . .

$$(a \text{ AND } b \text{ AND } d) \text{ OR } (a \text{ AND } (c \text{ OR } d) \text{ AND } e)$$
$$\equiv$$
$$((a \text{ AND } d) \text{ AND } b) \text{ OR } (a \text{ AND } c \text{ AND } e) \text{ OR } ((a \text{ AND } d) \text{ AND } e)$$

Query Rewriting

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$$(a \text{ AND } b \text{ AND } d) \text{ OR } (a \text{ AND } (c \text{ OR } d) \text{ AND } e)$$
$$\equiv$$
$$((a \text{ AND } d) \text{ AND } b) \text{ OR } (a \text{ AND } c \text{ AND } e) \text{ OR } ((a \text{ AND } d) \text{ AND } e)$$

Consider distributing so that we can compute intersect(L_a, L_d) once and store for reuse.

Exercise: Write the new plan as a sequence of instructions.
Exercise: Find cases where the new plan is more efficient.

Sublinear time intersection: Binary Search

Alternative: traverse one list and look up every docid in the other via binary search.

- Time: length of shortest list times log of length of longest.

Sublinear time intersection: Binary Search

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- Time: length of shortest list times log of length of longest.

If $|L1| \ll |L2|$

$$|L1| \cdot \log(|L2|) < |L1| + |L2|$$

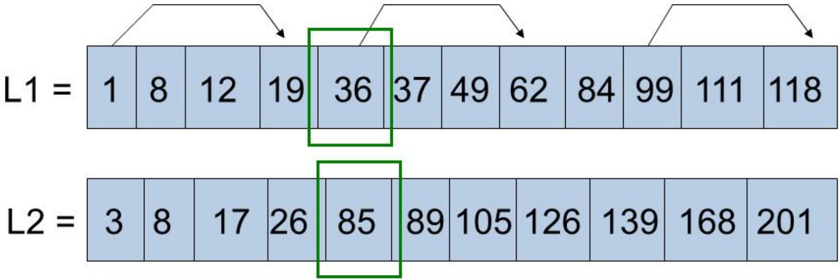
Query Optimization

Query Optimization

Example:

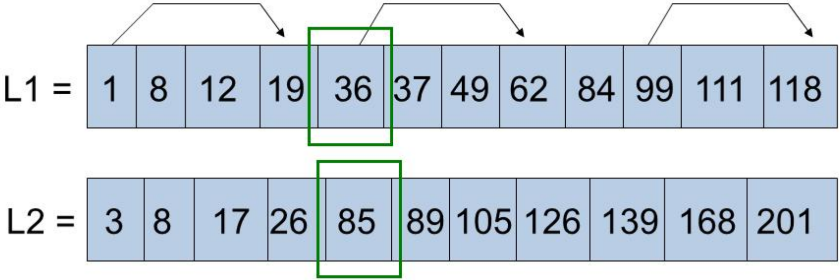
- $|L1| = 1000$, $|L2| = 1000$:
  - sequential scan: 2000 comparisons,
  - binary search: $1000 * 10 = 10,000$ comparisons.
- $|L1| = 100$, $|L2| = 10,000$:
  - sequential scan: $10,100$ comparisons,
  - binary search: $100 * \log(10,000) = 1400$ comparisons.

Sublinear time intersection: Skip pointers



- We've merged 1...19 and 3...26.
- We are looking at 36 and 85.
- Since pointer(36)=62 < 85, we can jump to 84 in L1.

Sublinear time intersection: Skip pointers



- Forward pointer from some elements.
- Either jump to next segment, or search within next segment (once).
- Optimal: in RAM, $\sqrt{|L|}$ pointers of length $\sqrt{|L|}$.
- Needs random access - not so easy if in disk.

Implementation of the Vector Model, I

Problem statement

Fixed similarity measure $sim(d, q)$:

Retrieve

documents d_i which have a similarity to the query q

- either
  - above a threshold sim_{min}, or
  - the top r according to that similarity, or
  - all documents,
- sorted by decreasing similarity to the query q.

Must react very fast (thus, careful to the interplay with disk!), and with a reasonable memory expense.

Implementation of the Vector Model

Obvious non-solution

```
for each d in D:
    sim(d,q) = 0
    get vector representing d
    for each w in q:
        sim(d,q) += tf(d,w) * idf(w)
    normalize sim(d,q) by |d|*|q|
sort results by similarity
```

idf_w and $|d|$ can be precomputed and stored in the index.
$|q|$ computed now.

. . . too inefficient for large D

Most documents include a small proportion of the available terms.

Queries usually include a humanly small number of terms.

Only a very small proportion of the documents will be relevant.

Inverted file available!

Implementation of the Vector Model

Idea: Invert the loops, use inverted file

```
for each w in q:
    L = posting list for w, from inverted file
    for each d in L:
        if d seen for first time:
            sim(d,q) = 0
        sim(d,q) += tf(d,w) * idf(w)
for each d seen:
    normalize sim(d,q) by |d|*|q|
sort results by similarity
```

Implementation of the Vector Model

Idea: Invert the loops, use inverted file

After a few outer loops:

- Instead of having all of $sim(d,q)$ for some d's
- We have partially computed $sim(d,q)$ for all d's

= scan the document-term matrix by columns, not by rows

Index compression, I

Why?

A large part of the query-answering time is spent

bringing posting lists from disks to RAM.

Need to minimize amount of bits to transfer.

Index compression schemes use:

- Docid's sorted in increasing order.
- Frequencies usually very small numbers.
- Can do better than e.g. 32 bits for each.

Index compression, II

Why?

A large part of the query-answering time is spent bringing posting lists from disks to RAM.

- Need to minimize amount of bits to transfer.

Easiest is to use "`int` type" to store docid's and frequencies

- 8 bytes, 64 bits per pair
- ... but want/can/need to do much better!

Index compression schemes use:

- Docid's sorted in increasing order.
- Frequencies usually very small numbers.

Index compression, III

Posting list is:

$$term \rightarrow [(id_1, f_1), (id_2, f_2), ..., (id_k, f_k)]$$

Can we compress frequencies f_i?:

Yes! Will use *unary self-delimiting* codes because frequencies typically very small

Can we compress docid's id_i?:

Yes! Will use *Gap compression* and *Elias Gamma* codes because docid's are sorted

Index compression, IV

Compressing frequencies

The distribution of frequencies is very biased towards small numbers, i.e., most f_i are very small

- Exercise: can you quantify this using Zipf's law?
- E.g. in files for lab session 1: 68% is 1, 13% is 2, 6% is 3, <13% is >3, <3% is >10, 0.6% is >20.

Unary code

Want encoding scheme that uses few bits for small frequencies

Index compression, V

Compressing frequencies: unary encoding

Unary encoding of x is $\overbrace{111 \ldots 1}^{x \text{ times}}$

- E.g. $unary(15) = 111111111111111$
- $|unary(x)| = x$
  - typical binary encoding: $|binary(x)| = \log_2(x)$
- variable length encoding

But..

want to encode *lists* of frequencies, where do we cut?

Index compression, VI

Compressing frequencies: self-delimiting unary encoding

- Make 0 act as a separator
- Replace last 1 in each number with a 0
- Example: $[3, 2, 1, 4, 1, 5]$ encoded as $110\ 10\ 0\ 1110\ 0\ 11110$
- This is a *self-delimiting* code: no prefix of a code is a code
- Self-delimiting *implies* unique decoding

Index compression, VII

Compressing frequencies: self-delimiting unary encoding

Recall example from lab session 1: 68% is 1, 13% is 2, 6% is 3, <13% is >3, <3% is >10, 0.6% is >20, the expected length would be (approx)

$$1 * 0.68 + 2 * 0.13 + 3 * 0.06 + 6^{1} * 0.13 = 1.91$$

Unary code works very well

- 1 bit when $f_i = 1$
- 1.3 to 2.5 bits per f_i on real corpuses
- 1 bit per term occurrence in document
  - Easy to estimate memory used!

[1] I put it something greater than 3 as an approximation

Index compression, VIII

Compressing docid's

Gap compression

Instead of compressing $[(id_1, f_1), (id_2, f_2), ..., (id_k, f_k)]$
Compress $[(id_1, f_1), (id_2 - id_1, f_2), ..., (id_k - id_{k-1}, f_k)]$

Example:

$(1000, 1), (1021, 2), (1037, 1), (1056, 4), (1080, 1), (1095, 3)$
compressed to:

$$(1000, 1), (21, 2), (16, 1), (19, 4), (24, 1), (15, 3)$$

Index compression, IX

Compressing docid's

- Fewer bits if gaps are small
- E.g.: $N = 10^6$, $|L| = 10^4$, then average gap is 100
  - So, could use 8 bits instead of 20 (or 32)
- .. but .. this is only on average! *Large gaps do exist*
  - Will need a *variable length, self-delimiting* encoding scheme
- Gaps are not biased towards 1, so unary not a good idea
  - Will use need a *variable length, self-delimiting, binary* encoding scheme

Index compression, X

Compressing docid's: Elias-Gamma code (self-delimiting binary code)

IDEA:

First say how long x is in binary, then send x

Pseudo-code for Elias-Gamma encoding:

- let $w = binary(x)$
- let $y = |w|$
- prepend $y - 1$ zeros to w, and return

Examples:

$EG(1) = 1, EG(2) = 010, EG(3) = 011, EG(4) = 00100, EG(20) = 000010100$

Index compression, XI

Compressing docid's: Elias-Gamma code (self-delimiting binary code)

- Elias-Gamma code is self-delimiting
  - Exercise: think how to decode uniquely
- Length of a code for x is about $2\log_2(x)$
  - Exercise: why?

Index compression, XII

Compressing docid's: easier alternative, *variable byte* codes

Easier alternative: byte-wise (8 bits) or nibble-wise (4 bits) encoding that make use of first bit to say whether it is the last byte or not (*continuation* bit).

- Encoding is also variable length, but much simpler
- Waste is not that much
- Better use of CPU by reading bytes instead of single bits
- First bit of byte is continuation bit, other 7 bits used to encode in binary
  - if 0, then last byte
  - if 1, number continues

Example:

10101001 11100111 01100111 is code for
0101001 1100111 1100111 (continuation bits in red)

Index compression, XIII

Bottom line

- Ratios of 20% to 25% routinely achieved
- Translates to similar speed-up at query time

Getting fast the top r results

The last line of the algorithm was

```
sort the documents in answer by value of sim(d,q)
```

Time $O(R \log R)$, where R = #docs with $sim(d, q) > sim_{min}$.
Noticeable if R is large (milions).

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Time $O(R \log R)$, where R = #docs with $sim(d,q) > sim_{min}$.
Noticeable if R is large (milions).

User usually wants really fast the top-r, where $r \ll R$.
E.g., $r = 10$.

Getting fast the top r results

Let $L = [d_1 ... d_R]$ be the answer (random order)

```
put [d_1, ..., d_r] in a minheap
for i = r+1..R:
    min_val = sim(d,q) for d = top of the heap
    if sim(d_i,q) > min_val:
        replace smallest element in heap with d_i
        reorganize heap
```

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Claim: If the similarities in L are randomly ordered, the expected running time of this algorithm is $O(R + r \cdot \ln(r) \cdot \ln(R/r))$.

Getting fast the top r results

Let $L = [d_1, \ldots, d_R]$ be the answer

- Time to put r elements in heap: $O(r)$
  - (recal why it is better than the obvious $O(r \log r)$)
- $Pr(d_i$ enters the heap$) =$

  $= Pr[d_i$ among r largest in $d_1, \ldots, d_i] = r/i$
- $E[$time to process $d_i] = \frac{r}{i} O(\log r) + \frac{i-r}{i} O(1)$
- E[Running time] = $O(r) + \sum_{i=r+1}^{R} \left(\frac{r}{i} O(\log r) + \frac{i-r}{i} O(1) \right)$

  $= \ldots$ (use $H(n) \simeq \ln(n)$, H harmonic function)

  $= O(R) + O(r \ln(r) \ln(R/r))$

Getting fast the top r results

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- Time to put r elements in heap: $O(r)$
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  $= O(R) + O(r \ln(r) \ln(R/r))$

For $r \ll R$, we go from $O(R \log R)$ to $O(R)$.

How to build the Index (offline)

Given document collection D, build the inverted file F

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Given document collection D, build the inverted file F

In python - in RAM:

```
F = {}
for doc in D:
    d = docid(doc)
    for w in doc:
        if w not in F:
            F[w] = {}
        if d not in F[w]:
            F[w][d] = 0
        F[w][d] += 1
```

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```

Large indices must go to disk, not RAM

Writing indices to disk

Without going to many details . . .

```
Initialize F to empty in disk
for docid in D in increasing order:
    for word in D[docid]:
       L = retrieve list F(word) from disk
       if (docid,c) in L:
          replace (docid,c) with (docid,c+1)
          # in disk list!
       else:
          append (docid,1) at the end of F(word)
          # this keeps lists sorted by docid
```

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Perhaps can be optimized to not read/write all of L, only parts

But the real problem is another: access to lists is random!

Disk technology

Traditional hard disks with moving parts

- Seek time veery slow - head movement.
- Once head is in place, sequential access is fast.
- = reads/writes with consecutive, large chunks of bits.
- N random read/writes muuuch slower than N sequential read/writes.
- Like $50\times$, easily.

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- Like $50\times$, easily.

[Things are different with new SSD drives - no moving parts, little difference between sequential and random access. They may have problems with small writes, as they read/write full pages. And they use large page sizes. Impact of this still not well studied.]

More efficient

1. Initialize disk index to be empty
2. Build index in RAM, up to allocated memory M
3. When RAM full:
    - append each list in RAM to end of corresponding list in disk
    - sequential writes to disk! fast!
    - clear the RAM index
    - goto to 2 to process more documents

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    - append each list in RAM to end of corresponding list in disk
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    - clear the RAM index
    - goto to 2 to process more documents

Observations:

- a RAMful of index is sometimes called a “barrel”
- many barrels can be built in concurently with a cluster
- merging barrels into disk index is done by a single machine