4 Transforms

4.1: 2D Discrete Fourier Transform

Transforms

4.1

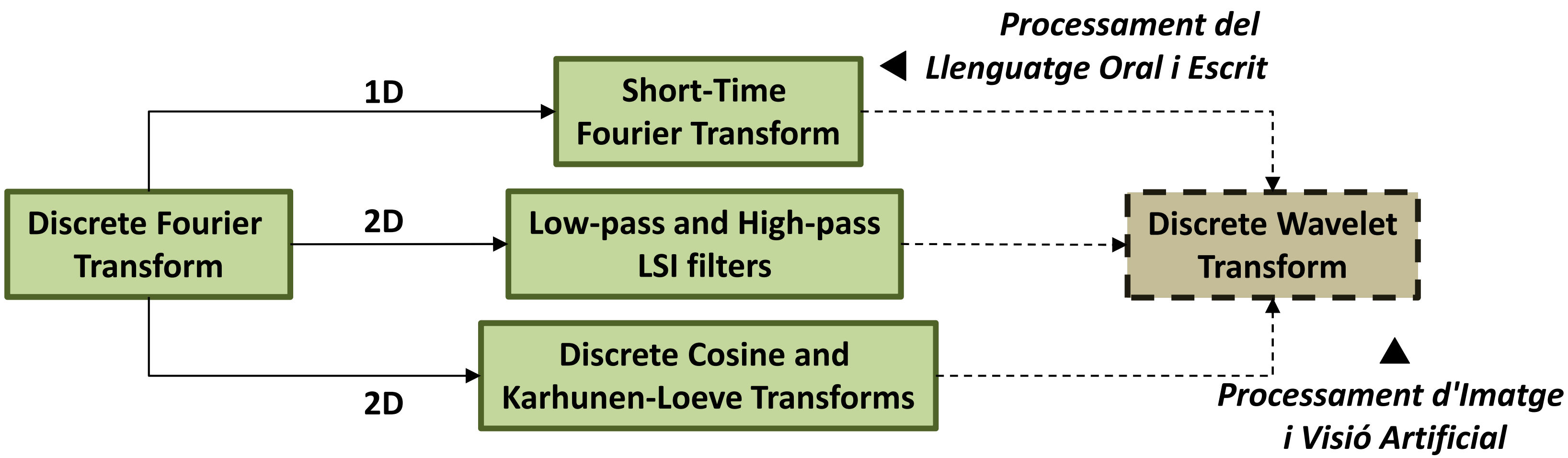
1. **2D Discrete Fourier Transform (2D-DFT)**
   - Basic properties
   - Basic signal transforms
2. **2D Linear Filtering**
   - Filter design in the spatial domain
   - Filter design in the frequency domain
3. **Other transforms**
   - Discrete Cosine Transform (DCT)
   - Karhunen-Loeve Transform (KLT)
4. **Short-Term Fourier Transform (STFT)**
   - STFT as a filter bank
   - Spectogram: Time-frequency analysis

Introduction

4.1

We are going to analyze **signals transforms** other than the 1D Discrete Fourier Transform to describe (audiovisual) signals:

- The **2D Discrete Fourier Transform** (2D-DCT), as basic frequency representation for images
- The **Discrete Cosine Transform** (DCT) and the **Karhunen-Loeve Transform** (KLT) as additional, specific representations for (audiovisual signals)
- The **Short-Time Fourier Transform** (STFT), as extension of the DFT for the case of non-stationary signals



Unit Structure

4.1

1. **Introduction**
2. **Definition of the 2D Discrete Fourier Transform**
   - 1D and 2D Discrete Fourier Transform
   - Relation between TF and DFT
3. **Basic properties**
   - DFT properties
   - Other 2D DFT features
4. **Some signal transforms**
   - Basic signal transforms
   - Complex signals transforms

Introduction

4.1

- Images can be modeled as **linear combinations** of simpler functions; typically:
  - **Impulse functions**, as in the canonical representation.
  - **Complex exponentials**, as in the Fourier representation.
  - …

- The Fourier representation (and its underlying space/frequency model) is a useful tool to define a natural set of operations such as **Linear Space-Invariant (LSI)** operators.
  - LSI operators linearly combine the pixel values in a given neighborhood of the pixel being processed (**impulse response** or **convolution mask**).

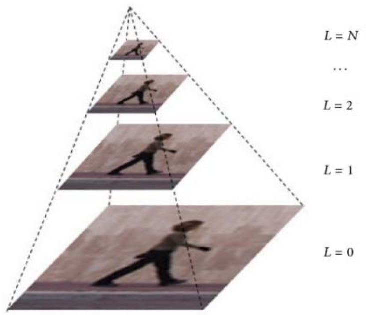
- Linear Space-Invariant (**LSI**) operators can be defined:
  - In the original (space) domain, through a **convolution**.
  - In the **transformed (frequency) domain**, through a **product**.

Space/Frequency Image Processing Tools

4.1

- If LSI operators involve large impulse responses, they can be **efficiently implemented** thanks to the use of fast transforms:
  1. The input image is (fast) transformed (FFT).
  2. The operation is performed in the transformed domain by a product.
  3. The output image is obtained through an inverse FFT.

- **Space/Frequency image processing tools** allow (among others):
  - Convolution operations
  - Linear filter design
  - Analysis of sampling
  - Multi-resolution analysis



- We need to define a **2D Discrete Fourier Transform**

Use of the DFT

4.1

The **Discrete Fourier Transform** is going to be used to analyze the signals in the frequency domain:

- The Fourier Transform provides a continuous function:
  - It is **not suitable** for its numeric treatment in a computer

✓ The Discrete Fourier Transform provides a **sampled representation** in the frequency domain:
  - It preserves the dual nature between original and transformed signals

✓ The Discrete Fourier Transform can be **implemented through the FFT**:
  - It reduces the computational load (from M^2 to $M \log_2 M$)

- How can we handle signals that may present a non-limited number of samples?
  ✓ Modeling the actual (N samples) signal through **windowing**
  ✓ Using a time-dependent DFT: **Short Time Fourier Transform**
  ✓ Processing the signal by **blocks**

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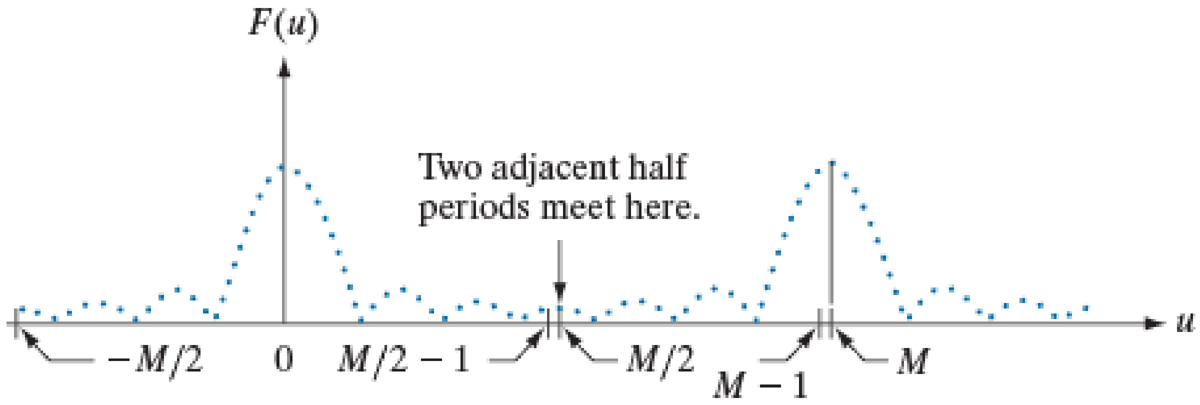
Definition of the 2D DFT

4.1

The **1D Discrete Fourier Transform** (**DFT**) is defined as:

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi\left(\frac{k}{N}n\right)} \qquad 0 \le k < N$$



Recall the **symmetries** and the **implicit periodicity** of DFT

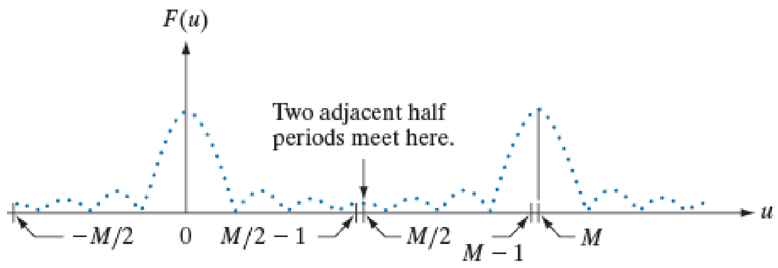
Digital Image Processing (4th Edition). Gonzalez, Rafael C.; Woods, Richard E.

Definition of the 2D DFT

4.1

The **1D DFT** is defined as:

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi\left(\frac{k}{N}n\right)} \qquad 0 \le k < N$$



and its **extension to the 2D** (image) case as:

$$X[k,l] = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x[m,n] e^{-j2\pi\left(\frac{k}{M}m + \frac{l}{N}n\right)} \qquad 0 \le k < M, \qquad 0 \le l < N$$

And, therefore, the **2D Inverse Discrete Fourier Transform** (**IDFT**) is defined as:

$$x[m,n] = \frac{1}{MN} \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} X[k,l] e^{j2\pi\left(\frac{k}{M}m + \frac{l}{N}n\right)} \qquad 0 \le m < M, \qquad 0 \le n < N$$

DFT versus FT of a sequence

4.1

The 2D-DFT **transforms a 2D signal of MxN samples** in the original domain into a set of MxN samples in the transformed domain:

- Do not mistake the DFT for the Fourier Transform (FT) of a sequence $X(F_1, F_2)$

$$X(F_1, F_2) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x[m,n] e^{-j2\pi(F_1 m + F_2 n)}$$

- $X(F_1, F_2)$ is a **periodic signal** in F_1 and F_2 with period $P = 1$

- In the **DFT case**:
  - Only data within a window is computed: $0 \leq m < M,\ 0 \leq n < N$
  - The transformed signal is sampled: $F_1 = \frac{k}{M}$ and $F_2 = \frac{l}{N}$

The DFT is a sampled version of the FT computed over a windowed signal

Definition of the 2D DFT

4.1

The 2D-DFT **transforms a 2D signal of MxN samples** in the original (space) domain into a set of MxN samples in the transformed (frequency) domain:

- Usually, samples in the spatial domain are real (or integer) whereas **samples in the frequency domain are complex**.
- The DFT of a signal can be represented in terms of its **magnitude** (modulus) and **phase**

$$|X[k,l]| = \sqrt{(X_R[k,l])^2 + (X_I[k,l])^2} \qquad \varphi_X[k,l] = \tan^{-1}\left[\frac{X_I[k,l]}{X_R[k,l]}\right]$$

- The original samples **can always be recovered** from the transformed ones:

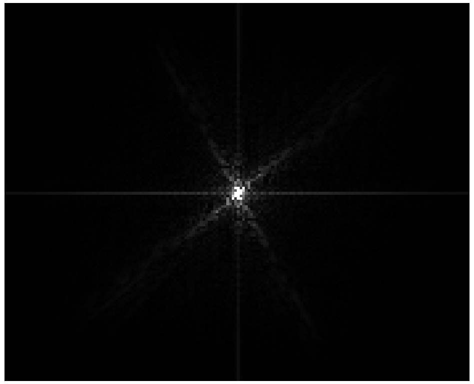
$$DFT^{-1}\{DFT\{x[m,n]\}\} = x[m,n] \qquad 0 \le m < M, \qquad 0 \le n < N$$

2D DFT: Magnitude and phase

The DFT of a signal can be represented in terms of its **magnitude** and **phase.**

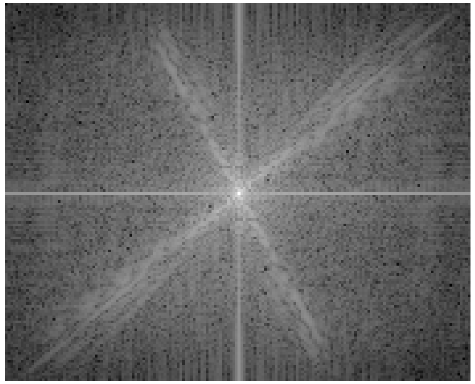
- Usually, only the magnitude is represented. To represent the magnitude, a **logarithmic transform** is commonly applied, due to its dynamic range

$|X[k,l]|$



Linear value scaling (8 bit representation)

$\log(1+|X[k,l]|)$



Logarithmic value scaling (8 bit representation)

- Actually, we are not representing the DFT but a **shifted version**.

Unit Structure

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DFT properties

4.1

Properties of DFT with respect to TF:

- Similar properties but, in the case of a DFT of length L (N in 1D and (M,N) in 2D), indexes should remain in the interval $[0, L-1]$
- This leads to circular convolution, displacement or time-reversal representations. Thus, properties can be expressed with the help of:

$$\tilde{t}[m,n] = \sum_{r=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} t[m + rM, n + sN] \qquad \begin{cases} 0 \leq m \leq M-1 \\ 0 \leq n \leq N-1 \end{cases}$$

- **Convolution**: Spatial convolution implies frequency product

$$\tilde{t}[m,n] \leftrightarrow X[k,l] \cdot Y[k,l]$$
$$\text{if } t[m,n] = x[m,n] * y[m,n]$$

- **Convolution and windowing**: Windowing in the spatial domain implies frequency convolution

$$x[m,n] \cdot y[m,n] \leftrightarrow \frac{1}{MN} \tilde{T}[k,l]$$
$$\text{if } T[k,l] = X[k,l] * Y[k,l]$$

DFT properties

4.1

- **Spatial shift or Translation**: A spatial shift only affects the phase of the transformed signal:

$$\tilde{x}[m-m',n-n'] \leftrightarrow X[k,l]e^{-j2\pi\left(\frac{k}{M}m'+\frac{l}{N}n'\right)}$$

- **Frequency shift or Modulation**: Multiplication of an image by a complex exponential implies a frequency shift:

$$x[m,n]e^{j2\pi\left(\frac{k'}{M}m+\frac{l'}{N}n\right)} \leftrightarrow \tilde{X}[k-k',l-l']$$

- **Separability**: Since the 2D-DFT kernel is separable, the 2D-DFT can be implemented as two 1D-DFT:

$$DFT_{image}^{2D}[\cdot] = DFT_{rows}^{1D}\left[DFT_{columns}^{1D}[\cdot]\right] = DFT_{columns}^{1D}\left[DFT_{rows}^{1D}[\cdot]\right]$$

DFT properties

4.1

- **Separability**: Since the 2D-DFT kernel is separable, the 2D-DFT can be implemented as two 1D-DFT:

$$DFT^{2D}_{image}[\cdot] = DFT^{1D}_{rows}\left[DFT^{1D}_{columns}[\cdot]\right] = DFT^{1D}_{columns}\left[DFT^{1D}_{rows}[\cdot]\right]$$

$$X[k,l] = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} x[m,n]\, e^{-j2\pi\left(\frac{k}{M}m + \frac{l}{N}n\right)} =$$

$$= \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} x[m,n]\, e^{-j2\pi \frac{k}{M}m}\, e^{-j2\pi \frac{l}{N}n} = \underbrace{\sum_{m=0}^{M-1} e^{-j2\pi \frac{k}{M}m} \underbrace{\sum_{n=0}^{N-1} x[m,n]\, e^{-j2\pi \frac{l}{N}n}}_{\text{COLUMNS / ROWS}}}_{\text{ROWS / COLUMNS}}$$

DFT properties

4.1

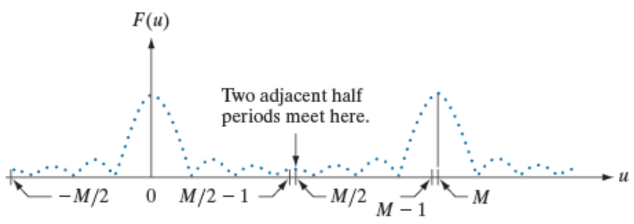
- **Rotation**: A spatial rotation corresponds to the same frequency rotation:

$$m = r\cos\theta \qquad n = r\sin\theta$$
$$k = \rho\cos\varphi \qquad l = \rho\sin\varphi$$
$$x[r, \theta + \theta_0] \leftrightarrow X[\rho, \varphi + \theta_0]$$

- **Hermitian symmetry**: If the image is **real**, its DFT presents Hermitian symmetry:

$$X[k, l] = X^*[-k, -l]$$

  - Note the implicit periodicity of the DFT
  - Useful for filter implementation



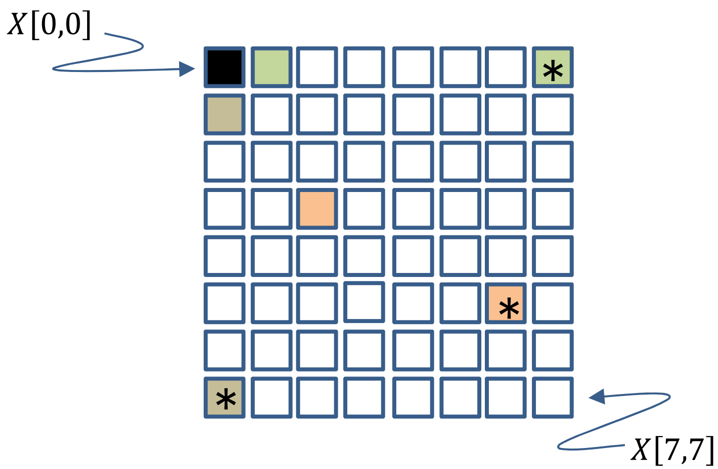
Symmetries in the MxN support:

$\rightarrow X[0,0] = X^*[0,0]$

$\rightarrow X[0, l] = X^*[0, N - l] \qquad 1 \le l < N$

$\rightarrow X[k, 0] = X^*[M - k, 0] \qquad 1 \le k < M$

$\rightarrow X[k, l] = X^*[M - k, N - l] \qquad 1 \le \begin{Bmatrix} k \\ l \end{Bmatrix} < \begin{Bmatrix} M \\ N \end{Bmatrix}$



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Other DFT features

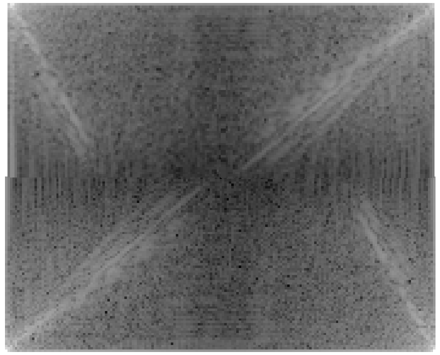
Centered representation:

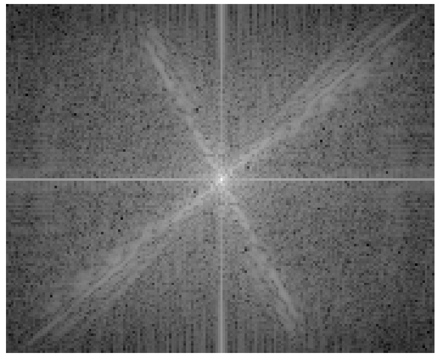
- Since images are generally positive signals, the highest value of the magnitude of their transform is at $(k,l) = (0, 0)$. Given the DFT symmetries, the four corners of the transformed image contain the highest values of the magnitude. In order **to help visualizing**, the transformed image is represented centered at $(M/2, N/2)$.
- This can be seen as an example of the **modulation property**

$$x[m,n]e^{j2\pi\left(\frac{k'}{M}m+\frac{l'}{N}n\right)} \leftrightarrow \tilde{X}[k-k', l-l']$$

$$\text{For } k' = \frac{M}{2}, l' = \frac{N}{2}: \; x[m,n]e^{j2\pi\left(\frac{m}{2}+\frac{n}{2}\right)} \leftrightarrow \tilde{X}\left[k-\frac{M}{2}, l-\frac{N}{2}\right]$$

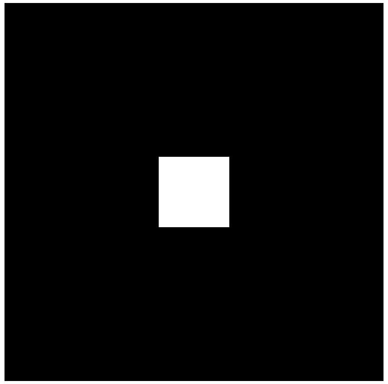
$$x[m,n]e^{j\pi(m+n)} = x[m,n](-1)^{(m+n)} \leftrightarrow \tilde{X}\left[k-\frac{M}{2}, l-\frac{N}{2}\right]$$

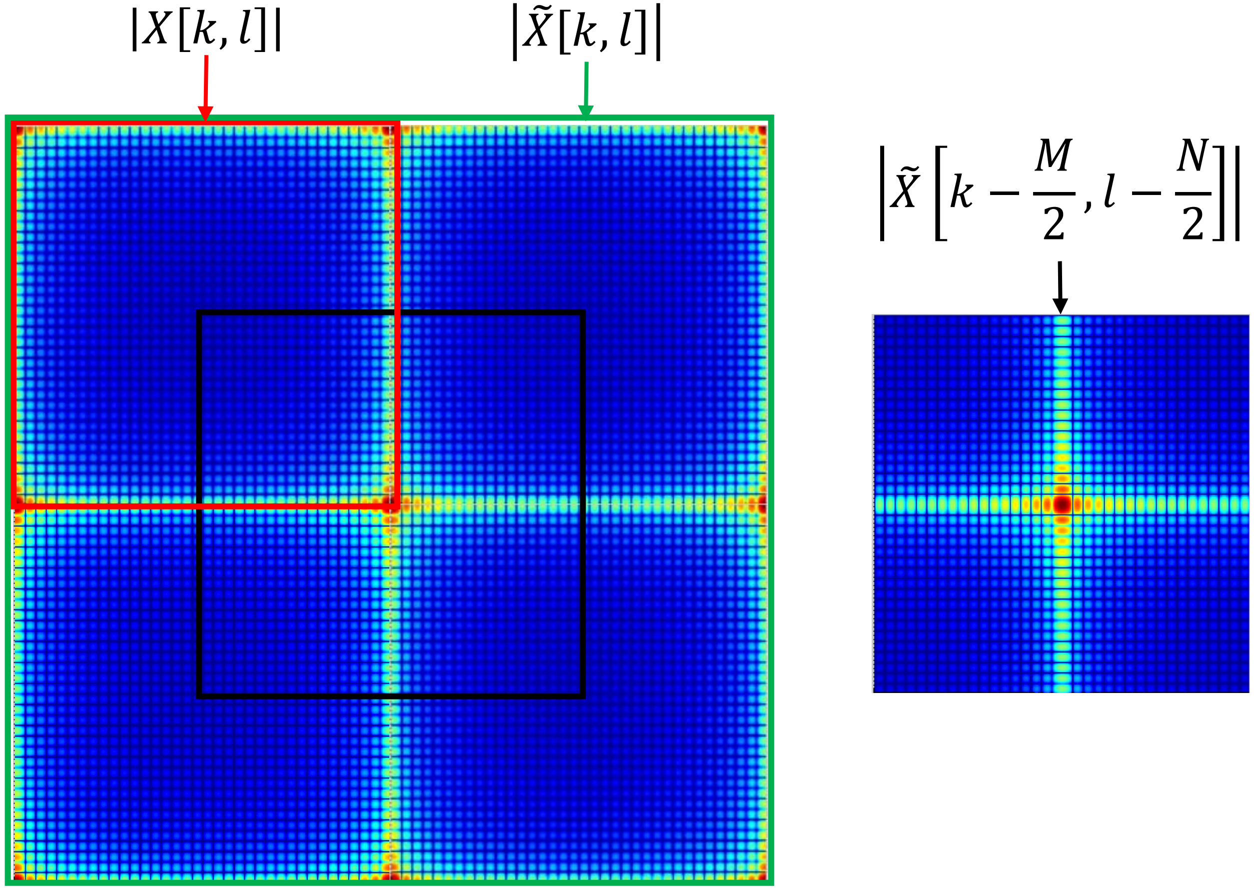




Centered representation

4.1

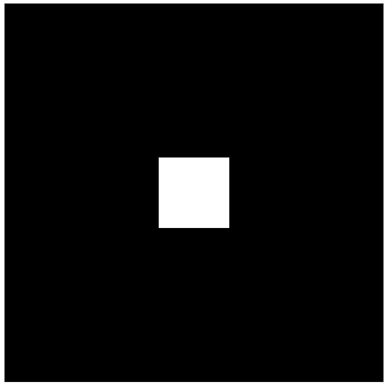

Original image



$$x[m,n]e^{j\pi(m+n)} = x[m,n](-1)^{(m+n)} \leftrightarrow \tilde{X}\left[k-\frac{M}{2}, l-\frac{N}{2}\right]$$

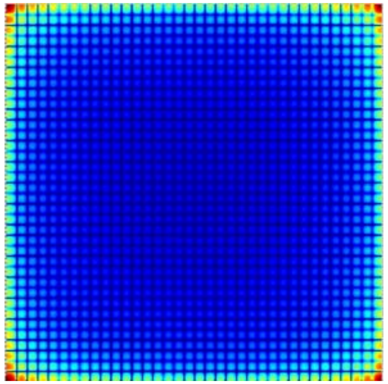
Centered representation

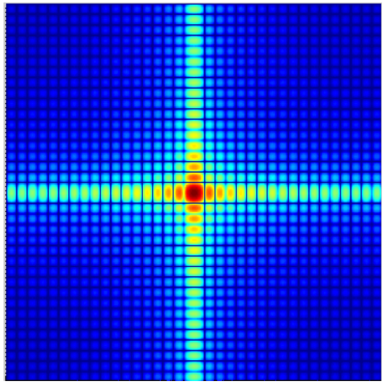
4.1



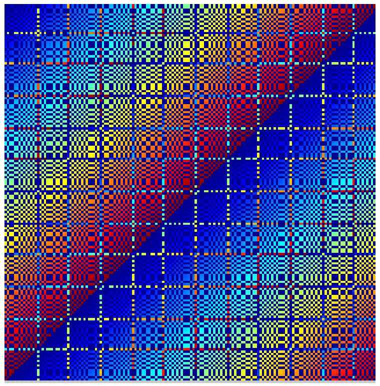
Original image

$|X[k,l]|$

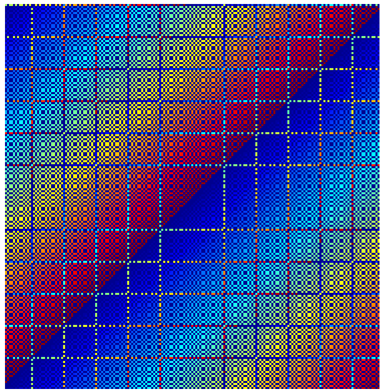




$$\left|\tilde{X}\left[k-\frac{M}{2},l-\frac{N}{2}\right]\right|$$



$\varphi_X[k,l]$



$$\varphi_{\tilde{X}}\left[k-\frac{M}{2},l-\frac{N}{2}\right]$$

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Basic signal transforms

- If a 2D function is separable into a product of 1D functions, so is its DFT

$$\boxed{x[m,n] = x_1[m]x_2[n] \;\; \leftrightarrow \;\; X[k,l] = X_1[k]X_2[l]}$$

$$X[k,l] = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} x[m,n]\, e^{-j2\pi\left(\frac{k}{M}m + \frac{l}{N}n\right)} =$$

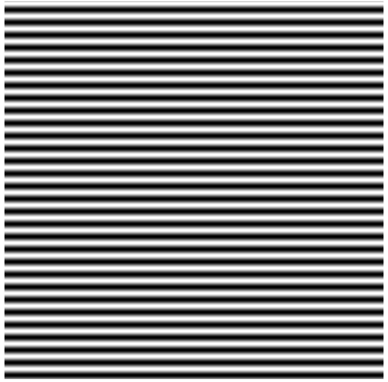
$$= \left[x[m,n] = x_1[m]\, x_2[n]\right] = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} x_1[m]\, x_2[n]\, e^{-j2\pi\left(\frac{k}{M}m + \frac{l}{N}n\right)} =$$

$$= \sum_{m=0}^{M-1} x_1[m]\, e^{-j2\pi\frac{k}{M}m} \sum_{n=0}^{N-1} x_2[n]\, e^{-j2\pi\frac{l}{N}n} = X_1[k]\, X_2[l]$$

Basic signal transforms

4.1

- The transform of simple images can be analyzed using basic DFT properties



$M = N = 300,$
$L = 30$

$$x[m,n] = \sin(2\pi F_1 m + 2\pi F_2 n) \qquad F_1 = 0,\ F_2 = 0.1$$

$$x[m,n] = \sin(2\pi F_2 n) = 1 \cdot \sin(2\pi F_2 n) = x_1[m] x_2[n]$$

$$x[m,n] = x_1[m]\, x_2[n] \longleftrightarrow X[k,l] = X_1[k] \cdot X_2[l]$$

$$x_1[m] = 1 \Rightarrow X_1[k] = \sum_{m=0}^{M-1} 1 \cdot e^{-i 2\pi \frac{k}{M} m} = \left[\sum_{n=0}^{q} a^n = \frac{1-a^{q+1}}{1-a} \quad \forall a \neq 1 \right] =$$

$$X_1[k] = \left\{ \begin{array}{ll} k = 0 \Rightarrow \sum_{m=0}^{M-1} 1 = M \\ k \neq 0 \Rightarrow \dfrac{1 - e^{-i 2\pi k \frac{M}{M}}}{1 - e^{-i 2\pi \frac{k}{M}}} = 0 \end{array} \right\} = M \delta[k]$$

Basic signal transforms

4.1

- The transform of simple images can be analyzed using basic DFT properties

$$x[m,n] = \sin(2\pi F_2 n) = 1 \cdot \sin(2\pi F_2 n) = x_1[m]x_2[n]$$

$$x_2[n] = \text{sen } 2\pi F_2 n = \left[F_2 = \frac{L_2}{N} \right] = \frac{e^{j2\pi \frac{L_2 n}{N}} - e^{-j2\pi \frac{L_2 n}{N}}}{2j}$$

MODULATION PROPERTY:
$$\begin{cases} 1 \longleftrightarrow N\,\delta[l] \\ e^{j2\pi \frac{L_2}{N} n} \longleftrightarrow N\,\delta[l-L_2] \end{cases}$$

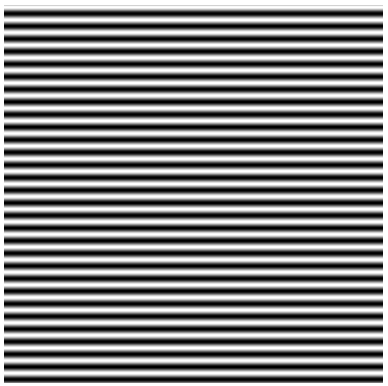
$$X_2[l] = \frac{N}{2j}\left[\delta[l-L_2] - \delta[l+L_2] \right]$$

$$X[k,l] = X_1[k]\,x_2[l] = \frac{MN}{2j}\left[\delta[k]\,\delta[l-L_2] - \delta[k]\,\delta[l+L_2] \right]$$

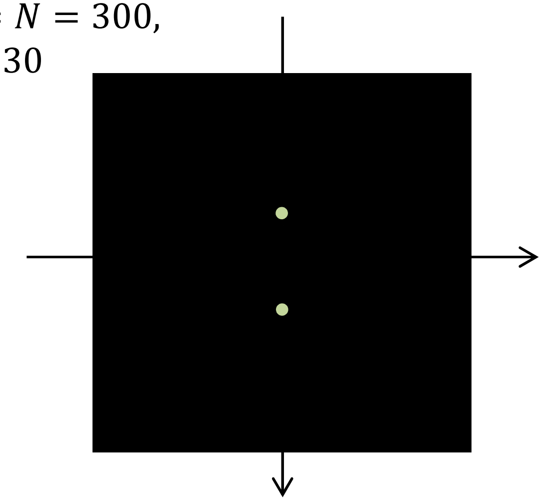
Basic signal transforms

4.1

- **The transform of simple images can be analyzed using basic DFT properties**



$M = N = 300,$
$L = 30$



$$x[m,n] = \sin(2\pi F_1 m + 2\pi F_2 n) \qquad F_1 = 0,\ F_2 = 0.1$$

$$x[m,n] = \sin(2\pi F_2 n) = 1 \cdot \sin(2\pi F_2 n) = x_1[m]x_2[n]$$

The image (2D function) can be represented as the product of two 1D functions. Applying **separability**:

$$DFT^{2D}_{m,n}[\cdot] = DFT^{1D}_{n}\left[DFT^{1D}_{m}[\cdot]\right]$$

If the 2D function is **separable** into a product of 1D functions:

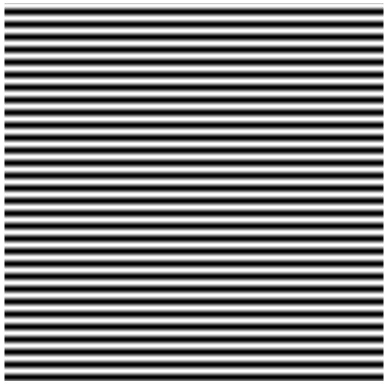
$$X[k,l] = DFT^{1D}_{n}\left[x_2[n]\right]DFT^{1D}_{m}\left[x_1[m]\right] = X_1[k]X_2[l]$$

$$\boxed{X[k,l] = \frac{MN}{2j}\delta[k](\delta[l-L] - \delta[l+L])}$$

Sinusoid signals

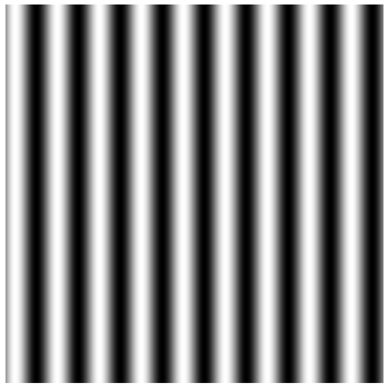
4.1

$F_1 = 0,\ F_2 = 0.1$

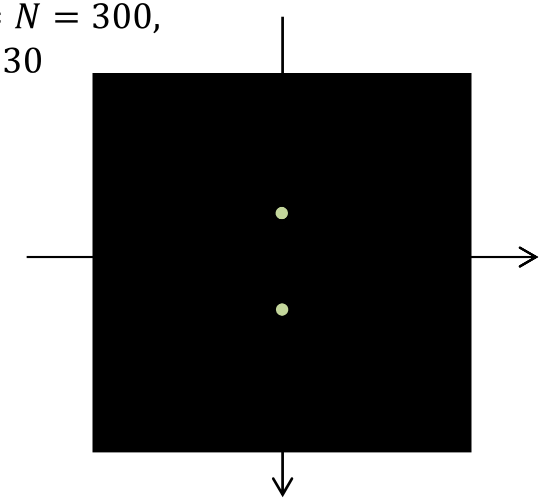


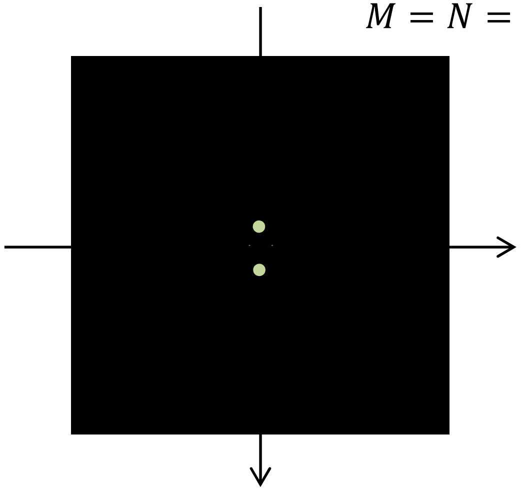
$F_1 = 0,\ F_2 = 0.03$

$F_1 = 0.03,\ F_2 = 0$

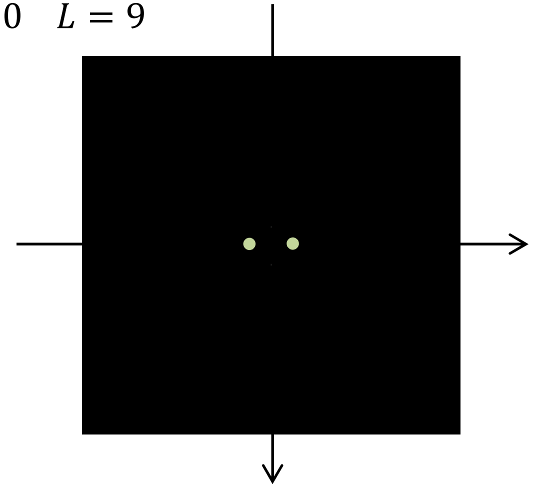


$M = N = 300,$
$L = 30$





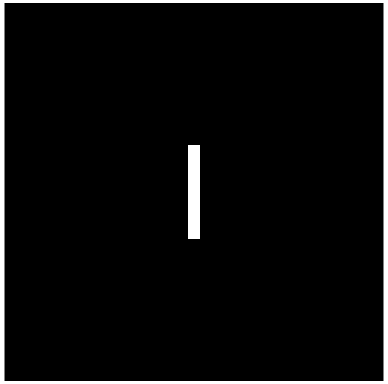
$M = N = 300 \quad L = 9$



Pulse signals

4.1

- The transform of simple images can be analyzed using basic DFT properties



$$x[m,n] = \prod\nolimits_{M'}[m-m_0]\prod\nolimits_{N'}[n-n_0] \quad (N' > M')$$

$$\prod\nolimits_{M'}[m] = \text{Pulse of length } M'$$

The image (2D function) can be represented as the product of two 1D functions. Applying **separability:**

$$DFT^{2D}_{m,n}[\cdot] = DFT^{1D}_{n}\left[DFT^{1D}_{m}[\cdot]\right]$$

If the 2D function is **separable** into a product of 1D functions:

$$X[k,l] = DFT^{1D}_{n}\left[x_2[n]\right]DFT^{1D}_{m}\left[x_1[m]\right] = X_1[k]X_2[l]$$

Pulse signals

4.1

AS A SHIFT ONLY AFFECTS THE PHASE:

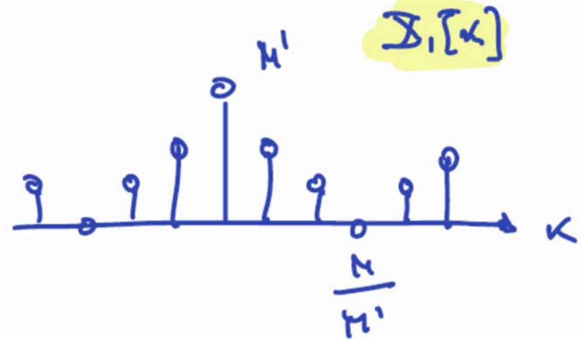
$$X_1[k] = \sum_{m=0}^{M-1} \Pi_{M'}[m]\, e^{-j 2\pi \frac{k}{M} m} = \sum_{m=0}^{M'-1} e^{-j 2\pi \frac{k}{M} m} =$$

$$= \frac{1 - e^{-j 2\pi \frac{k}{M} M'}}{1 - e^{-j 2\pi \frac{k}{M}}} = \frac{e^{-j \pi \frac{k}{M} M'}}{e^{-j \pi \frac{k}{M}}} \cdot \left[\frac{e^{j \pi \frac{k}{M} M'} - e^{-j \pi \frac{k}{M} M'}}{e^{j \pi \frac{k}{M}} - e^{-j \pi \frac{k}{M}}} \right] \Rightarrow$$

$$|X_1[k]| = \left| \frac{\operatorname{sen}\left[\pi k \frac{M'}{M}\right]}{\operatorname{sen}\left[\pi \frac{k}{M}\right]} \right| \leftarrow$$ DISCRETE SINC

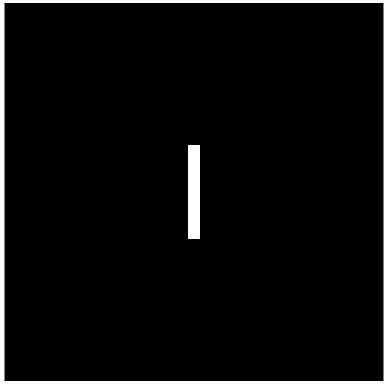
$X_1[k] = 0$ FOR $\pi k \frac{M'}{M} = \pi \Rightarrow k = \frac{M}{M'}$

$$X_1[0] \simeq \frac{\pi k\, M'/M}{\pi\, k/M} = M'$$



Pulse signals

- **The transform of simple images can be analyzed using basic DFT properties**



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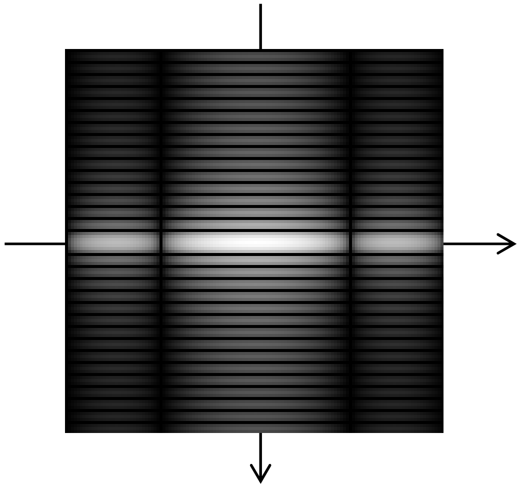
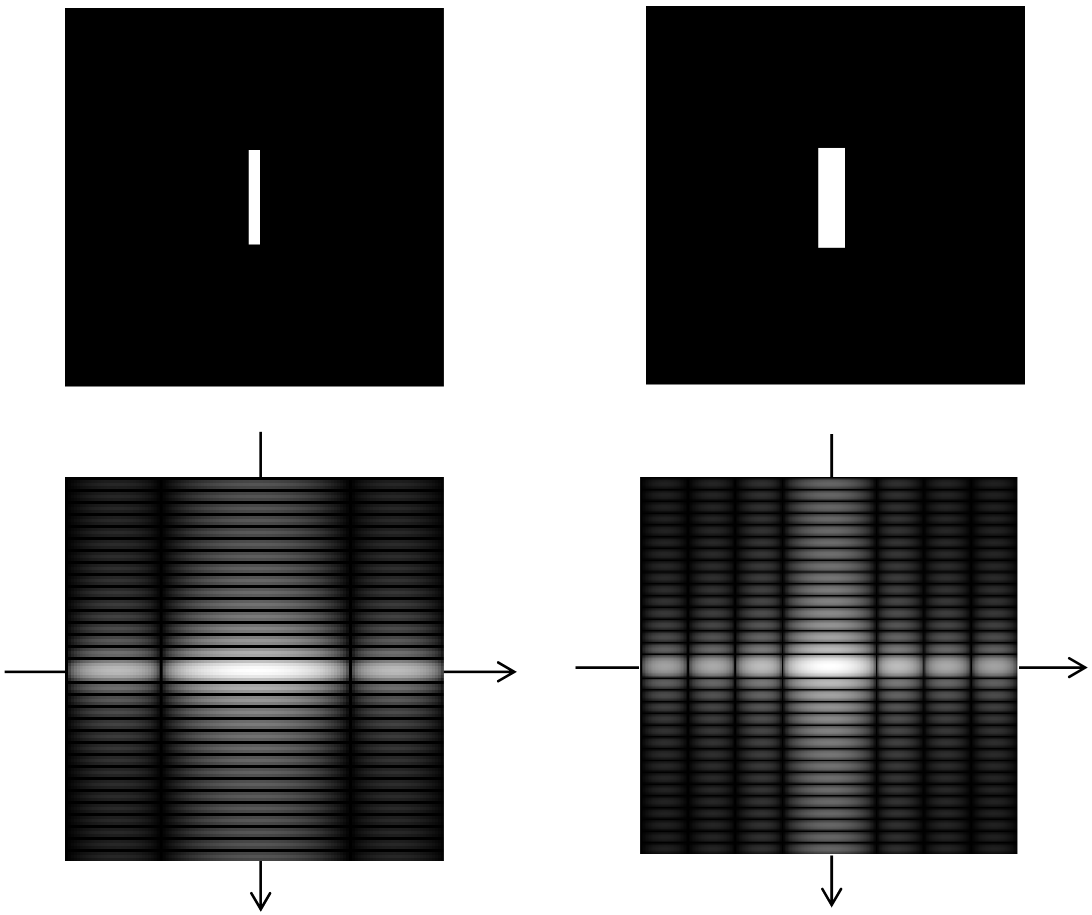


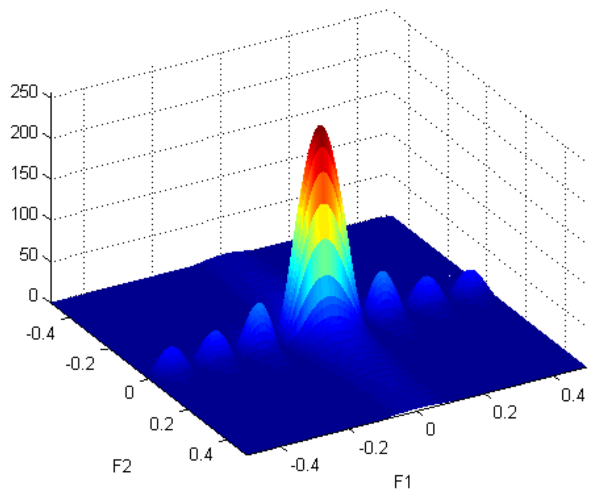
$$|X[k,l]| = \left|\frac{\sin\left[\pi k \frac{M'}{M}\right]}{\sin\left[\pi \frac{k}{M}\right]}\right|\left|\frac{\sin\left[\pi l \frac{N'}{N}\right]}{\sin\left[\pi \frac{l}{N}\right]}\right|$$

Pulse signals

4.1

$$N' > M'$$

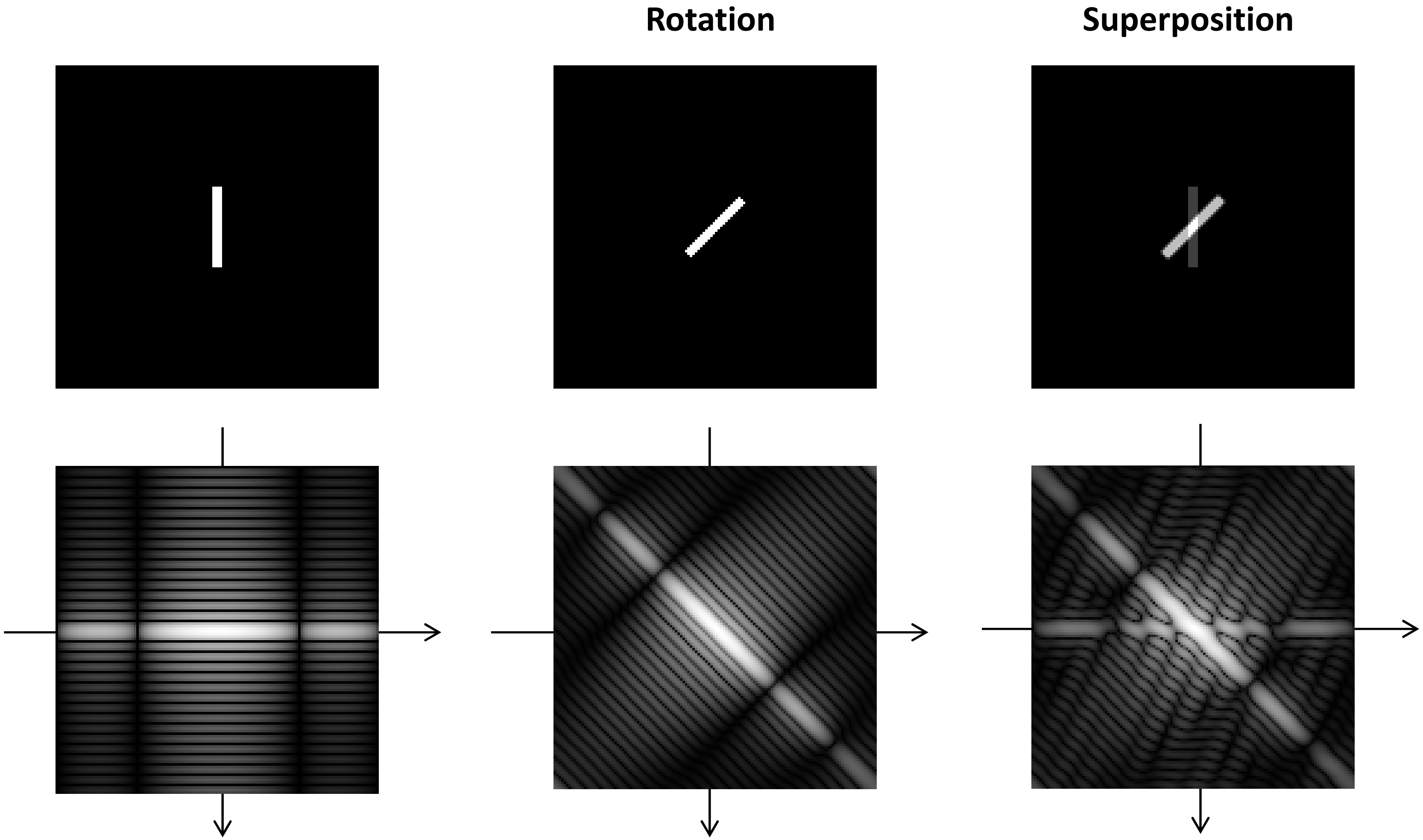




DFT magnitude

Pulse signals

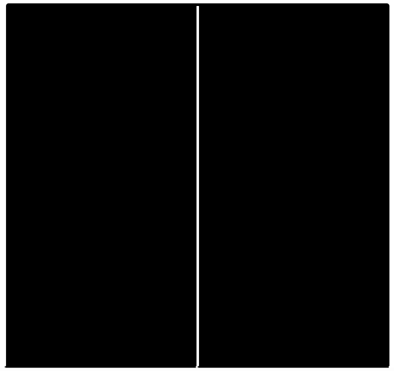
4.1



Line segments

4.1

- **Which is the transform of a line segment?**



- The problem can be analyzed as an **extreme case** of the previous example:

$$x[m,n] = \prod\nolimits_1 [m - m_0] \prod\nolimits_N [n - n_0]$$

$$|X[k,l]| = \left| \frac{\sin\left[\pi k \frac{M'}{M}\right]}{\sin\left[\pi \frac{k}{M}\right]} \right| \left| \frac{\sin\left[\pi l \frac{N'}{N}\right]}{\sin\left[\pi \frac{l}{N}\right]} \right|$$

$$x_1[m] = \Pi_1[m] \longleftrightarrow |X_1[k]| = \left| \frac{sen\left[\pi \frac{k}{M}\right]}{sen\left[\pi \frac{k}{M}\right]} \right| = 1$$

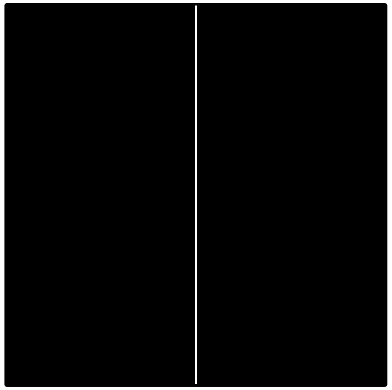
$$x_2[n] = \Pi_N[n] \longleftrightarrow |X_2[l]| = \left| \frac{sen\left[\pi l\right]}{sen\left[\frac{\pi l}{N}\right]} \right| = N\delta[l]$$

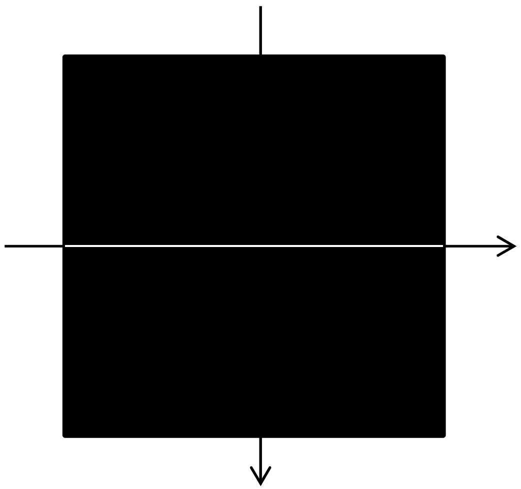
$$|X[k,l]| = |X_1[k]|\,|X_2[l]| = 1 \cdot N\delta[l] = N\delta[l]$$

Line segments

4.1

- **Which is the transform of a line segment?**





- The problem can be analyzed as an **extreme case** of the previous example:

$$x[m,n] = \prod\nolimits_1 [m - m_0] \prod\nolimits_N [n - n_0]$$

- A given line in the spatial domain is related to a **perpendicular line** in the frequency domain

$$\boxed{X[k,l] = N\delta[l]}$$

Unit Structure

4.1

1. **Introduction**
2. **Definition of the 2D Discrete Fourier Transform**
   - 1D and 2D Discrete Fourier Transform
   - Relation between TF and DFT
3. **Basic properties**
   - DFT properties
   - Other 2D DFT features
4. **Some signal transforms**
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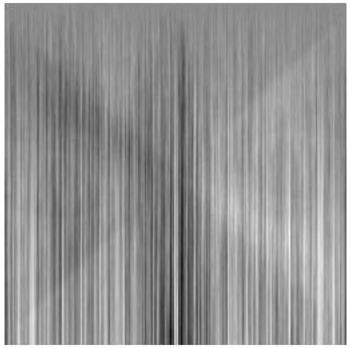
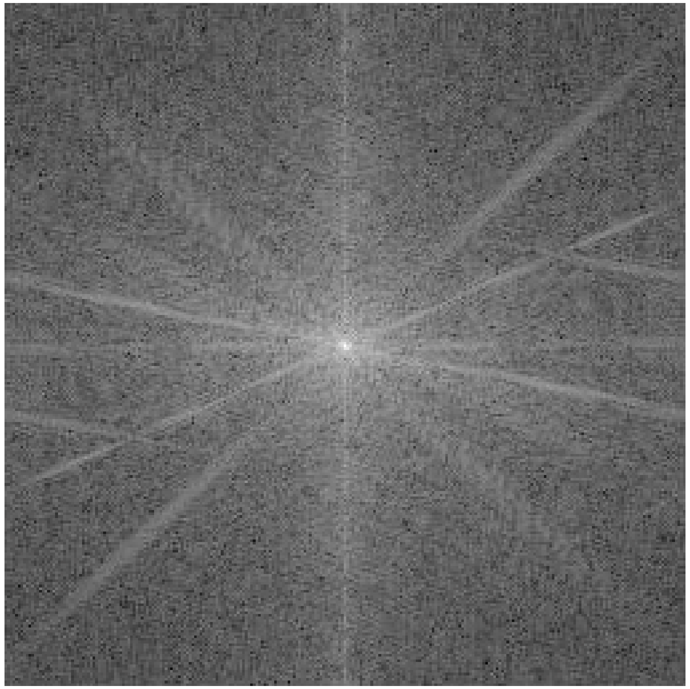
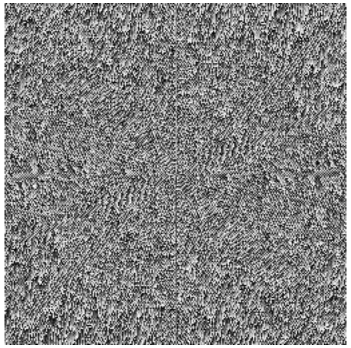
Complex signal transforms

4.1

DFT unwrapped phase







Original Image

DFT magnitude

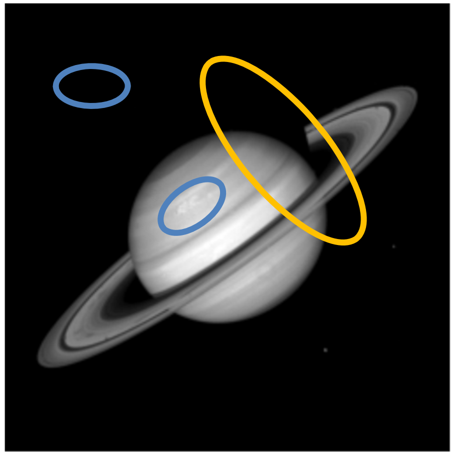
DFT wrapped phase

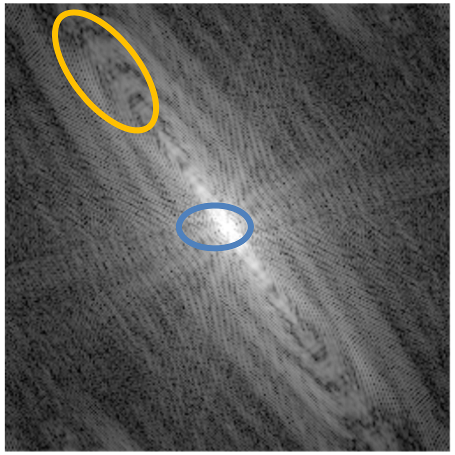
Complex signal transforms

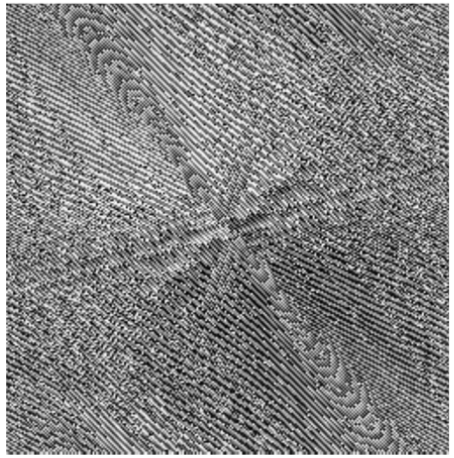
4.1

High frequency

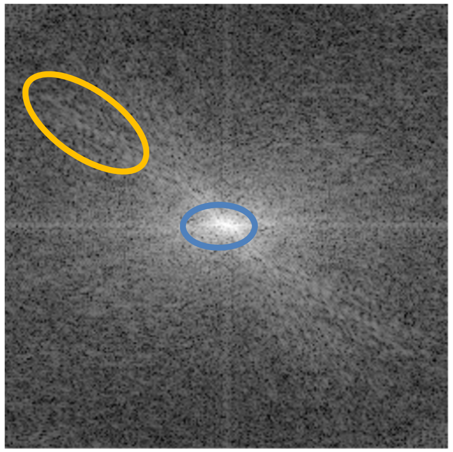
Low frequency

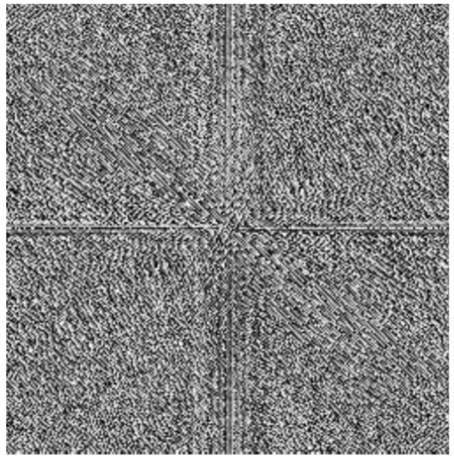






Signal **DFT modulus** **DFT phase**

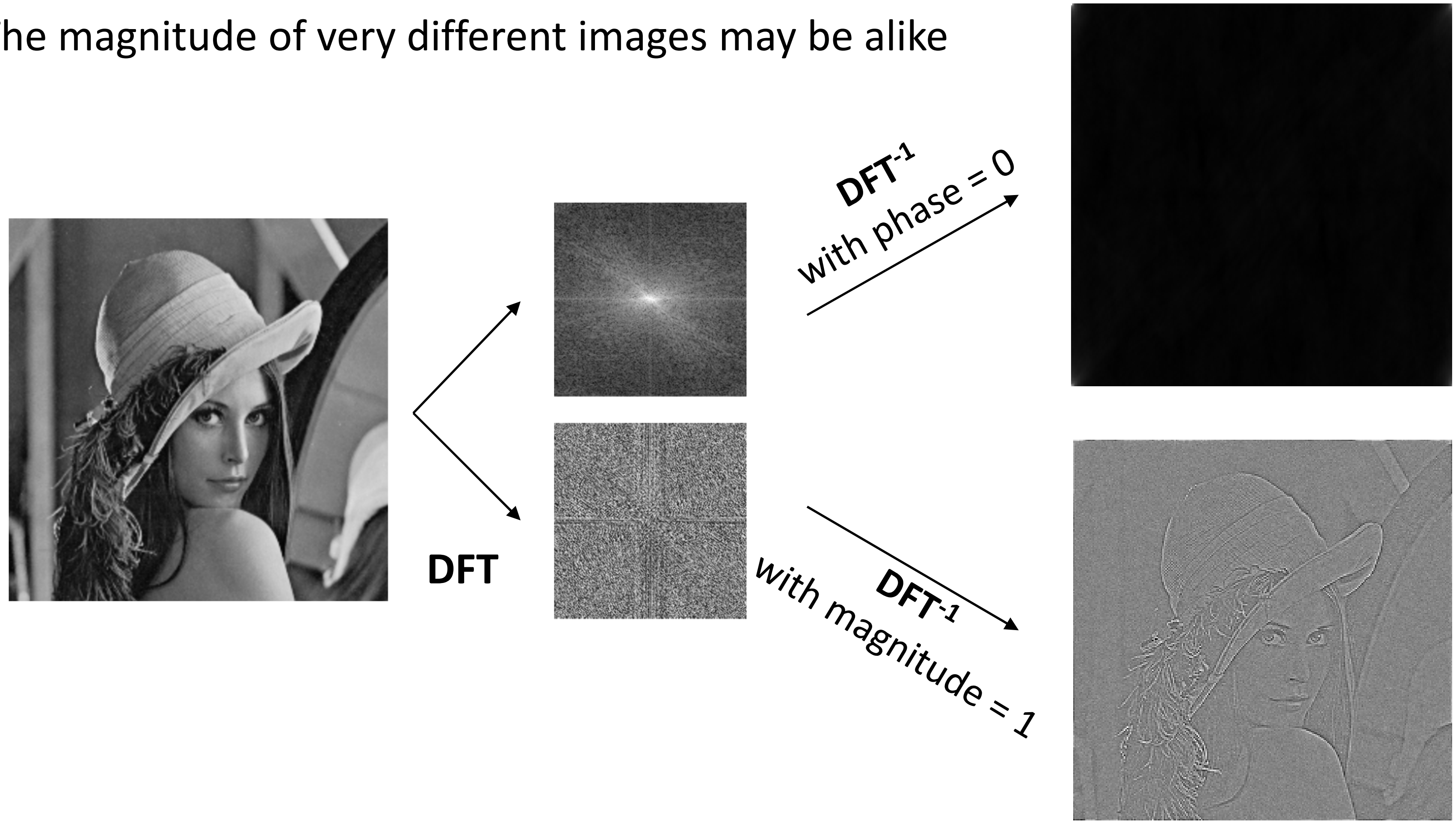




The importance of phase information

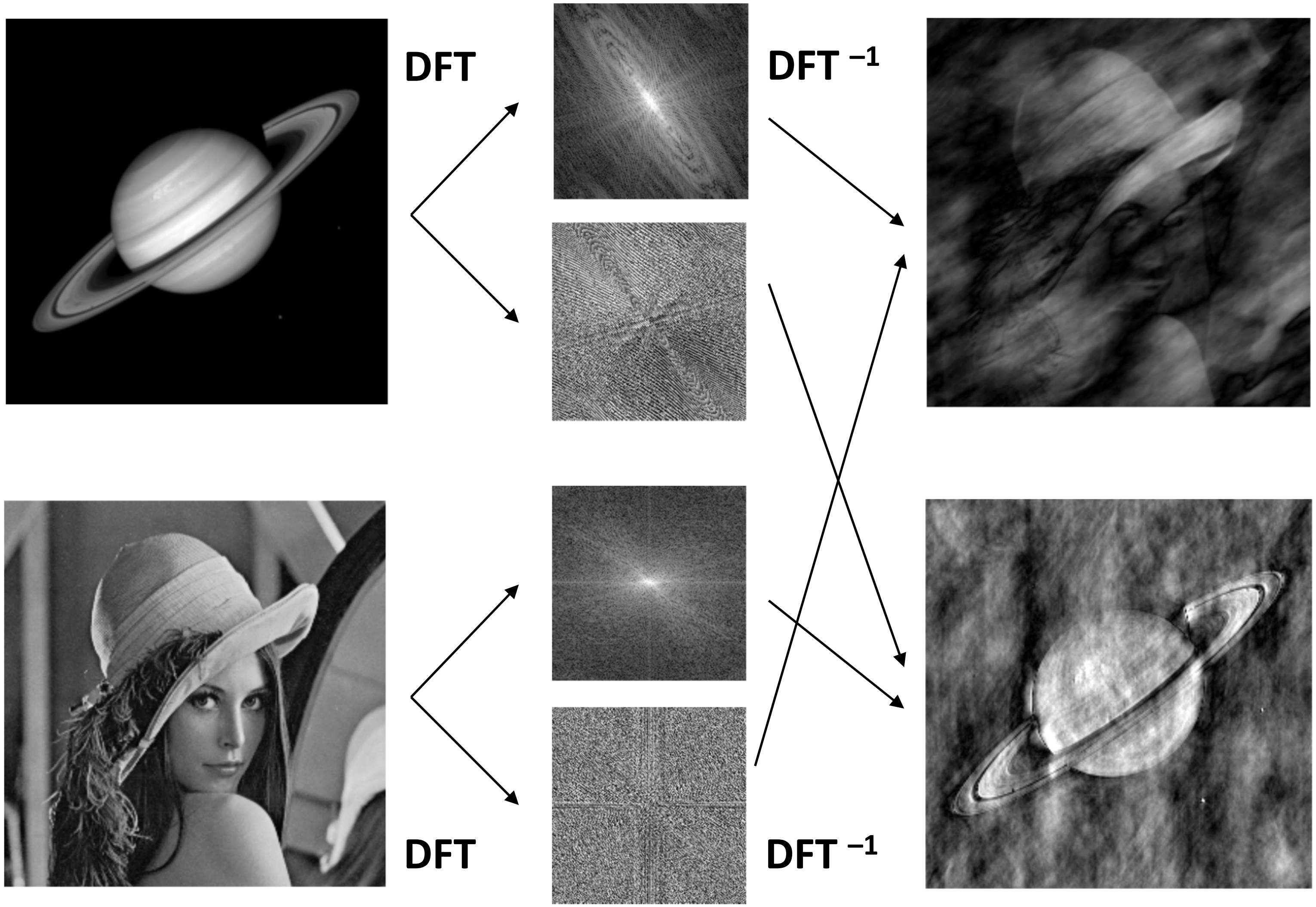
Phase is **more informative** than magnitude.

The magnitude of very different images may be alike



The importance of phase information

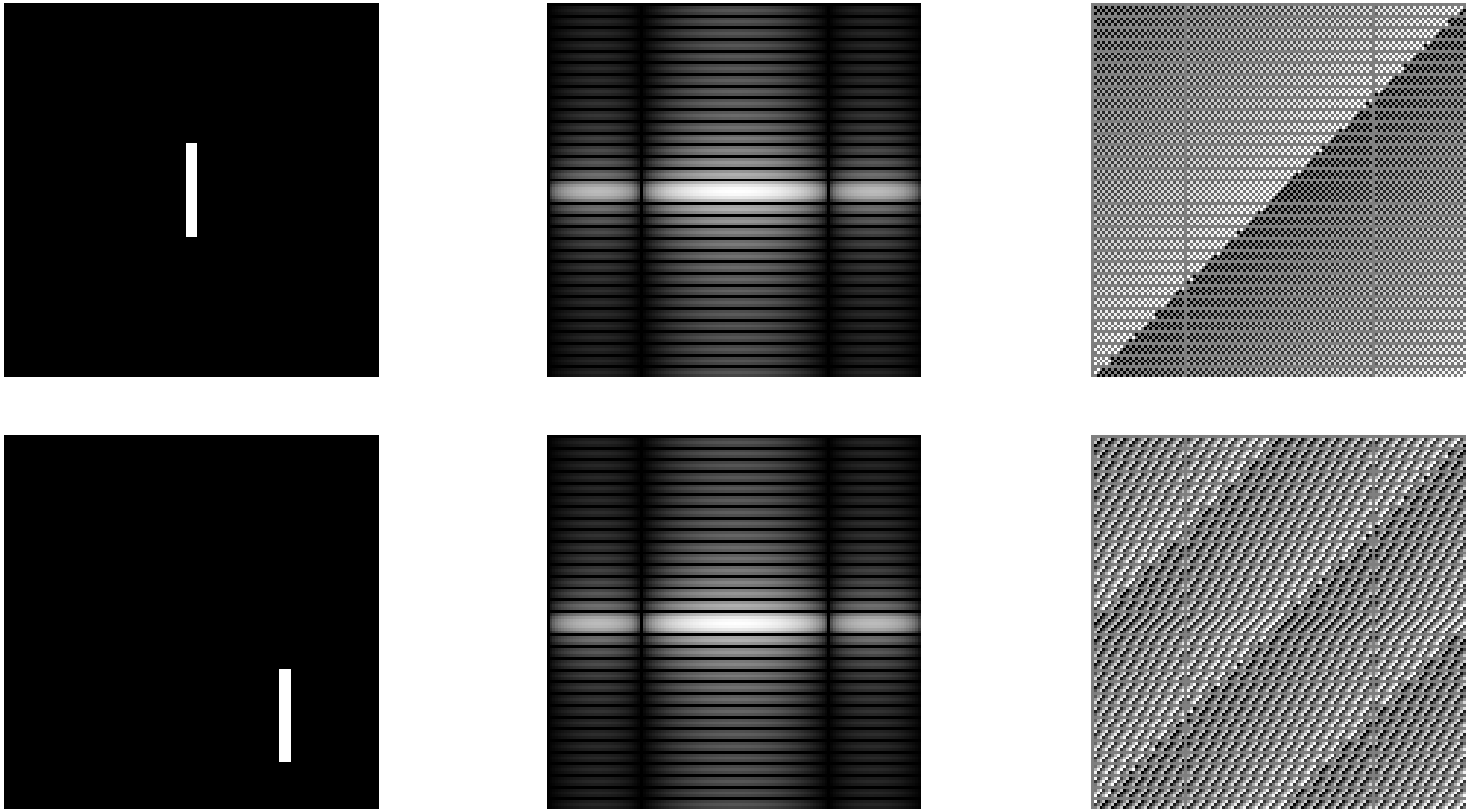
4.1



Phase and position of objects

4.1

Image | DFT magnitude | DFT phase



Shifted image | DFT magnitude | DFT phase

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