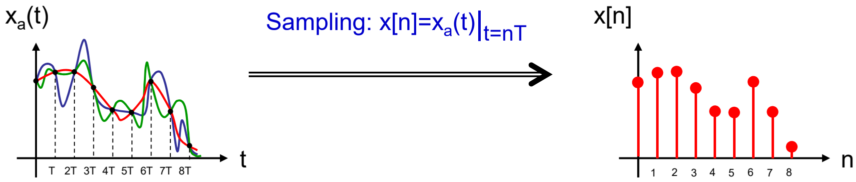
Sampling

U3



How does sampling affect the information contained in a signal?

We would like to sample in a way that preserves information: is that possible?

Sampling

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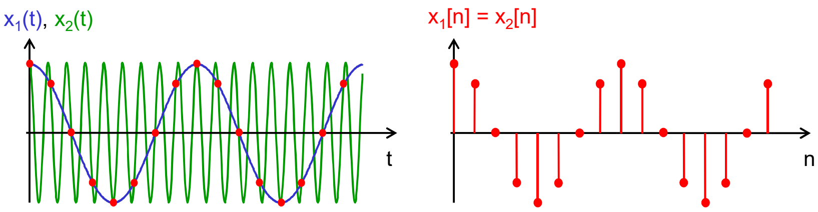
It does not seem so: information between samples is lost. Therefore, the same samples can represent multiple signals



But, actually we can sample <u>in a way</u> that information is preserved

Relationship between CTFT and DTFT

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$x_a(t)$ $\xrightarrow{\text{SAMPLING}}$ $x[n] = x_a(nT)$

Continuous-time Fourier Transform

Discrete-Time Fourier Transform

$$X_a(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t} dt \longleftrightarrow X(F) = \sum_{n=-\infty}^{\infty} x[n] e^{-j2\pi F n}$$

?

$x_p(t) = x_a\ (t) \cdot p(t)$

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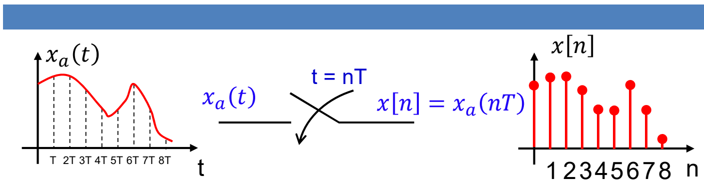
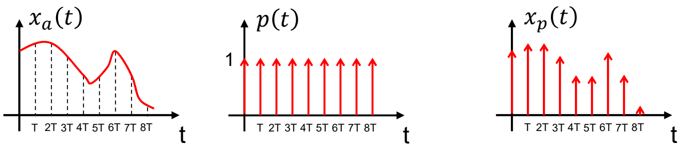


Another signal:

$x_a(t)$ → ⊗ → $x_p(t) = x_a\ (t) \cdot p(t) = \sum_{n=-\infty}^{\infty} x_a(t) \cdot \delta(t - nT)$

$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$

$= \sum_{n=-\infty}^{\infty} x_a(nT) \cdot \delta(t - nT)$



CTFT of $x_p(t) = x_a(t) \cdot p(t)$ (1)

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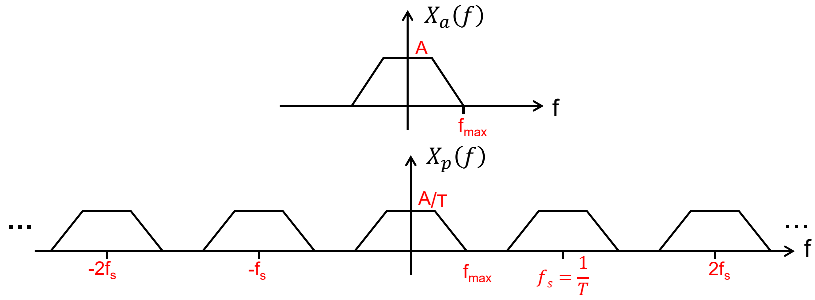
$$X_p(f) = X_a(f) * P(f) = X_a(f) * \left(\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)\right) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_a\left(f-\frac{k}{T}\right)$$

Multiplication by an impulse train in time is equivalent to convolution by an impulse train in frequency → generates multiple copies of original frequency content.



CTFT of $x_p(t) = x_a(t) \cdot p(t)$ (2)

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CTFT of x_p(t)

$$X_p(f) = \mathcal{F}\left\{\sum_{n=-\infty}^{\infty} x_a(nT)\delta(t-nT)\right\}$$

$$= \sum_{n=-\infty}^{\infty} x[n] \int_{-\infty}^{\infty} \delta(t-nT)e^{-j2\pi f t}dt$$

$$= \sum_{n=-\infty}^{\infty} x[n]e^{-j2\pi f nT}$$

Relationship between discrete and analog frequency

$$\boxed{F = fT = \frac{f}{f_s}}$$

DTFT of $x[n]$

$$X(F) = \sum_{n=-\infty}^{\infty} x[n]e^{-j2\pi F n}\Bigg|_{F=fT} = X_p(f)$$

Relationship between $X_a(f)$ and $X(F)$

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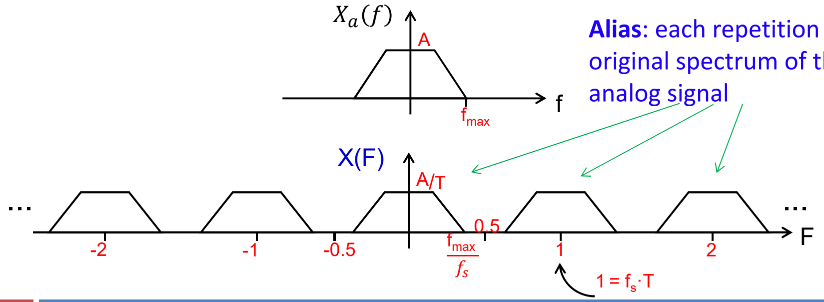
$$X(F)\big|_{F=\frac{f}{f_s}} = X_p(f) = f_s \sum_{k=-\infty}^{\infty} X_a(f - kf_s)$$

$$X(F) = X_p(f)\Big|_{f=F\cdot f_s} = f_s \sum_{k=-\infty}^{\infty} X_a\big((F-k)f_s\big)$$



Alias: each repetition of the original spectrum of the analog signal

Nyquist criterion

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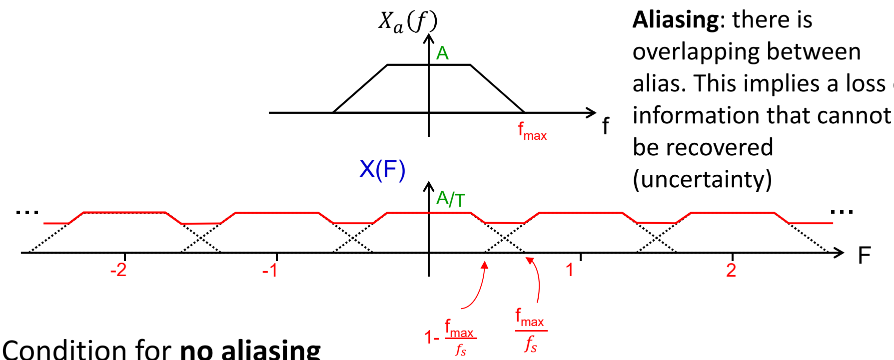

Aliasing: there is overlapping between alias. This implies a loss of information that cannot be recovered (uncertainty)

Condition for **no aliasing**

(no overlapping among alias): $\frac{f_{max}}{f_s} < 1 - \frac{f_{max}}{f_s} \Rightarrow \frac{f_{max}}{f_s} < 0.5 \Rightarrow \boxed{f_s > 2f_{max}}$

Nyquist's criterion
(Information is preserved)