Gamma I

Per ser GLM s'ha de complir

$$\log\left(f\left(y|\theta,\phi\right)\right) = \frac{y\theta - b\left(\theta\right)}{a\left(\phi\right)} + c\left(y,\phi\right)$$

Gamma II

$$\log\left(f\left(y|\theta,\phi\right)\right) = \frac{y\theta - b(\theta)}{a(\phi)} + c\left(y,\phi\right)$$

$$f_{Y_1}\left(y|\alpha,\beta\right) = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1} e^{-\beta y} = e^{-\beta y + (\alpha-1) log(y) + \log\left(\frac{\beta^\alpha}{\Gamma(\alpha)}\right)}$$

$$\log\left(f_{Y_1}\left(y|\alpha,\beta\right)\right) =$$

$$-\beta y + (\alpha - 1)\, log\left(y\right) + \alpha \log\left(\beta\right) - \log\left(\Gamma\left(\alpha\right)\right)$$

$$c\left(y,\phi\right) = (\alpha - 1)\, log\left(y\right) - \log\left(\Gamma\left(\alpha\right)\right) + c_1\left(y\right) + c_2\left(\phi\right)$$

$$\Longrightarrow \phi = \alpha$$

Gamma III

$$\log\left(f\left(y|\theta,\phi\right)\right)=\frac{y\theta-b(\theta)}{a(\phi)}+c\left(y,\phi\right)$$

$$\log\left(f_{Y_1}\left(y|\phi,\beta\right)\right)=$$

$$\frac{-\beta a(\phi)y+\phi a(\phi)\log(\beta)}{a(\phi)}+\left(\phi-1\right)log\left(y\right)-\log\left(\Gamma\left(\phi\right)\right)$$

$$\Longrightarrow\theta=-\beta a\left(\phi\right)\Rightarrow\beta=-\frac{\theta}{a\left(\phi\right)}$$

Gamma IV

$$\log\left(f\left(y|\theta,\phi\right)\right) = \frac{y\theta - b(\theta)}{a(\phi)} + c\left(y,\phi\right)$$

$$\log\left(f_{Y_1}\left(y|\theta,\phi\right)\right) =$$

$$\frac{\theta y + \phi a(\phi)\log\left(-\frac{\theta}{a(\phi)}\right)}{a(\phi)} + (\phi - 1)\, log\left(y\right) - \log\left(\Gamma\left(\phi\right)\right)$$

$$\log\left(f_{Y_1}\left(y|\theta,\phi\right)\right) =$$

$$\frac{\theta y + \phi a(\phi)\log(-\theta)}{a(\phi)} - \phi\log\left(a\left(\phi\right)\right) + (\phi - 1)\, log\left(y\right) - \log\left(\Gamma\left(\phi\right)\right)$$

$$b\left(\theta\right) = -\phi a\left(\phi\right)\log\left(-\theta\right) \Longrightarrow a\left(\phi\right) = \phi^{-1}$$

Gamma V

$$\log\left(f\left(y|\theta,\phi\right)\right) = \frac{y\theta - b(\theta)}{a(\phi)} + c\left(y,\phi\right)$$

$$\log\left(f_{Y_1}\left(y|\theta,\phi\right)\right) =$$

$$\frac{\theta y - (-\log(-\theta))}{\phi^{-1}} - \phi\log\left(\phi^{-1}\right) + \left(\phi - 1\right) log\left(y\right) - \log\left(\Gamma\left(\phi\right)\right)$$

$$\Longrightarrow \quad \begin{aligned} b\left(\theta\right) &= -\log\left(-\theta\right) \\ c\left(y,\phi\right) &= \phi\log\left(\phi\right) + \left(\phi - 1\right) log\left(y\right) - \log\left(\Gamma\left(\phi\right)\right) \end{aligned}$$

Gamma VI

Link canònic

$$\mathbb{E}\left(Y|\theta,\phi\right) = b'\left(\theta\right) = \left(-\log\left(-\theta\right)\right)' = \frac{-1}{\theta} = \mu$$

$$\Longrightarrow link_{canonic}\left(\mu\right) = \frac{1}{\mu} = -\theta$$

Gamma VII

funció de variància

$$Var\left(Y|\theta,\phi\right)=\frac{b''(\theta)}{\phi^{-1}}=\frac{\phi}{\theta^2}=\phi\mu^2$$

$$\Longrightarrow V\left(\mu\right)=\mu^2$$

Binomial I

Per ser GLM s'ha de complir

$$\log\left(f\left(y|\theta,\phi\right)\right)=\frac{y\theta-b\left(\theta\right)}{a\left(\phi\right)}+c\left(y,\phi\right)$$

Binomial II

$$\log\left(f\left(y|\theta,\phi\right)\right)=\frac{y\theta-b(\theta)}{a(\phi)}+c\left(y,\phi\right)$$

$f_{Y_2}\left(y\right)=\binom{n}{y}\pi^y\left(1-\pi\right)^{n-y},\ \ n\in\mathbb{N}^+ fix,\ \ y\in\{0,1,\ldots n\}\ \ ,\pi\in$

$(0,1)$

$$f_{Y_2}\left(y|\pi\right)=e^{\log(\pi)y+(n-y)\log(1-\pi)+\log\binom{n}{y}}$$

$$\log\left(f\right)=\left(\log\left(\pi\right)-\log\left(1-\pi\right)\right)y+n\log\left(1-\pi\right)+\log\binom{n}{y}$$

$$\Longrightarrow\phi=constant \text{ podem escollir } a\left(\phi\right)=1$$

Binomial III

$$\log\left(f\left(y|\theta,\phi\right)\right)=\frac{y\theta-b(\theta)}{a(\phi)}+c\left(y,\phi\right)$$

$$\log\left(f_{Y_2}\left(y|\pi\right)\right)=\frac{\log\left(\frac{\pi}{1-\pi}\right)a(\phi)y+a(\phi)n\log(1-\pi)}{a(\phi)}+\log\binom{n}{y}$$

$$\Longrightarrow \theta=\log\left(\frac{\pi}{1-\pi}\right)a\left(\phi\right)\Rightarrow\pi=\frac{e^{\frac{\theta}{a(\phi)}}}{1+e^{\frac{\theta}{a(\phi)}}}$$

Binomial IV

$$\log\left(f\left(y|\theta,\phi\right)\right) = \frac{y\theta - b(\theta)}{a(\phi)} + c\left(y,\phi\right)$$

$$\log\left(f_{Y_2}\left(y|\theta\right)\right) = \frac{\theta y - a(\phi) n \log\left(1 + e^{\frac{\theta}{a(\phi)}}\right)}{a(\phi)} + \log\binom{n}{y}$$

$$\textit{per simplificar} \Longrightarrow \phi = 1$$

Binomial V

$$\log\left(f\left(y|\theta,\phi\right)\right) = \frac{y\theta - b(\theta)}{a(\phi)} + c\left(y,\phi\right)$$

$$\log\left(f_{Y_2}\left(y|\theta\right)\right) = \theta y - n\log\left(1+e^{\theta}\right) + \log\binom{n}{y}$$

$$\implies \begin{array}{l} b\left(\theta\right) = n\log\left(1+e^{\theta}\right) \\ c\left(y,\phi\right) = \log\binom{n}{y} \end{array}$$

Binomial VI

Link canònic

$$\mathbb{E}\left(Y|\theta\right) = b'\left(\theta\right) = \left(n\log\left(1+e^{\theta}\right)\right)' = \frac{ne^{\theta}}{1+e^{\theta}} = \mu = n\pi$$

$$\Longrightarrow link_{canonic}\left(\mu\right) = \log\left(\frac{\mu}{n-\mu}\right) = \log\left(\frac{\pi}{1-\pi}\right) = \theta$$

funció de variància

$$Var\left(Y|\theta\right) = b''\left(\theta\right) = \frac{ne^{\theta}}{\left(1+e^{\theta}\right)^2} = \mu\left(1-\frac{\mu}{n}\right) = n\pi\left(1-\pi\right)$$

$$\Longrightarrow V\left(\mu\right) = \mu\left(1-\frac{\mu}{n}\right) = n\pi\left(1-\pi\right)$$

Binomial negativa I

Per ser GLM s'ha de complir

$$\log\left(f\left(y|\theta,\phi\right)\right)=\frac{y\theta-b\left(\theta\right)}{a\left(\phi\right)}+c\left(y,\phi\right)$$

$\log\left(f\left(y|\theta,\phi\right)\right)=\frac{y\theta-b(\theta)}{a(\phi)}+c\left(y,\phi\right)$

$f_{Y_3}\left(y\right)=\frac{\Gamma(y+\rho)}{y!\Gamma(\rho)}\pi^{y}\left(1-\pi\right)^{\rho},\ \ y\in\mathbb{N},\ \ \rho>0\ ,\pi\in\left(0,1\right)$

$\log\left(f_{Y_3}\left(y|\pi,\rho\right)\right)=$

$\log\left(\pi\right)y+\rho\log\left(1-\pi\right)-\log\left(\Gamma\left(\rho\right)\right)+\log\left(\Gamma\left(y+\rho\right)\right)-\log\left(y!\right)$

Binomial negativa II

$$\log\left(f\left(y|\theta,\phi\right)\right) = \frac{y\theta - b(\theta)}{a(\phi)} + c\left(y,\phi\right)$$

$$\log\left(f_{Y_3}\left(y|\pi,\rho\right)\right) =$$

$$\log\left(\pi\right)y + \rho\log\left(1-\pi\right) - \log\left(\Gamma\left(\rho\right)\right) + \log\left(\Gamma\left(y+\rho\right)\right) - \log\left(y!\right)$$

$$c\left(y,\phi\right) = -\log\left(\Gamma\left(\rho\right)\right) + \log\left(\Gamma\left(y+\rho\right)\right) - \log\left(y!\right) +$$

$$+c_1\left(y\right) + c_2\left(\phi\right) \Longrightarrow \phi = \rho$$

Binomial negativa III

$$\log\left(f\left(y|\theta,\phi\right)\right) = \frac{y\theta - b(\theta)}{a(\phi)} + c\left(y,\phi\right)$$

$$\log\left(f_{Y_3}\left(y|\pi,\phi\right)\right) =$$

$$\frac{a(\phi)\log(\pi)y + \phi a(\phi)\log(1-\pi)}{a(\phi)} - \log\left(\Gamma\left(\phi\right)\right) + \log\left(\Gamma\left(y+\phi\right)\right) - \log\left(y!\right)$$

$$\Longrightarrow \theta = \log\left(\pi\right) a\left(\phi\right) \Rightarrow \pi = e^{\frac{\theta}{a(\phi)}}$$

Binomial negativa IV

$$\log\left(f\left(y|\theta,\phi\right)\right)=\frac{y\theta-b(\theta)}{\phi}+c\left(y,\phi\right)$$

$$\log\left(f_{Y_3}\left(y|\theta,\phi\right)\right)=$$
$$\frac{a(\phi)\log\left(e^{\frac{\theta}{a(\phi)}}\right)y+\phi a(\phi)\log\left(1-e^{\frac{\theta}{a(\phi)}}\right)}{a(\phi)}-\log\left(\Gamma\left(\phi\right)\right)+\log\left(\Gamma\left(y+\phi\right)\right)-\log\left(y!\right)$$

$$\log\left(f_{Y_3}\left(y|\theta,\phi\right)\right)=$$
$$\frac{\theta y+\phi a(\phi)\log\left(1-e^{\frac{\theta}{a(\phi)}}\right)}{a(\phi)}-\log\left(\Gamma\left(\phi\right)\right)+\log\left(\Gamma\left(y+\phi\right)\right)-\log\left(y!\right)$$

$$\Longrightarrow b\left(\theta\right)=-\phi a\left(\phi\right)\log\left(1-e^{\frac{\theta}{a(\phi)}}\right)$$

Binomial negativa V

Condicions per a que sigui *GLM*

Per poder complir-se que $b(\theta)$ ***només depèn de*** θ***,***

aleshores

ϕ, $a(\phi)$ i $\rho = \phi$ ***han de ser constants conegudes***

$a(\phi) = 1$ *y* $\phi = \rho$ *constant*

Binomial negativa VI

$$\log\left(f\left(y|\theta,\phi\right)\right)=\frac{y\theta-b(\theta)}{a(\phi)}+c\left(y,\phi\right)$$

$$\log\left(f_{Y_3}\left(y|\theta\right)\right)=$$

$$\theta y+\rho\log\left(1-e^{\theta}\right)-\log\left(\Gamma\left(\rho\right)\right)+\log\left(\Gamma\left(y+\rho\right)\right)-\log\left(y!\right)$$

$$\Longrightarrow \begin{aligned} b\left(\theta\right)&=-\rho\log\left(1-e^{\theta}\right)\\ c\left(y,\phi\right)&=\log\left(\frac{\Gamma(y+\rho)}{y!\Gamma(\rho)}\right)\end{aligned}$$

Binomial negativa VII

Link canònic

$$\mathbb{E}\left(Y|\theta\right) = b'\left(\theta\right) = \left(-\rho \log\left(1 - e^{\theta}\right)\right)' = \frac{\rho e^{\theta}}{1-e^{\theta}} = \mu$$

$$\Longrightarrow link_{canonic}\left(\mu\right) = \log\left(\frac{\mu}{\rho + \mu}\right) = \theta$$

Binomial negativa VIII

funció de variància

$$Var\left(Y|\theta\right) = b''\left(\theta\right) = \frac{\rho e^{\theta}}{\left(1-e^{\theta}\right)^{2}} = \mu\frac{\mu+\rho}{\rho}$$

$$\Longrightarrow V\left(\mu\right) = \mu\left(1+\frac{\mu}{\rho}\right)$$