# Exercise sheet № 1

Practice with conditional probabilities and Bayes formula

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AA1 (GCED) 2019-2020

## Problem 1: The gambler (**)

Consider the game of tossing a coin in the air and guessing the result. The player wins or loses 1€ depending on whether or not he/she guesses the result. The starting capital is $x$ euros and the game goes until the player wins exactly $y$ euros ($y > x$). In other words, the game continues until either the player wins $y$ euros or loses all the starting capital. What is the probability that the player will go bankrupt?

## Problem 2: The taxi accident (*)

In a certain city 85% of the taxis are painted blue and the rest are painted green. There is an accident in which one taxi is involved. An eyewitness says that the taxi was green. This eyewitness is 80% reliable. What is the probability that the taxi was indeed green?

## Problem 3: The English-speaking tourist (*)

An English-speaking tourist visits a city in Europe whose language is not English. A local friend tells him that 1 in 10 natives speak English, 1 in 5 people in the streets are tourists and that half of the tourists speak English. Our visitor stops someone in the street and finds that this person speaks English. What is the probability that this person is a tourist?

## Problem 4: An ovarian cancer (OC) story (**)

One in every 2,500 women aged above 35 who visit the doctor do have this cancer. We have a wonderful OC test: it always detects OC when it is there. When it is not, the test is negative in 63/66 cases. How useful is the test?

## Problem 5: The pancake problem (*)

We have a bag with three pancakes. One is brown on both sides, another is golden on both sides, and the third has one brown and one golden side. We pick one pancake from the bag, look ant one side and found that it is brown. What is the probability that the other side is also brown?

## Problem 5: The factories problem (**)

Two factories $A, B$ produce the same product. Factory $A$ has a 2% of faulty products, whereas factory $B$ has a 1%. We order a batch of $X_1, \ldots, X_n$ products from the same factory (but we do not know which factory is it). We check the fist product $X_1$ and it is OK (not faulty). Assume that the correctness of each product for each factory is independent. What is the probability that the second product $X_2$ is also OK?

## Problem 6: The biased coin (**)

We suspect a coin is biased towards heads and specify a prior for $p$ (the probability of obtaining a head in a single toss) as: $P(p = 0,6) = 0,8$ and $P(p = 0,5) = 0,2$. We now toss the coin. We toss the coin $n = 10$ times and get $k = 4$ heads. What is the new belief in the coin being biased?

## Problem 7: Posterior mean for the Gaussian (***)

Let $\hat{M}$ be the sample mean of a i.i.d. sample of size $n$, the density of which is $\mathcal{N}(\mu, \sigma^2)$. The prior density for $\mu$ is $\mathcal{N}(\mu_0, \sigma_0^2)$. Calculate the posterior density $\mathcal{N}(\mu_1, \sigma_1^2)$ for $\mu$ given $\hat{M} = \hat{m}$.