

Workshop 1: Sets Solutions

SCC120 Fundamentals of
Computer Science
Solutions by Corina Sas

Exercise 1

Determine whether each of the following pairs of sets is equal:

a) $\{1, 3, 5\}$ and $\{5, 3, 1\}$

b) $\{1, 3, 5\}$ and $\{5, 1, 6\}$

Exercise1: Answer

- a) $\{1, 3, 5\}$ and $\{5, 3, 1\}$ are equal.
Remember, sets are unordered. Both sets contain the same number of elements, and the same elements.
- b) $\{1, 3, 5\}$ and $\{5, 1, 6\}$ are not equal. Both sets do have the same number of elements, but not the same elements, i.e., $3 \neq 6$.

Exercise 2

Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 3, 6\}$. Find

a) $A \cap B$

b) $A \cup B$

c) $A - B$

d) $B - A$

$U = \{$	0,	1,	2,	3,	4,	5,	6	$\}$
<hr/>								
$A = \{$		1,	2,	3,	4,	5		$\}$
$B = \{$	0,			3,			6	$\}$

Answer 2 (a)

- a) $A \cap B = \{3\}$

$U = \{$	0,	1,	2,	3,	4,	5,	6	$\}$
<hr/>								
$A = \{$		1,	2,	3,	4,	5		$\}$
$B = \{$	0,			3,			6	$\}$
$A \cap B = \{$				3				$\}$

Answer 2 (b)

- b) $A \cup B = \{0, 1, 2, 3, 4, 5, 6\}$

$U = \{$	0,	1,	2,	3,	4,	5,	6	$\}$
<hr/>								
$A = \{$		1,	2,	3,	4,	5		$\}$
$B = \{$	0,			3,			6	$\}$
$A \cup B = \{$	0,	1,	2,	3,	4,	5,	6	$\}$

Answer 2 (c)

- c) $A - B = \{1, 2, 4, 5\}$

$U = \{$	0,	1,	2,	3,	4,	5,	6	$\}$
<hr/>								
$A = \{$		1,	2,	3,	4,	5		$\}$
$B = \{$	0,			3,			6	$\}$
$A - B = \{$		1,	2,		4,	5		$\}$

Answer 2 (d)

- d) $B - A = \{0, 6\}$

$U = \{$	0,	1,	2,	3,	4,	5,	6	$\}$
<hr/>								
$A = \{$		1,	2,	3,	4,	5		$\}$
$B = \{$	0,			3,			6	$\}$
$B - A = \{$	0,						6	$\}$

Exercise 3

Let $A = \{0, 2, 4, 6, 8, 10\}$, $B = \{0, 1, 2, 3, 4, 5, 6\}$ and $C = \{4, 5, 6, 7, 8, 9, 10\}$. Find

a) $A \cap B \cap C$

b) $A \cup B \cup C$

c) $(A \cup B) \cap C$

d) $(A \cap B) \cup C$

$U = \{$	0,	1,	2,	3,	4,	5,	6,	7,	8,	9,	10	$\}$
$A = \{$	0,		2,		4,		6,		8,		10	$\}$
$B = \{$	0,	1,	2,	3,	4,	5,	6,					$\}$
$C = \{$					4,	5,	6,	7,	8,	9,	10	$\}$

Answer 3 (a)

- a) $A \cap B \cap C = \{4, 6\}$

$U = \{$	0,	1,	2,	3,	4,	5,	6,	7,	8,	9,	10	$\}$
$A = \{$	0,		2,		4,		6,		8,		10	$\}$
$B = \{$	0,	1,	2,	3,	4,	5,	6,					$\}$
$C = \{$					4,	5,	6,	7,	8,	9,	10	$\}$
					4,		6					$\}$

Answer 3 (b)

- b) $A \cup B \cup C = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

U = {	0,	1,	2,	3,	4,	5,	6,	7,	8,	9,	10	}	
A = {	0,		2,		4,		6,		8,		10	}	
B = {	0,	1,	2,	3,	4,	5,	6,					}	
C = {					4,	5,	6,	7,	8,	9,	10	}	
	{	0,	1,	2,	3,	4,	5,	6,	7,	8,	9,	10	}

Answer 3 (c)

- c) $(A \cup B) \cap C = \{4, 5, 6, 8, 10\}$

$U = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \}$

$A = \{ 0, 2, 4, 6, 8, 10 \}$

$B = \{ 0, 1, 2, 3, 4, 5, 6 \}$

$(A \cup B) = \{ 0, 1, 2, 3, 4, 5, 6, 8, 10 \}$

$C = \{ 4, 5, 6, 7, 8, 9, 10 \}$

$(A \cup B) \cap C = \{ 4, 5, 6, 8, 10 \}$

Answer 3 (d)

- d) $(A \cap B) \cup C = \{0, 2, 4, 5, 6, 7, 8, 9, 10\}$

$U = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \}$

$A = \{ 0, 2, 4, 6, 8, 10 \}$

$B = \{ 0, 1, 2, 3, 4, 5, 6 \}$

$(A \cap B) = \{ 0, 2, 4, 6 \}$

$C = \{ 4, 5, 6, 7, 8, 9, 10 \}$

$(A \cap B) \cup C = \{ 0, 2, 4, 5, 6, 7, 8, 9, 10 \}$

Exercise 4

if $A = \{1, 2, 3, a\}$, $B = \{1, 2, 3, 4, 5\}$, $C = \{a, b\}$ evaluate:

a) $A \cup (B \cap C)$

b) $(A \cup B) \cap C$

c) $C - A$

d) $(A - B) - C$

e) $A - (B - C)$

f) $(A \cap C) \cup B$

g) $A \cap (C \cup B)$

Answer 4 (a)

- (a) $A \cup (B \cap C) = A = \{1, 2, 3, a\}$
- $(B \cap C)$ is the empty set.

$U = \{$	$1,$	$2,$	$3,$	$4,$	$5,$	$a,$	b	$\}$
$A = \{$	$1,$	$2,$	$3,$			a		$\}$
$B = \{$	$1,$	$2,$	$3,$	$4,$	5			$\}$
$C = \{$						$a,$	b	$\}$
$(B \cap C) = \{$								$\}$
$A \cup (B \cap C) = \{$	$1,$	$2,$	$3,$			a		$\}$

Answer 4 (b)

- b) $(A \cup B) \cap C = \{a\}$

$$U = \{ 1, 2, 3, 4, 5, a, b \}$$

$$A = \{ 1, 2, 3, a \}$$

$$B = \{ 1, 2, 3, 4, 5 \}$$

$$(A \cup B) = \{ 1, 2, 3, 4, 5, a \}$$

$$(A \cup B) = \{ 1, 2, 3, 4, 5, a \}$$

$$C = \{ a, b \}$$

$$(A \cup B) \cap C = \{ a \}$$

Answer 4 (c)

- c) $C - A = \{b\}$

$$U = \{ 1, 2, 3, 4, 5, a, b \}$$

$$C = \{ \quad \quad \quad a, b \}$$

$$A = \{ 1, 2, 3, \quad \quad a \}$$

$$C - A = \{ \quad \quad \quad b \}$$

Answer 4 (d)

- d) $(A - B) - C = \emptyset$

$$U = \{ 1, 2, 3, 4, 5, a, b \}$$

$$A = \{ 1, 2, 3, a \}$$

$$B = \{ 1, 2, 3, 4, 5 \}$$

$$(A - B) = \{ a \}$$

$$(A - B) = \{ a \}$$

$$C = \{ a, b \}$$

$$(A - B) - C = \{ \}$$

Answer 4 (e)

- e) $A - (B - C) = \{a\}$
- Note that $(B - C) = B$.

$$U = \{ 1, 2, 3, 4, 5, a, b \}$$

$$B = \{ 1, 2, 3, 4, 5 \}$$

$$C = \{ a, b \}$$

$$(B - C) = \{ 1, 2, 3, 4, 5 \}$$

$$A = \{ 1, 2, 3, a \}$$

$$(B - C) = \{ 1, 2, 3, 4, 5 \}$$

$$A - (B - C) = \{ a \}$$

Answer 4 (f)

- f) $(A \cap C) \cup B = \{1, 2, 3, 4, 5, a\}$

$$U = \{ 1, 2, 3, 4, 5, a, b \}$$

$$A = \{ 1, 2, 3, a \}$$

$$C = \{ a, b \}$$

$$(A \cap C) = \{ a \}$$

$$(A \cap C) = \{ a \}$$

$$B = \{ 1, 2, 3, 4, 5 \}$$

$$(A \cap C) \cup B = \{ 1, 2, 3, 4, 5, a \}$$

Answer 4 (g)

- g) $A \cap (C \cup B) = A$

$$U = \{ 1, 2, 3, 4, 5, a, b \}$$

$$B = \{ 1, 2, 3, 4, 5 \}$$

$$C = \{ a, b \}$$

$$(C \cup B) = \{ 1, 2, 3, 4, 5, a, b \}$$

$$A = \{ 1, 2, 3, a \}$$

$$(C \cup B) = \{ 1, 2, 3, 4, 5, a, b \}$$

$$A \cap (C \cup B) = \{ 1, 2, 3, a \}$$

Exercise 5

Let A be the set of students who live within one mile of school and B the set of students who walk to classes. Describe the students in each of the following sets, in English.

a) $A \cap B$

b) $A \cup B$

c) $A - B$

d) $B - A$

Answer 5 (a)

Let A be the set of students who live within one mile of school and B the set of students who walk to classes. Describe the students in each of the following sets.

a) Answer: $A \cap B = ?$ (intersection)

The set of students who live within one mile of school and who walk to classes.

Answer 5 (b)

Let A be the set of students who live within one mile of school and B the set of students who walk to classes. Describe the students in each of the following sets.

b) Answer: $A \cup B = ?$ (union)

The set of students who live within one mile of school or who walk to classes (or who do both).

Answer 5 (c)

Let A be the set of students who live within one mile of school and B the set of students who walk to classes. Describe the students in each of the following sets.

c) $A - B = ?$ (difference : who appears in set A and not in set B)

The set of students who live within one mile of school but do not walk to classes.

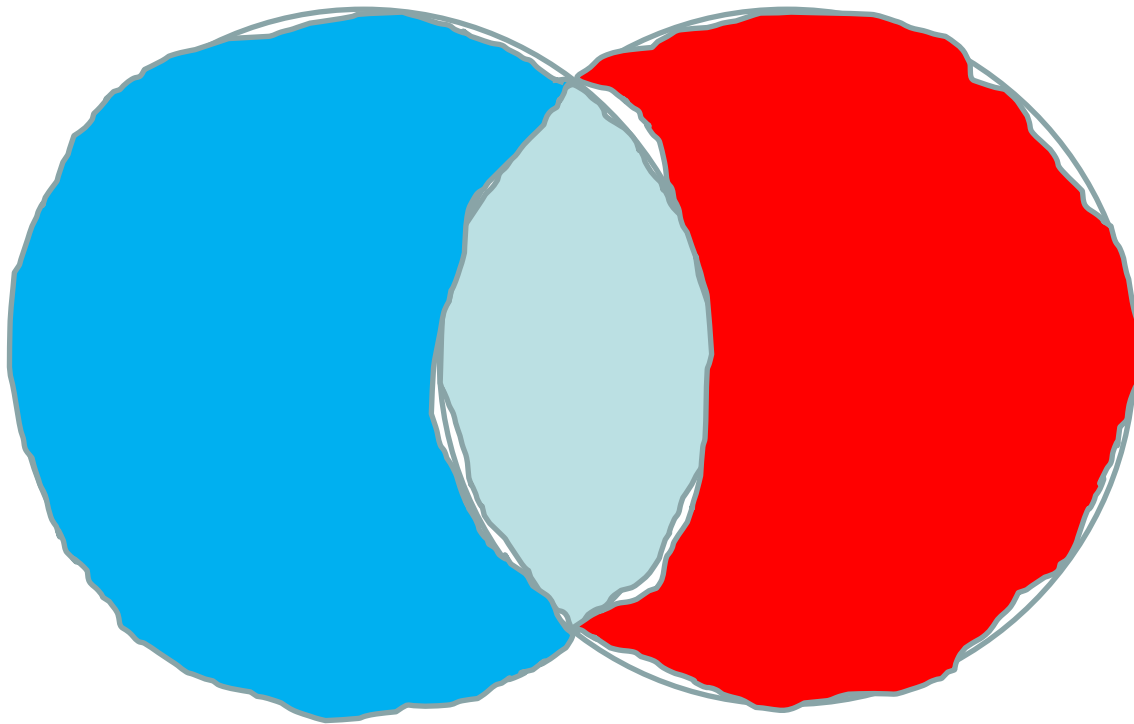
Answer 5 (d)

Let A be the set of students who live within one mile of school and B the set of students who walk to classes. Describe the students in each of the following sets.

d) $B - A = ?$

The set of students who walk to classes but live more than one mile away from school.

Sets A and B

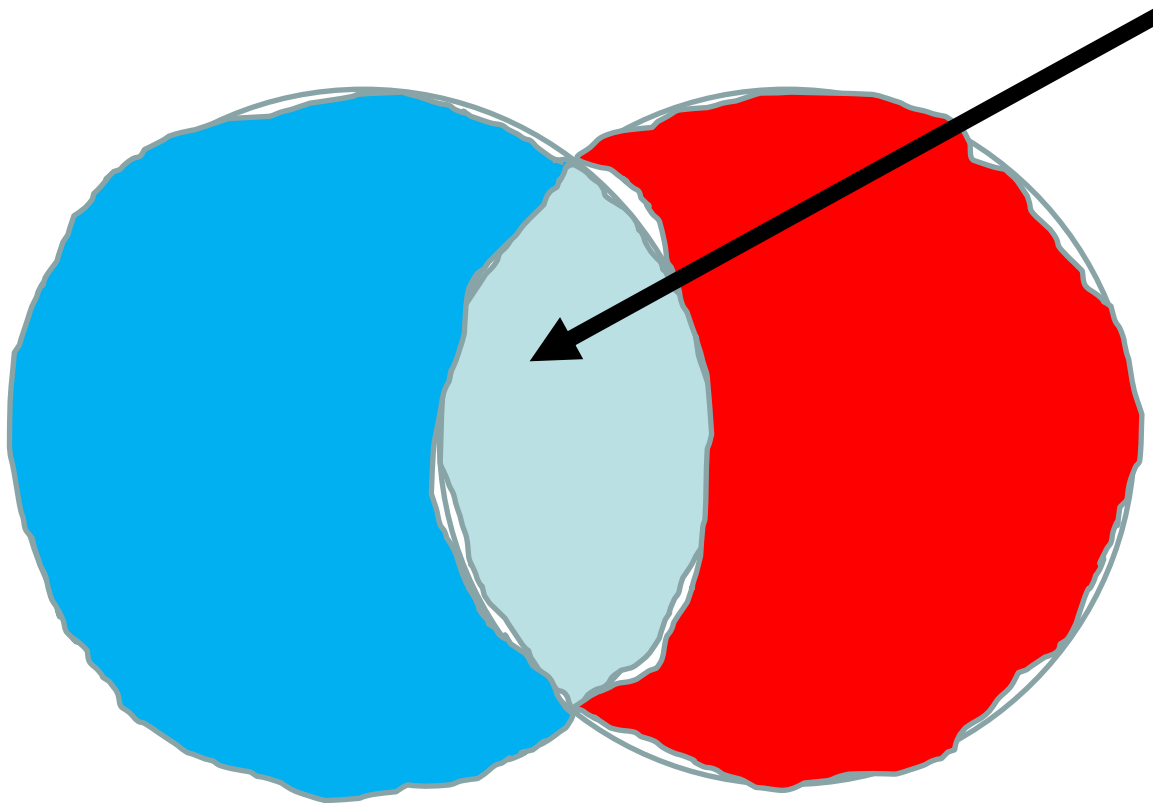


A : Students who live within
one mile of school

B : Students who walk to classes

Sets A and B: $A \cap B$

$A \cap B$: Students who live within
one mile of school AND walk to classes

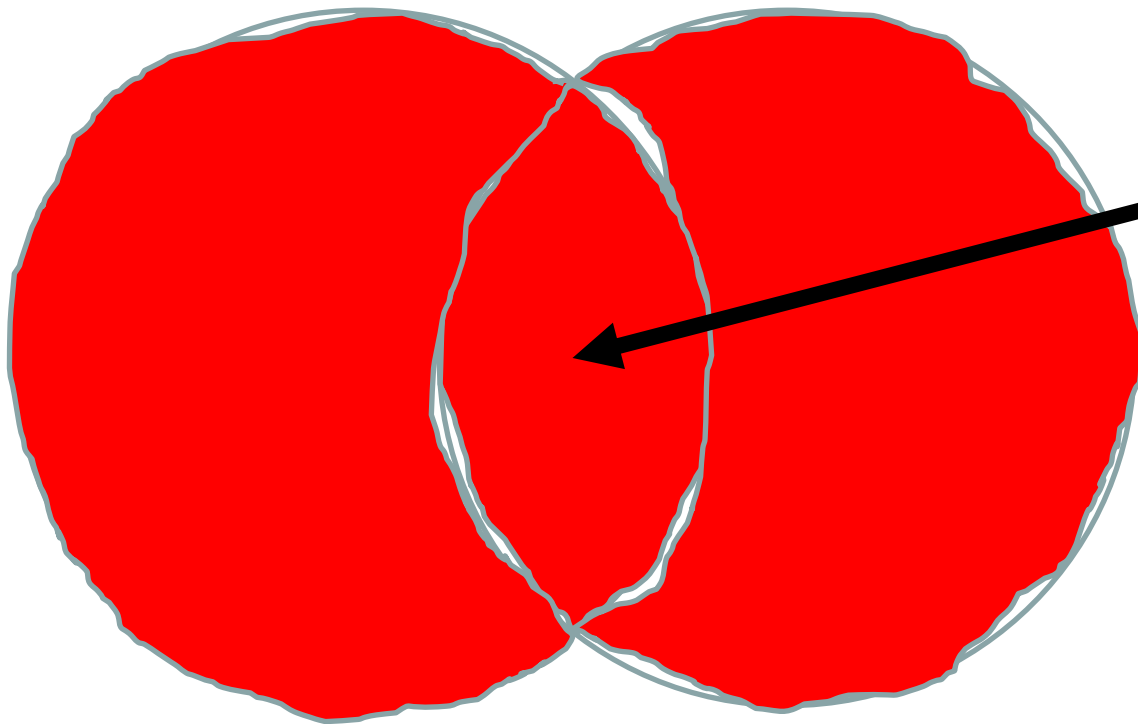


A : Students who live within
one mile of school

B : Students who walk to classes

Sets A and B: $A \cup B$

$A \cup B$: Students who live within
one mile of school OR students who walk to classes



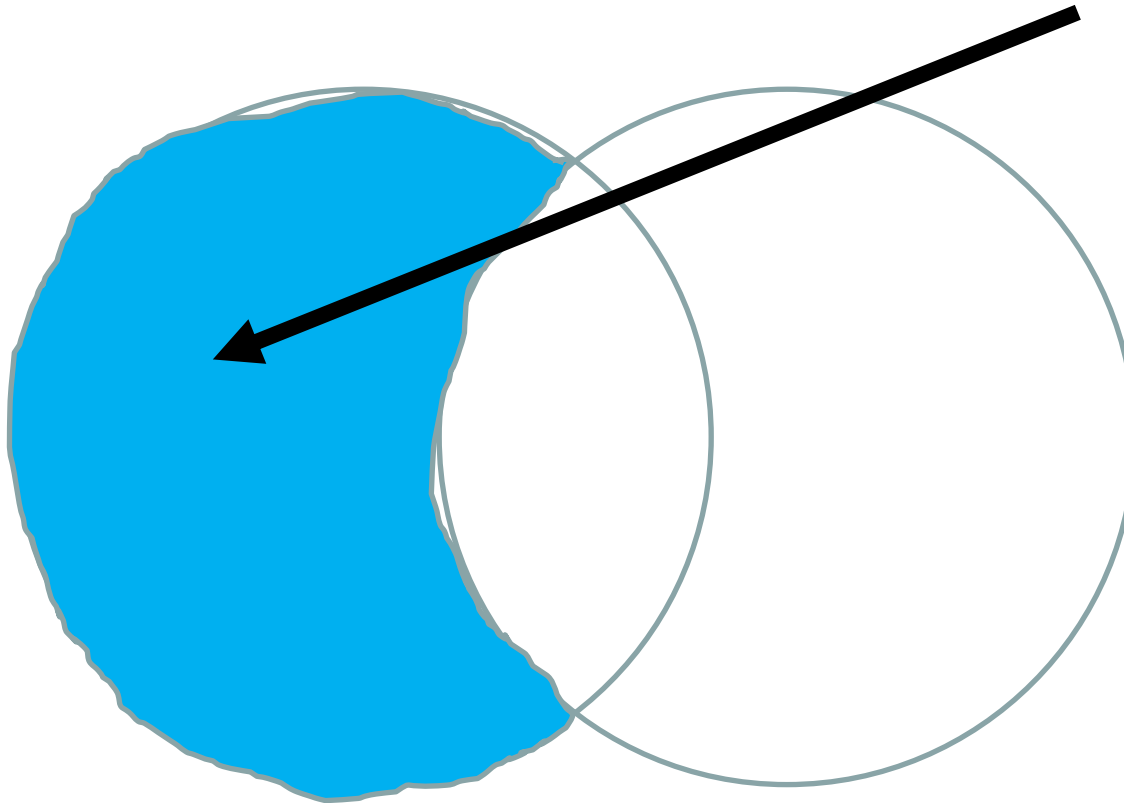
$A \cap B$: Students
who live within
one mile of school
AND walk to
classes

A : Students who live within
one mile of school

B : Students who walk to classes

Sets A and B: $A - B$

$A - B$: Students who live within
one mile of school but do not walk to classes

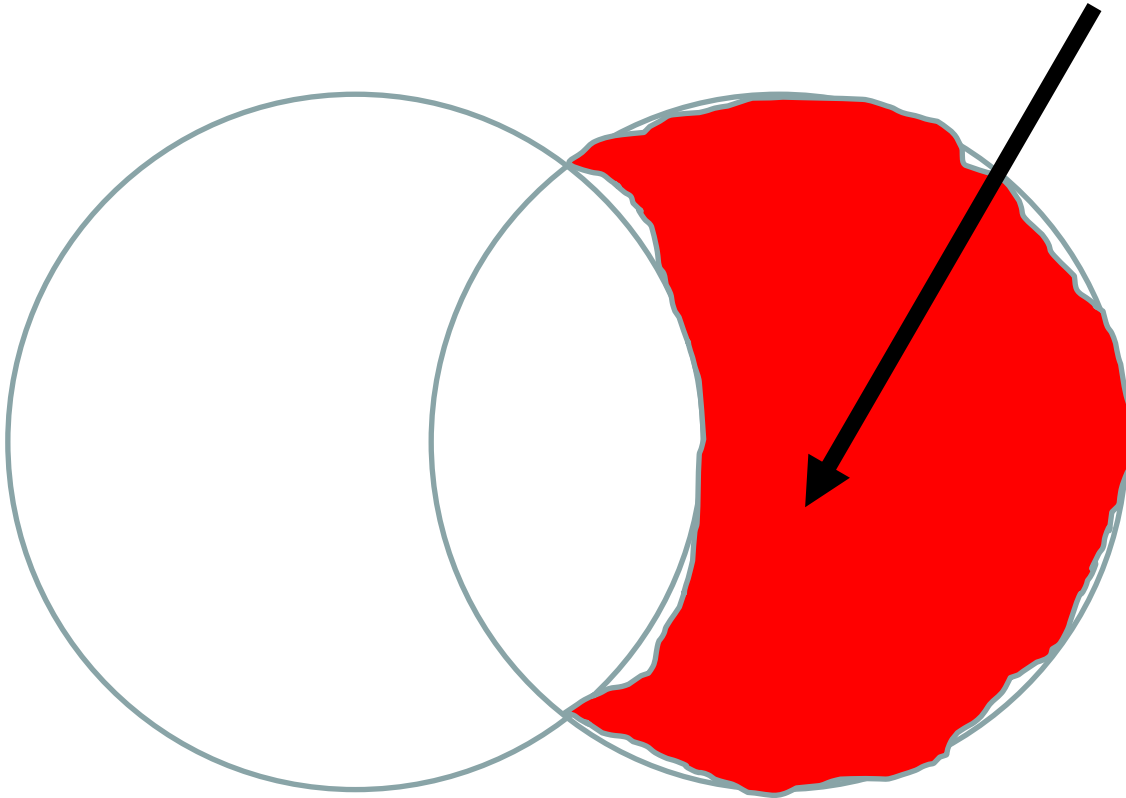


A : Students who live within
one mile of school

B : Students who walk to classes

Sets A and B: $B - A$

$B - A$: Students who walk to classes but
do not live within one mile of school



A : Students who live within
one mile of school

B : Students who walk to classes

Exercise 6

Let $A = \{a, b, c\}$ and $B = \{y, z\}$. Find

a) $A \times B$

b) $B \times A$

Answer 6 (a)

Let $A = \{a, b, c\}$ and $B = \{y, z\}$. Find
a) $A \times B$

Answer: $A \times B = \{ \langle a, y \rangle, \langle a, z \rangle, \langle b, y \rangle, \langle b, z \rangle, \langle c, y \rangle, \langle c, z \rangle \}$

	y	z
a	$\langle a, y \rangle$	$\langle a, z \rangle$
b	$\langle b, y \rangle$	$\langle b, z \rangle$
c	$\langle c, y \rangle$	$\langle c, z \rangle$

Answer 6 (b)

Let $A = \{a, b, c\}$ and $B = \{y, z\}$. Find
b) $B \times A$

Answer: $B \times A = \{ \langle y, a \rangle, \langle y, b \rangle, \langle y, c \rangle, \langle z, a \rangle, \langle z, b \rangle, \langle z, c \rangle \}$

	a	b	c
y	$\langle y, a \rangle$	$\langle y, b \rangle$	$\langle y, c \rangle$
z	$\langle z, a \rangle$	$\langle z, b \rangle$	$\langle z, c \rangle$

Exercise/Answer 7

How many different elements does $A \times B$ have if A has m elements and B has n elements?

Answer: $m \times n$

Exercise 8

List the members of the following sets

a) $\{x \mid x \text{ is a positive integer less than } 12\}$

b) $\{x \mid x \text{ is the square of an integer and } x < 100\}$

Answer 8 (a)

List the members of the following sets

a) $\{x \mid x \text{ is a positive integer less than } 12\}$

Answer: $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

Answer 8 (b)

List the members of the following sets

b) $\{x \mid x \text{ is the square of an integer and } x < 100\}$

The set consist of all squares less than 100

$$0^2 = 0 \quad 1^2 = 1 \quad 2^2 = 4 \quad 3^2 = 9 \quad 4^2 = 16$$

$$5^2 = 25 \quad 6^2 = 36 \quad 7^2 = 49 \quad 8^2 = 64 \quad 9^2 = 81$$

Answer: $\{0, 1, 4, 9, 16, 25, 36, 49, 64, 81\}$

Exercise 9

For each of the following sets, determine if 2 is an element of that set

- a) $\{x \in \mathbb{R} \mid x \text{ is an integer greater than } 1\}$
- b) $\{x \in \mathbb{R} \mid x \text{ is the square of an integer}\}$

Answer 9 (a)

For each of the following sets, determine if 2 is an element of that set

a) $\{x \in \mathbb{R} \mid x \text{ is an integer greater than } 1\}$

Answer: This set contains the element 2
(2 is an integer and $2 > 1$)

Answer 9 (b)

For each of the following sets, determine if 2 is an element of that set

b) $\{x \in \mathbb{R} \mid x \text{ is the square of an integer}\}$

Answer: This set does not contains the element 2 as 2 is not a square

Exercise 10

- Identify 2 sets whose cardinality is 1 and whose intersection's cardinality is also 1.
- Answer: any 2 identical sets with 1 element each.
- Cardinality of 1 means singleton sets with one element. If their intersection has also 1 element, the 2 sets are equal, i.e., $A = \{1\}$, and $B = \{1\}$

Exercise 11

Which of the following is not a proper subset of set A ? $A = \{4, 5, 6, x, y, z\}$

- $\{4, x, y\}$
- $\{3, 4, 5\}$
- $\{4, 5, 6\}$
- \emptyset

Answer: $\{3, 4, 5\}$ as $3 \notin A$

Exercise 12

Which is a subset of set A ?

$A = \{a, \{b\}, c, \{c\}, \{x, y\}\}$

- $\{a, \{b\}, c\}$
- $\{a, b, c\}$
- $\{c, x, y\}$
- $\{x, y\}$

Answer: $\{a, \{b\}, c\}$ is a subset as each of its elements are also in A

Exercise 13

Which is false?

- $\{1, 2\} \subseteq \{1, 2, 3\}$
- $\{1, 2\} \subseteq \{2, 3, 4\}$
- $\emptyset \subseteq \{1, 2, 3\}$

Answer: $\{1, 2\} \subseteq \{2, 3, 4\}$ as $1 \notin \{2, 3, 4\}$

Extension Exercise 14

Let Universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and its 3 sets:

$$E = \{2, 4, 6, 8, 10\},$$

$$O = \{x \mid x \text{ is an odd number}\}$$

$$\text{and } T = \{3\}.$$

- a) Enumerate the elements of set O
- b) What operation can be applied to set E and U to get set O ?
- c) Define set E through its elements' property
- d) Which of the sets above are subsets of set O ?
- e) Which of the sets above are proper subsets of set O ?
- f) Which of the sets above are proper supersets of empty set?

Answer 14 (a)

Let's us have four set:

Universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and its 3 sets:

$E = \{2, 4, 6, 8, 10\}$,

$O = \{x \mid x \text{ is an odd number}\}$

and $T = \{3\}$.

a) Enumerate the elements of set O

Answer: $O = \{1, 3, 5, 7, 9\}$

Answer 14 (b)

Let's us have four set:

Universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and its 3 sets:

$E = \{2, 4, 6, 8, 10\}$,

$O = \{x \mid x \text{ is an odd number}\}$

and $T = \{3\}$.

b) What operation can be applied to set E and U to get set O?

Answer: $\text{comp}(E) = O$

Answer 14 (c)

Let's us have four set:

Universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and its 3 sets:

$E = \{2, 4, 6, 8, 10\}$,

$O = \{x \mid x \text{ is an odd number}\}$

and $T = \{3\}$.

c) Define set E through its elements' property

Answer: $E = \{x \mid x \text{ is an even number}\}$

Without the Universe U, set E could be defined:

$E = \{x \text{ integer} \mid x \text{ is an even number and } 0 < x < 11\}$

Answer 14 (d)

Let's us have four set:

Universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and its 3 sets:

$E = \{2, 4, 6, 8, 10\}$,

$O = \{x \mid x \text{ is an odd number}\}$

and $T = \{3\}$.

d) Which of the sets above are subsets of set O ?

Answer: T is a subset of set O : $T \subseteq O$

O is subset of set O : $O \subseteq O$

Answer 14 (e)

Let's us have four set:

Universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and its 3 sets:

$E = \{2, 4, 6, 8, 10\}$,

$O = \{x \mid x \text{ is an odd number}\}$

and $T = \{3\}$.

e) Which of the sets above are proper subsets of set O ?

Answer: T is a proper subset of set O : $T \subset O$

Answer 14 (f)

Let's us have four set:

Universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and its 3 sets:

$E = \{2, 4, 6, 8, 10\}$,

$O = \{x \mid x \text{ is an odd number}\}$

and $T = \{3\}$.

f) Which of the sets above are proper supersets of empty set \emptyset ?

Answer: All non empty sets are proper supersets of \emptyset

$\emptyset \subset U$; $\emptyset \subset E$; $\emptyset \subset O$; $\emptyset \subset T$