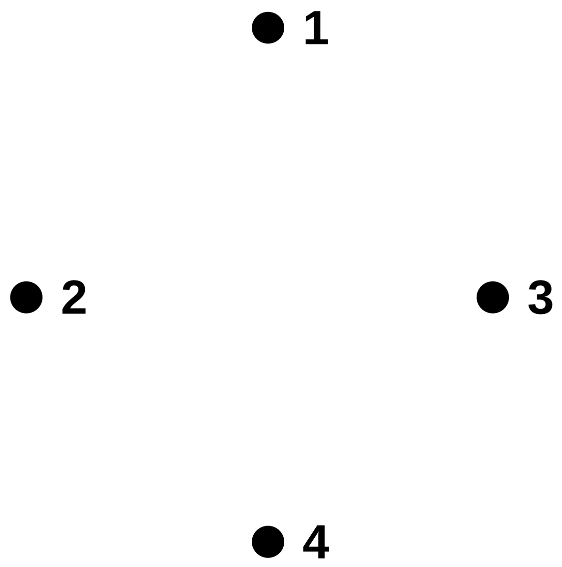
Workshop 2: Relations Sample Solutions

SCC120 Fundamentals of Computer Science

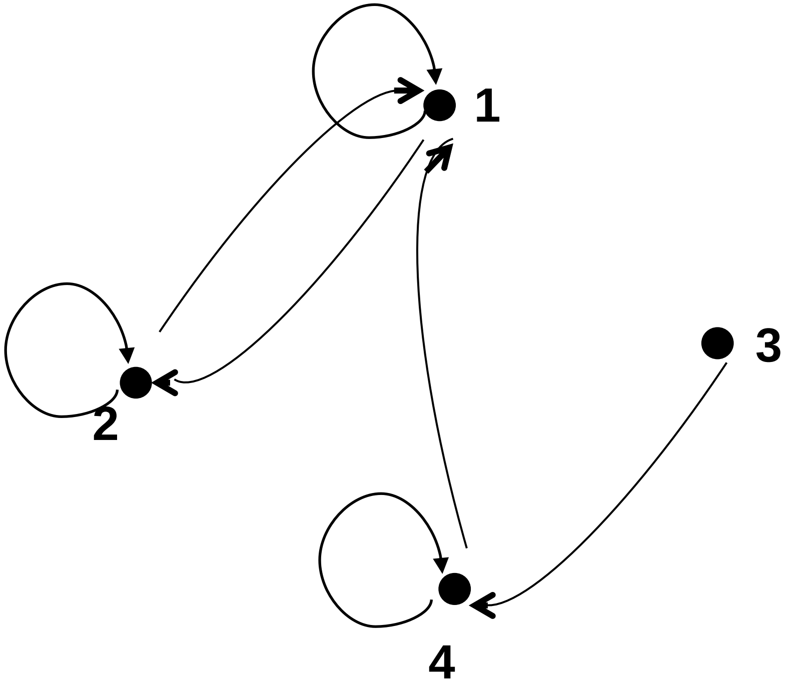
Solutions by Corina Sas

Exercise 1

Draw diagraphs of the 4 relations, using this as your starting point:

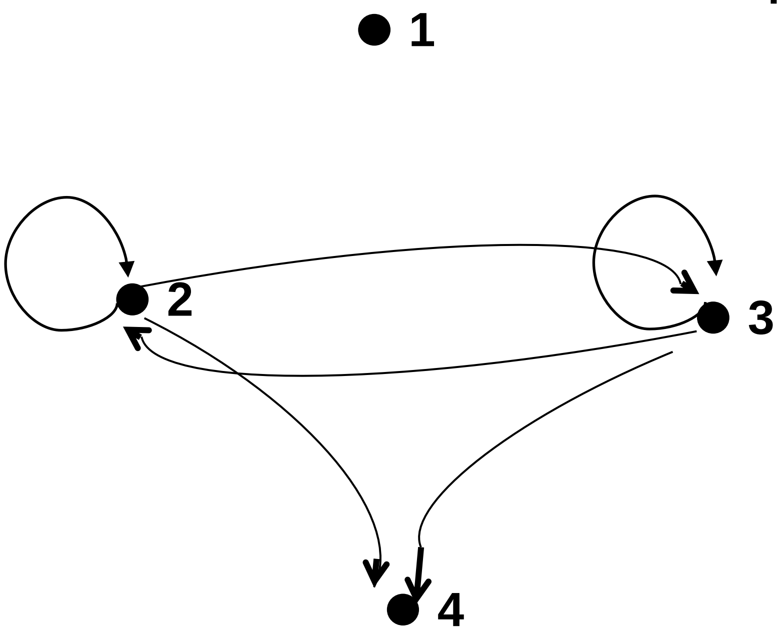


Answer 1: R1



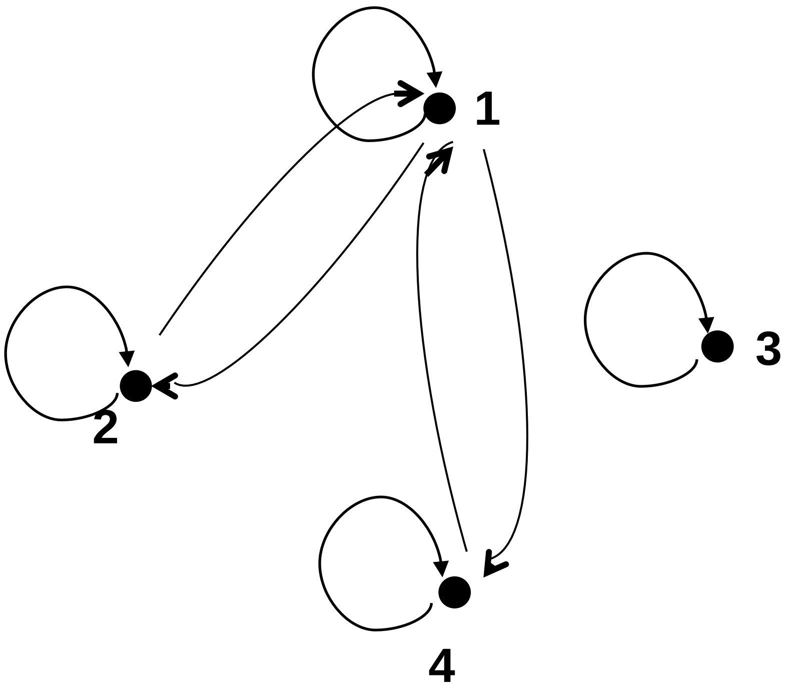
R1 = {<1, 1>, <1, 2>, <2, 1>,
<2, 2>, <3, 4>, <4,1>,
<4,4>}

Answer 1: R2



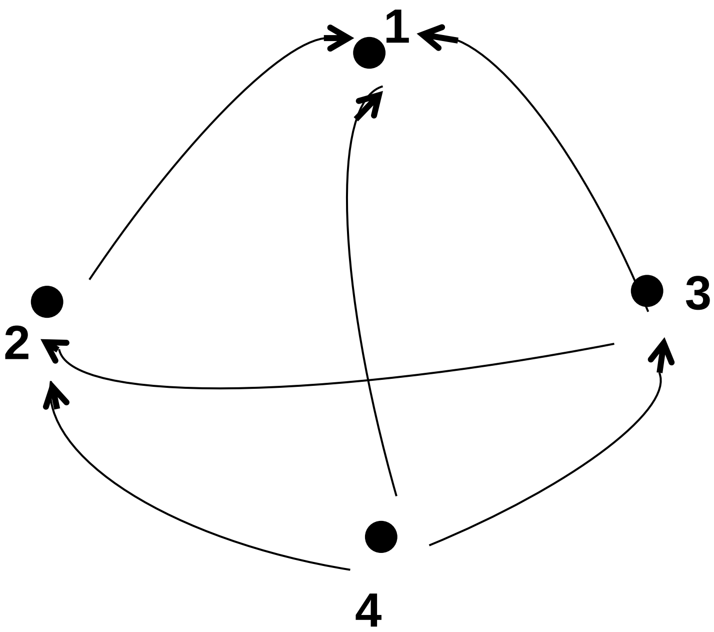
R2 = {<2, 2>, <2, 3>, <2, 4>,
<3, 2>, <3, 3>, <3, 4>}

Answer 1: R3



R3 = {<1, 1>, <1, 2>, <1, 4>,
<2, 1>, <2, 2>, <3, 3>,
<4,1 >, <4, 4>}

Answer 1: R4



R4 = {<2, 1>, <3, 1>,
<3, 2>, <4,1>,
<4, 2>, <4, 3 >}

Exercise 2

Consider the following relations on { 1, 2, 3, 4 }:

- R1 = {<1, 1>, <1, 2>, <2, 1>, <2, 2>, <3, 4>, <4,1>, <4,4>}
- R2 = {<2, 2>, <2, 3>, <2, 4>, <3, 2>, <3, 3>, <3, 4>}
- R3 = {<1, 1>, <1, 2>, <1, 4>, <2, 1>, <2, 2>, <3, 3>, <4,1 >, <4, 4>}
- R4 = {<2, 1>, <3, 1>, <3, 2>, <4,1>, <4, 2>, <4, 3 >}

Which of these relations are reflexive?

Answer 2: R3 reflexive

R3 since it contains all pairs of the form <a, a>, namely: <1, 1>, <2, 2>, <3, 3 >, <4, 4>.

- R3 = { <1, 1> , <1, 2>, <1, 4>, <2, 1>,
  <2, 2>, <3, 3> , <4,1 >, <4, 4> }

Answer 2: R1, R2, R4

Relations R1, R2 and R4 are not reflexive. Hint: find a pair <a, a> which is not in these relations.

- R1 does not contain <3, 3>
- R1 = { <1, 1> , <1, 2>, <2, 1>, <2, 2>, <3,4>, <4,1>, <4, 4> }

- R2 does not contain <1, 1> or <4, 4>
- R2 = { <2, 2>, <2, 3>, <2, 4>, <3, 2>, <3, 3>, <3, 4>}

- R4 does not contain <1,1>, <2, 2>, <3, 3> or <4, 4>
- R4 = {<2, 1>, <3, 1>, <3, 2>, <4,1>, <4, 2>, <4, 3 >}

Exercise 3

Consider the following relations on {1, 2, 3, 4}:

- R1 = {<1, 1>, <1, 2>, <2, 1>, <2, 2>, <3, 4>, <4,1>, <4,4>}
- R2 = {<2, 2>, <2, 3>, <2, 4>, <3, 2>, <3, 3>, <3, 4>}
- R3 = {<1, 1>, <1, 2>, <1, 4>, <2, 1>, <2, 2>, <3, 3>, <4,1 >, <4, 4>}
- R4 = {<2, 1>, <3, 1>, <3, 2>, <4,1>, <4, 2>, <4, 3 >}

Which of these relations are symmetric?

Answer 3: R3 symmetric

- R3, because in each case <b, a> belongs to the relation whenever <a, b> does.

- R3 = { <1, 1>, <1, 2>, <1, 4>, <2, 1>, <2, 2>, <3, 3>, <4,1 >, <4, 4> }

Answer 3: R3 symmetric

- R3 = { <1, 1>, <1, 2>, <1, 4>, <2, 1>, <2, 2>,
  <3, 3>, <4,1 >, <4, 4> }

	<a, b>	<b, a>	
a	<1,1>	<1,1>	(a)
b	<1,2>	<2,1>	(d)
c	<1,4>	<4,1>	(g)
d	<2,1>	<1,2>	(b)
e	<2,2>	<2,2>	(e)
f	<3,3>	<3,3>	(f)
g	<4,1>	<1,4>	(c)
h	<4,4>	<4,4>	(h)

Answer 3: R1, R2, R4

- The rest of relations are not symmetric: find a pair <a, b> so that it is in the relation but <b, a> is not.

- R1 = {<1, 1>, <1, 2>, <2, 1>, <2, 2>, <3, 4>, <4,1>, <4,4>}
- R2 = {<2, 2>, <2, 3>, <2, 4>, <3, 2>, <3, 3>, <3, 4>}
- R4 = { <2, 1>, <3, 1>, <3, 2>, <4,1>, <4, 2>, <4, 3 >}

Exercise 4

Consider the following relations on { 1, 2, 3, 4}:

- R1 = {<1, 1>, <1, 2>, <2, 1>, <2, 2>, <3, 4>, <4,1>, <4,4>}
- R2 = {<2, 2>, <2, 3>, <2, 4>, <3, 2>, <3, 3>, <3, 4>}
- R3 = {<1, 1>, <1, 2>, <1, 4>, <2, 1>, <2, 2>, <3, 3>, <4,1 >, <4, 4>}
- R4 = {<2, 1>, <3, 1>, <3, 2>, <4,1>, <4, 2>, <4, 3 >}
- Which of these relations are transitive?

Answer 4: R2 transitive

- R2 since if <a, b> and <b, c> is in relation, then <a, c> is.
- R2 : <2,3> and <3, 2> then <2,2>
- R2 = {<2, 2>, <2, 3>, <2, 4>, <3, 2>, <3, 3>, <3, 4>}

Answer 4: R2 transitive

R2 = {<2, 2>, <2, 3>, <2, 4>, <3, 2>, <3, 3>, <3, 4>}

Match?

a	<2,2>	<2,2>	∴	<2,2> (a)
	<2,2>	<2,3>	∴	<2,3> (b)
	<2,2>	<2,4>	∴	<2,4> (c)
b	<2,3>	<3,2>	∴	<2,2> (a)
	<2,3>	<3,3>	∴	<2,3> (b)
	<2,3>	<3,4>	∴	<2,4> (c)
c	<2,4>	no		
d	<3,2>	<2,2>	∴	<3,2> (d)
	<3,2>	<2,3>	∴	<3,3> (e)
	<3,2>	<2,4>	∴	<3,4> (f)
e	<3,3>	<3,3>	∴	<3,3> (e)
	<3,3>	<3,4>	∴	<3,4> (f)
f	<3,4>	no		

Answer 4: R4 transitive

- R4 since if <a, b> and <b, c> is in relation, then <a, c> is.
- R4 : <4,2> and <2, 1> then <4,1>
- R4 = { <2, 1>, <3, 1>, <3, 2>, <4,1>, <4, 2>, <4, 3 >}

Answer 4: R4 transitive

R4 = { <2, 1>, <3, 1>, <3, 2>, <4,1>, <4, 2>, <4, 3 >}

Match?

a	<2,1>	no		
b	<3,1>	no		
c	<3,2>	<2,1>	∴	<3,1> (b)
d	<4,1>	no		
e	<4,2>	<2,1>	∴	<4,1> (d)
f	<4,3>	<3,1>	∴	<4,1> (d)
	<4,3>	<3,2>	∴	<4,2> (e)

Answer 4: R1, R3

Why are the other relations R1 and R3 not transitive?

R1 = {<1, 1>, <1, 2>, <2, 1>, <2, 2>, <3, 4>, <4,1>, <4,4>}

- R1: <3, 4> and <4, 1> belong to it, while <3, 1> does not
- R3: <4, 1> and <1, 2> belong to it, while <4, 2> does not

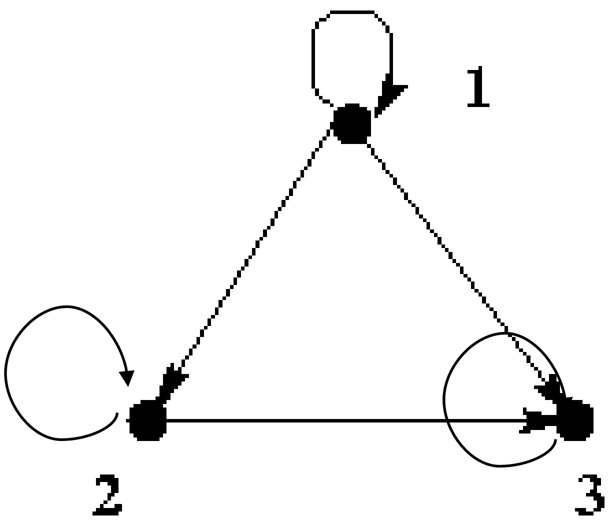
Answer 4: R1 not transitive

a	<1,1>	<1,2>	∴	<1,2> (b)
b	<1,2>	<2,1>	∴	<1,1> (a)
		<2,2>	∴	<1,2> (b)
c	<2,1>	<1,1>	∴	<2,1> (c)
		<1,2>	∴	<2,2> (d)
d	<2,2>	<2,1>	∴	<2,1> (c)
e	<3,4>	<4,1>	∴	<3,1> NO!
f	<4,1>	<1,1>	∴	<4,1> (f)
		<1,2>	∴	<4,2> NO!
g	<4,4>	<4,1>	∴	<4,1> (f)

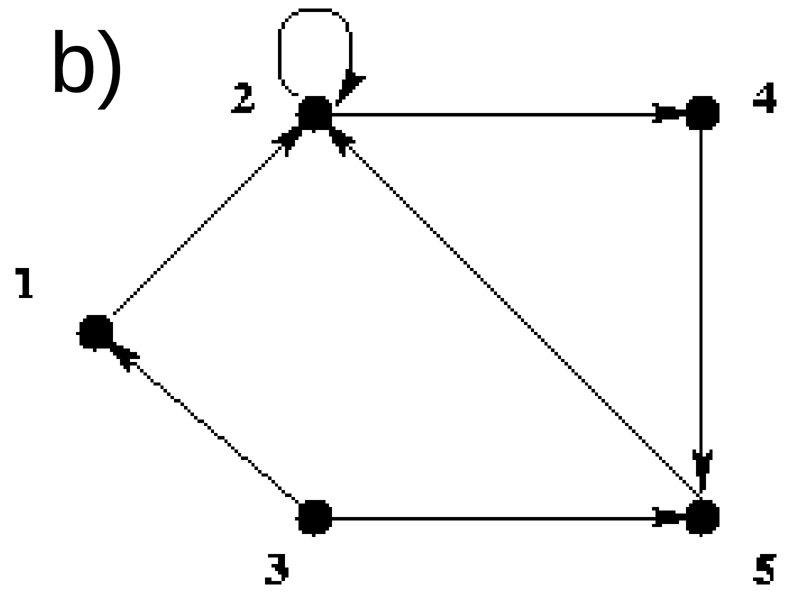
Exercise 5

- Identify the relation depicted by the following diagraphs:

a)

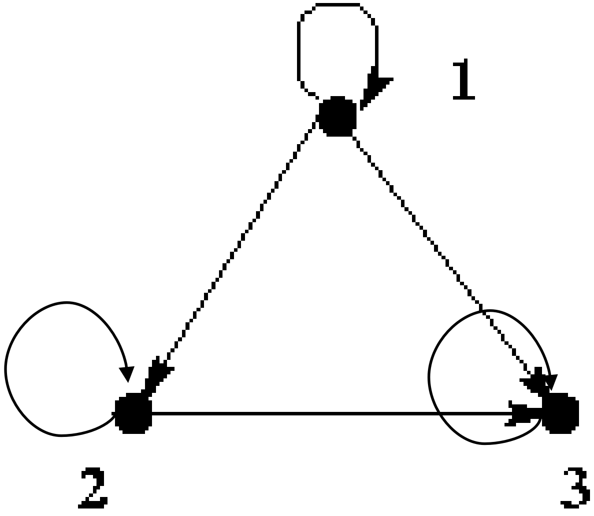


b)



Answer 5(a)

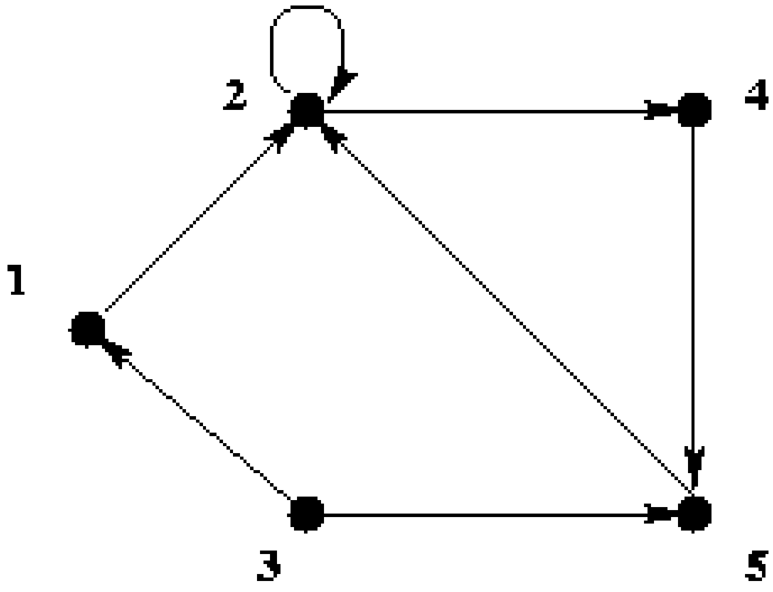
a)



R = {<1, 1>,<1,2>, <1,3>, <2, 2>, <2, 3>, <3, 3>}

Answer 5(b)

b)



R = {<1, 2>, <2, 2>, <2, 4>, <3, 1>, <3, 5>, <4, 5>, <5, 2>}

Exercise 6

Let R1 and R2 be two binary relations

- R1 = {<Ann, 22>, <Mary, 23>, <Laura, 20>}
- R2 = {<John, 19>, <Mike, 24>, <James, 21>, <Mary, 23>, <Laura, 20>}

What is the a) union, b) intersection and c) difference of the two relations?

Answer 6(a)

R1 = {<Ann, 22>, <Mary, 23>, <Laura, 20>}

R2 = {<John, 19>, <Mike, 24>, <James, 21>, <Mary, 23>, <Laura, 20>}

R1 ∪ R2 = {<Ann, 22>, <Mary, 23>, <Laura, 20>, <John, 19>, <Mike, 24>, <James, 21>}

Answer 6(b)

R1 = {<Ann, 22>, <Mary, 23>, <Laura, 20>}

R2 = {<John, 19>, <Mike, 24>, <James, 21>, <Mary, 23>, <Laura, 20>}

R1 ∩ R2 = {<Mary, 23>, <Laura, 20>}

Answer 6(c)

R1 = {<Ann, 22>, <Mary, 23>, <Laura, 20>}

R2 = {<John, 19>, <Mike, 24>, <James, 21>, <Mary, 23>, <Laura, 20>}

R1 - R2 = {<Ann, 22>}

Exercise 7

For each of the following relations defined on the positive integers:

$$>, <, =, \geq, \leq$$

justify whether the relation is:

- reflexive
- symmetric
- transitive

Answer 7

- Hint: build the 5 sets required where $R \subseteq A \times A$ and $A = \{1, 2, 3, 4, 5\}$.
- E for equal, L for less than, G for greater than, LE for less than or equal, GE for greater than or equal.
- Then test each set for the 3 qualities.

Answer 7 (A x A)

- A = { 1, 2, 3, 4, 5}
- A x A =

	1	2	3	4	5
1	<1, 1>	<1, 2>	<1, 3>	<1, 4>	<1, 5>
2	<2, 1>	<2, 2>	<2, 3>	<2, 4>	<2, 5>
3	<3, 1>	<3, 2>	<3, 3>	<3, 4>	<3, 5>
4	<4, 1>	<4, 2>	<4, 3>	<4, 4>	<4, 5>
5	<5, 1>	<5, 2>	<5, 3>	<5, 4>	<5, 5>

Answer 7 (=)

- E = { <1,1> , <2,2> , <3,3> , <4,4>, <5,5> }

E	1	2	3	4	5
1	**<1, 1>**	<1, 2>	<1, 3>	<1, 4>	<1, 5>
2	<2, 1>	**<2, 2>**	<2, 3>	<2, 4>	<2, 5>
3	<3, 1>	<3, 2>	**<3, 3>**	<3, 4>	<3, 5>
4	<4, 1>	<4, 2>	<4, 3>	**<4, 4>**	<4, 5>
5	<5, 1>	<5, 2>	<5, 3>	<5, 4>	**<5, 5>**

Answer 7 (<)

- L = { <1,2>, <1,3>, <1,4>, <1,5>, <2,3>, <2,4>, <2,5>, <3,4>, <3,5>,<4,5> }

L	1	2	3	4	5
1	<1, 1>	**<1, 2>**	**<1, 3>**	**<1, 4>**	**<1, 5>**
2	<2, 1>	<2, 2>	**<2, 3>**	**<2, 4>**	**<2, 5>**
3	<3, 1>	<3, 2>	<3, 3>	**<3, 4>**	**<3, 5>**
4	<4, 1>	<4, 2>	<4, 3>	<4, 4>	**<4, 5>**
5	<5, 1>	<5, 2>	<5, 3>	<5, 4>	<5, 5>

Answer 7 (>)

- G = {<2,1>, <3,1>, <3,2>, <4,1>, <4,2>, <4,3>,<5,1>, <5,2>, <5,3>, <5,4>}

G	1	2	3	4	5
1	<1, 1>	<1, 2>	<1, 3>	<1, 4>	<1, 5>
2	**<2, 1>**	<2, 2>	<2, 3>	<2, 4>	<2, 5>
3	**<3, 1>**	**<3, 2>**	<3, 3>	<3, 4>	<3, 5>
4	**<4, 1>**	**<4, 2>**	**<4, 3>**	<4, 4>	<4, 5>
5	**<5, 1>**	**<5, 2>**	**<5, 3>**	**<5, 4>**	<5, 5>

Answer 7

- LE = { <1,1>, <1,2>, <1,3>, <1,4>, <1,5>, <2,2>,<2,3>, <2,4>, <2,5>, <3,3>, <3,4>, <3,5>,<4,4,>,<4,5>, <5,5>}
- GE = {<1,1>, <2,1>, <2,2>, <3,1>, <3,2>, <3,3>, <4,1>, <4,2>,<4,3>, <4,4>, <5,1>, <5,2>, <5,3>, <5,4>, <5,5>}

Answer 7: Reflexive

R $\subseteq$ ***A x A*** is **reflexive** if and only if

< a, a > $\in$ ***R*** for every element ***a*** of ***A***

- every element of A is in relation with itself

- So for A = { 1, 2, 3, 4, 5} R *must* contain <1,1>, <2,2>, <3,3>, <4,4> and <5, 5>.

Answer 7: Reflexive?

E = { **<1,1> , <2,2> , <3,3> , <4,4>, <5,5>** }	yes
L = { <1,2>, <1,3>, <1,4>, <1,5>, <2,3>, <2,4>, <2,5>, <3,4>, <3,5>,<4,5> }	no
G = {<2,1>, <3,1>, <3,2>, <4,1>, <4,2>, <4,3>, <5,1>, <5,2>, <5,3>, <5,4>}	no
LE = { **<1,1>,** <1,2>, <1,3>,<1,4>, <1,5>, **<2,2>,** <2,3>,<2,4>, <2,5>, **<3,3>,** <3,4>, <3,5>, **<4,4>,** <4,5>, **<5,5>** }	yes
GE = { **<1,1>,** <2,1>, **<2,2>,** <3,1>, <3,2>, **<3,3>,** <4,1>, <4,2>,<4,3>, **<4,4>,** <5,1>, <5,2>, <5,3>, <5,4>, **<5,5>** }	yes

Answer 7: Symmetric

R $\subseteq$ ***A x A is* symmetric** if and only if for any ***a***, and ***b*** in ***A***, whenever **<*a, b*>** $\in$ ***R*** then **<*b, a*>** $\in$ ***R*****.**

Answer 7: Symmetric?

E = { **<1,1> , <2,2> , <3,3> , <4,4>, <5,5>** } yes

L = { <1,2>, <1,3>, <1,4>, <1,5>, <2,3>, <2,4>, <2,5>, <3,4>, <3,5>, <4,5> } i.e. <5,4> absent no

G = {<2,1>, <3,1>, <3,2>, <4,1>, <4,2>,<4,3>, <5,1>, <5,2>, <5,3>, <5,4> } i.e. <4,5> absent no

LE = { <1,1>, <1,2>, <1,3>,<1,4>, <1,5>, <2,2>, <2,3>,<2,4>, <2,5>, <3,3>, <3,4>, <3,5>, <4,4>, <4,5>, <5,5> } i.e. <5,4> absent no

GE = { <1,1>, <2,1>, <2,2>, <3,1>, <3,2>, <3,3>, <4,1>, <4,2>,<4,3>, <4,4>, <5,1>, <5,2>, <5,3>, <5,4>, <5,5> } i.e. <4,5> absent no

Answer 7: Transitive

- $R \subseteq A \times A$ is transitive if and only if
  for any a, b, and c $\in$ A,
  if $<a, b> \in R$, and $<b, c> \in R$
  then $<a, c> \in R$
- ‘=‘ is transitive, for if a = b and b = c then a = c. (1 = 1 and 1 = 1 then 1 = 1)
- ‘>’ is transitive, for if a > b and b > c then a > c. (5 > 4 and 4 > 3 then 5 > 3)
- ‘<‘ is transitive, for if a < b and b < c then a < c. (3 < 4 and 4 < 5 then 3 < 5)
- ‘>=‘ and ‘<=‘ are also transitive.

Answer 7

- All the sets are transitive.
- E is the only symmetric set.
- E, LE and GE are reflexive.

Exercise 8(a)

Consider the following relations on {1, 2, 3, 4 }:

- R1 = {<2, 2>, <2, 3>, <2, 4>, <3, 2>, <3, 3>, <3, 4>}
- R2 = {<1, 1>, <1, 2>, <2, 1>, <2, 2>, <3, 3>, <4, 4>}
- R3 = {<2, 4>, <4, 2>}

Which of these relations are reflexive?

Answer 8(a): R2 reflexive

R2 since it contains all pairs of the form <a, a>, namely: <1, 1>, <2, 2>, <3, 3 >, <4, 4>.

- R2 = { <1, 1>, <1, 2>, <2, 1>, <2, 2>, <3, 3>, <4, 4>}

Answer 8(a): R1, R3

Relations R1 and R3 are not reflexive. Hint: find a pair <a, a> which is not in these relations.

- R1 does not contain <1, 1> and <4, 4>
- R1 = {<2, 2>, <2, 3>, <2, 4>, <3, 2>, <3, 3>, <3, 4>}

- R3 does not contain <1, 1>, <2, 2>, <3, 3> and <4, 4>
- R3 = {<2, 4>, <4, 2>}

Exercise 8(b)

Consider the following relations on {1, 2, 3, 4 }:

- R1 = {<2, 2>, <2, 3>, <2, 4>, <3, 2>, <3, 3>, <3, 4>}
- R2 = {<1, 1>, <1, 2>, <2, 1>, <2, 2>, <3, 3>, <4, 4>}
- R3 = {<2, 4>, <4, 2>}

Which of these relations are symmetric?

Answer 8(b): R2 symmetric

Relation R2 is symmetric because in each case <b, a> belongs to the relation whenever <a, b> does.

R2 = {<1, 1>, <1, 2>, <2, 1>, <2, 2>, <3, 3>, <4, 4>}

Also for each pair of the form <a, a> which is in R, it also means that <a, a> is in R.

So R2 symmetric.

Answer 8(b): R3 symmetric

- Relation R3 is symmetric because in each case <b, a> belongs to the relation whenever <a, b> does.
- R3 = { <2, 4>, <4, 2> }

Answer 8(b): R1

- R1 is not symmetric: find a pair <a, b> so that it is in the relation but <b, a> is not.

- R1 = {<2, 2>, <2, 3>, <2, 4>, <3, 2>, <3, 3>, <3, 4>}
- For instance, the ordered pair <2, 4> is in R1, but <4, 2> is not.

Exercise 8(c)

Consider the following relations on {1, 2, 3, 4 }:

- R1 = {<2, 2>, <2, 3>, <2, 4>, <3, 2>, <3, 3>, <3, 4>}
- R2 = {<1, 1>, <1, 2>, <2, 1>, <2, 2>, <3, 3>, <4, 4>}
- R3 = {<2, 4>, <4, 2>}

- Which of these relations are transitive?

Answer 8(c): R1 transitive

- Relation R1 is transitive since if <a, b> and <b, c> is in relation, then <a, c> is.
- R1 = {<2, 2>, <2, 3>, <2, 4>, <3, 2>,
  <3, 3>, <3, 4>}

Answer 8(c): R1 transitive

R1 = {<2, 2>, <2, 3>, <2, 4>, <3, 2>, <3, 3>, <3, 4>}

		Match?		
a	<2,2>	<2,2>	∴	<2,2> (a)
	<2,2>	<2,3>	∴	<2,3> (b)
	<2,2>	<2,4>	∴	<2,4> (c)
b	<2,3>	<3,2>	∴	<2,2> (a)
	<2,3>	<3,3>	∴	<2,3> (b)
	<2,3>	<3,4>	∴	<2,4> (c)
c	<2,4>	no		
d	<3,2>	<2,2>	∴	<3,2> (d)
	<3,2>	<2,3>	∴	<3,3> (e)
	<3,2>	<2,4>	∴	<3,4> (f)
e	<3,3>	<3,3>	∴	<3,3> (e)
	<3,3>	<3,4>	∴	<3,4> (f)
f	<3,4>	no		

Answer 8(c): R2 transitive

- Relation R2 is transitive since if <a, b> and <b, c> is in relation, then <a, c> is.
- R2 = {<1, 1>, <1, 2>, <2, 1>, <2, 2>, <3, 3>, <4, 4>}

Answer 8(c): R2 transitive

R2 = {<1, 1>, <1, 2>, <2, 1>, <2, 2>, <3, 3>, <4, 4>}

		Match?		
a	<1, 1>	<1, 1>	∴	<1, 1> (a)
	<1, 1>	<1, 2>	∴	<1, 2> (b)
b	<1, 2>	<2, 1>	∴	<1, 1> (a)
	<1, 2>	<2, 2>	∴	<1, 2> (b)
c	<2,1>	<1, 1>	∴	<2, 1> (c)
	<2,1>	<1, 2>	∴	<2, 2> (d)
d	<2, 2>	<2, 1>	∴	<2, 1> (c)
	<2, 2>	<2, 2>	∴	<2, 2> (d)
e	<3, 3>	<3, 3>	∴	<3, 3> (e)
f	<4, 4>	<4, 4>	∴	<4, 4> (f)

Answer 8(c): R3

Why is relation R3 not transitive?

- R3 = {<2, 4>, <4, 2>}

<2, 4> and <4, 2> belong to R3, but <2, 2> does not belong to R3.

Another example: <4, 2> and <2, 4> belong to R3, but <4, 4> does not.

Exercise 9

Let A= {a, b, c, d} and R is the relation

R = {<a, a>}.

Is this relation:

a) reflexive

b) symmetric

c) irreflexive

d) transitive?

Answer 9(a)

- A= {a, b, c, d}

- Relation R = {<a, a>} is not reflexive as <b, b>, <c, c> and <d, d>

  do not belong to R.

Answer 9(b)

- A= {a, b, c, d}

- Relation R = {<a, a>} is symmetric as for <a, a> in R, <a, a> also belongs to R.

Answer 9(c)

- A= {a, b, c, d}

- Relation R = {<a, a>} is not irreflexive as <a, a> belongs to R.

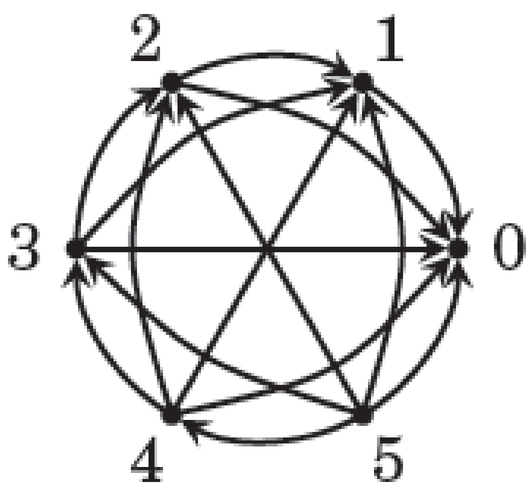
Answer 9(d)

- A= {a, b, c, d}

- Relation R = {<a, a>} is transitive as for <a, a> in R, and <a, a> in R, then <a, a> also belongs to R.

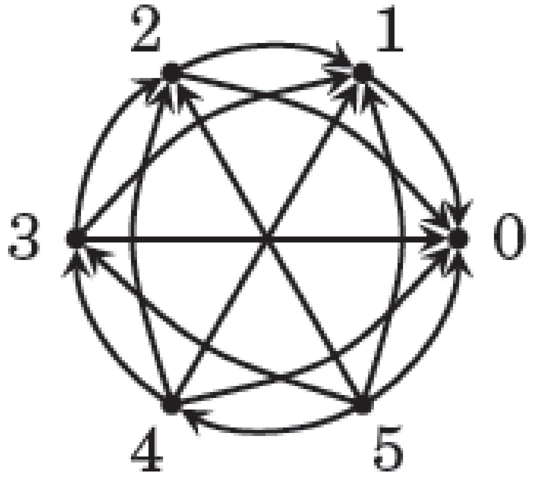
Exercise 10

- Specify the sets A and R, where A is the set on which the relation R is represented in this diagraph:



Answer 10

- A = {0, 1, 2, 3, 4, 5}
- R = {<5, 4>, <5, 3>, <5, 2>, <5, 1>, <5, 0>, <4, 3>, <4, 2>, <4,1>, <4, 0>, <3, 2>, <3, 1>, <3, 0>, <2, 1>, <2, 0>, <1, 0>}



Exercise 11

Let A = {a, b, c, d} and

R = { <a, a>, <b, b>, <c, c>, <d, d>}.

Is relation R

a) reflexive

b) symmetric

c) transitive?

If not, specify why.

Answer 11(a)

Let A = {a, b, c, d} and

R = { <a, a>, <b, b>, <c, c>, <d, d>}.

Relation R is reflexive because for each element x in A, <x, x> is also in R:

<a, a>, <b, b>, <c, c>, <d, d> are all in R

Answer 11(b)

Let A = {a, b, c, d} and

R = { <a, a>, <b, b>, <c, c>, <d, d>}.

Relation R is symmetric because for each <x, y> in R, <y, x> is also in R:

<a, a> in R, <a, a> also in R

<b, b> in R, <b, b> also in R

<c, c> in R, <c, c> also in R

<d, d> in R, <d, d> also in R

Answer 11(c)

Let A = {a, b, c, d} and

R = { <a, a>, <b, b>, <c, c>, <d, d>}.

Relation R is transitive because

- Relation R is transitive because if <a, a> in R, and <a, a> in R, then <a, a> also belongs to R.
- The same argument applies to the other 3 ordered pairs.

Exercise 12

Let R be the relation on the set of real numbers, described by $x = 2y + 1$.

Is relation R:

a) reflexive

b) symmetric

c) transitive?

Answer 12(a)

Let R be the relation on the set of real numbers, described by x = 2y + 1.

R = {<x, y> | x = 2y + 1}

R consist of order pairs like: <2y + 1, y>

R is not reflexive as for instance, <1, 1> is not in R

If x = 1, y = (x -1) / 2 = (1 – 1) / 2 = 0, so <**1**, 0> is in R

But <1, 1> is not in R, as y = (1 -1) /2 = 0 ≠ 1

Answer 12(b)

Let R be the relation on the set of real numbers, described by $x = 2y + 1$.

$R = \{<x, y> \mid x = 2y + 1\}$

$R = \{<2y + 1, y>\}$

R is not symmetric as for instance,

<3, 1> is in R, but <1, 3> is not in R

If $x = 3$, $y = (x - 1) / 2 = (3 - 1) / 2 = 1$, so <3, 1> is in R

If $x = 1$, $y = (x - 1) / 2 = (1 – 1) / 2 = 0$, so <1, 0> is in R

So <1, 3> is not in R, as $y = (1 - 1) / 2 = 0 \neq 3$

Answer 12(c)

Let R be the relation on the set of real numbers, described by $x = 2y + 1$.

$R = \{<x, y> \mid x = 2y + 1\}$

$R = \{<2y + 1, y>\}$

R is not transitive as for instance, <3, 1> and <1, 0> are in R, but <3, 0> is not in R

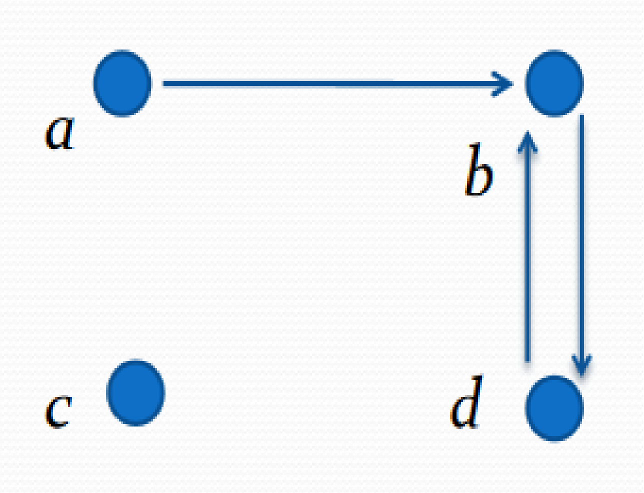
If $x = 3$, $y = (x - 1) / 2 = (3 - 1) / 2 = 1$, so <3, 1> is in R

If $x = 1$, $y = (x - 1) / 2 = (1 - 1) / 2 = 0$, so <1, 0> is in R

But <3, 0> is not in R, as for $x = 3$, $y = (3 - 1) / 2 = 1 \neq 0$

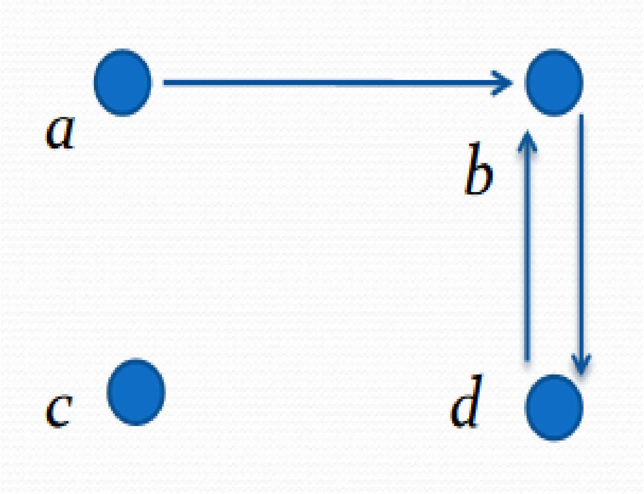
Exercise 13

Which properties the relation in the following diagraph has?



Answer 13

- *R* is not reflexive as the diagraph does not have a self-loop for each of its points
- *R* is not symmetric; for instance, there is an arc from *a* to *b*, but not from *b* to *a*
- R is not transitive*;* there is an arc from *a* to *b,* and another one from *b* to *d*, but there is no arc from *a* to *d*



Exercise 14

Let R = {<a, b>) | a = b + 1} on the set of real numbers.

Which properties this relation has?

a) reflexive

b) symmetric

c) transitive

Answer 14(a)

Let R = {<a, b>) | a = b + 1} on the set of real numbers.

Relation R is not reflexive.

For example: <3, 3> is not in R.

If <3, 3> would be in R, it would imply that a = 3, and b = 3; or 3 = 3 + 1, or 3 = 4 which is not true, so <3, 3> is not in R

Answer 14(b)

Let R = {<a, b>) | a = b + 1} on the set of real numbers.

Relation R is not symmetric.

For example: <4, 3> is in R, but <3, 4> is not

For b = 3, a = 3 + 1 = 4, so <4, 3> is in R

For a = 3, b = 2, and b ≠ 4 , so <3, 2 > is in R, but <3, 4> is not in R.

Answer 14(c)

Let R = {<a, b>) | a = b + 1} on the set of real numbers.

Relation R is not transitive.

For example: <4, 3> and <3, 2> are in R, but <4, 2> is not.

For b = 3, a = 3 + 1 = 4, so <4, 3> is in R

For b = 2, a = 2 + 1 = 3, so <3, 2> is in R

Regarding <4, 2>:

for a = 4, then b = 3, so b ≠ 2. Thus, <4, 2> is not in R