



Universidade Federal de Pernambuco

Centro de Informatica

Bacharelado em Engenharia da Computação

# **Space-efficient representations for de Bruijn graph for data streams**

Augusto Sales de Queiroz

Trabalho de Graduação

Recife

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# **Space-efficient representations for de Bruijn graph for data streams**

*Trabalho apresentado ao Programa de Bacharelado em Engenharia da Computação do Centro de Informatica da Universidade Federal de Pernambuco como requisito parcial para obtenção do grau de Bacharel em Engenharia da Computação.*

Orientador: *Paulo Gustavo Soares da Fonseca*  
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## CHAPTER 1

# Introduction

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# State of the Art

## 2.1 Outline

- Short-read sequencing
  - Sequencing errors
- Assembler Architecture
  - de Bruijn graph
  - Pre-processing of reads
    - \*  $k$ -mer counting & filtering
      - khmer[6]
- Navigational Data Structures (NDS)
  - [1]
- *FastEtch* [4]
  - Online counting → Filtering → Construction of de Bruijn graph



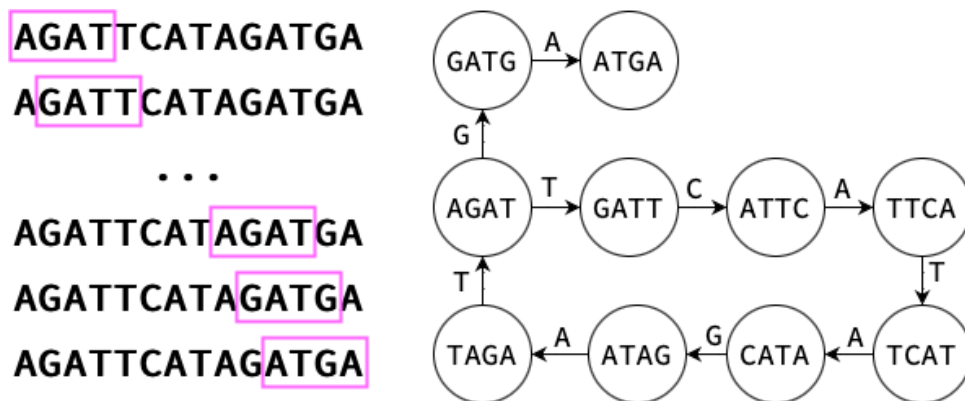


# Fundamentação teórica e metodologia

## 3.1 de Bruijn graph

The de Bruijn graph is a directed graph for which each node represents a sequence of symbols, and each edge between two nodes represents the overlap between the two sequences. That is, given two nodes on the graph, they each represent a distinct sequence of symbols  $S_1$  and  $S_2$ , and there is an edge between them if and only if the tail of  $S_1$  is the head of  $S_2$ .

Within the context of genome sequencing, de Bruijn graphs are used in the assembly process by storing the distinct  $k$ -mers identified in the read sequences. In the ideal case (when each  $k$ -mer is present only once in the original sequence, and there are no sequencing errors), the complete traversal of this graph would produce the original sequence. In practice, such a straightforward approach is not feasible, but the de Bruijn graph can still be used to produce longer sequences, called *contigs*, which can then be processed to assemble the original genome. Figure 3.1 presents an example of the de Bruijn graph within this context.



**Figure 3.1** Example of a de Bruijn graph.  $k = 4$

### 3.1.1 Representing a de Bruijn graph

→ asq says: Não é mais interessante manter a discussão sobre lower boundaries no capítulo de estado da arte? ←

Due to its nature, a de Bruijn graph can be represented by its set of nodes or edges independently, as one can be derived from the other. As such, a structure that can answer queries about the presence of a given node on the graph is enough to represent the graph<sup>1</sup>. In this regard, Conway & Bromage showed that a lower bound on the number of bits required to *exactly* represent a de Bruijn graph exists, and is  $\Omega(n \log n)$ , for  $n$  being the number of distinct  $k$ -mers present in the graph, and  $4^k > n$  [2].

In order to further improve space-efficiency, new forms of representation were created that trade exactness for a probabilistic approach, such as *Navigational Data Structures* (NDS), which have some probability of giving an erroneous answer to a membership query, but can be used to navigate the graph. This definition is useful due to the fact that a de Bruijn graph is not queried for the membership of randomly selected nodes, but rather only the neighborhood of a known member node is queried [1]. In the same paper where they introduce the idea of NDS, Chikhi *et al.* also present a lower bound for the number of bits needed to represent such a structure as  $3.24n$ . In sections 3.3 and 3.4 we will introduce two new NDS's. → asq says: É necessário reiterar os objetivos das duas estruturas aqui, visto que isso já seria feito na introdução e é feito nas próprias sessões dedicadas a cada estrutura? ←

### 3.1.2 Reverse Complements

One individuality of the genome sequencing context is the presence of *reverse complements*. When generating sequencing reads, a sequence of DNA can be read both in its forward form, or

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<sup>1</sup>Conversely, the same can be achieved with a structure that can answer the same kind of query but about edges. Such structures are called edge-centric, as opposed to the node-centric structures that we consider in this work. Both versions are trivially equivalent, however.

in its reverse complement. That is, the read is generated not from the original sequence  $S$ , but its complement  $\bar{S}$ , generated by swapping each base with its Watson-Crick complement ( $A \leftrightarrow T$ ,  $C \leftrightarrow G$ ) and reversing the string. As in [2], this will be treated by processing all reads in both directions, without, however, merging nodes representing reverse complements. As noted by Conway & Bromage: “This makes the graph symmetric; a forward traversal corresponds to a backwards traversal on the reverse complement path, and vice versa.” [2]

## 3.2 CountMin

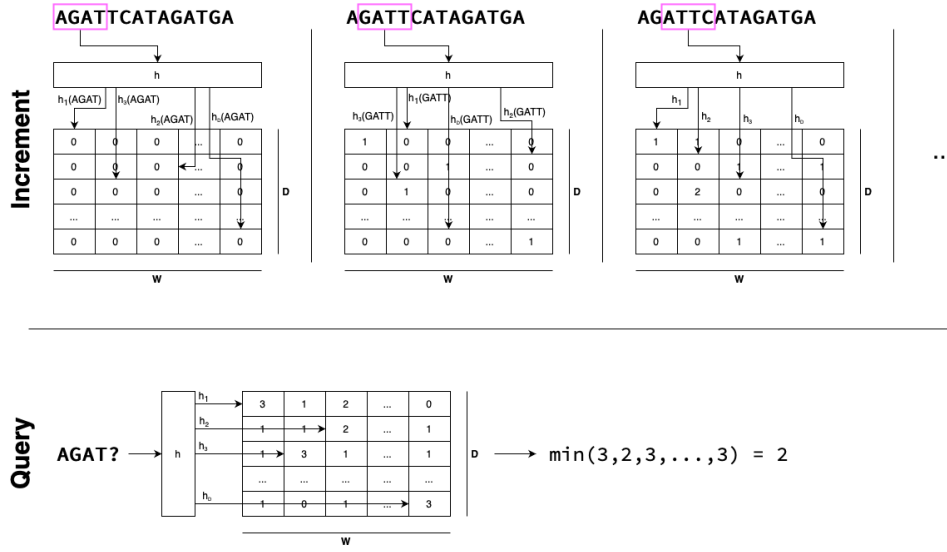
The CountMin sketch, first introduced in [3], is a sub-linear data structure intended to allow for event frequency mapping. In this way, it must allow for the query of the frequency of a given event, as well as the update of that frequency, through the *query* and *update* operations. The sketch is composed of a  $W$ -wide,  $D$ -deep matrix of counters. With each row in this matrix is associated a hash function that map the possible events to the  $W$  positions in that row. All  $D$  hash functions must be pair-wise independent.

Updating the frequency of a given event is done by passing it through the hash functions for each row, and then updating the counter in the resulting position accordingly. In this work we take interest in a simpler version of the CountMin sketch where the counters are only incremented by one.

Querying the structure consists of, similarly, retrieving the value of the counter associated with the key in each row, and then returning the minimum value among them.

→ asq says: É interessante colocar os algoritmos do CountMin mais formalmente ou só uma descrição "abstrata" é suficiente? ←

Figure 3.2 presents a visualization of the CountMin sketch.



**Figure 3.2** Example of a CountMin sketch being used to count  $k$ -mers.  $k = 4$

### 3.2.1 As a representation for a de Bruijn graph

→ asq says: Vale colocar isso aqui, ou é mais interessante deixar apenas para falar disso no capítulo anterior (State of the Art), quando falando sobre o *FastEch*? ←

A CountMin sketch can implement the membership query operation by querying the count for a given  $k$ -mer and comparing it to a presence threshold: if the count surpasses this threshold, the  $k$ -mer is considered to be present in the de Bruijn graph, and if the count is inferior to the threshold the  $k$ -mer is considered absent from the graph. This membership query is defined in Algorithm 1.

---

**Algorithm 1:** Membership operation on a CountMin sketch

---

**Data:**  $k$ -mer;  $sketch$ , the CountMin sketch,  $threshold$ , the threshold for the presence of a  $k$ -mer in the de Bruijn graph

**return**  $sketch.query(k\text{-mer}) \geq threshold$

---

### 3.3 de Bruijn CountMin

→ asq says: Como nós vamos falar das out-edges tanto no de Bruijn CountMin quanto no de Bruijn Hashtable, poderia ser interessante fazer uma sessão ou subessão acima falando sobre o conceito de forma que ele seja apenas referenciado nessas sessões? ←

In order to leverage the benefits of using a CountMin representation of the de Bruijn graph while also improving its navigability, we introduce a modified version of the CountMin sketch, which we call the de Bruijn CountMin, that allows the sketch to be queried not only for  $k$ -mer counts, but also for the out-edges from the de Bruijn graph associated with that  $k$ -mer. The goal with this modification is to reduce the number of false positives by querying only the neighbors that are known to possibly exist for that  $k$ -mer. This would avoid false paths to be traversed, improving the trustworthiness of the graph.

To store the additional information, we expand the CountMin sketch such that each cell in the matrix stores not only the counter, but also a set of out-edges. The structure, then, provides an interface to *increment* the counters associated with a given key by one, and an interface to *add an out-edge* to the sets associated with a given key. The increment operation is performed in the same way as in a regular CountMin sketch, and the algorithm for the add edge operation can be seen in Algorithm 2.

Furthermore, the de Bruijn CountMin must accomodate this new information in its query operation. In order to do this, the sketch returns not only the minimum value of the counters, but also the intersection of the sets of out-edges. The algorithm for the updated query operation is described in Algorithm 3.

---

**Algorithm 2:** Add an out-edge to a  $k$ -mer in a de Bruijn CountMin

---

**Data:**  $outEdge \in \{A, C, G, T\}$   
**for**  $i = 1, \dots, D$  **do**  
     $CountMin[i][h_i(k\text{-mer})].outEdges \leftarrow CountMin[i][h_i(k\text{-mer})].outEdges \cup outEdge;$   
**end**

---

From a practical perspective, due to a node only ever having 4 possible out-edges (corresponding to the 4 bases  $\{A, C, G, T\}$ ), the set storing them can be represented by a bit vector indicating whether each of these possible edges is present. An edge is added by setting the corresponding bit, and the intersection is obtained by performing the bitwise AND operation. This allows both set of out-edges and the counter to be stored together in a single integer. When using a  $k$  such that  $4^k > |S|$ , the each  $k$ -mer in  $S$  is expected to appear only once. As such, it is expected to appear a number of times equals to the coverage  $C$  in the reads. Considering that  $C$  is not a very high value (commonly below 200  $\rightarrow$  asq says: Citation Needed?  $\leftarrow$ ), each counter can be represented by a 12-bit integer, accounting for the expected count for all real  $k$ -mers, as well as any possible collisions.

---

**Algorithm 3:** Query a  $k$ -mer in a de Bruijn CountMin

---

```

count  $\leftarrow$  inf;
outEdges  $\leftarrow$   $\{A, C, G, T\}$ ;
for  $i = 1, \dots, D$  do
    | count  $\leftarrow$  min(count, CountMin[ $i$ ][ $h_i(k\text{-mer})$ ].count);
    | outEdges  $\leftarrow$  outEdges  $\cap$  CountMin[ $i$ ][ $h_i(k\text{-mer})$ ].outEdges;
end
return (count, outEdges)

```

---

### 3.3.1 As a representation of a de Bruijn graph

As with the CountMin sketch, a de Bruijn CountMin sketch can be queried for membership by comparing the count associated with a given  $k$ -mer to a threshold for presence (i.e. the  $k$ -mer is considered present if it appeared  $T$  or more times in the reads). This structure can be navigated from an initial set of  $k$ -mers by querying for their out-edges and then querying each of their neighbors, repeating this when a neighbor is found to be in the graph. By storing the out-edges, we expect some neighbors to never be explored, reducing the chances of finding a false path on the graph.

### 3.4 Hashtable

We also propose a new hashtable-based representation for the de Bruijn graph that is made more efficient by not storing the  $k$ -mer. Instead, a fingerprint generated from the  $k$ -mer is stored, along with the set of out edges as described in Section 3.3. When a  $k$ -mer is inserted into the hashtable, or queried from it, a hash value and a fingerprint are calculated in parallel. In case of an insertion, the fingerprint is written at the desired position and, on a query, the fingerprints are compared. Collisions are resolved by linear probing, such that if a key tries to insert in a position that is already occupied by a fingerprint that doesn't match its own, the  $k$ -mer is inserted in the next free position, unless its fingerprint is found before a free position is. During the query this process is repeated until the desired fingerprint is found, or a free position is reached (in which case the  $k$ -mer is considered to be absent from the structure).

This operation allows for the insertion of a node by adding the  $k$ -mer to the hashtable, as presented in Algorithm 4, and the insertion of an edge by updating the edge set associated with the given  $k$ -mer, presented in Algorithm 5. When queried, the structure returns the edge set associated with the given  $k$ -mer, provided the  $k$ -mer has been added to the structure. This operation is defined in Algorithm 6.

#### 3.4.1 Modular Fibonacci Hashing

The  $k$ -mer is hashed using the Fibonacci hashing algorithm, which is an algorithm that leverages a property of the golden ratio to produce a very uniform hash distribution while also having subsequent keys be hashed to values distant from each other. The function was modified, however, to use modular arithmetic instead of a bitwise shift operation to map the result to the intended range. This was done in order to allow for more freedom in the choice of the desired range and, therefore, the size of the hashtable. The algorithm for this modified version of the Fibonacci hashing function can be seen in Algorithm 7.

**Algorithm 4:** Insert  $k$ -mer in de Bruijn Hashtable

---

**Data:**  $k$ -mer  
 $hash \leftarrow fibonacci\_hash(k\text{-mer});$   
 $pos \leftarrow hash;$   
**while**  $!HT[pos].empty \wedge pos \neq hash - 1$  **do**  
  **if**  $HT[pos].fingerprint \neq fingerprint(k\text{-mer})$  **then**  
     $pos \leftarrow (pos + 1) \bmod |HT|;$   
  **else**  
    **return**  
  **end**  
**end**  
**if**  $HT[pos].empty$  **then**  
   $HT[pos].kmer \leftarrow k\text{-mer};$   
   $HT[pos].outEdges \leftarrow \emptyset;$   
   $HT[pos].fingerprint \leftarrow fingerprint(k\text{-mer});$   
**end**

---

**Algorithm 5:** Add out-edge to  $k$ -mer in de Bruijn Hashtable

---

**Data:**  $k$ -mer,  $outEdge \in \{A, C, G, T\}$   
 $hash \leftarrow fibonacci\_hash(k\text{-mer});$   
 $pos \leftarrow hash;$   
**while**  $!HT[pos].empty \wedge HT[pos].fingerprint \neq fingerprint(k\text{-mer}) \wedge pos \neq hash - 1$  **do**  
   $pos \leftarrow (pos + 1) \bmod |HT|;$   
**end**  
**if**  $!HT[pos].empty \wedge HT[pos].fingerprint = fingerprint(k\text{-mer})$  **then**  
   $HT[pos].outEdges.add(outEdge)$   
**end**

---

**Algorithm 6:** Query  $k$ -mer in de Bruijn Hashtable

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**Data:**  $k$ -mer,  $outEdge \in \{A, C, G, T\}$   
 $hash \leftarrow fibonacci\_hash(k\text{-mer});$   
 $pos \leftarrow hash;$   
**while**  $!HT[pos].empty \wedge HT[pos].fingerprint \neq fingerprint(k\text{-mer}) \wedge pos \neq hash - 1$  **do**  
   $pos \leftarrow (pos + 1) \bmod |HT|;$   
**end**  
**if**  $!HT[pos].empty \wedge HT[pos].fingerprint = fingerprint(k\text{-mer})$  **then**  
  **return**  $HT[pos].outEdges$   
**end**

---

**Algorithm 7:** Fibonacci Hash Function

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**Data:**  $key$ ;  $range\_max$   
**return**  $(11400714819323198485 \times key) \bmod range\_max$

---



### 3.4.2 Fingerprint

When performing the Fibonacci hashing, the most significant bits of the result of the multiplication are discarded by the modulo. Those bits are also distributed uniformly over the possible keys, such that they can be used as a fingerprint for the  $k$ -mer. As such, the fingerprint function can be defined as in Algorithm 8. → asq says: Existe uma forma melhor de formalmente escrever que os  $n$  bits mais significativos são escolhidos? ←

---

**Algorithm 8:** Fingerprint Function
 

---

**Data:**  $key$

**return**  $(11400714819323198485 \times key) \gg 61$

---

## 3.5 A pipeline using the de Bruijn CountMin and the de Bruijn Hashtable

Beyond the two isolated datastructures to represent the de Bruijn graph, we also propose a way to use both of them in tandem in order to obtain the benefits of both. In this pipeline, the sequencing reads are processed and inserted into the de Bruijn CountMin as they are made available, such that the de Bruijn graph can be constructed online. Once the reads are all processed in this manner, and a navigatable version of the graph has been constructed, it can be traversed, with all of its nodes being, then, inserted in a de Bruijn Hashtable. In this way, a de Bruijn CountMin is effectively compressed into a de Bruijn Hashtable. This pipeline is formalized in Algorithm 9.

---

**Algorithm 9:** Pipeline using a de Bruijn CountMin to construct a de Bruijn Hashtable
 

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**Data:**  $R$ , a set of sequencing reads  
 $dBCM \leftarrow$  Empty de Bruijn CountMin sketch;  
**for**  $read \in R$  **do**  
   $previous\_k\text{-mer} \leftarrow \emptyset$ ;  
  **for**  $kmer \in read$  **do**  
     $dBCM.increment(k\text{-mer})$ ;  
    **if**  $dBCM.query(k\text{-mer}).count \geq T$  **then**  
       $outEdge \leftarrow k\text{-mer}[-1]$ ;  
       $dBCM.addOutEdge(previous\_k\text{-mer}, outEdge)$ ;  
    **end**  
     $previous\_k\text{-mer} \leftarrow k\text{-mer}$ ;  
  **end**  
   $previous\_k\text{-mer} \leftarrow \emptyset$ ;  
  **for**  $kmer \in reverse\_complement(read)$  **do**  
     $dBCM.increment(k\text{-mer})$ ;  
    **if**  $dBCM.query(k\text{-mer}).count \geq T$  **then**  
       $outEdge \leftarrow k\text{-mer}[-1]$ ;  
       $dBCM.addOutEdge(previous\_k\text{-mer}, outEdge)$ ;  
    **end**  
     $previous\_k\text{-mer} \leftarrow k\text{-mer}$ ;  
  **end**  
**end**  
 $dBHT \leftarrow$  Empty de Bruijn Hashtable;  
**for**  $k\text{-mer} \in dBCM.k\text{-mers}$  **do**  
   $dBHT.insert(k\text{-mer})$ ;  
   $dBHT[k\text{-mer}].outEdges \leftarrow dBCM[k\text{-mer}].outEdges$ ;  
**end**  
**return**  $dBHT$

---

## 3.6 Experiments

### 3.6.1 Metrics

Through the experiments described further in this section, we evaluate different metrics for the de Bruijn CountMin and the de Bruijn Hashtable. This is due to the fact that both these structures have different goals and are used in different contexts.

As both of these structures are probabilistic in nature, however, there are certain metrics that are used in the evaluation of both. One such metric is the *false positive rate*. We define this rate based on the  $k$ -mers that are visited during a traversal of the graph. Let  $S$  be a genetic sequence that contains the set of  $k$ -mers  $K$ , and let  $G$  be the graph, represented either by a de Bruijn CountMin or a de Bruijn Hashtable, constructed from the sequencing reads of  $S$ . Further, let  $Q$  be the set of  $k$ -mers that were queried from  $G$  during its traversal, and  $P = \{k | k \in Q \wedge k \in graph\}$  be the set of queried  $k$ -mers that were in  $G$ . As such, we can define the set of false positives as  $F_P = P \setminus K$  (i.e. the set of  $k$ -mers that were queried and found to be in  $G$  but are not actually in the original sequence  $S$ ). Finally, the false positive rate is defined as  $fp = \frac{|F_P|}{|Q|}$  (i.e. the proportion of  $k$ -mers found during traversal of  $G$  that are not found in  $K$ ).

→ asq says: Realmente vale a pena tentar fazer isso ainda? Estamos na reta final do projeto já, e isso iria requerer a definição do que é uma mudança significativa nos resultados que justifique a mudança na estrutura ←

As we posit that adding the out-edge information to the structures will improve its navigability by reducing the chances of generating false branches on the graph, we also look at the number of out-edges found for each  $k$ -mer in the graph. We can look at the distribution of these results as a way to evaluate the possible impact of this additional information: the more  $k$ -mers that have fewer out-edges, the fewer forks there are, and the less likely that a false branch will be generated. → asq says: Provavelmente é válido gerar uma formalização desse conceito de distribuição da contagem de out-edges por  $k$ -mer ← . Beyond that, we also perform the traver-

sal of the graph with and without using the out-edge information, such that we can compare the results and measure the impact in this way.

### 3.6.1.1 de Bruijn CountMin

The de Bruijn CountMin was developed to be used directly with the sequencing reads without any pre-processing. It's goal is to build a reliable navigatable version of the de Bruijn graph as the reads are made available. In this context, not only do we expect a certain amount of false positives will appear, but as we must probabilistically filter the set of  $k$ -mers from the reads to remove all the spurious ones, we also expect the occurrence of *false negatives*, which are defined as  $k$ -mers from the original sequence  $S$  that are not present in the graph  $G$ . I.e.:  $F_N = K \setminus P$ . As such, the *false negative rate* can be defined as the ratio of false negatives to the total number of  $k$ -mers in the original sequence  $S$ , or  $fn = \frac{|F_N|}{|K|}$ .

### 3.6.1.2 de Bruijn Hashtable

## 3.6.2 *E. Coli*

The *E. Coli* genome is an established benchmark for new assemblers to compare against. We used the reference chromosomal dna genome made available by Ensembl Genomes for the strain k 12, substrain mg1655 of this species[?].

Three different experiments were performed. In the first we generated simulated perfect reads from the genome by taking substrings of the original sequence at random. We then used the *ART Illumina* toolkit → asq says: Citation needed ← to simulate realistic reads from this genome, including read errors and reverse complements. Finally, a dataset of real-world reads was downloaded from SRA [5] and used. → asq says: Eu mencionei esse último teste na esperança que dê tempo de realiza-lo ainda. ←

## 3.6.2.1 Synthetic reads without errors

Given the reference genome  $S$ , the length of each read,  $L$ , and the desired coverage  $C$ , the synthetic reads were generated by picking  $\frac{|S| \times C}{L}$  substrings of  $S$  at random. This is presented in algorithmic form in Algorithm 10.

**Algorithm 10:** Generate Reads

---

**Data:**  $S$ , the reference genome,  $L$  the read length,  $C$  the coverage  
 $\#reads \leftarrow \frac{|S| \times C}{L};$   
 $reads \leftarrow \emptyset;$   
**for**  $i \leftarrow 1, \dots, \#reads$  **do**  
     $j \leftarrow \text{random}(0, |S| - L);$   
     $reads.add(S[j : j + L]);$   
**end**  
**return**  $reads$

---

## 3.6.2.2 Synthetic reads with errors

In order to simulate the reads as they would be produced by the sequencing process, we used the ART Illumina toolkit to generate synthetic reads from the *E. Coli* genome. The reads were generated using the following parameters:

1. **Sequencing System:** Illumina MiSeq v3

2. **Read length:** 250bp

3. **Coverage:** 80x



## CHAPTER 4

**Results**





## CHAPTER 5

# Conclusion

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