CONTROL OF MECHATRONIC SYSTEMS

Theory and practice of control for packaging machines.

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ACRONYMS

AC	Alternate Current
MC	Motion Controller
PLC	Programmable Logical Controller
RMS	Root Mean Square

INTRODUCTION

Modern factories for the production and packaging of mass products, e.g. foodstuff and paper tissues, are organized in three main branches:

1. the processing branch, in which the product is made,
2. the packaging line, that prepares and wraps with packaging material every single piece of product to assure its sterility and integrity during the shipment and the final distribution to customers,
3. the factory logistic system, that handles the raw materials (packaging materials) and finished products in relation with the warehouse management, and for the truck loading and shipment.

Nowadays, product processing and packaging (i.e. manufacturing phases 1 and 2) have reached a high degree of automation, in which issues as energy consumption awareness, agile manufacturing and product customization are commonly fully addressed. For example, packaging lines are designed and deployed to allow a fast change in packaging size and pattern in order to respond promptly to production flexibility requirements. Notably, optimization in the electrical motors control of the machines in the packaging line has permitted a more efficient use of the energy during production.

In particular packaging machines have built to make and wrap with protective material goods, often foodstuff, for mass distribution and consume. Market of packaging machines was steadely incresing in the latter years, pushed by the request for production speed, product quality and lower cost of goods.

Such machinery employs electrical, mechanical, computer, and communication equipment in its design. An engineer who designs and maintains packaging machinery must be knowledgeable in both mechanical and electrical fields. In the past, packaging machinery manufacturers trained their own engineers in house, which took several years and many trial and error processes. Today's rapid growth in the packaging machinery sector requires a more systematic and scientific approach in training engineers. Training engineers in mechatronics is that systematic and scientific approach. It incorporates both electrical and mechanical engineering knowledge under one umbrella. Training in mechatronics lets engineers understand the mechanical as well as electrical requirements for design, operation, and maintenance of complicated machinery.

Packaging machinery requires precise, fast, and repeatable operation. Most new packaging machines utilize servo motors, in particular AC servo motor. The purpose of the motor is to provide controlled torque to move to a precise position the mechanical part of the machine, to reach the productive goal.

Servo motors employ digital control, which is versatile, repeatable, and reliable. In recent years servo motors are becoming more cost effective compared to other types of drives.

As servo motors continue to gain popularity in the world of packaging machinery, packaging professionals on the machinery side of things would do well to bone up on mechatronics. And if you're a packaging machinery builder, as you consider new hires in your engineering department, lean toward those who can demonstrate that they've been subjected to a systermatic and scientific approach to mechatronics training.

The aim of this book is to describe the process of developing a mechatronic approach to the development of the automatic machine control system.

PART I

MOTION CONTROL OF MECHATRONIC SYSTEMS

CHAPTER 1

MOTION CONTROL FOR PACKAGING MACHINES

Mechatronics 1.1 can be defined as the field where the following disciplines are necessary to be implemented and integrated:

- Control Theory
- Electronics (i.e. HW)
- Computer Science (i.e. SW)
- Mechanics

The applications of this new discipline are really many, including material processing, packaging, automotive, aerospace, bio-medical, defense, domotics, etc.

One industry branch that represents a fruitful but challenging application of motion control is the packaging machines sector.

The purpose of this chapter is to highlight motion control properties and challenges that are specific for this branch of industry.

A packaging machine is typically characterized by *highly coordinated* movements or functions that are repeated in a short time frame. Primary Packaging machines generally *shape* a packaging material into a container (e.g. a carton package, or an aluminum can) then *fills* the container with the product (e.g. liquid milk, or carbonated drink) and *close* it (e.g. seal the carton package, or close the can). In contrast, Secondary Packaging machines generally completes the packaging, sequencing and grouping the containers in patterns, to be inserted into trays or carton boxes and/or shrinked into film for examples. End-of-Line equipment finally places the boxes onto a pallet.

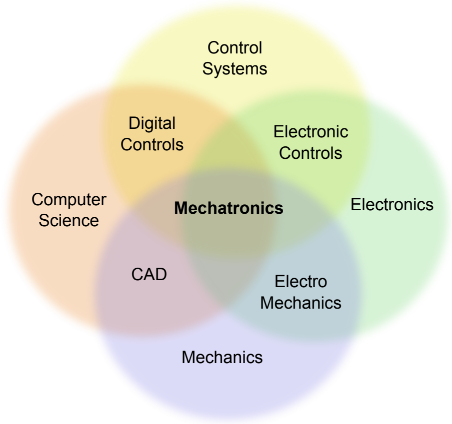

Figure 1.1: Mechatronics comes from the union of different and complementary disciplines.

The cycle times generally ranges from several milliseconds to few seconds, depending on the mechanical load size and function characteristics.

Packaging machines functions can be mainly divided into:

- *Indexed*, characterized by synchronous, repetetive functions, that can be achieved through servo (e.g. brushless) motors, providing high flexibility (e.g. format change) to the machine

- *Continuous*, characterized by continuous function, that can be achieved for example through rotating devices with velocity not passing through zero, providing high speed function

- *Robotic*, characterized by high flexibility, modularity and complex movements.

More and more the machine needs a mechatronic integration of means, i.e. a view that is considering at the same time the four different souls of Mechatronics discipline.

This in order to have higher performance (e.g. the through-put in terms of units per minute) and higher reliability (e.g. in terms of MTBF, Mean Time Between Failure).

This in turn needs a multidiscipline approach, and the necessity for a strict collaboration among different groups of designers.

Mechanical system control is undergoing a revolution in which the primary determinant is becoming the control software. This is enabled by developments in electronics and computer technology.

The word *Mechatronics* was indeed defined as (Tetsuro Mori, Yaskawa Electric, 1969):

> "*new kind of mechanical system where the electronics take the decision-making function formerly performed by mechanical components...*"

whilst later (Tomizuka, 2004) there has been a shift from electronics to software as primary decision-making, the definition thus becoming:

> "*Mechatronics is the synergetic integration of physical systems with information technology and complex-decision making in the design, manufacture and operation of industrial products and processes.*"

Real time software differs from conventional software in that its results must not only be *numerically* and *logically correct*: they must also be delivered at the *correct time*; it must embody the concept of duration.

Real-time software used in most mechanical system control is also safety-critical.

Software malfunction can result in injury and/or significant property damage.

Asynchronous operations, while uncommon in conventional software, are the heart and soul of real-time software.

The achievement of a successful mechatronic design environment essentially depends on the ability of the design team to:

1. Innovate,
2. Communicate,
3. Collaborate,
4. Integrate.

Indeed, a major role of the mechatronic engineer is often that of acting to bridge the communications gaps that can exist between more specialized colleagues, in order to ensure that the objectives of collaboration and integration are achieved.

This is important during the design phases of product development and particularly so in relation to requirements definition, where errors in interpretation of customer requirements can result in significant cost penalties.

The *Mechatronic Design* requires:

- System Perspective,
- System Interactions,
- System Modeling,
- System Stability.

The system perpsective is thus crucial in a true Mechatronic Design.

Mechatronics is simply the application of the latest, cost-effective technology in the areas of computers, electronics, controls, and physical systems to the design process to create more functional and adaptable products.

The means to achive this go through Virtual Simulation and Experimental Verification.

A representation of information, energy and material flows in a typical mechatronic system with a closed loop control action is shown in figure 1.2.

The Control Theory view of the scheme would be described as a control loop formed by (see figures 1.3 and 1.4) a *Controller*, a computer that implements the control laws for the mechatronic systems, the *actuator* and *sensors* that are the interface with the *plant*, which is the target of the control action.

Historically the packaging machines has been driven first by camwheel systems and indexing gears (see figure 1.5), in order to generate arbitrary motion profile.

In modern machine designs, electric servo motors are often used in place of mechanical solutions, providing advantages in terms of solution flexibility and reliability. Indeed a servo system, composed by a motor, feedback system and drive, can follow any position, speed or torque profile and even switch on-the-fly among them.

These profiles can also be driven, in turn, by a Master Axis (or Virtual Master), that is thus synchronizing all the axes or a subset of them.

These arbitrary motion profiles are defined as mathematical formulas that are continuous in time and values, whereas a digital system, such as the servo drive, is discrete in time and values. This raises a series of implications.

As can be seen in the following scheme, the controller, field bus board and drives can have different time discreteness and interpolation (see figure 1.6).

This in turn leads to an applied motion control curve that is similar but not exactly the same as the one from which the controller curve has been obtained (see figure 1.7).

As a consequence, the **Design Process** has to be iterative as described in the following algorithm

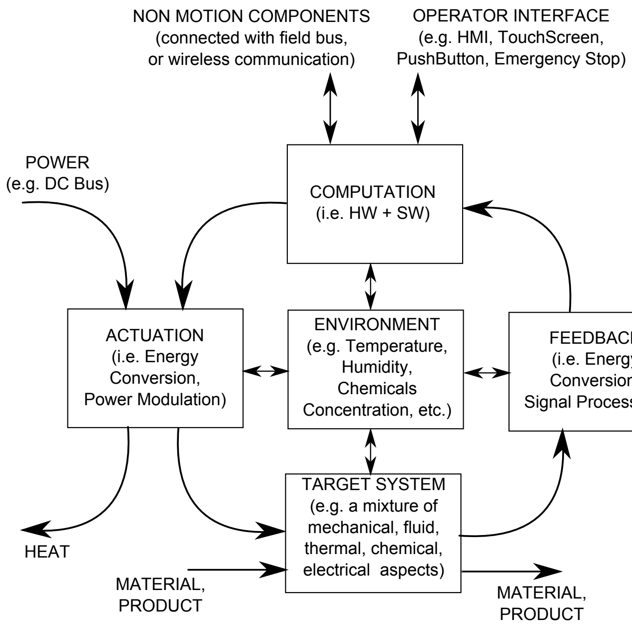

Figure 1.2: A representation of information, energy and material flows into a mechatronic system.

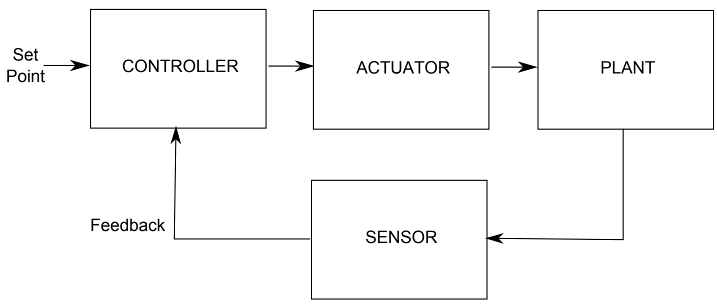

Figure 1.3: The Control Theory view.

1. Definition of coordinated multi-axes motion at load side
2. Definition of mechanical transmission system
3. Mechanical load calculation for each system
4. Definition of coordinated multi-axes motion at motor side. If the solutoin satisfies the specification go to next step, otherwise, go to step 1.

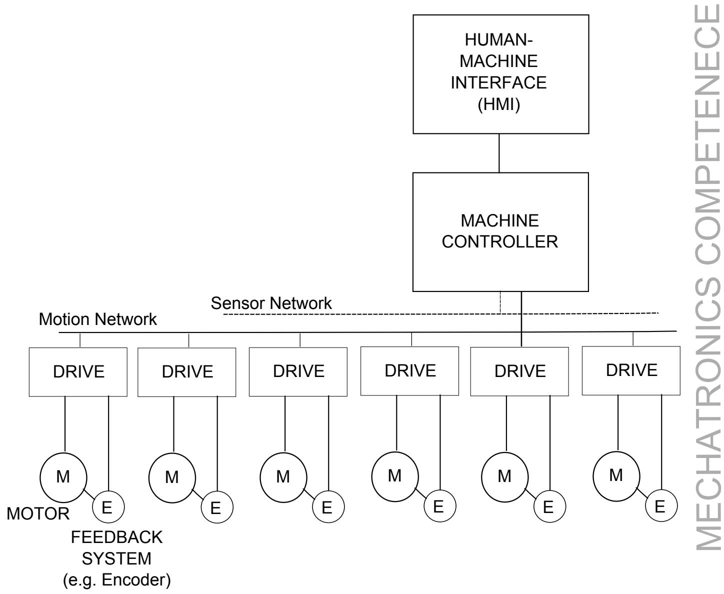

Figure 1.4: The components view of a system implementing a motion control loop.

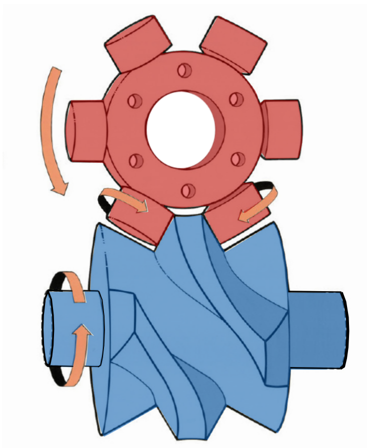

Figure 1.5: Camwheel systems and indexing gears.

5. Load simulation of single-axis servo units
6. Control simulation of single-axis servo units
7. Multi-axes simulation
8. Physical test
9. Verification of actual performance

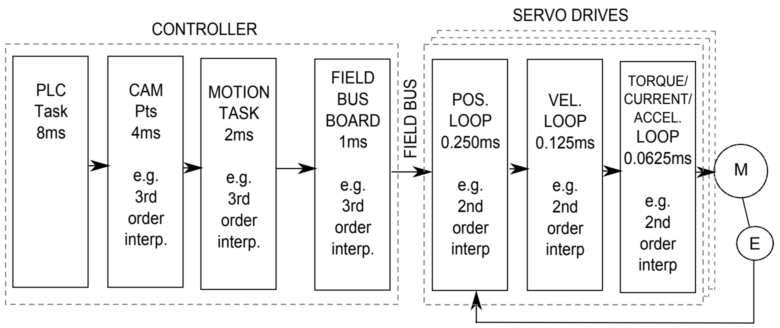

Figure 1.6: Logical and timing schemes for a motion control

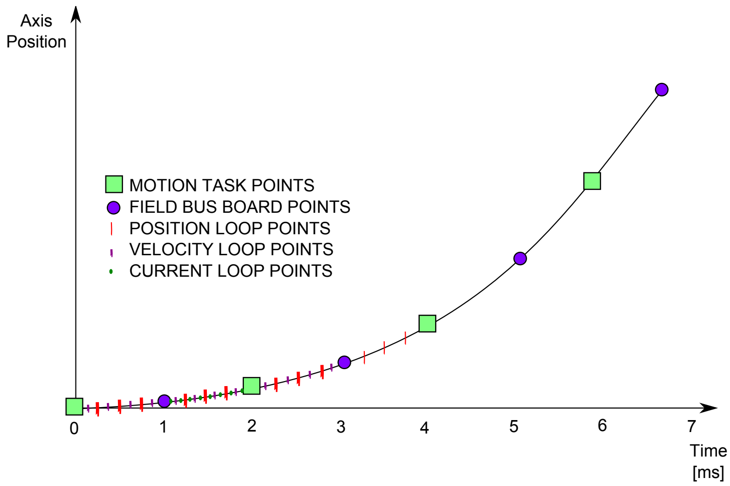

Figure 1.7: A plot of Cam Points for Controller, Field Bus and Drive.

Steps 1 to 4 constitute a highly iterative process that may become very complex for applications with critically high demands. These steps are usually done with help of specific mechatronic tools (e.g. Motion Analyzer^{TM} from Rockwell Automation^{TM}). The iterations are needed because of multiple constrains in different fields, that can lead to different optimal solution for the same variable depending on the step we are in. These application dependent constrains leads to requirements, that in turn can generally be expressed, for example, in terms of:

- Maximum cycle time
- Space occupancy
- Weight
- Materials
- Temperature

There are multiple challenges to be approached, such as:

Step 2. ***Definition of mechanical transmission system:*** A Direct Drive solution versus a Transmission based solution has to be chosen, making assumption on the motor. The first is generally chosen for position control applications where the motor power is low and bandwidth need is maximum, while the latter is generally chosen for velocity control applications with high motor power where bandwidth need is modest. In fact the gearbox transmission ratio N is optimized so to minimize the torque T given a certain requested acceleration a, load inertia J_L, motor inertia J_M. This result is reached looking at transmission ratio optimisation (see figure 1.8). We will see that a completely different result can be reached looking at optimizing bandwidth.

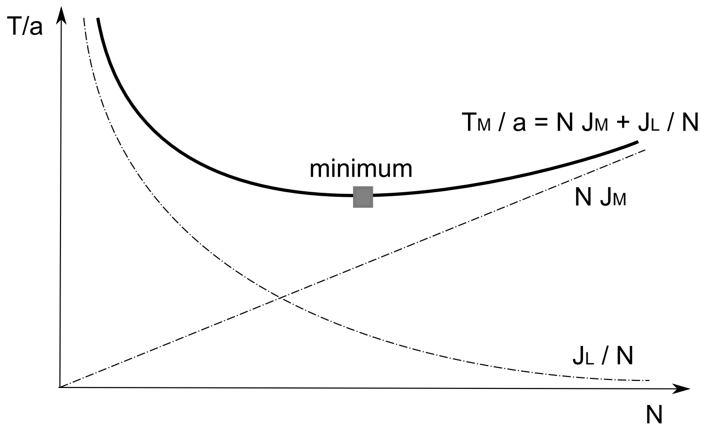

Figure 1.8: Optimal gearbox transmission ratio

Step 3. ***Mechanical load calculation for each motor.*** Static or Dry Friction, Viscous Friction, Inertia, Weights, Spring Forces, are estimated, making other assumptions (expecially on frictions).

Step 4. ***Definition of coordinated multi axes motion at motor side*** The servo motors and drives are chosen so to have a certain safety margin in terms of:

- Peak Velocity
- Peak Torque
- RMS Torque

The peak velocity and torque, if surpassed, would lead to a saturation of the velocity and torque respectively (see figure 1.9, that shown a typical motor torque versus speed characteristic), that in turn would lead to a deviation from the set point profile, causing a position error out of control.

The RMS torque, if surpassed, instead, would lead to motor burn, thus causing a permanent fault.

A Paradox to Step 4 is that motor, drive and motion profile are input variables, together with gearbox ratio, that could also be changed in order to find the optimal solution. Problem is that the gear ratio was already chosen at Step 2, leading to contradictory results that force the designer to do multiple iterations. This happens because a gearbox high ratio is particularly efficient in using at best the full motor torque characteristic, while the optimal gearbox ratio to minimize acceleration / torque ratio is generally a low gearbox ratio.

Moreover the full motion control system bandwidth is dependent on technological constrains all in series to each other, such as:

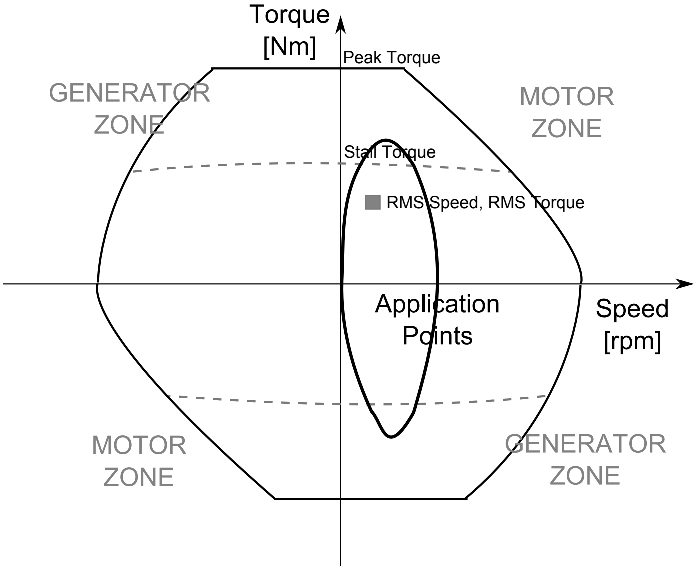

Figure 1.9: Motor torque characteristics.

- Controller Coarse Update Period
- Field Bus Update Period
- Drive Fine Update Period
- Drive PID Tuning
- Feedback Resolution
- Motor Stiffness
- Motor–Load Coupling Backlash
- Transmission Stiffness

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CHAPTER 2

DESIGN MOTION CONTROL FOR PACKAGING MACHINES.

2.1 introduction

The raw material is treated by a packaging machine in successive steps in order to obtain a finished product, for example it can be transported, sorted, cut, packaged and boxed in a sequence of operations that must be carefully designed in order to obtain the achievement of a production of good quality.

All the above tasks require the manipulation of the product through moving mechanical parts, whose planning and control are very important elements in the process of design the oontrol system of automatic machines.

Generally, the control of the automatic machine can be divided on two hierarchical levels like:

1. A series of logical operations that correspond to various stages of processing of the product. These operations can be represented as *finite-state automaton* that is usually implemented using simple combinatorial and sequential logic through a programmable logic controller (PLC).

2. In every state of the automaton, the automatic machine performs the process of the raw product through the coordinated movement of mechanical devices forming a *kinematic chain*. For example, a raw product can be cut, wrapped and printed through the movement of certain mechanical systems.

The motion can be implemented using an electrical motor at constant speed linked to a mechanism that transforms the constant speed motion into a varied motion profile (e.g. an alternate motion for driving the motion up and down of a printer, etc.).

However, the modern approach is to drive the varied motion by an electric motor controlled so as to ensure a desired motion profile which is necessary to achieve the desired functionality of the machine. In this case, the design phase of the kinematic or of the cam is translated in the preparation of a suitable program written for digital control of the movement of the electric motor.

In this latter case, the motion control system is formed by four parts (see figure 2.1):

1. A **trajectory generator**, which defines the position, velocity and acceleration of the final poit of the kinematic chain. The trajectory generator takes also into account various constraints into computation, such as velocity and acceleration limits beacuse physical constranints.

2. The **control system**, which drives the electrical motor to reach the motion profile defined by the trajectory generator.

3. The **electrical drive**, that controls the torque of the electrical motor to follow the reference set by the control system.

4. The **electrical motor**, which transform the electrical power into mechanical power required to move the kinematic chain of the automatic machine.

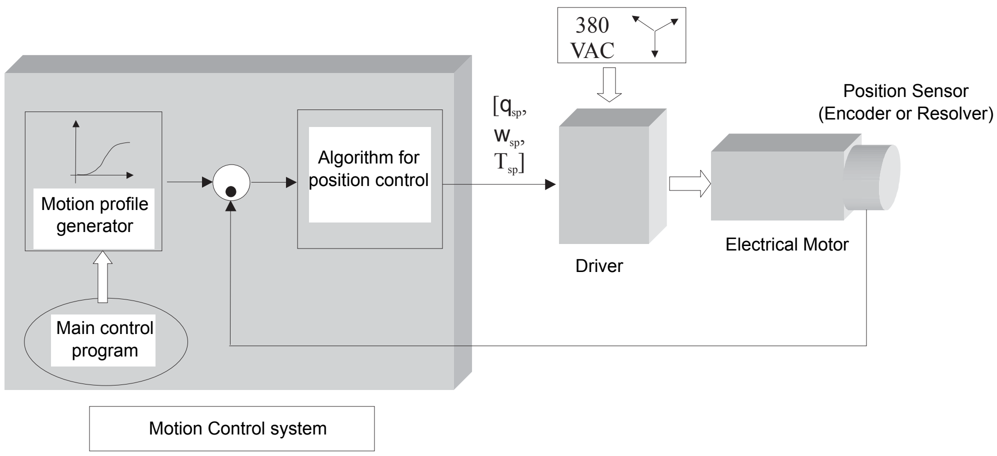


Figure 2.1: Conceptual scheme of a motion control system.

The description of typical kinematic chains you can find an automatic machine is presented in Section 2.2.

The kinematic characteristics of the motor such as position, speed and acceleration are modified by the kinematic chain, making it necessary for an accurate study to correctly design the motor motion profiles. This study takes the name of analysis of the kinematic chain and is treated in the section 2.3.

The motion profile should be generated in order to minimize the mechanical stresses of the component of thekinmatic chain which can, in the long run, lead to damage of the overall system. Section 2.6 presents the mathematica treatment for generating motion profiles with smooth properties.

2.2 Kinematic chain

A kinematic chain is an assemblage of links and joints, interconnected in away to provide a controlled output motion in response to a supplied input motion. As in the familiar use of the word chain, the rigid bodies, or links, are constrained by their connections to other links. An example is the simple open chain formed by links connected in series, like the usual chain, which is the kinematic model for a typical robot manipulator.

In an automatic machine, the first element of a kinematic chain is a motor, that acts as a source of mechanical power, and the last element is a device that is designed to handle the product, such as cutter, pusher, gripper, etc.

In the following, we will refer to the first element as *Actuator*, ACT in short, and the latter element as *Specific Handling Tool*, SHT in short.

The actuators are devices that generate the control motion of the mechanics. Actuators have always a control input and a power input (typically electrical or pneumatic) which is transformed into mechanical power.

The actuators are most widely used in the industrial field:

- DC electric motors (DC).
- Electric motors in alternating current (AC).
- Brushless Electric Motors
- Stepper Electric motors,
- Variable reluctance motors,
- linear electric motors,
- hydraulic motors,
- hydraulic and pneumatic pistons.

The SHT devices are mechanical devices or **kinematic** that perform the mechanical processing of the product. Their classification depends on the class of use, for example:

- tools and spindles in the area of numerical control machine,
- benders, gripping, belts, rollers etc. in the field of automatic machines,
- arms, clamps, fingers, etc. in the filed of robotics

The most complex kinematic chains have different mechanical devices connecting actuators and SHT . These devices, used to perform transformations on the kinematic parameters (eg. the position, velocity and acceleration of the SHT) with respect to the actuator, are commonly known as **kinematic**.

The simpler kinematic mechanisms have a single *degree of freedom* or **d.o.f.**, i.e. the output and the input are unique and it is unique the relationship transforming the input position in the output position.

In a linkage with one d.o.f. it can always be possibile to identify two sections:

The simpler kinematic mechanisms have a single *degree of freedom* or **d.o.f.**, i.e. the output and the input are unique and it is unique the transformation between the input position and the output position.

In a single d.o.f. kimeatics they are always two sections (see Figure 2.2):

- The **Driver link** or **Cam**, that is, the section which receives the motion from an actuator or by another mechanism. We will denote in the following with $p(t)$ the kinematic characteristic of driver link (position).
- The **Driven link** or **follower** is the pat of the linkage that makes available the motion transformed to SHT or to the Cam of other kinematic mechanisms. The kinematics characteristic (position) of the follower will be called $q(t)$.

The most common kinematic linkages are:

- **Gearings**: wheels, gears, pulleys,
- **Gears**: belts, chains, skids, racks, ball screws, madrevite,
- **Crank mechanisms**: Crank drives, Quadrilaterals, Pentalateri, eccentric thrust, Rocker,

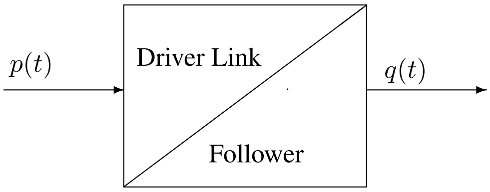


Figure 2.2: Driver Link and Follower

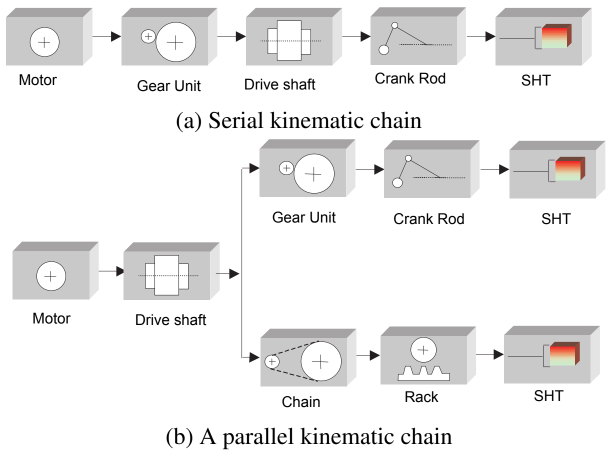


Figure 2.3: Examples of kinematic chains

- **Shafts**: Drive Shafts, Cardan Shaft,
- **Motion profile generators**: Cams, Intermittent mechanism, Geneva Drive, etc..

A kinematic chain (see figure 2.3) is a set of mechanisms to transfer the thurst from the actuators to the SHT , making the transformations of motion necessary to ensure that the profile of the motion SHT is appropriate for the target operational task.

The kinematic chain always starts with an actuator and ends with a SHT . Between actuators and SHT may be interposed some more kinematic mechanisms. A kinematic chain has a single d.o.f. if all the component blocks have a single d.o.f.

▽ *Example 2.1: An example of kinematic: the connecting rod–crank.*
Figure 2.4 shows a rod–crank system that is used in mechanical for transforming a rotary motion into a linear–alternating motion.

In such mechanisms one end of the crank is fixed to the frame via a hinge while the other is connected to the connecting rod. The second head of the latter is connected to the connecting rod (slide), normally arranged on a rectilinear guide. The rotary motion of the crank (driving link) in the reference plane, forced by a shaft inserted into the hinge of the frame, is converted into alternating linear movement of the connecting rod (follower link). In this mechanism the position of the mover is defined by the angle $p(t) = \theta(t)$ of rotation of the crank, the position of the originator is determined by the position of the linear slide $q(t) = x(t)$.

To calculate the kinematic function between Driver Link and the Follower, please refer to Figure 2.4, where r is the length of the crank and l of the rod.

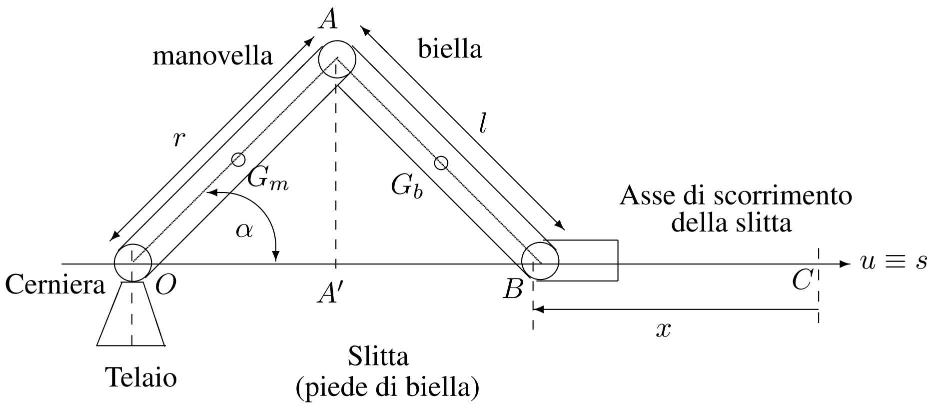


Figure 2.4: Computation of the kinematic transformation of the crank–rod mechanism (crankshaft thrust).

La retta u passa per la cerniera O ed è parallela all'asse di scorrimento s della slitta (che coincide con u nel caso del manovellismo semplice, mentre non coincide con u nel caso del manovellismo deviato). Essa rappresenta il riferimento per la posizione angolare della manovella, la quale è misurata in senso antiorario rispetto a tale retta. La posizione del piede di biella è valutata come la distanza x dal punto morto superiore C, ovvero rispetto al punto in cui la slitta inverte il proprio moto alla massima distanza dalla cerniera O. Il calcolo della funzione cinematica del manovellismo consiste nel calcolare il profilo di moto del piede di biella $q(t) = x(t)$ a partire dalla conoscenza del profilo di moto della manovella $p(t) = \alpha(t)$.

Con riferimeno alla figura 2.4, $x = \overline{(C-O) - (B-O)}$, e $C - O = r + l$ (massima estensione biella–manovella), mentre $B - 0 = r\cos(\alpha) + \sqrt{l^2 - r^2\sin(\alpha)^2}$ (come si può facilmente verificare applicando il teorema di Pitagora al triangolo rettangolo $A\hat{A}'B$), quindi:

$$x = f(\alpha) = r(1 - \cos(\alpha)) + l\left(1 - \sqrt{1 - \left(\frac{r}{l}\sin(\alpha)\right)^2}\right) \tag{2.1}$$

△

▽ *Example 2.2: Un sistema di produzione costituito da due cinematismi*

Consideriamo il sistema di produzione costituito da un nastro trasportatore ed una taglierina rappresentato in figura 2.12. Il nastro trasporta il prodotto avvolto da una pellicola che viene tagliato dalla taglierina che viene azionata in modo sincrono con l'avanzamento del prodotto.

I due sistemi, il nastro trasportatore e la taglierina, sono rappresentabili mediante semplici catene cinematiche raffigurate in figura 2.6a ed in figura 2.6b.

Lo schema rappresentante il nastro trasportatore comprende un motore elettrico, che funge da generatore del moto o attuatore, un riduttore che riduce la velocità di rotazione del carico rispetto a quella del motore la puleggia di trascinamento del nastro trasportatore ed il SHT . Dall'esempio risulta chiaro che il SHT è un dispositivo fittizio, corrispondente in questo caso al nastro a contatto con il prodotto.

Lo schema che rappresenta la taglierina è composto ancora da un motore elettrico e da un riduttore, quindi da un cinematismo detto "a cremagliera lineare", formato da un pignone (ruota dentata) che si innesta in una cremagliera ed infine dal SHT , costituito in questo caso dal coltello stesso.

△

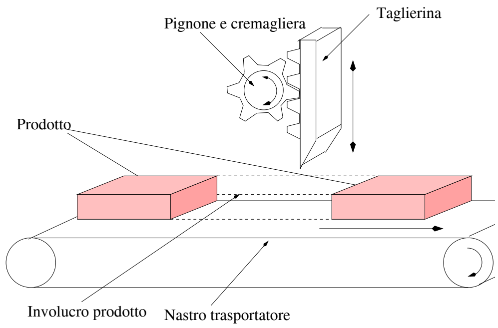


Figure 2.5: Un sistema di trattamento del prodotto costituito da due catene cinematiche.

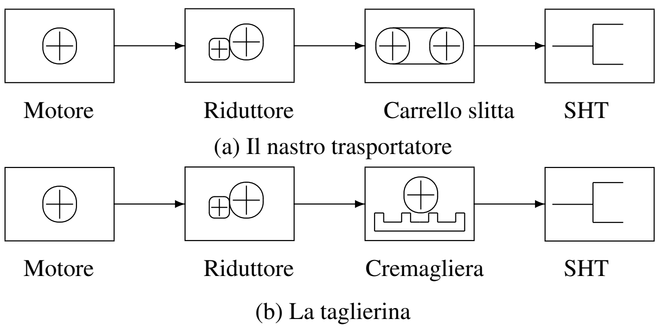


Figure 2.6: Esempi di catene cinematiche.

2.3 Kinematic transform

Un blocco cinematico rigido realizza una trasformazione geometrica di coordinate tra cedente e movente, esprimibile attraverso una relazione algebrica del tipo

$$q(t) = f(p(t)) \tag{2.2}$$

dove $q(t)$ e $p(t)$ rappresentano, rispettivamente, il valore della coordinata cinematica di posizione per il cedente ed il movente e $f(.)$ è la legge di trasformazione geometrica tra movente e cedente.

Nella seguente tabella 2.1 sono riportate alcune funzioni di trasformazione cinematica per i cinematismi più utilizzati.

Cinematismo	$q = f(p)$	Note
Cremagliera lineare	$x = R\alpha$	x: spostamento della cremagliera (in metri), R: Raggio del pignone (in metri), α: angolo di rotazione della ruota (in radianti).
Carrello slitta	$x = R\alpha$	x: spostamento del carrello (in metri), R: Raggio della ruota motrice (in metri), α: angolo di rotazione della ruota (in radianti).
Riduttori	$\theta = R\alpha$	θ: angolo di rotazione del cedente (in radianti), R: rapporto di riduzione del riduttore, α: angolo di rotazione del movente (in radianti).
Manovellismo di spinta semplice	$x = r(1 - cos(\alpha)) + l(1 - \sqrt{1 - (\frac{r}{l} sin(\alpha))^2})$	x: posizione lineare del cedente (in metri), r, l: lunghezza della manovella e della biella (in metri), α: angolo di rotazione del movente (in radianti).

Table 2.1: Le principali funzioni di trasfomazione cinematica diretta.

La composizione di n blocchi cinematici in cascata (fig. 2.7) si ottiene componendo le funzioni di trasformazione cinematica dei vari blocchi. Siano, ad esempio, f_i le funzioni di trasformazione cinematica di $i = 1, \ldots, n$ cinematismi collegati in serie tra di loro,

$$q_1(t) = f_1(p_1(t)), q_2(t) = f_2(p_2(t)), \ldots, q_n(t) = f_n(p_n(t))$$

ricordando che per due cinematismi in cascata il cedente del primo è il movente del secondo:

$$q_1(t) = p_2(t), \ldots, q_{n-1}(t) = p_n(t)$$

si ottiene:

$$q_n(t) = f_n(f_{n-1}(\ldots(f_1(p_1(t)))\ldots))$$

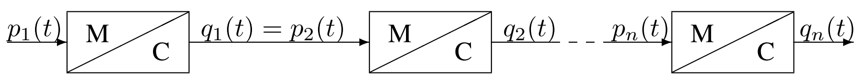
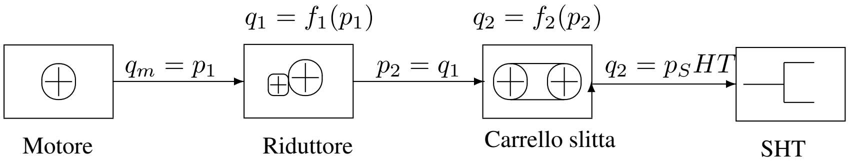


Figure 2.7: Schema a blocchi relativa ad una catena cinematica.

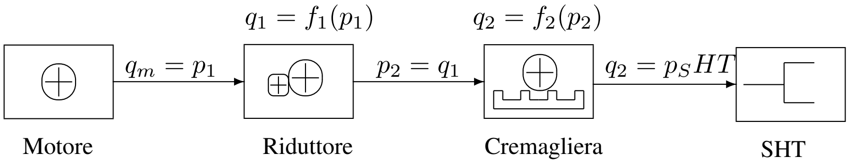
▽ *Example 2.3: Trasformazioni cinematiche composte.*

Consideriamo ancora l'esempio rappresentato dalle figure 2.6 e 2.12. Si supponga che i parametri dei cinematismi raffigurati siano come da tabella seguente:

Nastro trasportatore	
Rapporto di riduzione del riduttore	$Q_1 = 0.10$
Raggio della ruota	$Q_2 = 10$ cm
Taglierina	
Rapporto di riduzione del riduttore	$R_1 = 0.3$
Raggio del pignone	$R_2 = 5$ cm



(a) Trasformazioni cinematiche del nastro trasportatore



(b) Trasformazioni cinematiche della taglierina

Figure 2.8: Trasformazioni cinematiche.

Le catene cinematiche composte relative al nastro trasportatore ed alla taglierina sono raffigurate, rispettivamente, in fig. 2.8a, e fig. 2.8b.

Le funzioni cinematiche composte sono, sia per il nastro trasportatore che per la taglierina

$$q_1 = f_1(p_1) = f_1(q_m) \quad p_{SHT} = q_2 = f_2(p_2) = f_2(q_1) = f_2(f_1(q_m))$$

sostituendo i valori numerici delle funzioni, per il nastro trasportatore otteniamo

$$q_1 = f_1(p_1) \Rightarrow q_1 = Q_1 p_1; \quad q_2 = f_2(p_2) \Rightarrow q_2 = Q_2 p_2; \quad p_{SHT} = Q_2 Q_1 q_m \tag{2.3}$$

e nel caso della taglierina

$$q_1 = f_1(p_1) \Rightarrow q_1 = R_1 p_1; \quad q_2 = f_2(p_2) \Rightarrow q_2 = R_2 p_2; \quad p_{SHT} = R_2 R_1 q_m \tag{2.4}$$

Nel calcolo numerico occorre fare attenzione alle unità di misura. Consideriamo il caso del nastro trasportatore, se usassimo come unità di misura della posizione del motore e del nastro trasportatore rispettivamente i gradi ($[q_m] =^o$) e i millimetri ($[p_S HT] = mm$), otterremmo che la funzione cinematica composta non fornisce dei valori congruenti.

Infatti, si supponga che il motore fornisca uno spostamento di 3600^o (pari a dieci rivoluzioni complete), dalla relazione (2.3) si otterrebbe:

$$q_S HT = 10 \times 0.1 \times 3600 = 3600[mm]$$

cioè un valore palesemente errato.

Per poter eseguire correttemente i calcoli occorre omogeneizzare le unità di misura dei parametri dei cinematismi (nel caso in esame, il raggio della ruota, in quanto il rapporto di riduzione è un numero puro), ed in particolare risulta conveniente esprimere l'angolo di rotazione del motore in radianti[1], ed usare il *Sistema Internazionale* per le altre misure.

In tale caso si ottiene:

$$q_S HT = 0.010 \times 0.1 \times 2\pi \times 10 = 0.02\pi[m] \equiv 0.0628[m]$$

$\triangle$

[1]Si ricorderà che la misura di una circonferenza è esattamente $2\pi r$, r raggio della circonferenza, quindi se esprimiamo la misura dell'angolo in radianti, non occorre effettuare nessuna conversione per calcolare la misura della posizione del cedente. Occorre sottolineare che nella pratica industriale sono quasi sempre usati i gradi, quindi occorrerà usare il rapporto di conversione gradi/radianti: $K = 360/2\pi$

2.4 Analisi della catena cinematica

I problemi di maggiore interesse per il progetto di una catena cinematica sono essenzialmente di due tipi:

- Come progettare una traiettoria per un attuatore una volta definito il movimento desiderato per il SHT ?
- Come dimensionare un motore elettrico in coppia, velocità e potenza per consentire di raggiungere determinate prestazioni in termini dei velocità ed accelerazione del SHT secondo le traiettorie specificate?

La risoluzione di questi due problemi passa attraverso alle analisi della **cinematica inversa** e della **dinamica inversa (o cinetostatica)** del cinematismo.

Lo studio della cinematica consiste nella analisi delle trasformazioni geometriche delle posizioni, velocità ed accelerazioni indotte dai vincoli tra i cinematismi in ogni punto della catena cinematica. La cinematica viene detta **diretta** quando questo studio è propagato in modo ordinato dall'attuatore verso il MOS, mentre si dice **inversa** quando lo studio è condotto in senso contrario.

Nella analisi cinetostatica si legano le accelerazioni alle masse/inerzie, le velocità ai coefficienti di attrito e si considerano le coppie/forze di carico per determinare l'energia richiesta per eseguire il movimento desiderato. L'analisi cinetostatica porta al dimensionamento in potenza, coppia/forza e velocità degli attuatori e dei cinematismi.

2.5 Analisi cinematica

2.5.1 Analisi cinematica diretta

Nella analisi cinematica diretta, si suppongono noti i profili di moto agli attuatori e si compongono le funzioni rappresentanti i cinematismi fino a determinare i profili di moto fino ai SHT . Il problema, riassunto nella Tabella 2.2, è tipicamente di analisi o verifica.

Dati noti:	Profili di moto agli attuatori
Obbiettivo dello studio:	La verifica o l'analisi dei parametri cinematici in un qualunque punto della catena (in particolare sui SHT)
Direzione dei moti:	ACT $\Rightarrow$ SHT Moventi $\Rightarrow$ Cedenti
Difficoltà della analisi:	Nessuna, si applicano ripetutamente le funzioni che descrivono le trasformazioni cinematiche

Table 2.2: Tabella riassuntiva sulla analisi cinematica diretta.

Lo studio che ci si propone di eseguire consiste nel determinare le relazioni tra i parametri **cinematici** del movente $q(t)$ e del cedente $p(t)$ una volta nota la $f : p(t) \mapsto q(t)$, in modo di poter risolvere i problemi di cinematica diretta ed inversa.

In base alle proprietà della derivata di funzioni di funzioni, è immediato determinare le relazioni riassunte nella tabella 2.3 valida per la cinematica diretta:

Dalla tabella possiamo ricavare due osservazioni:

- La velocità fisica del cedente è data dal prodotto della velocità fisica del movente per la pendenza della legge di moto $f' = \frac{df}{dp}$. La pendenza della legge di moto prende il nome di **velocità geometrica** del profilo.

Parametro cinematico	Movente	Cedente
Posizione	$p(t)$	$q(t) = f(p(t))$
Velocità	$\dot{p}(t)$	$\dot{q}(t) = \dfrac{df}{dp}\dot{p}(t)$
Accelerazione	$\ddot{p}(t)$	$\ddot{q}(t) = \dfrac{d^2 f}{dp^2}\dot{p}(t)^2 + \dfrac{df}{dp}\ddot{p}(t)$

Table 2.3: Equazioni per il calcolo della cinematica diretta.

- L'accelerazione fisica del cedente è data dalla somma di due termini, il primo dipende dalla curvatura della legge di moto $f'' = \frac{d^2 f}{dp^2}$ (**accelerazione geometrica** del profilo di moto) e dal quadrato della velocità del movente, il secondo dipende dalla accelerazione del movente. Nel caso in cui il movente abbia velocità costante (tipico caso delle rotazioni uniformi) rimane solo il termine dipendente dal quadrato della velocità del movente.

▽ *Example 2.4: Analisi cinematica diretta di un cinematismo lineare*

Consideriamo un cinematismo descritto da una funzione cinematica lineare

$$q(t) = f(p(t)) = Kp(t)$$

in questa categoria troviamo i cinematismi semplici come la *cremagliera lineare*, il *carrello slitta*, ed i *riduttori*.

In questo caso la *velocità geometrica* è pari a K, mentre l'*accelerazione geometrica* è nulla. La velocità ed accelerazione del cedente sono pari a:

$$\dot{q}(t) = K\dot{p}(t)$$
$$\ddot{q}(t) = K\ddot{p}(t)$$

△

▽ *Example 2.5: Analisi cinematica di un sistema di produzione.*

Consideriamo ancora il sistema di produzione costituito da un nastro trasportatore e una taglierina (Fig. 2.12).

In base alle relazioni cinematiche determinate dalle Eq. (2.3) per il nastro trasportatore otteniamo:

$$\begin{aligned} p_{SHT}(t) &= Q_2 Q_1 q_m(t) \\ \dot{p}_{SHT}(t) &= Q_2 Q_1 \dot{q}_m(t) \\ \ddot{p}_{SHT}(t) &= Q_2 Q_1 \ddot{q}_m(t) \end{aligned} \tag{2.5}$$

e relazioni analoghe per il sistema a cremagliera che governa il moto della taglierina.

Supponiamo ora di assegnare una legge di moto in accelerazione costante al motore del nastro trasportatore per un tempo t_a, per poi assegnare una legge di moto a velocità costante.

$$\ddot{q}_m(t) = a, \quad t \in [0, t_a]$$

Integrando tale relazione otteniamo l'equazione della velocità:

$$\dot{q}_m(t) = \int_0^t a d\tau = [at]_{\tau=0}^{\tau=t}$$

imponendo velocità nulla all'inizio della traiettoria, otteniamo:

$$\dot{q}_m(t) = at, \quad t \in [0, t_a]$$

Integrando ancora la relazione precedente otteniamo l'equazione della posizione:

$$q_m(t) = \int_0^t at d\tau = \left[\frac{1}{2}at^2\right]_{\tau=0}^{\tau=t}$$

imponendo posizione nulla all'inizio della traiettoria, otteniamo:

$$q_m(t) = \frac{1}{2}at^2, \quad t \in [0, t_a]$$

I parametri cinematici del tratto a velocità costante vengono calcolati ragionando come nel caso precedente

$$\begin{aligned} &\dot{q}_m(t) = v, \quad t > t_a \\ &q_m(t) = v(t - t_a) + \tfrac{1}{2}at_a^2, \quad t > t_a \end{aligned}$$

Il tempo t_a e la velocità costante v sono legate fra di loro dal vincolo di continuità in t_a, in cui debbono valere entrambe le relazioni $\dot{q}_m(t_a) = v$:

$$v = at_a; \Rightarrow t_a = \frac{v}{a}$$

Le equazioni cinematiche possono quindi essere scritte come:

$$\begin{aligned} &p_{SHT}(t) = Q_2Q_1\tfrac{1}{2}at^2, \quad t \in [0, t_a] \\ &p_{SHT}(t) = Q_2Q_1(v(t - t_a) + \tfrac{1}{2}at_a^2), \quad t > t_a \\ \\ &\dot{p}_{SHT}(t) = Q_2Q_1at, \quad t \in [0, t_a] \\ &\dot{p}_{SHT}(t) = Q_2Q_1v, \quad t > t_a \\ \\ &\ddot{p}_{SHT}(t) = Q_2Q_1a, \quad t \in [0, t_a] \\ &\ddot{p}_{SHT}(t) = 0, \quad t > t_a \end{aligned} \tag{2.6}$$

△

▽ *Example 2.6: Cinematismo non lineare*

Effettuaiamo ora l'analisi cinematica diretta di un manovellismo di spinta del tipo "biella-manovella". Dalla (Eq. 2.1) otteniamo le relazioni che descrivono la *velocità* ed *accelerazioni geometriche* del cinematismo:

$$
\begin{aligned}
f = &\quad r\,(1-\cos(\alpha)) + l\left(1-\sqrt{1-\frac{r^2\sin(\alpha)^2}{l^2}}\right)\\
f' = &\quad r\,\sin(\alpha) + \frac{r^2\,\cos(\alpha)\,\sin(\alpha)}{l\sqrt{1-\frac{r^2\sin(\alpha)^2}{l^2}}}\\
f'' = &\quad r\left(\cos(\alpha) + \frac{l\,r\sqrt{1-\frac{r^2\sin(\alpha)^2}{l^2}}\,\left(l^2\cos(\alpha)^2 - l^2\sin(\alpha)^2 + r^2\sin(\alpha)^4\right)}{\left(l^2-r^2\sin(\alpha)^2\right)^2}\right)
\end{aligned}
\tag{2.7}
$$

In figura 2.9 sono visualizzate le precedenti relazioni, ottenute per $\alpha \in [0, 2\pi]$, calcolate ponendo $r = 1, l = 2$.

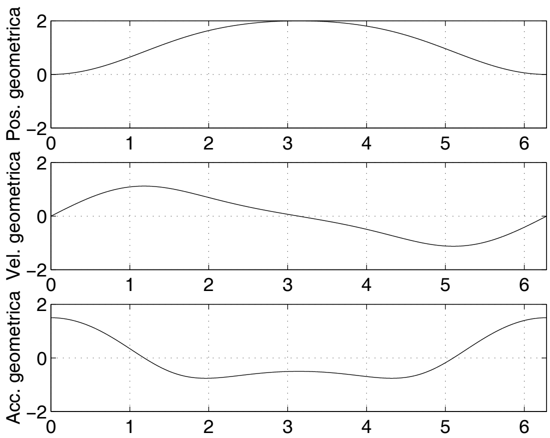


Figure 2.9: Trasformazioni cinematiche del manovellismo di spinta semplice.

Ipotizzando una velocità angolare costante di rotazione del movente $\dot{\alpha}(t) = \omega = \frac{2\pi}{T}$, T periodo di rivoluzione del movente, dalla tabella 2.3, otteniamo:

$$
\begin{aligned}
\dot{x} = &\quad \left[r\,\sin(\alpha) + \frac{r^2\,\cos(\alpha)\,\sin(\alpha)}{l\sqrt{1-\frac{r^2\sin(\alpha)^2}{l^2}}}\right]\omega\\
\ddot{x} = &\quad r\left(\cos(\alpha) + \frac{l\,r\sqrt{1-\frac{r^2\sin(\alpha)^2}{l^2}}\,\left(l^2\cos(\alpha)^2 - l^2\sin(\alpha)^2 + r^2\sin(\alpha)^4\right)}{\left(l^2-r^2\sin(\alpha)^2\right)^2}\right)\omega^2
\end{aligned}
\tag{2.8}
$$

In figura 2.10 sono mostrate la velocità ed accelerazione assolute del cedente per $\omega = 2rad/sec$. Come si può notare i grafici sono ottenuti moltiplicando i valori dei grafici di Fig. 2.9 rispettivamente per la velocità e per il quadrato della velocità.

△

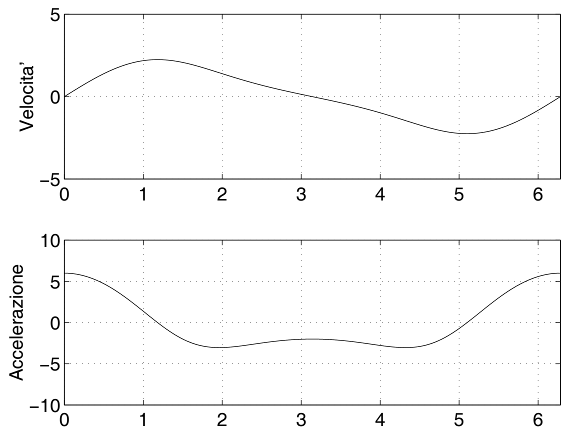


Figure 2.10: Velocità ed accelerazione assolute del cedente di un manovellismo di spinta semplice.

2.5.2 Analisi cinematica inversa

Nella cinematica inversa sono noti i profili di moto da imporre ai SHT e, tramite la conoscenza delle funzioni descriventi gli elementi della catena cinematica, si trasformano i profili indietro fino agli attuatori.

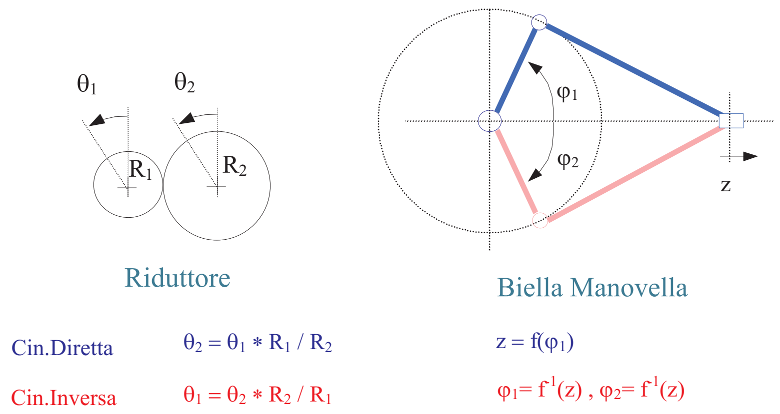
Il problema consiste nella sintesi delle le leggi di moto degli attuatori al fine di avere il moto desiderato ai SHT . La cinematica inversa è in generale più complessa della cinematica diretta a causa della non univocità delle relazioni inverse dei cinematismi. Il problema della analisi cinematica inversa è riassunto in Tabella 2.4.

Dati noti:	Profili di moto ai MOS
Obbiettivo dello studio:	Sintesi dei profili di moto agli attuatori
Direzione dei moti:	MOS ⇒ ATT Cedenti ⇒ Moventi
Difficoltà della a analisi:	Se nella catena cinematica sono presenti funzioni non invertibili, si ottengono dei punti singolari nelle funzioni di cinematica inversa

Table 2.4: Tabella riassuntiva sulla analisi cinematica inversa.

▽ *Example 2.7: Cinematica diretta ed inversa (Figura 2.11)*
Si consideri un riduttore di velocità meccanico. Il riduttore è un cinematismo biunivoco in quanto la relazione diretta tra cedente e movente ammette una unica relazione inversa, per cui la complessità del problema della cinematica inversa è la medesima della cinematica diretta. Invece nel caso della biella–manovella la relazione cinematica diretta è univoca, ossia nota la posizione della manovella è unica la posizione del piede di biella, invece la relazione inversa non è unica infatti per ogni posizione del piede di biella la manovella può trovarsi in due punti differenti.

I cinematismi composti da aste presentano punti di singolarità in cinematica inversa quando due o più aste sono allineate. Questi punti corrispondono a biforcazioni del moto.
△



	Riduttore	Biella-manovella
Cinematica Diretta	$\theta_2 = \theta_1 \times R_1/R_2$	$z = f(\phi_1)$
Cinematica Inversa	$\theta_1 = \theta_2 \times R_2/R_1$	$\phi_1 = f^{-1}(z), \phi_2 = f^{-1}(z)$

Figure 2.11: Relazioni cinematiche invertibili e non invertibili

In base alla conoscenza della funzione che esprime la relazione geometrica inversa tra movente e cedente, $p(t) = f^{-1}(q(t))$, è possibile stilare le funzioni valide per la cinematica inversa, mostrate in tabella 2.5, dove si è posto $f^{-1}(.) = g(.)$.

Parametro cinematico	Cedente	Movente
Posizione	$q(t)$	$p(t) = g(q(t))$
Velocità	$\dot{q}(t)$	$\dot{p}(t) = \frac{dg}{dq}\dot{q}(t)$
Accelerazione	$\ddot{q}(t)$	$\ddot{p}(t) = \frac{d^2g}{dq^2}\dot{q}(t)^2 + \frac{dg}{dq}\ddot{q}(t)$

Table 2.5: Equazioni per il calcolo della cinematica inversa.

Il calcolo della cinematica inversa può essere derivato completamente delle equazioni della cinematica diretta, $f(.)$ e delle sue derivate geometriche $f'(.)$ e $f''(.)$, consentendo una notevole semplificaizone computazionale. Infatti, in base ai teoremi sulla derivata inversa[2], è possibile scrivere: $g' = \frac{dg}{dq} = \frac{1}{f'} = \frac{dp}{df}$, e $g'' = \frac{d^2g}{dq^2} = -\frac{f''}{(f')^3}$, ottenendo la tabella 2.6.

[2]Sia $y = f(x)$ la funzione diretta e $x = g(y)$ la funzione inversa. La derivata della funzione inversa è definita come $g'(y) := \frac{dg(y)}{dy}$ che per il teorema della derivata della funzione inversa vale $g' = \frac{1}{f'(x)}$con $f'(x) = \frac{df}{dx}$.

La derivata seconda della funzione inversa vale $g''(y) := \frac{dg'}{dy}$, ma $dy = f'(x)dx$ (per definizione dalla funzione diretta) quindi $\frac{dg'}{dy} = \frac{dg'}{(f'dx)} = \frac{1}{f'(x)}\frac{dg'}{dx} = \frac{1}{f'}\frac{d(\frac{1}{f'(x)})}{dx}$ e quindi $g'' = -\frac{f''}{(f')^3}$.

Parametro cinematico	Cedente	Movente
Posizione	$q(t)$	$p(t) = g(q(t))$
Velocità	$\dot{q}(t)$	$\dot{p}(t) = \frac{\dot{q}(t)}{f'}$
Accelerazione	$\ddot{q}(t)$	$\ddot{p}(t) = -\frac{f''}{(f')^3}\dot{q}(t)^2 + \frac{\ddot{q}(t)}{f'}$

Table 2.6: Equazioni per il calcolo della cinematica inversa derivate dalla cinematica diretta.

$\bigtriangledown$ *Example 2.8: Analisi cinematica inversa di un cinematismo lineare*

Consideriamo ancora un cinematismo descritto da una funzione cinematica lineare

$$q(t) = f(p(t)) = Kp(t)$$

In questo caso l'inverso della *velocità geometrica* è pari a $1/K$, mentre l'*accelerazione geometrica* è nulla. La velocità ed accelerazione del cedente sono pari a:

$$\dot{p}(t) = \frac{1}{K}\dot{q}(t)$$
$$\ddot{p}(t) = \frac{1}{K}\ddot{q}(t)$$

$\bigtriangleup$

$\bigtriangledown$ *Example 2.9: Analisi cinematica inversa di un sistema di produzione.*

Consideriamo ancora il sistema di produzione costituito da un nastro trasportatore e una taglierina (Fig. 2.12).
In base alle relazioni cinematiche determinate dalle Eq. (2.3) ed in base alle relazioni espresse nella tabella 2.5 otteniamo:

$$\begin{aligned} q_m(t) &= \tfrac{1}{Q_2 Q_1} p_{SHT}(t) \\ \dot{q}_m(t) &= \tfrac{1}{Q_2 Q_1} \dot{p}_{SHT}(t) \\ \ddot{q}_m(t) &= \tfrac{1}{Q_2 Q_1} \ddot{p}_{SHT}(t) \end{aligned} \tag{2.9}$$

e relazioni analoghe per il sistema a cremagliera che governa il moto della taglierina.
Queste relazioni sono utilizzate per definire la legge di moto da assegnare al motore elettrico una volta fissata la legge di moto richiesta per il SHT .
Se, ad esempio, si volesse assegnare una legge di moto al SHT composta da un tratto iniziale ad accelerazione costante raccordato con un tratto a velocità costante, si potrebbe fare riferimento alle Eq. 2.6, riscritte secondo la cinematica inversa

$$
\begin{aligned}
&q_m(t) = \tfrac{1}{Q_2 Q_1}\tfrac{1}{2}at^2, \quad t \in [0, t_a] \\
&q_m(t) = \tfrac{1}{Q_2 Q_1}(vt + \tfrac{1}{2}at_a^2), \quad t > t_a \\
\\
&\dot{q}_m(t) = \tfrac{1}{Q_2 Q_1}at, \quad t \in [0, t_a] \\
&\dot{q}_m(t) = \tfrac{1}{Q_2 Q_1}v, \quad t > t_a \\
\\
&\ddot{q}_m(t) = \tfrac{1}{Q_2 Q_1}a, \quad t \in [0, t_a] \\
&\ddot{q}_m(t) = 0, \quad t > t_a
\end{aligned}
\tag{2.10}
$$

△

▽ *Example 2.10: Cinematismo non lineare*

L'analisi cinematica inversa che coinvolge cinematismi la cui trasformazione diretta è non biunivoca necessita di una trattazione particolare. In questo caso, infatti, ad una posizione del movente corrispondono due (o più) posizioni del cedente, e quindi la cinematica inversa conduce a considerare due (o più) trasformazioni equivalenti.
△

2.6 Motion profiles

Con il termine di progettazione di una **legge di moto** si intende la determinazione di un profilo di moto per il SHT (e quindi, per propagazione a ritroso nella catena cinematica, all'attuatore) adatto al raggiungimento della produzione a cui la macchina è destinata.

Nel funzionamento delle macchine automatiche è di interesse generale la generazione di moti periodici, cioè moti che si ripetono identicamente ad intervalli regolari di tempo[3]

In particolare, risultatono di interesse nella progettazione delle macchine automatiche la generazione di **moti coordinati** fra diversi dispositivi che concorrono al fine della produzione.

▽ *Example 2.11: Moto coordinato per un sistema di taglio*

Consideriamo ancora una linea di produzione composta da un nastro trasportatore che movimenta alcuni pezzi avvolti da un unico foglio di carta da incarto introdotto nell'esempio 2 di pagina 15. La carta viene separata da una taglierina che taglia l'incarto in modo sincrono con l'avanzamento del prodotto (Fig. 2.12)

Si supponga che il prodotto abbia una dimensione di 5 cm e che tra due pezzi successivi vi sia una distanza fissa di 5 cm [4]. Il coltello deve tagliare l'involucro che avvolge il prodotto in modo che il taglio lasci 2.5 cm di margine da entrambe le parti. All'inizio del moto la situazione è mostrata dalla figura 2.13, mentre a metà percorso la configurazione del sistema è mostrata in figura 2.14.

La legge di moto della taglierina deve essere progettata in modo da essere sempre sincrona alla velocità di avanzamento del nastro e commisurata alla dimensione del prodotto.

[3]La periodicità si riferisce all'andamento della velocità, e non necessariamente a quello dello spostamento, che risulta periodico solo per moti alternativi
[4]Ordinare il prodotto prima della lavorazione è una fase importante di ogni automatismo. In pratica occorre assicurare che il prodotto arrivi in modo regolare alla parte produttiva della macchina

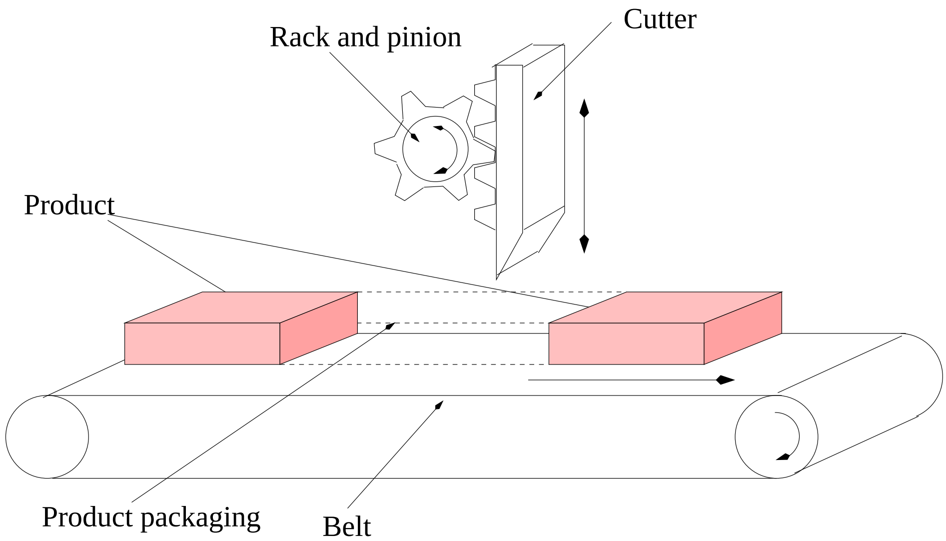


Figure 2.12: Schema della linea di produzione.

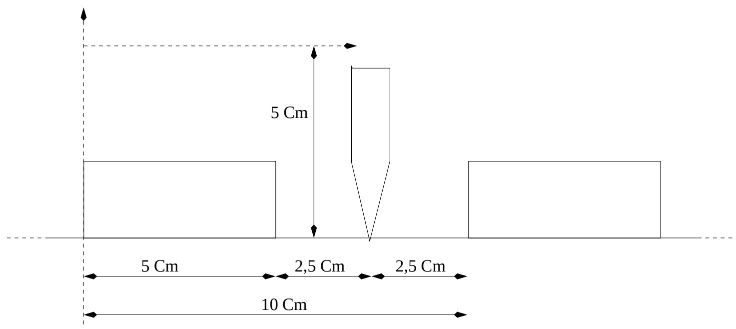


Figure 2.13: Situazione del sistema all'inizio (fine) dell'operazione di taglio.

△

In base a questo esempio, possiamo concludere che le esigenze di generazione di una legge di moto per una macchina automatica sono di due tipi:

1. Il moto è periodico, a ciascun periodo corrisponde, in genere, la lavorazione di una unità produttiva della macchina.

2. Il moto è coordinato, cioè occorre progettare le leggi di moto dei vari dispositivi che concorrono alla produzione in modo coordinato.

L'approccio comune utilizzato per risolvere il problema della generazione delle leggi di moto è riassunto in Fig. 2.15. Il sistema è composto da un asse "master", che genera una variabile di sincronizzazione $p(t)$, e n assi "slave", che generano le leggi di moto $q_1(p), \dots, q_n(p)$ progettate per ottenere la produzione richiesta in modo sincrono rispetto all'asse master utilizzando la variabile $p(t)$.

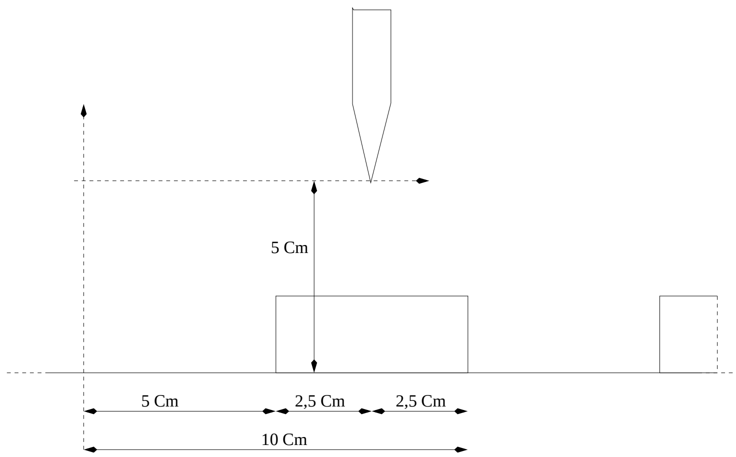


Figure 2.14: Situazione del sistema a metà dell'operazione di taglio.

Solitamente la legge di moto dell'asse master[5] è valutata in modulo Γ, essendo Γ la variazione di quota dell'asse master che corrisponde alla lavorazione di un singolo prodotto, mentre indicheremo nel seguito con il simbolo T il tempo di lavorazione del singolo prodotto. Ad esempio, una legge di moto a velocità costante per il master sarà descrivibile attraverso la relazione $p(t) = \omega\, t$, essendo $\omega = \Gamma/T$.

Il generico asse slave implementa una legge di moto del tipo:

$$q(t) = f(p(t))\ ,\ p \in [0, \Gamma] \tag{2.11}$$

La funzione $q = f(p)$ è l'espressione matematica della legge di moto che deve essere implementata per il raggiungimento della produzione desiderata, e viene anche chiamata con il nome di **diagramma delle alzate**.

La soluzione elettronica alla generazione di leggi di moto consente di eliminare gran parte della complessità nelle catene cinematiche di trasmissione del movimento, rimandando alla scrittura di software adeguato la soluzione del problema del progetto del diagramma delle alzate.

2.6.1 Progetto del diagramma delle alzate.

Il problema di progetto del diagramma delle alzate consiste nel determinare la legge di moto $q = f(p)$ tale che il dispositivo controllato abbia un funzionamento adeguato ai fini produttivi della macchina.

Il problema di generazione della legge di moto è quindi un problema vincolato, dove i vincoli sono:

- m vincoli sulla posizione $q(t_i), i = 1, \ldots, m$ rispetto all'asse master $p(t_i)$, $i = 1, \ldots, m$, in modo da ottenere la produzione desiderata.
- Vincoli sulle velocità per consentire di sincronizzare le velocità di più assi (traiettorie ad inseguimento).
- Vincoli sulla accelerazione massima e velocità massima dell'asse, in quanto i motori elettrici debbonono essere dimensionati sulle potenze e coppie massime richieste dai carichi meccanici.
- Vincoli sulle variazioni di velocità (accelerazione) e sulle variazioni di accelerazione (jerk), in quanto si vogliono evitare vibrazioni meccaniche che potrebbero portare alla distruzione della macchina.

[5] L'asse master può essere un sistema virtuale non fisicamente presente nella macchina, ma che viene implementato nel sistema di controllo della macchina automatica.

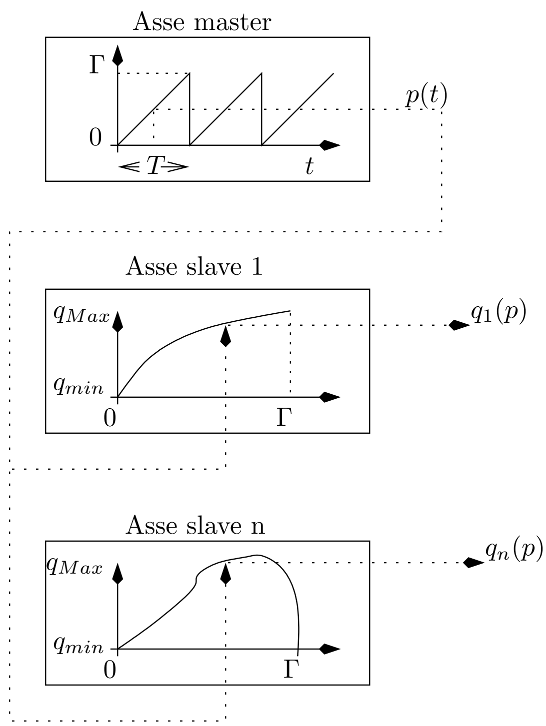


Figure 2.15: Configurazione di controllo del moto del tipo "Master-Slave".

Il progetto della legge di moto, quindi, consiste nel definire una funzione parametrizzata $q(p, a_0, a_1, \ldots, a_n)$ ed assegnare i valori numerici dei parametri $a_0, a_1, \ldots a_n$ in base alle specifiche da rispettare.

Nel seguito del capitolo vengono introdotte le principali famiglie di funzioni parametrizzate utilizzate comunemente nella pratica industriale.

2.6.2 Traiettorie polinomiali

La traiettoria polinomiale è descritta da un polinomio nella variabile p di ordine n:

$$q = f(p) = a_0 + a_1 t + a_2 t^2 + \ldots + a_n t^n \tag{2.12}$$

in cui i parametri incogniti $a_0, \ldots, a_n$ si determinano impostando un sistema di equazioni ottenuto imponendo vincoli di continuità nei valori iniziali e finali della variabile master t_i e t_f, sulla (2.12) e sulle sue derivate successive:

$$
\begin{array}{lcccccccl}
f(t_i) = & a_0 & + & a_1 t_i & + & a_2 t_i^2 & + & \ldots + a_n t_i^n \\
f(t_f) = & a_0 & + & a_1 t_f & + & a_2 t_f^2 & + & \ldots + a_n t_f^n \\
\\
f'(t_i) = & & & a_1 & + & 2a_2 t_i & + & \ldots + n a_n t_i^{n-1} \\
\ldots \\
f''(t_i) = & & & & & 2a_2 & + & \ldots + n(n-1) a_n t_i^{n-2} \\
\ldots
\end{array}
\tag{2.13}
$$

dove con $f', f'', \ldots$ si indicano le derivate prima, seconda, ecc. della funzione $f(p)$

La scelta del grado n del polinomio è fatta in base al numero di vincoli che si vuole rispettare. In particolare scegliendo un polinomio del 3^o ordine, possiamo imporre vincoli sulla posizione e la velocità agli estremi della traiettoria, mentre la scelta di un polinomio del 5^o ordine permette di fissare anche l'accelerazione agli estremi della traiettoria.

Traiettoria polinomiale del 3^o ordine La traiettoria polinomiale del terzo ordine è definita dalla funzione:

$$f(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \tag{2.14}$$

i cui parametri incogniti si ottengono risolvendo il sistema di equazioni per $t_i = 0$ e $t_f = \Gamma$:

$$
\begin{array}{lcl}
f(0) & = & a_0 \\
f'(0) & = & a_1 \\
f(\Gamma) & = & a_0 + a_1\Gamma + a_2\Gamma^2 + a_3\Gamma^3 \\
f'(\Gamma) & = & a_1 + 2a_2\Gamma + 3a_3\Gamma^2
\end{array}
\tag{2.15}
$$

$$
\begin{array}{lcl}
a_0 & = & f(0) \\
a_1 & = & f'(0) \\
a_2 & = & \dfrac{3(f(\Gamma) - f(0)) - \Gamma(2f'(0) + f'(\Gamma))}{\Gamma^2} \\
a_3 & = & -\dfrac{2(f(\Gamma) - f(0)) - \Gamma(f'(0) + f'(\Gamma))}{\Gamma^3}
\end{array}
$$

In figura 2.16 è mostrata una traiettoria polinomiale cubica ottenuta imponendo i vincoli $f(0) = 0, f'(0) = 0, f(\Gamma) = 20, f'(\Gamma) = 0$ utilizzando la funzione Matlab `poly3.m` con l'istruzione:

```
>> poly3(10,[0,0,20,0],'stampa')
```

È possibile notare, in figura, che l'accelerazione è discontinua in $t = 0$ e $t = \Gamma$ (all'inizio ed alla fine della traiettoria), mentre posizione e velocità sono continui. Ovviamente il jerk risulta infinito nei punti di discontinuità della accelerazione.

Calcolo semplificato. In molti casi il movimento richiesto ha velocià iniziale e finale nulle, onde per cui le equazioni 2.15 si modificano in

$$
\begin{array}{lcl}
f(0) & = & a_0 \\
0 & = & a_1 \\
f(0) + h & = & a_0 + a_1\Gamma + a_2\Gamma^2 + a_3\Gamma^3 \\
0 & = & a_1 + 2a_2\Gamma + 3a_3\Gamma^2
\end{array}
\tag{2.16}
$$

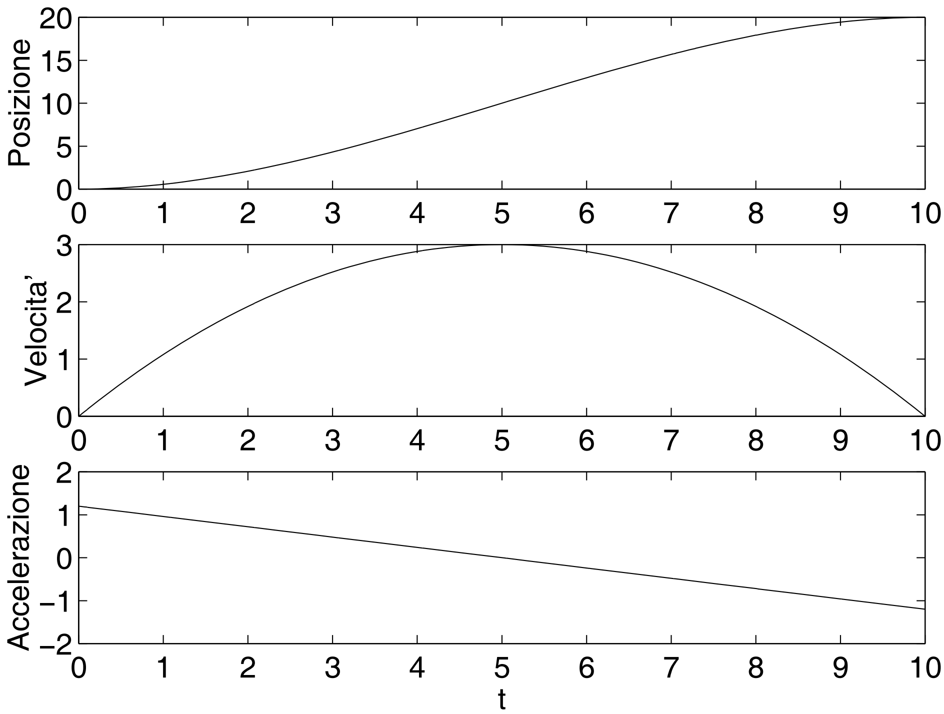


Figure 2.16: Traiettoria di tipo polinomiale cubico

avendo posto $h = f(\Gamma) - f(0)$. I coefficienti del polinomio si possono calcolare con le formule:

$$\begin{aligned} a_0 &= f(0) \\ a_1 &= 0 \\ a_2 &= \frac{3h}{\Gamma^2} \\ a_3 &= -\frac{2h}{\Gamma^3} \end{aligned}$$

e quindi il polinomio può essere scritto come

$$\frac{y}{h} = 3\left(\frac{t}{\Gamma}\right)^2 - 2\left(\frac{t}{\Gamma}\right)^3$$

avendo posto $y = f(t) - f(0)$

Traiettoria polinomiale del 5^o ***ordine*** La traiettoria polinomiale del quinto ordine è definita dalla funzione:

$$f(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 \tag{2.17}$$

i cui parametri incogniti si ottengono risolvendo il sistema di equazioni per $t_i = 0$ e $t_f = \Gamma$:

$$\begin{aligned}
f(0) &= a_0 \\
f'(0) &= a_1 \\
f''(0) &= 2a_2 \\
f(\Gamma) &= a_0 + a_1\Gamma + a_2\Gamma^2 + a_3\Gamma^3 + a_4\Gamma^4 + a_5\Gamma^5 \\
f'(\Gamma) &= a_1 + 2a_2\Gamma + 3a_3\Gamma^2 + 4a_4\Gamma^3 + 5a_5\Gamma^4 \\
f''(\Gamma) &= 2a_2 + 6a_3\Gamma + 12a_4\Gamma^2 + 20a_5\Gamma^3
\end{aligned} \tag{2.18}$$

$$\begin{aligned}
a_0 &= f(0) \\
a_1 &= f'(0) \\
a_2 &= \frac{f''(0)}{2} \\
a_3 &= \frac{-\left(20\,f(0) - 20\,f(\Gamma) + 3\,f''(0)\,\Gamma^2 - f''(\Gamma)\,\Gamma^2 + 12\,\Gamma\,f'(0) + 8\,\Gamma\,f'(\Gamma)\right)}{2\,\Gamma^3}, \\
a_4 &= \frac{-\left(-30\,f(0) + 30\,f(\Gamma) - 16\,f'(0)\,\Gamma - 14\,f'(\Gamma)\,\Gamma - 3\,f''(0)\,\Gamma^2 + 2\,f''(\Gamma)\,\Gamma^2\right)}{2\,\Gamma^4} \\
a_5 &= \frac{-\left(12\,f(0) - 12\,f(\Gamma) + 6\,f'(0)\,\Gamma + 6\,f'(\Gamma)\,\Gamma + f''(0)\,\Gamma^2 - f''(\Gamma)\,\Gamma^2\right)}{2\,\Gamma^5}
\end{aligned}$$

In figura 2.17 è mostrata una traiettoria polinomiale del quinto ordine ottenuta imponendo i vincoli $f(0) = 0, f'(0) = 0, f''(0) = 0, f(\Gamma) = 20, f'(\Gamma) = 0, f''(\Gamma) = 0$ utilizzando la funzione Matlab `poly5.m` con l'istruzione:

```
>> poly5(10,[0,0,0,20,0,0],'stampa')
```

È possibile notare, in figura, che la posizione, la velocità e l'accelerazione sono continue in $t = 0$ e $t = \Gamma$ (all'inizio ed alla fine della traiettoria). In generale, per una traiettoria polinomiale del quinto ordine, il Jerk risulta essere discontinuo ma limitato.

Calcolo semplificato. Nei casi in cui si richieda velocità ed accelerazioni iniziali e finali nulle, le equazioni 2.18 si semplificano in

$$\begin{aligned}
f(0) &= a_0 \\
0 &= a_1 \\
0 &= 2a_2 \\
f(0) + h &= a_0 + a_1\Gamma + a_2\Gamma^2 + a_3\Gamma^3 + a_4\Gamma^4 + a_5\Gamma^5 \\
0 &= a_1 + 2a_2\Gamma + 3a_3\Gamma^2 + 4a_4\Gamma^3 + 5a_5\Gamma^4 \\
0 &= 2a_2 + 6a_3\Gamma + 12a_4\Gamma^2 + 20a_5\Gamma^3
\end{aligned} \tag{2.19}$$

avendo posto $h = f(\Gamma) - f(0)$, con soluzioni

$$a_0 = f(0), a_1 = 0, a_2 = 0, a_3 = \frac{10h}{\Gamma^3}, a_4 = -\frac{15h}{\Gamma^4}, a_5 = \frac{6h}{\Gamma^5}$$

da cui l'espressione del polinomio:

$$\frac{y}{h} = 10\left(\frac{t}{\Gamma}\right)^3 - 15\left(\frac{t}{\Gamma}\right)^4 + 6\left(\frac{t}{\Gamma}\right)^5$$

avendo posto $y = f(t) - f(0)$

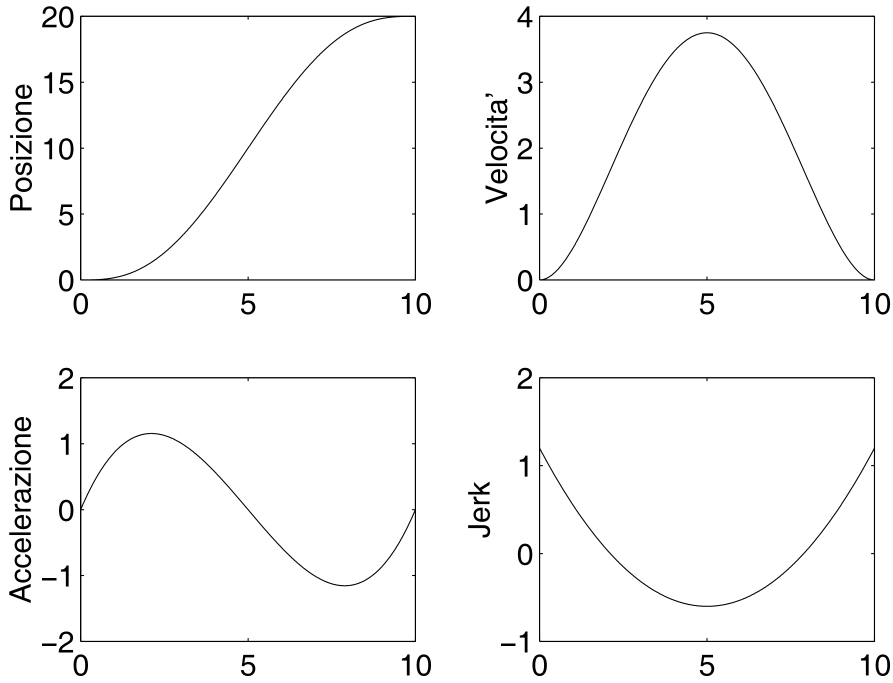


Figure 2.17: Traiettoria di tipo polinomiale del quinto ordine

Traiettoria parabolica (ad accelerazione costante) La traiettoria parabolica è definita attraverso l'unione di due traiettorie del secondo ordine (da cui il nome della legge di moto). Il moto è diviso in due parti simmetriche, una prima parte di accelerazione ed una seconda parte di decelerazione. Trattandosi di leggi descritte da polinomi del secondo ordine, abbiamo che l'accelerazione è costante in modulo e varia in segno nei due tratti.

Questa legge garantisce l'accelerazione minima possibile a parità di alzata e corsa dell'asse master, però non sempre viene scelta nella pratica in quanto i punti di discontinuità nella accelerazione possono condurre ad eccitare vibrazioni molto dannose per le parti meccaniche.

La legge di moto è costituita quindi da due tratti descritti da polinomi del secondo ordine:

$$\begin{aligned} f_1(t) &= a_0 + a_1 t + a_2 t^2 \quad t \in \left[0, \tfrac{\Gamma}{2}\right] \\ f_2(t) &= b_0 + b_1 t + b_2 t^2 \quad t \in \left[\tfrac{\Gamma}{2}, \Gamma\right] \end{aligned} \tag{2.20}$$

i cui sei parametri incogniti si determinano risolvendo il sistema di sei equazioni ottenute imponendo le condizioni relative alle posizioni e velocità iniziali e finali, ed alla continuità nel punto intermedio nella posizione e velocità.

Se, ad esempio, richiediamo posizione iniziale nulla, velocità iniziale e finale nulla, otteniamo il sistema di sei equazioni

$$\begin{array}{rclcl} f_1(0) & = & a_0 & = & 0 \\ f_1'(0) & = & a_1 & = & 0 \\ f_2(\Gamma) & = & b_0 + b_1\Gamma + b_2\Gamma^2 & = & h \\ f_2'(\Gamma) & = & b_1 + 2b_2\Gamma & = & 0 \\ f_1(\Gamma/2) & = & f_2(\Gamma/2) & & \\ f_1'(\Gamma/2) & = & f_2'(\Gamma/2) & & \end{array} \tag{2.21}$$

in cui le ultime due relazioni sono ottenute dai vincoli di continuità nella posizione intermedia.

Risolvendo il sistema si ottiene:

$$a_2 = \frac{2\,h}{\Gamma^2}, b_0 = -h, a_0 = 0, a_1 = 0, b_1 = \frac{4\,h}{\Gamma}, b_2 = \frac{-2\,h}{\Gamma^2}$$

per cui la prima delle eq. 2.20 diviene

$$\begin{array}{rcl} f_1(t) & = & 2\dfrac{h}{\Gamma^2}t^2 \\ f_1'(t) & = & 4\dfrac{h}{\Gamma^2}t \\ f_1''(t) & = & 4\dfrac{h}{\Gamma^2} \end{array} \tag{2.22}$$

mentre la seconda può essere scritta come:

$$\begin{array}{rcl} f_2(t) & = & -h - \dfrac{2\,h\,p^2}{\Gamma^2} + \dfrac{4\,h\,p}{\Gamma} \\ f_2'(t) & = & \dfrac{-4\,h\,p}{\Gamma^2} + \dfrac{4\,h}{\Gamma} \\ f_2''(t) & = & \dfrac{-4\,h}{\Gamma^2} \end{array} \tag{2.23}$$

È interessante notare che la velocità massima è raggiunta nel punto di passaggio dalla fase di accelerazione alla fase di decelerazione, ed è quindi facilmente determinabile da una delle due (2.22) o (2.23), calcolata in $t = \Gamma/2$.

In figura 2.18 è mostrata una traiettoria parabolica ottenuta imponendo i vincoli $f(0) = 0, f(\Gamma) = 20$ utilizzando la funzione Matlab `parabolica.m` con l'istruzione:

```
>> parabolica([1],[0,20],'s');
```

2.6.3 Trapezoidal motion profile

La legge di moto è suddivisa in sette tratti elementari, ciascuno dei quali è valido all'interno di un intervallo di posizioni dell'asse master Γ_i, $i = 1, \ldots, 7$. Negli intervalli $\Gamma_1, \Gamma_3, \Gamma_5$ e Γ_7 è valida una legge di moto polinomiale polinomiale cubica mentre nei tratti Γ_2, Γ_4 e Γ_6 viene utilizzata una legge di moto ad **accelerazione costante**.

Le leggi di moto nei tratti a jerk costante sono ottenuti come generalizzazione della legge cubica:

$$f_p''(t) = a_0 + t\,j_0 \tag{2.24}$$

essendo a_0 il valore dell'accelerazione iniziale e j_0 il valore costante del jerk. Le espressioni della velocità e della posizione sono determinate per successive integrazioni a partire dalla relazione (2.24):

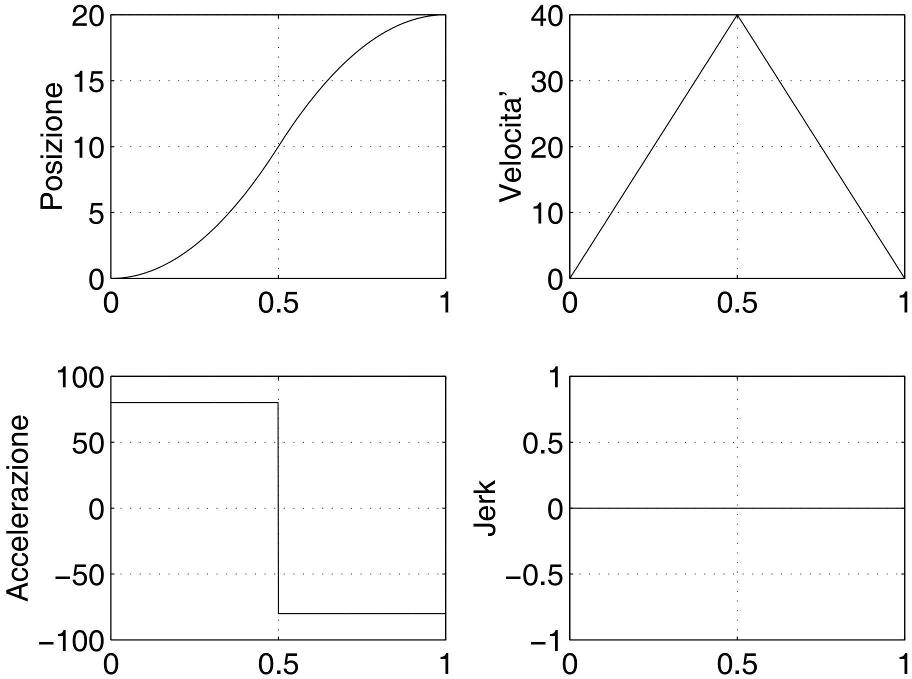


Figure 2.18: Traiettoria di tipo parabolico (ad accelerazione costante)

$$
\begin{aligned}
f_p'(t) &= \int_0^t f_a''(\tau)d\tau + v_0 = \frac{t^2\, j_0}{2} + t\, a_0 + v_0 \\
f_p(t) &= \int_0^t f_a'(\tau)d\tau + q_0 = \frac{t^3\, j_0}{6} + \frac{t^2\, a_0}{2} + t\, v_0 + p_0
\end{aligned}
\tag{2.25}
$$

La legge di moto complessiva è definita sull'intervallo Γ ottenuto come unione dei sottointervalli $\Gamma_i, i = 1, \ldots, 7$, imponendo opportune condizioni al contorno. La legge di moto totale è quindi completamente definita nella seguente tabella:

Intervallo	Funzione	Vincoli
Γ_1	$f_1(t) = f_p(t - t_1), t \in [p_1, p_1 + \Gamma_1]$	$q_0 = Q_0, v_0 = 0, a_0 = 0,$ $j_0 = J_0$
Γ_2	$f_2(t) = f_a(t - t_2), t \in [p_2, p_2 + \Gamma_2]$	$q_0 = f_1(p_2), v_0 = f_1'(p_2),$ $a_{p_2} = f_1''(p_2)$
Γ_3	$f_3(t) = f_p(t - t_3), t \in [p_3, p_3 + \Gamma_3]$	$q_0 = f_2(p_3), v_0 = f_2'(p_3),$ $a_{p_3} = f_2''(p_3), j_0 = J_1$
Γ_4	$f_4(t) = f_a(t - t_4), t \in [p_4, p_4 + \Gamma_4]$	$q_0 = f_3(p_4), v_0 = f_3'(p_4),$ $a_0 = f_3''(p_4)$
Γ_5	$f_5(t) = f_p(t - t_5), t \in [p_5, p_5 + \Gamma_5]$	$q_0 = f_4(p_5), v_0 = f_4'(p_5),$ $a_0 = f_4''(p_5), j_0 = J_2$
Γ_6	$f_6(t) = f_a(t - p_6), t \in [p_6, p_6 + \Gamma_6]$	$q_0 = f_5(p_6), v_0 = f_5'(p_6),$ $a_0 = f_5''(p_6)$
Γ_7	$f_7(t) = f_p(t - t_7), t \in [p_7, p_7 + \Gamma_7]$	$q_0 = f_6(p_7), v_0 = f_6'(p_7),$ $a_0 = f_6''(p_7), j_0 = J_3$

essendo J_0, J_1, J_3 e J_4 i valori di jerk nei tratti $\Gamma_1, \Gamma_3, \Gamma_5$ e Γ_7 della legge di moto. Tali valori si ottengono imponendo i seguenti vincoli di continuità:

$$
\begin{array}{ll}
f_7(p_7 + \Gamma_7) = h + Q_0, & \text{raggiungimento della quota finale} \\
f_7'(p_7 + \Gamma_7) = 0, & \text{velocità finale nulla} \\
f''(p_7 + \Gamma_7) = 0, & \text{accelerazione finale nulla} \\
f''(p_3 + \Gamma_3) = 0, & \text{accelerazione in } \Gamma_4 \text{ nulla}
\end{array}
$$

I passaggi matematici effettuati per ottenere la legge di moto complessiva sono omessi in quanto abbastanza complessi, tuttavia in figura 2.19 è mostrata una legge di moto a doppia "S" ottenuta imponendo i vincoli $h = 20$ e $Q_0 = 0$, utilizzando la funzione Matlab `doppiaesse.m` con l'istruzione:

```
>> doppiaesse([1,1,1,1,1,1,1],[0,20],'stampa')
```

2.7 Scelta della legge di moto

Frequentemente i moti della macchina automatica sono composti da una successione di tratti di salita, o discesa, eventualmente inframezzati da arresti (tratti a velocità nulla), ciascuno dei quali è caratterizzato dalla alzata h e dal corrispondente avanzamento dell'asse master Γ.

In quasi tutti i casi pratici sono assegnati i valori di Γ e h, mentre non sono assegnati vincoli sulla particolare legge di moto $f(t)$, che è determinata in base a vincoli sulle accelerazioni e velocità dell'asse slave.

In particolare, si consideri come esempio l'esecuzione di una alzata unitaria in un periodo Γ unitario. Il diagramma delle alzate ottenute applicando le leggi di moto **polinomiale** del 3^o e 5^o ordine, **cicloidale**, **trapezoidale** e **a doppia "esse"** è mostrato in figura 2.20.

È importante osservare che tutte le leggi di moto $f(t)$ provate hanno andamento abbastanza simile tra di loro, e quindi, in genere, il diagramma delle alzate non fornisce molti elementi per la scelta di una particolare legge di moto.

In genere, i criteri di scelta della legge di moto si basano su ben determinate specifiche, quali:

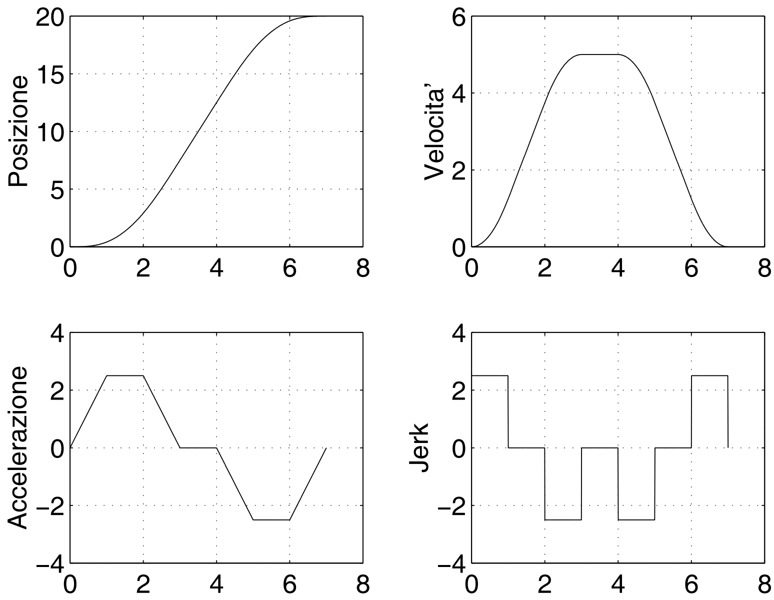


Figure 2.19: Traiettoria di tipo doppia "S".

- *Limitare la velocità massima.*
- *Limitare le accelerazioni massime e RMS.* Infatti, siccome la coppia dipende in modo proporzionale dalla accelerazione, bassi valori di accelerazione di picco e RMS conducono a scegliere motori di taglia più piccola (meno costosi).
- *Continuità fino alla accelerazione* (fino al jerk). Le discontinuità nelle accelerazioni comportano discontinuità nelle forze e nelle coppie, che conducono a sollecitazioni ed urti nella macchina controllata. Infatti discontinuità nelle accelerazioni producono movimenti irregolari nei meccanismi che, in presenza di giochi meccanici, comportano oscillazioni e collisioni delle parti a contatto negli organi di trasmissione del moto, con conseguenze usura e distruzione nel lungo periodo.

  Spesso il prodotto stesso non sopporta accelerazioni molto elevate. Ad esempio, un prodotto trasportato su un nastro tende a scivolare se sottoposto ad accelerazioni discontinue, o troppo elevate.
- *Limitare la banda dello spettro del segnale di accelerazione.* Questo si traduce in un limite di banda sulla corrente nel motore. Occorre infatti tenere presente che il sistema composto dall' azionamento elettrico, dal motore e dal carico meccanico, è un sistema con comportamento di tipo *passa–basso*, e quindi non può seguire fedelmente un segnale con banda spettrale troppo elevata. Quindi accelerazioni/correnti con banda troppo elevata producono:
  - Maggiori errori di inseguimento durante i transitori.
  - Eccitazione di frequenze di risonanza che possono innescare vibrazioni rumorose e dannose.

  Quindi i parametri da osservare per condurre una scelta della legge di moto sono:

  1. Velocità massima.
  2. Accelerazione massima.

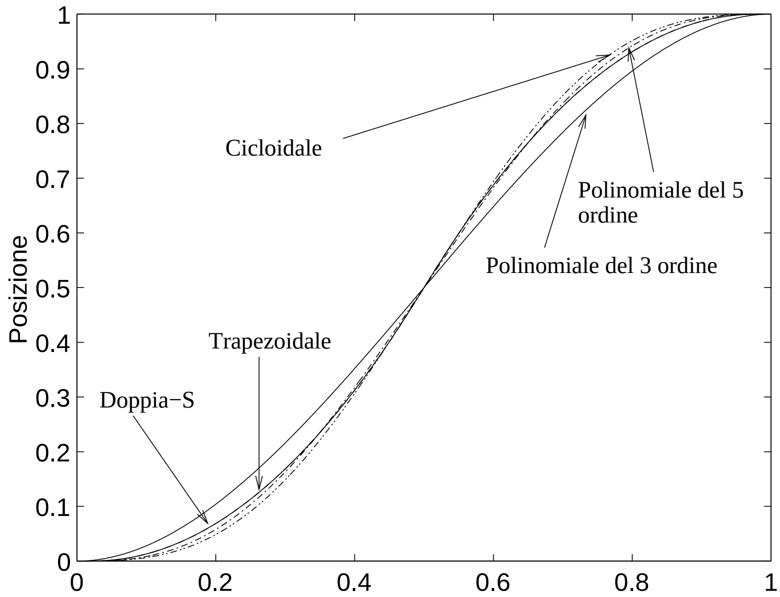


Figure 2.20: Comparazione del diagramma delle alzate delle leggi di moto **polinomiale** del 3^o e 5^o ordine, **cicloidale**, **trapezoidale** e **a doppia "esse"**.

3. Accelerazione RMS.
4. Diagramma spettrale delle accelerazioni.
5. Classe di derivabilità della legge di moto.
6. Rigidezza o elasticità della catena cinematica.

Il diagramma delle velocità $f'(t)$ per le varie leggi di moto è mostrato in figura 2.21.

In questo caso si nota una certa differenza tra le varie leggi di moto. In particolare, le leggi polinomiali richiedono una velocità massima inferiore delle leggi trapezoidali, cicloidali e a doppia S. In particolare la legge polinomiale del terzo ordine richiede una velocità massima inferiore del trenta per cento rispetto alle altre leggi, e quindi una specifica sulla limitazione della velocità massima può essere meglio soddisfatta da questo tipo di legge.

In figura 2.22 è mostrato il diagramma delle accelerazioni delle leggi considerate.

Dalla figura risulta evidente che la legge di moto di tipo polinomiale del terzo ordine presenta discontinuità nella accelerazione all'inizio ed alla fine della traiettoria. e quindi potrebbe introdurre sollecitazioni di tipo dinamiche non desiderabili.

Le leggi cicloidale e polinomiale del quinto ordine sono molto simili e presentano una buona "dolcezza" del diagramma delle accelerazioni, mentre le leggi trapezoidale e a doppia-S sono simili tra loro e presentano un valore di accelerazione massima minima tra le leggi considerate, il che risulta interessante se si vuole limitare la coppia richiesta agli attuatori.

In ultimo, è possibile analizzare lo spettro dei segnali di accelerazione delle leggi di moto viste, riportata in figura 2.23, da cui si può notare che la legge polinomiale del 3^o ordine presenta una dispersione spettrale molto evidente, mentre la legge cicloidale ha lo spettro più contenuto tra tutte le leggi di moto considerate.

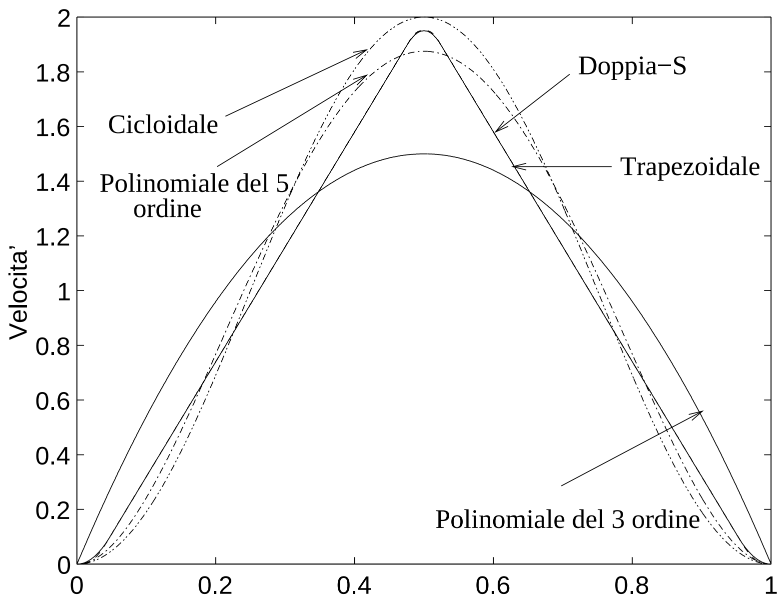


Figure 2.21: Comparazione del diagramma delle velocità delle leggi di moto **polinomiale** del 3^o e 5^o ordine, **cicloidale**, **trapezoidale** e **a doppia "esse"**.

Nella tabella 2.7 sono riportati i valori della velocità ed accelerazione massime, e della accelerazione RMS per le varie leggi di moto. Nella tabella compaiono oltre ai valori assoluti anche i valori normalizzati (velocità ed accelerazione massime normalizzate, accelerazione RMS normalizzata) rispetto ai corrispondenti valori della legge parabolica (o ad accelerazione costante).

La scelta di questa legge come riferimento è dettata dal fatto che la legge parabolica è la legge con accelerazione minima possibile. Inoltre, siccome l'accelerazione ha la forma di un'onda quadra simmetrica, il suo valore massimo ed RMS coincidono e quindi possono essere presi come valori di riferimento per la normalizzazione. Ricordiamo però che la legge parabolica è discontinua in accelerazione, per cui nella pratica viene scelta solamente se la catena cinematica non presenta elasticità che innescano risonanze.

Dalla analisi delle figure e della tabella possiamo trarre le seguenti conclusioni e consigli pratici.

- Considerazioni sulla **continuità delle derivate** della legge di moto e regolarità (*smoothness*) della traiettoria.
  - Il polinomio di 3^o grado possiede discontinuità sull'accelerazione agli estremi. quindi la legge impone coppie che variano bruscamente. La banda passante del segnale di accelerazione è ampia.
  - Il polinomio di 5^o grado, le leggi cicloidali, trapezoidale modificata e a doppia "S" sono invece continue nell'accelerazione e hanno jerk limitato. Le leggi non richiedono coppie impulsive, e, come si vede dal diagramma spettrale, spostano lo spettro dell'accelerazione verso la zona delle basse frequenze, aumentando però in modulo.
- Analisi della **velocità, accelerazione (massima e RMS)**.
  - Il polinomio di 3^o grado richiede la velocità ed accelerazione RMS minima, per contro presenta una accelerazione massima più elevata del 50% rispetto alla legge parabolica.

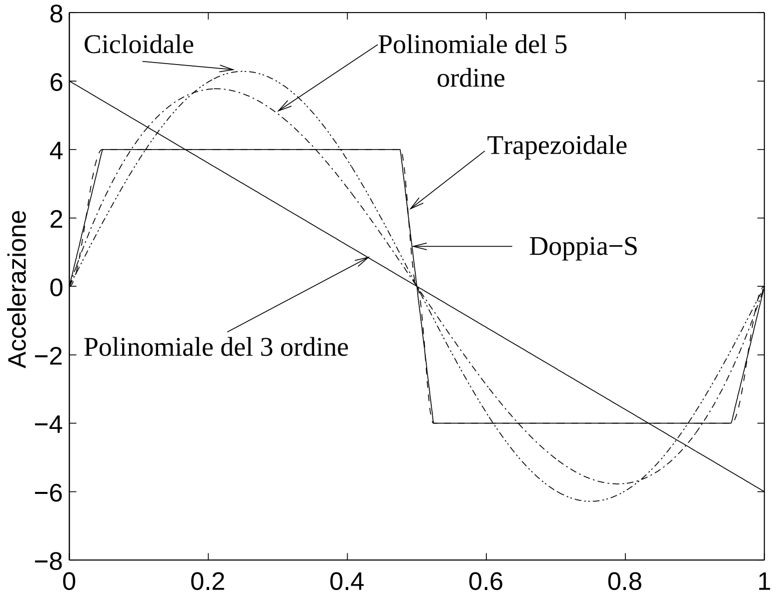


Figure 2.22: Comparazione del diagramma delle accelerazioni delle leggi di moto **polinomiale** del 3^o e 5^o ordine, **cicloidale**, **trapezoidale** e **a doppia "esse"**.

Tipo di legge	Velocità massima (normalizzata)	Accelerazione massima (normalizzata)	Accelerazione RMS (normalizzata)
Legge parabolica (o ad accelerazione costante)	2.00 (1.00)	4.00 (1.00)	4.00 (1.00)
Legge polinomiale del terzo ordine	1.50 (0.75)	6.00 (1.50)	3.47 (0.87)
Legge polinomiale del quinto ordine	1.87 (0.94)	5.77 (1.44)	4.14 (1.03)
Legge cicloidale	2.00 (1.00)	6.28 (1.57)	4.44 (1.11)
Legge trapezoidale modificata con parametri [0.05,0.45,0.025, 0.0,0.025,0.45,0.05]	1.95 (0.98)	4.00 (1.00)	3.82 (0.95)
Legge a doppia "esse" con parametri [0.05,0.45,0.025, 0.0,0.025,0.45,0.05]	1.95 (0.98)	4.00 (1.00)	3.80 (0.95)

Table 2.7: Indici di merito di natura cinematica per le varie leggi di moto.

- Il Polinomio di 5^o grado e la legge cicloidale presentano velocità ed accelerazioni più alte del polinomio di 3^o grado. L'accelerazione RMS è maggiore od uguale a quella del polinomio del 3^o grado, anche se ancora con valori non troppo elevati.
- La legge trapezoidale presenta alte velocità e bassi valori di accelerazione massima, e valori di accelerazione RMS intermedi

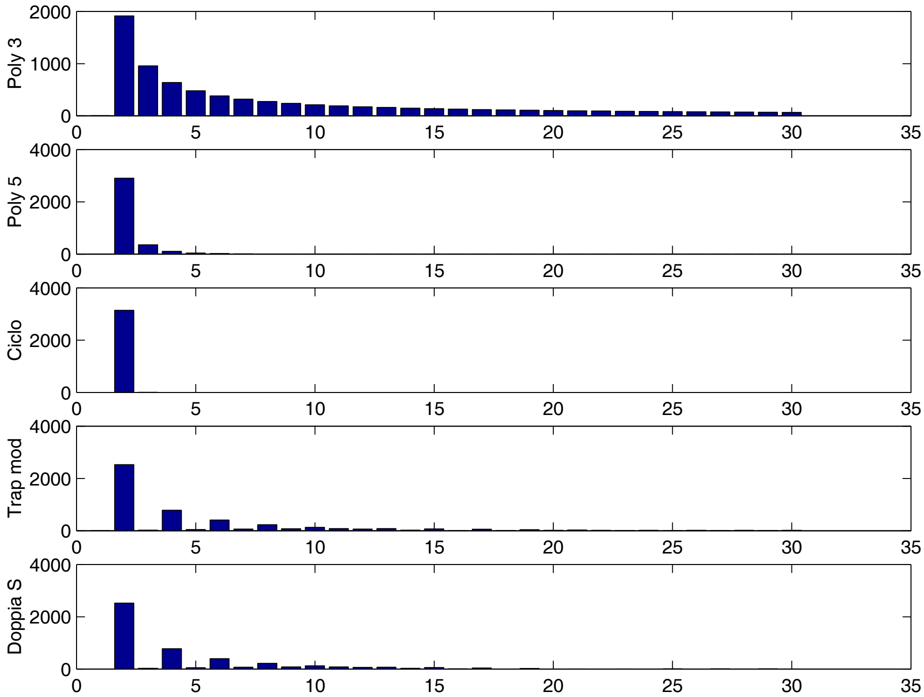


Figure 2.23: Spettro del segnale di accelerazione per le leggi polinomiale del 3^o e 5^o ordine, cicloidale, trapezoidale modificata e a doppia “S”.

Occorre poi considerare che è più importante minimizzare l’accelerazione RMS che l’accelerazione di picco, in quanto la taglia del motore viene scelta in base alla coppia nominale dello stesso, e la coppia nominale è spesso proporzionale alla accelerazione RMS. Comunque, una buona norma consiste nel limitare la coppia di picco a 3–5 volte la coppia nominale

Un fatto da considerare attentamente nella definizione della legge di moto consiste nel legame che esiste fra corsa Γ dell’asse master ed accelerazioni massime raggiunte dallo slave. In particolare consideriamo la parametrizzazione di una generica legge di moto $q(t)$ a partire da una legge di moto $f(t)$ con alzata e corsa unitarie:

$$q(t) = hf\left(\frac{t}{\Gamma}\right)$$

essendo h l’alzata e Γ corsa della legge di moto $q(t)$. Considerando le derivate di ordine superiore si ottiene:

$$q'(t) = \frac{h}{\Gamma}f'\left(\frac{t}{\Gamma}\right)$$
$$q''(t) = \frac{h}{\Gamma^2}f''\left(\frac{t}{\Gamma}\right)$$
$$q'''(t) = \frac{h}{\Gamma^3}f'''\left(\frac{t}{\Gamma}\right)$$

e quindi, aumentando o riducendo la corsa totale di un fattore Γ la velocità varia di un fattore Γ, mentre l’accelerazione varia di un fattore Γ^2. In base a questa considerazione si può affermare che è conveniente sviluppare l’alzata sulla massima corsa possibile, in modo da massimizzare Γ, e quindi ridurre velocità ed accelerazioni massime (nell'ultimo caso di un fattore quadratico).

In ultima analisi la scelta di legge di moto si esegue anche considerando l'accoppiamento tra carico e motore, infatti tale accoppiamento in genere presenta elasticità e giochi, le cui entità influenzano la scelta della possibile legge di moto. Consideriamo tre situazioni che si possono incontrare nella pratica:

- **Il carico ha alta inerzia ed è mosso da una cinghia**. In questo caso, l'elevata inerzia, l'elasticità e lo scarso fattore di smorzamento della cinghia portano a frequenze di risonanza nella banda passante dell'azionamento. È bene, quindi, non eccitare questi modi vibrazionali per cui si devono escludere tutte le leggi di moto con accelerazione discontinua (parabolica, e polinomiale del 3^o ordine), utilizzando invece leggi polinomiali del 5^o ordine, cicloidali e trapezoidali.

- **Il carico è accoppiato col motore tramite riduttori o cinematismi che presentano dei giochi**. In questo caso è bene usare leggi di moto a partenza molto morbida come polinomiali del 5^o ordine, cicloidali e trapezoidali, in modo da ridurre gli urti dovuti ai giochi nei punti di discontinuità delle accelerazioni. Quindi, anche in questo caso si devono scartare leggi tipo paraboliche e polinomiali del 3^o ordine.

- **Il carico ha bassa inerzia, l'accoppiamento col motore è sufficientemente rigido e non presenta giochi di rilievo**. In questo caso si possono usare senza problemi leggi ad accelerazione discontinua allo scopo di beneficiare delle loro modeste accelerazioni, che portano a scegliere motori di taglia più piccola. In genere si preferisce usare un polinomio del 3^o ordine e in seconda scelta, leggi paraboliche.

In conclusione si può affermare che quando occorre minimizzare la velocità massima e la coppia RMS del motore conviene scegliere una legge cubica (polinomiale del 3^o grado) a patto che si possano tollerare discontinuità nell'accelerazione. La legge parabolica può essere utilizzata per la sua semplicità realizzativa, anche se conduce ad avere velocità ed accelerazione RMS maggiori.

Invece, quando si deve ricorrere a leggi particolarmente "morbide" e con continuità fino alla accelerazione, conviene usare polinomi di 5^o grado o trapezoidali modificate (inclusa la legge a doppia "esse"), che ben rappresentano la sintesi tra "morbidezza" di moto e valori degli indici cinematici non troppo alti. La legge cicloidale ha un comportamanto molto regolare, però ha indici cinematici (in particolare accelerazione RMS) massimi, e quindi non particolarmente indicate per minimizzare la taglia dell'attuatore elettrico.

CHAPTER 3

ELECTRIC MOTOR SELECTION

3.1 Introduction

The first question that a designer needs to answer when selecting an electric actuator, is related to the type of electric motor technology he or she requires. That in turn corresponds to different performances (e.g. in terms of bandwidth) and costs.

The purpose of this chapter is indeed to go through the main different technologies for electric motors. The main ones covered in this chapter are:

- DC-Brush motors
- Stepper motors
- Induction motors
- Brushless motors

For now we refer to rotary motors, but we will see that the same technologies are available and used also for linear motors.

In the two next chapters, we will instead go deeper into the brushless category and inspect the subcategories and possibilities, and finally see the linear motors technologies. Other more exotic technologies will also be briefly touched. The following chapters will cover the Control Theory aspects of Mechatronics and finally the Mechanics point of view.

3.2 Electrical motors

To start with, we should first agree on the terms for the different electric motors types. In literature, but in particular in industry, different synonymous are used, leading to possible confusion. Here are some:

- DC-Brush motors are also called DC motors (referring to the electric use-case), Brush-motors, Commutator motors (referring to the electro-mechanical characteristical feature). Note that DC-Brush motors have very good control properties, being very easily controllable, and in principle able to perform any position, velocity, torque profile. In contrast the technology has serious limits in terms of heat dissipation, mechanical complexity, MTBF (Mean Time Between Failures), use in explosive environment (due to possible sparks in the commutator-brush system). Consequently a new type of electric motors, the brushless, have been developed, having the same control advantages of the DC-Brush ones, without the limits. This have been achieved basically inverting the rotor and stator functionalities, leaving the rotor as a passive element in the brushless, thus eliminating the role of the commutator.

- Stepper motors are sometimes referred to as Step motors referring to the use-case. Note that steppers are generally defined in terms of number of steps, and eventually torque at very low speed, since torque at high speed is soon meeting instabilities due to resonances, making it not suitable for such applications.

- Induction motors are often called AC motors (referring to the electric use-case), or Squirrel-Cage motors (referring to the rotor characteristical feature). Note that Induction motors are generally defined in terms of power, and not torque, since the torque would depend on speed and load.

- Brushless motors: in industry jargon "Servo" generally is used as a synonymous of "brushless", "synchronous motor", "permanent magnets motor", "axis" (plural: axes). In academic literature "Servo" is also used for any closed loop actuator system, including e.g. AC motors with encoder, pneumatic actuators with linear potentiometer. Note that brushless motors are generally defined in terms of torque, not power, since the stall torque is almost stably available from zero to nominal speed, while power increases proportionally with speed.

In the graphs shown in figure 3.1 the servo brushless torque characteristics is compared with induction motors controlled through the traditional "V/f constant" technique, and with induction motors controlled through the "Vector Control" technique (more similar to a brushless control algorithm).

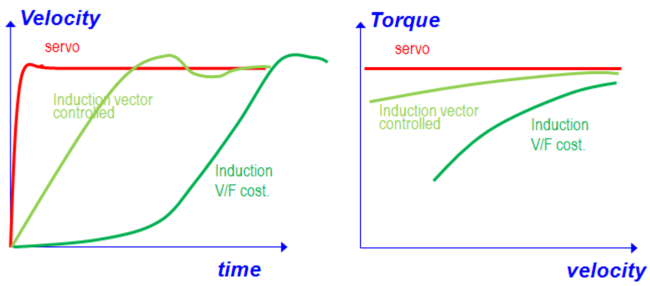


Figure 3.1: servo brushless torque characteristics

3.3 Mathematical modeling of electrical motors

The basic electrical principles for stepper motors are based on variable reluctance, while DC-Brush, Induction and Brushless motors basically share the same basic equations, based on Lentz and Lorentz physical laws. These are described in table 3.1.

Electrical Equations:	
$e = K_e\omega$	[units: VRMS]
$T = K_t I_q$	[units: Nm]
Mechanical Equations:	
$P = dE/dt = T\omega = \omega dE/d\omega$	[units: Nm/s = W]
$T = Jd\omega/dt$	[units: Nm]

Table 3.1: Basic physical principle for electrical motors.

Units are those of International System, in particular:

- V_{RMS} for voltage e,
- Nm, for torque T,
- A for current I_q,
- $Nm/s = W$ for Power P,
- J for Energy E,
- Rad/s for angular velocity ω,
- s for time t

Note that the two electrical equations have two electrical quantities (e, I_q) and two mechanical quantities (T, ω) criss-crossed in the two equations, i.e. the electrical quantities are in the first member in the first equation and second member in the second equation, and vice versa for the mechanical quantities. Moreover the two pairs of quantities, multiplied together deliver the electrical and mechanical power respectively. Equalling the electrical power P_a with the mechanical power P_m (neglecting losses), we can also find the link between the two electrical and mechanical constants K_t and K_e, considering the three phase system:

$$P_a = 3V_{p-n}I_{RMS}\cos(\phi) = \sqrt{3}V\text{p-p}I_{RMS}\cos(\phi) = \sqrt{3}K_e\omega I_{RMS}\cos(\phi) = P_m = T\omega = K_t I_{RMS}\omega$$

where V_{p-n} is Vphase-neutral, Vp-p is Vphase-phase.

That leads to:

$$\sqrt{3}K_e\omega I_{RMS}\cos(\phi) = K_t I_{RMS}\omega$$

And thus:

$$K_e = K_t/\sqrt{3}$$

That applied to the first electrical equation gives:

$$e = K_e\omega = K_t\omega/\sqrt{3}$$

Considering that the voltage "e" is obtained rectifying a three phase system, this leads to:

$$\text{DC Bus voltage} = \sqrt{2}e = \sqrt{2/3}K_t\omega$$

Where "e" is expressed in V_{RMS} units.

Finally note that the constant K_t is now the multiplier of both second members. This in turn means that, decreasing K_t higher angular velocity can be achieved with the same voltage, and lower current is necessary for achieving the same torque, but the total power, mechanical or electrical (we are neglecting losses), remains the same. We will see that this feature can be used, in different ways on the different motor types, for achieving field weakening technique or motor rewinding. The mechanical equations are clearly basic physics equations, while the electrical equations look less familiar, but are still directly derived from Lenz and Lorentz law. In particular, consider, for a copper segment of length "l" moving linearly through a perpendicular magnetic field B, the Lenz law effect:

$$e = \frac{d\phi_B}{dt} = Blv$$

That transformed into rotary frame gives directly:

$$e = K\omega$$

In equilibrium, for the same copper segment, we have thus a force due to the electrical field E expressed in V/m:

$$F = Eq = eq/l = Blvq/l = Bvq$$

With current "i" instead of single charge "q" we have the Lorentz law:

$$F = BlI_q$$

And transforming into rotary frame we have:

$$T = K_t I_q$$

Magnetic field B and current I_q delivers maximum force F, when perpendicular to each other. The force F, in turn, results perpendicular to both magnetic field B and current I_q, thus resulting in the tern $F - B - I_q$ associated to the first three fingers of the left hand. Electrical motors are indeed different technologies studied to keep two electro-magnetic fields misaligned to each other (ideally perpendicular), so to create torque by Lorentz Law. To better understand the behaviour of electrical and magnetic fields, we need to go at the root of electro-magnetism: Maxwell's Equations, which are summarized in figure 3.2.

Name	"Microscopic" equations	"Macroscopic" equations
Gauss's law	$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$	$\nabla \cdot \mathbf{D} = \rho_f$
Gauss's law for magnetism	$\nabla \cdot \mathbf{B} = 0$	
Maxwell–Faraday equation (Faraday's law of induction)	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	
Ampère's circuital law (with Maxwell's correction)	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$

Figure 3.2: Maxwell's equations

They can be explained in the following way:

- The Gauss law: the electrical field E diverges outward from plus charges and inward to minus charges. Both can thus exist separately.

- Gauss law for magnetism: magnetic field B never diverges, always loops around. That in turn means that where there's a North (N) magnetic pole, there's always a South (S) magnetic pole, and vice versa.

- Faraday law: electrical field E curls around a changing magnetic field B
- Ampere law: magnetic field B curls around a current (J is current density) and/or a changing electrical field E.

Summarizing: the Gauss laws are about electrical and magnetic field static behaviour, while Faraday and Ampere laws tells us how an electrical field can be generated by a changing magnetic field and vice versa.

3.3.1 Field Weakening

When increasing the speed of an electrical motor, following the first electrical equation, a maximum speed is reached when meeting the maximum voltage deliverable by the three phase system (see DC Bus voltage equation above). For increasing furthermore the speed it is necessary to lower the statoric flux with $1/\omega$ (and doing so also K_t will be lowered and so also $T = K_t I_q$).

In a brushless motor the effect can be obtained changing the phase of the statoric current I_s beyond $\pi/2$ with respect to the rotor position, so to reduce the total flux; the current thus staying maximum and thus avoiding quantization effects due to small digital vectors. A similar technique can be applied to use the motor as braking resistor heating it up.

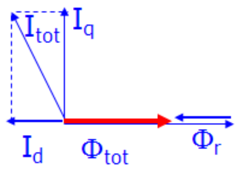


Figure 3.3: Field Weakening.

In a induction motor controlled with V/f constant (that increases voltage V and frequency f keeping the ratio constant) the technique can be applied simply by continuing to increase the frequency, keeping the voltage constant at the maximum value.

Having higher speed at lower torque we actually have $P = T\omega =$ constant.

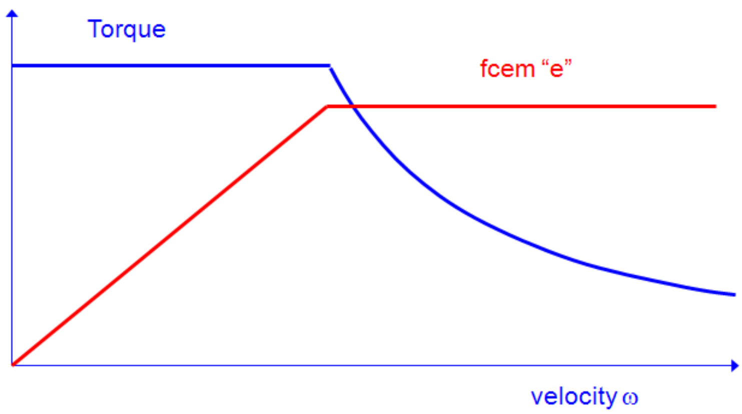


Figure 3.4: Electrical power in case of field weakening.

3.3.2 Rewinding

Another method to affect K_t is the rewinding of the copper coils. This, in contrast to field weakening, can of course be done only before to manufacture the motor. But it does not affect the general motor design: only the copper wire section is changed.

In a brushless motor for example, for increasing K_t, keeping the same motor structure, is sufficient to rewind stator slots with smaller section wire so to make more windings. Indeed the formula for K_t states that:

$$K_t = \Phi S_{rot} N/(4V_{rot})$$

Where Φ is the magnetic flux, S_{rot} the rotor surface, N the number of copper wire turns, V_{rot} the rotor volume.

We thus see that decreasing the wire section, we increase the number of turns N (being able to fit more wires in the same stator slot), leading to higher K_t.

With the same motor, we will thus have more torque at the same current I, but with a smaller maximum speed following the first electrical equation. For demonstrating the above formula for K_t we can say that, for a single wire we have:

$$e = \frac{d\phi_B}{dt} = Blv$$

and from the first electrical equation we have:

$$K_t = e/\omega = NBl$$

Now, defining the rotor cylinder dimensions as diameter "a", length "b", we have:

$$B = \phi B/(ab)$$

Considering all N turns in a motor, we thus have:

$$K_t = N\phi B(b/(ab)) = \Phi S_{rot} N/V_{rot}$$

Having defined:

$$S_{rot} = \pi ab$$
$$V_{rot} = \pi a^2 b/4$$

We can now have a look to different motor technologies, with a view on performances, that we can compare plotting the torque characteristics graph for each solution.

3.3.3 DC motors

In 1871 the Belgian engineer Zenobe Theophile Gramme patented the uniformly wound ring-armature dynamo. In the patent he recognized the work of the Italian Antonio Pacinotti. A different dynamo was also patented by Werner Von Siemens in 1866. The Gramme machine is a type of direct current dynamo, capable of generating smoother and much higher voltages than the dynamos known to that point.

In 1873 Gramme and Hippolyte Fontaine accidentally discovered that the device was reversible and would rotate when connected to any DC power supply, with a speed proportional to the voltage. The Gramme machine was actually the first usefully powerful electrical motor, that was industrially successful. Indeed before Gramme's invention, electric motors could achieve only low power and were mainly used as toys or laboratory curiosities.

The DC motor is thus the most mature electric motor technology, making it cheap. It has also the important advantage of being very easy to control, having the speed directly proportional to the DC voltage applied to the brushes, and in

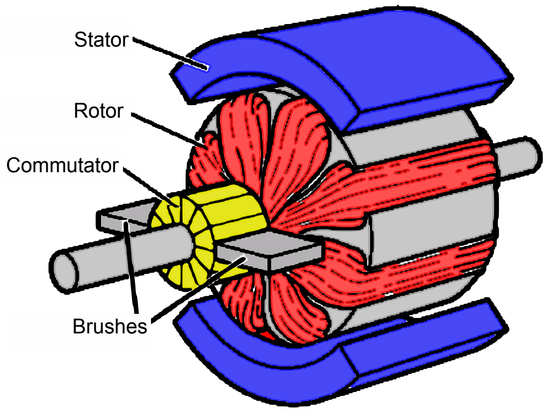


Figure 3.5: DC Motor

turn to the commutator. The latter is actually also the main intrinsic defect of the motor, leading to possible unreliability, sparks, wear.

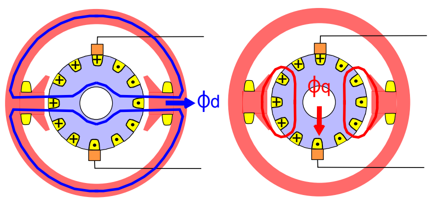


Figure 3.6: A cross sectional view of a DC motor.

For DC motors, as for other electric motors, the trick is to keep two electro-magnetic fields perpendicular to each other. In this case to job is done manufacturing the motor so to have the brushes insisting on the rotor coils at perpendicular angle with respect to the stator coils creating the statoric magnetic field. This design permits an ideal, almost flat, torque characteristic, which permits to have a nominal torque almost equal to the stall torque, as shown in figure 3.7.

Note that the shown torque characteristics, and the following ones, are shown for the first quadrant, i.e. where the motor acts as, indeed, motor, thus transforming electrical power into mechanical power. The same behaviour is present in the third quadrant, while the second and fourth, it shows a behaviour as generator (dynamo in case of DC motor, alternator in case of AC motors), i.e. transforming mechanical power into electrical power. Depending on coil electrical resistance, a small part of the second and fourth quadrant manifest a behaviour as an electrical brake where both electrical and mechanical power are wasted as heat.

3.3.4 Stepper motors

Steppers are incredibly simple in design as poor in performances. Nevertheless they are ideal for small loads, cheap applications that require no feedback. They are based on the magnetic circuit reluctance minimization, which leads to a moving rotor when current is applied. The design can differ depending on the number of phases, poles and rotor teeth.

The control strategies can also be diverse. For example we can have:

- One-Phase-On or Full Step: current is applied in sequence to each phase alternatively. This, for a 4 poles, 6 teeth, 2 phases motor leads to 12 steps per turn.

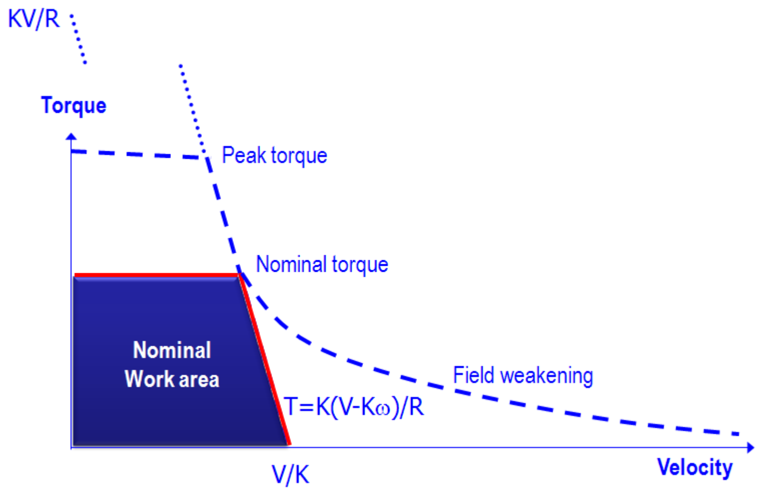


Figure 3.7: Torque characteristic of a DC motor.

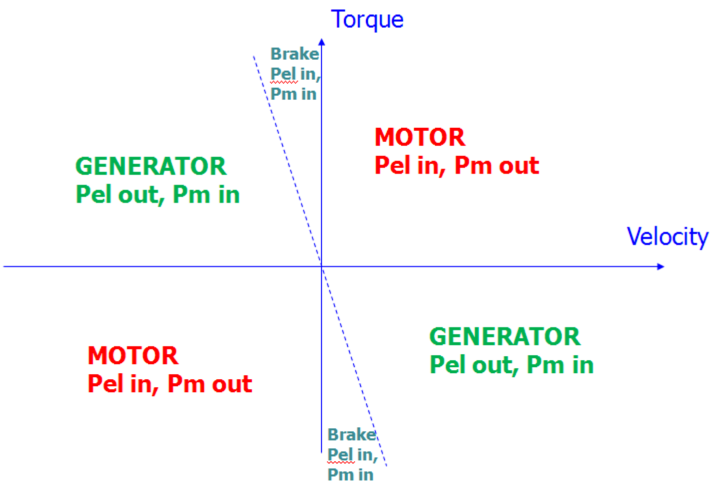


Figure 3.8: Four quadrant characteristics of a DC motor.

- Half Stepping: as the Full Step but with intermediate steps where half the current is applied to the phases. This leads of course to double the amount of steps per turn.

- Micro Stepping: between a step and the next the current is modulated so to reach many intermediate values, leading to hundreds of steps per turn. This technology needs a more sophisticated drive,

The number of steps per turn can be further increased increasing the number of teeth, or with multi stack axial design.

The torque can be improved mounting permanent magnets while the real time performances (i.e. the bandwidth) can be improved mounting a feedback device so to achieve close loop control, with an appropriate electronics to drive it. All these aspects of course increase the cost, which is the main advantage of the basic design stepper concept.

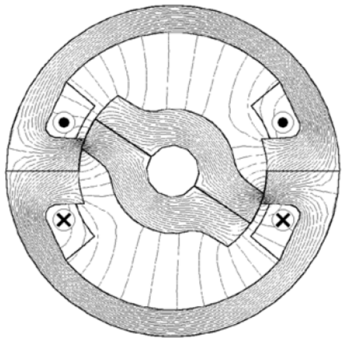


Figure 3.9: Stepper motor.

3.3.5 AC motors

Not many years after the DC motor technology invention, the AC motor one came to life thanks to three phase power transmission invention by John Hopkinson, King's College professor in London, and to the great intuition of the rotating field achieved with three phase electrical machines, by Nikola Tesla, work published about at the same time of the Italian Galileo Ferraris: 1888. A prototype of induction motor was indeed demonstrated by Galileo Ferraris already in Europe in 1885, while Nikola Tesla patented the first practicable AC motor in 1888 and with it the polyphase power transmission system. But the Serbian-American scientist already understood fully the concept in 1882 and demonstrated it systematically in 1883. After a brief period working for Thomas Edison (that instead, together with Guglielmo Marconi, believed in DC currents), he founded his own company, the Tesla Electric Company, in Manhattan, at 33-35 South Fifth Street, few blocks from Edison company. He then continued his work on the AC motor in the years to follow at the Westinghouse company, and later with the financial support of J.P. Morgan.

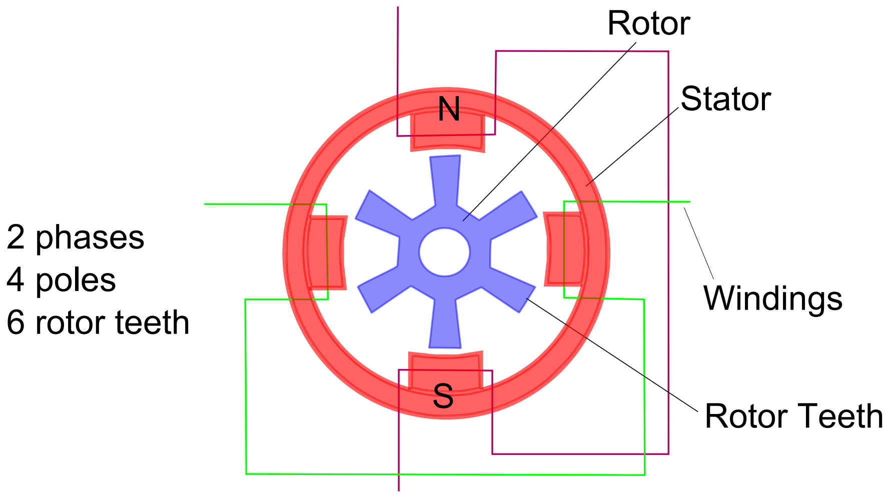
Frequency-controlled asynchronous (induction) motors are mostly used for simple drive functions, without feed-back, for example to regulate the speed. The motor is a squirrel-cage asynchronous motor, and the control unit a frequency converter. The squirrel-cage asynchronous motor is the absolutely most commonly used AC induction motor, and the most diffused electrical motor as installed base. This makes it cheap and standard product, also as dimensions.

The simple design (no commutator is needed) makes it very reliable, the bearings being the only contact point between stator and rotor.

The squirrel cage rotor is the key to the functional principle of the AC motor:

The coils in the stator are multiple of three being the system three phase, and thus generate a rotating magnetic field. This cuts the squirrel cage conducting bars and thus, by the Lenz law, generates a voltage potential on the bar that creates a current that finally generates force by Lorentz law. It is then clear that an intrinsic differential speed between rotor and statoric flux is required in order for the Lenz law to generate a voltage. This leads to an asynchronous behaviour that sees the rotor lagging behind, in speed, to the stator (synchronous) velocity, when commanded as constant. This speed difference is called "slip". The synchronous speed is indeed the rotation speed of the magnetic field, generated in the field windings when supplied with a three-phase AC voltage:

In a system equipped with a frequency converter the synchronous speed can be commanded to any value within the drives specifications. The actual, true, speed of the rotor is determined also by how great a load the motor is driving. This speed is the asynchronous speed, and the difference between the two is the above mentioned slip. Note that the AC induction motor (asynchronous) has always a physiological slip (in speed), while the AC brushless motor (synchronous) has always a physiological lag error (in position). From a construction point of view the stator of an AC induction motor and the one of an AC brushless are quite similar (both has a winding lay-out so to obtain a single sinusoidal rotating field from three sinusoidal pulsating fields).



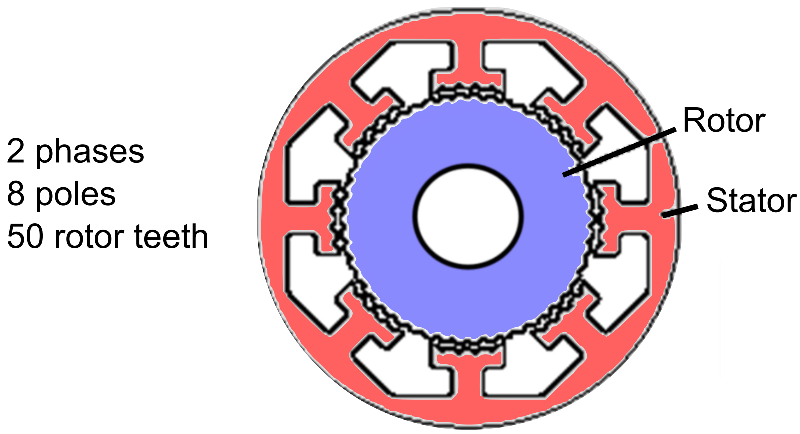


Figure 3.10: Control of a Stepper Motor.

The traditional control technique relies on increasing voltage and frequency keeping the ratio between the two constant. This until the voltage reach the maximum value, when a field weakening technique can be used, increasing further the frequency without changing the voltage. At this point the V/f ratio has reached its maximum value, that has to be below the iron saturation point (generally below $1.0T$), and starts to decrease with field weakening.

The nominal working area of an induction motor is with negative slope. Indeed with positive slope any accidental decrease in speed due to higher friction, would lead to lower torque and not higher as it would be necessary to surpass the obstacle. This in turn would lead to a sudden stop of the motor. For this reason the motor torque characteristics is gradually changed increasing voltage and frequency, so to keep the working point on the negative slope area.

Often an AC brushless drive can also control (with Vector Control techniques) an induction motor.

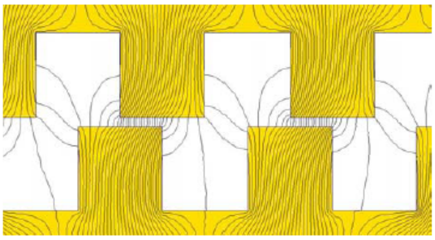


Figure 3.11: Flux distribution in a stepper motor.

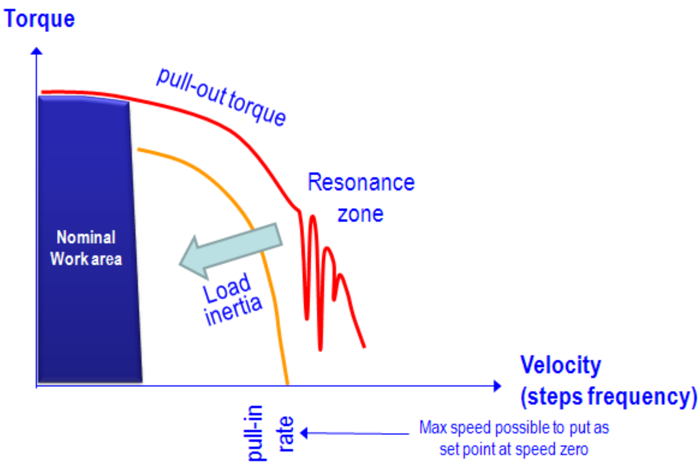


Figure 3.12: Torque characteristic of a stepper motor.

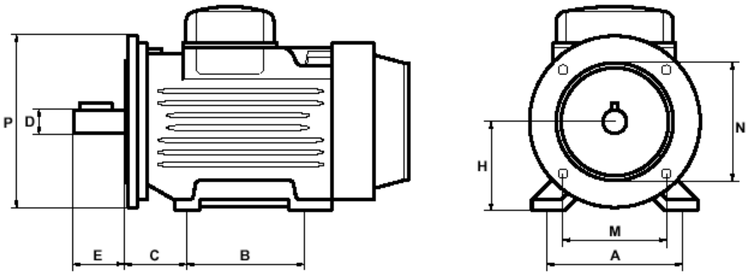


Figure 3.13: AC Motor

3.3.6 Brushless motors

They represent the leading edge technology in terms of performances, having the optimal control properties of DC motors without the disadvantages due to the commutator. In fact the brushless motor has a three phase stator with a rotating electro-magnetic field that synchronously drives a permanent magnet rotor.

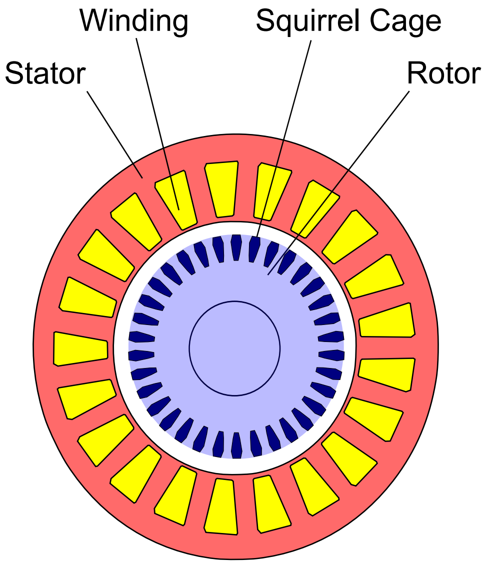


Figure 3.14: AC Motor with Squirrel Cage Rotor.

Number of poles	Frequency (Hz)			
	10	30	50	100
2	600	1800	3000	6000
4	300	900	1500	3000
6	200	600	1000	2000
8	150	450	750	1500

Figure 3.15: Characteristics of a AC motor.

The only contact point between rotor and stator are the bearings, and the design result to be optimized so to have heating created on the statoric part only, that is the most exposed to external environment so the optimally dissipate heat.

The design is thus optimal, compact, and maximises the power concentration.

The torque characteristics is the same as the DC motor, while the control technique is significantly more complex, since now the rotating statoric flux has to be obtained electronically switching the coils, according to the measured position of the rotor. The brushless motor is the natural choice for complex profiles and high performance applications, deserving a complete chapter.

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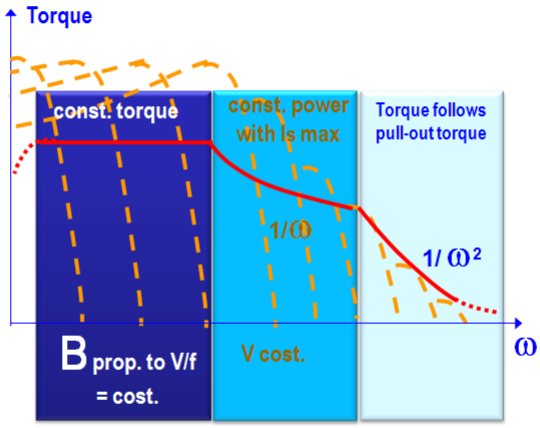

Figure 3.16: Torque characteristic of an AC motor.

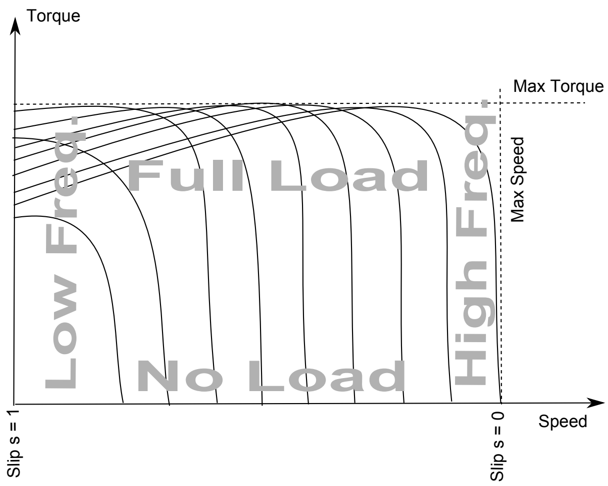

Figure 3.17: Characteristic of the Torque versus the Speed for an AC motor.

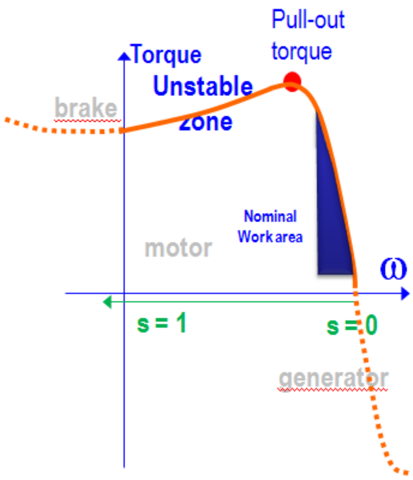

Figure 3.18: Torque characteristic for the V/f control of an AC motor.

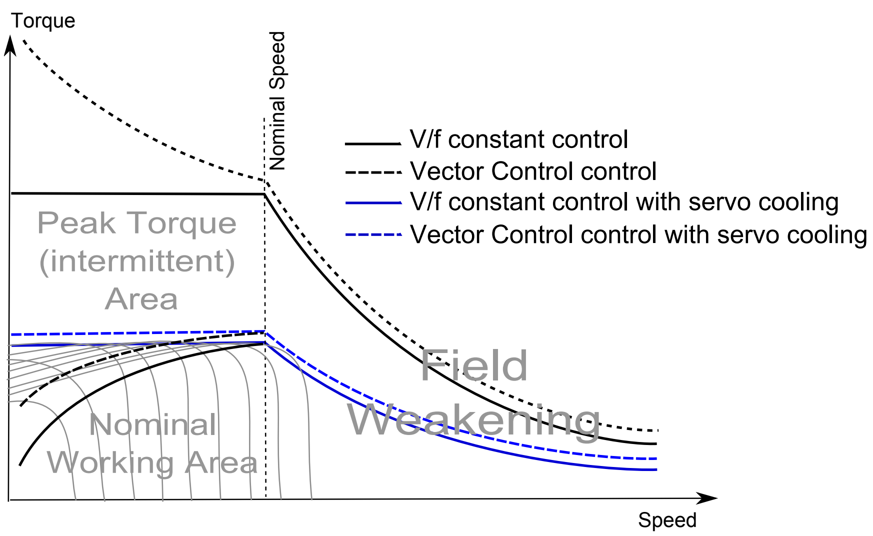

Figure 3.19: Characteristic of the Torque versus the Speed for an AC motor with cooling.pdf

CHAPTER 4

BRUSHLESS MOTORS

The Brushless motor technology, in terms of industrial applications, has been proved to be very versatile, powerful and high performance technology. A brushless motor is a synchronous electric motor, i.e. rotor mechanics and stator electro–magnetic field are rotating at the same speed, that is in contrast with Induction motors that are asynchronous.

This synchronous feature is an enabler for accurate position control. The function of keeping the stator and rotor fields perpendicular is made by servo drive electronics, that is in contrast with DC motors that has commutator and brushes to achive this. This allows the brushless to have all the copper, dissipating heat by Joule effect, in the stator, that is also in contact with external environment, enabling a good thermal characteristics, and in turn smaller dimension at the same power.

In order for the servo drive to function properly, the position of the rotor must be known, generally through a feedback system, e.g. an encoder. Its characteristics, in particular the resolution, is a critical parameter for the total bandwidth of the servo system.

The servo motor is composed by the following function-mean items:

- A stator with coils in multiple of three, being the system three phase,
- A rotor with permanent magnets in multiple of two (N-S) to provide rotoric flux,
- Copper windings inserted into the coils, whose current is provided by the servo drive switching transistors, to provide statoric flux. This flux is kept at 90 electrical degrees to the rotoric flux,
- An encoder or other feedback system to provide rotor position to the servo drive
- A mechanical housing, a front and a back flange. These are generally aluminum as standard, or stainless steel for wash-down hygenic version. They provide a mean to connect the motor to the machine body, and act as a protection

for the inner parts. Generally housed servo motors are IP65 protection in standard configuration, IP66 to IP69K for hygenic design versions.

- Electrical connections. These can be provided in a terminal box, in separate connectors for power to the windings, feedback and brake, or as a single connector for the single cable configuration. They provide connection to the servo cable(s).
- Bearings for Drive End (DE) and Non Drive End (NDE) side. These provide a support for the rotor. They are the only mechanical contact point between statoric and rotoric parts.
- A brake, generally electro-magnetic normally closed. This acts as a holding brake, i.e. it keeps the position when the servo is off, for example for vertical or spring loaded, systems. Generally the holding torque of a servo motor is lower than motor peak torque, so if mistakingly commanded to move with brake on, the motor would do so, damaging the brake.
- A Temperature sensor, generally a PTC (Positive Thermal Coefficient), that increases its resistance dramatically (non-linearly) when the maximum temperature is reached (generally around 110 - 150 C^o). This gives the possibility to the drive to stop the motor safely before to burn it. This mechanism is slow to react, so it's complemented inside the drive, by an efficient thermal model that estimates copper and iron temperature, based on given electrical variables

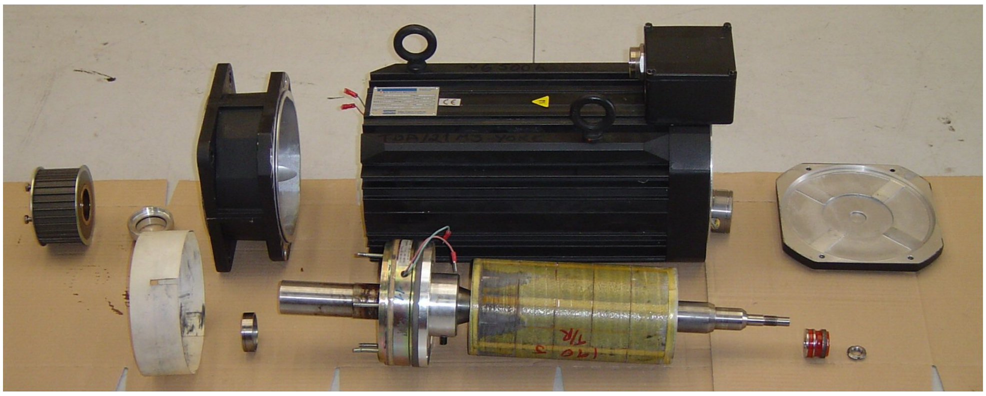

Figure 4.1: A picture of a Brushless motor with its components.

A servosystem is capable of turns any mathematical continuous function into a mechanical movement, so it can replace mechanical elements, such as cams and cam shafts, indexing gears, differentials, etc... A servosystem consists of a servomotor with its control unit (servo drive), and it can be used for:

- Positioning: the position, linear or angular, follows a predetermined position function.
- Speed control: the motor speed follows a predetermined speed function.
- Torque control: the torque of the motor follows a predetermined acceleration function.
- Hybrid control: the system alternates between different kinds of control

The effective electro magnetic field is generated by stator coil copper wires, runs through the stator iron, into South pole of magnets in the rotor, exits from North pole, runs through rotor iron, back into South pole of adjacent magnet, out of the North pole and back into the stator.

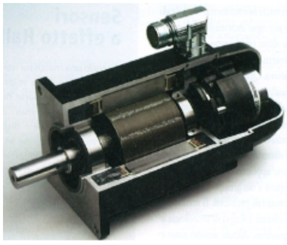

Figure 4.2: An open section of a Brushless Motor

In the figure 4.3 a 21 slots, 8 rotor poles configuration is shown. Generally the slots number and rotor poles number are chosen so to be not close multiple (i.e. 12 slots and 4 poles is avoided) so to mitigate cogging torque problems, as we will see. Brushless motors can technically be built with any pole pair number. A high pole pair number generally gives high torques. The technical limit is given by permanent magnets distance on the rotor, and by the diameter of the motor.

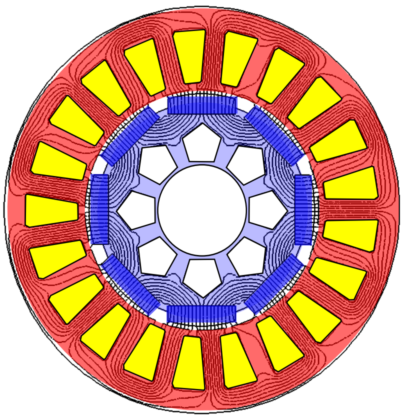

Figure 4.3: A description of the flux path in a Brushless motor with 8 rotor poles.

The *pros* of brushless motors are:

- Velocity (no sparks at the commutator)
- Efficiency (Torque/Inertia)
- Weight
- Dimensions
- Thermical Dissipation
- Acoustic Noise
- Maintenance
- MTBF

Figure 4.4: Rotor of a brushless motor with its permanent magnets.

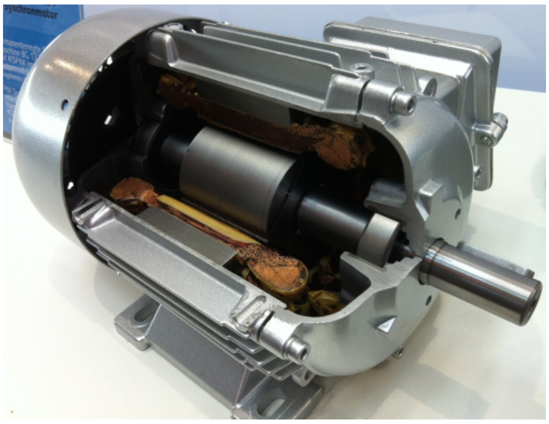

Figure 4.5: A cross–sectional view of a Brushless motor.

While the *con* is basically only one: cost. Nevertheless this has been generally driven down by cheaper servo drives, while servo motors are driven also by copper and rare earth (e.g. nodymium, dysprosium, samarium).

The advantage in terms of thermal dissipation is dictated by the fact that the rotor is basically a passive cold component, while the warm part is only the stator that can dissipate to the environment through air or water cooling. This is an aspect that reversed in the DC motor (where there's power in the copper wires on the rotor) and in the Induction motor (where there's current, thus power, induced in squirrel cage bars).

Two brushless motors types exist:

- AC brushless: with sinusoidal electro magnetic field

- DC brushless: with trapezoidal electro magnetic field

They may differ in stator windings, permanent magnets lay-down, statoric field wave shape. The main difference is anyhow the way they are controlled, i.e. the servo drive control strategy. The AC Brushless is driven with continuous sinusoidal change of the current through PWM (Pulse Width Modulation). The DC Brushless is instead driven with trapezoidal current shape, i.e. discontinuous current (at least in principle since the high inductance of the motor windings will anyhow round the shape of the current).

4.1 Permanent Magnets

The permanent magnets can be of different types. Among the most used are:

- Ferrite: they are low cost, low K_t, with torque loss: 0.2%/K, demagnetization temperature: 150C
- Samarium Cobalt (Sm_2Co_{17}): they are high cost, high K_t, torque loss: 0.04%/K, demagnetization temperature above 150C
- Neodimium Iron Boron (NdFeB): they are high cost, highest K_t (i.e. highest energy content), torque loss: 0.09%/K, demagnetization temperature above 150C. Other characteristics on NdFeB magnets are: highest energy content, not optimal thermal reversibility and Curie temperature, corrosion (not present with SmCo), bigger electrical resistance in Ω than SmCo magnets, which in turn limits eddy currents.

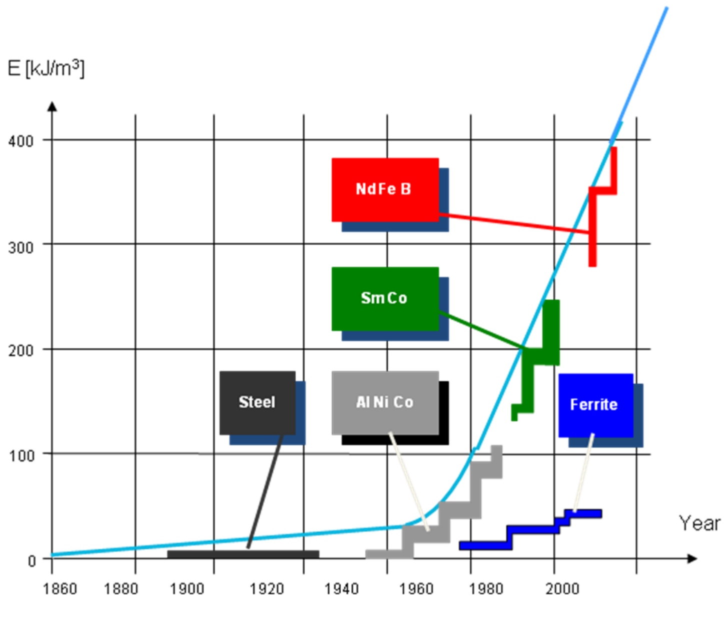

Figure 4.6: Energy content improvement of various type of permanent magnets for brushless motors.

Consider that a energy product above $300KJ/m^3$ is of little use since it would have a remanent flux-density above about $1T$ leading to possible iron teeth saturation.

The torque characteristics of Brushless motors is in principle a triangle with torque T vertex K_tV/R and as speed ω vertex V/K_t. The torque in reality is limited in the commercial torque characteristics provided in data sheets, before to reach the vertex. This is done for different reasons:

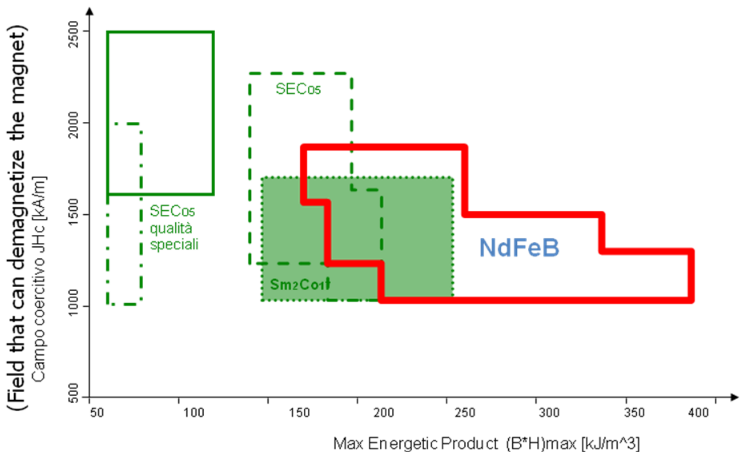

Figure 4.7: Characteristics of various type of permanent magnets for brushless motors.

	density [g/cm^3]	Curie Temp. [°C]	Br temp. coeff. [% 1/°C]
Sm_2Co_{17}	8,3	825	-0,03 (20°C ÷ 200°C)
Nd Fe B	7,4	315	-0,1 (20°C ÷ 150°C)

Figure 4.8: Permanent magnets Curie Temperatures.

- B-H characteristics bending, that can also lead to instability,
- Possible magnets demagnetization,
- Low K_t: indeed it is getting smaller with high temperature and current

Power (the hypotenuse of the triangular torque characteristics) is instead limited by:

- copper resistance,
- copper inductance $J\omega L$ at high speed ω,
- temperature

Finally the speed is limited by:

- commercial issues,
- system voltage,
- high K_t: indeed it is getting bigger with low current

Moreover everything is sized for worse voltage, e.g. $400V_{AC} - 10\%$.

We have mentioned the $\mathcal{B} - \mathcal{H}$ characteristics. That's a typical feature of ferromagnetic materials, that very large magnetizations can be obtained. The characteristics has an hysteresis, i.e. the magnetic field density $\mathcal{B}$, is not proportional to magnetic field intensity $\mathcal{H}$ as it is in most other materials. i.e., the value of $\mathcal{B}$ depends not only on the value of the

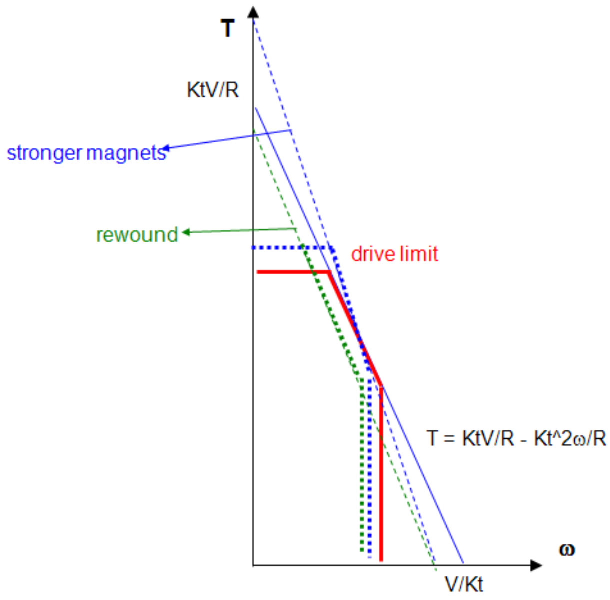

Figure 4.9: Brushless motors torque diagram variability on permanent magnets strenght.

applied field but also on the previous history of the sample. The magnet can thus retain its magnetization also in the absence of an external applied field. The behavior of a magnetic material during magnetization is what is commonly called a $\mathcal{B} - \mathcal{H}$ characteristics or plot.

The origin represents the unmagnetized condition of the magnet. When a magnetic field intensity $\mathcal{H}$ is applied, the plot proceeds along the line to the upper right point a as the magnetic polarization increases. This line is known as the *magnetization curve*. When the magnetic field intensity $\mathcal{H}$ is reduced, the magnetic field density $\mathcal{B}$ shows higher values than the magnetization curve. When magnetic field intensity $\mathcal{H}$ is reduced to zero, magnetic field density $\mathcal{B}$ still has a positive value called the remanence, a measure of its retentivity. With negative magnetic field intensity $\mathcal{H}$ the magnetic field density $\mathcal{B}$ can be brought to zero, point known the coercive force, a measure of its coercivity. The part of the loop which lies in the second quadrant is known as the *demagnetization curve*. This is generally the portion of interest in a discussion of permanent magnets.

The product $\mathcal{BH}$ has the dimensions of energy density, it is called the *energy product*, which is a good measure of how powerful a permanent magnet is. Permanent magnets are preferred with a large remanence to retain a great part of the magnetization, and a large coercive force so that the magnet will not easily be demagnetized.

4.2 Cogging Torque

Cogging torque can be defined as the position dependent and periodic torque ripple due to the interaction between the permanent magnets of the rotor and the stator slots of a Permanent Magnet (PM) machine. It is also known as detent or "no-current" torque. Its periodicity is related to the the number of magnetic poles and the number of stator slots. It's an undesirable feature, that nevertheless can be mitigated and limited. A continuous behaviour of the motor is expected through the 360 mechanical degrees, independently from the angle, when the motor is instead having non–continuous characteristics due to stator slots and rotor poles (permanent magnets). In theory, anyhow, if a machine has rotor or stator with no saliency, then there is a form of that machine that can produce constant, ripple-free torque when excited with pure AC sine wave.

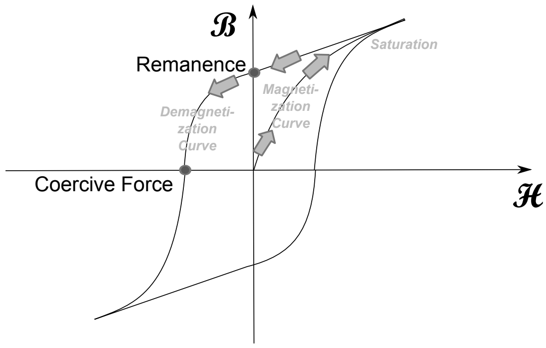

Figure 4.10: The magnetic charateristics for a typical brushless motor.

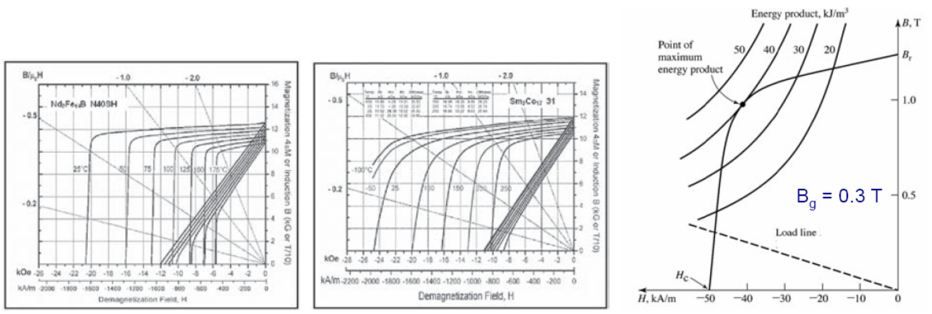

Figure 4.11: The magnetic charateristics for a typical brushless motor.

The problem is particularly evident at low speed, while at high speed the rotor and load inertia act as a low pass filter to the issue. Different design “tricks” can be adopted in order to mitigate the cogging torque:

- make the rotor magnetic field less discontinuous through:
  - skewed magnets or
  - bended magnets
  - stepped skewing of the rotor is axially divided in different cylinders
- create dummy poles so to virtually increase rotor poles through:
  - denting on stator teeths (dummy slots) or
  - dummy teeths
- avoid the molteplicity between stator and rotor discreteness through:
  - magnets of different length for each pole (the full number of magnets ion the 360 degrees are contributing to the torque, so this will not create a discontinuous torque, on the contrary can reduce cogging torque)
  - magnets not equally spaced
  - non symmetric saturation of the iron
  - slot and pole number choice so to avoid close multiples (e.g. 9 slots, 8 poles)

4.3 Thermal Protection

One of the main risk of a brushless motor is to suffer due to high temperature. This can lead to demagnetization of the magnets, insulation loss of the copper cable that in turn can lead to short circuit.

To avoid this different measures are taken, such as:

- Thermal Model of the motor: implemented in the drive, considering thermal dissipation from copper to iron to air
- PTC (Positive Thermal Coefficient). It's a cheap sensor composed by a resistor with nonlinear positive thermal coefficient. In other words, increasing the temperature, the electrical resistance of the PTC will increase violently upon reaching motor maximum temperature, thus allowing the drive to detect such condition. Generally PTC is anyhow slow to react, reason for having also the Thermal Model.
- The current can also be limited by the application program or in drive configuration, but this generally limits the peak current only, while it's the RMS (Root Mean Square) current that heats up the motor.

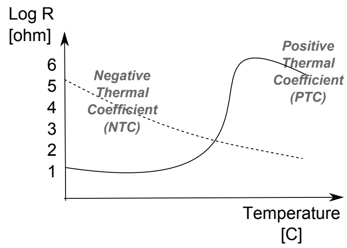

Figure 4.12: The characteristics of a PTC sensor.

Using a water cooling of the motor stator can increase the nominal current of the motor, up to the peak current, depending on water temperature and flux.

4.4 Internal Permanent Magnets (IPM) motors

An IPM motor is a brushless with different inductive properties along the d (direct) and q (quadrature) axes, in contrast with but the surface-magnet motor (SPM) that is rotationally symmetric apart from the magnetization of the magnets and the possibility of slight differences in permeability along the d and q axes.

So in IPMs is used as a feature what in SPM is actually a defect. Indeed the rotor and magnet position and shape of IPM motors are studied so to contribute to torque, in order to decrease the amount of magnets given a certain torque.

The torque is so produced by both the magnets and the rotor anisotropy.

$$\tau = \frac{3}{2} p[\lambda_m i_q + (L_d - L_q) i_d i_q]$$

4.5 Direct Drive Solutions

Automatic machines performance often is also related to machine capacity in terms of cycles per minute. This in turn is related to servo system bandwidth, that is an enabler for having quick and steep profiles driving the load. A way to achieve this is to adopt direct drive solutions, i.e. servo motors directly coupled, in a stiff way, with the load. This in

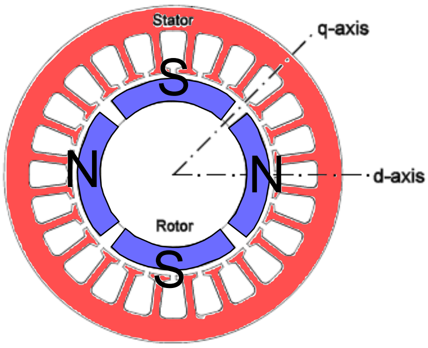

Figure 4.13: Surface permanent magnets (SPM) motor configuration, for a 4-pole motor, with d and q axes plotted

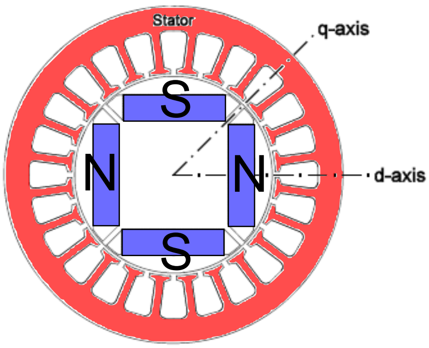

Figure 4.14: Internal permanent magnets (IPM) motor configuration, for a 4-pole motor, with d and q axes plotted

contrast with the traditional way to implement a servo solution, that is through a gearbox, in order to reduce the torque (and thus the size) of the servo motor, and the inertia of the load seen at servo motor shaft.

The opportunity with direct drive solution is to increase the servo drive loop gains, in order to have higher bandwidth of velocity and position that means high kinematic performances.

Nevertheless, doing a direct coupling othyer problems arises. Indeed high gains, leads to high bandwidth, that in turn can reach and cover a resonance frequency of the system leading to vibrations. The resonance frequency can be due to, simply, the motor shaft elastical torsion, i.e. its flexibility, if the shaft is long enough (that means a resonance frequency low enough).

Moreover each control block in the servo drive introduces a delay (integral action plays an important role in this respect), that leads to a lag error naturally different from zero. To minimize it the feed-forward could be useful: it bypasses closed loops regulating blocks (and thus it does not load the integral actions). The feed–forward action is dependent from the following compensating terms: velocity, inertia, acceleration, viscous friction, that thus have to be known with good accuracy, being the feedforward intrinsically open loop. We have said “high bandwith means high kinematic performance”. This because a steep, quick step response, means high frequency content in the frequency domain, that in turn means high bandwidth. Here is the heart of the mechatronics aspect of brushless implementation: loop gains (SW) implemented on the servo drive (HW), are strictly related to the overall bandwidth (Control Theory), given that the system stiffness (Mechanics) is high enough for moving the resonance frequency above the bandwidth. So all four disciplines of Mechatronics are involved, and necessary to be considered, at the same time.

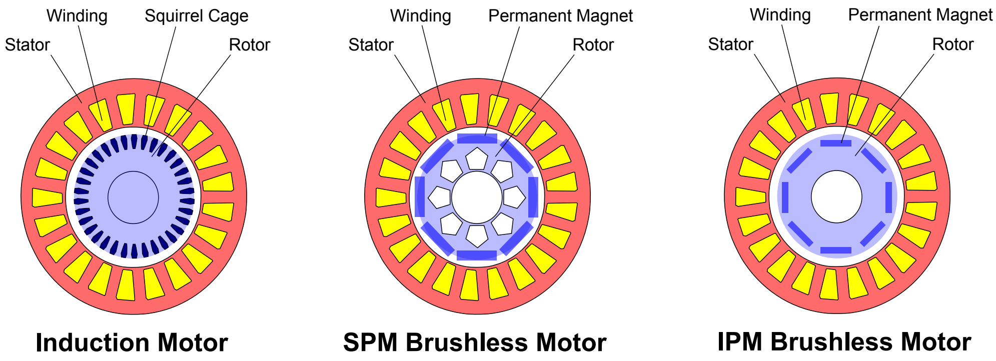

Figure 4.15: Induction vs. SPM vs IPM motors

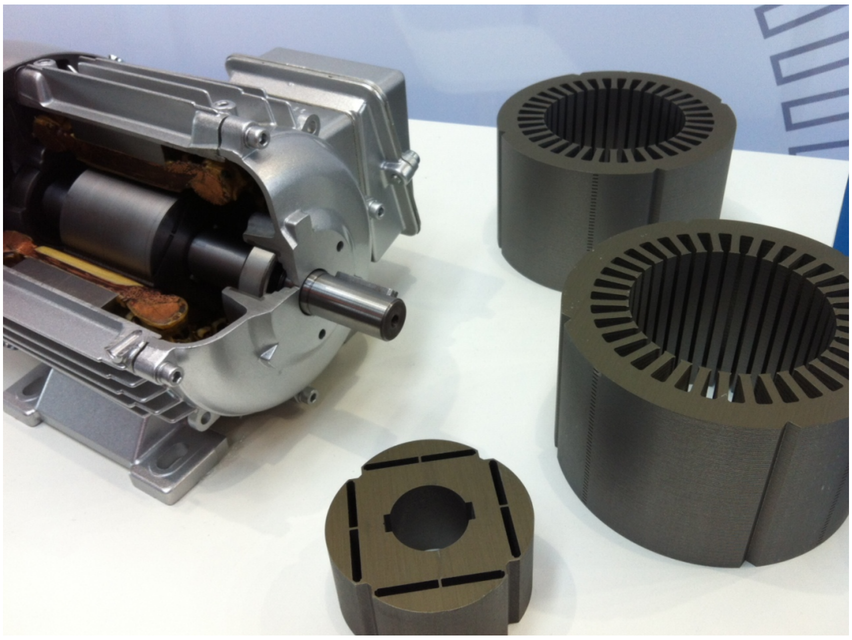

Figure 4.16: IPM rotor and stator

Not only the backlash can limit the bandwidth: other aspects such as a mechanical backlash or low encoder resolution, can lead to similar problem. Indeed a mechanical backlash, with feedback on motor shaft, means that for the backlash amount, the motor can be moving with very low load, leading to wrong servo reaction, that can affect performance once the backlash is eaten up and the real load encountered. If the feedback is on the load the result is not very different: the motor could be moving for a short while with encoder stopped, making the servo drive not realising that speed is above zero, that in turn can lead to wrong servo response once the backlash is eaten up.

A low resolution feedback, could mean that for a short while the encoder (and load) is moving but the output of the encoder is stable (due to the low resolution) leading to similar behaviour as in the backlash case. The non-linearity, due to the backlash or the low feedback resolution, causes a current peak that will take time to discharge, meanwhile causing abnormal behaviour. This, in turn, limits the steepness of the position curve, i.e. it limits the harmonics contents and

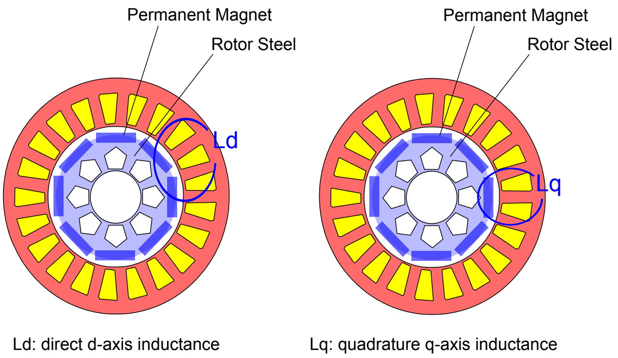

Figure 4.17: Direct (d) and Quadrature (q) inductances for SPM motor

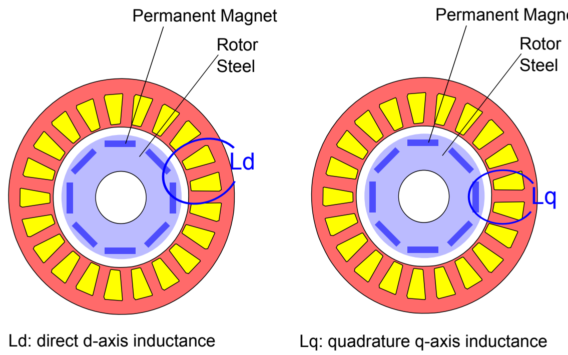

Figure 4.18: Direct (d) and Quadrature (q) inductances for IPM motor

thus the bandwidth. And again we see how aspects from disciplines contribute in similar way to affect mechatronics performance.

Note that to ignite resonance energy is needed. That is provided by the servo drive continuous position, velocity, current loop close. For instance, in a step response, the resonance frequency could be ingited during the steep part of the response, then kept ignited in the plateau part, but with less energy (the servo loop is always closed). In this plateau part the servo system will anyhow have more time to mount up the resonance leading to possible out of control vibrations. Summarizing, the vibration can be caused by:

- resonance frequency
- mechanical backlash
- feedback resolution
- position profile
- servo drive loop gains

These are the source of the problem, and can, of course, also be the solution to it. Other possible solutions could be:

- Digital Filters (only for constant resonance frequencies)

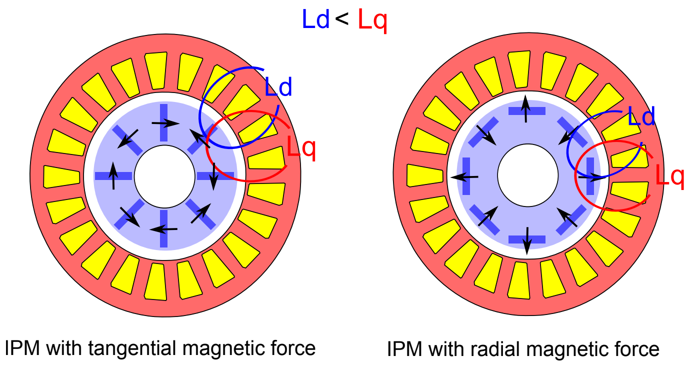

Figure 4.19: IPM concept

- High Stiffness Motors
- Torque Motors (low velocity, high stiffness, high torque direct drive motors)

Once the resonance frequency problem is solved, the control loop gains can be increased and thus a good accuracy in the position sensor becomes mandatory, leading to the choice of a good feedback resolution system. Different technologies are available:

- Resolvers has a resolution around 6 arc min = 0.1 deg can be defined
- SinCos Encoder has a resolution around 0.01 arc sec = 2.8 10-6 deg or 1nm for linear encoders

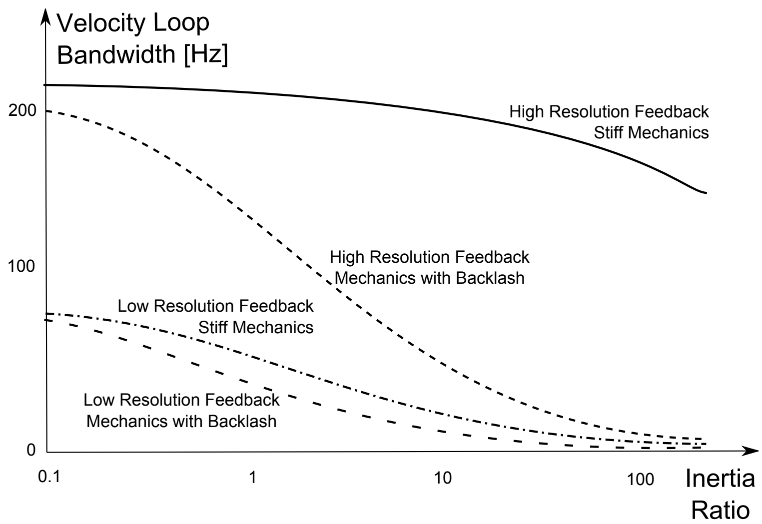

Figure 4.20: Velocity loop bandwith versus the backlash phenomenon.

The available bandwidth is actually also depending on inertia ratio between motor inertia and load inertia seen at motor shaft. In general the lower the better in terms of bandwidth of a direct drive system. That is: if, with a given motor, we have some degree of freedom on adjusting application load, then the lighter is the load the higher available bandwidth we will have.

The type of application (e.g. constant speed vs. index table vs. Delta robot application) is dictating the possible maximum inertia ratio: poor performance need (e.g. constant speed) will survive even with high inertia ratio (e.g. 1:200), while high (index table) or very high (Delta robot) performance application will require lower and lower inertia ratios (e.g. 1:10, 1:5 respectively).

In general a direct drive, high stiffness, application can be a remedy for different compliance problems such as:

- Belt or pulley transmission low stiffness
- Flexible coupling
- Gearbox backlash
- Long and/or thin transmission shaft
- Mounting brackets and other coupling

4.6 Resolvers

Resolvers, in principle, are rotating transformers with primary on stator, a secondary on rotor that in turn acts also as primary to emit toard the stator again where are situated two secondary coils at 90 electrical degrees to each other, called sine and cosine. Both are necessary to resolve ambiguity of position interpretation due to the nature of sine and cosine curves, that are passing through the same point twice.

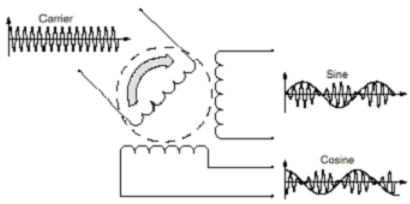

Figure 4.21: Resolver is conceptually similar to a transformer with a primary and two secondaries in quadrature.

Characteristics of a resolver are:

- linearity: 0. 1 – 0.5%
- resolution: 0. 1 – 0.5
- sensitivity: 5 – 10mV/^{o} (Vref =20V)
- frequency: 4 to 20KHz

Pros of resolvers are:

- absolute in one turn
- low cost
- robust (mechanically and electrically)

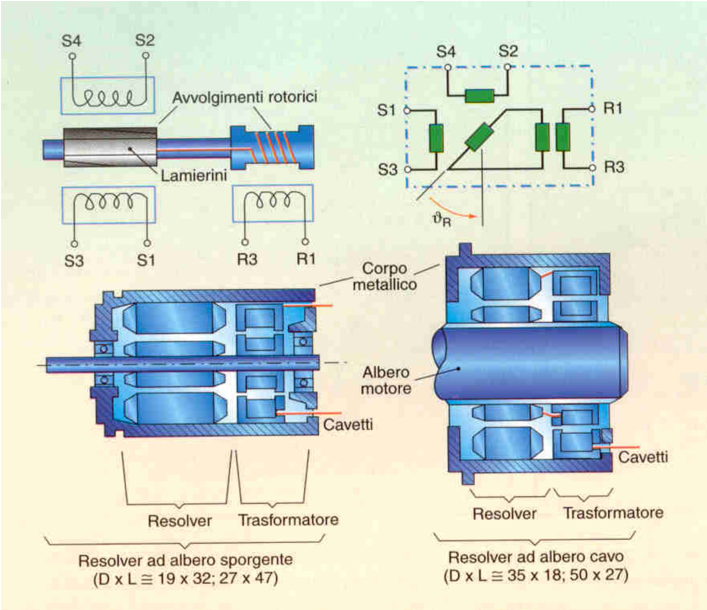

Figure 4.22: Resolver mechanical and electrical solutions

While cons are:

- sinusoidal 20KHz reference voltage is necessary
- non-linear output

It has been the standard position sensor on brushless motors for decades. But now, thanks to higher but acceptable cost, the standard is the encoder.

4.7 Optical Encoders

Optical encoders are angle transducers. They are composed of:

- an optical disc: made of glass or plastic, it hosts an optical pattern composed of transparent and opaque areas.
- a light source
- a photo detector array that reads the optical pattern that results from the position of the disc while it moves.
- a controlling device, such as a microprocessor or microcontroller, that decode the data received from the photo detector in order to determine the angle of the shaft.

They have generally three channels:

- channel A
- channel B at 90 electrical degrees to channel A

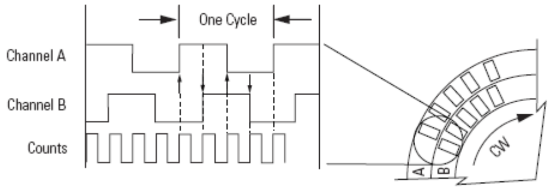

Figure 4.23: The operational principle of an Optical Encoder.

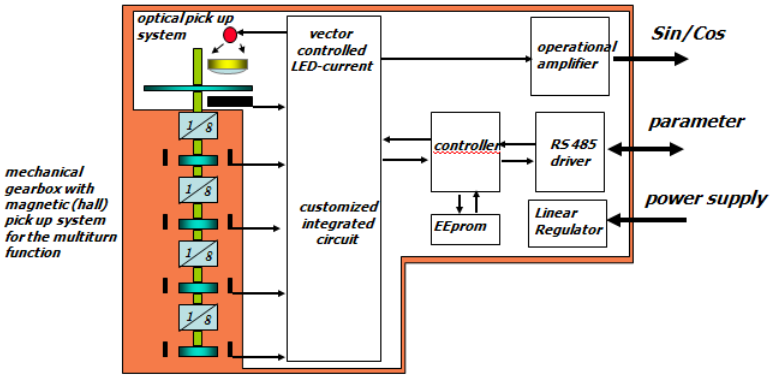

Figure 4.24: The signal acquisition scheme for an Optical Encoder.

- channel "0" for zeroing, i.e. for referencing the disc

Encoder can be divided into:

- Absolute:
  - Battery Back Up
  - One-Turn Absolute
  - Multi-Turn Absolute: the function is achieved through a micro gearbox than hold, mechanically, the multi turn position up to a certain number of turns (e.g. 4096, i.e. 12 bits, if we use for disks each reduced 1:8 by a gear)
- Incremental: only incremental pulses are provided so that position can be measured (counted) only from a predefined starting point.

And furthermore into:

- TTL: 5V outputs of the channel A, B (AquadB). It's a square wave. For instance it can have 1024 lines per revolution, that, after interpretation by decoding electronics, provides 4096 encoder counts per revolution, since every 360 electrical degrees 4 different AquadB edges can be detected.
- SinCos: it's a sin–cos output similiar to that of a resolver, but it provides a hundreds or thousands of sinusoidal cycle per turn. The decoding electronics can be then similar or the same of a resolver (Resolver–to–Digital Converter). And, most important, the resolution soars to millions of counts per revolution, thanks to the possibility to further

interpolate the sinusoid within each cycle. This can be interpolated a further thousand times or more depending on number of bits of the servo drive specific decoding electronics (e.g. 10 bits = 1024 times). Following the numeric examples we have given a typical multi turn absolute encoder could code the absolute position with a total of:

- 12 bit for counting multi turn functionality (4096 turns)
- 10 bit for counting the lines per revolution (1024)
- 10 bit for fine sin/cos interpolation (Resolver–to–Digital converter)

Total being 20 bit for coding the position inside a revoltion, i.e. about one million counts per revolution. The position is then extracted with the formula:

$$\Theta = tan^{-1}\frac{\sin\Theta}{\cos\Theta}$$

Where $\sin\Theta$ and $\cos\Theta$ are provided by the Resolver–to–Digital converter. Note that the top (heaviest) 2 bits for fine sin–cos interpolation, actually provide the counter of A–quad–B encoder counts, or, in other terms, quadrant count.

Modern encoders are completely digital at the motor connector interface, and use a single cable with also motor power included.

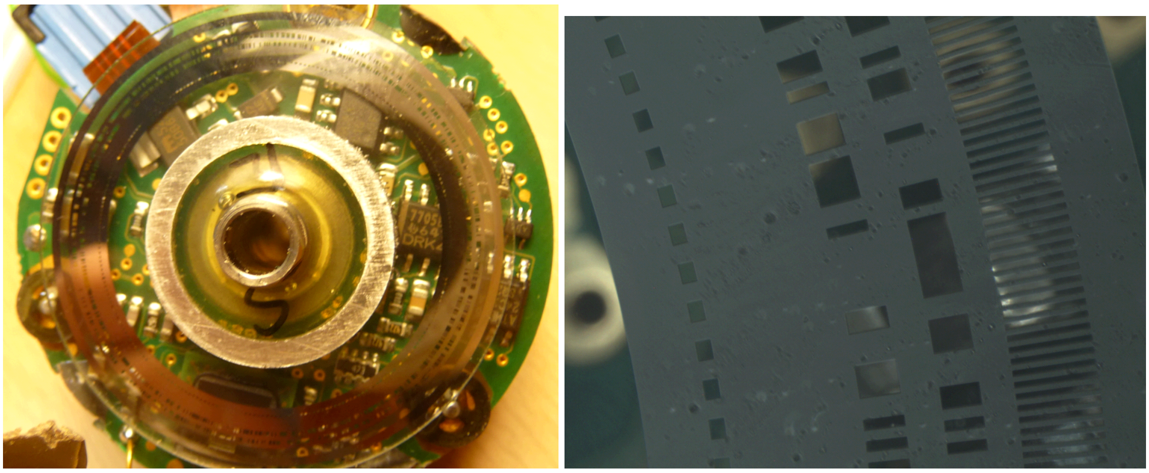

Figure 4.25: Optical encoder disc

Digital transmission rate of data can be 9 Mbd, with RS-485, data in a short span of 12.15 μs. Actually what is transmitted in so short time is only the delta–delta–position, i.e. the acceleration, that will be integrated two times to get the position, which will be sent periodically (for resetting cumulating integral error) at more coarse update, e.g. 195 μs. Transmission of a second channel absolute position for SIL3 redundant safety applications (IEC61508) is also often allowed. Finally a parameter channel for bidirectional general data communication is also often available, e.g. for electronic type label for identification of the motor-feedback system and for storage of drive-related data in the motor-feedback system.

4.8 Inductive Encoders

It is based on variable reluctance principle. It's composed of:

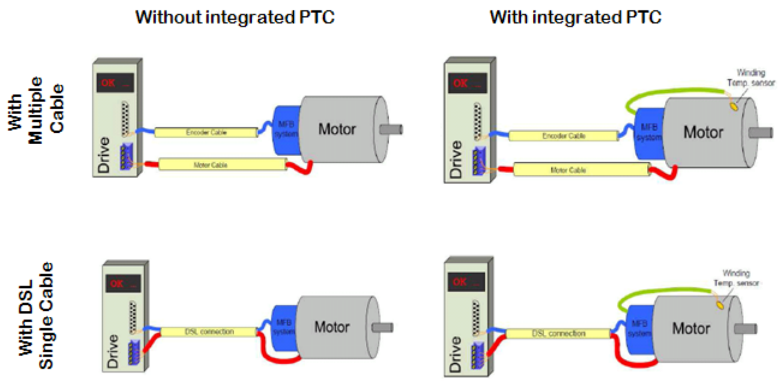

Figure 4.26: Motor to Drive connections possibilities with single vs. multiple cable

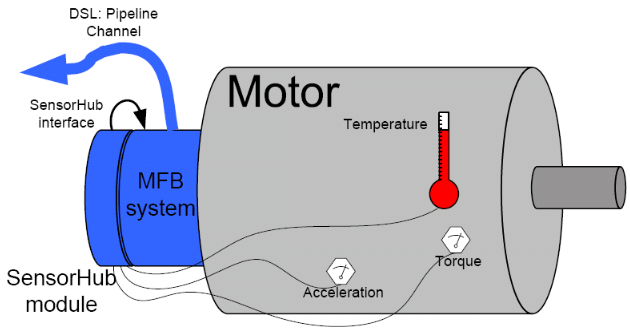

Figure 4.27: The motor encoder can act as a hub collecting information on motor status and send them digitally to the drive

- A measuring scale made of stainless-steel tape on which a periodic graduation of variable reluctance has been etched using photo-lithographic techniques
- A coil structure, with a number of coils aligned in the direction of measurement, made using micro-multi-layer technology.

The relative movement between the two provides changes in the mutual inductance of the individual coils, generating two sinusoidal signals with a 90 electrical degrees phase difference. Note that there's no optical or magnetic function in this technology, that enables it to be used in environment that are critical from these points of view.

4.9 Capacitive Encoders

It is based on variable capacitance. It's composed of:

- A stationary transmitter that emits electro magnetic waves
- A conductively patterned rotor disk that partially masks them following a certain pattern (e.g. daisy like)

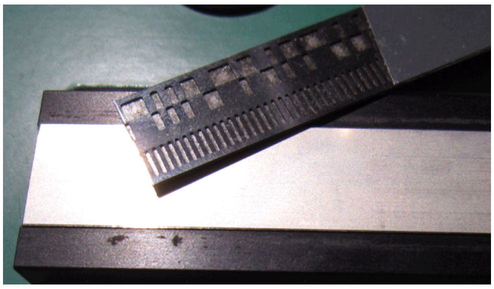

Figure 4.28: An inductive Encoder.

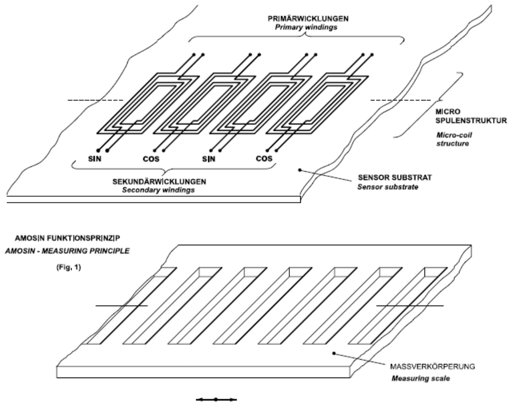

Figure 4.29: Indcuctive encoder principle

- A stationary receiver that receives the electr magnetic wave amplitude-modulated by the rotor disk.

The advantage is that it's a "holistic" rotor, i.e. unlike in other encoders, the whole area of the capacitive encoder rotor participates in signal generation. Multiple spatial periods are integrated that results in:

- Geometrical compensation
- Averaging of temperature and contaminations

4.10 Magnetostrictive Sensors

They are based on Magnetostriction, i.e. the tendency of some materials to change shape, constrict or expand in the presence of a magnetic field.

A magnetostrictive position sensor induces a mechanical wave or strain pulse in a specially designed magnetostrictive wire called waveguide. They are generally available in linear configuration, due to the nature of the technology.

Through the measure of the pulse time of flight, the distance can be deduced, since the wave speed is constant and repeatable. The pulse is created by momentarily causing the interaction of a magnetic field created by a permanent magnet and another created by a current pulse in a coil. A limit of the technology in the interrogation interval that is limited by the wave speed itself and distance of the moving slider. Typical interrogation rates are 1 to 3 KHz.

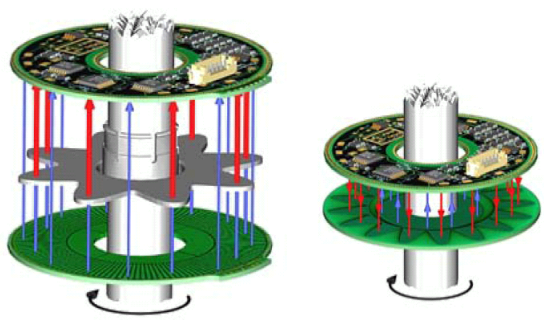

Figure 4.30: Capacitive encoder.

4.11 IP Protection Level

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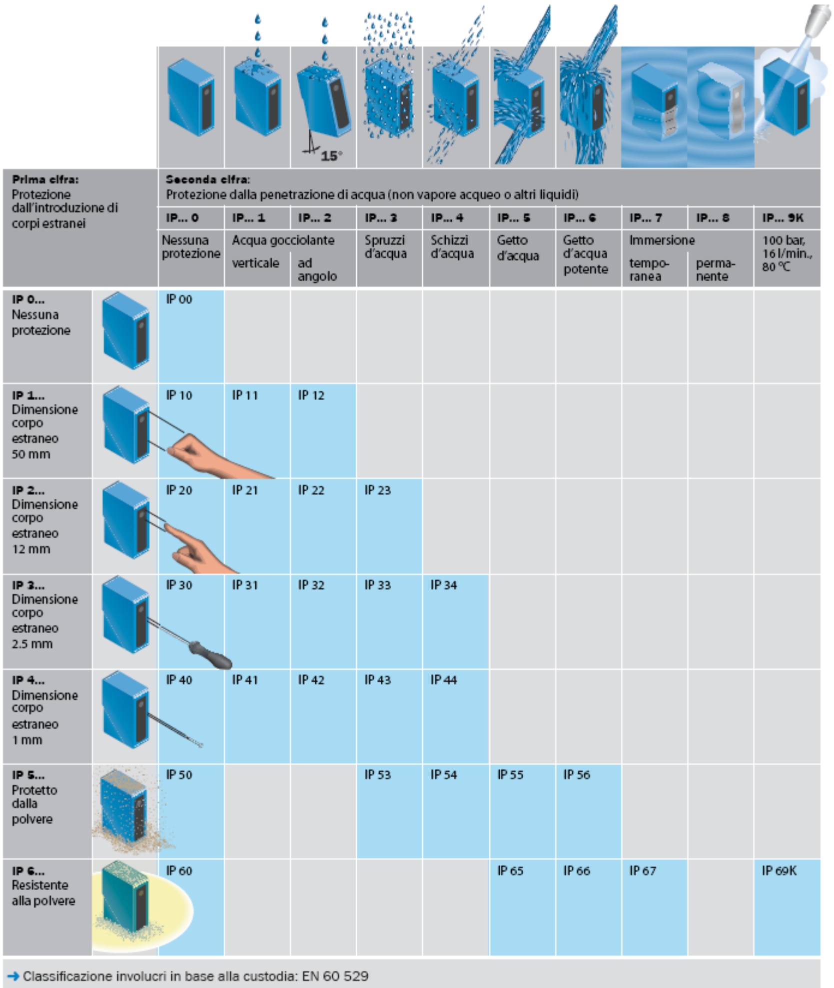

Prima cifra: Protezione dall'introduzione di corpi estranei	**Seconda cifra:** Protezione dalla penetrazione di acqua (non vapore acqueo o altri liquidi)									
	IP... 0	**IP... 1**	**IP... 2**	**IP... 3**	**IP... 4**	**IP... 5**	**IP... 6**	**IP... 7**	**IP... 8**	**IP... 9K**
	Nessuna protezione	Acqua gocciolante verticale	ad angolo	Spruzzi d'acqua	Schizzi d'acqua	Getto d'acqua	Getto d'acqua potente	Immersione tempo-ranea	perma-nente	100 bar, 16 l/min., 80 °C
IP 0... Nessuna protezione	IP 00									
IP 1... Dimensione corpo estraneo 50 mm	IP 10	IP 11	IP 12							
IP 2... Dimensione corpo estraneo 12 mm	IP 20	IP 21	IP 22	IP 23						
IP 3... Dimensione corpo estraneo 2.5 mm	IP 30	IP 31	IP 32	IP 33	IP 34					
IP 4... Dimensione corpo estraneo 1 mm	IP 40	IP 41	IP 42	IP 43	IP 44					
IP 5... Protetto dalla polvere	IP 50			IP 53	IP 54	IP 55	IP 56			
IP 6... Resistente alla polvere	IP 60					IP 65	IP 66	IP 67		IP 69K

→ Classificazione involucri in base alla custodia: EN 60 529

Figure 4.31: IP Protection Level

CHAPTER 5

DIRECT DRIVE MOTORS

Modern industrial automatic machines frequently have requirements expressed in terms of capacity (e.g. in products per minute) and real time performance. These, more and more often, are met by servo brushless system, that in turn, to be up to the task, need to have enough system bandwith. The best servo system bandwidth can be guaranteed by *Direct Drive Systems*, i.e. brushless motors (of different kinds) connected directly to the load, in order to:

- Have high stiffness, i.e. high resonance frequency, that allows the user to set high gains in the drive, an enabler for high bandwidth
- Eliminate mechanical backlashes, a non linear behaviour that would limit the gains
- Have high MTBF (Mean Time Between Failure), thanks to fewer component in the kinematic chain, due elimination of mechanical transmission components.

Being the kinematic chain composed by several elements in series, the choice of the following items, is critical for the servo system bandwidth:

- Servo position, velocity, acceleration, jerk profile
- Controller coarse update
- Drive gains
- Drive fine update loop
- Servo motor kind

- Feedback resolution
- Mechanical coupling

The focus of this chapter is on servo motor kinds for direct drive applications. Several kinds exists, that can be appriopriate or not, depending on applications. We will examine the followings:

- Linear Motors
- Torque Motors
- Iron Powder Core Motors

5.1 Linear Motors

Essential parts in a complete linear servo system are:

- a slider (or sliding element)
- a guide (with bearings, bushings or other sliding system)
- a stator
- a feedback system

Actually often the servo supplier is providing only the first three elements, while the feedback system has to be put together by the OEM, as well as other elements, if necessary, such as: case, brake, bumper for end-of-stroke. Note also that linear motors can come in two configurations:

- moving magnets
- moving coil

The first has the magnets in the slider, and the copper coils in the stator. Viceversa is true for the second, that, having coils in the slider, needs flexible cables for providing power to moving cart.

Advantages of linear electric motors are in terms of:

- high performances (bandwidth)
- precision (strictly related to feedback system resolution)
- reliability (due to few mechanical parts, that enables high MTBF)

while disadvantages can be:

- cost
- limited stroke
- space occupancy
- often new know-how is needed to mount it

Electric actuators are not alone in the linear systems arena. Other solutions can mitigate or solve the main cons, that is the cost. In particular:

- pneumatic cyclinders
- hydraulics
- solenoids or voice coils

Pneumatic cylinders in particular can be very cheap, easy to start up. While hydraulic systems can develop very big forces. Voice Coils or solenoids, that are generally single coils systems, are also cheap and compact but not accurate and repeatable. Nevertheless they can be useful for bang-bang (two positions) applications. In contrast electric actuators are clean solutions, with good force output, accurate, repeatable, any position-velocity-force profile, best environmental impact (in terms of energy efficiency, leaks, acustic noise, limited lubricating oil pollution that can be critical for hygenic design machines). Linear electric motors can be thought, in principle, as rotary motors whose stator and rotor has been split and laid down on a flat surface, as seen in Figure 5.1.

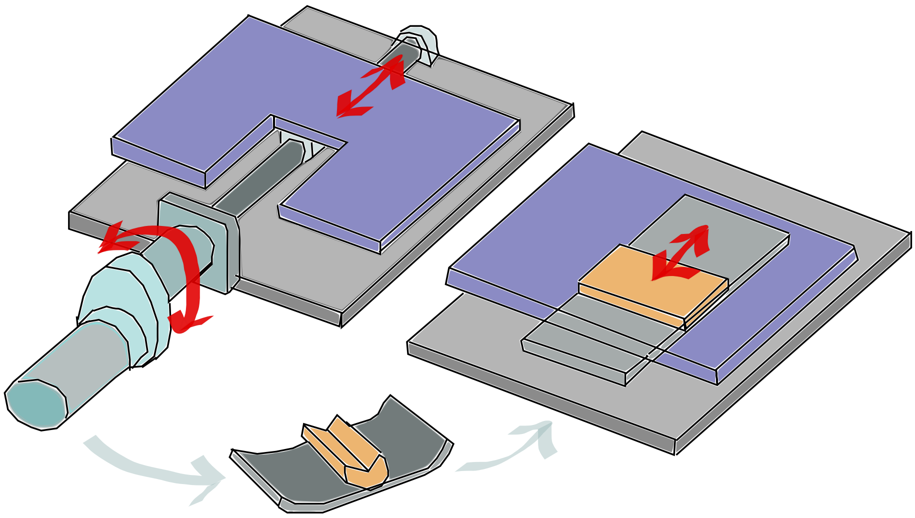

Figure 5.1: Linear motor principle.

But in reality they are a new technology with different challenges and opportunities than the rotary counterpart. For example, in a linear configuration, only the air gap between stator and slider is producing force, while in a rotary one the full air gap, 360 degrees, is producing torque. This, in a way, limits the force of the linear motor, given its total weight and cost. To have a rough idea, the force that can be extracted is generally equal or below 8 N/cm^2.

Linear electric motors are of course also in competition with rotary electric motors, that transforms the motion into linear through a transmissions such as a belt (low cost) or ball screw (high force). These are limited in performance, but still often cheap and common solutions.

The force limit of a linear motor can be overcome with multiple sliders or multiple linear motors, connected together with a rigid or flexible media. In this latter case, multiple feedback systems are needed, in order to close real time servo loops. Linear electric motors technologies are very similar to the rotary counterpart, in particular the following are available:

- Linear Induction Motors (LIM)
- Permanent Magnets Linear Synchronous Motors (PMLSM)
- Linear Step Motors (LSTM)

LIMs can be used for high torque, power and speed applications, while LSTMs are on opposite end. PMLSMs sit in between, and are most common on automatic machines.

5.2 Linear Step Motors (LSTM)

As for the rotary counterpart, they can come in different configurations (see Figures 5.2 and 5.3) such as:

- With or without permanent magnets. The magnets would be to reinforce the variable reluctance contribution.
- With different control strategy such as single step, half step, micro-stepping. These lead to increasing resolution contrasted by increasing drive cost.
- With single stator or double (bilateral) that delivers higher force at higher motor cost.
- With flat or tubular configuration that delivers balanced magnet attraction across the 360 degrees, leading to less stress on bearings.

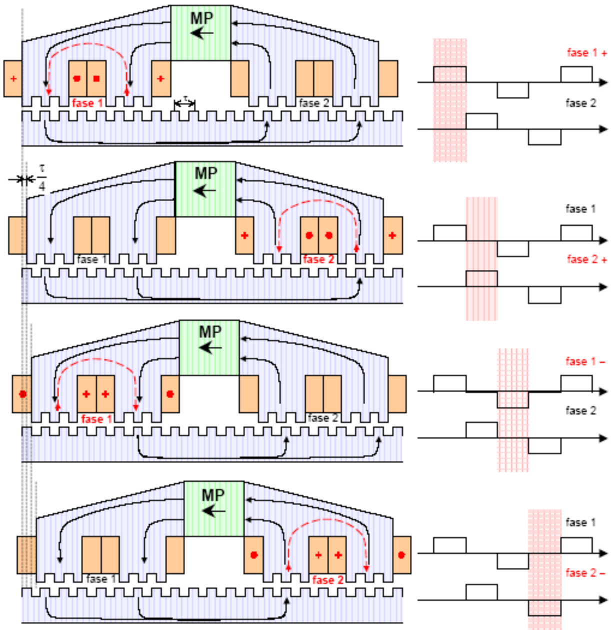

Figure 5.2: Principle of operation for a linear step motor.

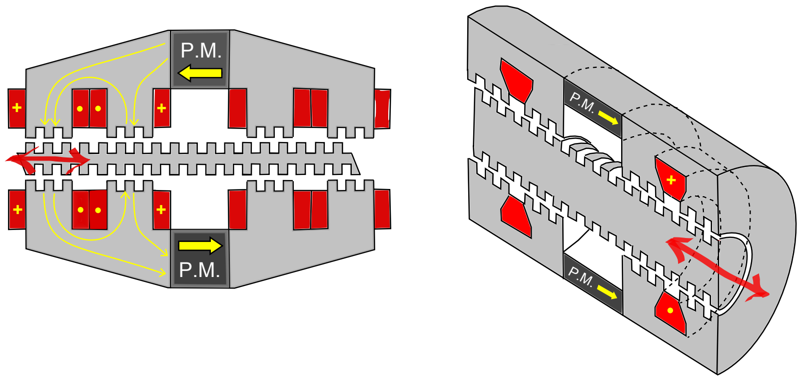

Figure 5.3: Operational concept for a Linear Step motor.

Figure 5.4: Linear step motors.

An exotic configuration of LSTMs is the two dimensions (2D) geometry, where a slider puck (moving coil) is placed on a flat bed of iron teeth (with or without permanent magnets) that provides the variable reluctance effect (and eventually magnetic field). With appriopriate control strategy on slider coils, this can be put in motion on the flat bed in X and/or Y direction. A common solution is for example a tetra phase coils system, where phases A, B, C, D are present.

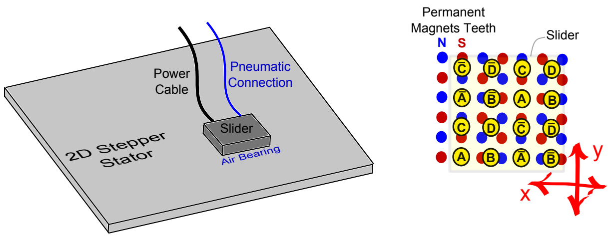

Figure 5.5: Control principle of a Linear 2D Step Motor.

5.3 Linear Induction Motors (LIM)

Linear induction motors (LIM) have a iron and coil primary that is inducing voltage, current and force in a linear (kind-of) squirrel cage secondary (see Figure 5.6). The physical laws in action are the same than in the rotary induction motors, i.e. Lenz and Lorentz Laws. The common configurations are:

- Short vs long primary (as opposed to long or short secondary respectively). The latter can have passive squirrel cage slider (i.e. no moving cables) at the price of more copper in the primary.
- Single face (SLIM) vs double face (DSLIM). The latter is providing more force at higher motor cost.
- Transversal Flux Single face (STFLIM) vs Transversal Flux Double face (DSTFLIM). The latter is providing more force at higher motor cost.

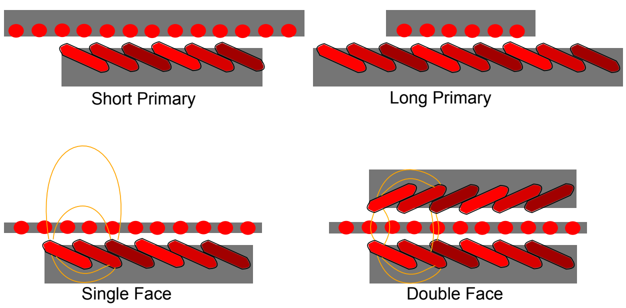

Figure 5.6: Linear Induction Motors (LIM)

5.4 Permanent Magnets Linear Synchronous Motors (PMLSM)

They represent the most common solution in industrial automatic machines. They have copper coils on one element and magnets in the other. The coils are producing a electro magnetic field that, as in the rotary solution, is kept at 90 electrical degrees to the magnetic field of the other element. This is done through the drive, that is reading the magnets position with the necessary feedback system.

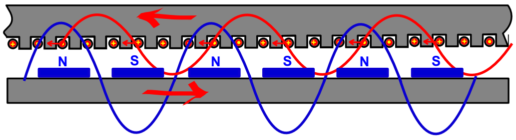

Figure 5.7: Linear Synchronous Motors Working Principle. A typical configuration is moving coil, monolateral, iron core.

The complete range of configurations is comprising:

- Planar vs tubolar shape
- Short vs long inductor
- Surface Permanent Magnets (SPM) vs Internal Permanent Magnets (IPM)
- Iron Core Slotted vs Slotless Iron Core vs Iron Less

This latter subdivision is worth particular interest.

The Iron Core solution is the most powerful in terms of K_t, thus delivered force given the drive current. This is possible thanks to the low magnetic reluctance of the iron that is present around the coils. The magnets on a single face causes a high attaction force that requests appriopriate bearings.

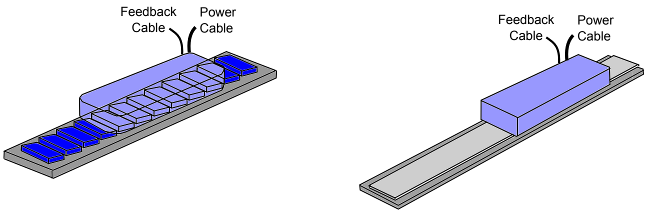

Figure 5.8: Components in a Linear Synchronous Motor

Slotless Iron Core motors (see Figure 5.9), in contrast, have iron only on the back of the copper coils, limiting the advantages of iron core implementation. They are often a convenient compromise in terms of cost.

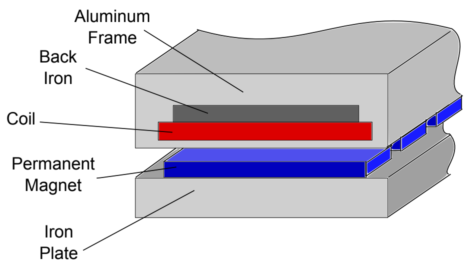

Figure 5.9: Linear Synchronous Motor Slotless with iron core.

Finally Iron Less motors (also known as Air Core or Epoxy Core) do not have iron at all on the coil element (see Figure 5.10), making it really light and applicable for low weight load, high dynamic, but low force application. The iron is replaced by epoxy or other light materials. The low K_t, caused by iron absence, is mitigated generally by double magnet row, that is also providing symmetric magnet attraction force thus reducing the mechanical stress on bearings.

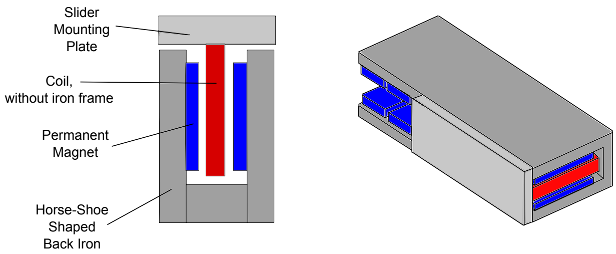

Figure 5.10: Iron Less Linear Synchronous Motor.

Another characteristics that distinguishes Iron Core and Iron Less motors, is the cogging torque, that is substantial in the first and negligible in the latter. This is indeed a way to recognize it once off and loose: moving the slider you can clearly feel the cogging torque only in the Iron Core case.

Tubolar motors are also quite common. They have the evident advantage of the symmetrical design, making the bearing design simpler or even not necessary, using bushings or autolubricated teflon guides. Magnets are placed in the cylinder North against North and South angainst South, so to force the magnetic lines to exit the cylinder, encircling the copper coils in the primary (see figure 5.12). They can be SPM or IPM magnets configuration.

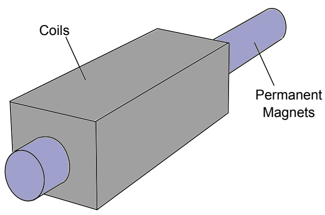

Figure 5.11: Tubolar motors.

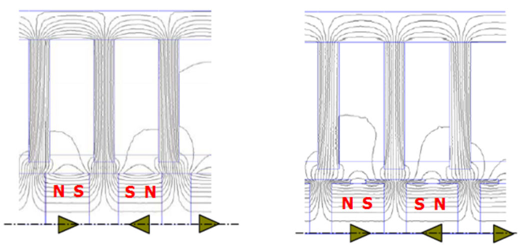

Figure 5.12: Flux of the magnetic field in the linear synchronous motor.

Common feedback systems are optical or magnetic encoders, while sometimes magnetostrictive systems can be the solution for washdown environments.

Some brushless linear motors, mainly for cost reasons, use Hall sensors to sense the magnet magnetic field. These are sometimes called sensorless, since they miss a continuous feedback system. Accuracy, not better than a 0.1 mm, is of course worse than above mentioned solutions, but still applicable for some automatic machines. It's important to stress that linear brushless motors suppliers sell force $[N]$, while suppliers of linear encoders sell accuracy [μ m] and bandwidth [Hz]. Also, not all applications are ready for linear brushless motors applications. In particular:

- If the machine is limited by insufficient stiffness that leads to low mechanical resonances, then the linear motor will not provide any improvement
- If the control system control loop is not fast enough, it can be the bottle neck of the full system making the linear motor not working or waste of money
- If the accuracy or bandwidth offered by the position sensor is not needed, then the linear motor is an expensive solution, that could be replaced by a more convenient ball screw or even (steel reinforced) belt system.

Nevertheless the price of linear brushless motors is more and more convenient and the right comparison should be done versus the rotary motor implementation counterpart. In other words, the full kinematic chain should be considered. That in the rotary motor implementation comprises a motor, a gearbox, and encoder, a mechanical transmission (i.e. rack and pinion), alignment and mounting procedure. In contrast the linear approach has only motor, encoder, alignment and mounting procedure (that once understood can simpler and cheaper that the rotary ones). Given the right comparison, sometimes we are positively surprised by cost convenient linear electric motor implementations.

5.5 Torque Motors

When the need goes in the direction of a direct drive solution, then, if we are in a linear application, a brushless linear motor may be the solution, while in a rotary application a torque motor may be the answer.

They look like big, short and large diameter rotary brushless motors, but they are, in practice, bended linear brushless motors, with cylindrical rotor. Indeed industrial suppliers of linear motors often supply also torque motors. They are characterized by:

- High number of pole pairs
- High stiffness
- Big diameter
- Short length

These aspects make them ideal for direct drive rotary applications, where high torque is needed in conditions of high stiffness and low, or very low, speed.

The stator design is slotted, with motor teeths that are wounded singularly, and then assembled into the stator. In other words the motor phases are wound onto the open teeths stack, and then closed in order to create the stator. This technique obtains very high slots filling efficiency, and is very much now used, not only for torque motors, but also for traditional brushless motors (see Figure 5.13).

5.6 Iron Powder Core Servo Motors

They are built with a powdered iron stator, which has been designed to take full advantage of the material properties. Indeed, there are a number of important design features, which are not possible with traditional laminated cores (or steel lamination) technology. For example, the coils are pre-pressed to form a solid component with a very high filling factor.

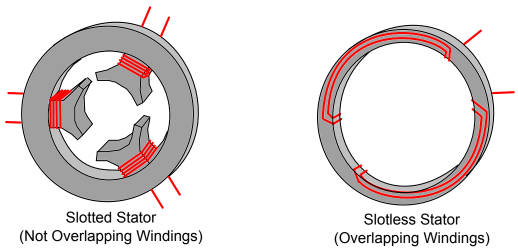

Figure 5.13: Slotted vs. slottless linear synchrnous motors.

Sintered Soft Magnetic Composite is used, instead of traditional laminated cores. In this way, it's possible to limit the space occupied by the copper winding, that in turns leads to the possibility to increase number of poles, thus obtaining higher power density. Indeed in a brushless motor magnetic, poles and winding compete for the same circular space.

The opportunity with Iron Powder Core technology is also to have a bell shaped external rotor, i.e. a hollow rotor that is turning outside the (inner) stator. This design goes also in favor of higher pole numbers, since we are using the outer motor space for magnets. Moreover magnets are placed in the inner part of the rotor bell, mitigating the problem of the centrifugal force that at high speed can remove the magnets in traditional SPM (non-salient poles) rotors. This problem can also be mitigated by another, already discussed, technology, that is the IPM (Internal Permanent Magnets), since the magnets can be conveniently placed inside the rotor. As a cons, these motors have a bad thermal characteristics, caused by the stator position, that is inside the motor and cannot dissipate heat easily. These motors can accomodate high pole number, leading to high torque and making them convenient for direct drive applications. As the torque motors they have a short, big diameter shape, to further increase torque and stiffness. The preferred path for heat dissipation is then the back cover, since the motor is very short but wide. Moreover the Joule losses in these motors are extremely low, due to low speed.

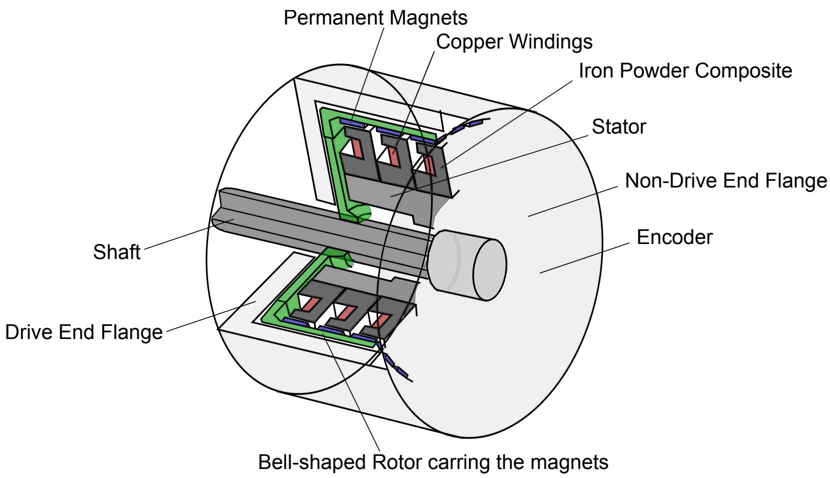

Figure 5.14: An example of Iron Powder Core Servo Motor

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CHAPTER 6

MOTOR-TRANSMISSION-DRIVE SIZING

In early 1990's, with the advent of modern IGBT power transistors, suddenly brushless motors control become industrially applicable. Brushless DC motors were introduced in 1962, when T.G. Wilson and P.H. Trickey unveiled what they called "a DC machine with solid state commutation". First applications were in computer disk drives, robotics and aerospace.

Later two milestone events paved the way for industrial acceptance of AC brushless motor:

- The development of high energy content, rare earth permanent magnets which significantly reduced motor size and weight
- Power IGBT transistors that are fast enough without heating up too much.

These technologies have solved the previous bottlenecks, that were in the motor power and servo drive respectively. So the performance limit was moved forward, and became, again, the transmission mechanics. This is the subject of this chapter.

Backlash non linearity is one aspect, that we have already covered. And can eventually be solved, for the shaft element, with conical mechanical coupling. We will see that, also in gear boxes, there are solutions that limit or solve the problem.

Another important aspect is mechanical resonance frequency. Looking for performances, high loop gains are selected when tuning the servo system, leading to a potentially high bandwidth, that in turn can cover a low mechanical resonance frequency, causing an unwanted vibration. This of course assuming that everything else in the servo loop is up to the task, e.g. feedback resolution, servo fine update loops.

A resonance can grow, due to load elasticity that cause oscillations, which behave as high-noise vibrations. The motor then overheats due to high frequency accelerations and decelerations. To solve the resonance frequency problem one can of course reduce the gains, but this can be paid in terms of lower machine performances in cycles per minute.

Another approach is an active response through a passive digital filter, which can be for example a Notch or Low Pass Filter. Note that active digital filters, such as State Observers (e.g. Kalman Filters) can also be used, generally with the different purpose of reducing position error.

The passive filter approach has anyhow the strong cons that we need to know the resonance frequency, and it has to be stable. This is not often the case, but when it happens a Notch filter can be the best solution.

The more robust solution is anyhow of course to design a more stiff mechanical transmission. This is moving the resonance frequency to higher values, enabling a higher servo bandwidth through higher gains.

The velocity accuracy of a servo system is basically motor–independent: it does depend only on the position sensor and regulator tuning.

The reaction times of a brushless motor can be as low as few ms: the limit is now the mechanics attached to it.

The mechanical kinematic chain of a rotary servo motor can be designed for converting rotational to rotational degree of freedom, or rotational to linear. As a consequence different technical solutions can be available. Some examples are:

- Rotational to Rotational:
  - *Gearbox*: high cost solution, a gearbox coupled with a servo motor needs a low backlash, typically below 10-15 arcmin.
  - *Belt* (with servo and load on two different pulleys of the belt system): low cost solution, but low band-width (from 10Hz to some hundred Hertz)
  - Worm-gear: very low efficiency at high speeds, high static friction, not ideal for varying speed applications
- Rotational to Linear:
  - *Belt* (with servo on one pulley of the belt system and load on a runner block attached to the belt): low cost solution, but low band-width (from 10Hz to some hundred Hertz); higher bandwidth can be obtained with metal core belt reinforcement or metallic tape-belt
  - Pinion-rack: big backlashes are generally present, band-width can be very low

Applications of servo motors, in terms of mechanical transmission need, in particular for gearbox case, can be thought as splitted in two categories:

- Power applications (e.g. mandrels, traction, winders): not relevant kinematic performances are needed since the application is often at constant, or quasi constant, speed; motor cost is a significant fraction of full system cost, and thus important. It's then generally useful a transmission with a reduction that allows for smaller motor for reducing torque requirement. Moreover typically these application do have high load inertia, that can be conveniently reduced, at motor shaft, by a gearbox.
- Positioning applications (e.g. indexed packaging applications): high kinematic performances can be needed; motor cost is a small part of full system cost, and thus less important than in power applications. It's then sometimes possible to use a direct drive, that generally means a bigger motor, or even a torque or linear servo motor. In case a gearbox is anyhow selected, with high transmission reduction ratio, the load inertia become less important, but the motor inertia becomes more important. When choosing the gearbox, the optimal ratio (with respect to minimum motor's torque given a certain acceleration need) is the one with load inertia seen at the rotor shaft, equal to rotor inertia, that is what is called a rotor-load inertia ratio of 1:1. Instead if a direct-drive solution is the way to go, then, given sufficiently high stiffness system, the best inertia is the lowest one, for rotor and load.

We have thus seen that load inertia seen at motor shaft, is a relevant parameter to consider for motion system performances. And indeed for a gearbox transmission the inertia J_{tot} at motor shaft would be given by the formula:

$$J_{tot} = J_{rot} + \frac{n_1^2}{n_2^2} J_{load}$$

Where J_{load} is the load inertia and J_{rot} is the rotor inertia, while n_1 and n_2 are the fast shaft (connected to the motor) and slow shaft gear (connected to the load) diameters (or number of teeth) respectively (see Figure 6.1).

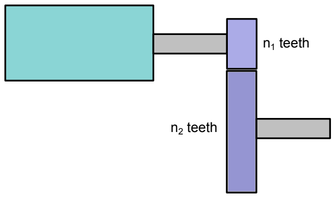

Figure 6.1: Motor transmission gear ratio.

For a rotational to linear belt transmission (see Figure 6.2), J_{tot} would be:

$$J_{tot} = mr^2 = m(v/\omega)^2$$

Where v is the linear speed, r is the pulley radius, m the load mass, and ω is the rotational speed.

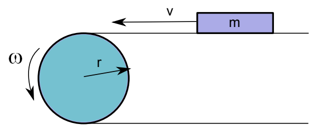

Figure 6.2: Rotational to linear belt transmission.

Finally for a screw system (see Figure 6.3), J_{tot} would be:

$$J_{tot} = m(s/(2\pi))^2$$

Where s is the screw pitch and m the load mass.

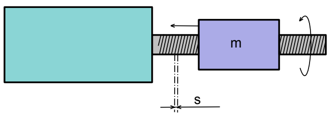

Figure 6.3: A screw system for rotational to linear transmission.

Low backlash gearboxes play a significant role in servo transmissions. They can be of different types, where the most common is the classical planetary gearbox solution (Figure 6.4). It is composed of a solar wheel, or pinion, and some planetary wheels that run into an internal crown. When the planetary rollers turn, they describe a geometrical profile called epicycloid.

They can be composed of one or more stages (Figure 6.5), where each stage typically provides up to a $1:10$ reduction.

Angular backlash, we have said, is a problem for mechanical transmission (see Figure 6.6), since it is introducing a significant non linearity that can seriously limit the overall system bandwidth.

For planetary gearboxes the problem cannot eliminated, due to intrinsic mechanical functionality of the teeth, but can be mitigated with precision mechanics.

When higher levels of stiffness are necessary, other mechanical transmissions technical solutions are available.

One so called zero-backlash solution is Cycloidal Gearboxes such as those produced by Sumitomo^{TM}. The concept has been invented in 1931 by Lorenz Braren that then founded Cyclo Getriebebau^{TM} GmbH in Munich, Germany,

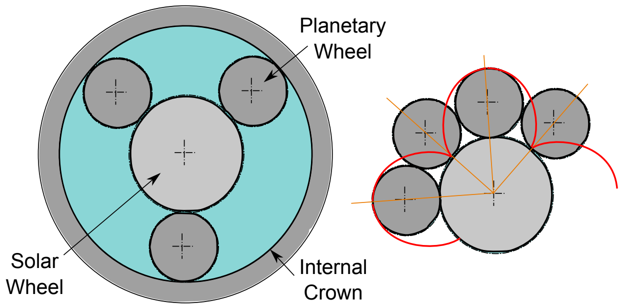

Figure 6.4: Planetary Gearbox

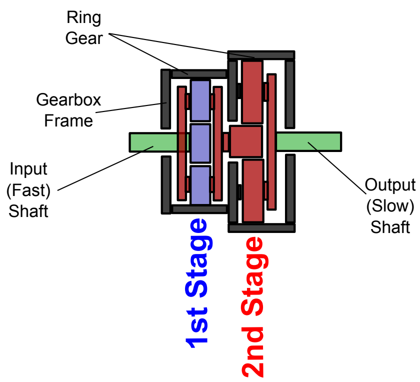

Figure 6.5: Planetary gearbox composed of two stages.

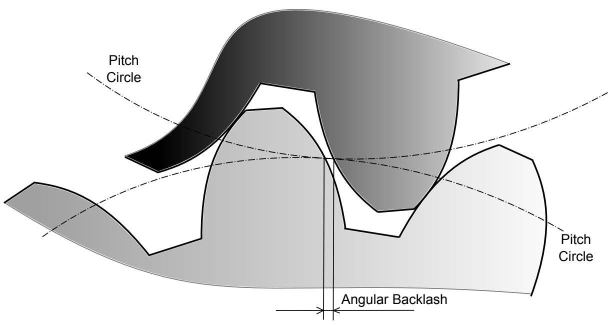

Figure 6.6: Angular backlash.

later bought by Sumitomo^{TM} in 1994. The gearbox is built with an inner eccentric wheel that is running inside a hollow internal crown. The inner is having at least one tooth (or petal since it has a smooth contour) less than its crown counterpart, so that, after one turn the inner wheel will be lagging behind of at least one teeth. This, for a 30 teeth wheels and 29 teeth in the inner wheel, means a 1:30 reduction. It can then be understood that these gearboxes can easily cover big or very big reduction ratios, but not the small ones (typically not below 1:20). They feature reduced backlash, hight torque possibility, high reliability and long life, smooth and quiet rolling. They do not operate in shear but in compression, that translates into a valuable benefit that is basically the avoidance of a catastrophic failure.

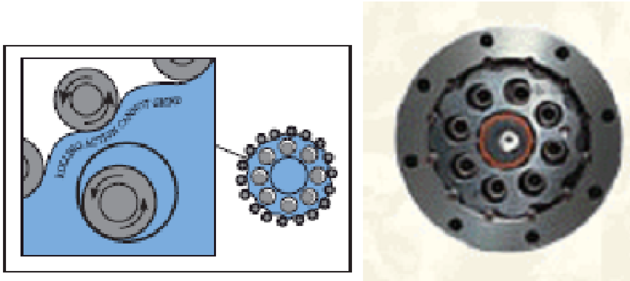

Figure 6.7: Cycloidal gearbox.

Similar advantages are found in the Strain Wave Gearing producved by Harmonics Drive^{TM}. The basic concept was introduced by american engineer C.W. Musser in his 1957 patent and first used successfully in 1964 by Hasegawa Gear Works^{TM} (later became Harmonic Drive Systems Inc. located in Japan) and USM^{TM} Co. (later Harmonic Drive Technologies Inc. located in U.S.A.).

The electrically-driven wheels of the Apollo Lunar Rover included Harmonic Drives gear boxes in their motion system. The concept is based on the metal flexibility of the inner spline, shaped like a shallow cup, that in this case is not eccentric but inserted in an elliptic wheel (or wave generator). In turn this is running into a rigid spline. Since the flex spline, meshing the tooth of the circular rigid spline, has an elliptical shape, its tooth only actually mesh with the tooth of the circular spline in two opposite sides areas of the flex spline. Again the key to the design of the harmonic drive is that there are fewer teeth on the flex spline than there are on the circular spline.

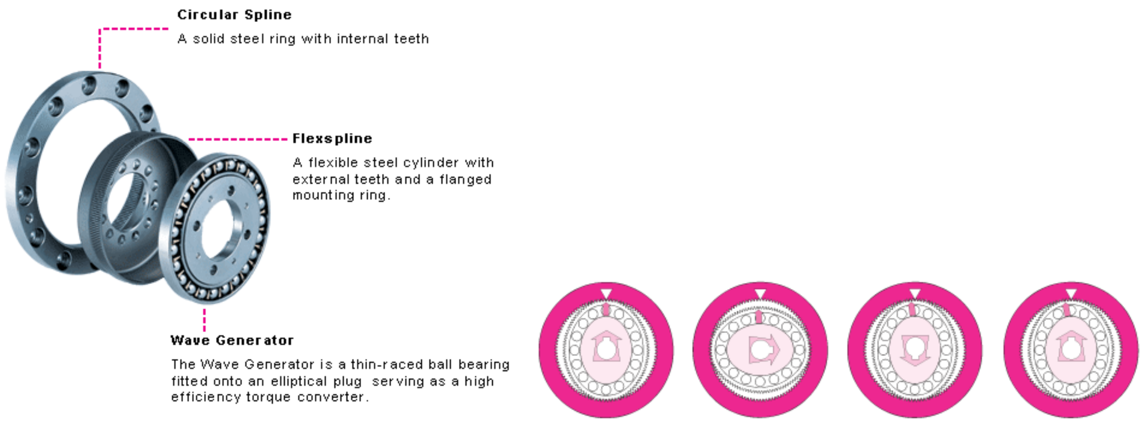

Figure 6.8: Strain wave gearbox.

Note that both these two concepts are considered zero-backlash, but still they are linked to a limited stiffness, that is still present. This, through material bending, leads to different load position than expected, upon high applied torque. This effect, present also in traditional planetary gearboxes, is then similar to a backlash in some respect, even if generally smaller in comparison.

Stiffness and backlash are thus two critical aspects, that can be limited (stiffness) or eliminated (backlash). Another one is Inertia Ratio.

With this term we mean Rotor-Load inertia ratio.

We have said that "When choosing the gearbox, the optimal ratio (with respect to minimum motor's torque given a certain acceleration need) is the one with load inertia seen at the rotor shaft, equal to rotor inertia, that is what is called a rotor-load inertia ratio of 1:1."

To demonstrate this we start from the total inertia seen at motor shaft with a transmission ratio of N, that is:

$$J_{tot} = J_{rot} + J_{load}/N^2$$

While the torque T is given by:

$$T_{tot} = J_{tot} d\omega/dt$$

And the rotor angle θ_{rot} relationship with load angle θ_{load}, is given by:

$$\theta_{rot} = N\theta_{load}$$

These three formulas lead to to:

$$T_{tot} = (NJ_{rot} + (J_{load}/N))\frac{d^2\theta_{load}}{dt^2}$$

Or:

$$T_{tot}/a = NJ_{rot} + (J_{load}/N)$$

Our optimization task, in order to limit as much as possible motor size, is to minimize the ratio T_{tot}/a, that in turn means to get out *minimum motor's torque given a certain acceleration need*, as previously stated.

The scenario we are exploring is in the case of "when choosing the gearbox", that means that the degree of freedom we want to / can use for optimisation, is the gearbox ratio N.

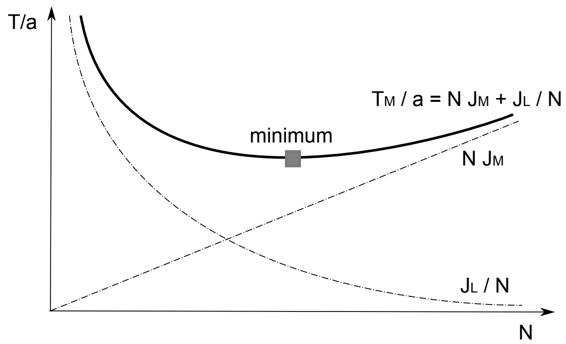

Figure 6.9: Motor-Transmission-Drive Sizing Optimal Gearbox

Note that the second equation member is composed by two terms: a proportional term NJ_{rot} (straight line passing through the origin) and an hyperbola J_{load}/N. The sum of the two is the function whose minimum gives the optimal N value in the abscissa.

Deriving with respect to N and equaling to zero, we get indeed:

$$N^2 = J_{load}/J_{rot}$$

That is:

$$J_{load}/N^2 = J_{rot}$$

That is a 1:1 inertia ratio, that we can consider the optimal solution for optimization of the motor size when choosing the gearbox.

Note, in fact, that this is for determining optimal N only.

This 1:1 inertia ratio indeed gives us:

- Optimal ratio N for maximizing load acceleration, given the motor and the load
- Robustness of system performance to variability of load condition (given by local minimum low curve steepness on both sides)
- Optimization of dynamic performances (given by small deformations between feedback and load).

If, instead, given the transmission ratio N, we can reduce J_{load}/N^2 and/or J_{rot} working directly on them, still having them different, we should try to do it.

Often, in this case, it's not convenient to increase the rotor's inertia to achieve the 1:1 inertia ratio. In other words: to increase rotor inertia could lead to a working solution, but we are not optimizing the motor size, actually the contrary: we are putting a bigger, more expensive motor to drive more inertia J_{tot}.

In general the way to achieve high servo system performances is still to go direct drive with high stiffness, and low inertia system. This because, in first approximation, being:

$$T = Js\omega$$

We have the open loop velocity bandwidth transfer function represented by:

$$\omega/T = 1/(Js)$$

That gives an open loop Bode plot that is a descending straight line. At a given point, depending on system stiffness and inertia, we have a resonance frequency.

At higher frequencies the frequency response continue with a descending straight line, that represents the rotor inertia only (i.e. the load is virtually decoupled).

At even higher frequencies, we have other poles (e.g. servo drive current loop) that become dominant and cut the bandwidth.

When going direct drive, given sufficient stiffness, also inertia ratios smaller than one (1), can optimize the bandwidth.

The gearbox cost is typically similar to the motor's cost, thus the optimal "motor plus gear" system sizing, is not necessarely and always the optimal sizing, under the cost point of view.

In general, a good thumb rule can be to avoid to put a gearbox when the ratio is:

$$N^2 > J_{load}/J_{rot}$$

because it reduces the bandwidth and increases the motor's energy consumption.

The ideal case for positioning applications is thus without gearbox, while an an important exception is when

$$J_{load} >> J_{rot}$$

so that the inertia of the rotor cannot compensate the load mechanical resonances. In this case a gearbox is needed, also for precise positioning applications.

When preparing for servo sizing is thus important to correctly choose and optimise the load masses and inertia. Once this is done, preliminary profiles can be designed.

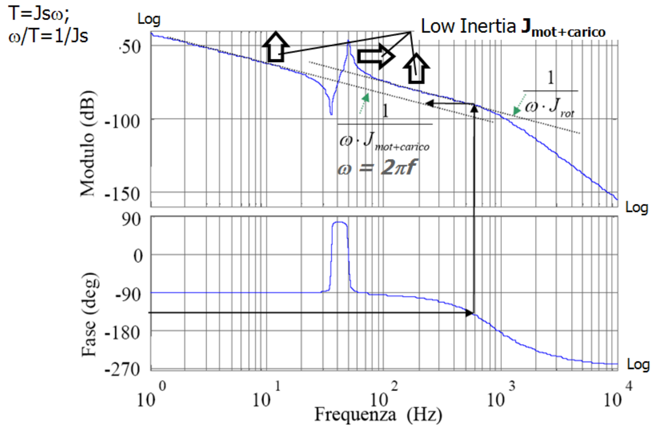

Figure 6.10: Motor-Transmission-Drive Bandwidth Example

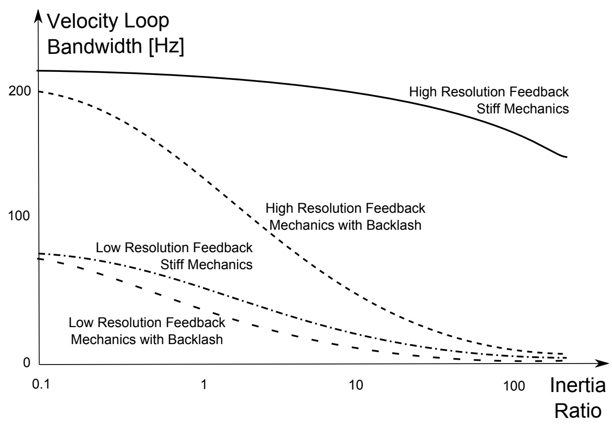

Figure 6.11: Motor-Transmission-Drive Sizing Velocity Loop Bandwidth

These are are the essential ingredients for attacking the servo sizing problem, often done with automatic software tools, that can be provided by the servo motor and/or gearbox supplier. These tools calculate the different torque contributions, in order to correctly choose (manually or automatically) the motor, the gearbox and the drive.

In order to do this, in the path, other quantities need to be calculated, such as:

- RMS (Root Mean Square) torque and speed, for correctly placing operating nominal point in the torque diagram, so to have a margin for not heating up the motor. Application RMS torque can be maintaned indefinetly if below motor's nominal torque. Torque above nominal torque and below peak torque, instead, can typically be mantained for limited time (fraction of a second or few seconds).

- Maximum torque and speed, for having enough margin in the torque diagram before to saturate the torque and/or speed. In other words the maximum torque required by the application cannot be bigger than the motor's peak available torque. On speed side: the maximum application speed cannot lead to a voltage that is bigger than minimum DC Bus voltage.

- Application nominal current: the drive shall be able to deliver it and it shall lead to a motor's torque that is bigger than what the motor can deliver (Nominal Torque at Nominal Speed or Stall Torque at zero speed).
- Application maximum or peak current: the drive shall be able to deliver it and it shall lead to a motor's torque that is bigger than what the motor can deliver (Peak Stall Torque at zero speed, then decreasing at higher speed).
- Inertia Ratio for bandwidth optimisation
- Energy stored in the capacitors for emergency power failure controlled stops (according to IEC/EN 60204-1)
- Optimal PWM (Pulse Width Modulation) drive switching frequency, related for example to motor inductance
- Optimal servo loops gains, through autotuning algorithms
- Effect of backlashes, if present, on performances
- Effect of feedback resolution on performances
- Effect of springs, if present
- Effect of frictions (static, viscous, etc.)
- In-Rush current need by drive when switching on (capacitors behave like short circuit), for correctly size in rush resistors, whose job is indeed to limit such current. These, in modern drives, are generally embedded onto the standard servo drive electronics
- EMC Filter need for conducted emissions (applied on 400VAC power supply to servo drive)
- EMC Filter need for conducted immunity (applied on 24VDC or 220VAC logics supply to servo drive)
- Effect of component tolerances for robustness assessment
- Lifetime estimation (e.g. L10 bearing life)
- 3D CAD integration for space occupancy checking and automatic masses and inertia inheritance

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CHAPTER 7

DRIVES: HARDWARE AND SOFTWARE

Servo Brushless motors, in contrast with AC Induction motors and DC Brush motors, always need a specific electronics to drive it, in order to correctly control the coils' currents so to keep stator and rotor electro magnetic field orthogonal to each other. This maximises the torque and make them more efficient than AC induction motors, and more practical, thanks to commutator absence) than DC Brush motors.

This specific electronics will be hereby called "Drive", and is generally composed by (see Figure 7.1):

- A rectifying bridge for achieving DC Bus voltage. This unit can be present only once, i.e. in only one drive, and the DC Bus be shared among multiple drives in line.
- A Capacitor for smoothing the DC Bus voltage. Same comment as above applies.
- Optionally a DC Bus discharge resistor, and corrispondent transistor to activate it. This is switched on when DC Bus voltage goes above a predefined limit, due to motors working in generator mode.
- Six IGBT transistors to drive the three phase motor.
- Six free-wheeling diodes to give continuity to coil current providing a way to close the circuit
- Digital electronics to drive the transistors (switch them ON and OFF)
- Digital electronics to interpret rotor position through feedback system reading (e.g. and encoder).

The coil current, we've just said, is achieved through switching the transistor ON and OFF, and not driving them in the linear area like in traditional amplifiers. This is done through a technique called Pulse Width Modulation (PWM).

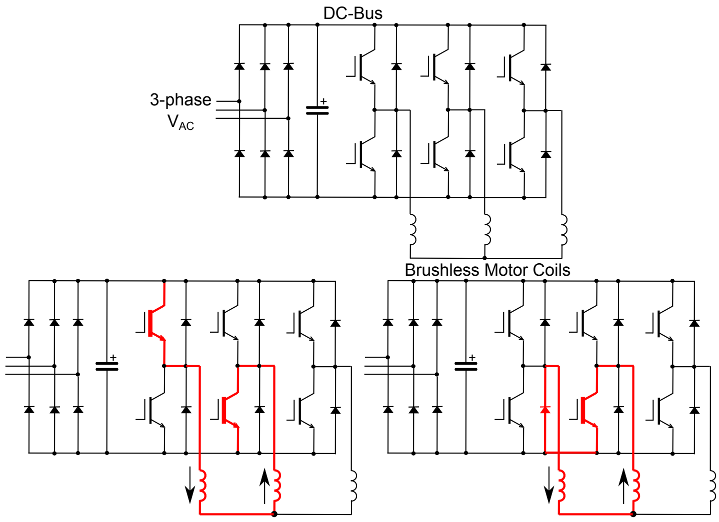

Figure 7.1: The switching scheme of a servo drive for AC motors.

The advantage of doing this has to do with the general advantages of digital over analog systems, such as independence of performance over:

- environment (temperature, humidity, etc.)
- ageing
- specific component characteristics (or defects, such as transistor linearity)

The PWM consists in switching transistors ON and OFF several times every current cycle, so that the coil's voltage will look like a rectangular wave while the current will change slowly due to the big inductance of motor coils. This results in a sinusoidal current driven by continually switching transistors (see Figure 7.2).

Note indeed that every coil of a brushless motor running, for example, at constant speed, will need a sinusoidal current to keep the two electro magnetic fields orthogonal to each other.

The way to implement this is generally through a triangular wave compared in real time with the sinusoidal set point, where the difference is digitalized so that if it is positive the result will be +V/2, while if it is negative the result will be -V/2, with V as DC Bus voltage (565VDC with 400 VAC three phase systems). This result is, for example, the E_{UO}, where U is one the three UVW motor's terminals. If we calculate, with the same triangular wave, also the E_{VO} and the E_{WO} we obtain similar wave forms. Now we can obtain the V_{UV}, V_{UW}, V_{VW} that we need to control the three UVW motor's terminals. These turn to be switching between +V and 0, and between -V and 0.

PWM frequency and current loop set point update rate are chosen so to match the motor inductance. In particular high PWM frequency (e.g. 10KHz or above) and high current loop rate (e.g. 50μs) are used for low inductance motors, while low PWM frequency (e.g. 4KHz or below) and low current loop rate (e.g. 125 μs or above) are used for high or very high inductance motors.

The three sinusoidal current set point waves have to be generated in real time starting from a single I_q current set point, proportional to torque, since I_d has to be kept equal to zero. Note that the two variables I_q and I_d are defined in a rotating reference frame, while the three variables I_U, I_V, I_W are defined in a statoric one.

The mathematical tranformation between the two vectors is called Clark & Park Transform (T), and, of course, make use of the feedback θ position information (Figure 7.3). This is the reason why servo systems always need feedback

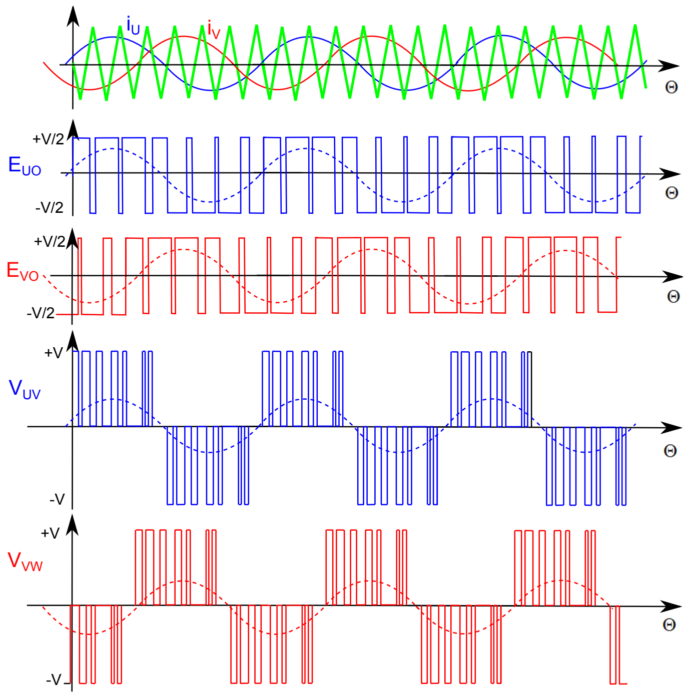

Figure 7.2: PWM operational principle.

position information. Note that this quantity can be generated by a feedback device, such as a resistor or an encoder, or it can be generated by an observer that is estimating it through an appriopriate model.

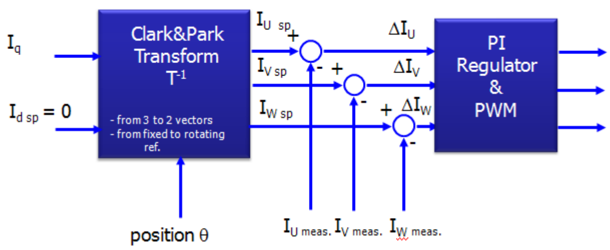

Figure 7.3: Current Control for an AC drive.

The Clark & Park Transform matrix T, actually transforms from 3 to 2 vectors and from fixed to rotary reference system, so the inverse is used. Also, to invert the matrix, we need a square matrix obtained adding a third current I_O of the neutral wire, that is actually zero if the motor is connected in Y configuration without access to neutral.

We can also split the Clark & Park Transform into a D transform for transforming between 3 and 2 phase statoric systems, and an E transform for transforming between statoric and rotoric systems.

With the above mentioned control we can then keep the two electro magnetic fields, statoric and rotoric, orthogonal to each other, through controlling I_q, and keeping I_d equal to zero. This is turn is actually making a brushless motor look like a DC brush motor from control point of view. We can appreciate this looking at the two models and putting $I_d = 0$ in the brushless one.

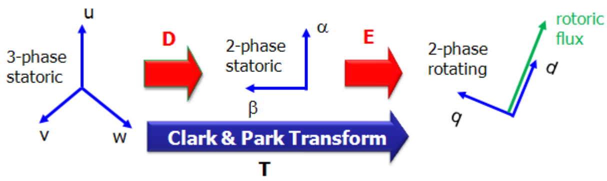

Figure 7.4: Clark & Park transform.

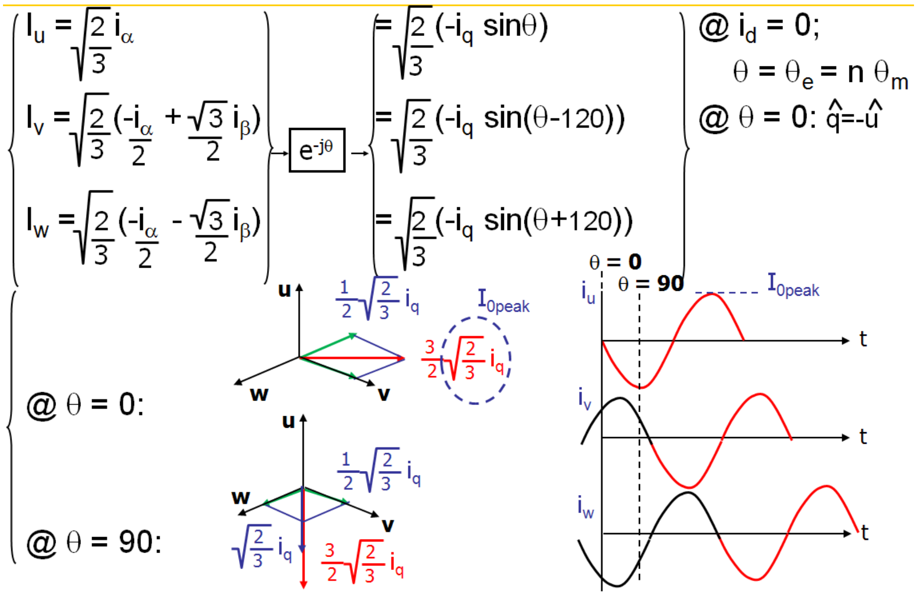

Figure 7.5: Clark & Park transform equations.

The feedback position interpretation is generally done through a Resolver to Digital converter that not only translates it in digital form, but actually hooks it and provides an estimate in closed negative loop.

Some drives can also achieve regenerative breaking function (see Figure 7.8), that is the possibility to drive energy toward the line upon excess DC bus voltage conditions. Note that the line is three phase AC while the DC Bus is, of course, one phase DC. We than need a transformation. This is achieved in a similar way of the motor control we just explained, since also the AC line can be modeled as a rotating vector. As in the preceding case, also here the feedback reading (that in this case is the three phase line voltage) is observed through a closed negative loop provided by a PLL (Phase Locked Loop) electronic function.

Safety aspects are also fundamental in a drive, for assuring:

- people safety
- machine safety

Fundamental safety aspects are:

- Stop Category (see EN60204-1 2006):

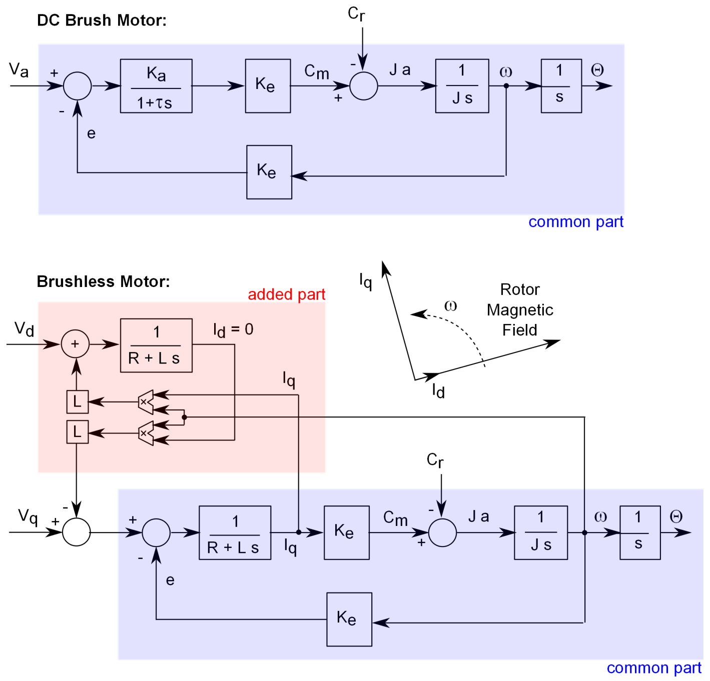

Figure 7.6: Comparison of the control system for a brush (DC) and bushless motors.

- Stop-Category 0: Immediate removal of power to actuators.
- Stop-Category 1: Controlled Stop and then power removal to actuators.
- Stop-Category 2: Controlled Stop with power available to actuators.

- Safety Stop Category (see IEC/EN62061, EN-ISO13849-1, -2)
- Safety Integration Level (SIL) related to Probability of Dangerous Failure per Hour (PFHd) (see IEC/EN61508 for components, IEC/EN62061 for machinery)
- Performance Level (see EN-ISO13849-1, -2)

A drive architecture is including some typical elements such as:

- A CPU (Central Processing Unit) managing motion commands and profiles
- A DSP (Digital Signal Processor) managing PIDs and Clark & Park Transform
- A Power Stage with the 6 IGBT Transistors, free-wheeling diodes
- Capacitor unit
- Eventually a rectifing diode bridge if the DC Bus has to be generated inside the drive
- A filed bus interface unit

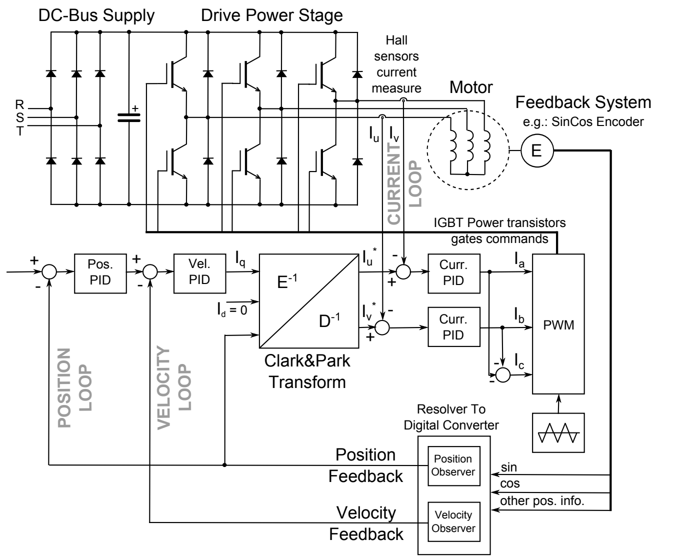

Figure 7.7: Drive logical scheme.

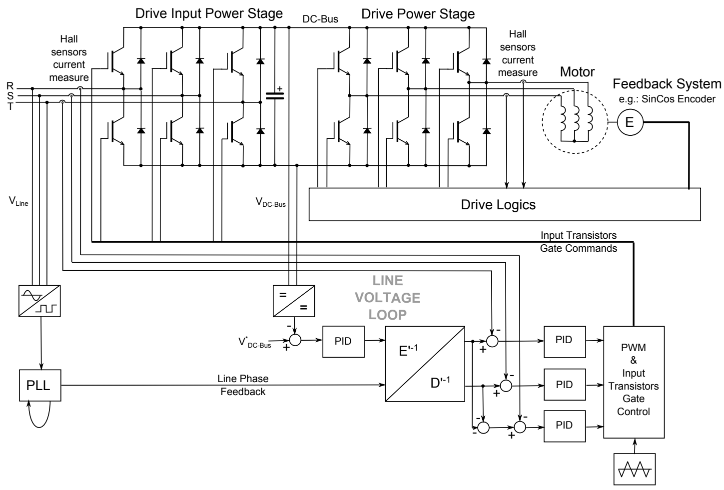

Figure 7.8: Drive with regenerative breaking.

- Fast input / output unit
- Display unit for diagnostics and troubleshooting
- A Brake Control unit
- A Safety unit
- A fan for cooling (some automation suppliers also have effective cold water cooling)
- A Heat Sink for more effective forced air cooling

A final aspect that must be mentioned for drives and motors is the Degree of Protection against solid and liquid means, the well known IPxy degree of protection, where x stands for protection against solid bodies and y for protection against water.

Most drives are IP20, that is a finger or bigger body cannot be introduced, and there's no protection against water. Some drives can be IP65 or even IP67 or IP69K. These means they are resistant to dust, and to a water jet (IP65), or temporary immersion (IP67), or even 100bar, 16l/min, 80C water jet with chemicals (IP69K). Drives of these types are also found mounted piggy back directly on servo motors to form a integrated drive motor configuration, that can be useful to save electrical cabinet space and cabling (i.e. complexity).

REFERENCES

[1] Davide Borghi, Progetto del Controllo di Macchine Automatiche per l'Impacchettamento, *http://www.automazione.ingre.unimore.it/pages/corsi/ProgettoControlloMacchineImpacchettamento.htm*

CHAPTER 8

PID: CONTROL THEORY AND TUNING

Servo systems are acting in closed loop systems, that includes:

- A computation unit (SW and HW, with control algorithms such as PID)
- An Actuation (energy conversion and/or power modulation)
- A target system (that can be a mixture of mechanical, fluid, thermal, chemicals, electrical aspects)
- A feedback (energy conversion and/or signal processing)

All this interact with the machine or process environment (Figure 8.1), and digitally communicates with the operator through a machine interface (equipped for example with a touch screen), and with all other machine components through a machine field bus and/or wireless communication.

This, from controller point of view only, can be schematically represented by the scheme shown in Figure 8.2:

The controller is often using a PID algorithm, that nowadays is done in a digital form as shown in Figure 8.3. The border between digital and analog worlds has been moving more and more from left to right in this scheme. And, as we have seen in the previous chapter, also PWM power control can be seen as a sort of digital control, thus in a certain way, moving the border further on the right.

The PID, or PIDs, for motor control can be placed on drive or on central controller, depending on motion control architecture and on automation supplier. This choice have an impact on field bus effective bandwidth since the amount of information to be exchanged is affected. One architecture example is shown in Figure 8.4. In this architecture the controller is running several SW tasks among which an application task that is loading the electronic position cam points, and a motion task providing equally time-spaced points, through interpolation at coarse update period, to following unit in the pipeline that can be a field bus board.

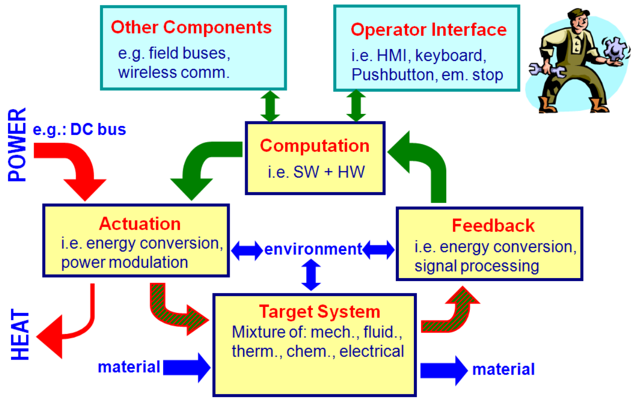

Figure 8.1: A comprehensive view over the machine, control system and the around environment.

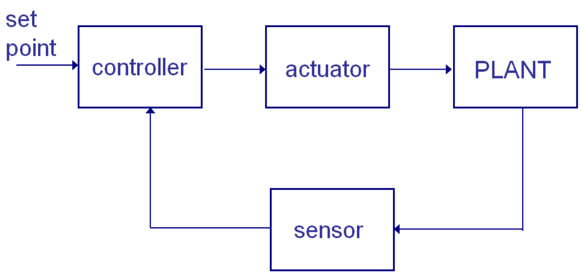

Figure 8.2: A control feedback scheme of a machine.

The full pipeline is composed by:

- A Controller (PLC) running the SW application and Motion Task
- A field Bus board interfacing with the specific bus protocol
- Drive loops:
  - Position loop
  - Velocity loop
  - Torque (or current, or acceleration) loop
- Motor
- Feedback system

The cam points on the SW application can be time-spaced by any time interval, provided that is bigger than coarse update period.

The interpolation in the SW application can be of a high order (e.g. fifth polynomial) as well as in the motion task (e.g. third polynomial).

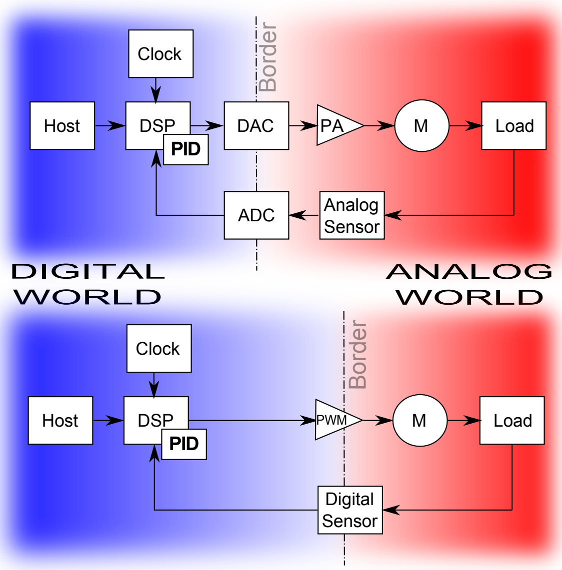

Figure 8.3: The division between the *analog* and the *digital* worlds.

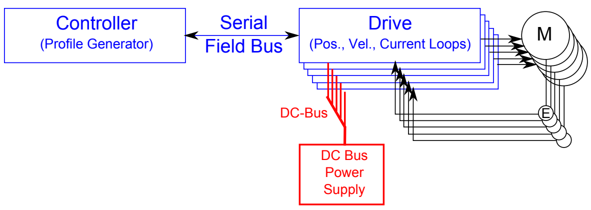

Figure 8.4: A possible solution in which the PIDs are placed on the drives.

The following interpolations in the pipeline are made at a tighter update rate (higher frequency) and thus generally have a lower interpolation order (e.g. third, second or first). The need for tighter and tighter update is because the target system is time continuous (analog in a way) while the controller, being a digital system, is time discrete: this creates the need for approaching the digital-analog border with tight update rate (see Figure 8.5).

Position, velocity, current loops can be closed, for example, with serial or parallel PID. Figure 8.13 shows an example of a serial PID.

The PID, as the word says, includes:

- A Proportional Action P that, if increased, can:
  - Make the system more reactive
  - Reduce the effect of noise: for high G values, the transfer function G/(1+GH) goes toward 1/H that is independent from G o Eventually lead to overshoot or even instability if increased too much
- Integral Action that, if increased, can:
  - Reduce steady state error

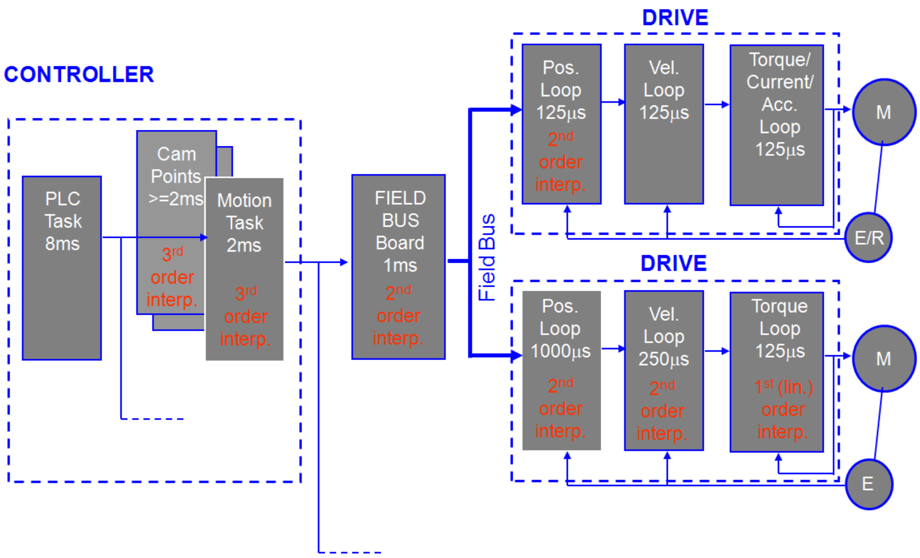

Figure 8.5: Different control loop timings.

  – Eventually lead to low frequency oscillations or even instability if increased too much

- Derivative Action that, if increased, can:

  – Act as a damper
  – Work against high slopes in the controller action
  – Eventually lead to high frequency oscillations or even instability if increased too much

The PID is applied on an (e.g. position) error that is created through comparison of set point and feedback data. The feedback sampling is decided examining the application real time needs, and considering Shannon Theorem. In particular strict real time applications requires tight sampling, that in turn acts as an enabler for high gains, high bandwidth and high performances. The Shannon Theorem tells us that once we know the frequency content of interest of the target system, then we have to set the sampling frequency at more than double that amount. Generally the choice is for five or ten times more, for avoiding complex modelling with Z-Transforms.

Stability of the closed loop system can be addressed with Nyquist Criterion or Root Locus. This latter one is giving added information such as how reactive the system is (poles on the right of the complex plane), if it oscillates (poles out of the real axis) and at which frequency (distance from the real axis).

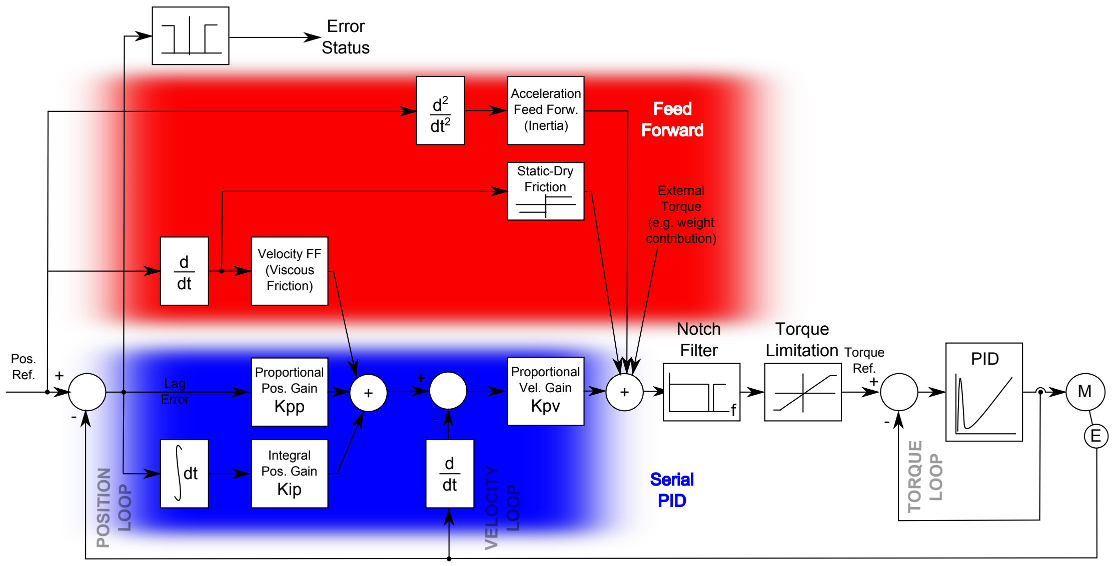

Figure 8.6: A serial scheme of a PID.

8.1 PID tuning

As we have seen PID gains, system bandwidth, system performances are linked. We can now visualize this in the closed loop system.

Let's assume that the set point is varying with a step.

The position (or lag) error will suddenly increase, given that the system has a finite energy (being of second order or above) to respond to the immediate set point change. Then the system will begin to move with a certain inertia and lag error will finally create a negative peak when the set point is reached, to decelerate the inertia. After this peak the lag error will somehow go back to zero once the motor moves to close the gap.

Let's focus on P action of PID, and neglect the I and D actions. P action indeed is directly acting on system bandwidth, as shown in Figure8.7. At the output of the P (PID) action there will be the same peaky waveform enhanced by a factor proportional to the P gain value. This, in a simplified scheme, with a parallel PID, is the current to the motor, that is it's acceleration. The motor will then accelerate to reach the set point (first peak, positive) and finally decelerates when the set point is reached (second peak, negative). In first approximation.

Now, if we increase the P gain, the the PID output peaks will be higher, than in turn will create more motor acceleration, quicker load response, i.e. higher bandwidth, i.e. higher system performances.

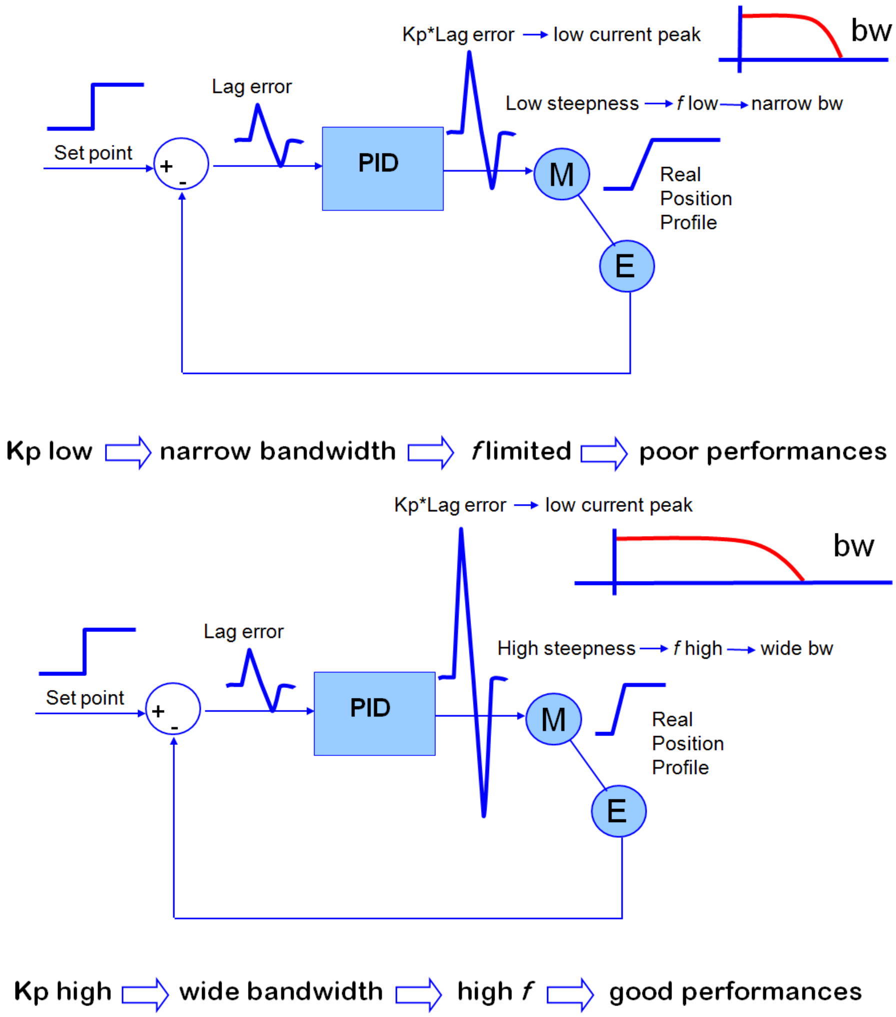

Figure 8.7: PID Proportional gain (“P” term) Tuning.

The system depicted above, we've said, is a simplified one. In practice there will be other blocks in series to the PID, such as a Notch Filter.

A Notch Filter eliminates a narrow frequency band around, for example, a load resonance frequency, without compromising the full system bandwidth (that a low pass filter would).

It's a Infinite Impulse Response (IIR) filter, i.e. with continuous, never-ending, adjustment of the output due to the closed loop feedback architecture. It adds together, weighed, the actual, previous, and previous-previous input together with the previous output value and the previous-previous one. In difference equations and in Z-transforms it leads to the Equation (8.1:

$$y_k = b_0 x_k + b_1 x_{k-1} + b_2 x_{k-2} - a_1 y_{k-1} - a_2 y_{k-2} \tag{8.1}$$

The corresponding block scheme is shown in Figure 8.8.

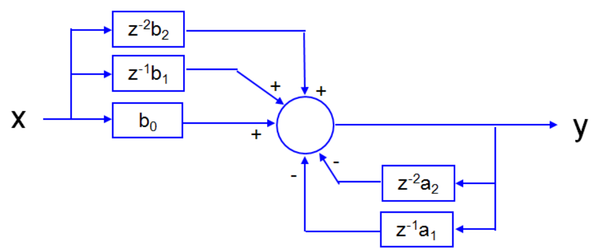

Figure 8.8: IIR filter scheme.

Generalizing the approach we can think at multiple N blocks for the input values, and multiple M blocks for the output values. This leads to the Basic Filtering Equation through which we can represent any digital filter:

$$y_n = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k) \tag{8.2}$$

To better understand the PID behavior we can also look at the PID response in open loop.

Given the same step change of the set point as above, we now open the loop, and we evaluate the result, this time, for each of the three contributions: P, I, D:

- The P would give a step response
- The I would give a ramp rising proportionally with time (that is the integral winding up)
- The D would give a sudden peak (a Dirac impulse in theory)

The combination of the three would give a sudden peak followed by a ramp starting from a value bigger than zero. This, comes out to be, graphically, the symbol of PID, summarizing the PID effect in open loop.

Note that, in practice, the derivative (D) contribution is treated in particular way:

- It does reduce the the steepness of the curve, so it has an intrinsic negative sign
- It could conveniently act only on feedback value (e.g. actual observed load position or velocity), instead of feedback error (e.g. position or velocity error), for a more robust design. This because, upon a sudden change in set point (due for example to operator intervention), the feedback error would have a sudden change (since the actuator has a limited energy and the load being at least a second order system). This would create in turn an unwanted, artificial,

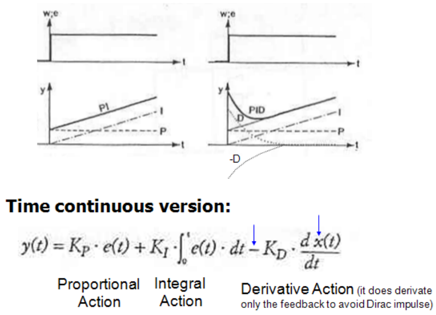

Figure 8.9: PID Control Tuning image12

sharp peak (a Dirac impulse in theory). Acting only on feedback guarantees that the signal is changing with a time constant that is not smaller than what the actuator-load system allows.

The Integral (I) action, as said, is killing the Steady State Error. Let's now consider a classical closed loop system with transfer functions G(s) (control function), and H(s) (feedback function). Let's also assume a set point that, after a steady state phase, begins to change as a step (zero order polynomial), as a ramp (first order polynomial) or as a parabola (second order polynomial). The Steady State Error would be dependent on number of poles in the origin of open loop transfer function $G(s) * H(s)$ (the so called Transfer Function Type), in the following way:

- Steady State Error (SSE) for a a step change set point (zero order polynomial):
  - Limited (bounded) SSE for a Type 0 GH Transfer Function
  - Zero SSE for a Type 1 GH Transfer Function
  - Zero SSE for a Type 2 GH Transfer Function
- Steady State Error (SSE) for change a ramp change set point (first order polynomial):
  - Infinite (unbounded) SSE for a Type 0 GH Transfer Function
  - Limited (bounded) SSE for a Type 1 GH Transfer Function
  - Zero SSE for a Type 2 GH Transfer Function
- Steady State Error (SSE) for parabola change set point (second order polynomial):
  - Infinite (unbounded) SSE for a Type 0 GH Transfer Function
  - Infinite (unbounded) SSE for a Type 1 GH Transfer Function
  - Limited (bounded) SSE for a Type 2 GH Transfer Function

8.2 Serial and parallel form for a PID

A PID can be studied in different forms. Typical, in servo systems, are the Parallel Implementation and the Serial Implementation. The first one has the P, I and D actions acting in parallel on the feedback error, and then added together to generate the current for the actuator. The latter is, instead, having first some action (e.g. Proportional and Integral) acting on position feedback error, in turn generating an output, that is used to create a velocity feedback error, on which the other actions (e.g. Derivative) is acting upon. In this case the first action (P, I) are on position loop, while other actions (D) are acting on velocity loop.

Feed Forward actions can be added on top of both Serial and Parallel Implementation. These actions are connected in open loop, bypassing completely the feedback information, to generate directly velocity set point correction (Velocity Feed Forward) and acceleration set point correction (Acceleration Set Point) based solely on position set point data. Note that a Velocity Feed Forward is actually correcting for load torque behavior proportional to speed, due, for example, to load viscous friction. While a Acceleration Feed Forward is actually correcting for load torque behavior proportional to acceleration, due, for example, to load inertia.

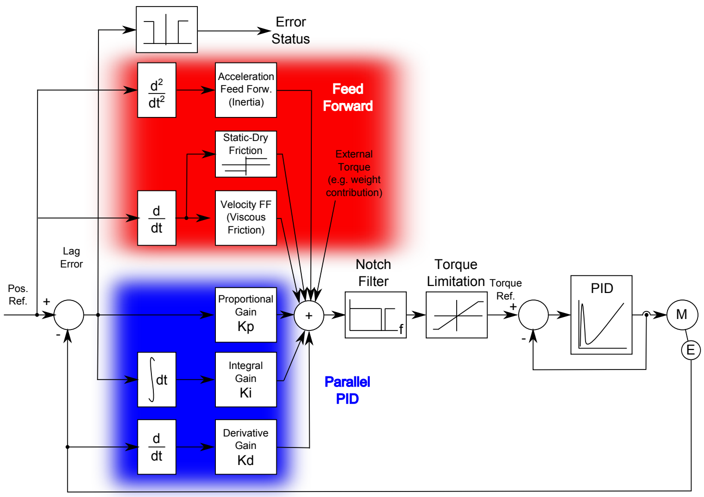

Figure 8.10: Parallel form for the PID.

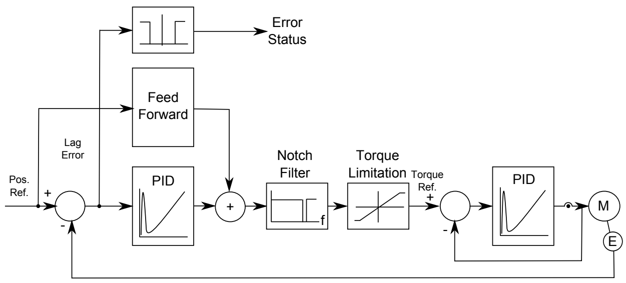

Figure 8.11: PID with feedforward.

An example of Parallel PID Implementation is shown in Figure 8.12.

And in Figure 8.13 is shown an example of Serial PID Implementation.

The PID can often also be configured in different configurations depending on application need. For example for applications with long kinematic chains, a secondary encoder assembled directly on load could be needed. This encoder, providing a real actual load position and velocity information, can be connected in different ways to the PID. Let's take for example a Serial PID Implementation. Here are some typical configurations for it:

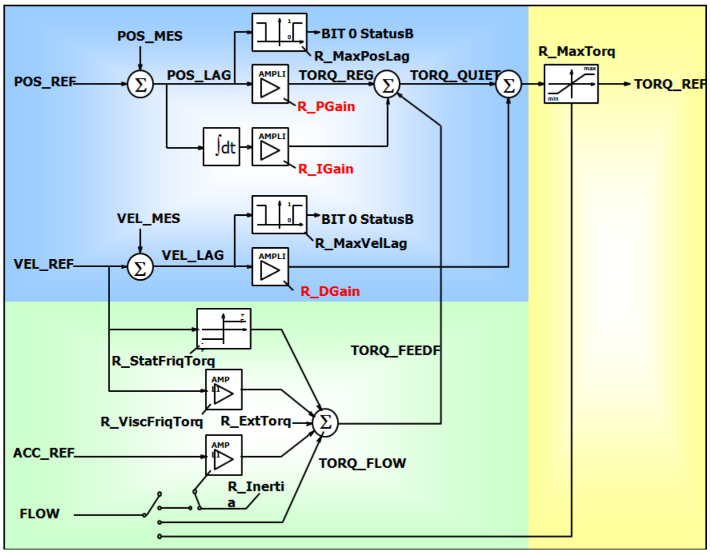

Figure 8.12: A complete scheme for a PID Controller.

- Position Servo Mode: the PID is acting solely on primary encoder position and velocity error. This is a classical servo configuration. It can be effective where smoothness and system stability are the target. Note that in this case the true load position is unknown and uncontrolled, so a high stiffness system is needed if positioning accuracy is a must.

- Secondary (or Auxiliary) Position Servo Mode: the PID is acting solely on secondary encoder position and velocity error, where secondary encoder is generally assembled very close to the load. This configuration is providing a very good positioning accuracy, compromising instead the smoothness and system stability, due to mechanical kinematic chain non-linearities. Note that the primary encoder is still needed, for the drive to solve Clark & Park Transform equations, to be able to correctly switch IGBT transistor in the power module, following rotor position.

- Dual Feedback Servo Mode: the position loop is acting on secondary encoder position error, while the velocity loop is acting on primary encoder velocity error. This configuration is taking advantage of good positioning performance and good smoothness and system stability, leveraging on typical correct tuning set up that would provide medium bandwidth position loop (in this configuration acting on slow changing secondary encoder) against a high bandwidth velocity loop (in this configuration acting on fast changing primary encoder).

- Velocity Servo Mode: The position loop is disabled, and the application program is providing directly the velocity set point to velocity loop. This configuration is using the servo for velocity control. Note that the position error is out of control, and actually not even defined, having no position set point at all.

- Torque Servo Mode: The position and velocity loops are disabled, and the application program is providing directly only the torque set point to the current (or acceleration or torque) loop. This configuration is using the servo for torque control. Note that the position and velocity errors are out of control, and actually not even defined, having no position or velocity set point at all. Typical application are paper web tensioner where a constant tension has to be maintained: the front servo motor can be controlled with a position servo mode, while the back servo motor can be controlled in torque servo mode.

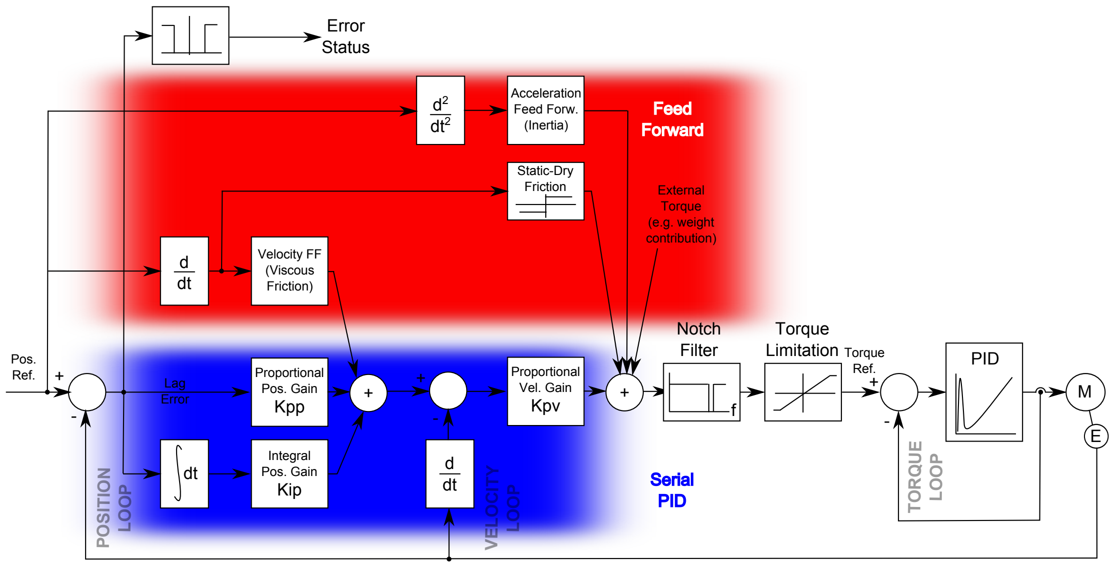

Figure 8.13: Serial PID implementation.

8.3 PID & Feed-Forward Regulator Tuning

Several approaches exists for PID and Feed-Forward Regulator tuning. For industrial use, we can group them in categories depending on use-case for the development engineer:

- PID:
  - Theoretical approach (e.g. Ziegler-Nichols Open Loop)
  - Experimental approach (e.g. Step Response and Zone Based)
- Feed-Forward:
  - Theoretical approach (i.e. CAD calculation of the contribution)
  - Semi-Experimental approach (e.g. torque diagram approach)
  - Experimental approach

For PID, the most used approach in industry is the experimental approach. In this category we focus on two methods: Step Response Tuning and Zone Based Tuning.

In the following tuning discussion we will refer to a parallel control. The same rule are anyhow in principle valid for a serial control substituting the wording “velocity P Gain” with “D Gain”. This because the velocity-P-Gain in a Serial PID Implementation is acting on velocity error as is the D Gain in a Parallel PID Implementation.

8.4 The Step Response PID Tuning

The basic approach used with step-response tuning is:

- Initialize the I term to zero, and set the D term to a small non-zero value.
- Use a step-response as reference:
  - Increase P from zero until the system substantially overshoots.
  - Then increase D until the oscillation is "critically damped."
  - Continue this iterative process until we get the right system performances or meet instability, in which case we step back for keeping a safety margin (typically at least 20%)

Although very easy to use, this method has the problem that increasing D will cause the optimum value of P to change, which in turn changes the optimum value of D, and so on. This requires a number of iterations to get to stable values.

In general terms, this is because the D term of a PID operates at the highest frequency zone, the P term at a middle zone, and the I term at the lowest frequency zone. What would be better is if we could first tune the highest frequency component, then move to the middle range value, and finish with the low frequency part. This second method, indeed based on this approach, is called "zone-based tuning".

8.5 The Zone Based PID Tuning

"Zone-based" refers to the "frequency zones" of the P, I, and D terms, and is adapted from George Ellis' book, Control System Design Guide. The basic approach used with zone based tuning is:

- Set the profile so that it accelerates instantaneously between a velocity of zero and a fixed velocity, and back to zero. Leave the P and I terms at zero,
- Increase D until the actual velocity profile closely matches the desired velocity profile or we have a vibration.
- Use a move profile with accelerations and velocities typical for our application profile
- Increase P. Continue this process until we get the right system performances (e.g. servo position error is minimized) or we meet a high position overshoot, or become unstable, at which point we should back off of this value by at least 20% for the final value, for keeping a safety margin

Zone-based tuning has the following advantages over step-response tuning:

- It is less iterative, because it tunes the PID terms in order of the frequency response zone.
- It allows the use of real application motion profiles

Using Step Response or Zone Based Tuning we then set the D and P gains.

In case we want to reduce or kill the Steady State Error, then we could use the Integral (I) action, increasing it until the SSE is under control. In case of I gain increased too much, low frequency oscillations could arise leading eventually even to instability. A step back of the I gain should then be performed to keep a safety margin. Note that these oscillations could be so low frequency that they are visible in the application load movement, without necessarily be audible (contrary to P and D vibrations that are often audible at medium and high frequency respectively).

On the Root Locus of a typical application with four complex poles, using for example the Zone Based Tuning, what happens is the following:

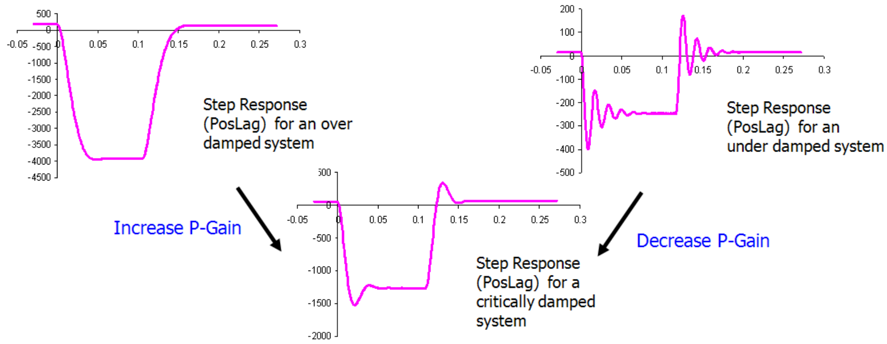

Figure 8.14: Tuning of Proportional Gain

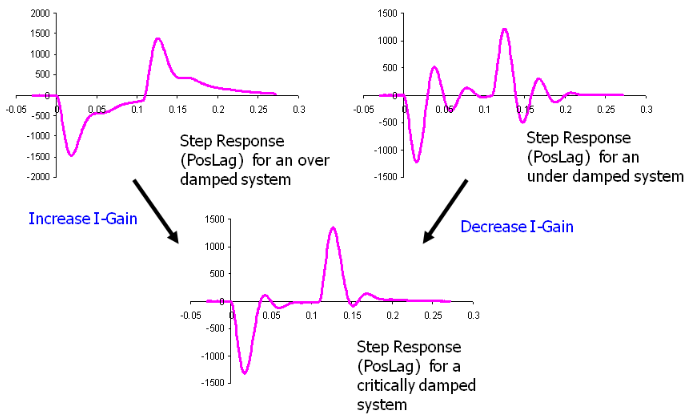

Figure 8.15: Tuning of Integral Gain

- First, increasing D, the high frequency poles (the most distant from Real axis) move toward the Complex axis until they meet it causing a vibration and we step back for safety margin, while the low frequency poles (the closest to Real axis) move away from Complex axis, i.e. to the left.

- Then, increasing P, the high frequency poles move significantly away from Complex axis, to the left, providing direct contribution to system real time performance and bandwidth, while the low frequency poles move toward from Complex axis, i.e. to the right, until they cross the Complex axis (and get back for safety margin) or we meet the performances.

As we have said increasing the three I, P, D gains we can eventually cause vibrations in three frequency zones, at increasing frequency values.

In terms of Bode Plot the bandwidth can be visualized in closed loop diagram of $G/(1+GH)$ as shown in Figure 8.18. Typically a phase misalignment of 150deg is considered to be the limit for the bandwidth, and the PID gains are set to keep it below this limit accordingly. A maximum position loop bandwidth of 100 to 200Hz and a velocity bandwidth of 200 to 300 Hz are typical for servo drives. Possible mechanical resonance within this range can be overcome with Notch filters.

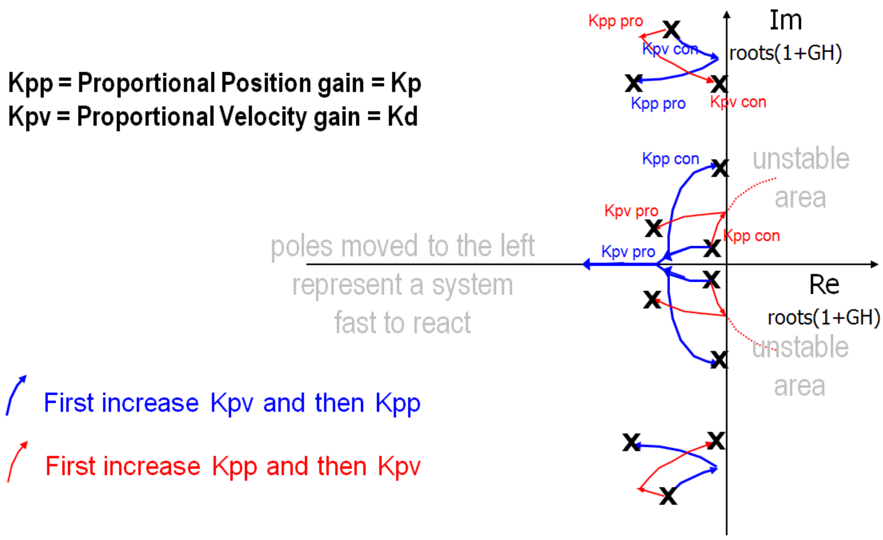

Figure 8.16: Zone Based Tuning: poles migration

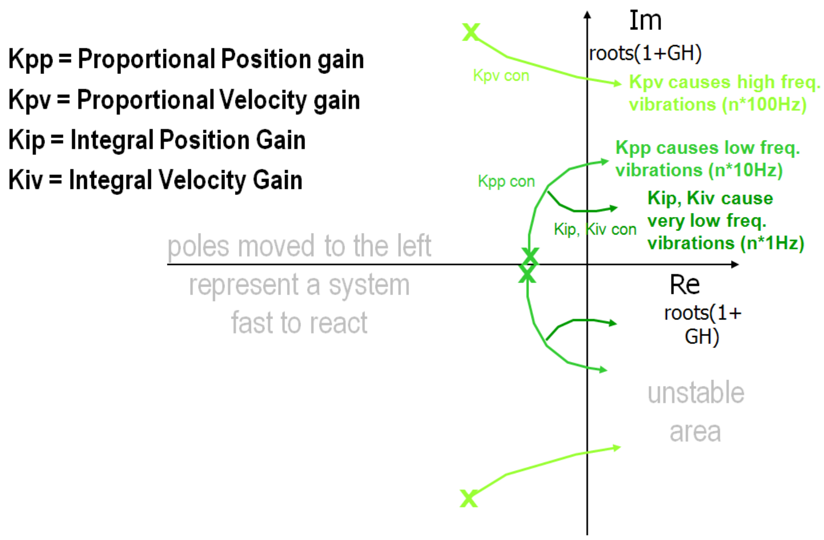

Figure 8.17: Vibration frequencies dependence on pole migrations

8.6 The Feed-Forward Experimental Approach Tuning

The full list of Feed-Forward Components is:

- External Force: e.g. gravity in a vertical load application. It is independent on the Motion and creates a constant torque value that is always present.

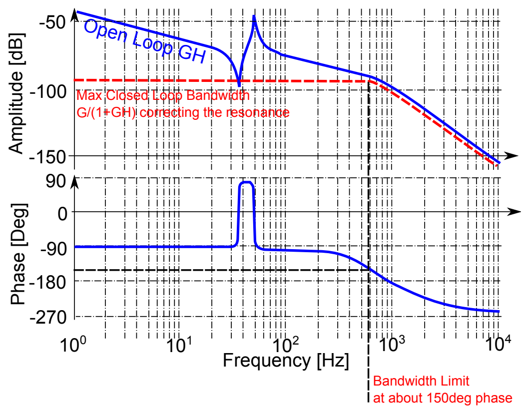

Figure 8.18: Bode Plot of the PID Controller

- Static / Dry Friction: depends heavily on material properties. It uses only the sign of the speed set point to create a constant torque value, whenever the speed different from zero.

- Viscous Friction (or Velocity Feed Forward): depends heavily on lubrication conditions. It is always proportional to the speed set point.

- Inertia (or Acceleration Feed Forward): depends on good mechatronic design, and used materials. It is always proportional to the acceleration set point and compensates the Inertia during acceleration and deceleration ($M = J * a$)

The experimental approach simply asks us to:

- Run the application profile or a profile that uses machine's nominal values for acceleration and velocity.

- Increase External Force Contribution until we get rid of the constant contribution that is always present independently from motion profile (e.g. gravity)

- Increase Inertia Contribution (or Acceleration Feed Forward) until we get rid of acceleration dependent position error or significantly reduce it. For positioning applications this is often the main position error contribution.

- Increase Viscous Friction Contribution (or Velocity Feed Forward) until we get rid of velocity dependent position error or significantly reduce it. This Feed Forward contribution can act in place of the Integral I gain in some respect, being able to kill the Steady State Error. Difference being that the the Feed Forward is of course acting in open loop leading to almost immediate fast reaction, but without any check of the result possibly causing unwanted overkilling when the lubrication conditions (or environment) are changing. The Integral action of PID instead is slow to react (the I contribution takes time to wind up) but it's closed loop, making it safe against changing lubrication conditions.

- Increase Static Friction Contribution until we get rid of the position error contribution that is present only when moving.

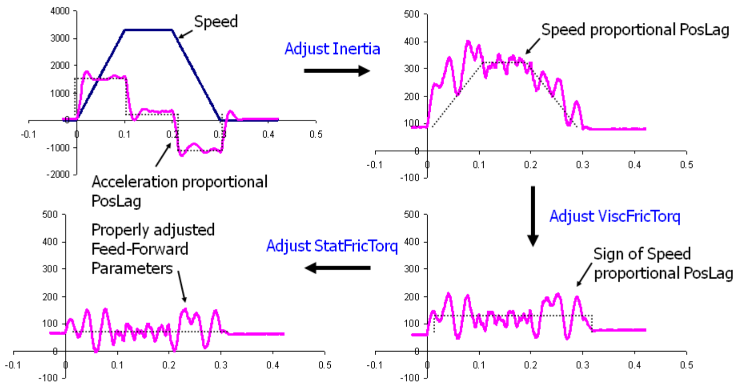

Figure 8.19: Feed Forward tuning: experimental approach

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CHAPTER 9

EXAMPLES OF PACKAGING MACHINES

Mechatronics Design implies to consider the application, and its functions and means, from multiple points of view, but still, at the same time. This means considering mutual implications among Mechanics, Control Engineering, Electronics, SW aspects, targeting for an optimal design according to the provided requirements (e.g. performances, MTBF, etc.).

Countless examples of mechatronics applications exists ranging, for example, from Formula 1 racing cars to military fighter jet planes, from tool machines to packaging machines.

In this chapter we would like to go through one example of Packaging Machine in order to explore some of the aspects that a Mechatronic Design implies.

When we say ”Packaging” we refer to the technology of enclosing products for subsequent distribution and selling.

Packaging functions and means can be distinguished between:

- Primary Packaging: functions used for enclosing the product, intended as the item to be sold.
- Secondary Packaging: functions used for sequencing, collating, grouping various pre-packaged products together onto another packaging container. Secondary packaging is, thus, not in direct contact with the product and plays different role in adding value to the product. Indeed it adds value in two main fields: Marketing (through branding and display), Logistics (through easy handling, transport and storage). Note that secondary package should protect not only the product as such, but also the primary package, that is the one on display on the retail store, eventually adding value to it. Common examples are cardboard cartons and boxes.

The package should provide protection to the product so to maintain its valuable characteristics during transportation and storage. Moreover it does also inform about its content and properties, and it can also advertise and promote through labels, printing or gifts.

Let’s consider a packaging machine whose purpose is to apply a glue pattern on the package, for application of a children gift on it. The primary package is represented by a potato chips can, made of carton, shaped like a cylinder.

Following are the requirements:

- The machine maximum capacity is of 300 units/min when applying two glue dots (or small elongated patterns)
- Machine capacity has to be variable from zero to maximum at will without stopping the equipment.
- The function is to apply two glue dots (or small elongated patterns) at maximum capacity or an arbitrary pattern of glue on the package side at lower machine capacity
- The glue dot application time can be neglected (i.e. time duration equal to zero)
- The primary package is a potato chips can of a given radius of 200mm and 200mm height
- The primary package is supplied through an asynchronous feeding unit composed by a belt system
- In case of ”n” contiguous packages the system shall process them without stopping
- Minimum process time (glue and gift application): 100ms
- The system shall be optimized in terms of reliability: in particular maximizing MTBF (Mean Time Between Failures) and MTTR (Mean Time To Repair).

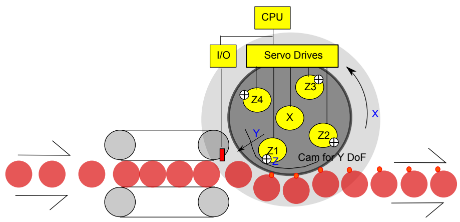

Figure 9.1: Glueing Application Example.

The function thus shall apply a glue pattern on the side of the package, so that a plastic gift can be pressed and attached on it, and add value and appeal to the potato chips can. Given the variability of the gift shape, that can be dependent on marketing seasonal trends (e.g. Christmas versus summer) or events (e.g. sport events), the glue pattern is not specified, since it could vary from a couple of glue dots to a line or even (at lower machine capacity) a small ”U” shaped pattern.

The mechatronic system design phase generally requires to go through such aspects like:

- Requirements Analysis
- Requirements Management using Customer Attributes (preparing a first Function-Means Tree) and Engineering Attributes
- Architecture Design filling in the Function Mean Tree and analyzing variants, common and optional modules.
- Robust Design Process can start with Module Definitions and Engineering Specifications from Requirements Management and Architecture Design. In this Design Process aspects such as the followings are considered:
  1. SW Application Architecture Design: e.g. State Machine or Flow Chart / Class Diagram
  2. Kinematics and Dynamics calculations
  3. Noise Factors evaluation
  4. Mechatronic System Architecture

5. Bill of Materials in terms of basic components (e.g. HW, actuators, drives, gearboxes, belts, pulleys, robots, sensors, vision system, sensors, HMI, etc.)
6. Safety aspects

- Virtual Verification: how to test and validate the concept, virtually (through simulation and/or emulation) and physically (through rigs). This part should be kept in mind from the start in the design process, in order to facilitate and speed up, eventually making it automatic or semi-automatic, the Virtual Verification.

In this chapter we would like to emphasize only the aspects that relates more closely to mechatronics and servo motors sizing. To do that we shall explore one of the possible solutions to this problem, without pretending it to be the only one nor the best. Some assumptions will also be made, since all the data are not made available on purpose, in order not to loose the focus on servo motors sizing.

One solution can be to guide the package through two parallel belts, driven by four pulleys with vertical axes. The actuators of these pulley is an asynchronous motor, that actuates both left and right side belts through a gearbox. The choice of an asynchronous motor versus a brushless motor is dictated by the fact that the packages, in this phase, are still not in a known position, being de-facto in an asynchronous state. An AC (asynchronous) motor is thus the logical choice, being cheaper than a brushless (synchronous) motor. This double belt system, thus, keeps the package controlled in attitude, and releases them at a given pace, or velocity control. It does this running at a fixed speed that is lower than the input and output conveyor speed. In this way the packages are first grouped together (by the lower belt speed) and then released at a given pace separating the packages by a certain gap (thanks to higher output conveyor speed). This gap provides a mean to detect different packages through a simple proximity sensor, later on in the process. As a consequence, a certain number of packages are released by the belt system in a given time interval, without having the specific knowledge of the exact time instant when each package is released. This piece of information (namely position data) has to be provided with a specific mean, that can be, for example, a proximity sensor positioned so to detect and distinguish each package. The package is then driven through a semicircular path where, a servo actuated wheel cylinder, applies the glue pattern with the necessary degrees of freedom, leveraging on the position data provided by the proximity sensor signal. The design choice that we are pursuing is to keep the packages in motion (i.e. not stopping it) in order to obtain the high machine capacity that is required. In fact an easier, and cheaper, machine design would be to stop each package (or a number of packages), and act on it (or them). But this approach would require an indexed kinematics, that in turn would require very high accelerations and un-acceptable forces or too long machine when very high machine capacity is the goal.

The wheel can be equipped with servo motors: for example one for vertical glue gun motion and one for transversal motion (this latter one, as we will see, can also be done by a cam). The wheel itself can be actuated by a torque servo motor or a geared servo motor. We can name these three servo Degrees of Freedom (DoFs) as X (the wheel torque servo motor), Y (the transversal or radial DoF), Z (the vertical or axial DoF) respectively. The DoFs actuated by servo motors shall be referred to as ”axes” from here on (singular: ”axis”).

Note that the system thus conceived is highly efficient for a given type of package, namely the potato chips can, but it cannot easily handle very different packages (an attribute, if present, that is commonly called ”Flexibility”), and indeed this was not among the requirements. In case Flexibility was among the high priority requirements, then different means should have been considered, such as several small and quick ”pick-and-place” robots (e.g. parallel geometries such as delta or H-Bot), handling single packages one-by-one without stopping the packages, or a big robot (such as serial geometry as an articulated 6 DoFs), handling several packages eventually stopping them for easier handling.

The sensor is conveniently placed close to the glue application point, in order to minimize position errors inserted by noise factors, and minimize amount of (electronic) cams to be buffered by the system; but it shall not be too close, in order to avoid the X axis to accelerate too violently to synchronize with the package (a process also called conveyor tracking or rendez-vous).

Assuming a minimum gap of 10mm between the packages, the wheel should have a tangential speed of 1.050m/s that is thus also the speed of the package during the glueing process. We can also assume that the wheel (X axis) angle, available for completing the process, is less than 90 degrees. This of course could imply to eventually have multiple Z actuators (depending on package gap and possible X axis acceleration), in order for the X axis to run at constant speed or limited acceleration, and not indexed violently (that could stretch the acceleration and torque need beyond reasonable values for an industrial application).

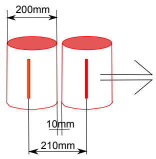

Figure 9.2: Package Pitch

The X axis could potentially be actuated by a torque servo motor, for maximizing the requirement of reliability, simplifying as much as possible the kinematic chain, avoiding any gearbox or belt - pulley systems. If the driver is instead the equipment cost (without considering maintenance costs), then the geared solution is often, but not always, the most indicated.

The Y axis has the task of pressing the gift onto the glue dots, thus it has to achieve a very small distance in a short time, resulting in small actuator size. Actually the actuator could also constitute of a single coil or, conveniently, a mechanical cam with roller bearings acting on it.

The Z axis is instead critical in terms of actuator weight: the function has to reach a long distance (200mm) in a short time (100ms), with limited but not negligible weight (the glue gun). Multiple Z actuators, acting and subsequent packages, can be necessary, depending on package gap and on possible X axis acceleration. In order to minimize the weight, and still be flexible, the Z axis can be actuated by rotary servo motors. We can assume a simple triangular profile for reaching the second (e.g. bottom) glue dot point after having applied the first (e.g. top) dot. This in order to minimize acceleration that in turn is proportional to the motor torque that, given the speed, is proportional to the motor weight. Then we can make assumptions in terms of kinematic coupling for achieving the linear Z axis, such as 180 motor degrees. The rotary servo is preferred with respect to a linear servo, for the sake of minimizing the weight. Indeed a linear servo has some mass that is not providing power at any time instant (that is the mass where the stator is not having the rotor in front, or viceversa). This is not true for a rotary servo that thus maximises the power/weight ratio efficiency. Actually if the requirement of arbitrary glue pattern on the package is dropped, the Z axis could actually be conveniently actuated by a mechanical cam with rollers.

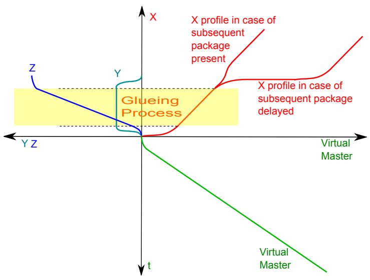

Figure 9.3: Example of Axes X, Y, Z Position Profiles driven by a Virtual Master Axis

Let's see the detail for Z axis in case of trapezoidal profile. The servo motor for the Z axis shall accelerate for 90 motor degrees and then decelerate for another 90 motor degrees, and finally stop. It will then have the rest of the X wheel turn in order to come back in position, if the glueing mechanisms requires it, or stay in place during the rest of the turn, if the glueing mechanism allows it (i.e. if it functions modulo 180 motor degrees). In this latter case this part is not critical in terms of peak torque, but it should be considered anyhow for the RMS torque calculation, to beneficially reduce the size of the motor, thanks to a reduced motor nominal torque. The equation for the Z angular position is given by:

$$\Theta_z = \frac{1}{2}\frac{d\omega_z}{dt}(t-t_0)^2 + \omega_{z0}(t-t_0) + s_{z0}$$

that leads to:

$$\frac{d\omega_z}{dt} = 1256\frac{rad}{s^2}$$

The maximum velocity that is reached by Z axis is given by:

$$\omega_{zmax} = \frac{d\omega_z}{dt}(t-t_0) + \omega_{z0} = 62.8\frac{rad}{s} = 600rpm$$

With a small load and a convenient leverage this set up of acceleration and velocity should lead to a torque - velocity diagram of a small servo motor, that may thus be used in direct drive configuration. A simulation of the tuning can also be made to ensure that the position error is under control.

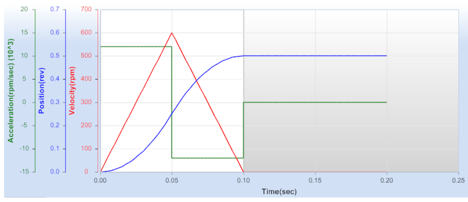

Figure 9.4: Position, Velocity, Acceleration for Z Axis

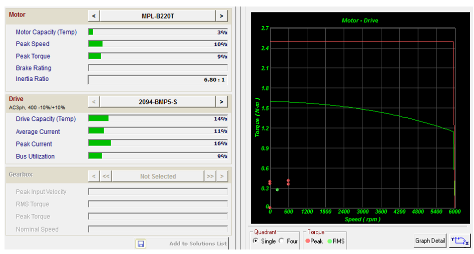

Figure 9.5: Example of Drive, Motor sizing done with Motion Analyzer^{TM} of Rockwell Automation^{TM} for Z Axis

For X axis, the radial velocity shall be dictated by the radius r of the wheel. Assuming 0.15m for r, we have:

$$\omega_x = \frac{v_x}{r} = 7.00\frac{rad}{s} = 66.85rpm$$

We can now verify how much angular space is needed, in X axis, in order to complete the glueing:

$$\theta_x = \omega_x(t-t_0) = 40deg$$

Now, the choice is between:

- Having multiple Z axes mounted on X axis that in turn would run at almost constant speed. This choice, potentially reduces torque need on X for accelerating, limits the size of X motor, but mutiplies the number of Z motors that

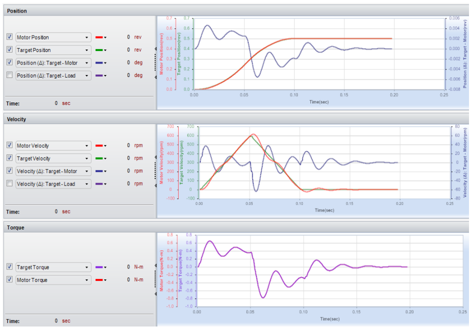

Figure 9.6: Tuning Simulation of Z Axis, using Motion Analyzer^{TM} of Rockwell Automation^{TM}

in turn have to be carried by X. In case of X running at constant speed, the extra torque would be needed only during acceleration and deceleration, such as product synchronization (i.e. rendez-vous) or start-stop sequences (e.g. emergency stop).

- Having single Z axis mounted on X axis, that in turn would need to accelerate violently in order to reach synchronization to achieve conveyor tracking (package rendez-vous) and glue the next package.

Let's take the second case, that potentially is also the simplest in term of number of items, thus going potentially in favour of component MTBF (but not necessarily function MTBF) and reliability, that are indeed among the requirements. In this scenario the X axis would have to move 360 - 40 = 320 degrees with high acceleration and subsequent deceleration, for finally resume constant rotational speed during glueing process. The time for completing this movement is bounded by the machine capacity of 300 products per minute, that leads to 200ms per package. This means that for completing 320 degrees the X axis has 200 - 100 = 100ms. The kinematic equation governing the motion is:

$$\Theta_x = \frac{1}{2}\frac{d\omega_x}{dt}(t - t_0)^2 + \omega_{x0}(t - t_0) + s_{x0}$$

that leads to:

$$\frac{d\omega_x}{dt} = 1952\frac{rad}{s^2}$$

The maximum velocity that is reached by X axis is given by:

$$\omega_{xmax} = \frac{d\omega_x}{dt}(t - t_0) + \omega_{x0} = 104\frac{rad}{s} = 994rpm$$

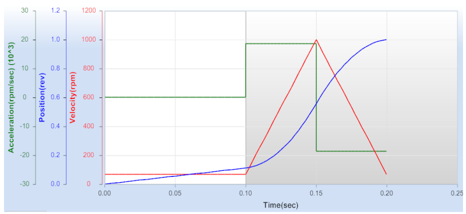

Figure 9.7: Position, Velocity, Acceleration for X Axis, in the case of a single Z motor on it

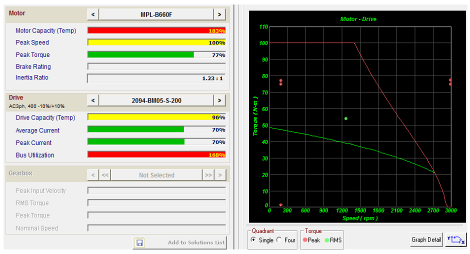

Figure 9.8: Example of a temptative of Drive, Motor sizing done with Motion Analyzer^{TM} of Rockwell Automation^{TM} for X Axis, with single Z motor. Note that both Peak Torque and Peak Speed limits are not met, as well as the Motor Capacity, that is identified in the Torque Diagram by the point: (RMS Speed, RMS Torque)

This would result in a rather demanding direct drive solution in terms of torque and speed, even with an aluminum, light load wheel where to mount the Z axis. This application limit, for this design path, is dictated by very high torque and too high speed, and so cannot be overcome with a gearbox, since the motor speed would grow even more.

As a result the first case of the above options should be followed, with multiple Z axes mounted on a constant speed X axis.

In this case, we can conveniently choose to split the 360 degrees turn of X axis into equal parts, each equipped with a Z axis. The Y DoF can be done with an axis per each Z station, or one only Y axis in total, pushing a cam, or also a fixed cam avoiding the Y axis at all. Having each package 200mm long and 10mm gap between them, we have a pitch of 210mm. Moreover the X wheel is 150mm in radius and we need 40 deg of X axis for completing the process per each package, that in turn equals to 0.702rad or 0.105mm. Considering that every 210mm we have one package to process, we could split the 2 π r into 4 parts (i.e. four Z axes) leading to 236mm distance on the circumference between each, equally spaced, Z servo motor axes. This means that, upon completion of each glueing, that lasts for 100ms, we have another 100ms for aligning (accelerating and decelerating) the next Z station for initiating the glueing process at constant speed. This movement of X axis should cover 236 - 210 = 26mm = 0.173rad in 100ms, that, in a reference system rotating at constant speed of ω_x, follows the formula:

$$\Theta_x = \frac{1}{2}\frac{d\omega_x}{dt}(t - t_0)^2$$

that leads to an acceleration (still with triangular acceleration profile) of:

$$\frac{d\omega_x}{dt} = 69.2\frac{rad}{s^2}$$

The maximum velocity that is reached by X axis is given by:

$$\omega_{xmax} = \frac{d\omega_x}{dt}(t - t_0) + \omega_{x0} = 10.46\frac{rad}{s} = 100rpm$$

This, as said, if each package is arriving separated by 10mm. In case the gap, package to package, is more, then the acceleration will be less.

We should now check the other demanding condition, in terms of acceleration and torque, that is the emergency stop. In fact, with respect to the start-up and normal-stop (step-down) sequence, it is usually more demanding. Assuming, for example, 0.5s time for decelerating from ω_x, constant production speed, down to standing still condition within one package pitch, i.e. 210mm = 1.4rad, we have:

$$\frac{d\omega_x}{dt} = -16.8\frac{rad}{s^2}$$

Considering four Z axes of 1Kg each, placed on an aluminum wheel of 150mm radius, the total inertia results 0.221Kg m^2. This in turn leads to a reasonable solution both in terms of direct drive and geared axis, keeping under control the inertia ratio in order to maximise performances.

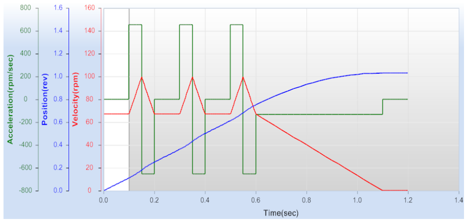

Figure 9.9: Position, Velocity, Acceleration for X Axis, in the case of four Z motors on it

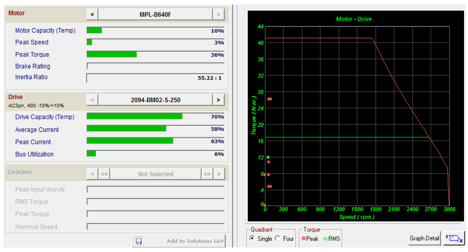

Figure 9.10: Example of Drive, Motor Direct Drive sizing done with Motion Analyzer^{TM} of Rockwell Automation^{TM} for X Axis, Direct Drive, in the case of four Z motors on it

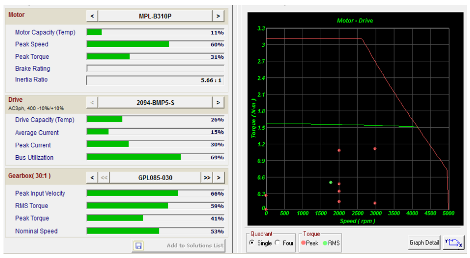

Figure 9.11: Example of Drive, Motor, Gear sizing done with Motion Analyzer^{TM} of Rockwell Automation^{TM} for X Axis, geared, in the case of a single Z motor on it

The Bill of Material for the motion control, could then consist of:

- 1 AC drive and cables for infeed conveyor motor
- 1 AC motor for infeed conveyor
- 2 belts for infeed section
- 1 AC motor for infeed section

- 4 servo drives and cables for Z axes
- 4 servo motors for Z axes
- 1 servo drive and cables for X axis
- 1 servo motors for X axis
- cam solution for Y DoF
- 1 proximity sensor for package detection
- 1 CPU for application program
- 1 field bus switch for drives connection
- 1 I/O board (with field bus adapter) for proximity sensor data and other data
- 1 HMI (Human Machine Interface) panel

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