

Forward Substitution: Example 2

Solve the following recurrence equation using the forward substitution method:

$$t_n = nt_{n-1} \text{ for } n \geq 1$$

$$t_0 = 1$$

Solution:

$$t_0 = 1$$

$$t_1 = n \times t_{n-1} = n \times t_{1-1} = n \times t_0 = 1 \times 1 = 1 \text{ (when } n = 1\text{)}$$

$$t_2 = n \times t_{n-1} = n \times t_{2-1} = n \times t_1 = 2 \times 1 = 2 \text{ (when } n = 2\text{)}$$

$$t_3 = n \times t_{n-1} = n \times t_{3-1} = n \times t_2 = 3 \times 2 = 6 \text{ (when } n = 3\text{)}$$

$$t_4 = n \times t_{n-1} = n \times t_{4-1} = n \times t_3 = 4 \times 6 = 24 \text{ (when } n = 4\text{)}$$

$$t_5 = n \times t_{n-1} = n \times t_{5-1} = n \times t_4 = 5 \times 24 = 120$$

$$\text{(when } n = 5\text{)}$$

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$$t_n = n \times t_{n-1} = n!$$

Alternatively,

Solve the following recurrence equation using the forward substitution method:

$$T(n) = n \times T(n - 1) \text{ for } n \geq 1$$

$$T(0) = 1$$

Solution:

$$T(0) = 1$$

$$\begin{aligned} T(1) &= n \times T(n - 1) = n \times T(1 - 1) = n \times T(0) = 1 \times 1 \\ &= 1 \text{ (when } n = 1) \end{aligned}$$

$$\begin{aligned} T(2) &= n \times T(n - 1) = n \times T(2 - 1) = n \times T(1) = 2 \times 1 \\ &= 2 \text{ (when } n = 2) \end{aligned}$$

$$\begin{aligned} T(3) &= n \times T(n - 1) = n \times T(3 - 1) = n \times T(2) = 3 \times 2 \\ &= 6 \text{ (when } n = 3) \end{aligned}$$

$$\begin{aligned} T(4) &= n \times T(n - 1) = n \times T(4 - 1) = n \times T(3) = 4 \times 6 \\ &= 24 \text{ (when } n = 4) \end{aligned}$$

$$\begin{aligned} T(5) &= n \times T(n - 1) = n \times T(5 - 1) = n \times T(4) = 24 \times 5 \\ &= 120 \text{ (when } n = 5) \end{aligned}$$

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$$T(n) = n \times T(n - 1) = n!$$

Hence, the solution of this recurrence equation is a factorial.
