Properties of Generating Functions – Addition and Multiplication Property

***Some of the essential properties of the generating functions are listed in the following subsections*:**

Addition and Multiplication Property

$$G(z) = a_0 + a_1 z + a_2 z^2 + \cdots + a_n z^n + \cdots$$

$$Let\ H(z) = b_0 + b_1 z + b_2 z^2 + \cdots + b_n z^n + \cdots$$

$$Then\ G(z) + H(z) = (a_0 + b_0) + (a_1 + b_1)z + \cdots$$
$$+(a_n + b_n)z_n + \cdots$$

$$= \sum_{i=0}^{\infty} (a_i + b_i) z^i$$

***The product of two generating functions*:**

$\boldsymbol{G(z)\ and\ H(z)\ is\ as\ follows}$:

$\boldsymbol{G(z)\ \times H(z) =}$

$$(a_0b_0) + (a_1b_0 + a_0b_1)z + \cdots + \sum_{i=1}^{k}(a_ib_{k-i})z^n + \cdots$$

$$= \sum_{k=0}^{\infty}\left\{\sum_{i=0}^{\infty}(a_ib_{k-i})z^i\right\}$$

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