

Polynomial Reduction

–Solving Homogeneous Equations

To solve a recurrence equation using the polynomial reduction method, the first step is to formulate the characteristic equation:

$$T(n) = a_1T(n-1) + a_2T(n-2)$$

Let us re – arrange the equation as follows:

$$T(n) - a_1T(n-1) - a_2T(n-2) = 0$$

Let $T(n) = x^n$

$\therefore T(n-1) = x^{n-1}$ and $T(n-2) = x^{n-2}$

This gives the following equation:

$$x^n + a_1x^{n-1} + a_2x^{n-2} = 0$$

As $x \neq 0$, divide the equation to get the following realtion:

$$\Rightarrow \frac{x^n}{x^{n-2}} + a_1 \times \frac{x^{n-1}}{x^{n-2}} + a_2 \times \frac{x^{n-2}}{x^{n-2}} = 0$$

$$\Rightarrow x^{n-(n-2)} + a_1 \times x^{n-1-(n-2)} + a_2 \times x^{n-2-(n-2)} = 0$$

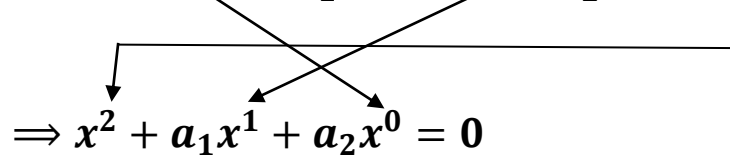
$$\Rightarrow x^2 + a_1 \times x^1 + a_2 \times x^0 = 0$$

$$\Rightarrow x^2 + a_1 x + a_2 = 0$$

Therefore it can be observed that the recurrence equation is transformed into a characteristic equation, which is as follows:

$$T(n) - a_1 T(n-1) - a_2 T(n-2) = 0$$

$$\Rightarrow T(n-0) - a_1 T(n-1) - a_2 T(n-2) = 0$$

$$\Rightarrow x^2 + a_1 x^1 + a_2 x^0 = 0$$


$$\Rightarrow x^2 + a_1 x + a_2 = 0$$

The order of the recurrence equation is 2; therefore, the characteristic equation is also of order 2.

One can generalize this in the form a theorem for the conversion of a recurrence equation into the characteristic equation.

Theorem

*Theorem: For a recurrence equation,
 $a_0t_n + a_1t_{n-1} + \dots + a_kt_{n-k} = 0$, having an order n ,
the characteristic equation will be of the following
form:*

$$a_0r^k + a_1r^{k-1} + \dots + a_kr^0 = 0$$
