Conditional Asymptotics

Sometimes , a recurrence equation may be associated with a constraint that \`n\` should be in powers or 2, 3, or some power of integer \`k\` in general. In other words, a recurrence equation is conditional . For example, consider the following recurrence equation:

$$T(n) = T\left(\frac{n}{2}\right) + 1, n \text{ is a power of } 2$$

In this example, the recurrence equation has a condition that the problem size \`n\` is a power of 2.
It can be observed that the solution for the recurrence equation is $T(n) = \{O(log(n)), \text{where } `n` \text{ is a power of } 2\}$.

Conditional asymptotics is a technique to solve a recurrence equation if the recurrence equation is associated with a constraint on \`n\`. This can be accomplished by applying the smoothness rule.

Formally, the smoothness rule can be stated as follows:

If $f(n)$ is an eventually non − decreasing and smooth function , then $T(n)$ is $\Theta(f(n))$ whenever $t(n)$ is $\Theta(f(n) \mid n$ is a power of $b)$.

The rule can be extended to all other asymptotic notations also. For applying the smoothness rule , the following two conditions must be satisfied:

1. $T(n)$ is an eventually non − decreasing function.
A function is called an eventually non decreasing function if for all values of \`n\` , the condition $f(a) \geq f(b)$ holds goods for $a > b$ after the point N.

2. $T(n)$ is a smooth function. The time complexity funcion $T(n)$ is, in general, said to be a \`b − smooth function\` if the function is eventually non − deceasing and also $T(bn) \epsilon \Theta(f(n))$.

Similarly, the smoothness rule can also be extended for all other asymptotic notations. Some of the examples of smooth functions are log_2 , n, n , $nlog_2n$, etc.

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