$\boldsymbol{Forward\ Substitution: Example\ 2}$

$\boldsymbol{Solve\ the\ following\ recurrence\ equation\ using\ the\ forward\ substitution\ method}$:

$t_n = nt_{n-1} \quad for\ n \geq 1$

$t_0 = 1$

<u>$\boldsymbol{Solution}$:</u>

$t_0 = 1$

$t_1 = n \times t_{n-1} = n \times t_{1-1} = n \times t_0 = 1 \times 1 = 1\ (when\ n = 1)$

$t_2 = n \times t_{n-1} = n \times t_{2-1} = n \times t_1 = 2 \times 1 = 2\ (when\ n = 2)$

$t_3 = n \times t_{n-1} = n \times t_{3-1} = n \times t_2 = 3 \times 2 = 6 (when\ n = 3)$

$t_4 = n \times t_{n-1} = n \times t_{4-1} = n \times t_3 = 4 \times 6 = 24 (when\ n = 4)$

$t_5 = n \times t_{n-1} = n \times t_{5-1} = n \times t_4 = 5 \times 24 = 120$

$(when\ n = 5)$

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$t_n = n \times t_{n-1} = n!$

$\boldsymbol{Alternatively,}$

$\boldsymbol{Solve\ the\ following\ recurrence\ equation\ using\ the\ forward\ substitution\ method:}$

$$\boldsymbol{T(n) = n \times T(n-1)\ for\ n \geq 1}$$

$$\boldsymbol{T(0)\ = 1}$$

$\underline{\boldsymbol{Solution:}}$

$$\boldsymbol{T(0) = 1}$$

$$\boldsymbol{T(1) = n \times T(n-1) = n \times T(1-1) = n \times T(0) = 1 \times 1}$$
$$\boldsymbol{= 1\ (when\ n = 1)}$$

$$\boldsymbol{T(2) = n \times T(n-1) = n \times T(2-1) = n\ \times T(1) = 2 \times 1}$$
$$\boldsymbol{= 2\ (when\ n = 2)}$$

$$\boldsymbol{T(3) = n \times T(n-1) = n \times T(3-1) = n\ \times T(2) = 3 \times 2}$$
$$\boldsymbol{= 6\ (when\ n = 3)}$$

$$\boldsymbol{T(4) = n \times T(n-1) = n \times T(4-1) = n\ \times T(3) = 4 \times 6}$$
$$\boldsymbol{= 24\ (when\ n = 4)}$$

$$\boldsymbol{T(5) = n \times T(n-1) = n \times T(5-1) = n\ \times T(4) = 24 \times 5}$$
$$\boldsymbol{= 120\ (when\ n = 5)}$$

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$$\boldsymbol{T(n) = n \times T(n-1) = n!}$$

$\boldsymbol{Hence, the\ solution\ of\ this\ recurrence\ equation\ is\ a\ factorial.}$

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