$\boldsymbol{Backward\ Substitution: Example\ 1}$

$\boldsymbol{Example\ 1: Solve\ the\ following\ recurrence\ equation.}$

$$t_n = t_{n-1} + 3$$
$$t_1 = 4$$

<u>$\boldsymbol{Solution}$</u>:

$\boldsymbol{Substituting\ the\ values\ of\ t_{n-1}\ in\ the\ recurrence\ equation,}$
$\boldsymbol{one\ gets\ the\ following\ equations:}$

$$
\begin{aligned}
t_n &= t_{n-1} + 3 \\
&= (t_{n-1-1} + 3) + 3 \\
&= (t_{n-2} + 3) + 3\ (plug)
\end{aligned}
$$

$\boldsymbol{We\ can\ write\ it\ as:}$

$$= t_{n-2} + 2 \times 3\ (chug)$$

$\boldsymbol{Repeat\ the\ process\ by\ substituting\ the\ value\ of\ t_{n-2}:}$

$$= (t_{n-2-1} + 3 + 3) + 3$$
$$= (t_{n-3} + 3 + 3) + 3$$

$$= (t_{n-3} + 3) + (2 \times 3)(plug)$$
$$= t_{n-3} + (3 \times 3)(chug)$$

$Repeat\ the\ process\ by\ substituting\ the\ value\ of\ t_{n-3}$:

$$(t_{n-3-1} + 3 + 3 + 3) + 3$$
$$= (t_{n-4} + 3 + 3 + 3) + 3$$

$$= (t_{n-4} + 3) + (3 \times 3)(plug)$$
$$= t_{n-4} + (4 \times 3)(chug)$$

$By\ Repeating\ the\ process, one\ can\ observe\ that\ at\ the\ ith\ iteration, this\ equation\ be\ as\ follows$:

$$t_i = t_{n-i} + i \times 3$$

$When\ i = n - 1, the\ resulting\ equation\ would\ be\ as\ follows$:

$$t_i = t_{n-i} + i \times 3$$

$\boldsymbol{Note: Here\ we\ will\ only\ change\ the\ right - hand\ side}$
$\boldsymbol{only\ according\ to\ backward\ substution\ method.}$

$$= t_{n-(n-1)} + (n-1) \times 3 (plug)$$
$$= t_1 + 3(n-1)(chug)$$

$\boldsymbol{Since\ t_1 = 4}$,

$$t_n = 4 + 3(n-1)$$
$$= 4 + 3n - 3$$
$$= 3n + 1$$

$\boldsymbol{Therefore\ , the\ solution\ of\ this\ recurrence\ equation}$
$\boldsymbol{is\ 3n + 1.}$

$\boldsymbol{We\ can\ also\ relate\ the\ above\ problem\ as}$:

$$T(n) = T(n-1) + 3$$
$$T(1) = 4$$

<u>$\boldsymbol{Solution}$</u>:

$$T(n) = T(n-1) + 3$$

$\boldsymbol{Substituting\ the\ values\ of\ T(n-1) in\ the\ recurrence\ equation, one\ gets\ the\ following\ equations}$:

$\boldsymbol{T(n-1-1+3)+3}$

$\boldsymbol{= T(n-2)+3+3}$

$\boldsymbol{= T(n-2)+2\times 3}$

$\boldsymbol{Repeat\ the\ process\ by\ substituting\ the\ value\ of\ T(n-2)}$:

$\boldsymbol{T(n-2-1)+3+3+3}$

$\boldsymbol{= T(n-3)+3+3+3}$

$\boldsymbol{= T(n-3)+3\times 3}$

$\boldsymbol{Repeat\ the\ process\ by\ substituting\ the\ value\ of\ T(n-3)}$:

$\boldsymbol{T(n-3-1)+3+3+3+3}$

$\boldsymbol{= \ T(n-4)+3+3+3+3}$

$\boldsymbol{= \ T(n-4)+4\times 3}$

$\boldsymbol{By\ Repeating\ the\ process, one\ can\ observe\ that\ at\ the\ ith\ iteration, this\ equation\ be\ as\ follows}$:

$$T(i) = T(n - i) + i \times 3$$

When $i = n - 1$, the resulting equation would be as follows:

$$T(i) = T(n - i) + i \times 3$$

Note: Here we will only change the right-hand side only according to backward substution method.

$$= T\big(n - (n - 1)\big) + (n - 1) \times 3 (plug)$$
$$= T(1) + 3(n - 1)\ (chug)$$

We know $T(1) = 4$,

$$T(n) = 4 + 3(n - 1).$$
$$= 4 + 3n - 3.$$
$$= 3n + 1.$$

Therefore, the solution of this recurrence equation is $3n + 1$.

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