

Generating Functions – Partial Fractions – Example

Resolve the following function into partial fraction:

$$\frac{5}{x^2 + x - 6}$$

Solution:

Factor the polynomial $x^2 + x - 6$:

$$x^2 + 3x - 2x - 6$$

$$\Rightarrow x(x + 3) - 2(x + 3)$$

$$\Rightarrow (x + 3)(x - 2)$$

Therefore the partial fraction can be written as follows:

$$\frac{5}{x^2 + x - 6} = \frac{A}{(x + 3)} + \frac{B}{(x - 2)}$$

Clear the fraction: $5 = A(x - 2) + B(x + 3)$

Now find A and B

Note: $x - 2 = 0 \Rightarrow x = 2$ and $x + 3 = 0 \Rightarrow x = -3$.

i. e. $x = 2, -3$.

Put $x = -3$ in the identity and equate it to 5

$$\therefore 5 = A(-3 - 2)$$

$$\Rightarrow 5 = -5A$$

$$\Rightarrow A = -1$$

Put $x = 2$ in the identity and equate it to 5.

$$\therefore 5 = B(2 + 3)$$

$$\Rightarrow 5 = 5B$$

$$\Rightarrow B = 1$$

$$\text{This shows that } \Rightarrow \frac{5}{x^2 + x + 6} = -\frac{1}{(x + 3)} + \frac{1}{(x - 2)}$$

These are the partial fractions of the given problem.

One can verify if these partial fractions are correct

or not by simplifying them in the following way,

to check whether the original equation is obtained:

$$-\frac{1}{(x+3)} + \frac{1}{(x-2)} = \frac{-(x-2) + (x+3)}{(x+3)(x-2)}$$

$$= \frac{-x+2+x+3}{x^2-2x+3x-6}$$

$$= \frac{5}{x^2+x-6}$$

Therefore, the partial fractions are correct.
