Cases where Master theorem fails

***Although both the simplified and the generalized master theorems are quite useful , these may fail under certain circumstances . The following examples show the cases where the theorem fails*:**

$$1. T(n) = 3^n T\left(\frac{n}{2}\right) + n^3$$

Comparing this with the standard form , one can see that $a = 3^n$ ***. The master theorem is not applicable as `a` is not constant and is dependent on the value of `n` .***

$$2. T(n) = 0.3\left(\frac{n}{2}\right) + n$$

The master theorem cannot be applied when a is less than **1**, ***and in this case a is less than*** **1**.

$$3. T(n) = T\left(\frac{n}{2}\right) - n^4$$

$Comparing\ this\ with\ the\ standard\ from, it\ can\ be\ observed\ that\ f(n)\ is\ negative\ . The\ master\ theorem\ is\ not\ applicable\ if\ f(n)\ is\ negative\ .$

$$4. T(n) = 4T\ \left(\frac{n}{4}\right) + \frac{n}{log\ n}$$

$When\ comparing\ f(n)\ and\ n^{log_b a}, in\ many\ cases\ the\ factors\ are\ not\ exactly\ polynomially\ lesser\ or\ greater. In\ these\ cases\ , the\ factors\ are\ said\ to\ fall\ in\ a\ gap\ \ and\ the\ generalized\ master\ theorem\ would\ not\ work\ .$

$Considering\ the\ preceeding\ recurrence\ equation\ and\ talking\ the\ limit\ , one\ obtain\ the\ following\ equation:$

$$\frac{f(n)}{n^{log_4^4}} = \frac{n}{\frac{(log\ n)}{n}} = \frac{n}{nlogn} = \frac{1}{log\ n}$$

$It\ can\ be\ seen\ that \frac{1}{log\ n} is\ less\ than\ n^{\varepsilon}\ for\ any\ value\ of\ c\ , but\ the\ factors\ are\ not\ polynomially\ comparable\ . Therefore, the\ master\ theorem\ would\ not\ be\ applicable\ for\ this\ recurrence\ equation.$

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