

B. B. 14. A. Domain Transformation – Example – 1

Example 1: Solve the recurrence equation

$T(n) = 7T\left(\left\lfloor \frac{n}{2} \right\rfloor\right)$ subjected to the initial condition

$T(1) = 1.$

Solution:

***Substitute $n = 2^k$ in the given equation. Since $n = 2^k$
and $k = \log n$, the equation takes the following form:***

$$T(2^k) = 7T\left(\frac{2^k}{2}\right) = 7T(2^{k-1}).$$

Rewrite this equation by substituting $T(k) = 7T(k - 1).$

***Now, the equation is of a familiar form and can be solved
in the traditional manner. This problem has already been
solved and the solution of this equation can be verified
to be 7^k .***

Next, the solution must be reconverted to the original form. Now substitute this in the original solution:

$$T(2^k) = 7^k$$

As $k = \log_2 n$, substituting this in the preceding equation, we conclude the following:

$$T(n) = 7^{\log_2 n}$$

As discussed earlier, one can rewrite this equation as:

$$T(n) = n^{\log_2 7} = n^{2.81} .$$
