$Guess\ And\ Verify\ Method - Example - 1$

$Example\ 1$: $Solve\ the\ recurrence\ equation\ t_n = t_{n-1} + 2$ $using\ the\ guess - and - verify\ method.$

<u>$Solution$</u>:

<u>$Guess$</u>: $For\ making\ a\ guess, use\ the\ different\ values\ of$ $`n`\ in\ the\ recurrence\ equation\ as\ follows$:

$t_0 = 1$
$t_1 = t_{1-1} + 2 = t_0 + 2 = 1 + 2 = 3$
$t_2 = t_{2-1} + 2 = t_1 + 2 = 3 + 2 = 5$
$t_3 = t_{3-1} + 2 = t_2 + 2 = 5 + 2 = 7$
....

$The\ sequence\ we\ found\ is\ (1, 3, 5, 7, \ldots\ldots) indicates\ that\ every$ $term\ differs\ from\ the\ previous\ one\ by\ 2\ . This\ is\ an$ $odd - number\ series.$

$Hence\ we\ can\ guess\ the\ recurrence\ will\ be: 2n + 1.$

$i.e. for\ n = 0, 2 \times 0 + 1 = 0 + 1 = 1$

$, for\ n = 1, 2 \times 1 + 1 = 2 + 1 = 3$

$, for\ n = 2, 2 \times 2 + 1 = 4 + 1 = 5$

$, for\ n = 3, 2 \times 3 + 1 = 6 + 1 = 7$

$\ldots\ldots$

And we see this equation : $2n + 1$ ***is non − recursive formula.***

if we represent it through function: $f(n) = 2n + 1$,
then the function will be called as general solution for recurrence equation : $t_n = t_{n-1} + 2$.

<u>Verify</u>: ***The next step is to verify whether the guess is right. Let us use the mathematical induction .***

<u>Mathematical Induction</u>:

Suppose $P(n)$ ***is a mathematical relation which is to be proved for positive integral values of n . If we can prove that:***

$(i) P(1)$ ***is true for*** $n = 1$.

(ii) ***if*** $P(m)$ ***is true then*** $P(m + 1)$ ***is true.***

Hence continuing with the verification:

Therefore ,

$P(n) = 2n + 1.$

For $2n + 1$ ***we cannot just compare it with*** $P(1)$***only,***

but series of n values.

When $n = 1$, $2 \times 1 + 1 = 2 + 1 = 3.$

$P(1) = 2 \times 1 + 1 = 3$,

Hence, $P(1)$ ***is true , when*** $n = 1.$

For $P(m) = 2m + 1$ ***is true then,***

For $P(m + 1) = 2(m + 1) + 1$ ***is true.***

If we see for values of m and $m + 1.$

$m = 0, 2 \times 0 + 1 = 1$

$m = 1, 2 \times 1 + 1 = 3$

$m = 2, 2 \times 2 + 1 = 5$

$m = 3, 2 \times 3 + 1 = 7$

$m + 1 = 0 + 1 = 1 , 2(0 + 1) + 1 = 3$

$m + 1 = 1 + 1 = 2 , 2(1 + 1) + 1 = 5$

$m + 1 = 2 + 1 = 3 , 2(2 + 1) + 1 = 7$

$\dots\dots$

$\dots \dots$

$$\textit{Hence } P(m) \textit{ is true then } P(m+1) \textit{ is also true. Hence the recurrence equation is verified.}$$