$\textbf{\textit{Backward Substitution}}: \textbf{\textit{Example 6}}$
$(\textbf{\textit{Based on Theorem}})$

$Example: Let\ t_n = kt_{n-1}\ for\ n \geq 0. Let\ t_3 = 343\ and\ t_4 = 2401.$

$What\ is\ the\ value\ of\ assuming\ the\ initial\ condition\ t_0 = 1?$

$\underline{Solution:}$

$It\ can\ be\ observed\ that\ the\ solution\ for\ this\ recurrence\ equation\ is\ t_n = k^n t_0 (Using\ the\ Theorem: t_n = r^n t_0)$

$Therefore,$

$$t_3 = k^3 t_0\ and\ t_4 = k^4 t_0$$

$$\therefore \frac{t_4}{t_3} = \frac{k^4 \times t_0}{k^3 \times t_0} = k$$

$$This\ implies\ that\ k = \frac{2401}{343} = 7.$$

$Therefore, the\ value\ of\ k\ in\ the\ recurrence\ equation\ is\ 7.$

$Alternative,$

$Example: Let\ T(n) = k \times T(n-1)\ for\ n \geq 0.$

$Let\ T(3) = 343\ and\ T(4) = 2401.$

$What\ is\ the\ value\ of\ assuming\ the\ initial\ condition$

$T(0) = 1?$

<u>$Solution$:</u>

$It\ can\ be\ observed\ that\ the\ solution\ for\ this\ recurrence$

$equation\ is\ T(n) = k^n T(0)(Using\ the\ Theorem:$

$T(n) = r^n T(0))$

$Therefore,$

$T(3) = k^3 T(0)\ and\ T(4) = k^4 T(0)$

$$\therefore \frac{T(4)}{T(3)} = \frac{k^4 \times T(0)}{k^3 \times T(0)} = k$$

$$This\ implies\ that\ k = \frac{T(4)}{T(3)} = \frac{2401}{343} = 7.$$

$Therefore, the\ value\ of\ k\ in\ the\ recurrence\ equation\ is\ 7.$

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