$\boldsymbol{Generating\ Function -}$
$\boldsymbol{Example - 1}$

Find the generating function for the sequence
$\boldsymbol{a_n = \{4, 16, 64, \dots\}}$

Solution:

It can be observed that the sequence can be denoted as $\boldsymbol{a_n = 4^n}$. ***The generating function is given as follows***:

$$\boldsymbol{G(z) = 1 + 4z + (4z)^2 + (4z)^3 + \cdots}$$

$$\boldsymbol{G(z) - 1 = 4z + (4z)^2 + (4z)^3 + \cdots}$$

$$\boldsymbol{= 4z(1 + 4z + (4z)^2 + \cdots)}$$

One can rewrite this equation of $\boldsymbol{G(z)}$ ***by simplifying the preceeding equation***:

$$\boldsymbol{\frac{G(z) - 1}{4z} = 1 + 4z + (4z)^2 + \cdots}$$

$$\Rightarrow \frac{\boldsymbol{G(z)-1}}{\boldsymbol{4z}} = \ \boldsymbol{G(z)}$$

$$\Rightarrow \boldsymbol{G(z)-1} = \ \boldsymbol{4z\big(G(z)\big)}$$

$$\Rightarrow \boldsymbol{G(z)} - \ \boldsymbol{4z\big(G(z)\big)} = \boldsymbol{1}$$

$$\Rightarrow \boldsymbol{G(z)(1-4z)} = \boldsymbol{1}$$

$$\Rightarrow \boldsymbol{G(z)} = \frac{\boldsymbol{1}}{\boldsymbol{1-4z}}$$

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