$Difference\ Method - Example\ 2$

$Solve\ the\ following\ recurrence\ equation\ using\ the$
$difference\ method\ and\ find\ the\ term\ t_{40.}$

$$t_n = t_{n-1} + 6n, n \geq 1\ and\ t_1 = 6$$

<u>$Solution$:</u>

$As\ per\ the\ method\ of\ difference\ , one\ can\ rearrange$
$the\ equation\ as\ t_n - t_{n-1} = 6n\ .$

$If\ n = 2\ ,$
$we\ get\ t_2 - t_1 = 6 \times 2;$

$Repeating\ this\ procedure\ , we\ get\ the\ following\ telescope$:

$t_2 - t_1 = 6 \times 2$
$t_3 - t_2 = 6 \times 3$
.
.
.
$t_{40} - t_{39} = 6 \times 40$
$- -$

$$\therefore t_{40} - t_1 = 6 \times (2 + 3 + 4 + \cdots + 40)$$

$$= t_1 + 6 \times (2 + 3 + 4 + \cdots + 40)$$

$$= (6 \times 1) + 6 \times (2 + 3 + 4 + \cdots + 40)\ [t_1 = 6 \times 1]$$

$$= 6(1 + 2 + \cdots + 40)$$

$$We\ know: \sum_{k=1}^{n} k = \frac{n(n+1)}{2}\ , Hence:$$

$$= 6 \times \frac{40(40+1)}{2}$$

$$= 6 \times \frac{40 \times 41}{2}$$

$$= 6 \times 820$$

$$= 4920.$$

$Therefore, the\ element\ t_{40}\ of\ the\ series\ would\ be$
$4920.$

$The\ aforementioned\ methods\ are\ useful\ for\ solving$
$first - order\ recurrence\ equations\ and\ are\ not$
$suitable\ for\ solving\ equations\ of\ higher\ order.$

$Higher - order\ recurrence\ equations\ are\ solved\ by$
$polynomial\ reduction\ method$.

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