$\boldsymbol{Forward\ Substitution: Example\ 1}$

$\boldsymbol{Solve\ the\ following\ recurrence\ equation\ using\ the\ forward\ substitution\ method: t_n = t_{n-1} + 3\ with\ initial\ condition\ t_0 = 4.}$

$\underline{\boldsymbol{Solution:}}$

$\boldsymbol{t_0 = 4}$

$\boldsymbol{t_1 = t_{1-1} + 3 = t_0 + 3 = 4 + 3(plug)}$

$\boldsymbol{t_2 = t_{2-1} + 3 = t_1 + 3(plug)}$

$\boldsymbol{Hence,}$

$\boldsymbol{= [(t_0 + 3)] + 3 = [(4 + 3)] + 3 = 4 + 2 \times 3(chug)}$

$\boldsymbol{t_3 = t_2 + 3(plug)}$

$\boldsymbol{= (t_1 + 3) + 3\ (plug)}$

$\boldsymbol{= \big((t_0 + 3) + 3\big) + 3(plug\ )}$

$\boldsymbol{= \big((4 + 3) + 3\big) + 3}$

$\boldsymbol{= 4 + 3 \times 3(chug)}$

.

.

.

$\boldsymbol{= 4 + (n) \times 3 = 4 + 3n}$

$\boldsymbol{Therefore, one\ could\ say\ that\ the\ solution\ of\ this\ recurrence\ is\ 4 + 3n.}$

$\boldsymbol{Alternatively,}$

$\boldsymbol{Solve\ the\ following\ recurrence\ equation\ using\ the\ forward\ substitution\ method: T(n) = T(n-1) + 3\ with\ initial\ condition\ T(0) = 4.}$

<u>$\boldsymbol{Solution:}$</u>

$\boldsymbol{T(0) = 4}$

$\boldsymbol{T(1) = T(1-1) + 3 = T(0) + 3 = 4 + 3(plug)}$

$\boldsymbol{T(2) = T(2-1) + 3 = T(1) + 3(plug)}$

$\boldsymbol{Hence,}$

$\boldsymbol{= [(T(0) + 3)] + 3 = [(4 + 3)] + 3 = 4 + 2 \times 3(chug)}$

$\boldsymbol{T(3) = T(2) + 3(plug)}$

$\boldsymbol{= (T(1) + 3) + 3\ (plug)}$

$\boldsymbol{= \big((T(0) + 3) + 3\big) + 3(plug\ )}$

$\boldsymbol{= \big((4 + 3) + 3\big) + 3}$

$\boldsymbol{= 4 + 3 \times 3(chug)}$

.

.

.

$\boldsymbol{= 4 + (n) \times 3 = 4 + 3n}$

$\boldsymbol{Therefore, one\ could\ say\ that\ the\ solution\ of\ this\ recurrence\ is\ 4 + 3n.}$