# Properties of Generating Functions – Shifting Property

Let $G(z) = a_0 + a_1 z + a_2 z^2 + \cdots + a_n z^n + \cdots$.

This sequence is given as $(a_0, a_1, \ldots., a_n, \ldots.)$.

Multiply this sequence by `z` .

This new sequence is given as:

$$z \times G(z) = (a_0 z^1 + a_1 z^2 + a_2 z^3 + \cdots + a_n z^{n+1} + \cdots).$$

This gives the sequence as $(0\ , a_0, a_1, \ldots, a_n, \ldots)$, that is, the sequence is shifted by $1$.

Similarly, $z^2 \times G(z)$ generates the sequence $(0, 0, a_0, a_1, \ldots., a_n, \ldots)$ .

In general , $k$ times $G(z)$ generates sequence $(\underbrace{0, 0, 0, \ldots., 0}_{\boxed{k\ times}}, a_0, a_1, \ldots, a_n, \ldots)$, where $k$ zeroes precede the

$sequence.$

$The\ left\ shift\ can\ be\ obtained\ as\ follows:$

$Let\ G(z) - a_0 = a_1 z + a_2 z + a_2 z^2 + \cdots + a_n z^n + \cdots$ .
$This\ leads\ to\ the\ sequence\ (0, a_0, a_1, \ldots, a_n, \ldots .).$

$Again\ , G(z) - a_0 - a_1 z\ leads\ to\ the\ sequence$
$(0, 0, a_0, a_1, \ldots ., a_n, \ldots)$

$Again\ , G(z) - a_0 - a_1 z - \cdots - a_k z^{k-1}\ leads\ to\ the\ sequence$
$(0, 0, 0, \ldots ., 0, a_k, a_{k+1}, \ldots ., a_{k+n}, \ldots). That\ is\ `k`\ zeroes\ precede$
$the\ sequence.$

$Similarly, a\ division\ by\ z\ leads\ to\ shifting\ of\ the\ sequence$
$to\ the\ left\ , that\ is\ , \dfrac{G(z) - a_0}{z}\ leads\ to\ the\ sequence:$
$(a_1, \ldots, a_n, \ldots).$

$Similarly, \dfrac{G(z) - a_0 - a_1 z}{z^k}\ generates\ the\ sequence$
$(a_2, a_3, \ldots, a_n, \ldots .).$

$\boldsymbol{In\ general}$ ,

$$\frac{G(z) - a_0 - a_1 z - \cdots - a_{k-1} z^{k-1}}{z^k}$$ , $\boldsymbol{generates\ the\ sequence}$

$(a_k, a_{k+1}, \dots)$ $\boldsymbol{for}$ $k \geq 1$.

$\boldsymbol{These\ properties\ are\ used\ in\ converting\ a\ sequence\ into\ a\ generating\ function.}$

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