

Divide And Conquer – Continuous Master Theorem or Generalized Master Theorem –Introduction

***A generalized version of the Akra – Bazzi theorem
has been given as follows:***

$$T(n) = \begin{cases} d & n \leq n_0 \\ aT\left(\frac{n}{b}\right) & n > n_0 \end{cases}$$

***and $a \geq 1, b > 1, n_0 \geq 1, d > 0$ and $f(n)$ is positive for
 $n > n_0$, or***

***Let $a > 0$ and $b > 1$ be constants, and let numbers $n \geq 1$.
Then the recurrence:***

$$T(n) = \begin{cases} \Theta(1) & \text{if } 0 \leq n < 1 \\ aT\left(\frac{n}{b}\right) + f(n) & \text{if } n \geq 1 \end{cases}$$

Then the asymptotic behavior of $T(n)$ can be characterized as follows:

1. If there exists a constant $\varepsilon > 0$ such that

$f(n) = O(n^{\log_b a - \varepsilon})$, then $T(n) = \Theta(n^{\log_b a})$.

2. If there exists a constant $k \geq 0$ such that

$f(n) = \Theta(n^{\log_b a} \log^k n)$, then $T(n) = \Theta(n^{\log_b a} \log^{k+1} n)$.

3. If there exists a constant $\varepsilon > 0$ such that

$f(n) = \Omega(n^{\log_b a + \varepsilon})$, and if $f(n)$ additionally

satisfies the regularity condition

$af\left(\frac{n}{b}\right) \leq cf(n)$ for some constant $c < 1$ and

all sufficiently large n , then $T(n) = \Theta(f(n))$.
