$Backward\ Substitution$: $Example\ 3$

$Solve\ the\ following\ recurrence\ equation\ using\ the\ backward\ substitution\ method$:

$t_n = nt_{n-1} \quad for\ n > 1$

$t_0 = 1$

$\underline{Solution}$:

$Let\ us\ apply\ the\ backward\ substitution\ method$:

$t_n = nt_{n-1}$

$= n[(n-1)t_{n-2}]$

$= n(n-1)[(n-2)t_{n-3}]$

$= n(n-1)(n-2)[(n-3)t_{n-4}]$

… ….

$At\ the\ ith\ \ step\ this\ leads\ to$

$= n(n-1)(n-2)(n-3)\ldots(n-i)$

$When\ i = n-1$,

$= n(n-1)(n-2)(n-3)\ldots.\big(n-(n-1)\big)$

$= n(n-1)(n-2)(n-3)\ldots.1$

$= n!$

$\boldsymbol{We\ can\ re-write\ it\ as:}$

$$T(n) = n \times T(n-1)\ for\ n > 1$$

$$T(0) = 1$$

<u>***Solution***:</u>

$$T(n) = n \times \big(T(n-1)\big)$$

$$= n[(n-1)T(n-2)]$$

$$= n(n-1)[(n-2)T(n-3)]$$

$$= n(n-1)(n-2)[(n-3)T(n-4)]$$

… ….

$\boldsymbol{At\ the\ ith\ \ step\ this\ leads\ to}$

$$= n(n-1)(n-2)(n-3)\ldots(n-i)$$

$\boldsymbol{When\ i = n-1,}$

$$= n(n-1)(n-2)(n-3)\ldots.\big(n-(n-1)\big)$$

$$= n(n-1)(n-2)(n-3)\ldots.1$$

$$= n!$$

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