

Guess And Verify Method – Example – 1

Example 1: Solve the recurrence equation $t_n = t_{n-1} + 2$ using the guess – and – verify method.

Solution:

Guess: For making a guess, use the different values of n in the recurrence equation as follows:

$$t_0 = 1$$

$$t_1 = t_{1-1} + 2 = t_0 + 2 = 1 + 2 = 3$$

$$t_2 = t_{2-1} + 2 = t_1 + 2 = 3 + 2 = 5$$

$$t_3 = t_{3-1} + 2 = t_2 + 2 = 5 + 2 = 7$$

....

The sequence we found is (1, 3, 5, 7,) indicates that every term differs from the previous one by 2. This is an odd – number series.

Hence we can guess the recurrence will be: $2n + 1$.

i. e. for $n = 0, 2 \times 0 + 1 = 0 + 1 = 1$

, for $n = 1, 2 \times 1 + 1 = 2 + 1 = 3$

, for $n = 2, 2 \times 2 + 1 = 4 + 1 = 5$

, for $n = 3, 2 \times 3 + 1 = 6 + 1 = 7$

....

And we see this equation : $2n + 1$ is non – recursive formula.

if we represent it through function: $f(n) = 2n + 1$, then the function will be called as general solution for recurrence equation : $t_n = t_{n-1} + 2$.

Verify: The next step is to verify whether the guess is right. Let us use the mathematical induction .

Mathematical Induction:

Suppose $P(n)$ is a mathematical relation which is to be proved for positive integral values of n . If we can prove that:

(i) $P(1)$ is true for $n = 1$.

(ii) if $P(m)$ is true then $P(m + 1)$ is true.

Hence continuing with the verification:

Therefore,

$$P(n) = 2n + 1.$$

For $2n + 1$ we cannot just compare it with $P(1)$ only, but series of n values.

$$\text{When } n = 1, 2 \times 1 + 1 = 2 + 1 = 3.$$

$$P(1) = 2 \times 1 + 1 = 3,$$

Hence, $P(1)$ is true, when $n = 1$.

For $P(m) = 2m + 1$ is true then,

For $P(m + 1) = 2(m + 1) + 1$ is true.

If we see for values of m and $m + 1$.

$$m = 0, 2 \times 0 + 1 = 1$$

$$m = 1, 2 \times 1 + 1 = 3$$

$$m = 2, 2 \times 2 + 1 = 5$$

$$m = 3, 2 \times 3 + 1 = 7$$

$$m + 1 = 0 + 1 = 1, 2(0 + 1) + 1 = 3$$

$$m + 1 = 1 + 1 = 2, 2(1 + 1) + 1 = 5$$

$$m + 1 = 2 + 1 = 3, 2(2 + 1) + 1 = 7$$

... ..

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Hence $P(m)$ is true then $P(m + 1)$ is also true. Hence the recurrence equation is verified.