

Conditional Asymptotics – Example

***Example: Let $T(n)$ be a conditional asymptotic
 $T(n) \in \{\Theta(\log_2 n \mid n \text{ is a power of } 2)\}$.***

Apply smoothness rule.

***Solution: Check whether $T(n)$ is an eventually
non – decreasing function. The function $\log_2 n$ is an
eventually decreasing function.***

However , is it a smooth function?

Let us verify this now.

***It is clear that $\log n \in \Theta(\log n)$. Therefore, check
whether this condition holds good for a value
of $b = 2$. It can be shown as a smooth function as
follows:***

$$\begin{aligned}\log_2 2n &= \log_2 2 + \log_2 n \\ &= 1 + \log_2 n \\ &\in O(\log n)\end{aligned}$$

$\therefore T(n) \in O(\log n)$ for all values of n .

It is established that $\log n$ is a smooth function.

Therefore, the recurrence equation is applicable for all values of n .
