Backward Substitution: Example 2

Q) Find the compound interest for the principal amount Rs. 100 if the interest given by a bank is 3% . Formulate the recurrence equation and solution using the substitution method. What would be the principal amount after the 50th month?

Solution:

The principal amount for the current year would be the principal amount of the previous year plus 3% of that amount. Therefore , the recurrence equation can be written as follows:

$$t_n = t_{n-1} + \left(\frac{3}{100}\right) t_{n-1} = t_{n-1} + 0.03t_{n-1}$$

$$= (1 + 0.03)t_{n-1} = 1.03t_{n-1} .$$

The initial investment t_0 would be Rs. 100. For determining the solution, let us first attempt to find the sequence for this recurrence equation.

The compound interest for the first year would be as follows:

$$t_1 = 1.03t_{1-1} = 1.03t_0$$
$$t_2 = 1.03t_{2-1} = 1.03t_1 = (1.03)^2 t_0$$
$$t_3 = 1.03t_{3-1} = 1.03t_2 = (1.03)^3 t_0$$
....
$$t_n = (1.03)^n t_0 .$$

Now, the investement at the 50th month can be calculated as

$$t_{50} = (1.03)^{50} t_0 .$$

This is the solution of the recurrence equation.

We can also write it as:

$$T(n) = T(n-1) + \left(\frac{3}{100}\right) T(n-1)$$
$$= T(n-1) + 0.03 \times T(n-1)$$
$$= (1 + 0.03) T(n-1)$$
$$= 1.03 T(n-1)$$

Now,

$$T(1) = 1.03\, T(1-1) = 1.03\, T(0)$$

$$T(2) = (1.03)T(2-1) = (1.03)T(1) = 1.03^2T(0)$$
$$T(3) = (1.03)T(3-1) = (1.03)T(2) = 1.03^3T(0)$$

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$$T(n) = 1.03^nT(0)$$

Now, the investment at the 50th month can be calculated as $t_{50} = 1.03^{50}T(0)$.

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