

Divide And Conquer – Akra – Bazzi Theorem

In 1998 , two Lebanon – based researchers provided the solutions for the generalized form of the master theorem, which is as follows:

$$T(n) = \begin{cases} h(n) \\ aT\left(\frac{n}{b^k}\right) + f(n) \end{cases}$$

Here , $a > 0$, $b > 1$ and $n_0 \geq b$ are integers ; $h(n)$ is a function that is in the range $d_1 \leq h(n) \leq d_2$ for two constant d_1 and d_2 and $1 \leq n \leq n_0$; and $f(n)$ is a polynomial that is in the range $c_1g(n) \leq f(n) \leq c_2g(n)$ for all $x > 0$ and $u \in \left[\frac{n}{b}, n\right]$. if all these conditions are satisfied and the condition $\frac{a}{b^p} = 1$ is true , then the solution of the recurrence is given as follows:

$$T(n) = \Theta\left(n^p \left(1 + \int_1^u \frac{f(u)}{u^{p+1}}\right)\right)$$

This is a powerful theorem and solves almost all those recurrences that cannot be solved easily by other methods.
