Divide And Conquer −
Akra − Bazzi Theorem −
Example − 1.

Example: Solve the following recurrence equation:

$$T(n) = 3T\left(\frac{n}{3}\right) + n$$

<u>Solution</u>

Here $a = 3$, $b = 3$. $\therefore \frac{3}{3^p} = 1$. **This is true when** $p = 1$.

Therefore, as per Akra − Bazzi theorem ,
the solution can be given as follows:

$$T(n) = 3T\left(\frac{n}{3}\right) + n$$

$u = n$, $f(n) = n$ **and** $p = 1$

$$\Theta\left(n^p\left(1 + \int_1^n \frac{f(n)}{n^{p+1}}\, dn\right)\right)$$

$$= \boldsymbol{\Theta}\left(n^{1}\left(1 + \int_{1}^{n} \frac{n}{n^{1+1}}\, dn\right)\right)$$

$$= \boldsymbol{\Theta}\left(n^{1}\left(1 + \int_{1}^{n} \frac{n}{n^{2}}\, dn\right)\right) \Longrightarrow \left[\frac{n}{n^{2}} = \frac{1}{n}\right]$$

$$= \boldsymbol{\Theta}\left(n^{1}\left(1 + \int_{1}^{n} \frac{1}{n}\, dn\right)\right) \Longrightarrow \left[\frac{n}{n^{2}} = \frac{1}{n}\right]$$

$$= \boldsymbol{\Theta}\big(n(1 + [\log n]_{1}^{n})\big)\ \left[\int \frac{1}{n} dn = \log n\right]$$

$$= \boldsymbol{\Theta}\big(n(1 + [\log n]_{1}^{n})\big)$$

Now compute boundaries of $\log n$:

$$\int_{b}^{a} f(x)d(x) = F(b) - F(a) = \lim_{x\to b-} F(x) - \lim_{x\to a+} F(x)$$

$\lim\limits_{n\to 1+} (\log n) =$ ***plugging 1 in place of n we get***:

$$= \log(1) = 0$$

$\lim_{n \to n-} (\log n) = As \ \ \lim_{x \to x} f(x) = f(x)$, $hence\ we\ get$:

$\log n\ \ only$.

$Hence$:

$= \ \Theta\big(n(1 + (\log n - 0)\ )\big)$

$= \ \Theta\big(n(1 + \log n)\big)$

$= \ \Theta(n + n \log n)$

$= \ \Theta(n \log n)$

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