# *Theorem on Backward Substitution*

**Theorem**: ***For the recurrence equation of the form,***

$t_n = rt_{n-1} \;, n > 0$

$t_0 \; = \; \alpha$

***the solution is given as*** $t_n = \; \alpha r^n$ .

**Proof** :

***This is a geometric sequence and r is called a ratio.***

***Let us apply the substitution method for the equation:***

$t_n \; = r \times t_{n-1}$

$\quad = r \times [rt_{n-2}] = r^2 t_{n-2}$

$\quad = r^2 \times [rt_{n-3}] = r^3 t_{n-3}$

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$= r^n t_0$

***Since*** $t_0 = \alpha$, ***substituting this in the solution yields the following:*** $t_n = \; \alpha r^n$

$We\ could\ rewrite\ the\ above\ proof\ as:-$

$T(n) = r \times T(n-1)\ , n > 0$

$T(0)\ =\ \alpha$

$the\ solution\ is\ given\ as\ T(n) =\ \alpha r^n$ .

$Proof$ :

$This\ is\ a\ geometric\ sequence\ and\ r\ is\ called\ a\ ratio.$

$Let\ us\ apply\ the\ substitution\ method\ for\ the\ equation:$

$T(0)\ = r \times T(n-1)$

$= r \times [rT(n-2)] = r^2T(n-2)$

$= r^2 \times [rT(n-3)] = r^3T(n-3)$

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$= r^nT(0)$

$Since\ T(0) = \alpha, substituting\ this\ in\ the\ solution\ yields\ the$

$following: T(n) =\ \alpha r^n$

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