$Generating\ Function -$
$Example - 2$

$Example\ 2$: $Solve\ the\ following\ recurrence\ equation$
$using\ a\ generating\ function$:

$t_n = 5t_{n-1}\ for\ n = 1, 2, 3, \ldots$
$t_0 = 2$

<u>$Solution$:</u>

$The\ solution\ is\ formulated\ by\ converting\ the\ given\ recurrence\ equation\ to\ a\ generating\ function.$

$Here, substitute\ G(z)\ for\ t_n\ and\ represent\ the\ given\ recurrence\ as\ t_n - 5t_{n-1} = 0$, $thereby\ \ converting\ the\ recurrence\ equation\ into\ a\ generating\ function\ as\ follows$:

$$G(z) \quad = \quad a_0 + a_1 z + a_2 z^2 + a_3 z^3 + \cdots$$
$$5z \times G(z) \quad = \quad 5a_{1-1}z^1 + 5a_{2-1}z^2 + 5a_{3-1}z^3$$

$$G(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + \cdots$$
$$5z \times G(z) = 5a_0 z + 5a_1 z^2 + 5a_2 z^3$$

$$G(z) - \left(5z \times G(z)\right) = a_0 + (a_1 z - 5a_0 z) + (a_2 z^2 - 5a_1 z^2)$$
$$+(a_3 z^3 - 5a_2 z^3) + \cdots$$

$$(1 - 5z)\left(G(z)\right) = a_0 + (a_1 - 5a_0)z + (a_2 - 5a_1)z^2 +$$
$$(a_3 - 5a_2)z^3 + \cdots$$

Now, this generating function should be simplified.

The initial condition is given as 2 and one can observed that the difference in the recurrence equation is 0 that is, $t_n - 5t_{n-1} = 0$.

Therefore all the terms such as:

$$(a_1 - 5a_0) = 0$$
$$(a_2 - 5a_1) = 0$$

and

$$(a_3 - 5a_2) = 0 \ldots..$$

$\textit{Therefore, this equation is reduced as follows}$:

$$(1 - 5z)G(z) = 2 \; [as\ t_0 = 2]$$

$$G(z) = \frac{2}{(1 - 5z)}$$

$\textit{The final step is to get the solution in terms of sequence}$:

$$2(1 - 5z)^{-1} = 2(1 + (5z) + (5z)^2 + \cdots) = 2 \times 5^n$$

$\textit{This is the solution of the given recurrence equation.}$

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