$\boldsymbol{Backward\ Substitution}$: $\boldsymbol{Example\ 4}$

$\boldsymbol{Solve\ the\ following\ recurrence\ equation\ using\ the}$

$\boldsymbol{backward\ substitution\ method}$:

$\boldsymbol{t_n = 7t_{n-1}\ with\ the\ initial\ condition\ t_0 = 1}$

<u>$\boldsymbol{Solution}$:</u>

$\boldsymbol{t_n = 7t_{n-1}}$

$\quad \boldsymbol{= 7 \times 7t_{n-2}\ (plug)}$

$\quad \boldsymbol{= 7^2 t_{n-2} (chug)}$

$\boldsymbol{= 7^2\ \times 7t_{n-3}\ (plug)}$

$\boldsymbol{= 7^3 t_{n-3}\ (chug)}$

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$\boldsymbol{At\ step\ i, this\ corresponds\ to = 7^i [t_{n-i}]}$

$\boldsymbol{when\ i = n, this\ corresponds\ to}$

$\boldsymbol{t_n = 7^n [t_{n-n}] = 7^n t_0\ , As\ t_0 = 1\ , then}$:

$\boldsymbol{We\ get}$: $\boldsymbol{7^n}$.

$$We\ can\ re-write\ it\ as:$$

$$T(n) = 7T(n-1)\ with\ the\ initial\ condition\ T(0) = 1$$

<u>***Solution***</u>:

$$T(n) = 7T(n-1)$$

$$= 7 \times 7T(n-2)\ (plug)$$

$$= 7^2T(n-2)(chug)$$

$$= 7^2 \times 7T(n-3)\ (plug)$$

$$= 7^3T(n-3)\ (chug)$$

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$$At\ step\ i, this\ corresponds\ to = 7^i[T(n-i)]$$

$$when\ i = n, this\ corresponds\ to$$

$$T(n) = 7^n[T(n-n)] = 7^nT(0)\ , As\ T(0) = 1\ , then:$$

$$We\ get: 7^n\ .$$

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