$$\boldsymbol{Divide\ And\ Conquer -}$$

$$\boldsymbol{Akra - Bazzi\ Theorem -}$$

$$\boldsymbol{Example - 2.}$$

$\boldsymbol{Example}$: $\boldsymbol{Solve\ the\ following\ recurrence\ equation}$:

$$T(n) = 8T\left(\frac{n}{2}\right) + n^2$$

$\boldsymbol{Solution}$:

$\boldsymbol{Here\ a = 8\ , hence \frac{8}{2^3} = 1\ , hence\ p = 3\ . Therefore\ by}$

$\boldsymbol{Akrabazzi\ Theorem}$:

$\boldsymbol{u = n, f(n) = n^2\ and\ p = 3}$

$$\Theta\left(n^p\left(1 + \int_1^n \frac{f(n)}{n^{p+1}}\ dn\right)\right)$$

$$= \ \Theta\left(n^3\left(1 + \int_1^n \frac{n^2}{n^{3+1}}\ dn\right)\right)$$

$$= \ \Theta\left(n^3\left(1 + \int_1^n \frac{n^2}{n^4}\ dn\right)\right)$$

$$= \boldsymbol{\Theta}\left(n^3\left(1+\int_1^n \frac{1}{n^2}\,dn\right)\right)$$

Now,

$$= \boldsymbol{\Theta}\left(n^3\left(1+\int_1^n n^{-2}\,dn\right)\right)$$

***Now apply power rule of integeral*:**

$$\int x^a dx = \frac{x^{a+1}}{a+1}\ , hence,$$

$$= \boldsymbol{\Theta}\left(n^3\left(1+\left[\frac{n^{-2+1}}{-2+1}\right]_1^n\,dn\right)\right)$$

$$= \boldsymbol{\Theta}\left(n^3\left(1+\left[\frac{n^{-1}}{-1}\right]_1^n\,dn\right)\right)$$

$$= \boldsymbol{\Theta}\left(n^3\left(1+\left[-\frac{1}{n}\right]_1^n\,dn\right)\right)$$

$$\boldsymbol{Now\ compute\ boundaries}:$$

$$\int_{b}^{a} f(x)d(x) = F(b) - F(a) = \lim_{x \to b-} F(x) - \lim_{x \to a+} F(x)$$

$$\lim_{n \to 1+} \left(-\frac{1}{n}\right) = plugging\ 1\ in\ place\ of\ n\ we\ get = -\frac{1}{1} = -1$$

$$\lim_{n \to n-} \left(-\frac{1}{n}\right) = As\ \lim_{x \to x} f(x) = f(x)\ , hence\ we\ get = -\frac{1}{n}\ only.$$

$$\therefore\ \Theta\left(n^3\left(1 + \left(-\frac{1}{n} - (-1)\right)\right)\right)$$

$$= \Theta\left(n^3\left(1 + \left(-\frac{1}{n} - (-1)\right)\right)\right)$$

$$= \Theta\left(n^3(1 - \frac{1}{n} + 1\right)$$

$$= \Theta\left(n^3\left(2 - \frac{1}{n}\right)\right)$$

$$= \boldsymbol{\Theta}\left(2n^3 - \frac{n^3}{n}\right)$$

$$= \boldsymbol{\Theta}(2n^3 - n^2)$$

$$= \boldsymbol{\Theta}(n^3)$$

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