$Difference\ Method - Example\ 1$

Example* 1**: ***Solve the following recurrence equation using the difference method:

$$t_n = t_{n-1} + 4 \text{ with } t_1 = 2$$

Solution:

One can rewrite the equation $t_n - t_{n-1} = 4$. ***Therefore , the difference between two successive elements is*** 4.

Rewriting this for all elements yields the following set of equations:

$$\begin{cases} t_n - t_{n-1} = 4 \\ t_{n-1} - t_{n-2} = 4 \\ t_{n-2} - t_{n-3} = 4 \\ \vdots \\ t_2 - t_1 = 4 \end{cases}$$

This is a set of $(n-1)$ ***equations***.

$\boldsymbol{Therefore, n-1\ equation\ can\ be\ written\ by\ taking\ advantage\ of\ the\ fact\ that\ the\ difference\ between\ two\ successive\ terms\ is\ 4.}$

$\boldsymbol{After\ adding\ n-1\ equations, one\ can\ observe\ that\ only\ the\ first\ and\ last\ terms\ remain\ and\ all\ other\ terms\ are\ cancelled\ .The\ final\ results\ would\ be\ as\ follows:}$

$$t_n - t_1 = 4(n-1)$$
$$t_n = t_1 + 4(n-1)$$
$$= 2 + 4(n-1)\ [t_1 = 2]$$
$$= 2 + 4n - 4$$
$$= 4n - 2$$

$\boldsymbol{Therefore, the\ solution\ to\ the\ recurrence\ equation\ is\ 4n-2.}$

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