Generating Functions – Partial Fraction.

Many a times a partial fraction has to be found for a given generating function to solve a recurrence equation.

What is a partial fraction?

A partial fraction is a way to decompose a fraction into parts that can be solved easily.

Lets consider two polynomials P and Q.

The ratio of P and Q , that is $\frac{P}{Q}$, is called a rational fraction.

A rational fraction is one whose numerator as well as denominator are polynomials.

$$i.e.\frac{x^2+x+1}{x^3+x^2+x+1}$$

If the degree of the polynomial P is greater than that of the polynomial Q, then it is called a proper fraction.

Hence,

$$\frac{x^3 + x^2 + x + 1}{x^2 + x + 1}$$

As degree of numerator P: $x^3 + x^2 + x + 1$ ***is*** 3 ***and degree of denominator Q:*** $x^2 + x + 1$ ***is*** 2, ***then*** $3 > 2$, ***hence proper fraction.***

If the degree of the polynomial P is less than that of the polynomial Q, then it is called a improper fraction.

$$\frac{x^2 + x + 1}{x^3 + x^2 + x + 1}$$

As degree of numerator P: $x^2 + x + 1$ ***is*** 2 ***and degree of denominator Q:*** $x^3 + x^2 + x + 1$ ***is*** 3, ***then*** $2 < 3$, ***hence improper fraction.***

Many times , partial fractions need to be solved.

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