Polynomial Reduction

So, far , linear recurrences have been solved using methods such as guess and verify, substitution, and recurrence tree.

However these methods are not useful for solving recurrence equations of a higher order, for which the polynomial reduction method is used.

The idea is to reduce the recurrence equation to a characteristic equation and express its solution in terms of its roots.

A linear recurrence equation of the order `k` can be expressed in the following form:

$$a_0t_n + a_1t_{n-1} + \cdots + a_kt_{n-k} = 0$$

This equation is linear because it does not involve any square, square root, or cubic terms.

In addition, the order of this linear recurrence is `k`

as the difference between the highest and the smallest suffix is :

$$a_0 t_n + a_1 t_{n-1} + \cdots + a_k t_{\underbrace{n-k}} = 0$$

$$n - (n - k) = k \, .$$

The following are the steps involved in the polynomial reduction procedure used for solving a recurrence equation:

1. ***Form a characteristic equation for the given recurrence equation***.

2. ***Find the roots of the characteristic equation***.

3. ***Find a general solution with unknown coefficients***.

4. ***Solve the equations with respect to the initial conditions to get a specific solution***.

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