

B. B. 14. B. Domain Transformation – Example – 2

Solve the following recurrence equation:

$$T(n) = 2T(\sqrt{n})$$

Solution

Substituting $n = 2^k$, the equation becomes:

$$T(2^k) = 2T\left(2^{\frac{k}{2}}\right)$$

\therefore This is in a familiar form. We can compare this with the earlier problems and see that the solution is $T(k) = 2^k$.

Now this solution has to be reconvereted into a solution of the original problem. Substituting $k = \log_2 n$ into the solution, we get the following relation:

$$2^{\log_2 n} = n^{\log_2 2} = n .$$

Similarly , for the simple recurrences the smaller terms can be dropped. For example, consider the following recurrence form:

$$T(n) = 2T - (n + \sqrt{n})$$

Since n is large and dominates like other terms like \sqrt{n} , the preceding recurrences can roughly be approximated as $T(n) = 2T(n) \times \sqrt{n}$.
