$Divide\ And\ Conquer -$

$Akra - Bazzi\ Theorem$

$In\ 1998$, $two\ Lebanon - based\ researchers\ provided\ the\ solutions\ for\ the\ generalized\ form\ of\ the\ master\ theorem,\ which\ is\ as\ follows$:

$$T(n) = \begin{cases} h(n) \\ \\ aT\left(\frac{a}{b^k}\right) + f(n) \end{cases}$$

$Here$, $a > 0$, $b > 1\ and\ n_0 \geq b\ are\ integers$; $h(n)\ is\ a\ function\ that\ is\ in\ the\ range\ \ d_1 \leq h(n) \leq d_2$
$for\ two\ constant\ d_1\ and\ d_2\ and\ 1 \leq n \leq n_0; and\ f(n)\ is\ a\ polynomial\ that\ is\ in\ the\ range\ c_1 g(n) \leq f(n) \leq c_2 g(n)$
$for\ all\ x > 0\ and\ u\ \in \left[\frac{n}{b}\ , n\right]. if\ all\ these\ conditions\ are\ satisfied\ and\ the\ condition\ \frac{a}{b^p} = 1\ is\ true\ , then\ the\ solution\ of\ the\ recurrence\ is\ given\ as\ follows$:

$$T(n) = \Theta\left(n^p\left(1 + \int_1^u \frac{f(u)}{u^{p+1}}\, du\right)\right)$$

$$\textit{This is a powerful theorem and solves almost all those recurrences that cannot be solved easily by other methods.}$$

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