$Divide\ And\ Conquer -$
$Continuous\ Master\ Theorem\ or$
$Generalized\ Master\ Theorem$
$-Example - 2$

$Example\ 2$: $Solve\ the\ following\ recurrence\ using\ the\ generalized\ master\ theorem$:

$$T(n) = 2T\left(\frac{n}{2}\right) + n$$

$Solution$:

$f(n) = n$

$a = 2$

$b = 2$

$\therefore n^{log_b a} = n^{log_2 2} = n^{log_2 2} = n^1 = n[as\ log_a a = 1]$

$hence, \varepsilon = n^1 - n^1 = n^{1-1} = n^0\ , hence\ \varepsilon = 0.$

$Hence\ \ \varepsilon = 0$

$\boldsymbol{And}$

$$\boldsymbol{n = \Theta\left(n^{log_2 2}\right) = \Theta(n)}$$

$\boldsymbol{Hence\ there\ exists\ both\ O(n) and\ \Omega(n)\ and}$

$$\boldsymbol{i.e. n = n \implies n^{log_a b - \varepsilon} = n^{log_a b}}$$

$\boldsymbol{This\ implies\ that\ the\ given\ complexity\ function\ belongs\ to\ Case\ 2\ of\ the\ generalized\ master\ theorem.}$

$\boldsymbol{Now\ we\ can\ rewrite\ the\ equation\ f(n) = \Theta(n)}$
$\boldsymbol{as}$:

$$\boldsymbol{f(n) = \Theta(nlog_2 2\ log^0 n)\ and\ log^0 n \implies (logn)^0 = 1}\ .$$

$\boldsymbol{Hence\ k = 0}$,

$$\boldsymbol{\therefore T(n) = \Theta(nlog_2 2\ log^{0+1} n\ ) = \Theta(n\ log^1 n\ ) = \Theta(n\ log\ n\ )}$$

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