UNIVERSITY OF THESSALY
SCHOOL OF ENGINEERING

DEPARTMENT OF ELECTRICAL
& COMPUTER ENGINEERING

Νευρο-Ασαφής Υπολογιστική
Χειμερινό Εξάμηνο 2023-2024
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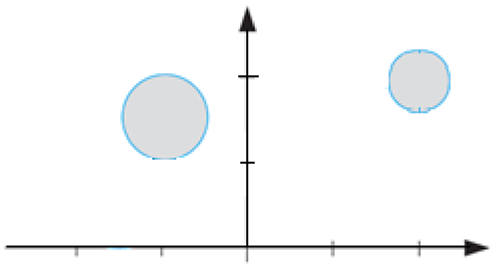
Σειρά προβλημάτων: 3η: ΟΜΑΔΙΚΕΣ (2-ΑΤΟΜΩΝ) ΕΡΓΑΣΙΕΣ

Ημέρα ανανέωσης: Thursday, February 15, 2024
Προθεσμία παράδοσης: Κυριακή, Μάρτιος 03, 2024

SECTION 1: Radial Basis Function neural networks

Problem-01

Design by hand (i.e., select manually the layers, the number of neurons per layer, the weights and biases per layers) an RBF network to perform the classification illustrated in the following figure. The network should produce a positive output whenever the input vector is in the shaded region and a negative output otherwise. Provide arguments for the selection of weights and biases.



Problem-02

Write a MATLAB/python/… program to implement the steepest descent algorithm for the 1-S^1-1 RBF network. Train the network to approximate the function:

$$g(p) = 1 + \sin(p\pi/8) \text{ for } -4 \leq p \leq 4.$$

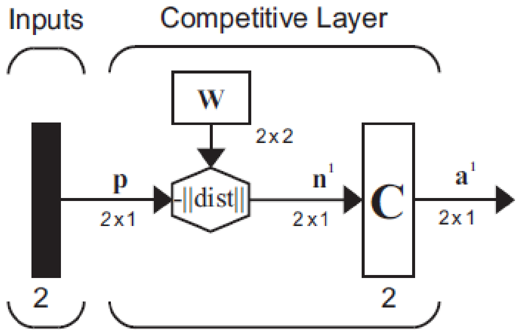
- To train the network, select 30 data points at random from the interval $-4 \leq p \leq 4$.
- Initialize all parameters (weights and biases in both layers) as small random numbers, and then train the network to convergence. (Experiment with the learning rate α, to determine a stable value.) Plot the network response for $-4 \leq p \leq 4$, and show the training points on the same plot. Compute the sum squared error over the training set. Use 4, 8, 12 and 20 centers. Try different sets of initial weights, different sampling methods for selecting training pairs and record your observations.

SECTION 2: Hebbian and Competitive learning

Problem-03

The net input expression for LVQ networks calculates the distance between the input and each weight vector directly, instead of using the inner product. The result is that the

LVQ network does not require normalized input vectors. This technique can also be used to allow a competitive layer to classify non-normalized vectors. Such a network is shown in figure below.



$$n^1_i = -\|{}_i\mathbf{w}-\mathbf{p}\|$$
$$\mathbf{a}^1 = \mathbf{compet}(\mathbf{n}^1)$$

Use this technique to train a two-neuron competitive layer on the (non-normalized) vectors below, using a learning rate, α, of 0.5

$$\mathbf{p}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ \mathbf{p}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix},\ \mathbf{p}_3 = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

Present the vectors in the following order: p_1, p_2, p_3, p_2, p_3, p_1.
Here are the initial weights of the network:

$${}_1\mathbf{w} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ {}_2\mathbf{w} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Problem-04

Repeat Problem-03 for the following inputs and initial weights. Show the movements of the weights graphically for the first six each steps. If the network is trained for a large number of iterations, how will the three vectors be clustered in the final configuration?

$$\mathbf{p}_1 = \begin{bmatrix} 2 \\ 0 \end{bmatrix},\ \mathbf{p}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ \mathbf{p}_3 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$${}_1\mathbf{w} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ {}_2\mathbf{w} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}.$$

SECTION 3: Recurrent neural networks

Problem-05

You are asked to generate an Auto Regressive (AR) model and then create an RNN (such as LSTM, or GRU) that predicts it. Generate samples of an Auto Regressive model of the form:

$$X_t = a_1X_{t-1} + a_2X_{t-2} + a_3X_{t-3} + U_t$$

where $a_1 = 0.5$, $a_2 = -0.1$, $a_3 = 0.2$ and U_t is independent-identically distributed Uniform in the interval (-0.25, 0.25). Now train an RNN of your choice that predicts the sequence.

A. Apply the training algorithm on new samples and calculate the averaged cost square error cost function. Investigate different number of training samples. (You are not allowed to use your model knowledge when you design the RNN.)
B. [**Optional part**: You may use the Yule-Walker equations to compare your RNN model.]

Problem-06

Consider the following Moving Average (MA) process:

$$X_t = U_t + a_1U_{t-1} + a_2U_{t-2} + a_3U_{t-3} + a_4U_{t-4} + a_5U_{t-5} + a_6U_{t-6}$$

where $a_1 = a_2 = 5$, $a_3 = a_4 = a_5 = a_6 = -0.5$ and U_t independent-identically distributed Uniform (0, 0.5).

Repeat the exercise above with a moving average model.

SECTION 4: Fuzzy subsets theory

Problem-07

Model the following expressions as fuzzy subsets:

A. Large integers.
B. Very small numbers.
C. Medium-weight men.
D. Numbers approximately between 10 and 20.

Problem-08

During the lectures, we have defined the concept of ordinary subset of level α of a fuzzy subset. The analogous definition, i.e., the *ordinary relation of level a of a fuzzy relation*, is straightforward. Determine analytically and show graphically the ordinary relation of level 0.3 for the fuzzy relation with membership function: $\mu_{\backslash\text{utilde}\{R\}}(x,y) = 1 - 1/(1+x^2+y^2)$.

Problem-09

Consider the reference set E= [0, α] ⊂ R. If \utilde{A} is the fuzzy subset defined by $\mu_{\backslash\text{utilde}\{A\}}(x)$, give the index ν of fuzziness of \utilde{A} για:

A. $\mu_{\backslash\text{utilde}\{A\}}(x) = x^2/\alpha^2$, $x \in [0, \alpha]$.
B. $\mu_{\backslash\text{utilde}\{A\}}(x) = 4x^2/\alpha^2$ if $0 \le x \le \alpha/2$ and $\mu_{\backslash\text{utilde}\{A\}}(x) = 4(x-\alpha)^2/\alpha^2$ if $\alpha/2 < x \le \alpha$.

Problem-10

Determine analytically and show graphically the *max-min composition* of the two fuzzy relations \utilde{R_1} and \utildc{R_2} defined by the following membership functions:

$$\mu_{\backslash\text{utilde}\{R1\}} = e^{-k(x-y)^2}, k \ge 1$$

$$\mu_{\backslash\text{utilde}\{R2\}} = e^{-k(y-z)^2}, k \ge 1$$

<u>Χρηστικές πληροφορίες</u>:

Η προθεσμία παράδοσης είναι αυστηρή. Είναι δυνατή η παροχή παράτασης (μέχρι 3 ημέρες), αλλά μόνο αφού δώσει ο διδάσκων την έγκρισή του και αυτή η παράταση στοιχίζει 10% ποινή στον τελικό βαθμό της συγκεκριμένης Σειράς Προβλημάτων. Η παράδοση γίνεται με email (στο dkatsar@e-ce.uth.gr) του αρχείου λύσεων σε μορφή pdf (ιδανικά typeset σε LATEX). Θέμα του μηνύματος πρέπει να είναι το: CE418-Problem set 03: AEM1-AEM2

<u>Ερμηνεία συμβόλων</u>:

Δεν απαιτεί την χρήση υπολογιστή ή/και την ανάπτυξη κώδικα.

Απαιτεί κώδικα σε όποια γλώσσα προγραμματισμού ή Matlab. Το παραδοτέο θα περιέχει:

- Την λύση της άσκησης
- Τον πηγαίο κώδικα υλοποίησης