ME 4501 Lab 10: Position Control Using Quanser Qube

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Due: A week after the lab. I will be grading your lab report individually in the beginning of the class.

Learning Objectives:

- Design a PD controller so that the closed loop system have the desired transient response specifications
- Create MATLAB Simulink model to test the controller using real time controller with Quanser Qube
- Measure the corresponding performance specifications
- Determine the effect of proportinal and derivative controller on system performance

A. Theory

A1. Transfer Function of A System

Quanser Qube transfer function from input Voltage (V_m) to Angular Position (θ_m) is

$$G(s) = \frac{\theta_m(s)}{V_m(s)} = \frac{K}{s(\tau s + 1)} \qquad (1)$$

where K = 22.4 rad/(Vs) is the model steadystate gain, τ = 0.166 is the model time constant. You might want to use the same K and τ values in Lab 9 (Step response of First Order Systems).

A2. PID Control

The proportional, integral, and derivative control can be expressed mathematically as follows

$$u(t) = k_p e(t) + k_i \int_0^t e(t)\mathbf{dt} + k_d \frac{\mathbf{de}(t)}{\mathbf{dt}} \qquad (2)$$

where $k_p, k_i,$ and k_d are proportional, integral and derivative controller coefficients that need to be adjusted to achieve desired tasks from the controlled system and e(t) is the error between the reference and the output. The control action is a sum of three terms referred to as proportional (P), integral (I) and derivative (D) control gain.

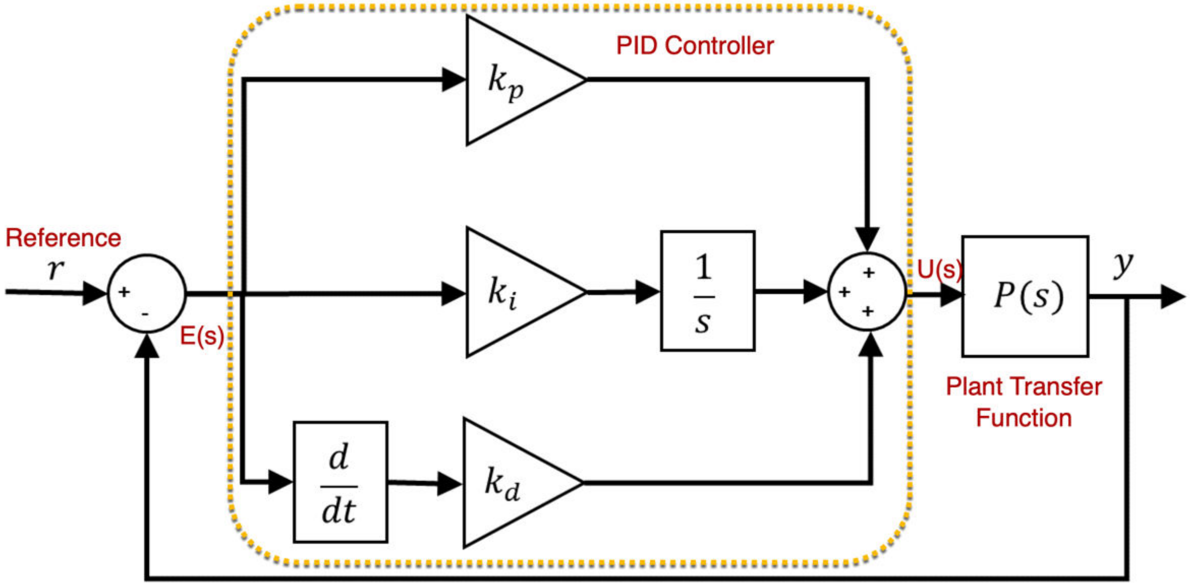
Taking the Laplace Transform of both sides of Eqn. 2 and taking the ratio

$$U(s) = k_p E(s) + k_i \frac{E(s)}{s} + k_d s\mathbf{E}(s) \qquad (3)$$

Then the controller transfer function becomes

$$G_c(s) = \frac{U(s)}{E(s)} = k_p + \frac{k_i}{s} + k_d s \qquad (4)$$

The corresponding block diagram is shownin the below figure.



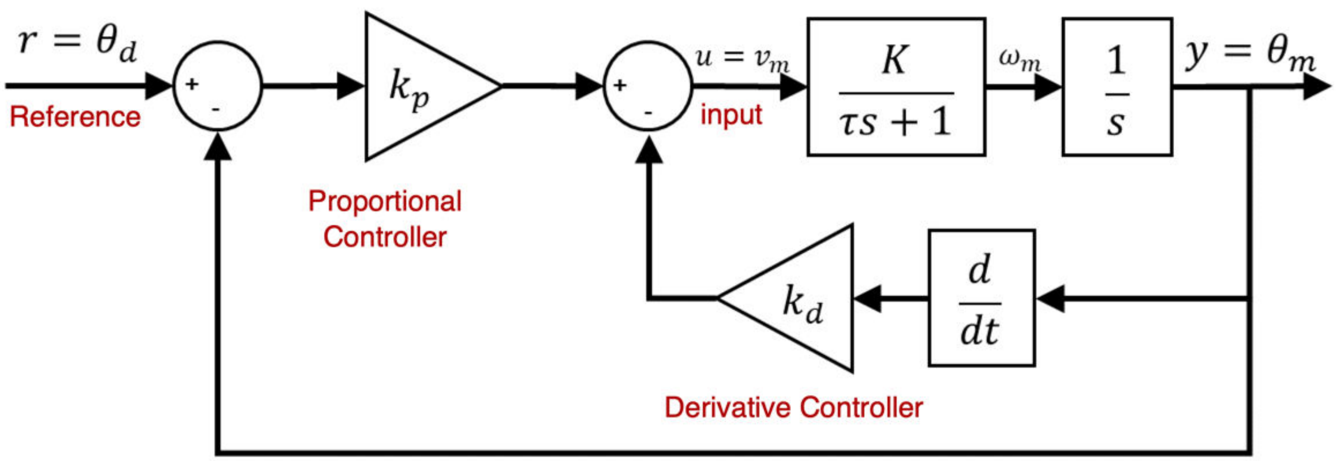
The functionality of the PID controller can be summarized as follows. The proportional term is based on the present error, the integral term depends on past errors, and the derivative term is a prediction of future errors.

The PID controller described by Equation 2-4 is an ideal PID controller. However, attempts to implement such a controller may not lead to a good system response for realworld system. The main reason for this is that measured signals often includes measurement noise. Differentiating a noisy measured signal (i.e. in the derivative control) can produce a control signal with highfrequency components that yields an undesirable response and, over time, may even damage the actuator (e.g. DC motor).

A3. PD Position Control

For this laboratory, the integral term will not be used to control the servo position. A variation of the classic PD control will be used: the proportional plus rate feedback control illustrated in the below figure, also known as proportional plus velocity or PV control.

Unlike the standard PD, only the negative velocity is feedback (i.e. not the velocity of the error). Further, in the hardware implementation of the control a lowpass filter will be used inline with the derivative term to suppress measurement noise.



The combination of a first order low pass filter, $\frac{\omega_f}{s+\omega_f}$ and the derivative term, s, will be used instead of a direct derivative as

$$\boldsymbol{D}(\boldsymbol{s}) = \frac{\omega_f \boldsymbol{s}}{\boldsymbol{s}+\omega_f} \qquad (5)$$

where ω_f is the cut-off frequency.

The proportional plus rate feedback control has the following structure

$$\boldsymbol{u}(\boldsymbol{t}) = \boldsymbol{k_p}(\boldsymbol{r}(\boldsymbol{t}) - \boldsymbol{y}(\boldsymbol{t})) - \boldsymbol{k_d}\dot{\boldsymbol{y}}(\boldsymbol{t}) \qquad (6)$$

where k_p is the proportional gain, k_d is the derivative (velocity) gain, $t(t) = \theta_{\text{desired}}(t) = \theta_d(t)$ is the setpoint or reference motor / load angle, $y(t) = \theta_{\text{motor}}(t) = \theta_m(t)$ is the measured load shaft angle, and $u(t) = V_m(t)$ is the control input (applied motor voltage).

The closed loop system transfer function is denoted by $\frac{Y(s)}{R(s)} = \frac{\theta_m(s)}{\theta_d(s)}$. Assume all the initial conditions are 0, taking the Laplace Transform of Eqn. 6 yields

$$U(s) = k_p(R(s) - Y(s)) - k_d \mathrm{s}\mathrm{Y}(s) \qquad (7)$$

Substituting Eqn. 7 into Eqn. 1 result in

$$Y(s) = \frac{K}{s(\tau s+1)}\left[k_p(R(s) - Y(s)) - k_d \mathrm{s}\mathrm{Y}(s)\right] \qquad (8)$$

$$Y(s) = \frac{K}{s(\tau s+1)}k_p R(s) - \frac{K}{s(\tau s+1)}Y(s) - \frac{K}{s(\tau s+1)}k_d \mathrm{s}\mathrm{Y}(s) \qquad (9)$$

$$Y(s) + \frac{K}{s(\tau s+1)}Y(s) + \frac{K}{s(\tau s+1)}k_d \mathrm{s}\mathrm{Y}(s) = \frac{K}{s(\tau s+1)}k_p R(s) \qquad (10)$$

$$\left[1 + \frac{K}{s(\tau s+1)} + \frac{\mathrm{Kk}_d s}{s(\tau s+1)}\right]Y(s) = \frac{\mathrm{Kk}_p}{s(\tau s+1)}R(s) \qquad (11)$$

Solving for $Y(s)/R(s)$, we obtain the closed loop system equation as

$$T(s) = \frac{Y(s)}{R(s)} = \frac{\mathrm{Kk}_p}{\tau s^2 + (1+\mathrm{Kk}_d)s + \mathrm{Kk}_p} \qquad (12)$$

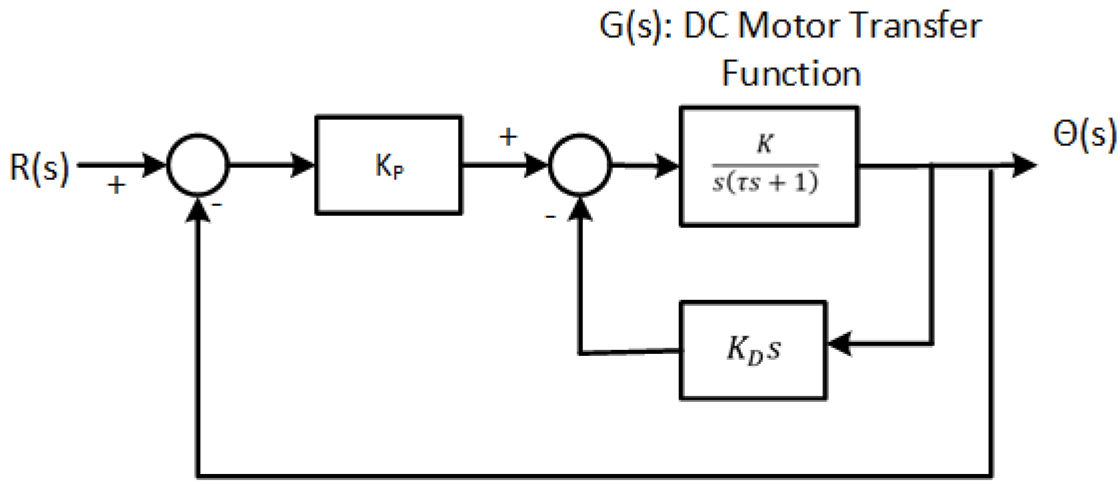
This is a secondorder transfer function. Recall the standard form of a second order transfer function is

$$T(s) = \frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2} \qquad (13)$$

where ω_n is the natural frequency and ς is damping ratio.

Lab Procedures

1. Find the closed loop system transfer function of the given block diagram using symbolic transfer function.



where G(s) is the DC motor transfer function. We found the transfer function in the Step Response Lab as below

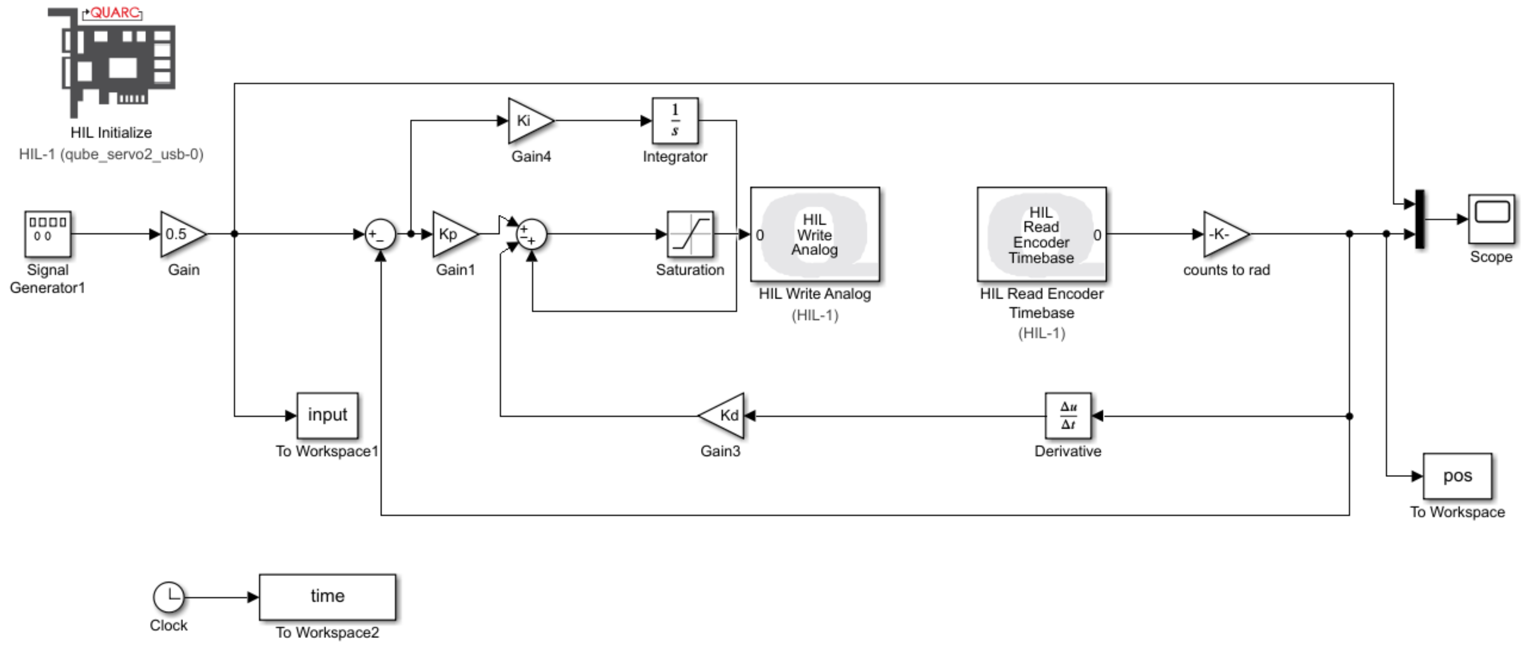
$$G(s) = \frac{K}{s(\tau s + 1)} = \frac{23}{s(0.166s + 1)}$$

<u>**Write your script below:**</u>

2. Find the proportional (kp) and Derivative (kd) controller constants so that the system transient response will exhibit 10% overshoot and 0.3 sec settling time.

<u>**Write your script below to show your solution:**</u>

3. Create the Simulink model in MATLAB as shown below and run the simulations for 10 seconds using Quanser Qube. The signal genereator amplitude is 1 V and the frquany is 0.5 Hz. The model without the controller is in D2L under Lab 10 folder.



4. Plot the response of the system and calculate the maximum overshoot and settling time.

5. Tune the parameters of the controller (kp and kd) to see their effects on the system response and create a table below including the Kp values, Overshoot, Settling Time, and steady-state error.

Discussion

1. What is the effect of propotional gain (kp) on the servo position?

2. What is the effect of derivative controller (kd) on the servo position?

3. Simulate the same system using transfer function and compare the two outputs (simulated angular position and Quanser Qube angular position).

4. Can we meet all the design specifications using only proportional or proportional and derivative controller?