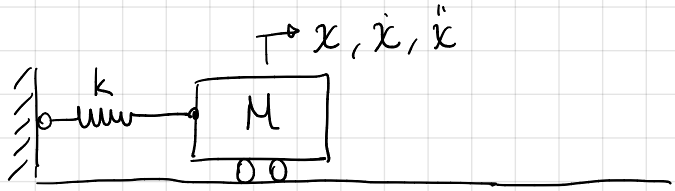
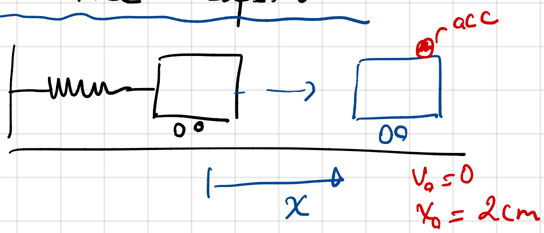
Natural Frequency of SDOF System



→ undamped

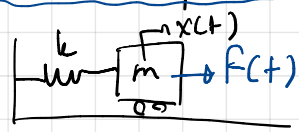
→ damped → underdamped ($\zeta < 1$)
→ critically damped ($\zeta = 1$)
→ overdamped ($\zeta > 1$)

→ Free Response



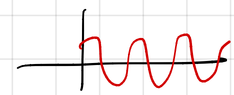
$$x(t) = A\sin(\omega_n t)$$

→ Forced Response



→ step input:

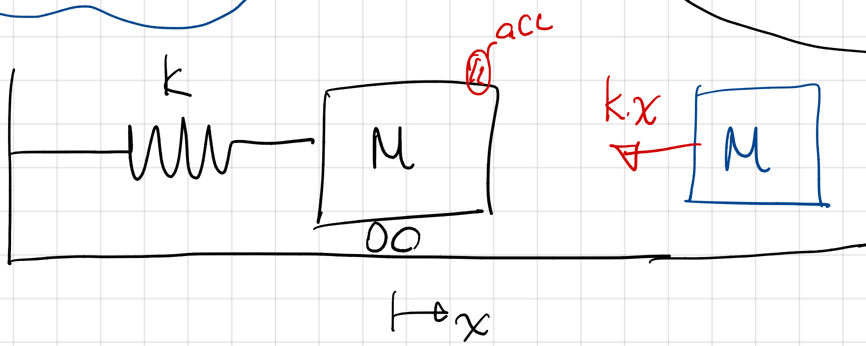
→ sinusoidal input:



$$F(t) = A\sin(\omega t)$$

ω → forcing freq., excitation freq.

$$x(t) = x_t(t) + x_{ss}(t)$$



$$M = m_c + m_t + m_{thick} + m_{acc}$$

$$\Sigma F = M \cdot \ddot{x} \Rightarrow -kx = M \cdot \ddot{x} \Rightarrow M\ddot{x} + kx = 0$$

The equation of motion: $M\ddot{x}(t) + kx(t) = 0$

$$\ddot{x}(t) + \frac{k}{M}x(t) = 0 \Rightarrow \ddot{x}(t) + \omega_n^2 x(t) = 0$$

$$x(t) = A\sin\omega_n t, \quad \dot{x}(t) = A\omega_n \cos\omega_n t, \quad \ddot{x}(t) = -A\omega_n^2 \sin\omega_n t$$

$$\ddot{x}(t) = -\omega_n^2 x(t)$$

$$\ddot{x}(t) + \frac{k}{M} \cdot x(t) = 0 \Rightarrow -\omega_n^2 x(t) + \frac{k}{M}x(t) = 0$$

$$-\omega_n^2 \, x(t) + \frac{k}{m} x(t) = 0$$

$$\underbrace{\left[-\omega_n^2 + \frac{k}{m}\right]}_{0} \cdot \underbrace{x(t)}_{x \neq 0} = 0$$

$$a \cdot b = 0$$
$$a = 0 \qquad b = 0$$

$x = 0$ (stationary)

$$-\omega_n^2 = -\frac{k}{m}$$

$$\boxed{\omega_n^2 = \frac{k}{m}}$$

→ EOM: $\ddot{x}(t) + \frac{k}{m} x(t) = 0$

$$\ddot{x}(t) + \omega_n^2 \, x(t) = 0$$

$$\omega_n^2 = \frac{k}{m} \Rightarrow \omega_n = \sqrt{\frac{k}{m}}$$

ω_n = Natural Freq. in rd/sec

$\omega_n = 2\pi f_n$

f_n = Natural Freq. in Hz

$f_n = \frac{1}{T}$, T: period (sec)

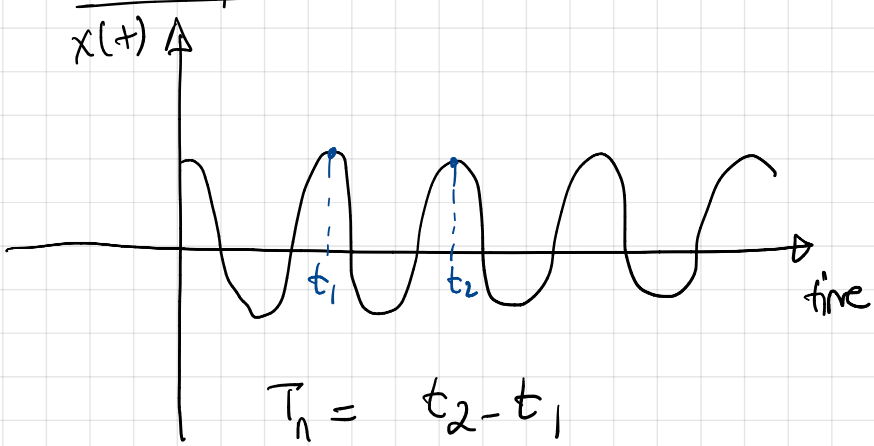
1 * If you know the total mass and stiffness:

$$\omega_n = \sqrt{\frac{k}{M}}$$

$$f_n = \frac{1}{2\pi}\omega_n$$

2 * If you have the free response:

<u>undamped</u>

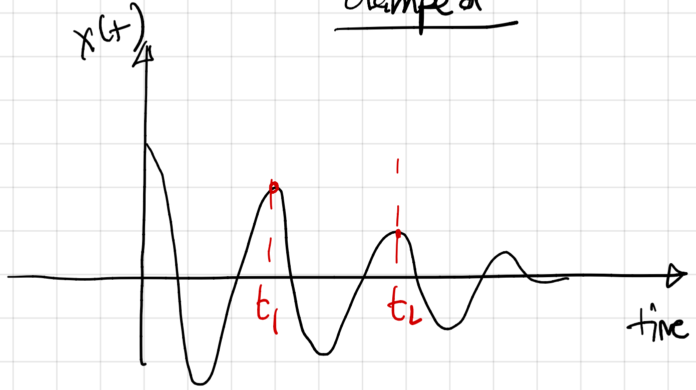


$$T_n = t_2 - t_1$$

$$f_n = \frac{1}{T}$$

$$\omega_n = 2\pi f$$

<u>damped</u>



$$T_d = t_2 - t_1$$

$$f_d = \frac{1}{T}$$

$$\omega_d = 2\pi f$$