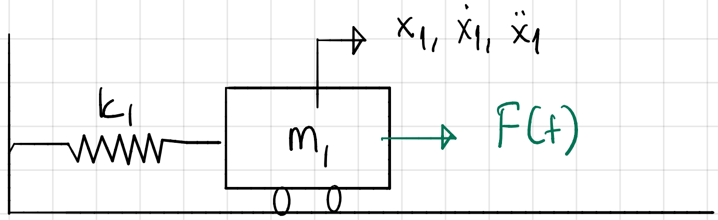
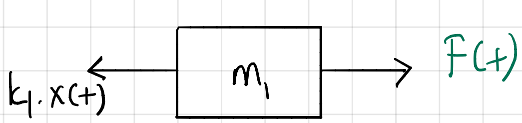
VIBRATION ISOLATION





$$\sum \vec{F} = m . \ddot{x}(t)$$

$$-k_1 x(t) + F(t) = m_1 \ddot{x}_1(t)$$

The equation of motion (EOM) of SDOF is :

$$m_1 \ddot{x}_1(t) + k_1 x_1(t) = F(t)$$

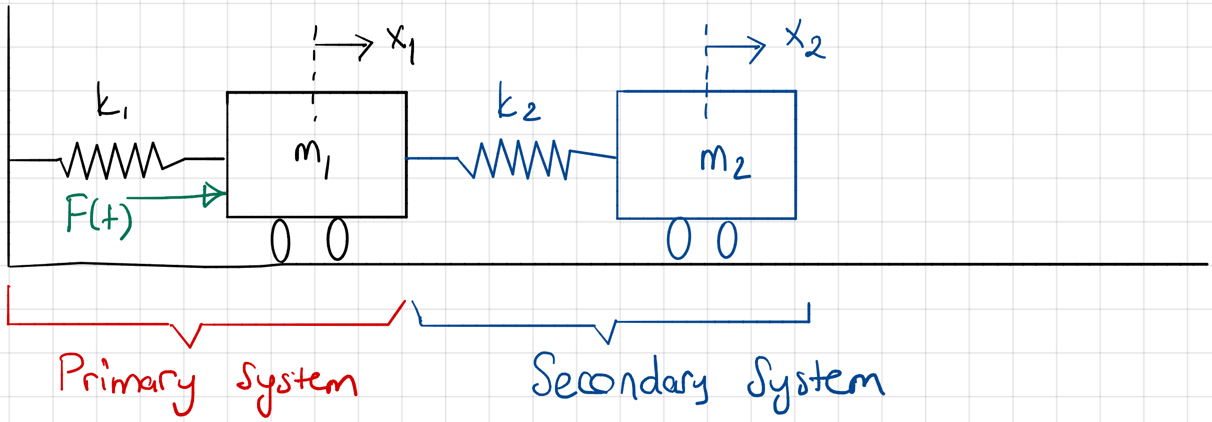
The natural frequency of the system is

$$\omega_{n_1} = \sqrt{\frac{k_1}{m_1}} \quad , \quad f_{n_1} = 2\pi f_{n_1}$$

$F(t) = A . \sin(\omega t)$, where A is the magnitude and ω is the forcing/excitation frequency.

If $\omega = \omega_{n_1}$ $\Rightarrow$ Resonance $\Rightarrow$ large amplitudes of oscillations are expected.

How can we control the vibrations when the system is forced at its own natural frequency?



Since we have translational 2 DOF System:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F(t) \\ 0 \end{bmatrix} \qquad (1)$$

Let's say $x_1(t) = X_1 \cos(\omega t + \phi)$ and $x_2(t) = X_2 \cos(\omega t + \phi)$

$\dot{x}_1(t) = -X_1 . \omega . \sin(\omega t + \phi)$ and $\dot{x}_2(t) = -X_2 \omega \sin(\omega t + \phi)$

$\ddot{x}_1(t) = -X_1 . \omega^2 \cos(\omega t + \phi)$ and $\ddot{x}_2(t) = -X_2 \omega^2 \cos(\omega t + \phi)$

So, $\ddot{x}_1(t) = -\omega^2 x_1(t) \qquad (2)$

$\ddot{x}_2(t) = -\omega^2 x_2(t) \qquad (3)$

If Eqns. 2 and 3 are plugged in Eqn. 3, then

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} - \omega^2 \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} F(t) \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} k_1+k_2-m_1\omega^2 & -k_2 \\ -k_2 & k_2-m_2\omega^2 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} F(t) \\ 0 \end{bmatrix} \qquad (4)$$

Here, ω is the forcing frequency.

$$\begin{bmatrix} k_1+k_2-m_1\omega^2 & -k_2 \\ -k_2 & k_2-m_2\omega^2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} F(t) \\ 0 \end{bmatrix} \quad (4)$$

Solving Eqn. 4 for $x_1(t)$ and $x_2(t)$ yields:

$$x_1(t) = \frac{F(k_2 - m_2\omega^2)}{(k_1+k_2-m_1\omega^2)(k_2-m_2\omega^2)-k_2^2} \quad (5)$$

$$x_2(t) = \frac{Fk_2}{(k_1+k_2-m_1\omega^2)(k_2-m_2\omega^2)-k_2^2} \quad (6)$$

(*) Our goal was to dampen the oscillations of the first cart, to make $x_1(t)=0$ if possible or as small as possible.

From Eqn. 5, if we set $x_1(t)=0$

$$x_1(t) = \frac{F(k_2 - m_2\omega^2)}{(k_1+k_2-m_1\omega^2)(k_2-m_2\omega^2)-k_2^2} = 0,$$

then the numerator should be "0".

$$F(k_2-m_2\omega^2)=0$$

Since $F \neq 0$, then $k_2-m_2\omega^2=0$; So,

$$\omega^2 = \frac{k_2}{m_2} \quad (7)$$

$$\omega^2 = \frac{k_2}{m_2} \Rightarrow \omega = \sqrt{\frac{k_2}{m_2}}$$

$$F(t) = A \cdot \sin(\omega t)$$

if $\omega = \omega_{n_1} \Rightarrow \omega = \sqrt{\frac{k_1}{m_1}}$

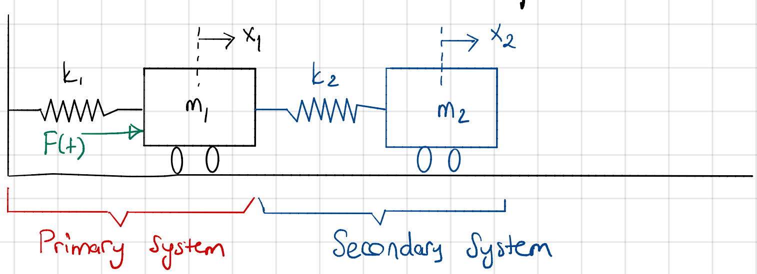
$$\sqrt{\frac{k_1}{m_1}} = \sqrt{\frac{k_2}{m_2}} \Rightarrow \boxed{\frac{k_1}{m_1} = \frac{k_2}{m_2}} \quad (8)$$

If we design a secondary system so that

$$\frac{k_1}{m_1} = \frac{k_2}{m_2}$$

where $k_1 > k_2$ and $m_1 > m_2$, then it's expected that the oscillations on the primary cart is significantly dampened.

Ex



① $k_1 = 600$ N/m, $m_1 = 2$ kg $\Rightarrow \omega_{n_1} = \sqrt{\frac{k_1}{m_1}} = 17.3205$ rd/sec

② If $\frac{k_1}{m_1} = \frac{k_2}{m_2} \Rightarrow \frac{k_2}{m_2} = \frac{600}{2} = 300$

I selected $k_2 = 300$ N/m and $m_1 = 1$ kg.