27 Equations

Area Vectors:

Area vectors are obtained by taking the area under the relevant segment of the median VCG ($\mathbf{V}(t) = [X(t), Y(t), Z(t)]$) QRST complex using the trapezoidal rule:

$$\mathbf{QRS}_{\text{area}} = \int_{Q_{\text{on}}}^{Q_{\text{off}}} \mathbf{V}(t)\,dt = \left[\int_{Q_{\text{on}}}^{Q_{\text{off}}} X(t)\,dt, \int_{Q_{\text{on}}}^{Q_{\text{off}}} Y(t)\,dt, \int_{Q_{\text{on}}}^{Q_{\text{off}}} Z(t)\,dt\right] \tag{1}$$

$$\mathbf{T}_{\text{area}} = \int_{Q_{\text{off}}}^{T_{\text{off}}} \mathbf{V}(t)\,dt = \left[\int_{Q_{\text{off}}}^{T_{\text{off}}} X(t)\,dt, \int_{Q_{\text{off}}}^{T_{\text{off}}} Y(t)\,dt, \int_{Q_{\text{off}}}^{T_{\text{off}}} Z(t)\,dt\right] \tag{2}$$

Peak Vectors:

Peak vectors are obtained by taking the value of relevant segment of $\mathbf{V}$ at the time point which is of maximum distance from the origin:

$$\mathbf{QRS}_{\text{peak}} = [X(t_{Qmax}), Y(t_{Qmax}), Z(t_{Qmax})] \tag{3}$$

$$\mathbf{T}_{\text{peak}} = [X(t_{Tmax}), Y(t_{Tmax}), Z(t_{Tmax})] \tag{4}$$

where t_{Qmax} is the time of max distance of the QRS loop from the origin, and t_{Tmax} is the time of max distance of the T loop from the origin. These times correspond to the maximum values of the QRS complex and T wave in the VM lead, respectively.

Spatial Ventricular Gradient:

The Spatial Ventricular Gradient (SVG) is the vector created by the QRST integrals in $X, Y,$ and Z:

$$\mathbf{SVG} = \mathbf{QRS}_{\text{area}} + \mathbf{T}_{\text{area}} = \int_{Q_{\text{on}}}^{T_{\text{off}}} \mathbf{V}(t)dt = \left[\int_{Q_{\text{on}}}^{T_{\text{off}}} X(t)\,dt, \int_{Q_{\text{on}}}^{T_{\text{off}}} Y(t)\,dt, \int_{Q_{\text{on}}}^{T_{\text{off}}} Z(t)\,dt\right] \tag{5}$$

Vector Magnitude:

Vector magnitude (VM) is defined as the Euclidean norm of the VCG:

$$\text{VM} = \sqrt{X^2 + Y^2 + Z^2} \tag{6}$$

Azimuth Angle:

Azimuth is defined as the angle in the transverse (XZ) plane with negative angles pointing anterior, and positive angles pointing posterior (see **Figure 71**). Azimuth can take values from 0 to ±180 degrees, with 0 degrees pointing to towards the left, and ±180 degrees pointing towards the right.

$$\text{azimuth} = \arctan\left(\frac{Z}{X}\right) \tag{7}$$

Elevation Angle:

Elevation is defined as the angle up from pointing straight down towards the ground (see **Figure 71**). Values range from 0 to 180 degrees, with 0 degrees pointing towards the feet and 180 degrees pointing towards the head.

$$\text{elevation} = \arccos\left(\frac{Y}{\text{VM}}\right) \tag{8}$$

Absolute Integrals:

The sum absolute integral (SAI) is defined as the area under the absolute value of the QRST complex:

$$\text{SAI}_i = \int_{Q_{\text{on}}}^{T\text{off}} |V_i(t)|\, dt \qquad \text{for } i = X, Y, Z, \text{ or VM} \tag{9}$$

SAI QRST is defined as:

$$\text{SAI QRST} = \text{SAI}_x + \text{SAI}_y + \text{SAI}_z = \int_{Q_{\text{on}}}^{T\text{off}} |X(t)|\, dt + \int_{Q_{\text{on}}}^{T\text{off}} |Y(t)|\, dt + \int_{Q_{\text{on}}}^{T\text{off}} |Z(t)|\, dt \tag{10}$$

QRST Angles:

The spatial QRST angle is the 3-dimensional angle between QRS and T vectors:

$$\text{QRST Angle} = \arccos\left(\frac{\mathbf{QRS} \cdot \mathbf{T}}{|\mathbf{QRS}|\,|\mathbf{T}|}\right) \tag{11}$$

where the peak QRST angle uses $\mathbf{QRS}_{\text{peak}}$ and $\mathbf{T}_{\text{peak}}$, and the area QRST angle (also known as mean QRST angle) uses $\mathbf{QRS}_{\text{area}}$ and $\mathbf{T}_{\text{area}}$.

Total Cosine R to T (TCRT):

TCRT was calculated as previously described using singular value decomposition [15].

VCG Loop Length:

Loop length is calculated as the sum of distances covered as the VCG loop increments by each sample For example, QRS loop length is calculated as:

$$\sum_{i=Q_{\mathrm{on}}}^{Q_{\mathrm{off}-1}} \sqrt{(X_{i+1}-X_i)^2+(Y_{i+1}-Y_i)^2+(Z_{i+1}-Z_i)^2} \tag{12}$$

and T loop length is calculated as:

$$\sum_{i=Q_{\mathrm{off}}}^{T_{\mathrm{off}-1}} \sqrt{(X_{i+1}-X_i)^2+(Y_{i+1}-Y_i)^2+(Z_{i+1}-Z_i)^2} \tag{13}$$

VCG Loop Speed:

The instantaneous speed of the QRS or T loops (v_i) in mV/s for a VCG with frequency f is calculated as the distance traveled in a sample of time:

$$v_i = \sqrt{(X_{i+1}-X_i)^2+(Y_{i+1}-Y_i)^2+(Z_{i+1}-Z_i)^2}\,/\,\Delta t \qquad \text{where } \Delta t = 1000/f \tag{14}$$

Left Ventricular Hypertrophy (LVH):

LVH metrics are calculated as previously described: The Cornell Voltage is calculated as the sum of the S wave in lead V3 + the R wave in lead aVL [16]. The Sokolow-Lyon LVH criteria is calculated as the sum of the S wave in lead V1 and the maximum of the R wave in either lead V5 or V6 [17].

QRS Electrical Axis:

The ECG electrical axis (E_A) in the frontal plane is calculated as:

$$E_A = \arctan\left(\frac{2F}{\sqrt{3}I}\right) \tag{15}$$

where F is the R wave - S wave in the lead aVF median beat, I is the R wave - S wave in the lead I median beat, and arctan is the 2-argument arctangent (results are $-180\,\text{deg}$ to $+180\,\text{deg}$) to assign the positive and negative values correctly based on established conventions. Notably, the factor of $2/\sqrt{3}$ is included to account for variations in signal amplitude when calculating the electrical axis using a combination of a bipolar ECG lead (Lead I) and an augmented unipolar lead (aVF). See [18] for further information.

VCG Loop Morphology and Singular Value Decomposition:

The singular value decomposition (SVD) is used to find the best fit planes for the QRS and T loops separately. Let $X' = X - c_x$, $Y' = Y - c_y$, and $Z' = Z - c_z$ where the centroid $\mathbf{c} = (c_x, c_y, c_z)$ is the mean QRS or T vector. Then let the matrix $\mathbf{M}'$ have the centroid-subtracted leads as columns. The SVD of $\mathbf{M}'$ is:

$$\mathbf{M}' = \mathbf{U}\mathbf{S}\mathbf{V}^T \tag{16}$$

After SVD, unit vectors that span the best fit plane for the VCG loop are given by the columns of $\mathbf{V}$ (right singular vectors).

The 3 singular values along the diagonal of matrix $\mathbf{S}$, when squared, give the proportion of mean-squared error or variance (σ^2) in the direction of the 3 corresponding basis vectors in matrix $\mathbf{V}$:

$$S = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix} \qquad \sigma^2 = \begin{bmatrix} S_1^2 & 0 & 0 \\ 0 & S_2^2 & 0 \\ 0 & 0 & S_3^2 \end{bmatrix} \tag{17}$$

The columns are arranged such that $S_1 > S_2 > S_3$. Note that if the the VCG loop is perfectly coplanar, then $S_3 = 0$. Assuming that the VCG loop is approximately planar, the third column of matrix $\mathbf{V}$ ($\mathbf{V}_3$) is normal to the best fit plane; let this vector be denoted by $\mathbf{n}$.

The major and minor axes of the VCG loop are given by the first and second columns of $\mathbf{V}$, $\mathbf{V}_1$ and $\mathbf{V}_2$, respectively (the first and second right singular vectors). The signs of the components of $\mathbf{V}$ are arbitrary, and to enforce consistency, we require that the z-component of $\mathbf{V}_3$ ($= \mathbf{n}$) be negative and the x-component of $\mathbf{V}_1$ be positive. To ensure a right-handed coordinate system, $\mathbf{V}_2$ is set to $\mathbf{V}_3 \times \mathbf{V}_1$, which enforces the correct sign convention.

Of note, the squares of the singular values are equivalent to the respective eigenvalues in principle component analysis.

VCG Loop Coplanarity:

The degree of VCG loop coplanarity is assessed in 2 ways. First, the mean squared error (MSE) in the direction normal to the best fit plane (S_3^2) is obtained from squaring the 3rd singular value as noted above. If $S_3^2 = 0$ then all points in the loop are coplanar. Coplanarity is also assessed by calculating RMSE for points in the VCG loop compared to the points projected onto the best fit plane:

For a given point in the QRS or T loops $\mathbf{s} = [s_x, s_y, s_z]$ let $\mathbf{s}' = \mathbf{s} - \mathbf{c}$. Then the projection s'_{proj} relative to the centroid $\mathbf{c}$ onto the best-fit plane is found by subtracting out the component of the vector in the direction of the normal vector $\mathbf{n}$ defined above:

$$\mathbf{s}'_{\text{proj}} = \mathbf{s}' - (\mathbf{s}' \cdot \mathbf{n})\mathbf{n} \tag{18}$$

RMSE is calculated as the square root of the mean squared distance between corresponding points $\mathbf{s}'$ and $\mathbf{s}'_{\text{proj}}$. Using this metric, if all points are coplanar, RMSE $= 0$.

VCG Loop Dihedral Angle:

VCG loop dihedral angle is defined as the 3-dimensional angle between the unit normal vectors that define the QRS loop best fit plane (N_{QRS}) and the T loop best fit plane (N_{T}). By convention, the dihedral angle is an acute angle:

$$\text{Dihedral Angle} = \arccos(|\mathbf{N}_{\text{QRS}} \cdot \mathbf{N}_{\text{T}}|) \tag{19}$$

VCG Loop Roundness:

Assuming that the MSE in the direction normal to the best fit plane should be relatively small compared to the MSE in the plane itself, the “roundness" (R) of the VCG loop is defined as the ratio of the largest to second largest singular values:

$$R = S_1/S_2 \tag{20}$$

If $R = 1$ the VCG loop is a perfect circle, and as the value of R increases above 1 the loop is more oval or oblong.

VCG Loop Perimeter:

QRS and T loop perimeter are calculated as the length of the QRS or T loop projected into the best fit plane with the set of points (x'_i, y'_i) defined as $\mathbf{s'}_{\text{proj}}$ above, and is different than the QRS and T loop length defined above, which is the length of the loop without projection. The perimeter of the set of points defined by $\mathbf{s'}_{\text{proj}}$ is calculated using Matlab polyshapes.

VCG Loop Area:

QRS and T loop area are calculated as the area of the QRS or T loop projected into the best fit plane with the set of points (x'_i, y'_i) defined as $\mathbf{s'}_{\text{proj}}$ above. The area of the set of points defined by $\mathbf{s'}_{\text{proj}}$ is calculated using Matlab polyshapes which implement the "shoelace formula":

$$A = \frac{1}{2}\left|\sum_{i=1}^{n}(x'_i y'_{i+1} - x'_{i+1} y'_i)\right| \tag{21}$$

where indices are taken modulo n (so $x'_{n+1} = x'_1$ and $y'_{n+1} = y'_1$) to close the loop.

T Wave Mechanical Dispersion (TMD):

T Wave Mechanical Dispersion (TMD) quantifies variation in T wave morphology between ECG leads and is calculated as previously described using singular value decomposition with some slight modifications to signal filtering [19, 15]. In brief, the T wave vector loops are projected into the subspace represented by the first 2 left singular vectors and weighting based on the first 2 singular values. TMD is then calculated as the mean angle (θ_{ij}) between all 21 pairs of independent ECG leads projected into the new subspace ($\mathbf{u_i}$), excluding lead V1:

$$\text{TMD} = \frac{1}{21}\sum_{i\neq j}\theta_{ij} \qquad \text{where } \theta_{ij} = \arccos\left(\frac{\mathbf{u_i}\cdot\mathbf{u_j}}{|\mathbf{u_i}|\,|\mathbf{u_j}|}\right) \qquad \text{for } i,j = \text{I}, \text{II}, \text{V2}, \text{V3}, \text{V4}, \text{V5}, \text{V6} \tag{22}$$

T Wave Residuum (TWR):

T Wave Residuum (TWR) quantifies the non-dipolar components of the T wave with the concept that the energy in the first 3 leads of the decomposed T wave represent the dipolar components, and the small amount of energy contained remaining 5 leads represent the non-dipolar components. There are some variations in how TWR is calculated. BRAVEHEART uses the following equation for the absolute TWR (TWR_{abs}) which is the sum of the squares of the 4th through 8th singular values (equivalent to the sum of the 4th through 8th eigenvalues) [20, 21]:

$$\mathrm{TWR}_{\mathrm{abs}} = \sum_{i=4}^{8} S_i^2 \tag{23}$$

The relative TWR ($\mathrm{TWR}_{\mathrm{rel}}$) is the percent of the absolute TWR divided by the sum of the squares of the 1st through 8th singular values (equivalent to the sum of the 1st through 8th eigenvalues):

$$\mathrm{TWR}_{\mathrm{rel}} = 100 \times \frac{\sum_{i=4}^{8} S_i^2}{\sum_{i=1}^{8} S_i^2} \tag{24}$$

QRS Loop Rotation Direction:

To determine if a QRS loop is rotating clockwise (CW) or counterclockwise (CCW), a viewing plane is determined as a normal unit vector to a plane, and the QRS points are projected into that plane as noted above. the signed area of the loop is calculated using the "shoelace formula" as noted above in the information on how VCG loop area is calculated. For loop direction, however, the area can be positive or negative:

$$A = \frac{1}{2} \sum_{i=1}^{n} (x'_i y'_{i+1} - x'_{i+1} y'_i) \tag{25}$$

where indices are taken modulo n (so $x'_{n+1} = x'_1$ and $y'_{n+1} = y'_1$) to close the loop.

Based on our coordinate system conventions, if $A > 0$ the rotation is "CCW", and if $A < 0$ the rotation is "CW". If the absolute value of the signed area is very small (nominally < 0.1, although this threshold can be adjusted), the rotation is considered "Indeterminate", as it is not clear if the is loop just small, or a figure of 8/crossing over itself with a complex path, in which case it is difficult to define what is "CCW" or "CW". BRAVEHEART reports the QRS loop rotation when viewed in the XY plane (frontal) from the front, the XZ plane (transverse) viewed from below with the patient facing upwards, the ZY plane (left sagital) viewed from the left, and the best fit plane viewed from the direction of the assigned normal vector $\mathbf{n}$. Note that a "CCW" loop will appear "CW" (and vice versa) when viewed from the opposite direction.