

五、实验数据处理

1. 计算光栅常数d，并计算不确定度u(d)

(1) 原始数据记录表格

测量级次 测量次数	-1级		+1级		$2\theta_1 = \frac{1}{2}[(\alpha_1 - \beta_1) - (\alpha_2 - \beta_2)]$
	α_1	β_1	α_2	β_2	
1	156.47°	336.5°	136.37°	316.3°	20.250°
2	219.05°	39.04°	198.42°	18.42°	20.375°
3	290.59°	110.55°	270.37°	90.33°	20.367°
4	353.24°	173.23°	333.01°	153.0°	20.383°
5	52.05°	232.08°	31.45°	211.46°	20.350°

-2级		+2级		$2\theta_2 = \frac{1}{2}[(\alpha_1 - \beta_1) - (\alpha_2 - \beta_2)]$
α_1	β_1	α_2	β_2	
62.35°	242.29°	21.1°	201.12°	41.35°

(2) 计算光栅常数d

$$\overline{2\theta_1} = \frac{\sum_{k=1}^5 2\theta_1}{5} = 20.345^\circ$$

$$\overline{\theta_1} = \frac{1}{2} \overline{2\theta_1} = 0.1775 \text{ rad}$$

$$\overline{\theta_2} = \frac{1}{2} \overline{2\theta_2} = 0.3608 \text{ rad}$$

$$\text{由 } d \sin \theta = k\lambda \text{ , 取 } k = 1 \text{ 得 } d = \frac{\lambda}{\sin \theta_1}$$

又钠黄光 $\lambda = 589.3 \text{ nm}$

$$\therefore d = \frac{\lambda}{\sin \theta_1} = 3.337 \mu\text{m}$$

$$\text{取 } k = 2 \text{ 得 } d' = \frac{2\lambda}{\sin \theta_2} = 3.338 \mu\text{m}$$

(3) 计算不确定度u(d)

1. ± 1 级d的不确定度

$$u_a(\overline{2\theta}) = \sqrt{\frac{\sum_{i=1}^5 (2\theta_i - \overline{2\theta_1})^2}{5 \times 4}} = 0.000426 \text{ rad}$$

$$u_b(\overline{2\theta}) = \frac{1}{\sqrt{3}} = 0.000168 \text{ rad}$$

$$\text{不确定度合成为 } u(\overline{2\theta}) = \sqrt{u_a^2(\overline{2\theta}) + u_b^2(\overline{2\theta})} = 0.000457 \text{ rad}$$

$$u(\overline{\theta_1}) = \frac{1}{2} u(\overline{2\theta_1}) = 2.287 \times 10^{-4} \text{ rad}$$

$$\text{由 } d = \frac{\lambda}{\sin \theta_1} \text{ 有 } \ln d = \ln \lambda - \ln \sin \theta_1$$

$$\text{相对不确定度 } \frac{u(d)}{d} = \sqrt{\left[\frac{\partial \ln \sin \theta_1}{\partial \theta_1} u(\theta_1) \right]^2} = \sqrt{\left[\frac{u(\theta_1)}{\tan \theta_1} \right]^2} = 1.275 \times 10^{-3} \text{ rad}$$

$$\therefore u(d) = d \frac{u(d)}{d} = 0.00425 \mu\text{m}$$

2. ± 2 级d的不确定度

$$\text{由 } d' = \frac{\lambda}{\sin \theta_2} \text{ 有 } \ln d' = \ln \lambda - \ln \sin \theta_2$$

$$\text{而 } u(2\theta_2) = u_b(2\theta_2) = \frac{1'}{\sqrt{3}} = 0.00962^\circ = 0.000168 \text{ rad}$$

$$\therefore u(\theta_2) = \frac{1}{2} u(2\theta_2) = 0.00481^\circ = 8.395 \times 10^{-5} \text{ rad}$$

$$\therefore \text{相对不确定度 } \frac{u(d')}{d'} = \sqrt{\left[\frac{\partial \ln \sin \theta_2}{\partial \theta_2} u(\theta_2) \right]^2} = \sqrt{\left[\frac{u(\theta_2)}{\tan \theta_2} \right]^2} = 2.225 \times 10^{-4}$$

$$\therefore u(d') = d' \frac{u(d')}{d'} = 0.0007428 \mu\text{m}$$

(4) 测量结果加权平均求d最佳值

测量结果:

$$d \pm u(d) = (3.337 \pm 0.004) \mu\text{m}$$

$$d' \pm u(d') = (3.338 \pm 0.001) \mu\text{m}$$

$$\bar{d} = \frac{\frac{d}{u^2(d)} + \frac{d'}{u^2(d')}}{\frac{1}{u^2(d)} + \frac{1}{u^2(d')}} = 3.338 \mu\text{m}$$

$$u^2(\bar{d}) = \frac{1}{\frac{1}{u^2(d)} + \frac{1}{u^2(d')}} = 5.354 \times 10^{-7} \mu\text{m}^2$$

$$\therefore u(\bar{d}) = 7.3 \times 10^{-4} \mu\text{m}$$

$$\therefore \text{光栅常数d的最终表达式为 } \bar{d} \pm u(\bar{d}) = (3.338 \pm 0.001) \mu\text{m}$$

2. 计算氢原子的里德伯常数 $R_H + u(R_H)$; 并通过加权平均获得 R_H 的最佳值 $\overline{R_H} \pm u(\overline{R_H})$

巴耳末系:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right) (n = 3, 4, 5, 6 \dots)$$

当 $n = 3$ 时, 光谱颜色为红光; 当 $n = 5$ 时, 光谱颜色为蓝光; 当 $n = 6$ 时, 光谱颜色为紫光;

以下将分别计算红光, 蓝光, 紫光对应的 R_H :

(1)红光

测量级次 测量次数	-1级		+1级		$2\theta_\gamma = \frac{1}{2}[(\alpha_1 - \beta_1) - (\alpha_2 - \beta_2)]$
	α_1	β_1	α_2	β_2	
1	53.15°	233.19°	30.33°	210.34°	22.725°
2	117.55°	297.59°	95.17°	275.16°	22.675°
3	183.11°	3.12°	160.29°	340.31°	22.692°
4	249.01°	69.0°	226.19°	46.18°	22.700°
5	311.31°	131.29°	288.49°	108.47°	22.700°

1.

$$\overline{2\theta_\gamma} = \frac{\sum_{k=1}^5 2\theta_\gamma}{5} = 0.3962 \text{ rad}$$

$$\text{由 } d \sin \theta = \lambda \text{ 得 } \lambda_\gamma = d \sin \theta_\gamma = d \sin \frac{\overline{2\theta_\gamma}}{2} = 656.904 \text{ nm}$$

$$\text{在巴耳末系中对应 } n \text{ 取 } 3, \text{ 有 } \frac{1}{\lambda_\gamma} = R_{H_1} \left(\frac{1}{2^2} - \frac{1}{3^2} \right)$$

$$\therefore R_{H_1} = \frac{1}{\lambda_\gamma} \left(\frac{1}{2^2} - \frac{1}{3^2} \right) = 1.096051 \times 10^7 \text{ m}^{-1}$$

2. 不确定度的计算

$$u_a(\overline{2\theta_\gamma}) = \sqrt{\frac{\sum_{i=1}^5 (2\theta_{\gamma_i} - \overline{2\theta_\gamma})^2}{5 \times 4}} = 1.4101 \times 10^{-4} \text{ rad}$$

$$u_b(\overline{2\theta}) = \frac{1}{\sqrt{3}} = 9.6225 \times 10^{-3} = 1.679 \times 10^{-4} \text{ rad}$$

$$\therefore \text{不确定度合成 } u(\overline{2\theta_\gamma}) = \sqrt{u_a^2(\overline{2\theta_\gamma}) + u_b^2(\overline{2\theta_\gamma})} = 2.1929 \times 10^{-4} \text{ rad}$$

$$u(\overline{\theta_\gamma}) = \frac{1}{2} u(\overline{2\theta_\gamma}) = 1.0965 \times 10^{-4} \text{ rad}$$

$$\therefore \theta_\gamma \pm u(\theta_\gamma) = (0.1981 \pm 0.0001) \text{ rad}$$

$$\text{而 } R_{H_1} = \frac{1}{\lambda_\gamma} \left(\frac{1}{2^2} - \frac{1}{3^2} \right) = \frac{7.2}{d \sin \theta_\gamma}$$

$$\therefore \ln R_{H_1} = \ln 7.2 - \ln d - \ln d \sin \theta_\gamma$$

$$\therefore \frac{u(R_{H_1})}{R_{H_1}} = \sqrt{\left[\frac{\partial \ln d}{\partial d} u(d) \right]^2 + \left[\frac{\partial \ln \sin \theta_\gamma}{\partial \theta_\gamma} u(\theta_\gamma) \right]^2} = \sqrt{\left[\frac{u(d)}{d} \right]^2 + \left[\frac{u(\theta_\gamma)}{\tan \theta_\gamma} \right]^2} = 5.8863 \times 10^{-4}$$

$$\therefore u(R_{H_1}) = R_{H_1} \frac{u(R_{H_1})}{R_{H_1}} = 6.45167818 \times 10^3$$

$$R_{H_1} \pm u(R_{H_1}) = (1.0961 \pm 0.0006) \times 10^7 \text{ m}^{-1}$$

测量级次 测量次数	-1级		+1级		$2\theta_b = \frac{1}{2}[(\alpha_1 - \beta_1) - (\alpha_2 - \beta_2)]$
	α_1	β_1	α_2	β_2	
1	50.18°	230.2°	33.31°	213.33°	16.783°
2	114.57°	295.0°	98.1°	278.12°	16.792°
3	180.15°	0.14°	163.28°	343.29°	16.767°
4	246.04°	66.02°	229.18°	49.16°	16.767°
5	308.32°	128.31°	291.48°	111.42°	16.775°

(2) 蓝光(深绿)

1.

$$\overline{2\theta_b} = \frac{\sum_{k=1}^5 2\theta_b}{5} = 0.29281 \text{ rad}$$

$$\text{由 } d \sin \theta = \lambda \text{ 得 } \lambda_b = d \sin \theta_b = d \sin \frac{\overline{2\theta_b}}{2} = 486.973 \mu\text{m}$$

$$\text{在巴耳末系中对应 } n \text{ 取 } 4, \text{ 有 } \frac{1}{\lambda_b} = R_{H_2} \left(\frac{1}{2^2} - \frac{1}{4^2} \right)$$

$$\therefore R_{H_2} = \frac{1}{\lambda_b} \left(\frac{1}{2^2} - \frac{1}{4^2} \right) = 1.0952 \times 10^7 \text{ m}^{-1}$$

2. 不确定度的计算

$$u_a(\overline{2\theta_b}) = \sqrt{\frac{\sum_{i=1}^5 (2\theta_{b_i} - \overline{2\theta_b})^2}{5 \times 4}} = 8.48078 \times 10^{-5} \text{ rad}$$

$$u_b(\overline{2\theta}) = \frac{1}{\sqrt{3}} = 9.6225 \times 10^{-3} = 1.679 \times 10^{-4} \text{ rad}$$

$$\therefore \text{不确定度合成为 } u(\overline{2\theta_b}) = \sqrt{u_a^2(\overline{2\theta_b}) + u_b^2(\overline{2\theta_b})} = 1.88143 \times 10^{-4} \text{ rad}$$

$$u(\overline{\theta_b}) = \frac{1}{2} u(\overline{2\theta_b}) = 9.40713 \times 10^{-5} \text{ rad}$$

$$\therefore \theta_b \pm u(\theta_b) = (0.14640 \pm 0.00009)$$

$$\text{而 } R_{H_2} = \frac{1}{\lambda_b} \left(\frac{1}{2^2} - \frac{1}{4^2} \right) = \frac{5.333}{d \sin \theta_b}$$

$$\therefore \ln R_{H_2} = \ln 5.333 - \ln d - \ln d \sin \theta_b$$

$$\therefore \frac{u(R_{H_2})}{R_{H_2}} = \sqrt{\left[\frac{\partial \ln d}{\partial d} u(d) \right]^2 + \left[\frac{\partial \ln \sin \theta_b}{\partial \theta_b} u(\theta_b) \right]^2} = \sqrt{\left[\frac{u(d)}{d} \right]^2 + \left[\frac{u(\theta_b)}{\tan \theta_b} \right]^2} = 6.7456 \times 10^{-4}$$

$$\therefore u(R_{H_2}) = R_{H_2} \frac{u(R_{H_2})}{R_{H_2}} = 7.3877809 \times 10^3 \text{ m}^{-1}$$

$$R_{H_2} \pm u(R_{H_2}) = (1.0952 \pm 0.0007) \times 10^7 \text{ m}^{-1}$$

测量级次 测量次数	-1级		+1级		$2\theta_p = \frac{1}{2}[(\alpha_1 - \beta_1) - (\alpha_2 - \beta_2)]$
	α_1	β_1	α_2	β_2	
1	49.22°	229.25°	34.28°	214.29°	14.917°
2	114.0°	294.04°	99.04°	279.08°	14.933°
3	179.2°	359.2°	164.23°	344.23°	14.950°
4	245.09°	65.05°	230.12°	50.1°	14.933°
5	307.39°	127.37°	292.42°	112.39°	14.958°

(3)紫光(青)

1.

$$\overline{2\theta_p} = \frac{\sum_{k=1}^5 2\theta_p}{5} = 0.13036 \text{ rad}$$

$$\text{由 } d \sin \theta = \lambda \text{ 得 } \lambda_p = d \sin \theta_p = d \sin \frac{\overline{2\theta_p}}{2} = 433.93306 \text{ nm}$$

$$\text{在巴耳末系中对应 } n \text{ 取 } 5, \text{ 有 } \frac{1}{\lambda_p} = R_{H_3} \left(\frac{1}{2^2} - \frac{1}{5^2} \right)$$

$$\therefore R_{H_3} = \frac{1}{\lambda_p} \left(\frac{1}{2^2} - \frac{1}{5^2} \right) = 1.097382 \times 10^7 \text{ m}^{-1}$$

2. 不确定度的计算

$$u_a(\overline{2\theta_p}) = \sqrt{\frac{\sum_{i=1}^5 (2\theta_{p_i} - \overline{2\theta_p})^2}{5 \times 4}} = 1.2680 \times 10^{-4} \text{ rad}$$

$$u_b(\overline{2\theta}) = \frac{1}{\sqrt{3}} = 9.6225 \times 10^{-3} = 1.679 \times 10^{-4} \text{ rad}$$

$$\therefore \text{不确定度合成 } u(\overline{2\theta_p}) = \sqrt{u_a^2(\overline{2\theta_p}) + u_b^2(\overline{2\theta_p})} = 2.1043 \times 10^{-4} \text{ rad}$$

$$u(\overline{\theta_p}) = \frac{1}{2} u(\overline{2\theta_p}) = 1.0522 \times 10^{-4} \text{ rad}$$

$$\therefore \theta_p \pm u(\theta_p) = (0.1304 \pm 0.0001) \text{ rad}$$

$$\text{而 } R_{H_3} = \frac{1}{\lambda_p} \left(\frac{1}{2^2} - \frac{1}{5^2} \right) = \frac{1}{0.21} \frac{1}{d \sin \theta_p}$$

$$\therefore \ln R_{H_3} = \ln \frac{1}{0.21} - \ln d - \ln d \sin \theta_p$$

$$\therefore \frac{u(R_{H_3})}{R_{H_3}} = \sqrt{\left[\frac{\partial \ln d}{\partial d} u(d) \right]^2 + \left[\frac{\partial \ln \sin \theta_p}{\partial \theta_p} u(\theta_p) \right]^2} = \sqrt{\left[\frac{u(d)}{d} \right]^2 + \left[\frac{u(\theta_p)}{\tan \theta_p} \right]^2} = 8.3194 \times 10^{-4}$$

$$\therefore u(R_{H_3}) = R_{H_3} \frac{u(R_{H_3})}{R_{H_3}} = 1.09738234 \times 10^7 \text{ m}^{-1}$$

$$R_{H_3} \pm u(R_{H_3}) = ((1.0974 \pm 0.0009) \times 10^7) \text{ m}^{-1}$$

3. 加权平均求 R_H 的最佳值

$$\overline{R_H} = \frac{\frac{R_{H1}}{u^2 R_{H1}} + \frac{R_{H2}}{u^2 R_{H2}} + \frac{R_{H3}}{u^2 R_{H3}}}{\frac{1}{u^2 R_{H1}} + \frac{1}{u^2 R_{H2}} + \frac{1}{u^2 R_{H3}}} = 1.0961 \times 10^7 m^{-1}$$

$$u^2(\overline{R_H}) = \frac{1}{\frac{1}{u^2 R_{H1}} + \frac{1}{u^2 R_{H2}} + \frac{1}{u^2 R_{H3}}} = m^{-1}$$

$$\therefore u(\overline{R_H}) = 4289.66027208 m^{-1}$$

$$\therefore \text{最佳测量值 } \overline{R_H} \pm u(\overline{R_H}) = (1.0961 \pm 0.0004) \times 10^7 m^{-1}$$

3. 分别计算钠黄光 $k=1,2$ 级的角散射率和分辨本领，并由此说明钠黄光双线能否被分开

(1) 色分辨本领

$$\therefore N = \frac{D}{d} = 659.05$$

$$\therefore R = \frac{\lambda}{\delta\lambda} = kN = \begin{cases} 659.05, & k=1 \\ 1318.10, & k=2 \end{cases}$$

(2) 角色散率

由前面实验， $\overline{\theta_1} = 0.1775 rad$, $\overline{\theta_2} = 0.3608 rad$ 由公式 $D_\theta = \frac{k}{ds \sin \theta}$, 求解可得

$$k=1 \text{ 时, } D_{\theta_1} = \frac{1}{d \sin \overline{\theta_1}} = 1.69619 \times 10^0 rad/m$$

$$k=2 \text{ 时, } D_{\theta_2} = \frac{2}{d \sin \overline{\theta_2}} = 1.69695 \times 10^0 rad/m$$

(3) 钠黄光双线

$$\theta_1 = \arcsin \frac{\lambda_1}{d} = 0.177 rad$$

$$\theta_2 = \arcsin \frac{\lambda_2}{d} = 0.178 rad$$

$$\Delta\theta = \theta_1 - \theta_2 = 0.000183^\circ$$

根据谱线的半角宽度计算公式可得

$$\delta_\theta = \arcsin \frac{2\lambda N_0}{Nd} = 0.000027^\circ$$

$$\therefore \Delta\theta > \delta_\theta$$

\therefore 本实验可将钠黄光的双线分开。