五、实验数据处理

1.计算光栅常数d，并计算不确定度u(d)

(1)原始数据记录表格

测量级次 / 测量次数	−1级		+1级		$2\theta_1=\frac{1}{2}[(\alpha_1-\beta_1)-(\alpha_2-\beta_2)]$
	α_1	β_1	α_2	β_2	
1	4.0°	4.0°	4.0°	4.0°	0.000°
2	4.0°	44.0°	4.0°	44.0°	0.000°
3	4.0°	4.0°	44.0°	4.0°	160.000°
4	4.0°	4.0°	44.0°	4.0°	160.000°
5	4.0°	4.0°	44.0°	4.0°	160.000°

−2级		+2级		$2\theta_2=\frac{1}{2}[(\alpha_1-\beta_1)-(\alpha_2-\beta_2)]$
α_1	β_1	α_2	β_2	
4.0°	44.0°	4.0°	4.0°	20.00°

(2)计算光栅常数d

$$\overline{2\theta_1}=\frac{\sum_{k=1}^{5}2\theta_1}{5}=96.000^\circ$$

$$\overline{\theta_1}=\frac{1}{2}\,\overline{2\theta_1}=0.8378rad$$

$$\overline{\theta_2}=\frac{1}{2}\,\overline{2\theta_2}=0.1745rad$$

由 $d\sin\theta=k\lambda$,取 $k=1$ 得 $d=\dfrac{\lambda}{\sin\theta_1}$

又钠黄光 $\lambda=589.3mm$

$$\therefore\ d=\frac{\lambda}{\sin\theta_1}=0.793\mu m$$

取 $k=2$得 $d'=\dfrac{2\lambda}{\sin\theta_2}=6.787\mu m$

(3)计算不确定度u(d)

1. ±1级d的不确定度

$$u_a(\overline{2\theta})=\sqrt{\frac{\sum_{i=1}^{5}(2\theta_i-\overline{2\theta_1})^2}{5\times 4}}=0.684027rad$$

$$u_b(\overline{2\theta})=\frac{1}{\sqrt{3}}=0.000168rad$$

$$不确定度合成为\ u(\overline{2\theta}) = \sqrt{u_a^2(\overline{2\theta}) + u_b^2(\overline{2\theta})} = 0.684027rad$$

$$u(\overline{\theta_1}) = \frac{1}{2}\ u(\overline{2\theta_1}) = 3.420\times10^{-1}rad$$

$$由\ d = \frac{\lambda}{\sin\theta_1}有\ \ln d = \ln\lambda - \ln\sin\theta_1$$

$$相对不确定度\ \frac{u(d)}{d} = \sqrt{\left[\frac{\partial \ln\sin\theta_1}{\partial\theta_1}\ u(\theta_1)\right]^2} = \sqrt{\left[\frac{u(\theta_1)}{\tan\theta_1}\right]^2} = 3.080\times10^{-1}rad$$

$$\therefore\ u(d) = d\ \frac{u(d)}{d} = 0.24420\mu m$$

2. ±2级d的不确定度

$$由\ d' = \frac{\lambda}{\sin\theta_2}有\ \ln d' = \ln\lambda - \ln\sin\theta_2$$

$$而\ u(2\theta_2) = u_b(2\theta_2) = \frac{1'}{\sqrt{3}} = 0.00962^\circ = 0.000168rad$$

$$\therefore u(\theta_2) = \frac{1}{2}\ u(2\theta_2) = 0.00481^\circ = 8.395\times10^{-5}rad$$

$$\therefore 相对不确定度\ \frac{u(d')}{d'} = \sqrt{\left[\frac{\partial \ln\sin\theta_2}{\partial\theta_2}\ u(\theta_2)\right]^2} = \sqrt{\left[\frac{u(\theta_2)}{\tan\tan_2}\right]^2} = 4.762\times10^{-4}$$

$$\therefore\ u(d') = d'\ \frac{u(d')}{d'} = 0.0032323\mu m$$

(4)测量结果加权平均求d最佳值

测量结果：

$$d \pm u(d) = (0.8 \pm 0.2)\mu m$$

$$d' \pm u(d') = (6.787 \pm 0.003)\mu m$$

$$\overline{d} = \frac{\frac{d}{u^2(d)} + \frac{d'}{u^2(d')}}{\frac{1}{u^2(d)} + \frac{1}{u^2(d')}} = 6.786\mu m$$

$$u^2(\overline{d}) = \frac{1}{\frac{1}{u^2(d)} + \frac{1}{u^2(d')}} = 1.045\times10^{-5}\mu m^2$$

$$\therefore\ u(\overline{d}) = 3.2\times10^{-3}\mu m$$

$$\therefore 光栅常数d的最终表达式为\ \overline{d} \pm u(\overline{d}) = (6.786 \pm 0.003)\mu m$$

2.计算氢原子的里德伯常数$R_H + u(R_H)$;并通过加权平均获得R_H的最佳值$\overline{R_H} \pm u(\overline{R_H})$

巴耳末系:

$$\frac{1}{\lambda} = R_H\ \left(\frac{1}{2^2} - \frac{1}{n^2}\right)(n = 3,4,5,6\dots)$$

当 $n = 3$ 时，光谱颜色为红光; 当 $n = 5$ 时，光谱颜色为蓝光; 当 $n = 6$ 时，光谱颜色为紫光;

以下将分别计算红光，蓝光，紫光对应的R_H:

(1)红光

测量级次 / 测量次数	-1级		$+1$级		$2\theta_\gamma=\frac{1}{2}[(\alpha_1-\beta_1)-(\alpha_2-\beta_2)]$
	α_1	β_1	α_2	β_2	
1	4.0°	44.0°	4.0°	4.0°	20.000°
2	4.0°	44.0°	4.0°	4.0°	20.000°
3	4.0°	44.0°	4.0°	4.0°	20.000°
4	4.0°	44.0°	4.0°	4.0°	20.000°
5	4.0°	44.0°	4.0°	4.0°	20.000°

1.

$$\overline{2\theta_\gamma}=\frac{\sum_{k=1}^{5}2\theta_\gamma}{5}=0.3491rad$$

$$\text{由 } d\sin\theta=\lambda \text{ 得 } \lambda_\gamma=d\sin\theta_\gamma=d\sin\frac{\overline{2\theta_\gamma}}{2}=1178.418nm$$

$$\text{在巴耳末系中对应n取3，有 } \frac{1}{\lambda_\gamma}=R_{H_1}\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$$

$$\therefore\ R_{H_1}=\frac{1}{\lambda_\gamma}\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=6.109888\times10^6m^{-1}$$

2. 不确定度的计算

$$u_a(\overline{2\theta_\gamma})=\sqrt{\frac{\sum_{i=1}^{5}(2\theta_{\gamma_i}-\overline{2\theta_\gamma}])^2}{5\times4}}=0.0000\times10^0rad$$

$$u_b(\overline{2\theta})=\frac{1}{\sqrt{3}}=9.6225\times10^{-3}=1.679\times10^{-4}rad$$

$$\therefore \text{不确定度合成为 } u(\overline{2\theta_\gamma})=\sqrt{u_a^2(\overline{2\theta_\gamma})+u_b^2(\overline{2\theta_\gamma})}=1.6794\times10^{-4}rad$$

$$u(\overline{\theta_\gamma})=\frac{1}{2}\ u(\overline{2\theta_\gamma})=8.3972\times10^{-5}rad$$

$$\therefore \theta_\gamma\pm u(\theta_\gamma)=(0.17453\pm0.00008)rad$$

$$\text{而 } R_{H_1}=\frac{1}{\lambda_\gamma}\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=\frac{7.2}{d\sin\theta_\gamma}$$

$$\therefore \ln R_{H_1}=\ln 7.2-\ln d-\ln d\sin\theta_\gamma$$

$$\therefore \frac{u(R_{H_1})}{R_{H_1}}=\sqrt{\left[\frac{\partial\ln d}{\partial d}\ u(d)\right]^2+\left[\frac{\partial\ln\sin\theta_\gamma}{\partial\theta_\gamma}\ u(\theta_\gamma)\right]^2}=\sqrt{\left[\frac{u(d)}{d}\right]^2+\left[\frac{u(\theta_\gamma)}{\tan\theta_\gamma}\right]^2}=6.7351\times10^{-4}$$

$$\therefore\ u(R_{H_1})=R_{H_1}\ \frac{u(R_{H_1})}{R_{H_1}}=4.11509201\times10^3$$

$$R_{H_1}\pm u(R_{H_1})=(6.110\pm0.004)\times10^6m^{-1}$$

测量次数 \ 测量级次	−1级		+1级		$2\theta_b = \dfrac{1}{2}\left[(\alpha_1-\beta_1)-(\alpha_2-\beta_2)\right]$
	α_1	β_1	α_2	β_2	
1	44.0°	4.0°	4.0°	4.0°	20.000°
2	44.0°	4.0°	4.0°	4.0°	20.000°
3	4.0°	44.0°	4.0°	4.0°	20.000°
4	44.0°	4.0°	4.0°	4.0°	20.000°
5	44.0°	4.0°	4.0°	4.0°	20.000°

(2)蓝光(深绿)

1.

$$\overline{2\theta_b} = \frac{\sum_{k=1}^{5} 2\theta_b}{5} = 0.34907rad$$

$$\text{由 } d\sin\theta = \lambda \text{ 得 } \lambda_b = d\sin\theta_b = d\sin\frac{\overline{2\theta_b}}{2} = 1178.418\mu m$$

$$\text{在巴耳末系中对应n取4，有 } \frac{1}{\lambda_b} = R_{H_2}\left(\frac{1}{2^2}-\frac{1}{4^2}\right)$$

$$\therefore\ R_{H_2} = \frac{1}{\lambda_b}\left(\frac{1}{2^2}-\frac{1}{4^2}\right) = 4.5258{\times}10^6 m^{-1}$$

2. 不确定度的计算

$$u_a(\overline{2\theta_b}) = \sqrt{\frac{\sum_{i=1}^{5}(2\theta_{b_i}-\overline{2\theta_{\gamma]}})^2}{5\times 4}} = 0.00000{\times}10^0 rad$$

$$u_b(\overline{2\theta}) = \frac{1}{\sqrt{3}} = 9.6225\times 10^{-3} = 1.679\times 10^{-4} rad$$

$$\therefore \text{不确定度合成为 } u(\overline{2\theta_b}) = \sqrt{u_a^2(\overline{2\theta_b}) + u_b^2(\overline{2\theta_b})} = 1.67944{\times}10^{-4} rad$$

$$u(\overline{\theta_b}) = \frac{1}{2}\,u(\overline{2\theta_b}) = 8.39722{\times}10^{-5} rad$$

$$\therefore \theta_b \pm u(\theta_b) = (0.17453 \pm 0.00008)$$

$$\text{而 } R_{H_2} = \frac{1}{\lambda_b}\left(\frac{1}{2^2}-\frac{1}{4^2}\right) = \frac{5.333}{d\sin\theta_b}$$

$$\therefore \ln R_{H_2} = \ln 5.333 - \ln d - \ln d\sin\theta_b$$

$$\therefore \frac{u(R_{H_2})}{R_{H_2}} = \sqrt{\left[\frac{\partial \ln d}{\partial d}\ u(d)\right]^2 + \left[\frac{\partial \ln\sin\theta_b}{\partial\theta_b}\ u(\theta_b)\right]^2} = \sqrt{\left[\frac{u(d)}{d}\right]^2 + \left[\frac{u(\theta_b)}{\tan\theta_b}\right]^2} = 6.7351{\times}10^{-4}$$

$$\therefore\ u(R_{H_2}) = R_{H_2}\ \frac{u(R_{H_2})}{R_{H_2}} = 3.0482163{\times}10^3 m^{-1}$$

$$R_{H_2} \pm u(R_{H_2}) = (4.526 \pm 0.003){\times}10^6 m^{-1}$$

测量次数 \ 测量级次	−1级		+1级		$2\theta_p=\frac{1}{2}\left[(\alpha_1-\beta_1)-(\alpha_2-\beta_2)\right]$
	α_1	β_1	α_2	β_2	
1	44.0°	4.0°	4.0°	4.0°	20.000°
2	44.0°	4.0°	4.0°	4.0°	20.000°
3	44.0°	4.0°	4.0°	4.0°	20.000°
4	4.0°	4.0°	44.0°	4.0°	160.000°
5	4.0°	4.0°	44.0°	4.0°	160.000°

(3)紫光(青)

1.

$$\overline{2\theta_p}=\frac{\sum_{k=1}^{5}2\theta_p}{5}=0.66323rad$$

$$\text{由 } d\sin\theta=\lambda \text{ 得 } \lambda_p=d\sin\theta_p=d\sin\frac{\overline{2\theta_p}}{2}=4178.02460nm$$

$$\text{在巴耳末系中对应n取5，有 } \frac{1}{\lambda_p}=R_{H_3}\left(\frac{1}{2^2}-\frac{1}{5^2}\right)$$

$$\therefore\ R_{H_3}=\frac{1}{\lambda_p}\left(\frac{1}{2^2}-\frac{1}{5^2}\right)=1.139750{\times}10^6m^{-1}$$

2. 不确定度的计算

$$u_a(\overline{2\theta_p})=\sqrt{\frac{\sum_{i=1}^{5}(2\theta_{p_i}-\overline{2\theta_{\gamma]}})^2}{5\times 4}}=5.9852{\times}10^{-1}rad$$

$$u_b(\overline{2\theta})=\frac{1}{\sqrt{3}}=9.6225\times10^{-3}=1.679\times10^{-4}rad$$

$$\therefore \text{不确定度合成为 } u(\overline{2\theta_p})=\sqrt{u_a^2(\overline{2\theta_p})+u_b^2(\overline{2\theta_p})}=5.9852{\times}10^{-1}rad$$

$$u(\overline{\theta_p})=\frac{1}{2}\ u(\overline{2\theta_p})=2.9926{\times}10^{-1}rad$$

$$\therefore \theta_p\pm u(\theta_p)=(0.7\pm0.3)rad$$

$$\text{而 } R_{H_3}=\frac{1}{\lambda_p}\left(\frac{1}{2^2}-\frac{1}{5^2}\right)=\frac{1}{0.21}\ \frac{1}{d\sin\theta_p}$$

$$\therefore \ln R_{H_3}=\ln\frac{1}{0.21}-\ln d-\ln d\sin\theta_p$$

$$\therefore \frac{u(R_{H_3})}{R_{H_3}}=\sqrt{\left[\frac{\partial\ln d}{\partial d}\ u(d)\right]^2+\left[\frac{\partial\ln\sin\theta_p}{\partial\theta_p}\ u(\theta_p)\right]^2}=\sqrt{\left[\frac{u(d)}{d}\right]^2+\left[\frac{u(\theta_p)}{\tan\theta_p}\right]^2}=3.8304{\times}10^{-1}$$

$$\therefore\ u(R_{H_3})=R_{H_3}\ \frac{u(R_{H_3})}{R_{H_3}}=1.13975029{\times}10^6m^{-1}$$

$$R_{H_3}\pm u(R_{H_3})=((1.1\pm0.4){\times}10^6)m^{-1}$$

3. 加权平均求R_H的最佳值

$$\overline{R_H}=\frac{\frac{R_{H_1}}{u^2R_{H_1}}+\frac{R_{H_2}}{u^2R_{H_2}}+\frac{R_{H_3}}{u^2R_{H_3}}}{\frac{1}{u^2R_{H_1}}+\frac{1}{u^2R_{H_2}}+\frac{1}{u^2R_{H_3}}}=5.0869\times10^6m^{-1}$$

$$u^2(\overline{R_H})=\frac{1}{\frac{1}{u^2R_{H_1}}+\frac{1}{u^2R_{H_2}}+\frac{1}{u^2R_{H_3}}}=m^{-1}$$

$$\therefore\ u(\overline{R_H})=2449.37758449m^{-1}$$

$$\therefore \text{最佳测量值}\ \overline{R_H}\pm u(\overline{R_H})=(5.087\pm0.002)\times10^6m^{-1}$$

3.分别计算钠黄光k=1,2级的角散射率和分辨本领，并由此说明钠黄光双线能否被分开

(1)色分辨本领

$$\because\ N=\frac{D}{d}=324.19$$

$$\therefore\ R=\frac{\lambda}{\delta_\lambda}=kN=\begin{cases}324.19, & k=1\\648.37, & k=2\end{cases}$$

(2)角色散率

由前面实验，$\overline{\theta_1}=0.8378rad,\overline{\theta_1}=0.1745rad$ 由公式$D_\theta=\dfrac{k}{ds\sin\theta}$,求解可得

$$k=1\ \text{时},\ D_{\theta_1}=\frac{1}{d\sin\overline{\theta_1}}=1.98289\times10^{-1}rad/m$$

$$k=2\ \text{时},\ D_{\theta_2}=\frac{2}{d\sin\overline{\theta_2}}=1.69719\times10^{0}rad/m$$

(3)钠黄光双线

$$\theta_1=\arcsin\frac{\lambda_1}{d}=0.087rad$$

$$\theta_2=\arcsin\frac{\lambda_2}{d}=0.087rad$$

$$\Delta\theta=\theta_1-\theta_2=0.000089rad$$

根据谱线的半角宽度计算公式可得

$$\delta_\theta=\arcsin\frac{2\lambda N_0}{Nd}=0.000027rad$$

$\because \Delta_\theta>\delta_\theta$

$\therefore$本实验可将钠黄光的双线分开。