五、实验数据处理

1.计算光栅常数d，并计算不确定度u(d)

(1)原始数据记录表格

测量级次 / 测量次数	−1级		+1级		$2\theta_1=\frac{1}{2}[(\alpha_1-\beta_1)-(\alpha_2-\beta_2)]$
	α_1	β_1	α_2	β_2	
1	156.47°	336.5°	136.37°	316.3°	20.250°
2	219.05°	39.04°	198.42°	18.42°	20.375°
3	290.59°	110.55°	270.37°	90.33°	20.367°
4	353.24°	173.23°	333.01°	153.0°	20.383°
5	52.05°	232.08°	31.45°	211.46°	20.350°

−2级		+2级		$2\theta_2=\frac{1}{2}[(\alpha_1-\beta_1)-(\alpha_2-\beta_2)]$
α_1	β_1	α_2	β_2	
62.35°	242.29°	21.1°	201.12°	41.35°

(2)计算光栅常数d

$$\overline{2\theta_1}=\frac{\sum_{k=1}^{5}2\theta_1}{5}=20.345°$$

$$\overline{\theta_1}=\frac{1}{2}\overline{2\theta_1}=0.1775rad$$

$$\overline{\theta_2}=\frac{1}{2}\overline{2\theta_2}=0.3608rad$$

$$\text{由 } d\sin\theta=k\lambda \text{ ,取 } k=1 \text{ 得 } d=\frac{\lambda}{\sin\theta_1}$$

又钠黄光 $\lambda=589.3mm$

$$\therefore\ d=\frac{\lambda}{\sin\theta_1}=3.337\mu m$$

$$\text{取 } k=2\text{得 } d'=\frac{2\lambda}{\sin\theta_2}=3.338\mu m$$

(3)计算不确定度u(d)

1. ±1级d的不确定度

$$u_a(\overline{2\theta})=\sqrt{\frac{\sum_{i=1}^{5}(2\theta_i-\overline{2\theta_1})^2}{5\times 4}}=0.000426rad$$

$$u_b(\overline{2\theta})=\frac{1}{\sqrt{3}}=0.000168rad$$

$$不确定度合成为\ u(\overline{2\theta}) = \sqrt{u_a^2(\overline{2\theta}) + u_b^2(\overline{2\theta})} = 0.000457rad$$

$$u(\overline{\theta_1}) = \frac{1}{2}\ u(\overline{2\theta_1}) = 2.287{\times}10^{-4}rad$$

$$由\ d = \frac{\lambda}{\sin\theta_1}有\ \ln d = \ln\lambda - \ln\sin\theta_1$$

$$相对不确定度\ \frac{u(d)}{d} = \sqrt{\left[\frac{\partial \ln\sin\theta_1}{\partial\theta_1}\ u(\theta_1)\right]^2} = \sqrt{\left[\frac{u(\theta_1)}{\tan\theta_1}\right]^2} = 1.275{\times}10^{-3}rad$$

$$\therefore\ u(d) = d\ \frac{u(d)}{d} = 0.00425\mu m$$

2. ±2级d的不确定度

$$由\ d' = \frac{\lambda}{\sin\theta_2}有\ \ln d' = \ln\lambda - \ln\sin\theta_2$$

$$而\ u(2\theta_2) = u_b(2\theta_2) = \frac{1'}{\sqrt{3}} = 0.00962^\circ = 0.000168rad$$

$$\therefore u(\theta_2) = \frac{1}{2}\ u(2\theta_2) = 0.00481^\circ = 8.395\times10^{-5}rad$$

$$\therefore 相对不确定度\ \frac{u(d')}{d'} = \sqrt{\left[\frac{\partial \ln\sin\theta_2}{\partial\theta_2}\ u(\theta_2)\right]^2} = \sqrt{\left[\frac{u(\theta_2)}{\tan\tan_2}\right]^2} = 2.225{\times}10^{-4}$$

$$\therefore\ u(d') = d'\ \frac{u(d')}{d'} = 0.0007428\mu m$$

(4)测量结果加权平均求d最佳值

测量结果：

$$d \pm u(d) = (3.337 \pm 0.004)\mu m$$

$$d' \pm u(d') = (3.338 \pm 0.001)\mu m$$

$$\overline{d} = \frac{\frac{d}{u^2(d)} + \frac{d'}{u^2(d')}}{\frac{1}{u^2(d)} + \frac{1}{u^2(d')}} = 3.338\mu m$$

$$u^2(\overline{d}) = \frac{1}{\frac{1}{u^2(d)} + \frac{1}{u^2(d')}} = 5.354{\times}10^{-7}\mu m^2$$

$$\therefore\ u(\overline{d}) = 7.3{\times}10^{-4}\mu m$$

$$\therefore 光栅常数d的最终表达式为\ \overline{d} \pm u(\overline{d}) = (3.338 \pm 0.001)\mu m$$

2.计算氢原子的里德伯常数$R_H + u(R_H)$;并通过加权平均获得R_H的最佳值$\overline{R_H} \pm u(\overline{R_H})$

巴耳末系:

$$\frac{1}{\lambda} = R_H\ \left(\frac{1}{2^2} - \frac{1}{n^2}\right)(n = 3, 4, 5, 6\dots)$$

当 $n = 3$ 时，光谱颜色为红光; 当 $n = 5$ 时，光谱颜色为蓝光; 当 $n = 6$ 时，光谱颜色为紫光;

以下将分别计算红光，蓝光，紫光对应的R_H:

(1)红光

测量级次 / 测量次数	−1级		+1级		$2\theta_\gamma=\dfrac{1}{2}\left[(\alpha_1-\beta_1)-(\alpha_2-\beta_2)\right]$
	α_1	β_1	α_2	β_2	
1	53.15°	233.19°	30.33°	210.34°	22.725°
2	117.55°	297.59°	45.17°	275.16°	47.675°
3	183.11°	3.12°	160.29°	340.31°	22.692°
4	249.01°	69.0°	226.19°	46.18°	22.700°
5	311.31°	131.29°	288.49°	108.47°	22.700°

1.

$$\overline{2\theta_\gamma}=\frac{\sum_{k=1}^{5}2\theta_\gamma}{5}=0.4834rad$$

$$\text{由 } d\sin\theta=\lambda \text{ 得 } \lambda_\gamma=d\sin\theta_\gamma=d\sin\frac{\overline{2\theta_\gamma}}{2}=3200.651nm$$

$$\text{在巴耳末系中对应n取3，有 }\frac{1}{\lambda_\gamma}=R_{H_1}\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$$

$$\therefore\ R_{H_1}=\frac{1}{\lambda_\gamma}\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=2249542.653510m^{-1}$$

2. 不确定度的计算

$$u_a(\overline{2\theta_\gamma})=\sqrt{\frac{\sum_{i=1}^{5}(2\theta_{\gamma_i}-\overline{2\theta_\gamma}])^2}{5\times4}}=4.5928\times10^{-2}rad$$

$$u_b(\overline{2\theta})=\frac{1}{\sqrt{3}}=9.6225\times10^{-3}=1.679\times10^{-4}rad$$

$$\therefore \text{不确定度合成为 } u(\overline{2\theta_\gamma})=\sqrt{u_a^2(\overline{2\theta_\gamma})+u_b^2(\overline{2\theta_\gamma})}=4.5928\times10^{-2}rad$$

$$u(\overline{\theta_\gamma})=\frac{1}{2}\ u(\overline{2\theta_\gamma})=2.2964\times10^{-2}rad$$

$$\therefore \theta_\gamma\pm u(\theta_\gamma)=(13.85\pm0.02)rad$$

$$\text{而 } R_{H_1}=\frac{1}{\lambda_\gamma}\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=\frac{7.2}{d\sin\theta_\gamma}$$

$$\therefore \ln R_{H_1}=\ln 7.2-\ln d-\ln d\sin\theta_\gamma$$

$$\therefore \frac{u(R_{H_1})}{R_{H_1}}=\sqrt{\left[\frac{\partial\ln d}{\partial d}\ u(d)\right]^2+\left[\frac{\partial\ln\sin\theta_\gamma}{\partial\theta_\gamma}\ u(\theta_\gamma)\right]^2}=\sqrt{\left[\frac{u(d)}{d}\right]^2+\left[\frac{u(\theta_\gamma)}{\tan\theta_\gamma}\right]^2}=6.8063\times10^{-3}$$

$$\therefore\ u(R_{H_1})=R_{H_1}\ \frac{u(R_{H_1})}{R_{H_1}}=1.53110484\times10^4$$

$$R_{H_1}\pm u(R_{H_1})=(2.25\pm0.02)\times10^6m^{-1}$$

测量级次 / 测量次数	-1级		$+1$级		$2\theta_b = \dfrac{1}{2}\left[(\alpha_1-\beta_1)-(\alpha_2-\beta_2)\right]$
	α_1	β_1	α_2	β_2	
1	50.18°	230.2°	33.31°	213.33°	16.783°
2	114.57°	296.0°	98.1°	278.12°	17.292°
3	180.15°	0.14°	163.28°	343.29°	16.767°
4	246.04°	66.02°	229.18°	49.16°	16.767°
5	308.32°	128.31°	291.48°	111.42°	16.775°

(2)蓝光(深绿)

1.

$$\overline{2\theta_b} = \frac{\sum_{k=1}^{5} 2\theta_b}{5} = 0.29455rad$$

$$\text{由 } d\sin\theta = \lambda \text{ 得 } \lambda_b = d\sin\theta_b = d\sin\frac{\overline{2\theta_b}}{2} = 489.854\mu m$$

$$\text{在巴耳末系中对应n取4，有 } \frac{1}{\lambda_b} = R_{H_2}\left(\frac{1}{2^2}-\frac{1}{4^2}\right)$$

$$\therefore\ R_{H_2} = \frac{1}{\lambda_b}\left(\frac{1}{2^2}-\frac{1}{4^2}\right) = 1.0888{\times}10^{7}m^{-1}$$

2. 不确定度的计算

$$u_a(\overline{2\theta_b}) = \sqrt{\frac{\sum_{i=1}^{5}(2\theta_{b_i}-\overline{2\theta_{\gamma]}})^2}{5\times 4}} = 1.03796{\times}10^{-1}rad$$

$$u_b(\overline{2\theta}) = \frac{1}{\sqrt{3}} = 9.6225\times 10^{-3} = 1.679\times 10^{-4}rad$$

$$\therefore \text{不确定度合成为 } u(\overline{2\theta_b}) = \sqrt{u_a^2(\overline{2\theta_b})+u_b^2(\overline{2\theta_b})} = 1.04241{\times}10^{-1}rad$$

$$u(\overline{\theta_b}) = \frac{1}{2}\ u(\overline{2\theta_b}) = 5.21205{\times}10^{-2}rad$$

$$\therefore \theta_b \pm u(\theta_b) = (0.15 \pm 0.05)$$

$$\text{而 } R_{H_2} = \frac{1}{\lambda_b}\left(\frac{1}{2^2}-\frac{1}{4^2}\right) = \frac{5.333}{d\sin\theta_b}$$

$$\therefore \ln R_{H_2} = \ln 5.333 - \ln d - \ln d\sin\theta_b$$

$$\therefore \frac{u(R_{H_2})}{R_{H_2}} = \sqrt{\left[\frac{\partial \ln d}{\partial d}\ u(d)\right]^2 + \left[\frac{\partial \ln\sin\theta_b}{\partial\theta_b}\ u(\theta_b)\right]^2} = \sqrt{\left[\frac{u(d)}{d}\right]^2 + \left[\frac{u(\theta_b)}{\tan\theta_b}\right]^2} = 3.5133{\times}10^{-1}$$

$$\therefore\ u(R_{H_2}) = R_{H_2}\ \frac{u(R_{H_2})}{R_{H_2}} = 3.8251705{\times}10^{6}m^{-1}$$

$$R_{H_2} \pm u(R_{H_2}) = (1.1 \pm 0.4){\times}10^{7}m^{-1}$$

测量次数 \ 测量级次	−1级		+1级		$2\theta_p = \frac{1}{2}[(\alpha_1-\beta_1)-(\alpha_2-\beta_2)]$
	α_1	β_1	α_2	β_2	
1	49.22°	229.25°	34.28°	214.29°	14.917°
2	114.0°	294.04°	99.04°	229.08°	39.933°
3	179.2°	359.2°	164.23°	344.23°	14.950°
4	245.09°	65.05°	230.12°	50.1°	14.933°
5	307.39°	127.37°	292.42°	112.39°	14.958°

(3)紫光(青)

1.

$$\overline{2\theta_p} = \frac{\sum_{k=1}^{5} 2\theta_p}{5} = 0.17399rad$$

$$\text{由 } d\sin\theta = \lambda \text{ 得 } \lambda_p = d\sin\theta_p = d\sin\frac{\overline{2\theta_p}}{2} = 577.89200nm$$

$$\text{在巴耳末系中对应n取5，有 } \frac{1}{\lambda_p} = R_{H_3}\left(\frac{1}{2^2}-\frac{1}{5^2}\right)$$

$$\therefore\ R_{H_3} = \frac{1}{\lambda_p\left(\frac{1}{2^2}-\frac{1}{5^2}\right)} = 8.240129{\times}10^6 m^{-1}$$

2. 不确定度的计算

$$u_a(\overline{2\theta_p}) = \sqrt{\frac{\sum_{i=1}^{5}(2\theta_{p_i}-\overline{2\theta_{\gamma}]})^2}{5\times 4}} = 4.9988{\times}10^0 rad$$

$$u_b(\overline{2\theta}) = \frac{1}{\sqrt{3}} = 9.6225\times 10^{-3} = 1.679\times 10^{-4} rad$$

$$\therefore \text{不确定度合成为 } u(\overline{2\theta_p}) = \sqrt{u_a^2(\overline{2\theta_p})+u_b^2(\overline{2\theta_p})} = 4.9988{\times}10^0 rad$$

$$u(\overline{\theta_p}) = \frac{1}{2}\ u(\overline{2\theta_p}) = 2.4994{\times}10^0 rad$$

$$\therefore \theta_p \pm u(\theta_p) = (0\pm 2)rad$$

$$\text{而 } R_{H_3} = \frac{1}{\lambda_p}\left(\frac{1}{2^2}-\frac{1}{5^2}\right) = \frac{1}{0.21}\ \frac{1}{d\sin\theta_p}$$

$$\therefore \ln R_{H_3} = \ln\frac{1}{0.21} - \ln d - \ln d\sin\theta_p$$

$$\therefore \frac{u(R_{H_3})}{R_{H_3}} = \sqrt{\left[\frac{\partial \ln d}{\partial d}\ u(d)\right]^2 + \left[\frac{\partial \ln\sin\theta_p}{\partial\theta_p}\ u(\theta_p)\right]^2} = \sqrt{\left[\frac{u(d)}{d}\right]^2+\left[\frac{u(\theta_p)}{\tan\theta_p}\right]^2} =$$

$$\therefore\ u(R_{H_3}) = R_{H_3}\ \frac{u(R_{H_3})}{R_{H_3}} = 8.24012918{\times}10^6 m^{-1}$$

$$R_{H_3} \pm u(R_{H_3}) = ((8\pm 117){\times}10^6)m^{-1}$$

3. 加权平均求R_H的最佳值

$$\overline{R_H}=\frac{\frac{R_{H_1}}{u^2R_{H_1}}+\frac{R_{H_2}}{u^2R_{H_2}}+\frac{R_{H_3}}{u^2R_{H_3}}}{\frac{1}{u^2R_{H_1}}+\frac{1}{u^2R_{H_2}}+\frac{1}{u^2R_{H_3}}}=2.2497\times10^6m^{-1}$$

$$u^2(\overline{R_H})=\frac{1}{\frac{1}{u^2R_{H_1}}+\frac{1}{u^2R_{H_2}}+\frac{1}{u^2R_{H_3}}}=m^{-1}$$

$$\therefore\ u(\overline{R_H})=15310.92559275m^{-1}$$

$$\therefore \text{最佳测量值}\ \overline{R_H}\pm u(\overline{R_H})=(2.25\pm0.02)\times10^6m^{-1}$$

3.分别计算钠黄光k=1,2级的角散射率和分辨本领，并由此说明钠黄光双线能否被分开

(1)色分辨本领

$$\because\ N=\frac{D}{d}=0.66$$

$$\therefore\ R=\frac{\lambda}{\delta_\lambda}=kN=\begin{cases}0.66, & k=1\\ 1.32, & k=2\end{cases}$$

(2)角色散率

由前面实验，$\overline{\theta_1}=0.1775°,\overline{\theta_1}=0.3608°$ 由公式$D_\theta=\dfrac{k}{ds\sin\theta}$,求解可得

$$k=1\ \text{时},\ D_{\theta_1}=\frac{1}{d\sin\overline{\theta_1}}=1.69619\times10^0rad/m$$

$$k=2\ \text{时},\ D_{\theta_2}=\frac{2}{d\sin\overline{\theta_2}}=1.69695\times10^0rad/m$$

(3)钠黄光双线

$$\theta_1=\arcsin\frac{\lambda_1}{d}=0.177°$$

$$\theta_2=\arcsin\frac{\lambda_2}{d}=0.178°$$

$$\Delta\theta=\theta_1-\theta_2=0.000°$$

根据谱线的半角宽度计算公式可得

$$\delta_\theta=\arcsin\frac{2\lambda N_0}{Nd}=0.000183°$$

$\because\ \Delta_\theta>\delta_\theta$

$\therefore$本实验可将钠黄光的双线分开。