五、实验数据处理

1.计算光栅常数d，并计算不确定度u(d)

(1)原始数据记录表格

测量级次 \ 测量次数	−1级		+1级		$2\theta_1 = \dfrac{1}{2}\left[(\alpha_1 - \beta_1) - (\alpha_2 - \beta_2)\right]$
	α_1	β_1	α_2	β_2	
1	156.47°	336.5°	136.37°	316.3°	20.250°
2	219.05°	39.04°	198.42°	18.42°	20.375°
3	290.59°	110.55°	270.37°	90.33°	20.367°
4	353.24°	173.23°	333.01°	153.0°	20.383°
5	52.05°	232.08°	31.45°	211.46°	20.350°

−2级		+2级		$2\theta_2 = \frac{1}{2}\left[(\alpha_1 - \beta_1) - (\alpha_2 - \beta_2)\right]$
α_1	β_1	α_2	β_2	
62.35°	242.29°	21.1°	201.12°	41.35°

(2)计算光栅常数d

$$\overline{2\theta_1} = \frac{\sum_{k=1}^{5} 2\theta_1}{5} = 20.345°$$

$$\overline{\theta_1} = \frac{1}{2}\,\overline{2\theta_1} = 0.1775\,rad$$

$$\overline{\theta_2} = \frac{1}{2}\,\overline{2\theta_2} = 0.3608\,rad$$

由 $d\sin\theta = k\lambda$,取 $k = 1$ 得 $d = \dfrac{\lambda}{\sin\theta_1}$

又钠黄光 $\lambda = 589.3\,mm$

$$\therefore\ d = \frac{\lambda}{\sin\theta_1} = 3.337\,\mu m$$

取 $k = 2$ 得 $d' = \dfrac{2\lambda}{\sin\theta_2} = 3.338\,\mu m$

(3)计算不确定度u(d)

1. ±1级d的不确定度

$$u_a(\overline{2\theta}) = \sqrt{\frac{\sum_{i=1}^{5}(2\theta_i - \overline{2\theta_1})^2}{5 \times 4}} = 0.000426\,rad$$

$$u_b(\overline{2\theta}) = \frac{1}{\sqrt{3}} = 0.000168\,rad$$

$$不确定度合成为\ u(\overline{2\theta}) = \sqrt{u_a^2(\overline{2\theta}) + u_b^2(\overline{2\theta})} = 0.000457rad$$

$$u(\overline{\theta_1}) = \frac{1}{2}\ u(\overline{2\theta_1}) = 2.287 \times 10^{-4}rad$$

$$由\ d = \frac{\lambda}{\sin\theta_1}有\ \ln d = \ln\lambda - \ln\sin\theta_1$$

$$相对不确定度\ \frac{u(d)}{d} = \sqrt{\left[\frac{\partial\ln\sin\theta_1}{\partial\theta_1}\ u(\theta_1)\right]^2} = \sqrt{\left[\frac{u(\theta_1)}{\tan\theta_1}\right]^2} = 1.275 \times 10^{-3}rad$$

$$\therefore\ u(d) = d\ \frac{u(d)}{d} = 0.00425\mu m$$

2. ±2级d的不确定度

$$由\ d' = \frac{\lambda}{\sin\theta_2}有\ \ln d' = \ln\lambda - \ln\sin\theta_2$$

$$而\ u(2\theta_2) = u_b(2\theta_2) = \frac{1'}{\sqrt{3}} = 0.00962° = 0.000168rad$$

$$\therefore u(\theta_2) = \frac{1}{2}\ u(2\theta_2) = 0.00481° = 8.395 \times 10^{-5}rad$$

$$\therefore 相对不确定度\ \frac{u(d')}{d'} = \sqrt{\left[\frac{\partial\ln\sin\theta_2}{\partial\theta_2}\ u(\theta_2)\right]^2} = \sqrt{\left[\frac{u(\theta_2)}{\tan\tan_2}\right]^2} = 2.225 \times 10^{-4}$$

$$\therefore\ u(d') = d'\ \frac{u(d')}{d'} = 0.0007428\mu m$$

(4)测量结果加权平均求d最佳值

测量结果：

$$d \pm u(d) = (3.337 \pm 0.004)\mu m$$

$$d' \pm u(d') = (3.338 \pm 0.001)\mu m$$

$$\overline{d} = \frac{\frac{d}{u^2(d)}\ +\ \frac{d'}{u^2(d')}}{\frac{1}{u^2(d)}\ +\ \frac{1}{u^2(d')}} = 3.338\mu m$$

$$u^2(\overline{d}) = \frac{1}{\frac{1}{u^2(d)}\ +\ \frac{1}{u^2(d')}} = 5.354 \times 10^{-7}\mu m^2$$

$$\therefore\ u(\overline{d}) = 7.3 \times 10^{-4}\mu m$$

$$\therefore 光栅常数d的最终表达式为\ \overline{d} \pm u(\overline{d}) = (3.338 \pm 0.001)\mu m$$

2.计算氢原子的里德伯常数$R_H + u(R_H)$;并通过加权平均获得R_H的最佳值$\overline{R_H} \pm u(\overline{R_H})$

巴耳末系：

$$\frac{1}{\lambda} = R_H\ \left(\frac{1}{2^2} - \frac{1}{n^2}\right)(n = 3, 4, 5, 6\ldots)$$

当 $n = 3$ 时，光谱颜色为红光；当 $n = 5$ 时，光谱颜色为蓝光；当 $n = 6$ 时，光谱颜色为紫光；

以下将分别计算红光，蓝光，紫光对应的R_H：

(1)红光

测量级次 / 测量次数	−1级		+1级		$2\theta_\gamma = \dfrac{1}{2}\left[(\alpha_1 - \beta_1) - (\alpha_2 - \beta_2)\right]$
	α_1	β_1	α_2	β_2	
1	53.15°	233.19°	30.33°	210.34°	22.725°
2	117.55°	297.59°	95.17°	275.16°	22.675°
3	183.11°	3.12°	160.29°	340.31°	22.692°
4	249.01°	69.0°	226.19°	46.18°	22.700°
5	311.31°	131.29°	288.49°	108.47°	22.700°

1.

$$\overline{2\theta_\gamma} = \frac{\sum_{k=1}^{5} 2\theta_\gamma}{5} = 0.3962\,rad$$

由 $d\sin\theta = \lambda$ 得 $\lambda_\gamma = d\sin\theta_\gamma = d\sin\dfrac{\overline{2\theta_\gamma}}{2} = 656.904\,nm$

在巴耳末系中对应n取3，有 $\dfrac{1}{\lambda_\gamma} = R_{H_1}\left(\dfrac{1}{2^2} - \dfrac{1}{3^2}\right)$

$$\therefore R_{H_1} = \frac{1}{\lambda_\gamma}\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.096051\times10^7 m^{-1}$$

2. 不确定度的计算

$$u_a(\overline{2\theta_\gamma}) = \sqrt{\frac{\sum_{i=1}^{5}\left(2\theta_{\gamma_i} - \overline{2\theta_\gamma]}\right)^2}{5\times4}} = 1.4101\times10^{-4}\,rad$$

$$u_b(\overline{2\theta}) = \frac{1}{\sqrt{3}} = 9.6225\times10^{-3} = 1.679\times10^{-4}\,rad$$

$\therefore$ 不确定度合成为 $u(\overline{2\theta_\gamma}) = \sqrt{u_a^2(\overline{2\theta_\gamma}) + u_b^2(\overline{2\theta_\gamma})} = 2.1929\times10^{-4}\,rad$

$$u(\overline{\theta_\gamma}) = \frac{1}{2}\,u(\overline{2\theta_\gamma}) = 1.0965\times10^{-4}\,rad$$

$$\therefore \theta_\gamma \pm u(\theta_\gamma) = (0.1981 \pm 0.0001)\,rad$$

而 $R_{H_1} = \dfrac{1}{\lambda_\gamma}\left(\dfrac{1}{2^2} - \dfrac{1}{3^2}\right) = \dfrac{7.2}{d\sin\theta_\gamma}$

$$\therefore \ln R_{H_1} = \ln 7.2 - \ln d - \ln d\sin\theta_\gamma$$

$$\therefore \frac{u(R_{H_1})}{R_{H_1}} = \sqrt{\left[\frac{\partial\ln d}{\partial d}\,u(d)\right]^2 + \left[\frac{\partial\ln\sin\theta_\gamma}{\partial\theta_\gamma}\,u(\theta_\gamma)\right]^2} = \sqrt{\left[\frac{u(d)}{d}\right]^2 + \left[\frac{u(\theta_\gamma)}{\tan\theta_\gamma}\right]^2} = 5.8863\times10^{-4}$$

$$\therefore u(R_{H_1}) = R_{H_1}\,\frac{u(R_{H_1})}{R_{H_1}} = 6.45167818\times10^3$$

$$R_{H_1} \pm u(R_{H_1}) = (1.0961 \pm 0.0006)\times10^7 m^{-1}$$

测量级次 测量次数	−1级		+1级		$2\theta_b = \frac{1}{2}\left[(\alpha_1 - \beta_1) - (\alpha_2 - \beta_2)\right]$
	α_1	β_1	α_2	β_2	
1	50.18°	230.2°	33.31°	213.33°	16.783°
2	114.57°	295.0°	98.1°	278.12°	16.792°
3	180.15°	0.14°	163.28°	343.29°	16.767°
4	246.04°	66.02°	229.18°	49.16°	16.767°
5	308.32°	128.31°	291.48°	111.42°	16.775°

(2)蓝光(深绿)

1.

$$\overline{2\theta_b} = \frac{\sum_{k=1}^{5} 2\theta_b}{5} = 0.29281 rad$$

由 $d\sin\theta = \lambda$ 得 $\lambda_b = d\sin\theta_b = d\sin\frac{\overline{2\theta_b}}{2} = 486.973 \mu m$

在巴耳末系中对应n取4，有 $\frac{1}{\lambda_b} = R_{H_2}\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$

$$\therefore R_{H_2} = \frac{1}{\lambda_b}\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = 1.0952 \times 10^7 m^{-1}$$

2. 不确定度的计算

$$u_a(\overline{2\theta_b}) = \sqrt{\frac{\sum_{i=1}^{5}(2\theta_{b_i} - \overline{2\theta_{\gamma]}})^2}{5 \times 4}} = 8.48078 \times 10^{-5} rad$$

$$u_b(\overline{2\theta}) = \frac{1}{\sqrt{3}} = 9.6225 \times 10^{-3} = 1.679 \times 10^{-4} rad$$

$\therefore$ 不确定度合成为 $u(\overline{2\theta_b}) = \sqrt{u_a^2(\overline{2\theta_b}) + u_b^2(\overline{2\theta_b})} = 1.88143 \times 10^{-4} rad$

$$u(\overline{\theta_b}) = \frac{1}{2} u(\overline{2\theta_b}) = 9.40713 \times 10^{-5} rad$$

$$\therefore \theta_b \pm u(\theta_b) = (0.14640 \pm 0.00009)$$

而 $R_{H_2} = \frac{1}{\lambda_b}\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{5.333}{d\sin\theta_b}$

$$\therefore \ln R_{H_2} = \ln 5.333 - \ln d - \ln d\sin\theta_b$$

$$\therefore \frac{u(R_{H_2})}{R_{H_2}} = \sqrt{\left[\frac{\partial\ln d}{\partial d} u(d)\right]^2 + \left[\frac{\partial\ln\sin\theta_b}{\partial\theta_b} u(\theta_b)\right]^2} = \sqrt{\left[\frac{u(d)}{d}\right]^2 + \left[\frac{u(\theta_b)}{\tan\theta_b}\right]^2} = 6.7456 \times 10^{-4}$$

$$\therefore u(R_{H_2}) = R_{H_2} \frac{u(R_{H_2})}{R_{H_2}} = 7.3877809 \times 10^3 m^{-1}$$

$$R_{H_2} \pm u(R_{H_2}) = (1.0952 \pm 0.0007) \times 10^7 m^{-1}$$

测量级次 测量次数	−1级		+1级		$2\theta_p = \dfrac{1}{2}\left[(\alpha_1 - \beta_1) - (\alpha_2 - \beta_2)\right]$
	α_1	β_1	α_2	β_2	
1	49.22°	229.25°	34.28°	214.29°	14.917°
2	114.0°	294.04°	99.04°	279.08°	14.933°
3	179.2°	359.2°	164.23°	344.23°	14.950°
4	245.09°	65.05°	230.12°	50.1°	14.933°
5	307.39°	127.37°	292.42°	112.39°	14.958°

(3)紫光(青)

1.

$$\overline{2\theta_p} = \frac{\sum_{k=1}^{5} 2\theta_p}{5} = 0.13036 rad$$

由 $d\sin\theta = \lambda$ 得 $\lambda_p = d\sin\theta_p = d\sin\dfrac{\overline{2\theta_p}}{2} = 433.93306 nm$

在巴耳末系中对应n取5，有 $\dfrac{1}{\lambda_p} = R_{H_3}\left(\dfrac{1}{2^2} - \dfrac{1}{5^2}\right)$

$$\therefore\ R_{H_3} = \frac{1}{\lambda_p}\left(\frac{1}{2^2} - \frac{1}{5^2}\right) = 1.097382\times10^7 m^{-1}$$

2. 不确定度的计算

$$u_a(\overline{2\theta_p}) = \sqrt{\frac{\sum_{i=1}^{5}(2\theta_{p_i} - \overline{2\theta_{\gamma]}})^2}{5\times4}} = 1.2680\times10^{-4} rad$$

$$u_b(\overline{2\theta}) = \frac{1}{\sqrt{3}} = 9.6225\times10^{-3} = 1.679\times10^{-4} rad$$

$\therefore$ 不确定度合成为 $u(\overline{2\theta_p}) = \sqrt{u_a^2(\overline{2\theta_p}) + u_b^2(\overline{2\theta_p})} = 2.1043\times10^{-4} rad$

$$u(\overline{\theta_p}) = \frac{1}{2}\,u(\overline{2\theta_p}) = 1.0522\times10^{-4} rad$$

$$\therefore \theta_p \pm u(\theta_p) = (0.1304 \pm 0.0001) rad$$

而 $R_{H_3} = \dfrac{1}{\lambda_p}\left(\dfrac{1}{2^2} - \dfrac{1}{5^2}\right) = \dfrac{1}{0.21}\,\dfrac{1}{d\sin\theta_p}$

$$\therefore \ln R_{H_3} = \ln\frac{1}{0.21} - \ln d - \ln d\sin\theta_p$$

$$\therefore \frac{u(R_{H_3})}{R_{H_3}} = \sqrt{\left[\frac{\partial\ln d}{\partial d}\,u(d)\right]^2 + \left[\frac{\partial\ln\sin\theta_p}{\partial\theta_p}\,u(\theta_p)\right]^2} = \sqrt{\left[\frac{u(d)}{d}\right]^2 + \left[\frac{u(\theta_p)}{\tan\theta_p}\right]^2} = 8.3194\times10^{-4}$$

$$\therefore\ u(R_{H_3}) = R_{H_3}\,\frac{u(R_{H_3})}{R_{H_3}} = 1.09738234\times10^7 m^{-1}$$

$$R_{H_3} \pm u(R_{H_3}) = ((1.0974 \pm 0.0009)\times10^7) m^{-1}$$

3. 加权平均求R_H的最佳值

$$\overline{R_H} = \frac{\frac{R_{H_1}}{u^2 R_{H_1}} + \frac{R_{H_2}}{u^2 R_{H_2}} + \frac{R_{H_3}}{u^2 R_{H_3}}}{\frac{1}{u^2 R_{H_1}} + \frac{1}{u^2 R_{H_2}} + \frac{1}{u^2 R_{H_3}}} = 1.0961 \times 10^7 m^{-1}$$

$$u^2(\overline{R_H}) = \frac{1}{\frac{1}{u^2 R_{H_1}} + \frac{1}{u^2 R_{H_2}} + \frac{1}{u^2 R_{H_3}}} = m^{-1}$$

$$\therefore \ u(\overline{R_H}) = 4289.66027208 m^{-1}$$

$$\therefore 最佳测量值 \ \overline{R_H} \pm u(\overline{R_H}) = (1.0961 \pm 0.0004) \times 10^7 m^{-1}$$

3.分别计算钠黄光k=1,2级的角散射率和分辨本领，并由此说明钠黄光双线能否被分开

(1)色分辨本领

$$\because \ N = \frac{D}{d} = 659.05$$

$$\therefore \ R = \frac{\lambda}{\delta_\lambda} = kN = \begin{cases} 659.05, & k = 1 \\ 1318.10, & k = 2 \end{cases}$$

(2)角色散率

由前面实验，$\overline{\theta_1} = 0.1775 rad, \overline{\theta_1} = 0.3608 rad$ 由公式$D_\theta = \dfrac{k}{ds \sin \theta}$,求解可得

$$k = 1 \ 时, D_{\theta_1} = \frac{1}{d \sin \overline{\theta_1}} = 1.69619 \times 10^0 rad/m$$

$$k = 2 \ 时, D_{\theta_2} = \frac{2}{d \sin \overline{\theta_2}} = 1.69695 \times 10^0 rad/m$$

(3)钠黄光双线

$$\theta_1 = \arcsin \frac{\lambda_1}{d} = 0.177 rad$$

$$\theta_2 = \arcsin \frac{\lambda_2}{d} = 0.178 rad$$

$$\Delta \theta = \theta_1 - \theta_2 = 0.000183°$$

根据谱线的半角宽度计算公式可得

$$\delta_\theta = \arcsin \frac{2\lambda N_0}{Nd} = 0.000183°$$

$\because \Delta_\theta > \delta_\theta$

$\therefore$本实验可将钠黄光的双线分开。