

五、实验数据处理

1. 计算光栅常数d，并计算不确定度u(d)

(1) 原始数据记录表格

| 测量级次 测量次数 | -1级 | | +1级 | | $2\theta_1 = \frac{1}{2}[(\alpha_1 - \beta_1) - (\alpha_2 - \beta_2)]$ |
|--------------|------------|-----------|------------|-----------|--|
| | α_1 | β_1 | α_2 | β_2 | |
| 1 | 156.47° | 336.5° | 136.37° | 316.3° | 20.250° |
| 2 | 219.05° | 39.04° | 198.42° | 18.42° | 20.375° |
| 3 | 290.59° | 110.55° | 270.37° | 90.33° | 20.367° |
| 4 | 353.24° | 173.23° | 333.01° | 153.0° | 20.383° |
| 5 | 52.05° | 232.08° | 31.45° | 211.46° | 20.350° |

| -2级 | | +2级 | | $2\theta_2 = \frac{1}{2}[(\alpha_1 - \beta_1) - (\alpha_2 - \beta_2)]$ |
|------------|-----------|------------|-----------|--|
| α_1 | β_1 | α_2 | β_2 | |
| 62.35° | 242.29° | 21.1° | 201.12° | 41.35° |

(2) 计算光栅常数d

$$\overline{2\theta_1} = \frac{\sum_{k=1}^5 2\theta_1}{5} = 20.345^\circ$$

$$\overline{\theta_1} = \frac{1}{2} \overline{2\theta_1} = 0.1775 \text{ rad}$$

$$\overline{\theta_2} = \frac{1}{2} \overline{2\theta_2} = 0.3608 \text{ rad}$$

$$\text{由 } d \sin \theta = k\lambda, \text{ 取 } k = 1 \text{ 得 } d = \frac{\lambda}{\sin \theta_1}$$

又钠黄光 $\lambda = 589.3 \text{ nm}$

$$\therefore d = \frac{\lambda}{\sin \theta_1} = 3.337 \mu\text{m}$$

$$\text{取 } k = 2 \text{ 得 } d' = \frac{2\lambda}{\sin \theta_2} = 3.338 \mu\text{m}$$

(3) 计算不确定度u(d)

1. ± 1 级d的不确定度

$$u_a(\overline{2\theta}) = \sqrt{\frac{\sum_{i=1}^5 (2\theta_i - \overline{2\theta_1})^2}{5 \times 4}} = 0.000426 \text{ rad}$$

$$u_b(\overline{2\theta}) = \frac{1}{\sqrt{3}} = 0.000168 \text{ rad}$$

$$\text{不确定度合成为 } u(\overline{2\theta}) = \sqrt{u_a^2(\overline{2\theta}) + u_b^2(\overline{2\theta})} = 0.000457 \text{ rad}$$

$$u(\overline{\theta_1}) = \frac{1}{2} u(\overline{2\theta_1}) = 2.287 \times 10^{-4} \text{ rad}$$

$$\text{由 } d = \frac{\lambda}{\sin \theta_1} \text{ 有 } \ln d = \ln \lambda - \ln \sin \theta_1$$

$$\text{相对不确定度 } \frac{u(d)}{d} = \sqrt{\left[\frac{\partial \ln \sin \theta_1}{\partial \theta_1} u(\theta_1) \right]^2} = \sqrt{\left[\frac{u(\theta_1)}{\tan \theta_1} \right]^2} = 1.275 \times 10^{-3} \text{ rad}$$

$$\therefore u(d) = d \frac{u(d)}{d} = 0.00425 \mu\text{m}$$

2. ± 2 级d的不确定度

$$\text{由 } d' = \frac{\lambda}{\sin \theta_2} \text{ 有 } \ln d' = \ln \lambda - \ln \sin \theta_2$$

$$\text{而 } u(2\theta_2) = u_b(2\theta_2) = \frac{1'}{\sqrt{3}} = 0.00962^\circ = 0.000168 \text{ rad}$$

$$\therefore u(\theta_2) = \frac{1}{2} u(2\theta_2) = 0.00481^\circ = 8.395 \times 10^{-5} \text{ rad}$$

$$\therefore \text{相对不确定度 } \frac{u(d')}{d'} = \sqrt{\left[\frac{\partial \ln \sin \theta_2}{\partial \theta_2} u(\theta_2) \right]^2} = \sqrt{\left[\frac{u(\theta_2)}{\tan \theta_2} \right]^2} = 2.225 \times 10^{-4}$$

$$\therefore u(d') = d' \frac{u(d')}{d'} = 0.0007428 \mu\text{m}$$

(4) 测量结果加权平均求d最佳值

测量结果:

$$d \pm u(d) = ((3.337 \pm 0.004)) \mu\text{m}$$

$$d' \pm u(d') = ((3.3382 \pm 0.0007)) \mu\text{m}$$

$$\bar{d} = \frac{\frac{d}{u^2(d)} + \frac{d'}{u^2(d')}}{\frac{1}{u^2(d)} + \frac{1}{u^2(d')}} = 3.338 \mu\text{m}$$

$$u^2(\bar{d}) = \frac{1}{\frac{1}{u^2(d)} + \frac{1}{u^2(d')}} = 5.354 \times 10^{-7} \mu\text{m}^2$$

$$\therefore u(\bar{d}) = 7.3 \times 10^{-4} \mu\text{m}$$

$$\therefore \text{光栅常数d的最终表达式为 } \bar{d} \pm u(\bar{d}) = ((3.3381 \pm 0.0007)) \mu\text{m}$$

2. 计算氢原子的里德伯常数 $R_H + u(R_H)$; 并通过加权平均获得 R_H 的最佳值 $\overline{R_H} \pm u(\overline{R_H})$

巴耳末系:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right) (n = 3, 4, 5, 6 \dots)$$

当 $n = 3$ 时, 光谱颜色为红光; 当 $n = 5$ 时, 光谱颜色为蓝光; 当 $n = 6$ 时, 光谱颜色为紫光;

以下将分别计算红光, 蓝光, 紫光对应的 R_H :

(1)红光

| 测量级次 测量次数 | -1级 | | +1级 | | $2\theta_\gamma = \frac{1}{2}[(\alpha_1 - \beta_1) - (\alpha_2 - \beta_2)]$ |
|--------------|------------|-----------|------------|-----------|---|
| | α_1 | β_1 | α_2 | β_2 | |
| 1 | 53.15° | 233.19° | 30.33° | 210.34° | 22.725° |
| 2 | 117.55° | 297.59° | 45.17° | 275.16° | 47.675° |
| 3 | 183.11° | 3.12° | 160.29° | 340.31° | 22.692° |
| 4 | 249.01° | 69.0° | 226.19° | 46.18° | 22.700° |
| 5 | 311.31° | 131.29° | 288.49° | 108.47° | 22.700° |

1.

$$\overline{2\theta_\gamma} = \frac{\sum_{k=1}^5 2\theta_\gamma}{5} = 0.0084 \text{ rad}$$

$$\text{由 } d \sin \theta = \lambda \text{ 得 } \lambda_\gamma = d \sin \theta_\gamma = d \sin \frac{\overline{2\theta_\gamma}}{2} = 799.039 \text{ nm}$$

$$\text{在巴耳末系中对应 } n \text{ 取 } 3, \text{ 有 } \frac{1}{\lambda_\gamma} = R_{H_1} \left(\frac{1}{2^2} - \frac{1}{3^2} \right)$$

$$\therefore R_{H_1} = \frac{1}{\lambda_\gamma} \left(\frac{1}{2^2} - \frac{1}{3^2} \right) = 9010828.698089 \text{ m}^{-1}$$

2. 不确定度的计算

$$u_a(\overline{2\theta_\gamma}) = \sqrt{\frac{\sum_{i=1}^5 (2\theta_{\gamma_i} - \overline{2\theta_\gamma})^2}{5 \times 4}} = 4.5928 \times 10^{-2} \text{ rad}$$

$$u_b(\overline{2\theta}) = \frac{1}{\sqrt{3}} = 9.6225 \times 10^{-3} = 1.679 \times 10^{-4} \text{ rad}$$

$$\therefore \text{不确定度合成为 } u(\overline{2\theta_\gamma}) = \sqrt{u_a^2(\overline{2\theta_\gamma}) + u_b^2(\overline{2\theta_\gamma})} = 4.5928 \times 10^{-2} \text{ rad}$$

$$u(\overline{\theta_\gamma}) = \frac{1}{2} u(\overline{2\theta_\gamma}) = 2.2964 \times 10^{-2} \text{ rad}$$

$$\therefore \theta_\gamma \pm u(\theta_\gamma) = () \text{ rad}$$

$$\text{而 } R_{H_1} = \frac{1}{\lambda_\gamma} \left(\frac{1}{2^2} - \frac{1}{3^2} \right) = \frac{7.2}{d \sin \theta_\gamma}$$

$$\therefore \ln R_{H_1} = \ln 7.2 - \ln d - \ln d \sin \theta_\gamma$$

$$\therefore \frac{u(R_{H_1})}{R_{H_1}} = \sqrt{\left[\frac{\partial \ln d}{\partial d} u(d) \right]^2 + \left[\frac{\partial \ln \sin \theta_\gamma}{\partial \theta_\gamma} u(\theta_\gamma) \right]^2} = \sqrt{\left[\frac{u(d)}{d} \right]^2 + \left[\frac{u(\theta_\gamma)}{\tan \theta_\gamma} \right]^2} = 9.3148 \times 10^{-2}$$

$$\therefore u(R_{H_1}) = R_{H_1} \frac{u(R_{H_1})}{R_{H_1}} = 8.39337918 \times 10^5$$

$$R_{H_1} \pm u(R_{H_1}) = ((9.0 \pm 0.8) \times 10^6) \text{ m}^{-1}$$

| 测量级次 测量次数 | -1级 | | +1级 | | $2\theta_b = \frac{1}{2}[(\alpha_1 - \beta_1) - (\alpha_2 - \beta_2)]$ |
|--------------|------------|-----------|------------|-----------|--|
| | α_1 | β_1 | α_2 | β_2 | |
| 1 | 50.18° | 230.2° | 33.31° | 213.33° | 16.783° |
| 2 | 114.57° | 296.0° | 98.1° | 278.12° | 17.292° |
| 3 | 180.15° | 0.14° | 163.28° | 343.29° | 16.767° |
| 4 | 246.04° | 66.02° | 229.18° | 49.16° | 16.767° |
| 5 | 308.32° | 128.31° | 291.48° | 111.42° | 16.775° |

(2)蓝光(深绿)

1.

$$\overline{2\theta_b} = \frac{\sum_{k=1}^5 2\theta_b}{5} = 0.29455 \text{ rad}$$

$$\text{由 } d \sin \theta = \lambda \text{ 得 } \lambda_b = d \sin \theta_b = d \sin \frac{\overline{2\theta_b}}{2} = 489.854 \mu\text{m}$$

$$\text{在巴耳末系中对应n取4, 有 } \frac{1}{\lambda_b} = R_{H_2} \left(\frac{1}{2^2} - \frac{1}{4^2} \right)$$

$$\therefore R_{H_2} = \frac{1}{\lambda_b} \left(\frac{1}{2^2} - \frac{1}{4^2} \right) = 1.0888 \times 10^7 \text{ m}^{-1}$$

2. 不确定度的计算

$$u_a(\overline{2\theta_b}) = \sqrt{\frac{\sum_{i=1}^5 (2\theta_{b_i} - \overline{2\theta_b})^2}{5 \times 4}} = 1.03796 \times 10^{-1} \text{ rad}$$

$$u_b(\overline{2\theta}) = \frac{1}{\sqrt{3}} = 9.6225 \times 10^{-3} = 1.679 \times 10^{-4} \text{ rad}$$

$$\therefore \text{不确定度合成为 } u(\overline{2\theta_b}) = \sqrt{u_a^2(\overline{2\theta_b}) + u_b^2(\overline{2\theta_b})} = 1.04241 \times 10^{-1} \text{ rad}$$

$$u(\overline{\theta_b}) = \frac{1}{2} u(\overline{2\theta_b}) = 5.21205 \times 10^{-2} \text{ rad}$$

$$\therefore \theta_b \pm u(\theta_b) =$$

$$\text{而 } R_{H_2} = \frac{1}{\lambda_b} \left(\frac{1}{2^2} - \frac{1}{4^2} \right) = \frac{5.333}{d \sin \theta_b}$$

$$\therefore \ln R_{H_2} = \ln 5.333 - \ln d - \ln d \sin \theta_b$$

$$\therefore \frac{u(R_{H_2})}{R_{H_2}} = \sqrt{\left[\frac{\partial \ln d}{\partial d} u(d) \right]^2 + \left[\frac{\partial \ln \sin \theta_b}{\partial \theta_b} u(\theta_b) \right]^2} = \sqrt{\left[\frac{u(d)}{d} \right]^2 + \left[\frac{u(\theta_b)}{\tan \theta_b} \right]^2} = 3.5133 \times 10^{-1}$$

$$\therefore u(R_{H_2}) = R_{H_2} \frac{u(R_{H_2})}{R_{H_2}} = 3.8251705 \times 10^6 \text{ m}^{-1}$$

$$R_{H_2} \pm u(R_{H_2}) = () \text{ m}^{-1}$$

| 测量级次 测量次数 | -1级 | | +1级 | | $2\theta_p = \frac{1}{2}[(\alpha_1 - \beta_1) - (\alpha_2 - \beta_2)]$ |
|--------------|------------|-----------|------------|-----------|--|
| | α_1 | β_1 | α_2 | β_2 | |
| 1 | 49.22° | 229.25° | 34.28° | 214.29° | 14.917° |
| 2 | 114.0° | 294.04° | 99.04° | 229.08° | 39.933° |
| 3 | 179.2° | 359.2° | 164.23° | 344.23° | 14.950° |
| 4 | 245.09° | 65.05° | 230.12° | 50.1° | 14.933° |
| 5 | 307.39° | 127.37° | 292.42° | 112.39° | 14.958° |

(3)紫光(青)

1.

$$\overline{2\theta_p} = \frac{\sum_{k=1}^5 2\theta_p}{5} = 0.17399 \text{ rad}$$

$$\text{由 } d \sin \theta = \lambda \text{ 得 } \lambda_p = d \sin \theta_p = d \sin \frac{\overline{2\theta_p}}{2} = 577.89200 \text{ nm}$$

$$\text{在巴耳末系中对应 } n \text{ 取 } 5, \text{ 有 } \frac{1}{\lambda_p} = R_{H_3} \left(\frac{1}{2^2} - \frac{1}{5^2} \right)$$

$$\therefore R_{H_3} = \frac{1}{\lambda_p} \left(\frac{1}{2^2} - \frac{1}{5^2} \right) = 8.240129 \times 10^6 \text{ m}^{-1}$$

2. 不确定度的计算

$$u_a(\overline{2\theta_p}) = \sqrt{\frac{\sum_{i=1}^5 (2\theta_{p_i} - \overline{2\theta_p})^2}{5 \times 4}} = 4.9988 \times 10^0 \text{ rad}$$

$$u_b(\overline{2\theta}) = \frac{1}{\sqrt{3}} = 9.6225 \times 10^{-3} = 1.679 \times 10^{-4} \text{ rad}$$

$$\therefore \text{不确定度合成 } u(\overline{2\theta_p}) = \sqrt{u_a^2(\overline{2\theta_p}) + u_b^2(\overline{2\theta_p})} = 4.9988 \times 10^0 \text{ rad}$$

$$u(\overline{\theta_p}) = \frac{1}{2} u(\overline{2\theta_p}) = 2.4994 \times 10^0 \text{ rad}$$

$$\therefore \theta_p \pm u(\theta_p) = () \text{ rad}$$

$$\text{而 } R_{H_3} = \frac{1}{\lambda_p} \left(\frac{1}{2^2} - \frac{1}{5^2} \right) = \frac{1}{0.21} \frac{1}{d \sin \theta_p}$$

$$\therefore \ln R_{H_3} = \ln \frac{1}{0.21} - \ln d - \ln d \sin \theta_p$$

$$\therefore \frac{u(R_{H_3})}{R_{H_3}} = \sqrt{\left[\frac{\partial \ln d}{\partial d} u(d) \right]^2 + \left[\frac{\partial \ln \sin \theta_p}{\partial \theta_p} u(\theta_p) \right]^2} = \sqrt{\left[\frac{u(d)}{d} \right]^2 + \left[\frac{u(\theta_p)}{\tan \theta_p} \right]^2} =$$

$$\therefore u(R_{H_3}) = R_{H_3} \frac{u(R_{H_3})}{R_{H_3}} = 8.24012918 \times 10^6 \text{ m}^{-1}$$

$$R_{H_3} \pm u(R_{H_3}) = () \text{ m}^{-1}$$

3. 加权平均求 R_H 的最佳值

$$\overline{R_H} = \frac{\frac{R_{H1}}{u^2 R_{H1}} + \frac{R_{H2}}{u^2 R_{H2}} + \frac{R_{H3}}{u^2 R_{H3}}}{\frac{1}{u^2 R_{H1}} + \frac{1}{u^2 R_{H2}} + \frac{1}{u^2 R_{H3}}} = 9.0970 \times 10^6 m^{-1}$$

$$u^2(\overline{R_H}) = \frac{1}{\frac{1}{u^2 R_{H1}} + \frac{1}{u^2 R_{H2}} + \frac{1}{u^2 R_{H3}}} = m^{-1}$$

$$\therefore u(\overline{R_H}) = 819813.46864194 m^{-1}$$

$$\therefore \text{最佳测量值 } \overline{R_H} \pm u(\overline{R_H}) = () m^{-1}$$

3. 分别计算钠黄光 $k=1,2$ 级的角散射率和分辨本领，并由此说明钠黄光双线能否被分开

(1) 色分辨本领

$$\therefore N = \frac{D}{d} = 0.66$$

$$\therefore R = \frac{\lambda}{\delta\lambda} = kN = \begin{cases} 0.66, & k=1 \\ 1.32, & k=2 \end{cases}$$

(2) 角色散率

由前面实验， $\overline{\theta_1} = 0.1775^\circ$, $\overline{\theta_2} = 0.3608^\circ$ 由公式 $D_\theta = \frac{k}{ds \sin \theta}$, 求解可得

$$k=1 \text{ 时, } D_{\theta_1} = \frac{1}{d \sin \overline{\theta_1}} = 1.69619 \times 10^0 \text{ rad/m}$$

$$k=2 \text{ 时, } D_{\theta_2} = \frac{2}{d \sin \overline{\theta_2}} = 1.69695 \times 10^0 \text{ rad/m}$$

(3) 钠黄光双线

$$\theta_1 = \arcsin \frac{\lambda_1}{d} = 0.177^\circ$$

$$\theta_2 = \arcsin \frac{\lambda_2}{d} = 0.178^\circ$$

$$\Delta\theta = \theta_1 - \theta_2 = 0.000^\circ$$

根据谱线的半角宽度计算公式可得

$$\delta_\theta = \arcsin \frac{2\lambda N_0}{Nd} = 0.000183^\circ$$

$$\therefore \Delta\theta > \delta_\theta$$

\therefore 本实验可将钠黄光的双线分开。