HAR

keywords # # # # # #

HAR-type volatility models

1. HAR-type volatility models [@wenForecastingVolatilityCrude2016]

2. “ ” ETF

   - ETF Exchange Traded Funds
   -
   -

3. Realized Volatility

   Integrated Volatility,

$$IV = \int_{t-1}^{t} \sigma_s^2 dx$$

σ Realized Volatility Integrated Volatility

$$RV_t^d = \sum_{i=1}^{M} r_{t,i}^2$$

M $r_{t,i} = lnP_{t,0} - lnP_{t-1,N}$ $P_{t,i}$ t i

5 240/5=48 [1]

RV IV

$$RV_t = IV_t + \eta_t, \eta_t \sim MN(0, 2\Delta\$IQ_t)$$

η_t IV RV RV MN(Mixed Normal)

1.
2.
3. [@wenForecastingVolatilityCrude2016] 5 Volatility components
4. HAR-type volatility model /
5. [@wenForecastingVolatilityCrude2016] rolling window prediction method 16 HAR-type volatility model 3 R2

baostock “ ” “ ” “ ” “ ”

$P_{t,i}$ Table index “ ” column

t t-1

Δ $M = 240/\Delta + 1$ $N = M - 1$

[@wenForecastingVolatilityCrude2016]

- overnight return variance

$$r_{(t,0)} = r_{(t,n)} = 100(lnP_{t,0} - lnP_{t-1,N})$$

- i^{th} intraday return at day t

$$r_{t,i} = 100(lnP_{t,i} - lnP_{t,i-1})), i = 1, \dots, N$$

Volatility components

[@wenForecastingVolatilityCrude2016] 5 Volatility components

1. daily realized volatility

$$RV_t^d = RV_t^{d0} + r_{(t,0)}^2$$
$$RV_t^{d0} = \sum_{i=1}^{N} r_{t,i}^2$$

2. daily discontinuous jump variation daily continuous sample path variation

$$J_t^d = I(Z_t > \phi(\alpha))(RV_t - RBV_t)$$
$$C_t^d = I(Z_t \leq \phi(\alpha))RV_t + I(Z_t > \phi(\alpha))RBV_t$$

   $\phi(\alpha)$ is the appropriate critical value from the standard normal distribution, $\alpha = 0.99$.

3. Realized semivariance

$$RSV_t^{d+} = \sum_{j=1}^{M} \{r_{t,j} \geq 0\} r_{t,j}^2$$
$$RSV_t^{d-} = \sum_{j=1}^{M} \{r_{t,j} < 0\} r_{t,j}^2$$

4. Signed jump

$$SJ_t^d = RSV_t^{d+} - RSV_t^{d-}$$

5. Signed semi-jump

$$SSJ_t^{d+} = I\{SJ_t^d \geq 0\} SJ_t^d$$
$$SSJ_t^{d-} = I\{SJ_t^d < 0\} SJ_t^d$$

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$$RV_t^d = \sqrt{RV_t^d}$$

1. HAR-RV model

$$RV_{t+1}^d = c + \alpha_1 RV_t^d + \alpha_2 RV_t^w + \alpha_3 RV_t^m + \epsilon_{t+1}$$

2. HAR-RV-J model

$$RV_{t+1}^d = c + \alpha_1 RV_t^d + \alpha_2 RV_t^w + \alpha_3 RV_t^m + \beta_1 J_t^d + \epsilon_{t+1}$$

3. HAR-CJ model

$$RV_{t+1}^{d} = c + \alpha_1 RV_t^d + \alpha_2 RV_t^w + \alpha_3 RV_t^m + \beta_1 J_t^d + \beta_2 J_t^w + \beta_3 J_t^m + \epsilon_{t+1}$$

4. HAR-RSV model

$$RV_{t+1}^{d} = c + \alpha_1 RSV_t^{d+} + \alpha_2 RSV_t^{w+} + \alpha_3 RSV_t^{m+} + \beta_1 RSV_t^{d-} + \beta_2 RSV_t^{w-} + \beta_3 RSV_t^{m-} + \epsilon_{t+1}$$

5. HAR-RSV-J model

$$RV_{t+1}^{d} = c + \alpha_1 RSV_t^{d+} + \alpha_2 RSV_t^{w+} + \alpha_3 RSV_t^{m+} + \beta_1 RSV_t^{d-} + \beta_2 RSV_t^{w-} + \beta_3 RSV_t^{m-} + \phi_1 J_t^d + \epsilon_{t+1}$$

6. HAR-RV-SJ model

$$RV_{t+1}^{d} = c + \alpha_1 RV_t^d + \alpha_2 RV_t^w + \alpha_3 RV_t^m + \beta_1 SJ_t^d + \epsilon_{t+1}$$

7. HAR-RV-SSJ(1) model

$$RV_{t+1}^{d} = c + \alpha_1 RV_t^d + \alpha_2 RV_t^w + \alpha_3 RV_t^m + \beta_1 SSJ_t^{d+} + \phi_1 SSJ_t^{d-} + \epsilon_{t+1}$$

8. HAR-RV-SSJ(2) model

$$\begin{aligned} RV_{t+1}^{d} &= c + \alpha_1 RV_t^d + \alpha_2 RV_t^w + \alpha_3 RV_t^m + \beta_1 SSJ_t^{d+} + \beta_2 SSJ_t^{w+} + \beta_3 SSJ_t^{m+} \\ &\quad + \phi_1 SSJ_t^{d-} + \phi_2 SSJ_t^{w-} + \phi_3 SSJ_t^{m-} + \epsilon_{t+1} \end{aligned}$$

scikit-learn 3

scikit-learn.linear_model[2] Lasso

1.

$$J(\beta) = \sum (y - X\beta)^2$$

2. Lasso L1 $ESS(\beta)$ $\lambda l_1(\beta)$

$$\begin{aligned} J(\beta) &= \sum (y - X\beta)^2 + \lambda \|\beta\|_1 \\ &= \sum (y - X\beta)^2 + \sum \lambda |\beta| \\ &= ESS(\beta) + \lambda l_1(\beta) \end{aligned}$$

3. L2

$$\begin{aligned} J(\beta) &= \sum (y - X\beta)^2 + \lambda \|\beta\|_2^2 \\ &= \sum (y - X\beta)^2 + \sum \lambda \beta^2 \end{aligned}$$

rolling window prediction method

L realized volatility

$\{X_{t-L+1}, \dots, X_t\}$ HAR X_t $\hat{RV}^d_{t+1}$

t

$$L \leq t \leq N-1, t \in Z^+$$

N

N $\{\hat{RV}^d_{L+1}, \dots, \hat{RV}^d_N\}$ N-L

realized volatility R2

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1. 9:30-11:30 13:00-15:00
2. https://scikit-learn.org/stable/modules/linear_model.html