$$\begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -0.5 \\ 5 \end{bmatrix}$$

3x2 2x2

$$A \quad x \quad = b$$

Optimization concept $\quad x = (A^T \cdot A)^{-1} A^T \cdot b$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \left(\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 3 & 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -0.5 \\ 5 \end{bmatrix}$$

2x3 3x2 2x3 3x1

$$\begin{bmatrix} \overset{1}{1\times1} + \overset{4}{2\times2} + \overset{9}{3\times3} = 14 & 3 \\ 3 & 1 \end{bmatrix} \qquad \begin{bmatrix} \overset{1}{1\times1} + \overset{-1}{2\times(0.5)} + \overset{15}{3\cdot5} \\ 1\times5 \end{bmatrix}$$

$$\begin{bmatrix} 14 & 3 \\ 3 & 1 \end{bmatrix} \qquad \begin{bmatrix} 15 \\ 5 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}A}{|A|}$$

$$|A| = 14\cdot1 - 9 = 5$$

adjA for $14_{(1,1)} = 1\cdot(-1)^{1+1}$

$3_{(1,2)} = 3\cdot(-1)^{1+2}$

$3_{(2,1)} = 3\,(-1)^{2+1}$

$1_{(2,2)} = 14(-1)^{2+2}$

$$\begin{bmatrix} +1 & -3 \\ -3 & +14 \end{bmatrix}$$

$$\frac{\text{adj}A}{|A|} = \frac{\begin{bmatrix} 1 & -3 \\ -3 & 14 \end{bmatrix}}{5} \Rightarrow \begin{bmatrix} 1/5 & -3/5 \\ -3/5 & 14/5 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0.2 & -0.6 \\ -0.6 & 2.8 \end{bmatrix} \begin{bmatrix} 15 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} \overset{3.0}{0.2\cdot15} + \overset{3.0}{0.6\cdot5} \\ -0.6\cdot15 + 2.8\cdot5 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$$\frac{-9.0 \quad 14.0}{5}$$

$$\begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} \neq \begin{bmatrix} 1 \\ -0.5 \\ 5 \end{bmatrix}$$

3x2 2x1 Solved

Obtained Solution $\neq$ b (given)

Obtained sol. for #3 & couldn't for #1 & #2

However, this is the best solution in collective minimization error which we defined as

// Sum of squared of error.

Observe that # equations > # variables

$$\begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

$$A \quad x \quad = b$$

$x_1 = 1$ — #1

$\cancel{2}x_1 = \cancel{2}$ — #2

$3x_1 + x_2 = 5$ — #3

$x_2 = 5 - 3$
$= 2$

hence

$x_1 = 1$
$x_2 = 2$ $\Rightarrow \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ \underbrace{3\cdot 1 + 2\cdot 1}_{5} \end{bmatrix}$$

✓ Satisfied..!!

Now lets check what our optimization approach gives...

$$x = (A^T \cdot A)^{-1} A^T \cdot b$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \left(\underset{2\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}} \underset{3\times 2}{\begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 3 & 1 \end{bmatrix}}\right)^{-1} \underset{2\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}}$$

$$\begin{bmatrix} \overset{14}{1+4+9} & \overset{3}{0+0+3} \\ \underbrace{0+0+3}_{3} & 1 \end{bmatrix} \qquad \begin{bmatrix} \overset{20}{1\cdot 1 + 4 + 15} \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 14 & 3 \\ 3 & 1 \end{bmatrix} \qquad \begin{bmatrix} 20 \\ 5 \end{bmatrix}$$

$$\text{adj}A = \begin{bmatrix} 1 & -3 \\ -3 & 14 \end{bmatrix}$$

$$|A| = 5 \qquad A^{-1} = \frac{\text{adj}A}{|A|} = \frac{\begin{bmatrix} 1 & -3 \\ -3 & 14 \end{bmatrix}}{5} = \begin{bmatrix} 0.2 & -0.6 \\ -0.6 & 2.8 \end{bmatrix} \equiv \begin{bmatrix} 1/5 & -3/5 \\ -3/5 & 14/5 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underset{2\times 2}{\begin{bmatrix} 0.2 & -0.6 \\ -0.6 & 2.8 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} 20 \\ 5 \end{bmatrix}}$$

$$= \begin{bmatrix} \overset{1}{\overbrace{\frac{1}{5}\times 20 + \frac{3}{5}\times 5}} \\ \underbrace{-\frac{3}{5}\times 20 + \frac{14}{5}\cdot 5}_{2} \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Optimal Solution ~~as via optimized~~ method

✓ Same as the solution obtained via "Solving via Substitution" method.