Data Science for Engineers
Linear algebra

1. What would be the size of the matrix 'M'?
   Answer: b) 150x7
   Feedback:
   In the data matrix M, rows refer to 150 cricket players and the columns refer to 7 attributes of each cricket player, so it is a 150x7 matrix.

2. Rank of the matrix 'M' is 3 then what would be the nullity (number of equations) for the matrix 'M'?
   Answer: d) 4
   Feedback:
   According to the rank nullity theorem,
   Nullity of 'M' = Total number of variables -Rank of 'M'
   =7-3
   Nullity of 'M'=4

3. Rank of the matrix, A= $\begin{bmatrix} 1 & 4 & 3 & 5 & 6 \\ 2 & 5 & 0 & 5 & 1 \\ 8 & 6 & 5 & 1 & 0 \\ 2 & 8 & 6 & 10 & 12 \\ 9 & 7 & 6 & 5 & 1 \end{bmatrix}$ is _____
   Answer: 3
   Feedback:
   Row (4) = 2* Row (1)
   which means there are 4 independent rows in the matrix 'A'.
   So, the rank is 4.

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> A=matrix(c(1,4,3,5,6,2,5,0,5,1,8,6,5,1,0,2,8,6,10,12,9,7,6,5,1),
+          nrow=5, byrow=TRUE)
> Rank(A)
[1] 4
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4. Eigen values of the matrix, D= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is
   Answer: **3, 2, 1**

   Feedback:

$$\begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

   The determinant of the obtained matrix is $-\lambda^3 + 6\lambda^2 - 11\lambda + 6$
   Eigen values are $\lambda_1 = 3, \lambda_2 = 2, \lambda_3 = 1$

5. Eigen vectors of the given matrix D is

Answer: $\begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}$

Feedback:

$$\lambda_1 = 3$$

$$\begin{bmatrix}1-\lambda & 0 & 0\\ 0 & 2-\lambda & 0\\ 0 & 0 & 3-\lambda\end{bmatrix} = \begin{bmatrix}-2 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$\lambda_2 = 2$$

$$\begin{bmatrix}1-\lambda & 0 & 0\\ 0 & 2-\lambda & 0\\ 0 & 0 & 3-\lambda\end{bmatrix} = \begin{bmatrix}-1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\lambda_3 = 1$$

$$\begin{bmatrix}1-\lambda & 0 & 0\\ 0 & 2-\lambda & 0\\ 0 & 0 & 3-\lambda\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 2\end{bmatrix}$$

$$\begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}$$

6. The product of roots of characteristic equation of a square matrix ***A*** is equal to
   Answer: A) |A|
   Feedback:
   The product of roots of characteristic equation of a square matrix ***A*** is equal to |A|

7. Which of the following vector(s) is / are orthogonal?
   Answer:
   b) V1 = (1 4 -2) ^{T}, V2 = (12 −2 2) ^{T}

   d) V1= (-2 6 1) ^{T}, V2 = (4 1 2) ^{T}

   Feedback:

   $$\begin{bmatrix}1\\4\\-2\end{bmatrix}[12\ -2\ 2] = 12 - 8\ -4 = 0$$

   $$\begin{bmatrix}-2\\6\\1\end{bmatrix}[4\ 1\ 2] = -8\ +6+2 = 0$$

8. If ***A*** and ***B*** are any two square matrices of SAME dimensions such that ***AB*** = **0** and if ***A*** is non-singular, then

Answer: a) B=0

Feedback: If $\boldsymbol{A}$ and $\boldsymbol{B}$ are any two square matrices of SAME dimensions such that $\boldsymbol{AB}$ = 0 and if $\boldsymbol{A}$ is non-singular, then B=0

9. The point $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 6 \\ 3 \end{pmatrix}$ is in _____ half space of the hyper plane $x_1 - 9x_2 + 3x_3 + 2x_4 = 8$

Answer: Negative

Feedback:

$x_1 - 9x_2 + 3x_3 + 2x_4 = 8$

1- 9(4) +3(6) + 2(3)- 8= 20

10. The trace of a matrix A can be found by

Answer:

a) Sum of its eigenvalues

b) Sum of its diagonals

The trace of a given matrix can be found by the sum of the **leading diagonal elements**. It can also be found out by the sum of its **eigenvalues**.