Opgave 1

Beregn volumen af det område, der er beskrevet ved ulighederne:

$x \leq 0,\ y \geq 0,\ z \geq 0,\ x^2 + y^2 \leq 4$ og $z \leq -x^2 + 4$

To cylindrical coordinates

$x = r \cdot \cos\theta \quad y = r\sin\theta \quad z = z \quad \mathrm{dxdydz} = r \cdot \mathrm{d}\theta\mathrm{drdz}$

$$x^2 + y^2 = a^2 \Rightarrow r = a$$

$$\begin{cases} x \leq 0 \text{ and } 0 \leq y \Rightarrow 0 \leq \theta \leq \frac{\pi}{2} \\ x^2 + y^2 \leq 4 \Rightarrow 0 \leq a \leq 4 \\ z \leq -x^2 + 4 \Rightarrow z \leq -(r \cdot \cos(\theta))^2 + 4 = r^2 \cdot \cos^2(\theta) + 4 \end{cases}$$

$$\Rightarrow \int_0^4 \int_0^{\frac{\pi}{2}} \int_0^{r^2 \cdot \cos^2(\theta) + 4} r \, dz \, d\theta \, dr = \int_0^4 \int_0^{\frac{\pi}{2}} \left[rz \right]_0^{r^2 \cdot \cos^2(\theta) + 4} d\theta \, dr$$

$$= \int_0^4 \int_0^{\frac{\pi}{2}} r^3 \cdot \cos^2(\theta) + 4r - \cancel{r \cdot 0} \, d\theta \, dr = \int \left[r^3 \cdot \left(\frac{\theta}{2} + \frac{1}{4} \sin(2\theta) \right) + 4r\theta \right]_0^{\frac{\pi}{2}} dr$$

$$= \int_0^4 \left[r^3 \cdot \left(\frac{\theta}{2} + \frac{1}{4} \sin(2\theta) \right) + 4r\theta \right]_0^{\frac{\pi}{2}} dr = \int_0^4 r^3 \cdot \left(\frac{\pi}{4} + \cancel{\frac{1}{4} \underbrace{\sin\left(2 \cdot \frac{\pi}{2}\right)}_{0}} \right) + 4 \cdot \frac{\pi}{2} \cdot r \, dr$$

$$= \int_0^4 r^3 \cdot \frac{\pi}{4} + 2\pi r = \left[\frac{\pi}{4} \cdot r^4 + \pi r^2 \right]_0^4 = \frac{\pi}{4} \cdot 4^4 + \pi 4^2 = 64\pi + 16\pi = 80\pi$$

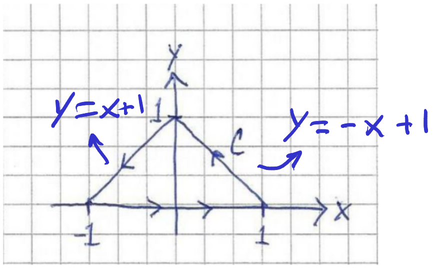
Opgave 2

Betragt vektorfeltet

$$\mathbf{F} = \begin{pmatrix} xy^2 \\ xy^2 - xy \end{pmatrix} \begin{matrix} = f_1 \\ = f_2 \end{matrix}$$

og kurven C, der er beskrevet ved skitsen herunder.

$$\iint_R \frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y} \, \mathrm{dA}$$



Beregn

$$\oint_C \mathbf{F} \cdot \mathbf{dr}$$

$$\frac{\partial f_2}{\partial x} - \frac{\partial \cdot f_1}{y} = y^2 - y - xy$$

$$\Rightarrow \oint \vec{F} \, dr = \iint_R y^2 - y - xy \, dR$$

$$\left.\begin{matrix} y = x + 1 \Rightarrow x = y - 1 \\ y = -x + 1 \Rightarrow x = -y + 1 \end{matrix}\right\} \; y - 1 \leq x \leq -y + 1$$

$$\Rightarrow \int_0^1 \int_{y-1}^{-y+1} y^2 - y - xy \, dx \, dy = \int_0^1 \left[y^2 x - yx - \frac{yx^2}{2} \right]_{y-1}^{-y+1} dy$$

$$= \int_0^1 y^2(-y+1) - y(-y+1) - \frac{y \cdot (-y+1)^2}{2} - y^2(y-1) + y(y-1) + \frac{y(y+1)^2}{2} \, dy$$

$$= \int_0^1 \underline{-y^3} + \underline{y^2} + \underline{y^2} \underline{-y} + \underline{\tfrac{1}{2} y^2} \underline{- \tfrac{1}{2} y} - \underline{y^3} + \underline{y^2} + \underline{y^2} \underline{-y} + \underline{\tfrac{1}{2} y^2} + \underline{\tfrac{1}{2} y} \, dy$$

$$= \int_0^1 -2y^3 + 5y^2$$