Opgave 1.1 (Fourierrækker)

Find Fourierkoefficienterne og Fourierrækken for firkant-signalet defineret som

$$f(x) = \begin{cases} 0 & \text{hvis } -1 \le x < 0 \\ 1 & \text{hvis } 0 \le x < 1 \end{cases}$$

og

$$f(x+2) = f(x)$$

Løsning:

$$f(x) = \frac{1}{2} + \sum_{n=1,3,5,\ldots} \frac{2}{n\pi}\sin(n\pi x)$$

$$-1 \le x < 1 \Rightarrow T = 2 \Rightarrow L = 1$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cdot\cos\left(\frac{n\pi x}{L}\right)dx = \int_{-1}^{0}\underbrace{f(x)}_{0}\cdot\cos(n\pi x)\,dx + \int_{0}^{1}\underbrace{f(x)}_{1}\cdot\cos(n\pi x)\,dx$$

$$= \int_0^1 \cos(n\pi x)\,dx = \int_{x=0}^{x=1}\cos(u)\,\frac{1}{n\pi}\,du = \left[\frac{1}{n\pi}\cdot\sin(u)\right]_{x=0}^{x=1} = \left[\frac{1}{n\pi}\cdot\sin(n\pi x)\right]_0^1$$

$$u = n\pi x \Rightarrow \frac{du}{dx} = n\pi \Rightarrow dx = \frac{1}{n\pi}du$$

$$= \frac{2}{n\pi}\cdot\underbrace{\sin(n\pi)}_{0} - \frac{2}{n\pi}\cdot\underbrace{\sin(0)}_{0} = 0$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\cdot\sin\left(\frac{n\pi x}{L}\right)dx = \int_{-1}^{1} f(x)\cdot\sin(n\pi x)\,dx$$

$$= \cancel{\int_{-1}^{0}\underbrace{f(x)}_{0}\cdot\sin(n\pi x)\,dx} + \int_0^1 \underbrace{f(x)}_{1}\cdot\sin(n\pi x)\,dx = \int_0^1 \sin(\underbrace{n\pi x}_{u})\,dx$$

$$\frac{du}{dx} = n\pi \Rightarrow dx = \frac{1}{n\pi}$$

$$= \int_{x=0}^{x=1}\sin(u)\cdot\frac{1}{n\pi}\,du = \frac{1}{n\pi}\cdot\left[-\cos(u)\right]_{x=0}^{x=1} = \frac{1}{\pi n}\cdot\left[\cos(n\pi)\right]_0^1 = \frac{1}{\pi n}\cdot\left(\underbrace{\cos(\pi)}_{1} + \underbrace{\cos(0)}_{1}\right) = \frac{2}{\pi n}$$

$$a_0 = \int_0^1 f(x)\cdot\cos(0)\,dx = \int_0^1 f(x)\,dx = \int_0^1 1\,dx = [x]_0^1 = 1$$

$$g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cdot\cos\left(\frac{n\pi x}{L}\right) + b_n\cdot\sin\left(\frac{n\pi x}{L}\right)\right) = \frac{1}{2} + \sum_{n=1}^{\infty}\left(\frac{2}{\pi n}\cdot\sin(n\pi x)\right)$$

Opgave 1.2 (Fouriertransformation)

Enhedstrinfunktionen defineret som

$$u(t-a) = \begin{cases} 1 \text{ for } t-a > 0 \\ 0 \text{ for } t-a < 0 \end{cases}$$

benyttes til at definere en firkantimpuls

$$x(t) = u(t-a) - u(t-b)$$

hvor $a < b$.

- Tegn grafen for firkantimpulsen $x(t)$
- Udregn den Fouriertransformerede af $x(t)$

Løsning:

$$X(\omega) = \frac{e^{-j\omega a} - e^{-j\omega b}}{j\omega}$$