Problem 1:

$$\iiint xy^2 \cos(z) \; dydzdx \quad R:\left\{x,y,z \;\middle|\; \begin{matrix} 0 \le x \le 2 \\ 0 \le y \le 3 \; and \; 0 \le z \le \pi/2 \end{matrix}\right.$$

Answer: 18

$$\int_0^2 \int_0^{\frac{\pi}{2}} \int_0^3 xy^2 \cdot \cos(z) \; dy \, dz \, dx = \int_0^2 \int_0^{\frac{\pi}{2}} \left[x \cdot \cos(z) \cdot \frac{y^3}{3} \right]_0^3 dz \, dx$$

$$= \int_0^2 \int_0^{\frac{\pi}{2}} x \cdot \cos(z) \cdot \frac{3^3}{3} \; dz \, dx = 9 \cdot \int_0^2 \int_0^{\frac{\pi}{2}} x \cdot \cos(z) \; dz \, dx$$

$$= 9 \cdot \int_0^2 \left[x \cdot \sin(z) \right]_0^{\frac{\pi}{2}} dx = 9 \cdot \int_0^2 x \; dx = 9 \cdot \left[\frac{x^2}{2} \right]_0^2 = 9 \cdot \frac{4}{2} = \underline{18}$$

Problem 2:

$$\iiint 2\sqrt{y}\,e^{-x^2}\,dz\,dx\,dy \quad R:\left\{x,y,z \,\Big|\, \begin{matrix} 0 \le x \le 1 \\ 0 \le y \le 4 \text{ and } 0 \le z \le x \end{matrix}\right.$$

Answer: $\frac{16}{3}(1 - e^{-1})$

$$\int_0^4 \int_0^1 \int_0^x 2\sqrt{y}\,e^{-x^2}\,dz\,dx\,dy = 2 \cdot \int_0^4 \sqrt{y} \int_0^1 e^{-x^2} \int_0^x 1\,dz\,dx\,dy$$

$$= 2 \cdot \int_0^4 \sqrt{y} \int_0^1 e^{-x^2}\left[z\right]_0^x dx\,dy = 2 \cdot \int_0^4 \sqrt{y} \int_0^1 e^{-x^2}\cdot x\,dx\,dy$$

$$u = -x^2 \Rightarrow \frac{du}{dx} = -2x \Rightarrow dx = \frac{-1}{2x}\,du$$

$$= 2 \cdot \int_0^4 \sqrt{y} \int_{x=0}^{x=1} e^u \cdot x \cdot \frac{-1}{2x}\,du\,dy = -\int_0^4 \sqrt{y} \int_{x=0}^{x=1} e^u\,du\,dy$$

$$= -\int_0^4 \sqrt{y}\cdot\left[e^u\right]_{x=0}^{x=1} d\,y = -\int_0^4 \sqrt{y}\cdot\left[e^{-x^2}\right]_0^1 d\,y = -\int_0^4 \sqrt{y}\cdot\left(e^{-1}-e^{-0}\right)dy$$

$$= -\left(e^{-1}-1\right)\cdot\int_0^4 \sqrt{y}\,dy = \left(1-e^{-1}\right)\cdot\int_0^4 y^{\frac{1}{2}}\,dy = \left(1-e^{-1}\right)\cdot\int_0^4 y^{\frac{1}{2}}\,dy$$

$$= \left(1-e^{-1}\right)\cdot\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 = \left(1-e^{-1}\right)\cdot\left[\frac{2}{3}\cdot y^{\frac{3}{2}}\right]_0^4 = \left(1-e^{-1}\right)\cdot\frac{2}{3}\left[\left(\sqrt{y}\right)^3\right]_0^4$$

$$= \left(1-e^{-1}\right)\cdot\frac{2}{3}\cdot\left(\left(\sqrt{4}\right)^3 - \left(\sqrt{0}\right)^3\right) = \left(1-e^{-1}\right)\cdot\frac{2}{3}\cdot 2^3 = \left(1-e^{-1}\right)\cdot\frac{16}{3}$$

Problem 3:

$$\iiint 1 + 2x - 3y \, dxdydz \qquad R: \left\{ x, y, z \,\middle|\, \begin{array}{l} -a \le x \le a \\ -b \le y \le b \text{ and } -c \le z \le c \end{array} \right.$$

Answer: 8abc

$$\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} 1 + 2x - 3y \, dx \, dy \, dz = \int_{-c}^{c} \int_{-b}^{b} \left[x + x^2 - 3yx \right]_{-a}^{a} dy \, dz$$

$$= \int_{-c}^{c} \int_{-b}^{b} a + a^2 - 3ya - \left(-a + (-a)^2 - 3y(-a) \right) \, dy \, dz$$

$$= \int_{-c}^{c} \int_{-b}^{b} a + a^2 - 3ya + a - a^2 - 3ay \, dy \, dz = \int_{-c}^{c} \int_{-b}^{b} 2a - 6ay \, dy \, dz$$

$$= \int_{-c}^{c} \left[2ay - 3ay^2 \right]_{-b}^{b} dz = \int_{-c}^{c} 2ab - 3ab^2 - \left(2a(-b) - 3a(-b)^2 \right) \, dz$$

$$= \int_{-c}^{c} 2ab - 3ab^2 + 2ab + 3ab^2 \, dz = \int_{-c}^{c} 4ab \, dz = \left[4abz \right]_{-c}^{c}$$

$$= 4abc - 4ab(-c) = \underline{\underline{8abc}}$$

$$\int_1^2 \int_0^z \int_0^{y+z} \frac{1}{(x+y+z)^3}\,dx\,dy\,dz = \int_1^2 \int_0^z \int_0^{y+z} u \cdot \frac{1}{(x+y+z)2} \cdot \left(-(x+y+z)^2\right)\,du\,dy\,dx$$

$$u = \frac{1}{x+y+z} \Rightarrow \frac{du}{dx} = \frac{-1}{(x+y+z)^2} \cdot 1 \Rightarrow dx = -(x+y+z)^2\,du$$

$$= \int_1^2 \int_0^z \int_0^{y+z} u \cdot (-1)\,du\,dy\,dx = -\int_1^2 \int_0^z \int_0^{y+z} u\,du\,dy\,dx = -\int_1^2 \int_0^z \left[\frac{u^2}{2}\right]_0^{y+z}\,dy\,dx$$

$$= \frac{-1}{2} \int_1^2 \int_0^z \left[u^2\right]_0^{y+z}\,dy\,dx = \frac{-1}{2} \int_1^2 \int_0^z \left[\left(\frac{1}{x+y+z}\right)^2\right]_0^{y+z}\,dy\,dz = \frac{-1}{2}\int_1^2 \int_0^z \left[\frac{1}{(x+y+z)^2}\right]_0^{y+z}\,dy\,dz$$

$$= \frac{-1}{2} \int_1^2 \int_0^z \frac{1}{(y+z+y+z)^2} - \frac{1}{(y+z)^2}\,dy\,dz = \frac{-1}{2} \int_1^2 \int_0^z \frac{1}{(2y+2z)^2} - \frac{1}{(y+z)^2}\,dy\,dz$$

$$\frac{1}{(2y+2z)^2} = \frac{1}{(2\cdot(y+z))^2} = \frac{1}{4\cdot(y+z)^2} = \frac{1}{4}\cdot\frac{1}{(y+z)^2}$$

$$= \frac{-1}{2} \int_1^2 \int_0^z \frac{1}{4}\cdot\frac{1}{(y+z)^2} - \frac{1}{(y+z)^2}\,dy\,dz$$

$$= \frac{-1}{2} \int_1^2 \int_0^z \frac{-3}{4}\cdot\frac{1}{(y+z)^2}\,dy\,dz = \frac{3}{8} \int_1^2 \int_0^z \frac{1}{(y+z)^2}\,dy\,dz = \frac{3}{8} \int_1^2 \int_{y=0}^{y=z} u^{-2}\,du\,dz$$

$$u = y - z \Rightarrow du = dy$$

$$= \frac{3}{8} \int_1^2 \left[-u^{-1}\right]_{y=0}^{y=z}\,dz = \frac{-3}{8} \int_1^2 \left[(y+z)^{-1}\right]_0^z\,dz = \frac{3}{8} \int_1^2 (z+z)^{-1} - (0+z)^{-1}\,dz = \frac{3}{8} \int_1^2 \frac{1}{2z} - \frac{1}{z}\,dz$$

$$= \frac{3}{8}\cdot \int_1^2 \frac{1}{z}\left(\frac{1}{2}-1\right)\,dz = \frac{-3}{16}\cdot\int_1^2 \frac{1}{z}\,dz = \frac{-3}{16}\left[\ln(z)\right]_1^2 = \frac{-3}{16}\left(\ln(2) - \ln(1)\right) = \frac{-3}{16}\ln(2)$$

almost right...

Problem 5:

determinant of the

Find Jacobian for x=2u+w y=$-v^2$ *and* $z = u + v^2 - 2w^2$

Answer: 16vw+~~2v~~+2v

$$f(u, v, w) = (\overset{f_1}{2u+w}, \overset{f_2}{-v^2}, \overset{f_3}{u+v^2-2w^2})$$

$$J = \begin{bmatrix} \frac{f_1}{\partial u} & \frac{f_1}{\partial v} & \frac{f_1}{\partial w} \\ \frac{f_2}{\partial u} & \frac{f_2}{\partial v} & \frac{f_2}{\partial w} \\ \frac{f_3}{\partial u} & \frac{f_3}{\partial v} & \frac{f_3}{\partial w} \end{bmatrix} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & -2v & 0 \\ 1 & 2v & -4w \end{bmatrix}$$

$$|J| = 2(8vw-0) - 0 + 1 \cdot (0-(-2v)) = \underline{\underline{16vw+2v}}$$

I choose to believe that this is the correct answer!

Problem 6:

Find the volume of solid bounded by $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 = 8z$

Answer: $\frac{40}{3}\pi$

Convert to cylindrical coordinates

$$
\begin{aligned}
x &\to r \cdot \cos\theta \\
y &\to r\sin\theta \\
z &\to z \\
dxdydz &\to r \cdot d\theta drdz \\
x^2 + y^2 = a^2 &\to r = a
\end{aligned}
$$

$$r^2(\cos^2(\theta) + \sin^2(\theta)) = 1$$

$$x^2 + y^2 = 8z \Rightarrow r^2 \cdot \cos^2(\theta) + r^2 \cdot \sin^2(\theta) = 8z \Rightarrow r^2 = 8z \Rightarrow z = \frac{r^2}{8}$$

$$x^2 + y^2 + z^2 = 9 \Rightarrow r^2 + z^2 = 9 \Rightarrow z^2 = 9 - r^2 \Rightarrow z = \sqrt{9-r^2}$$

$$\Rightarrow \int_0^{2\pi} \int_?^{\sqrt{9-r^2}} \int_{\frac{r^2}{8}}^{\sqrt{9-r^2}} r \, dz \, dr \, d\theta$$

Try to find the limits for r

$$\begin{cases} z = \frac{r^2}{8} \\ z = \sqrt{9-r^2} \end{cases} \Rightarrow \left(\frac{r^2}{8} = \sqrt{9-r^2}\right)^2 \Rightarrow \frac{1}{64}r^4 = 9 - r^2 \Rightarrow \frac{1}{64}r^4 + r^2 = 9 \Rightarrow r^4 + 64r^2 - 9 \cdot 64 = 0$$

$$9 \cdot 64 = 8 \cdot 72$$

$$\Rightarrow r^4 + 64r^2 - 9 \cdot 64 = 0 \Rightarrow (r^2 - 8)(r^2 + 72) = 0$$

$$\Rightarrow \begin{cases} r^2 - 8 = 0 \Rightarrow r^2 = 8 \Rightarrow r_1 = \sqrt{8} \\ r^2 + 72 = 0 \Rightarrow r^2 = -72 \Rightarrow r_2 = \sqrt{-72} \end{cases} \Rightarrow r = \sqrt{8} = 2\sqrt{2}$$

Imaginary part is thrown away   This is the boundary for r.

Rewriting the integral

$$\int_0^{2\pi}\int_0^{\sqrt{8}}\int_{\frac{r^2}{8}}^{\sqrt{9-r^2}} r \, dz \, dr \, d\theta = \int_0^{2\pi}\int_0^{\sqrt{8}} \left[rz\right]_{\frac{r^2}{8}}^{\sqrt{9-r^2}} dr \, d\theta = \int_0^{2\pi}\int_0^{\sqrt{8}} r\sqrt{9-r^2} - \frac{1}{8}r^3 \, dr \, d\theta$$

From table: $\int x\sqrt{-x^2 \pm a^2}\, dx = -\frac{1}{3}\left(x^2 \pm a^2\right)^{3/2}$

$$\int_0^{2\pi}\left[-\frac{1}{3}(9-r^2)^{\frac{3}{2}} - \frac{1}{32}r^4\right]_0^{\sqrt{8}} d\theta = \int_0^{2\pi} -\frac{1}{3}(9-(\sqrt{8})^2)^{\frac{3}{2}} - \frac{1}{32}(\sqrt{8})^4 + \frac{1}{3}(9-0)^{\frac{3}{2}} + \frac{1}{32} \cdot 0^4 \, d\theta$$

$$= \int_0^{2\pi} -\frac{1}{3}(9-8)^{\frac{3}{2}} - \frac{1}{32}\cdot 8^2 + \frac{1}{3}\cdot 9^{\frac{3}{2}} \, d\theta = \int_0^{2\pi} -\frac{1}{3}\cdot(\sqrt{1})^3 - \frac{64}{32} + \frac{1}{3}\cdot(\sqrt{9})^3 \, d\theta$$

$$= \int_0^{2\pi} -\frac{1}{3} - 2 + (\sqrt{9})^2 \, d\theta = \int_0^{2\pi} -\frac{1}{3} - 2 + 9 \, d\theta = \int_0^{2\pi} -\frac{1}{3} + 7 \, d\theta$$

$$= \int_0^{2\pi} \frac{20}{3} \, d\theta = \left[\frac{20\theta}{3}\right]_0^{2\pi} = \frac{40\pi}{3}$$