Opgave 7.1

Design et båndpasfilter med maksimal flad pasbånd, der overholder følgende specifikation

- Centerfrekvens $f_c = 1,5$ kHz.
- Pasbåndsbredde (-3 dB) $\Delta f_3 = 250$ Hz.
- Stopbåndsbredde (-40 dB) $\Delta f_{40} \leq 1,1$ kHz.

Det digitale filter skal findes ved brug af bilineær z-transformation og have en samplefrekvens på 10 kHz.

$$f_s = 10\,000 \text{ Hz}$$

1. Prewarping konstanten bestemmes

$$C = \cot\left(\frac{w_c T}{2}\right) = \cot\left(\frac{1500 \cdot \frac{1}{10000} \cdot 2\pi}{2}\right) \approx 1.9626$$

2. Prewarpede stopbåndsfrekvens

Find stopbåndsfrekvenser og afskæringsfrekvenser

$$Q = \frac{1500}{250} = 6$$

$$Q = \frac{f_c}{\Delta f}$$

$$f_1 = f_c \cdot \left(\sqrt{1 + \frac{1}{4Q^2}} - \frac{1}{2Q}\right)$$

$$f_2 = f_c \cdot \left(\sqrt{1 + \frac{1}{4Q^2}} + \frac{1}{2Q}\right)$$

$$\Rightarrow \begin{cases} f_{a1} = 1380 \text{ Hz}, & f_{a2} = 1630 \text{ Hz} \\ f_{s1} = 1047 \text{ Hz}, & f_{s2} = 2147 \text{ Hz} \end{cases}$$

Prewarp them bitcheeeessssssssssssss!

$$\Omega_a = C \cdot \tan\left(\frac{w_a T}{2}\right)$$

$$\Omega_{a1} = 0.908512$$
$$\Omega_{a2} = 1.10316$$
$$\Omega_{s1} = 0.669882$$
$$\Omega_{s2} = 1.5693$$

Finder formfaktoren for at finde filter ordenen

$$W_a = \frac{\Delta f_a}{f_c} \qquad W_s = \frac{\Delta f_s}{f_c}, \qquad F = \frac{W_s}{W_a}$$

$$f_c = 1$$

$$W_a = \frac{\Omega_{a2} - \Omega_{a1}}{1 \cdot 2\pi}, \qquad W_s = \frac{\Omega_{s2} - \Omega_{s1}}{1 \cdot 2\pi}$$

$$F = 4,62071$$

Butterworth filter bruges da det har den mest konstante forstærkning

Dette er et lavpas filter

$$H_4(s) = \frac{1}{(s^2 + 0.7654s + 1)(s^2 + 1.8478s + 1)}$$

Transformer til båndpas

$$s \rightarrow \frac{1}{W_a} \cdot \left(s + \frac{1}{s}\right)$$

$$H_{bp}(s) = H_{4lp}(s) \Big|_{s = \frac{1}{W_a} \cdot \left(s + \frac{1}{s}\right)}$$

$$= \frac{1}{1 + 84.3524\left(\frac{1}{s} + s\right) + 3557.56\left(\frac{1}{s} + s\right)^2 + 87891.4\left(\frac{1}{s} + s\right)^3 + 1.08567 \times 10^6\left(\frac{1}{s} + s\right)^4}$$

Vi denormerer filtret

$$s \rightarrow \frac{s}{\omega_c}$$

$$(1.10768 \times 10^{-18}\, s^4) / (9488.53 + 0.0815034\, s + 0.000427634\, s^2 + 2.75356 \times 10^{-9}\, s^3 + 7.22337 \times 10^{-12}\, s^4 + 3.09993 \times 10^{-17}\, s^5 + 5.41986 \times 10^{-20}\, s^6 + 1.16292 \times 10^{-25}\, s^7 + 1.52416 \times 10^{-28}\, s^8)$$

z-tranformation af filter

$$s \rightarrow c \cdot \frac{z-1}{z+1}$$

$$H(z) = (1.73196 \times 10^{-21}\,(-1. + z^2)^4) / (0.999966 + 7.99976\, z + 27.9993\, z^2 + 55.9988\, z^3 + 69.9988\, z^4 + 55.9993\, z^5 + 27.9998\, z^6 + 7.99997\, z^7 + 1.\, z^8)$$