Fysik 1 – lektion 9

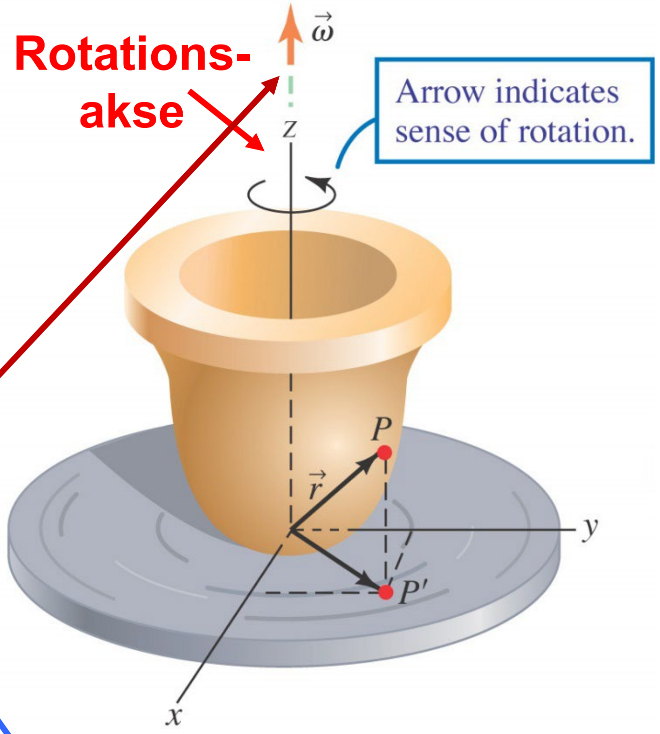
Dagens emne: Rotation af stive legemer, kap. 10.

- Rotation – vinkelhastighed, vinkelacceleration
- Rotationsenergi (K_{rot})
- Masse-inertimoment (*rotational inertia*)
- Parallel-akse teoremet – flytningsformlen
- Kraftmoment (torque)
- Frit-legeme analyse (igen)
- Arbejde, energi og effekt i rotation
- Analogi mellem translations- og rotationsbevægelse
- Opgaveregning

Rotation – vinkelhastighed

Rotation om én akse kan beskrives ved én variabel: vinklen $\theta(t)$

Rotations-akse



Middel-vinkelhastigheden bliver:

$$\omega_{av} = \frac{\theta(t+\Delta t)-\theta(t)}{\Delta t}$$

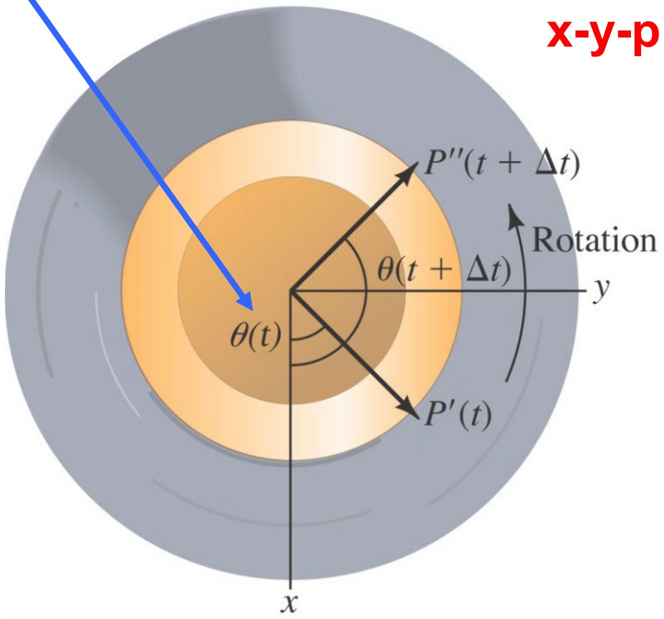
Den øjeblikkelige vinkelhastighed bliver:

Vektor!

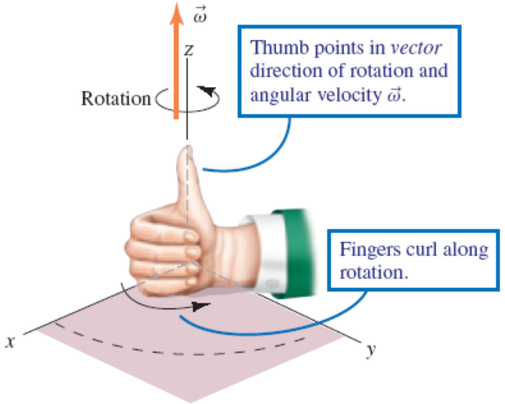
$$\boxed{\vec{\omega} = \frac{d\theta(t)}{dt}\hat{\omega} \quad \left[s^{-1}\right]}$$

retning

x-y-planet



Retningen af denne vektor findes ved højrehåndsreglen:



Er vinkelhastigheden konstant: $\omega(t) = \omega_0$ bliver:

$$\boxed{\theta(t) = \theta_0 + \omega_0 t}$$

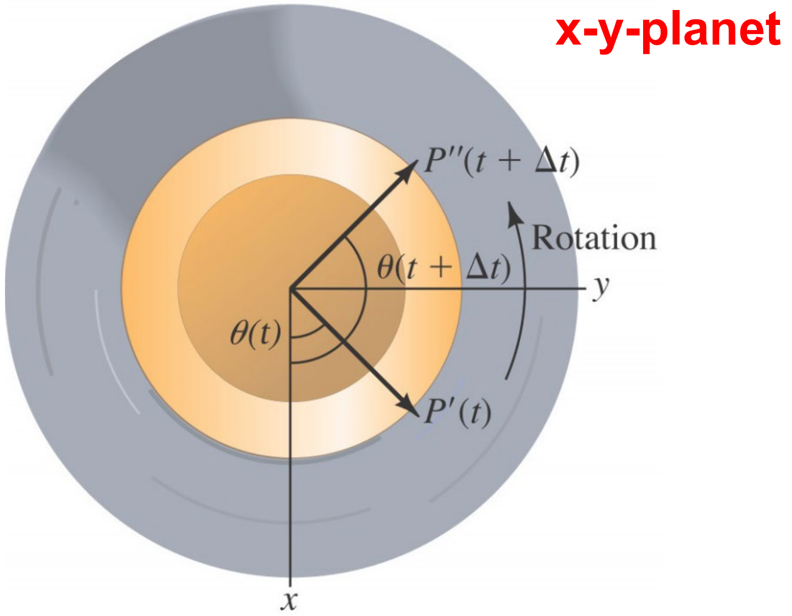
Rotation – vinkelacceleration

Middel-vinkelaccelerationen bliver:

$$\alpha_{av} = \frac{\omega(t+\Delta t) - \omega(t)}{\Delta t}$$

Den øjeblikkelige vinkelacceleration bliver:

$$\boxed{\vec{\alpha} = \frac{d\omega(t)}{dt}\hat{\omega} = \frac{d^2\theta}{dt^2}\hat{\omega} \quad \left[s^{-2}\right]}$$



For rotationer om én akse med <u>konstant vinkelacceleration</u> bliver rotationsligningerne <u>analoge</u> til ligningerne for én-dimensionale lineære bevægelser med konstant acceleration:

(1) $\omega(t) = \omega_0 + \alpha t$

(2) $\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

(3) $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$\Leftrightarrow$

$v(t) = v_0 + at$

$s(t) = s_0 + v_0 t + \frac{1}{2}at^2$

$v^2 = v_0^2 + 2a(s - s_0)$

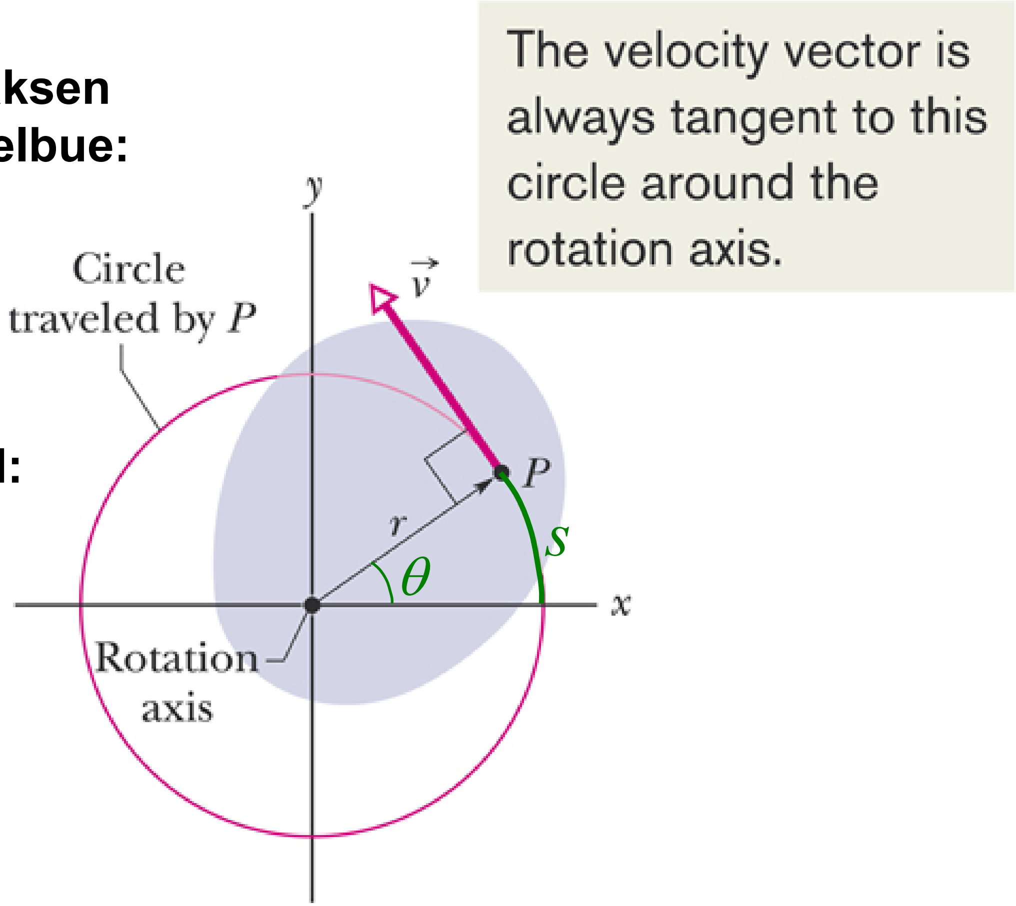
Relation mellem lineære og angulære størrelser

Et punkt *P* i afstanden *r* ifht. rotationsaksen bevæger sig afstanden *s* langs en cirkelbue:

$$s = r\theta \quad (\theta \text{ er målt i radianer!})$$

Punktets lineære hastighed findes ved:

$$v = \frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega$$



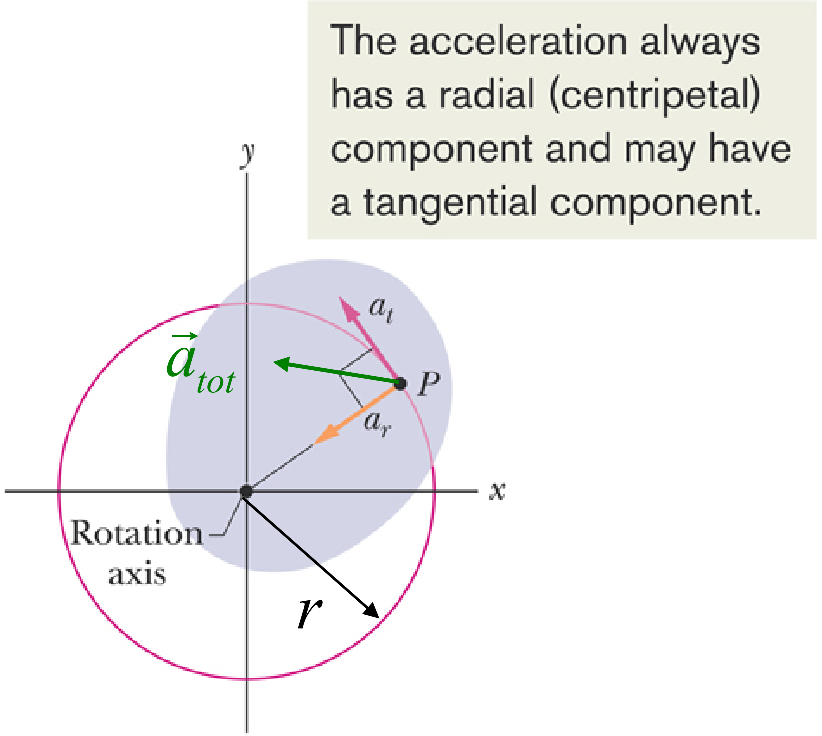
Retningen af den lineære hastighed, v, er vinkelret på punktet P's cirklebevægelse

Retningen af den lineære acceleration, a, er lidt mere udfordrende, da der er to bidrag til accelerationen!

Rotation - acceleration

Betragter vi igen et punkt P på det roterende legeme, og finder først den tangentiale acceleration:

$$\boxed{a_{\mathrm{t}} = \frac{dv}{dt} = \frac{d}{dt}\left(\omega r\right) = \frac{d\omega}{dt} r = \alpha r}$$

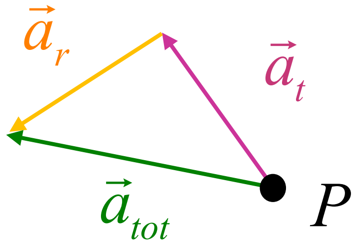


Den radiale (centripetale) acceleration er ved cirkelbevægelse som bekendt :

$$\boxed{a_r = \frac{v^2}{r} = \omega^2 r}$$

Den totale acceleration:

$$\boxed{\vec{a}_{tot} = \vec{a}_r + \vec{a}_{\mathrm{t}}}$$



Translation og rotation

Translation		**Rotation**
Konstant acceleration, *a*		Konstant vinkelacceleration, *α*

Linear Equation	Missing Variable		Angular Equation
$v = v_0 + at$	$x - x_0$	$\theta - \theta_0$	$\omega = \omega_0 + \alpha t$
$x - x_0 = v_0 t + \frac{1}{2}at^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	t	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	α	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
$x - x_0 = vt - \frac{1}{2}at^2$	v_0	ω_0	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$

$$s = \theta r$$

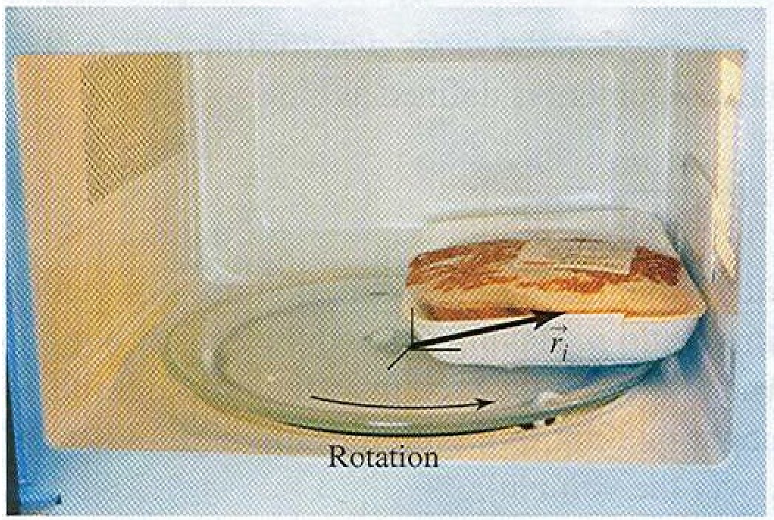
$$v = \frac{ds}{dt} = \frac{d(\theta r)}{dt} = \omega r$$

$$T = \frac{2\pi}{\omega}$$

$$a = \frac{dv}{dt} = \frac{d(\omega r)}{dt} = \alpha r$$

Rotationsenergi

Den kinetiske energi i rotationen kan findes ved at summere den kinetiske energi for hver del af et legeme:

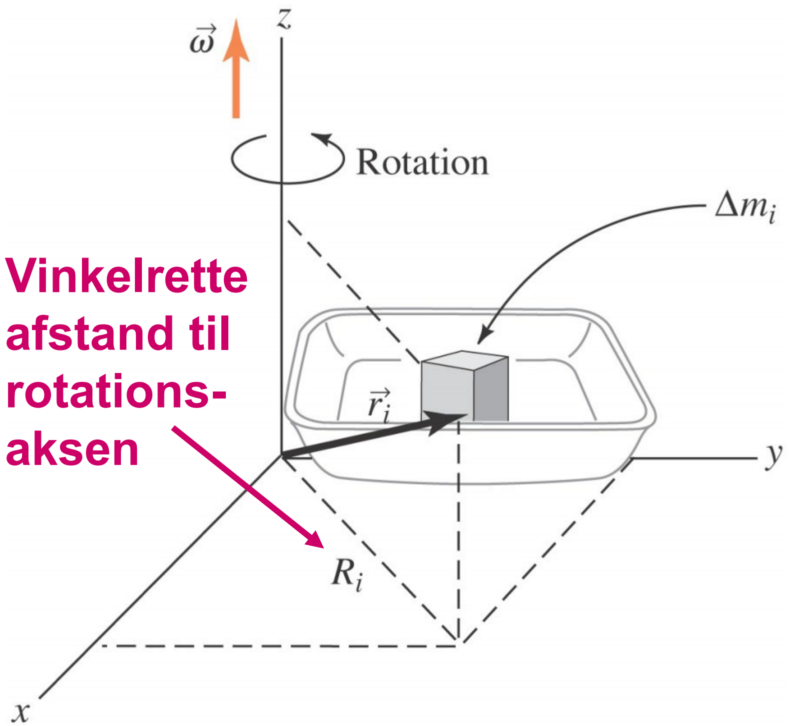


$$K = \sum_i K_i = \frac{1}{2}\sum_i \Delta m_i v_i^2 \;,\; v_i = R_i \omega$$

$$K = \frac{1}{2}\left(\sum_i \Delta m_i R_i^2\right)\omega^2 \Rightarrow \boxed{K = \frac{1}{2} I \omega^2}$$

Hvor *I* kaldes legemets (masse-) inertimoment mht. den specifikke rotationsakse:

$$\boxed{I = \sum_i \Delta m_i R_i^2} \quad \left[\mathrm{kg} \cdot \mathrm{m}^2\right]$$

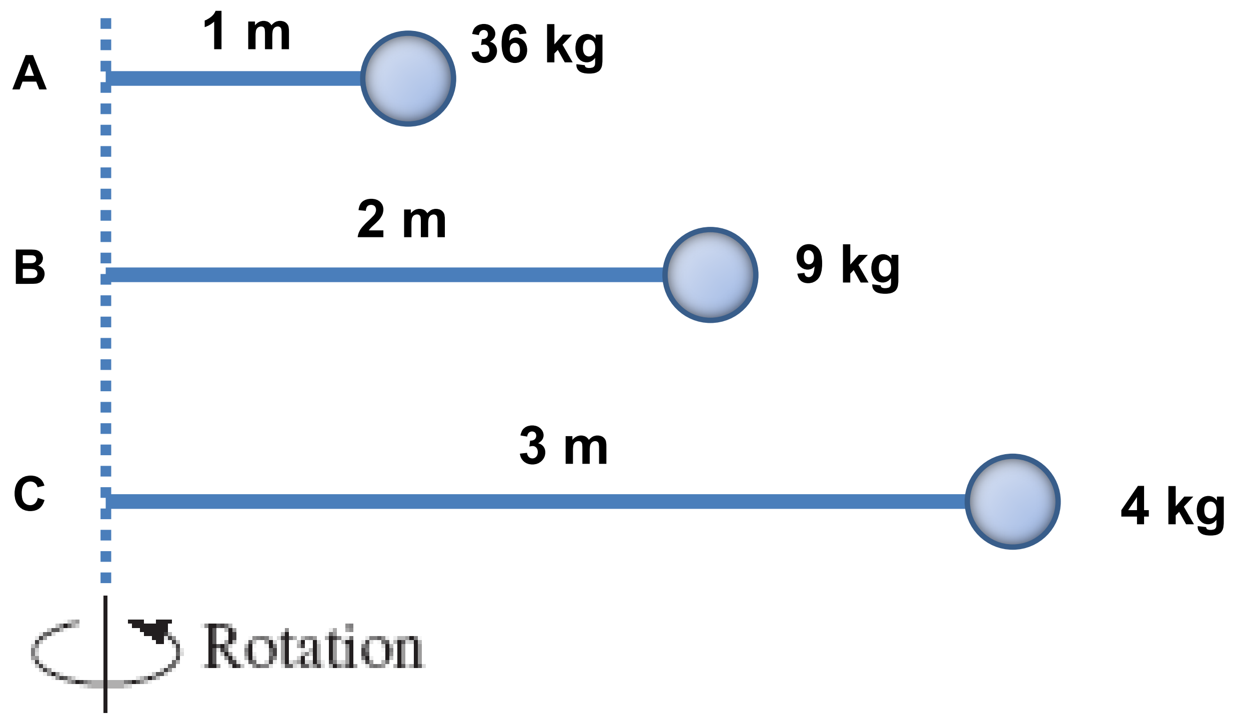


Vinkelrette afstand til rotations-aksen

Masseinertimoment

Figuren viser tre <u>små</u> kugler der roterer omkring en lodret akse.
Den vinkelrette afstand mellem aksen og centrum af hver kugle er vist.

Sorter de tre <u>små</u> kugler efter deres masseinertimoment,
den mindste skrives først



Masseinertimoment

Ønsker vi at finde et kontinuert legemes inertimoment, benyttes integration:

dm's vinkelrette afstand til rotationsaksen

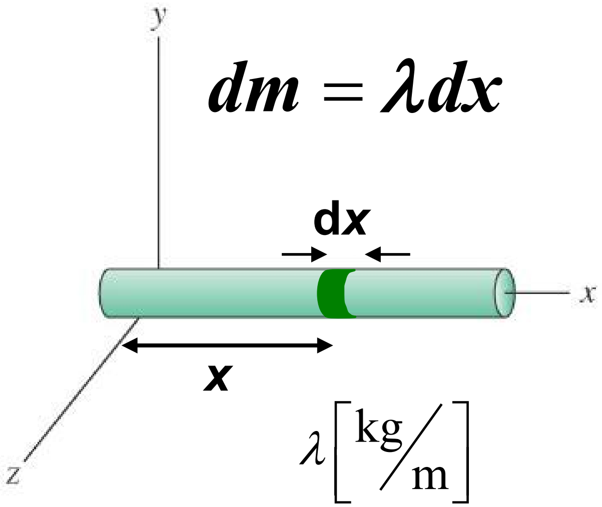
$$I = \lim_{\Delta m \to 0}\left(\sum_i R_i^2 \Delta m_i\right) = \int_{legeme} R^2 dm$$

For 1-, 2- og 3-dimensionale legemer findes inertimomentet ud fra:

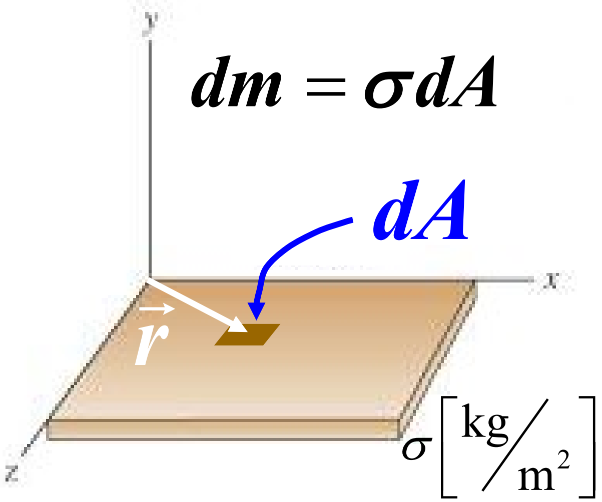
x, r og R er afstanden ifht. rorationsasken!

$$I = \int_{linie} x^2 dm = \int_{linie} \lambda x^2 dx \qquad I = \int_{flade} \sigma r^2 dA \qquad I = \int_{volumen} \rho R^2 dV$$

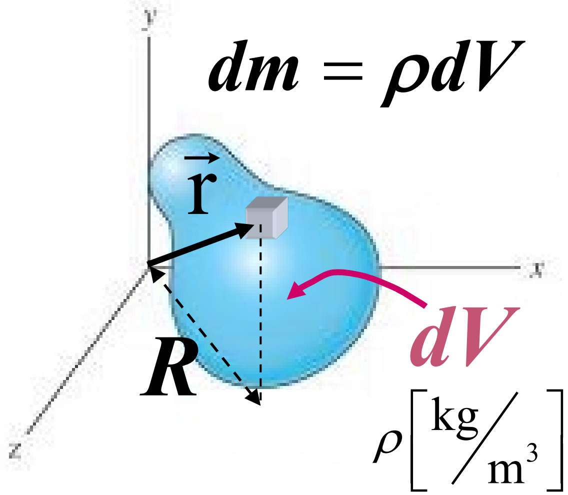
$$dm = \lambda dx$$



$$dm = \sigma dA$$



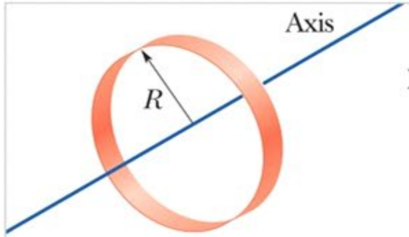
$$dm = \rho dV$$



Masseinertimoment for simple legemer

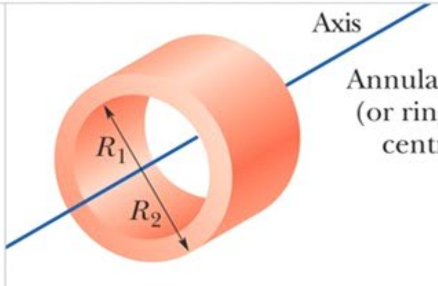
Tabel 10.2

Table 10-2 Some Rotational Inertias



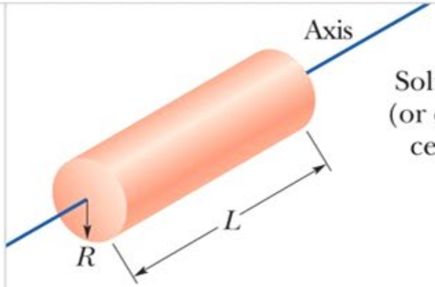
Hoop about central axis

$$I = MR^2$$ (a)



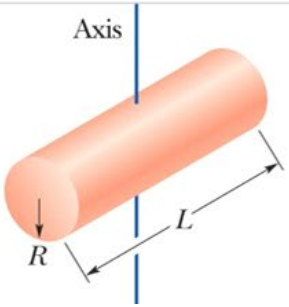
Annular cylinder (or ring) about central axis

$$I = \tfrac{1}{2}M\left(R_1{}^2 + R_2{}^2\right)$$ (b)



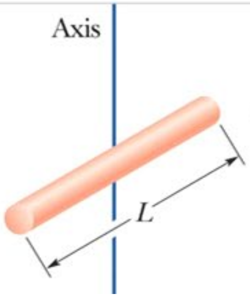
Solid cylinder (or disk) about central axis

$$I = \tfrac{1}{2}MR^2$$ (c)



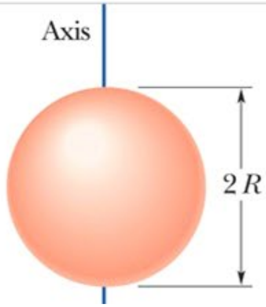
Solid cylinder (or disk) about central diameter

$$I = \tfrac{1}{4}MR^2 + \tfrac{1}{12}ML^2$$ (d)



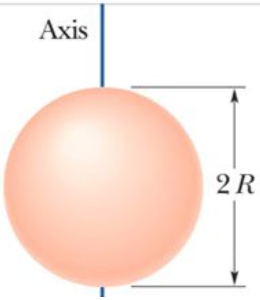
Thin rod about axis through center perpendicular to length

$$I = \tfrac{1}{12}ML^2$$ (e)



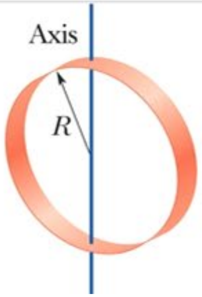
Solid sphere about any diameter

$$I = \tfrac{2}{5}MR^2$$ (f)



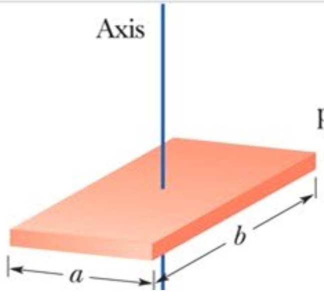
Thin spherical shell about any diameter

$$I = \tfrac{2}{3}MR^2$$ (g)



Hoop about any diameter

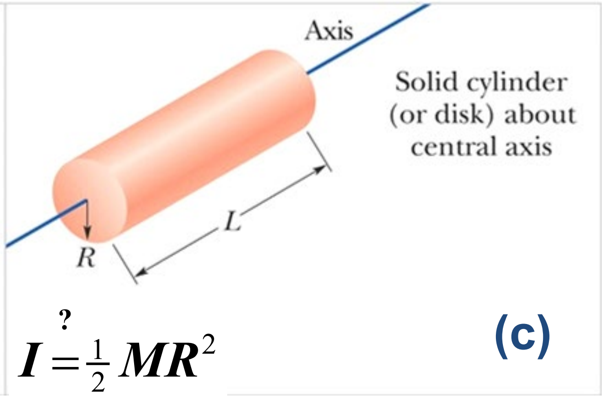
$$I = \tfrac{1}{2}MR^2$$ (h)

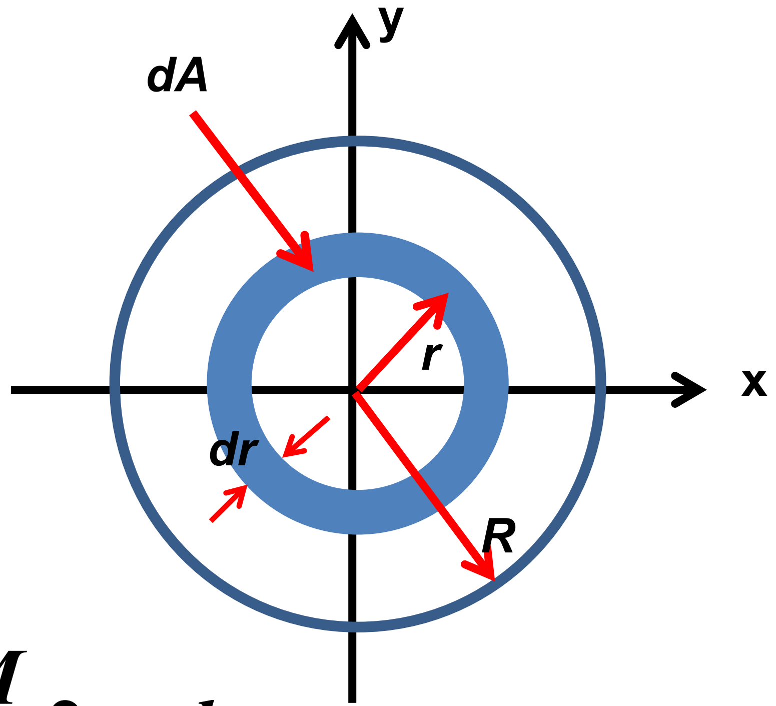


Slab about perpendicular axis through center

$$I = \tfrac{1}{12}M\left(a^2 + b^2\right)$$ (i)

Masseinertimoment for massiv cylinder





$$\boxed{I = \int r^2 dm}$$

$$\frac{dm}{M} = \frac{dA}{A} \quad \Rightarrow dm = \frac{M}{A} dA \; = \frac{M}{A} 2\pi r dr$$

$$A \; = \pi r^2 \quad \Rightarrow dA = 2\pi r dr$$

$$I = \int_0^R r^2 \frac{M}{A} 2\pi r dr \;\; = \frac{2\pi M}{A} \int_0^R r^3 dr \;\; = \frac{2\pi M}{\pi R^2} \left(\frac{r^4}{4} \right)_0^R = \frac{1}{2} MR^2$$

Flytningsformlen

Kendes et legemes inertimoment, I_0, mht. en rotationsakse igennem legemets massemidtpunkt (CM), kan inertimomentet, I_p, mht. en parallelforskudt rotationsakse findes ud fra flytningsformlen:

$$\boxed{I_p = I_0 + Md^2}$$

M = legemets totale masse

d = afstanden mellem de to rotationsakser *O* og *P*

Eftervisning:

$$I_P = \int r'^2 dm\,, \quad hvor\ r'^2 = d^2 + r^2 - 2rd\cos\theta$$

$$I_P = \int (r^2 + d^2 - 2rd\cos\theta)dm \Rightarrow$$

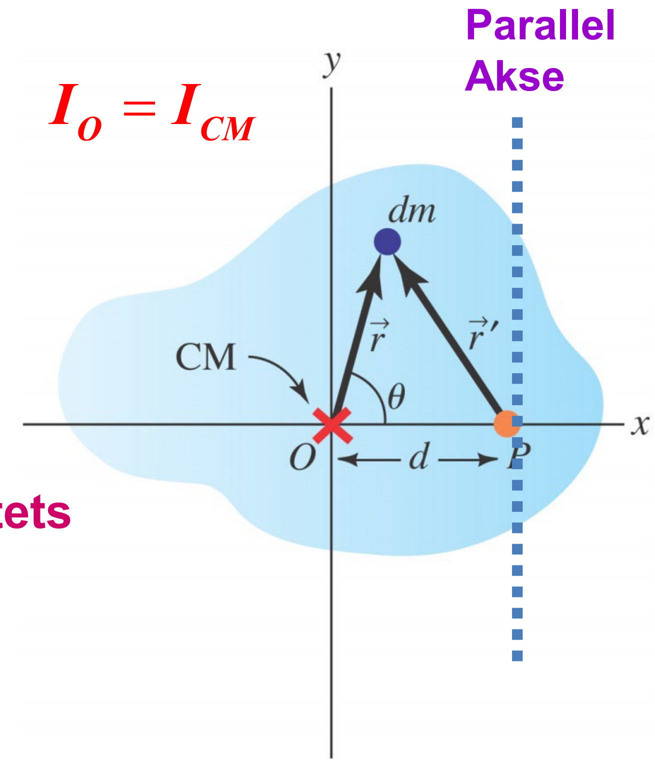
$$I_P = \int r^2 dm + d^2\int dm - 2d\int r\cos\theta dm \Rightarrow$$

$$I_P = I_{CM} + Md^2 - 2d\boxed{\int xdm}$$

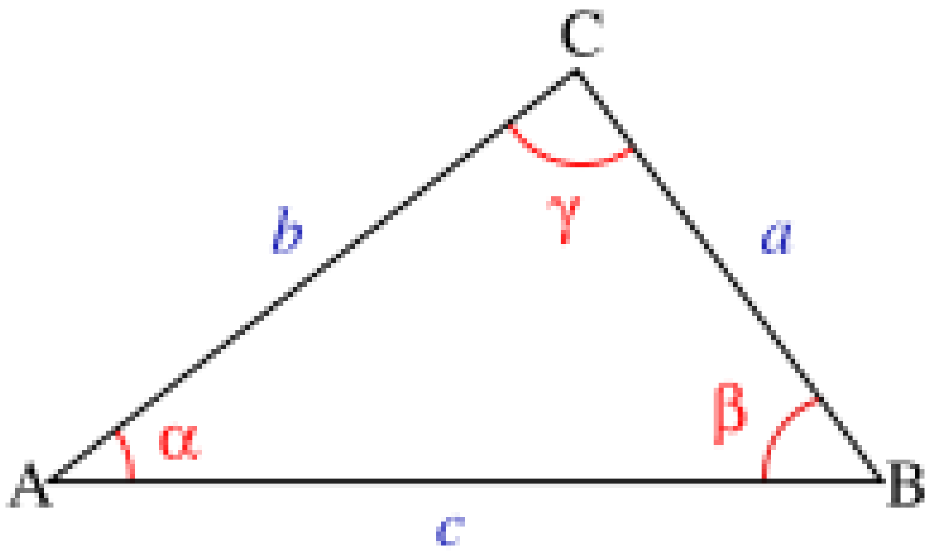
Massemidtpunktets x-position = 0

$$\boxed{I_P = I_{CM} + Md^2}$$

$I_O = I_{CM}$



Cosinus relationer (hvis man har glemt dem!)



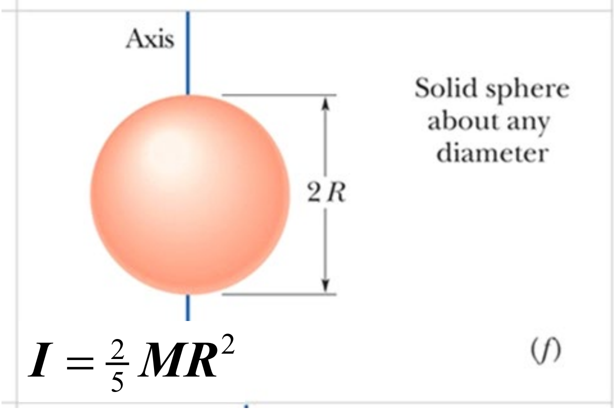
$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

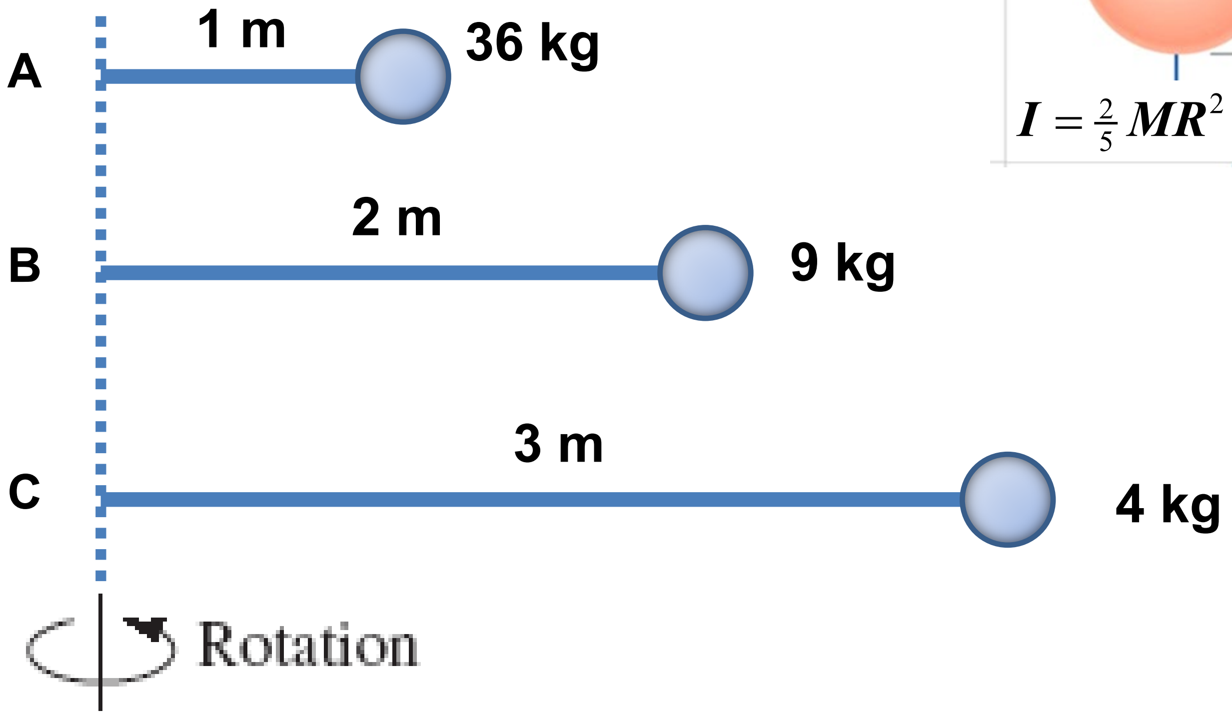
$$\boxed{a^2 = b^2 + c^2 - 2bc\cos\alpha}$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta.$$

Masseinertimoment (igen)

Figuren viser tre homogene kugler (radius 5 cm) der roterer omkring en lodret akse. Den vinkelrette afstand mellem aksen og centrum af hver kugle er vist.
Sorter de tre kugler efter deres masseinertimoment, det mindste skrives først





A: 36+2/5*36*R^2

B: 36+2/5*9*R^2

C: 36+2/5*4*R^2

Kraftmoment

Årsagen til et legemes rotation er, at det bliver påvirket af et kraftmoment, $\vec{\tau}$.

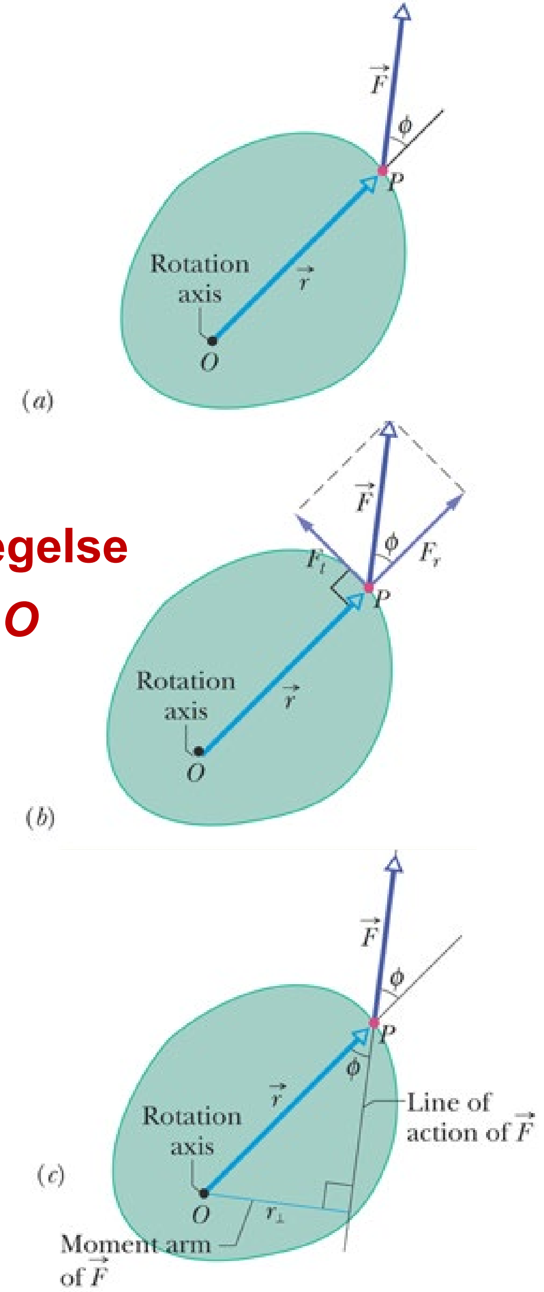
Størrelsen af et kraftmoment er bestemt af en kraft, en 'momentarm' og vinklen mellem disse to:

$$\tau = rF_t = r(F\sin\phi) = rF\sin\phi$$

$$\tau = r_{\perp}F = (r\sin\phi)F = rF\sin\phi$$

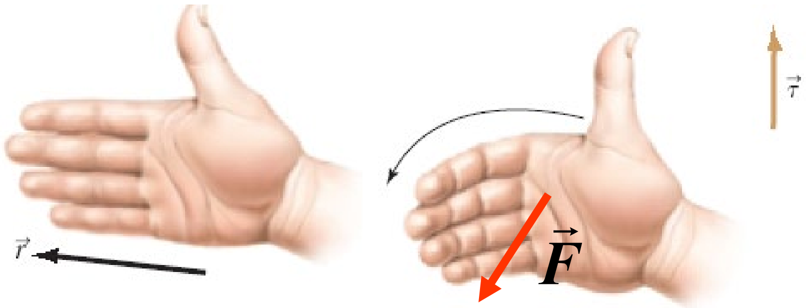
$F_r \rightarrow$ lineær bevægelse

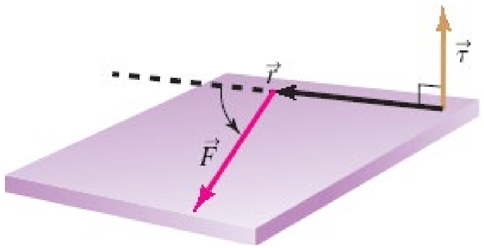
$F_t \rightarrow$ rotation om O



Eller som vektorligning, hvor retningen af kraftmomentet er givet ved højrehåndsreglen:

$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$





Kraftmoment - bevægelsesligninger

En ekstern kraft, $\vec{F}_i$, virker på et element, *i*, af en stang. Vi antager at: $\vec{F}_i \perp \vec{r}_i$

Opskrives Newton's 2. lov for det i'te element, som får tagentialaccelerationen, $\vec{a}_i$:

$$F_i = m_i a_i = m_i(\alpha r_i) \Rightarrow$$

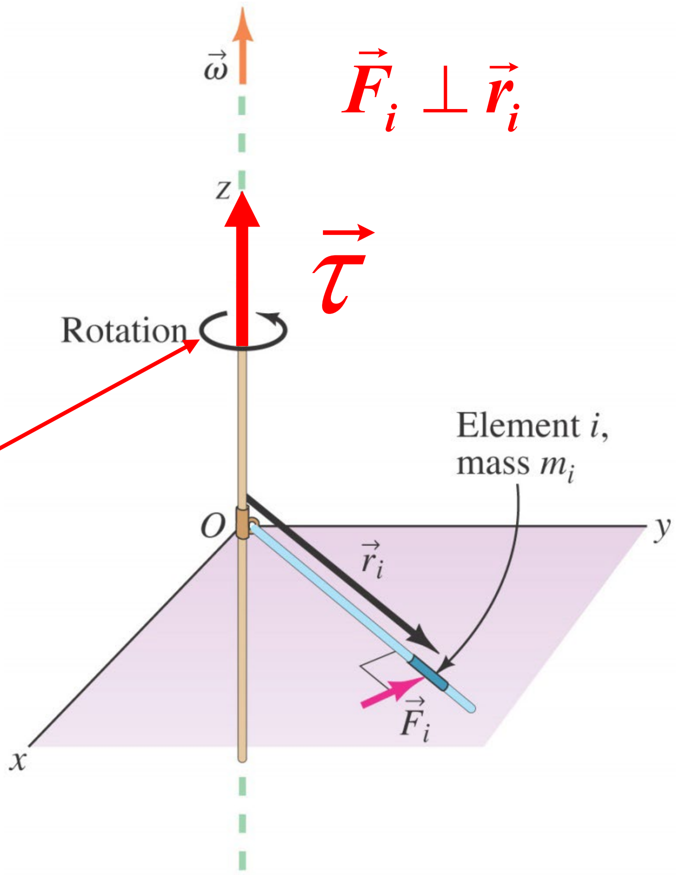
$$\tau_i = r_i F_i = m_i r_i^2 \alpha \Rightarrow$$

$$\tau = \sum_i \tau_i = \left(\sum_i m_i r_i^2 \right) \alpha$$

$$\boxed{\vec{\tau} = I\vec{\alpha}}$$

$$I = \sum_i m_i r_i^2$$

Newton's 2-lov for roterende systemer

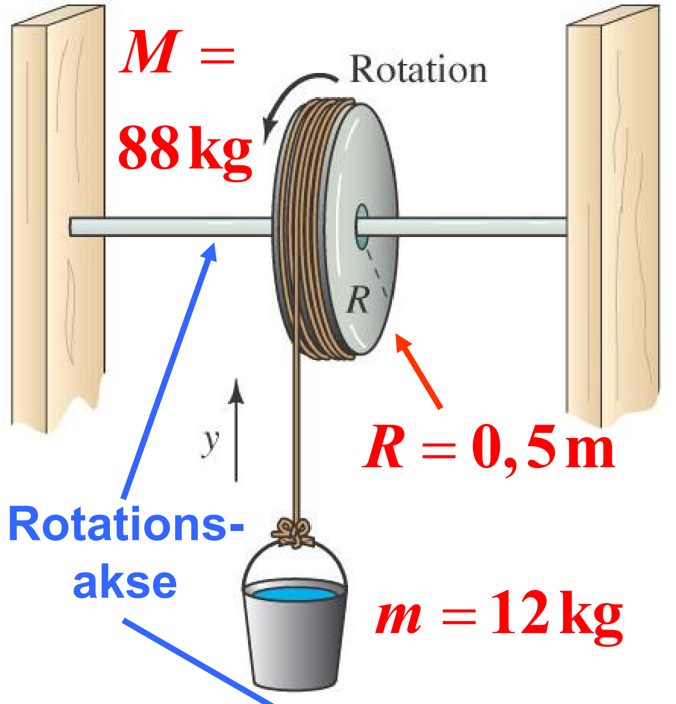


Vektorsummen af alle kraftmomenter = masseinertimoment gange vinkelaccelerationen

Frit-legemeanalyse - igen

1. Identificér og isolér legemet, som de eksterne kræfter virker på.
2. Identificér de eksterne kræfter, som virker på legemet, og hvor kræfterne virker på legemet!
3. Skitsér et frit-legemediagram, hvor alle kræfter og <u>deres angrebspunkt</u> er indtegnet. Sidstnævnte er meget vigtigt mht. beregning af kraftmomentet.
   Bemærk: Tyngdekraften indtegnes med angrebspunkt i masse-midtpunktet.
4. Identificér en enkelt akse, som normalt vil være rotationsaksen for legemet.
5. Find kraftmomenterne omkring denne akse hidrørende fra de eksterne kræfter. Retningen af rotationen findes ud fra højrehåndsreglen.
6. Opskriv bevægelsesligningerne som indeholder netto-kraftmomentet. (dette indebærer ofte en udregning af legemets masseinertimoment, *I*, mht. rotationsaksen).
7. Netto-kraften bestemmer massemidtpunktets bevægelse!

Eksempel på Frit-legemeanalyse



Find trissens vinkelhastighed efter at spanden er faldet i *t=5 s*.

Frit-legemediagrammer med indtegnede kræfter skitseres.

For spanden i y'-retningen: $mg - T = ma$

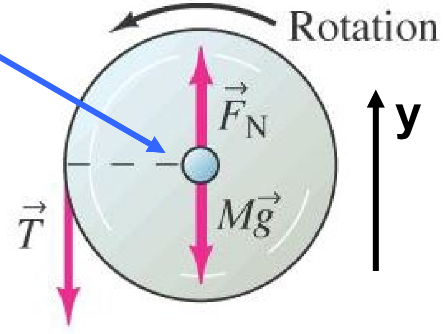
(For trissen i y-retningen: $F_N - T - Mg = 0$ **)**

Kraftmomentet på trissen: $\tau = TR = I_{trisse}\alpha$

Trissens inertimoment: $I_{trisse} = \frac{1}{2}MR^2$

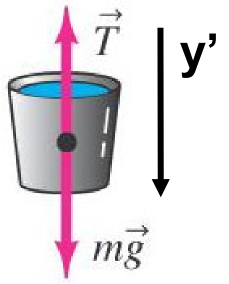
Endvidere er sammenhængen mellem spandens acceleration, *a*, og trissens vinkelacceleration, *α*:

$$a = \alpha R$$



$$\tau = TR = m(g - \alpha R)\cancel{R} = \frac{1}{2}MR^{\cancel{2}}\alpha \Leftrightarrow \alpha = \frac{mg}{\left[m + (M/2)\right]R}$$

$$\omega = \cancel{\omega_0} + \alpha t = \frac{mgt}{\left[m + (M/2)\right]R} \Rightarrow \boxed{\omega = \underline{\underline{21\ \mathrm{rad/s}}}}$$



Arbejde og energi i rotation

Arbejde-kinetisk energi-teorem for et roterende system:

$$\boxed{\Delta K = K_f - K_i = \tfrac{1}{2} I\omega_f^2 - \tfrac{1}{2} I\omega_i^2 = W}$$

Det udførte arbejde i en rotation omkring en fast akse:

$$W = \int_{s_i}^{s_f} \vec{F}\cdot d\vec{s} = \int_{\theta_i}^{\theta_f} \frac{\vec{\tau}}{R}\cdot R\cdot d\vec{\theta} \Rightarrow \boxed{W = \int_{\theta_i}^{\theta_f} \vec{\tau}\cdot d\vec{\theta}}$$

Som for konstant kraftmoment reduceres til:

$$\boxed{W = \tau\left(\theta_f - \theta_i\right)}$$

Effekt i rotationsbevægelse

Sammenhængen mellem arbejde og effekt er:

$$\boxed{P = \frac{dW}{dt}}$$

Og dermed bliver:

$$\boxed{P = \frac{\vec{\tau} \cdot d\vec{\theta}}{dt} = \vec{\tau} \cdot \frac{d\vec{\theta}}{dt} = \vec{\tau} \cdot \vec{\omega}}$$

Analogi mellem fysiske størrelser i translations- og rotationsbevægelse

Table 10-3 Some Corresponding Relations for Translational and Rotational Motion

Pure Translation (Fixed Direction)		Pure Rotation (Fixed Axis)	
Position	x	Angular position	θ
Velocity	$v = dx/dt$	Angular velocity	$\omega = d\theta/dt$
Acceleration	$a = dv/dt$	Angular acceleration	$\alpha = d\omega/dt$
Mass	m	Rotational inertia	I
Newton's second law	$F_{net} = ma$	Newton's second law	$\tau_{net} = I\alpha$
Work	$W = \int F\,dx$	Work	$W = \int \tau\,d\theta$
Kinetic energy	$K = \frac{1}{2}mv^2$	Kinetic energy	$K = \frac{1}{2}I\omega^2$
Power (constant force)	$P = Fv$	Power (constant torque)	$P = \tau\omega$
Work–kinetic energy theorem	$W = \Delta K$	Work–kinetic energy theorem	$W = \Delta K$

Hooks lov: $\boldsymbol{F_{fj} = -k \cdot x}$ $\boldsymbol{\tau_{fj} = -\kappa \cdot \theta}$

Opsummering

Stift legemes rotation

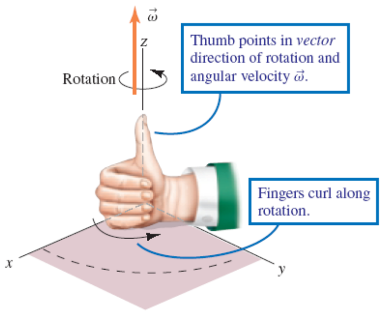
Rotationsvinkel: $\vec{\theta}(t) = \theta(t)\,\hat{\theta} \quad [\text{radianer}]$

Vinkelhastighed: $\vec{\omega} = \dfrac{d\theta(t)}{dt}\,\hat{\omega} \quad [s^{-1}]$

Vinkelacceleration: $\vec{\alpha} = \dfrac{d\omega(t)}{dt}\,\hat{\alpha} = \dfrac{d^2\theta}{dt^2}\,\hat{\alpha} \quad [\text{s}^{-2}]$

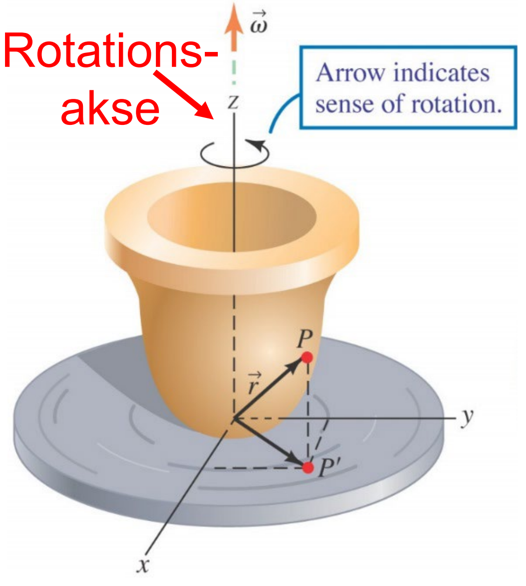
Enhedsvektorerne:

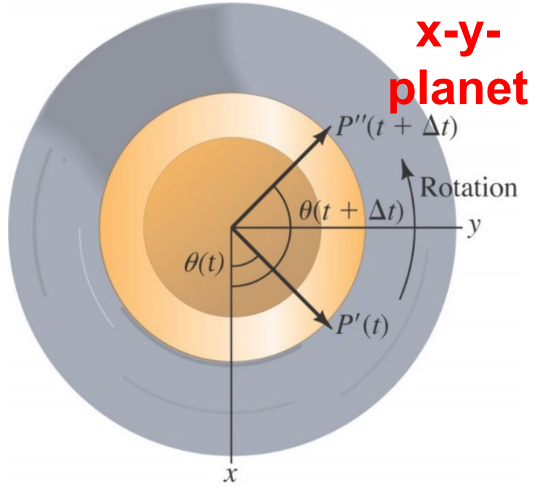
$\hat{\theta}$, $\hat{\omega}$ og $\hat{\alpha}$ er defineret således at:



Rotation mod urets retning: Positiv

Rotation med urets retning: Negativ





Opsummering

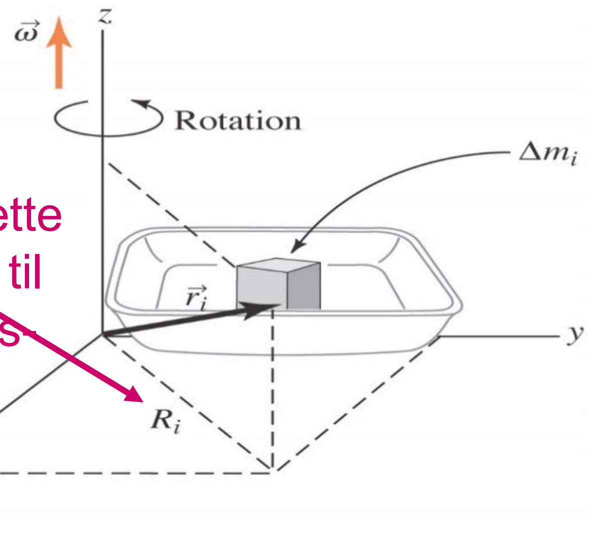
Analogi mellem translation og rotation ved konstant acceleration og konstant vinkelacceleration

Linear Equation	Missing Variable		Angular Equation
$v = v_0 + at$	$x - x_0$	$\theta - \theta_0$	$\omega = \omega_0 + \alpha t$
$x - x_0 = v_0 t + \frac{1}{2}at^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	t	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	α	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
$x - x_0 = vt - \frac{1}{2}at^2$	v_0	ω_0	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$

Inertimoment

$$I \equiv \sum_i \Delta m_i R_i^2 \qquad \left[\mathrm{kg} \cdot \mathrm{m}^2\right]$$

Vinkelrette afstand til rotations-aksen



Opsummering

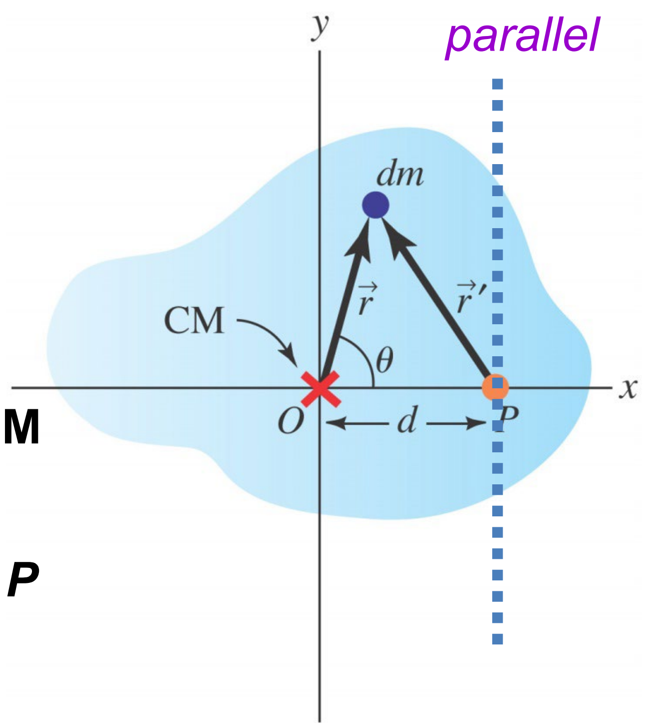
Flytningsformlen

$$I_p = I_{CM} + Md^2$$

I_{CM} = Inertimoment ifht. rotationsakseigennem CM

M = legemets totale masse

d = afstanden mellem de to rotationsakser O og P



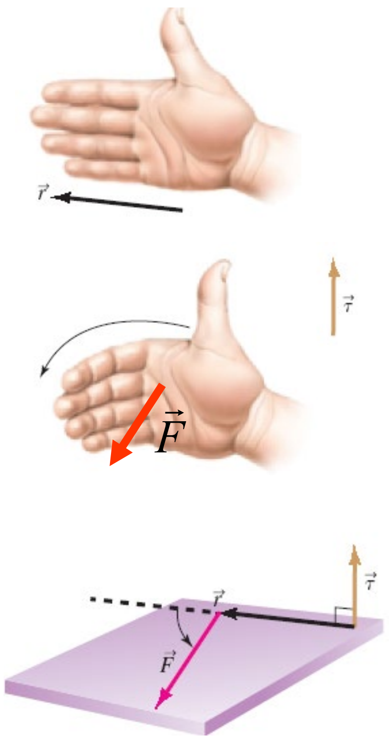
Rotationsenergi

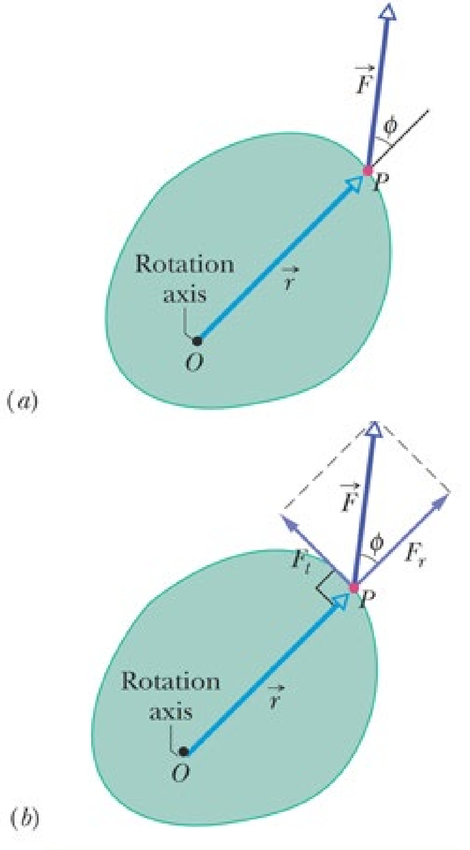
$$K = \frac{1}{2} I\omega^2$$

Opsummering

Kraftmoment

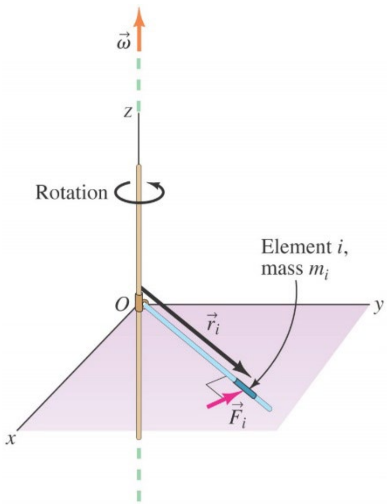
$$\vec{\tau} = \vec{r} \times \vec{F}$$





Newton's 2. lov for roterende bevægelse

$$\vec{\tau}_{net} = I\vec{\alpha}$$



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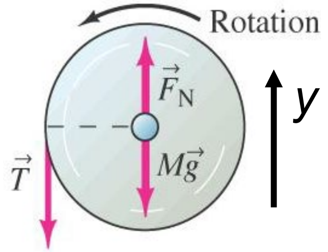
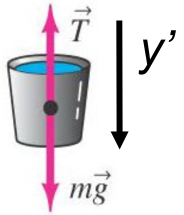


Frit-legemeanalyse inklusiv rotation



1. Identificér og isolér legemet, som de eksterne kræfter virker på.
2. Identificér de eksterne kræfter, som virker på legemet, og hvor kræfterne virker på legemet!
3. Skitsér et frit-legemediagram, hvor alle kræfter og deres angrebs-punkt er indtegnet. Sidstnævnte er meget vigtigt mht. beregning af kraftmomentet. Bemærk: Tyngdekraften indtegnes med angrebspunkt i masse-midtpunktet.
4. Identificér en enkelt akse, som normalt vil være rotationsaksen for legemet.
5. Find kraftmomenterne omkring denne akse hidrørende fra de eksterne kræfter. Retningen af rotationen findes ud fra højre-håndsreglen.
6. Opskriv bevægelsesligningerne som indeholder netto-kraftmomentet. (dette indebærer ofte en udregning af legemets inertimoment, *I*, mht. rotationsaksen).
7. Netto-kraften bestemmer massemidtpunktets bevægelse!

Opsummering

Arbejde-energi teorem

$$\Delta K = K_f - K_i = \tfrac{1}{2} I\omega_f^2 - \tfrac{1}{2} I\omega_i^2 = W$$

$$W = \int_{\theta_i}^{\theta_f} \vec{\tau} \cdot d\vec{\theta}$$

Mekanisk effekt i rotationsbevægelse

$$P = \frac{dW}{dt} = \frac{\vec{\tau} \cdot d\vec{\theta}}{dt} = \vec{\tau} \cdot \vec{\omega}$$

Opsummering

Analogi mellem fysiske størrelser i translation og rotation

Table 10-3 Some Corresponding Relations for Translational and Rotational Motion

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Hooks lov: $\boldsymbol{F_{fj} = -k \cdot x}$ $\boldsymbol{\tau_{fj} = -\kappa \cdot \theta}$

Fysik 1 – lektion 10

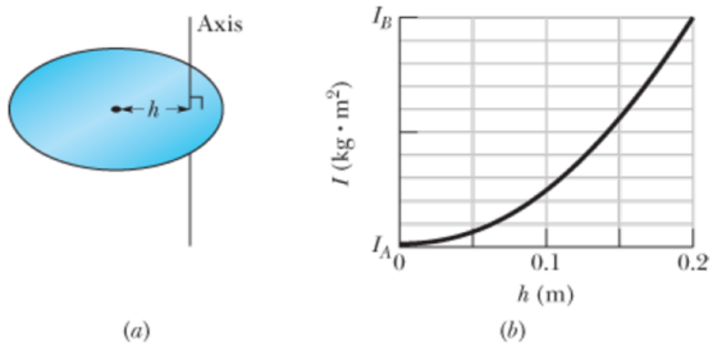
Næste lektion:
Pendulbevægelsen og rulning.

- Torsionspendul
- Det matematiske pendul
- Det fysiske pendul
- Fysisk versus matematisk pendul
- Pendul med *store* udsving
- Rulning

$$\boxed{T = 2\pi\sqrt{\frac{I}{Mgr}}}$$

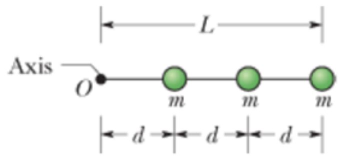
Hjemmeforberedelse opgaver:

Opgave 1



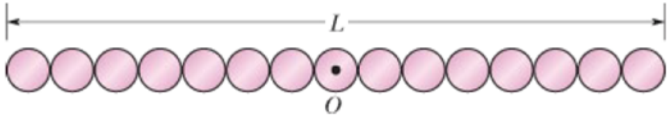
The figure (a) shows a disk that can rotate about an axis at a radial distance h from the center of the disk. Figure (b) gives the rotational inertia I of the disk about the axis as a function of that distance h, from the center out to the edge of the disk. The scale on the I axis is set by I_A=0.050 kg·m^2 and I_B=0.150 kg·m^2. What is the mass of the disk?

Opgave 2



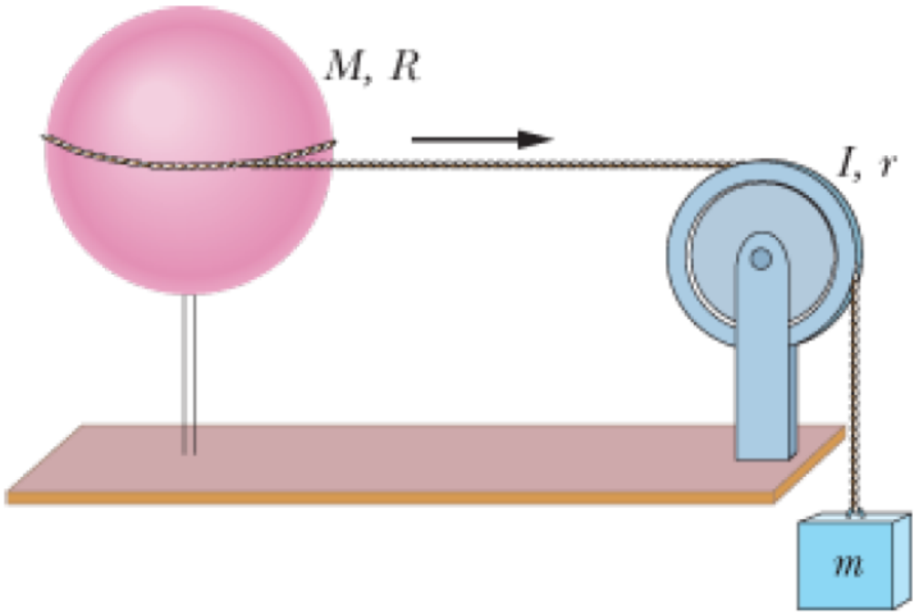
The figure shows three 0.0100 kg particles that have been glued to a rod of length L=8.00 cm and negligible mass. The assembly can rotate around a perpendicular axis through point O at the left end. If we remove one particle (that is, 33% of the mass), by what percentage does the rotational inertia of the assembly around the rotation axis decrease when that removed particle is (a) the innermost one and (b) the outermost one?

Opgave 3



The figure shows an arrangement of 15 identical disks that have been glued together in a rod-like shape of length L= 1.0000 m and (total) mass M= 100.0 mg . The disks are uniform, and the disk arrangement can rotate about a perpendicular axis through its central disk at point O. (a) What is the rotational inertia of the arrangement about that axis? (b) If we approximated the arrangement as being a uniform rod of mass M and length L, what percentage error would we make in using the formula in Table 10-2e to calculate the rotational inertia?

Opgave 4

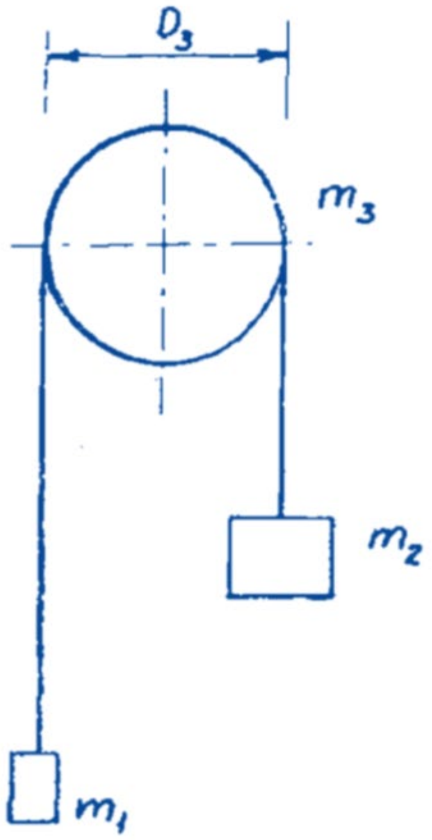


A uniform spherical shell of mass M=4.5 kg and radius R=8.5 cm can rotate about a vertical axis on frictionless bearings (See the figure). A massless cord passes around the equator of the shell, over a pulley of rotational inertia $I = 3.0 \times 10^{-3}\ kg \cdot m^2$ and radius r= 5.0 cm, and is attached to a small object of mass m=0.60 kg. There is no friction on the pulley's axle; the cord does not slip on the pulley. What is the speed of the object when it has fallen 82 cm after being released from rest? Use energy considerations.

Opgave 5

Opgave 12 *(IOT-02-I-3)*

Opgave på klassen



m_1 = 2,0 kg

m_2 = 4,0 kg

m_3 = 12.0 kg

D_3 = 0,28 m

Figuren viser to lodder, som er forbundet med en let bøjelig, masseløs snor.
Snoren føres over en cylindrisk skive med massen m_3 og udvendig diameter D_3.
Systemet regnes friktionsfrit, og der tages ikke hensyn til luftmodstanden.
Til tiden $t_0 = 0$ slippes systemet, som da er i hvile, dvs. $v_0 = 0$ m/s.

a) Beregn massen m_1's acceleration, a_1
b) Beregn snorekraften umiddelbart over loddet med massen m_2.
c) Beregn systemets kinetiske energi efter 5 sekunders forløb.