Introduction to Control

Control Engineering (Reguleringsteknik)

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University of Southern Denmark

Agenda

- Introduction
  - Course Overview
  - Motivation
- Open Loop Control
- Feedback Control
  - Stability
  - Disturbance Rejection
  - Sensitivity Analysis
  - Linearization
- Steady State Tracking
- Cascade Control

Introduction

Curriculum for Reguleringsteknik (REG)

Matematiske og grafiske metoder til syntese af lineære tidsinvariante systemer:[1]

- diskret og kontinuert tilstandsbeskrivelse
- analyse i tid og frekvens
- stabilitet, reguleringshastighed, følsomhed og fejl
- digitale PI, PID, LEAD og LAG regulatorer (serieregulatorer)
- tilstandsregulering, pole-placement og tilstands-estimering (observer)
- optimal regulering (least squares) og optimal tilstands-estimation (Kalman-filter)

Færdigheder:
Efter gennemførelse af kurset kan den succesfulde studerende:

- kunne analysere, dimensionere og implementere såvel kontinuert som tidsdiskret regulering af lineære tidsinvariante og stokastiske systemer

Kompetencer:
Efter gennemførelse af kurset kan den succesfulde studerende:

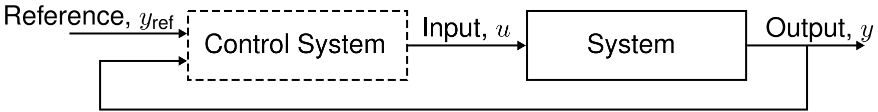
- anvende og implementere klassiske og moderne reguleringsteknikker for at kunne styre og regulere en robot hurtig og præcist

[1] Based on https://fagbesk.sam.sdu.dk/?fag_id=39673

The twelve lectures of the course are

- **Lecture 1**: Introduction to Linear Time-Invariant Systems
- **Lecture 2**: Stability and Performance Analysis
- **Lecture 3**: Introduction to Control
- **Lecture 4**: Design of PID Controllers
- **Lecture 5**: Root Locus
- **Lecture 6**: The Nyquist Plot
- **Lecture 7**: Dynamic Compensators and Stability Margins
- **Lecture 8**: Implementation
- **Lecture 9**: State Feedback
- **Lecture 10**: Observer Design
- **Lecture 11**: Optimal Control (Linear Quadratic Control)
- **Lecture 12**: The Kalman Filter

Task: Design a cruise control for a car.



- **Control Input**: Throttle position u
- **Measured Output**: Velocity of the car y
- **Reference Input**: Desired velocity of the car y_{ref}

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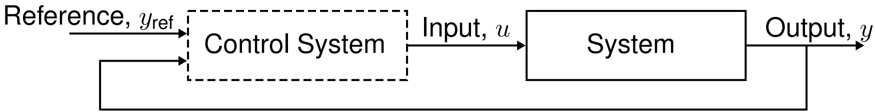

Today, answers to the following questions are provided:

1. How should the control input u be designed such that a desired velocity y_{ref} is reached?

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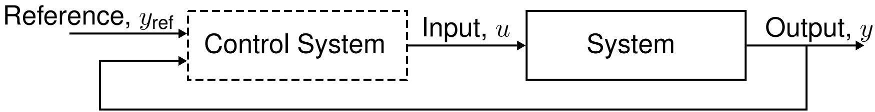

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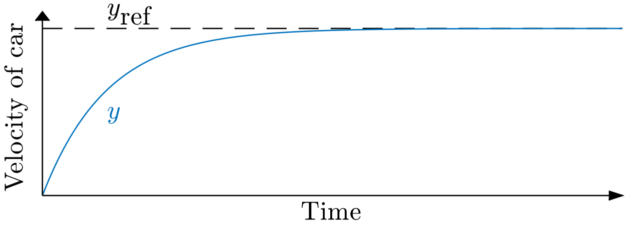

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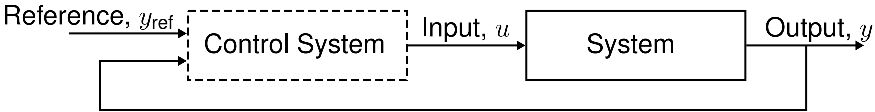

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2. Can y_{ref} be reached despite uncertainties in the system?

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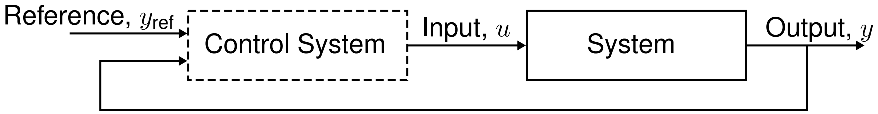

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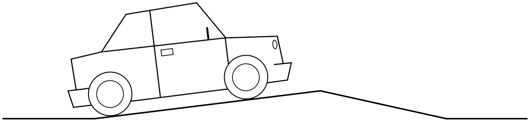


Open Loop Control

Example (1)

How should $D(s)$ be chosen to ensure that the desired constant velocity r (in km/h) is reached?

$$r(s) \longrightarrow \boxed{D(s)} \xrightarrow{u(s)} \boxed{G(s)} \xrightarrow{y(s)}$$

Open Loop Control
Example (1)

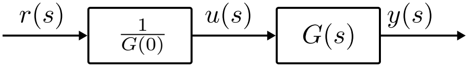
How should $D(s)$ be chosen to ensure that the desired constant velocity r (in km/h) is reached?

$$r(s) \longrightarrow \boxed{D(s)} \xrightarrow{u(s)} \boxed{\frac{150}{10s+1}} \xrightarrow{y(s)}$$

Consider an example where the step response of the system is given as follows, where u is the throttle position and y is the speed in km/h.

Open Loop Control
Example (2)

A first approach to controlling the system could be to use $D(s) = \frac{1}{G(0)}$. Then the desired constant velocity r is reached.



Steady-State Value of Time Function

Possible Final Values

Suppose that $Y(s)$ is the Laplace transform of $y(t)$. Then the final value of $y(t)$ is either

- **Unbounded**.
- **Undefined**.
- **Constant**.

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- **Unbounded**. If $Y(s)$ has any poles in the open right half-plane.
- **Undefined**. If $Y(s)$ has a pole pair on the imaginary axis.
- **Constant**. If all poles of $Y(s)$ are in the open left half-plane, except for one at $s = 0$.

If all poles of $sY(s)$ are in the open left half of the s-plane, then

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s).$$

Steady-State Value of Time Function

The Final Value Theorem

If all poles of $sY(s)$ are in the open left half of the s-plane, then

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s).$$

The ***Final Value Theorem*** determines the constant value that the impulse response of a stable system converges to.

Steady-State Value of Time Function

DC Gain

The Final Value Theorem can also by used to determine the DC gain of a system, i.e., the output when a step input is applied to the system.

Steady-State Value of Time Function

DC Gain

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Recall that the impulse response of the transfer function $1/s$ is a constant (this is also seen from the Final Value Theorem).

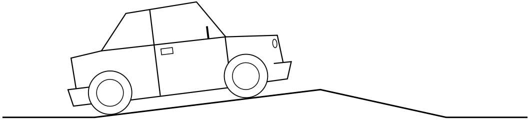
The Final Value Theorem can also by used to determine the DC gain of a system, i.e., the output when a step input is applied to the system.

Recall that the impulse response of the transfer function $1/s$ is a constant (this is also seen from the Final Value Theorem). Thus, the DC gain of a system $G(s)$ is

$$\lim_{s\to 0} sG(s)\frac{1}{s} = \lim_{s\to 0} G(s).$$

Open Loop Control
Example (3)

Disturbances affect all systems; as an example, the inclination of the road changes.



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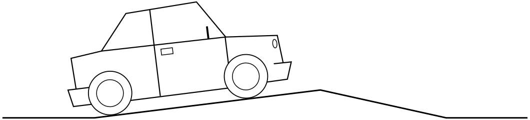
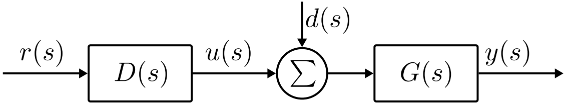


Block diagram of the system including a disturbance.



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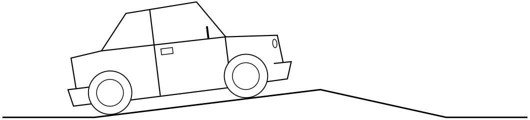

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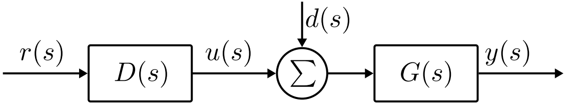
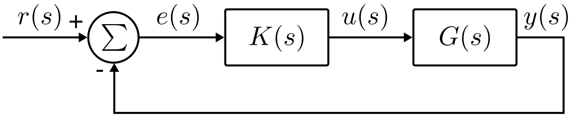


The open loop control system, with $D(s) = \frac{1}{G(0)}$, only reaches the reference if $d(s) = 0$!

Feedback Control

Definition

The connection of a controller $K(s)$ and a system (also called plant) $G(s)$ is called a closed-loop system.



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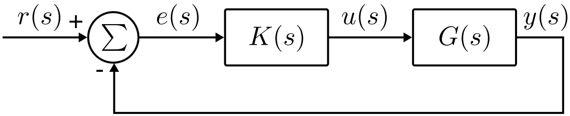

The closed-loop transfer function is derived in the following

$$\begin{cases} y(s) = G(s)u(s) \\ u(s) = K(s)(r(s) - y(s)) \end{cases}$$

$$\begin{aligned} y(s) &= G(s)K(s)(r(s) - y(s)) \\ (1 + G(s)K(s))y(s) &= G(s)K(s)r(s) \\ \frac{y(s)}{r(s)} &= \frac{G(s)K(s)}{1 + G(s)K(s)} \end{aligned}$$

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$$\frac{y(s)}{r(s)} = \frac{G(s)K(s)}{1 + G(s)K(s)}$$

The closed loop system is stable when the closed-loop poles are in the open left half plane.
The ***loop gain*** is defined as $L(s) = G(s)K(s)$. Then the closed-loop poles are given by

$$1 + L(s) = 0.$$

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$$1 + L(s) = 0.$$

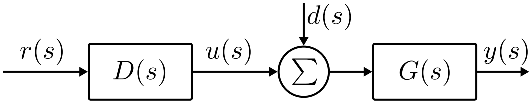
The closed loop poles satisfy

$$L(s) = -1$$

Disturbance Rejection

How does feedback affect the disturbance rejection properties of a control system?

Open loop control system



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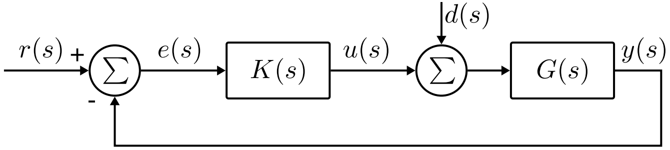

The output $y(s)$ is given as a function of inputs $d(s)$ and $r(s)$ in the following.

Open loop control system
The output $y(s)$ is given by

$$y(s) = G(s)d(s) + G(s)D(s)r(s)$$

Closed loop control system
The output $y(s)$ is given by (superposition can be applied for deriving the expression)

$$y(s) = \frac{G(s)}{1 + G(s)K(s)}d(s) + \frac{G(s)K(s)}{1 + G(s)K(s)}r(s)$$

Disturbance Rejection

Example (1)

Consider the speed control of an engine, which is affected by an unknown load torque τ_l and has dynamics given by

$$J\dot{\omega} + b\omega = Au + A\tau_l$$

where J is the inertia, b is a viscous friction, u is the control torque, and A is a constant parameter.

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The dynamics of the system can be written as

$$\omega(s) = \underbrace{\frac{A}{\tau s + 1}}_{=G(s)} u(s) + \frac{A}{\tau s + 1}\tau_l(s)$$

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Apply the feedback control $K(s)$ ($u(s) = -K(s)\omega(s)$) then

$$\omega(s) = \frac{G(s)}{1 + G(s)K(s)}\tau_l(s)$$

Disturbance Rejection
Example (2)

We aim to compare the steady state error of the open loop and closed loop system.

#ERROR!

Disturbance Rejection

Example (2)

We aim to compare the steady state error of the open loop and closed loop system.

Example of system parameters

- **System**: DC gain of system is given by $G(0) = 1$.
- **Controller**: The controller is given by $K(s) = 99$.
- **Disturbance**: The controller is assumed to be a step of amplitude $w_0 = 1$.

By the *final value theorem*, we compute the steady state value of open-loop and closed-loop systems.

Open-loop control

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sG(s)\frac{w_0}{s} = 1$$

Closed-loop control

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\frac{G(s)}{1+G(s)K(s)}\frac{w_0}{s} = \frac{1}{100}$$

Disturbance Rejection
Example (2)

We aim to compare the steady state error of the open loop and closed loop system.

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$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sG(s)\frac{w_0}{s} = 1$$

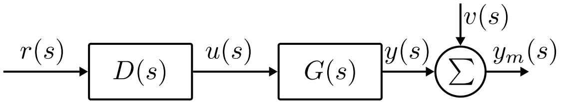
Closed-loop control

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\frac{G(s)}{1+G(s)K(s)}\frac{w_0}{s} = \frac{1}{100}$$

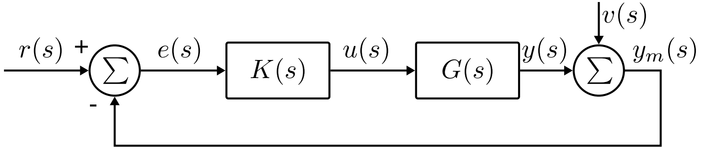
Conclusion: The sensitivity of the disturbance is reduced to 1/100 by feedback control.

How does feedback affect the noise rejection properties of a control system?

Open loop control system



Closed loop control system



The output $y_m(s)$ is given as a function of inputs $v(s)$ and $r(s)$ in the following.

Open loop control system
The measured output $y_m(s)$ is given by

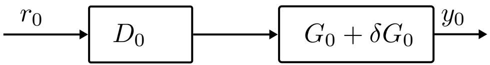
$$y_m(s) = G(s)D(s)r(s) + v(s)$$

Closed loop control system
The output $y_m(s)$ is given by (superposition can be applied for deriving the expression)

$$y_m(s) = \frac{1}{1 + G(s)K(s)}v(s) + \frac{G(s)K(s)}{1 + G(s)K(s)}r(s)$$

How does feedback affect the steady state gain when the DC gain of the plant changes?

Open loop control system



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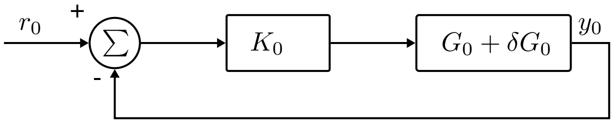

The output y_0 is given as a function of inputs r_0 and $G_0 + \delta G_0$, where $G_0 = G(s)$ for $s = 0$.

Open loop control system
The output y_0 is given by

$$y_0 = (G_0 + \delta G_0) D_0 r_0 = (G_0 D_0 + \delta G_0 D_0) r_0 = (T_{\text{ol}} + \delta T_{\text{ol}}) r_0$$

The ration of $\delta T_{\text{ol}} / T_{\text{ol}}$ to $\delta G_0 / G_0$ is the **sensitivity**, S. For the open-loop system $S = 1$.

Closed loop control system
The output y_0 is given by

$$y_0 = \frac{(G_0 + \delta G_0) K_0}{1 + (G_0 + \delta G_0) K_0} r_0$$

The sensitivity of the closed-loop system is $1/(1 + G_0 K_0)$.

Consider a nonlinear nth order differential equation

$$z^{(n)} = f(z, \ldots, z^{(n-1)}, u)$$

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To approximate the differential equation in the neighborhood of a point $p = (\bar{z}, \dots, \bar{z}^{(n-1)}, \bar{u})$, first-order Taylor approximation is applied

$$z^{(n)} \approx f(\bar{z}, \dots, \bar{z}^{(n-1)}, \bar{u}) + \frac{\partial f}{\partial z}|_p \hat{z} + \cdots + \frac{\partial f}{\partial z^{(n-1)}}|_p \hat{z}^{(n-1)} + \frac{\partial f}{\partial u}|_p \hat{u}$$

where $\hat{z} = z - \bar{z}$ and all partial derivatives are evaluated at the point $p = (\bar{z}, \dots, \bar{z}^{(n-1)}, \bar{u})$.

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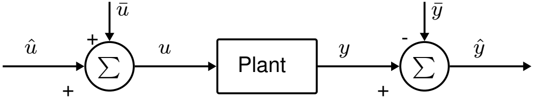
where $\hat{z} = z - \bar{z}$ and all partial derivatives are evaluated at the point $p = (\bar{z}, \ldots, \bar{z}^{(n-1)}, \bar{u})$.

The approximation is **linear** in $\hat{z}^{(i)}$.

The transfer function from $\hat{u}$ to $\hat{y}$ of the linearized model is linear.



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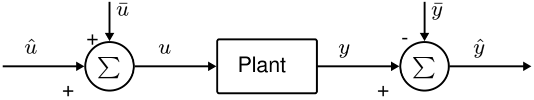

To transform the linearized model into state space form, it is seen that

$$\hat{z}^{(n)} \approx \frac{\partial f}{\partial z}|_p \hat{z} + \cdots + \frac{\partial f}{\partial z^{(n-1)}}|_p \hat{z}^{(n-1)} + \frac{\partial f}{\partial u}|_p \hat{u}$$

is on the standard form

$$z^{(n)} - a_{n-1} z^{(n-1)} - \cdots - a_1 \dot{z} - a_0 z = b_1 u_1(t) + \cdots + b_m u_m(t)$$

with

$$a_i = \frac{\partial f}{\partial z^{(i)}}|_p \quad \text{and} \quad b_i = \frac{\partial f}{\partial u_i}|_p$$

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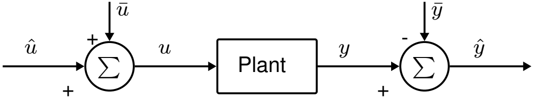

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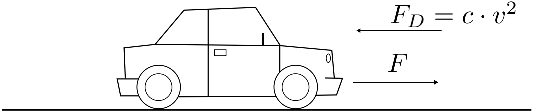
with

$$a_i = \frac{\partial f}{\partial z^{(i)}}|_p \quad \text{and} \quad b_i = \frac{\partial f}{\partial u_i}|_p$$

Then the following procedure from Lecture 1 can be followed, by defining the state variables x_i according to $x_1 = \hat{z}$, $x_2 = \hat{z}^{(1)}, \ldots, x_n = \hat{z}^{(n-1)}$.

Consider a car affected by drag.



The dynamics of the system is

$$m\dot{v} = F - \underbrace{\frac{1}{2}\rho C_D A}_{=c}\, v^2$$

where ρ is the air density [kg/m^3], A is the projected area [m^2], and C_D is the drag coefficient.

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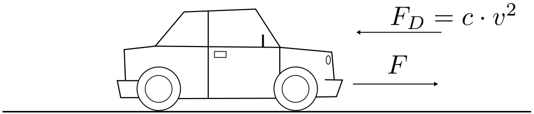

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$$m\dot{v} = F - \underbrace{\frac{1}{2}\rho C_D A}_{=c} v^2$$

where ρ is the air density [kg/m^3], A is the projected area [m^2], and C_D is the drag coefficient.

Objective: Design a controller that lets the vehicle drive at 25 m/s.

Stable systems can be classified according to its ***system type***, defined to be the degree of the polynomial for which the steady-state error is a nonzero finite constant.

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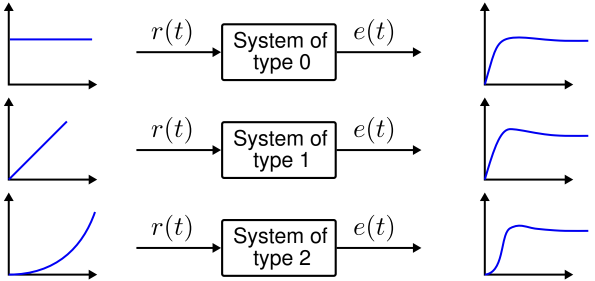

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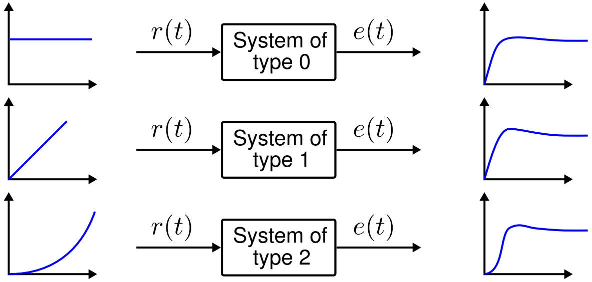

The *final value theorem* can be used to determine the system type.

An expression for the tracing error is computed to determine the steady-state error

$$e(s) = \frac{1}{1 + L(s)} r(s)$$

where $r(s)$ is the reference signal and $L(s)$ is the loop gain.

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Final value theorem is used to determine the steady state error when the reference signal is a polynomial $r(t) = t^k 1(t)$

$$\begin{aligned} e_{ss} = \lim_{t\to\infty} e(t) &= \lim_{s\to 0} e(s)s \\ &= \lim_{s\to 0} s\frac{1}{1+L(s)} r(s) \\ &= \lim_{s\to 0} s\frac{1}{1+L(s)} \frac{1}{s^{k+1}} \end{aligned}$$

The expression for the steady state error e_{ss} is simplified by rewriting the loop gain as

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where n is the number of poles at the origin of $L(s)$.

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From the above expression it is seen that the system type of a system is equal to the number of poles at the origin of the system's loop gain $L(s)$.

Steady State Tracking

	Input		
Type	Step (Position)	Ramp (Velocity)	Parabola (Acceleration)
Type 0	$\frac{1}{1+K_p}$	∞	∞
Type 1	0	$\frac{1}{K_v}$	∞
Type 2	0	0	$\frac{1}{K_a}$

Consider the system

$$G(s) = \frac{k}{\tau s + 1}$$

controlled by the PI controller

$$K(s) = k_p + \frac{k_I}{s}$$

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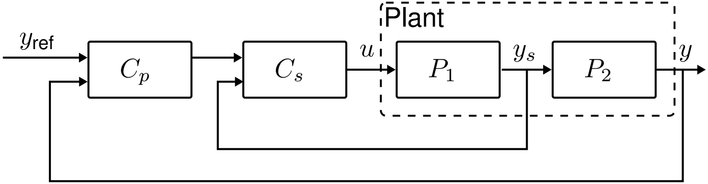
The loop gain has one pole at the origin, i.e., the system type is one.
The steady state error for ramp input is given by the velocity constant

$$K_v = \lim_{s \to 0} L_0(s) = \lim_{s \to 0} sL(s) = k \cdot k_I$$

Cascade Control

A **cascade control** uses the output from one controller as the input to another.



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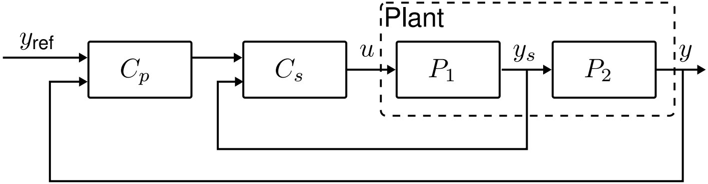

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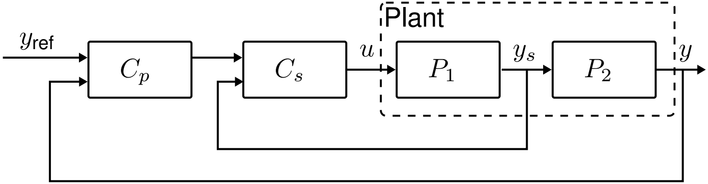

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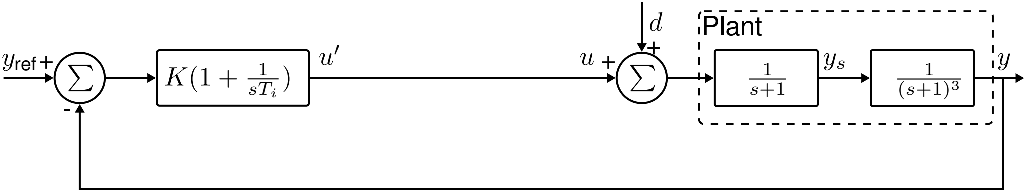
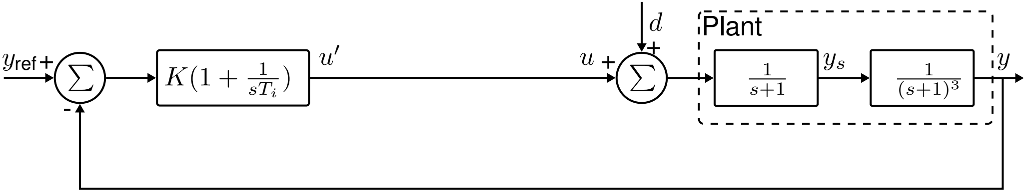
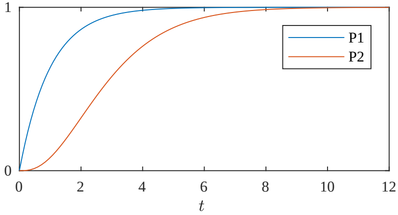
A fundamental reason for applying cascade control is to obtain ***better disturbance rejection*** and ***lower sensitivity to parameter variations***.

Single loop control



Single loop control

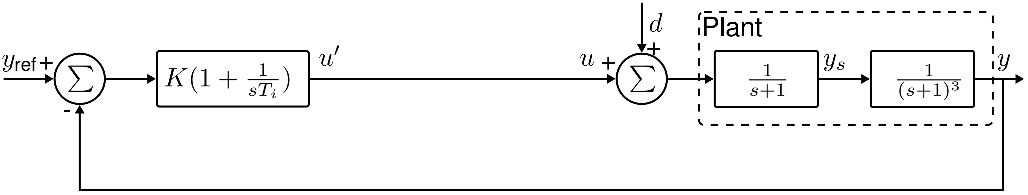




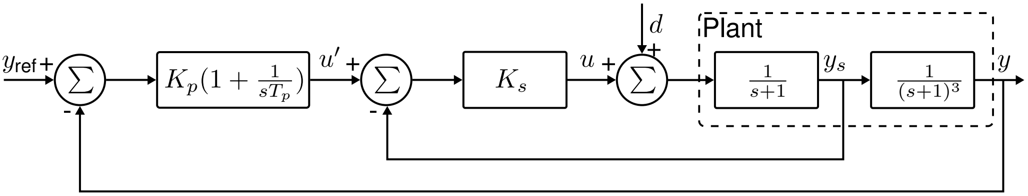
Cascade Control

Example of Disturbance Rejection (1)

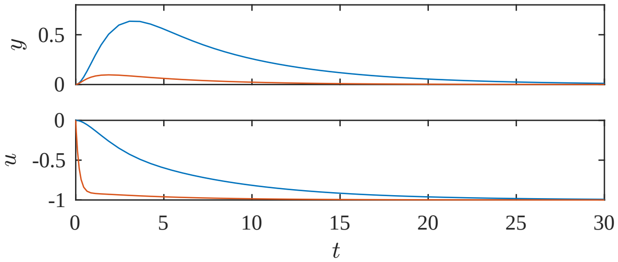
Single loop control



Cascade control



Response for step disturbance ($d = 1$)



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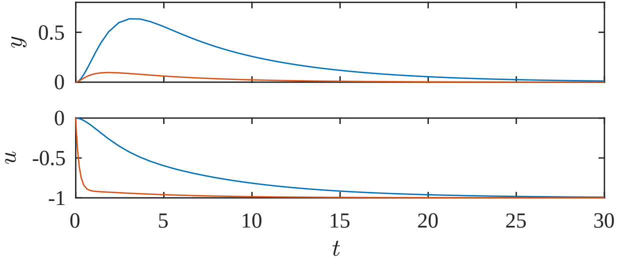

Single loop control

$$y(s) = P_2(s)P_1(s)d(s) + P_2(s)P_1(s)u'(s)$$

Cascade control

$$y(s) = \frac{P_2(s)P_1(s)}{1 + P_1(s)C_s(s)}d(s) + \frac{P_2(s)P_1(s)C_s(s)}{1 + P_1(s)C_s(s)}u'(s)$$

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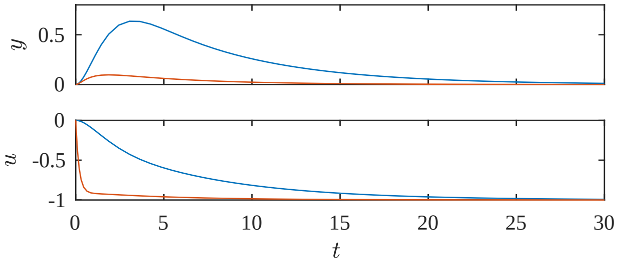

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Cascade control

$$y(s) = \underbrace{\frac{P_2(s)P_1(s)}{1 + P_1(s)C_s(s)}}_{\text{Small for large } C_s} d(s) + \underbrace{\frac{P_2(s)P_1(s)C_s(s)}{1 + P_1(s)C_s(s)}}_{\approx P_2 \text{ for large } C_s} u'(s)$$

Cascade control can be used when

- There should be a well defined relation between the primary and secondary measured variable
- Essential disturbances should act in the inner loop
- The inner loop should be faster than the outer loop. A rule of thumb is that the average residence times should have a ratio of at least five.
- should be possible to have a high gain in the inner loop.

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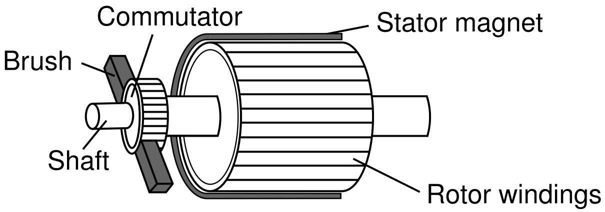
The following procedure should be followed to tune a cascade control

1. Tune inner-loop controller (C_s) such that the transfer function from u' to y_s is critically damped or over damped (unity DC gain is not a requirement).
2. Tune the outer-loop controller (C_p) such that the desired performance is obtained.


Commutator
Stator magnet
Brush
Shaft
Rotor windings

En børstet DC motors kraftmoment τ_m har størrelsen

$$\tau_m(t) = KI_a(t) \qquad [\text{Nm}]$$

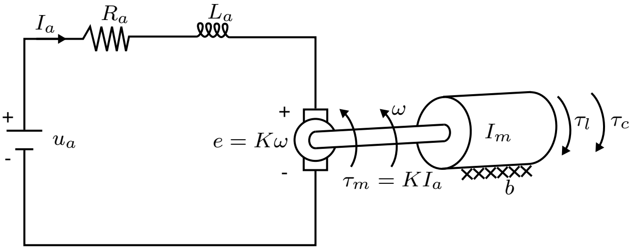
hvor K er den mekaniske motorkonstant [Nm/A] og I_a er ankerstrømmen [A].

Den elektromotoriske kraft e for en børstet DC motor er

$$e(t) = K\omega(t) \qquad [\text{V}]$$

hvor K er den elektriske motorkonstant [V/(rad/s)] og ω er motorens vinkelhastighed [rad/s].

Nu haves udtryk for motorens kraftmoment og spændingen over motoren (givet af den elektromotoriske kraft). Dermed kan følgende diagram anvendes til bestemmelse af motorens dynamik.



Cascade Control

Exercise: Motor Control

In the exercise, we consider the following cascade control of a DC motor.