

Opgave 7.1

Design et båndpasfilter med maksimal flad pasbånd, der overholder følgende specifikation

- Centerfrekvens $f_c = 1,5$ kHz.
- Pasbåndsbredde (-3 dB) $\Delta f_3 = 250$ Hz.
- Stopbåndsbredde (-40 dB) $\Delta f_{40} \leq 1,1$ kHz.

Det digitale filter skal findes ved brug af bilineær z -transformation og have en samplefrekvens på 10 kHz.

$$f_s = 10000 \text{ Hz}$$

1. Prewarping konstanten bestemmes

$$C = \cot\left(\frac{\omega_c T}{2}\right) = \cot\left(\frac{1500 \cdot \frac{1}{10000} \cdot 2\pi}{2}\right) \approx 1.9626$$

2. Prewarpede stopbåndsfrekvens

Find stopbåndsfrekvenser og afskæringsfrekvenser

$$Q = \frac{1500}{250} = 6$$

$$\Rightarrow \begin{cases} f_{a1} = 1380 \text{ Hz}, & f_{a2} = 1630 \text{ Hz} \\ f_{s1} = 1047 \text{ Hz}, & f_{s2} = 2147 \text{ Hz} \end{cases}$$

$$Q = \frac{f_c}{\Delta f}$$

$$f_1 = f_c \cdot \left(\sqrt{1 + \frac{1}{4Q^2}} - \frac{1}{2Q} \right)$$

$$f_2 = f_c \cdot \left(\sqrt{1 + \frac{1}{4Q^2}} + \frac{1}{2Q} \right)$$

Prewarp them bitcheeeeeeeeeeeeeeeeeeeeee!

$$\Omega_a = C \cdot \tan\left(\frac{\omega_a T}{2}\right)$$

$$\begin{aligned} \Omega_{a1} &= 0.908512 \\ \Omega_{a2} &= 1.10316 \\ \Omega_{s1} &= 0.669882 \\ \Omega_{s2} &= 1.5693 \end{aligned}$$

Finder formfaktoren for at finde filter ordenen

$$W_a = \frac{\Delta f_a}{f_c} \quad W_s = \frac{\Delta f_s}{f_c}, \quad F = \frac{W_s}{W_a} \quad \underline{f_c = 1}$$

$$W_a = \frac{\Omega_{a2} - \Omega_{a1}}{1 \cdot 2\pi}, \quad W_s = \frac{\Omega_{s2} - \Omega_{s1}}{1 \cdot 2\pi}$$

$$\underline{F = 4,62071}$$

Butterworth filter bruges da det har den mest konstante forstærkning

Dette er et lavpas filter

$$H_4(s) = \frac{1}{(s^2 + 0.7654s + 1)(s^2 + 1.8478s + 1)}$$

Transformer til båndpas

$$s \rightarrow \frac{1}{w_a} \cdot (s + \frac{1}{s})$$

$$H_{bp}(s) = H_{lp}(s) \Big|_{s = \frac{1}{w_a} \cdot (s + \frac{1}{s})}$$

$$= \frac{1}{1 + 84.3524 \left(\frac{1}{s} + s\right) + 3557.56 \left(\frac{1}{s} + s\right)^2 + 87891.4 \left(\frac{1}{s} + s\right)^3 + 1.08567 \times 10^6 \left(\frac{1}{s} + s\right)^4}$$

Vi denormerer filtret

$$s \rightarrow \frac{s}{w_c}$$

$$\frac{(1.10768 \times 10^{-18} s^4)}{(9488.53 + 0.0815034 s + 0.000427634 s^2 + 2.75356 \times 10^{-9} s^3 + 7.22337 \times 10^{-12} s^4 + 3.09993 \times 10^{-17} s^5 + 5.41986 \times 10^{-20} s^6 + 1.16292 \times 10^{-25} s^7 + 1.52416 \times 10^{-28} s^8)}$$

z-transformation af filter

$$s \rightarrow \zeta \cdot \frac{z-1}{z+1}$$

$$H(z) = \frac{(1.73196 \times 10^{-21} (-1 + z^2)^4)}{(0.999966 + 7.99976 z + 27.9993 z^2 + 55.9988 z^3 + 69.9988 z^4 + 55.9993 z^5 + 27.9998 z^6 + 7.99997 z^7 + 1 \cdot z^8)}$$

Create a HIGH-pass filter with $f_s = 8000$ and a cutoff at 2500 Hz and a stop band at 3500 Hz with at least -40db.

1. Find prewarping constant

$$C = \cot\left(\frac{\omega_a T}{2}\right) = \cot\left(\frac{2500 \cdot 2\pi \cdot \frac{1}{8000}}{2}\right) \approx 0,668$$

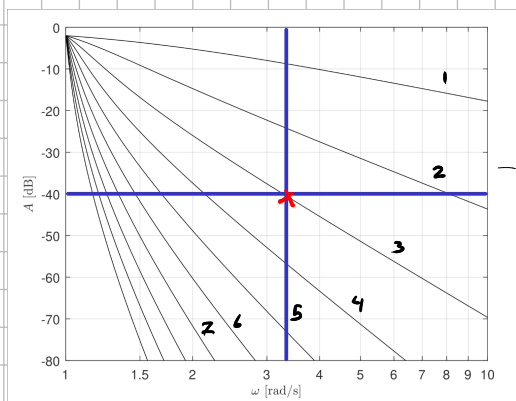
2. Prewarp and normalized frequencies

$$\Omega = C \cdot \tan\left(\frac{\omega T}{2}\right) \Rightarrow \begin{cases} \Omega_a = 1 \quad \text{We are normalizing around the cutoff frequency} \\ \Omega_s = C \cdot \tan\left(\frac{f_s \cdot 2\pi \cdot 1}{2}\right) = C \cdot \tan(3500\pi \cdot \frac{1}{8000}) \approx 3,359 \end{cases}$$

3. Choose filter with order

I choose a Chebyshev for fun!

$$F = \frac{\Omega_s}{\Omega_a} = \Omega_s = 3,359$$



$N = 3$

4. We lookup the transfer function in a table

n	Chebyshev Polynomials
1	$s + 1.965$
2	$s^2 + 1.097s + 1.102$
3	$s^3 + 0.7378s^2 + 1.0222s + 0.3269$
4	$s^4 + 0.952s^3 + 1.453s^2 + 0.742s + 0.275$
5	$s^5 + 0.7064s^4 + 1.4995s^3 + 0.6935s^2 + 0.4594s + 0.0817$

$$\Rightarrow H_n(s) = s^3 + 0,7378s^2 + 1,0222s + 0,3269$$

5. Convert to high-pass: $H_{hp}(s) = H_n(s) \Big|_{s=\frac{1}{s}} = 0,3269 + \frac{1}{s^3} + \frac{0,7378}{s^2} + \frac{1,0222}{s}$

6. De-normalize filter: $H(s) = H_{hp}(s) \Big|_{s=\frac{s}{\omega_a}} = 0,3269 + \frac{343\,000\,000\,000\,000\,\pi^3}{s^3} + \frac{3,56808 \times 10^8}{s^2} + \frac{22\,479,4}{s}$

7. Do the z-transformation

$$H(z) = H(s) \Big|_{s=C \cdot \frac{z-1}{z+1}} = 0.3269 + \frac{33642.7 (1+z)}{-1+z} + \frac{7.99189 \times 10^8 (1+z)^2}{(-1+z)^2} + \frac{3.56505 \times 10^{13} (1+z)^3}{(-1+z)^3}$$

Not sure if this is correct...