Opgave 4

Betragt differentialligningen

$$y''(t) - y'(t) = \exp(-t) \; = e^{-t}$$

hvor $y(0) = 0$ og $y'(0) = 0$.
Brug nu Laplace transformation til at løse ligningen og således bestemme $y(t)$.

$$Y(s) \cdot s^2 - Y(s) \cdot s = \frac{1}{s+1} \Rightarrow Y(s) = \frac{1}{s+1} \cdot \frac{1}{s^2 - s} = \frac{1}{s+1} \cdot \frac{1}{s-1} \cdot \frac{1}{s} = \frac{1}{(s+1)(s-1)s}$$

$$\Rightarrow Y(s) = \frac{A}{s+1} + \frac{B}{s-1} + \frac{C}{s}$$

$$k_i = (x - p_i)\, Y(x) \,|_{x=p_i}$$

$$A = (s+1) \cdot \left(\frac{1}{(s+1)(s-1)s} \right) \Big|_{s=-1} = \frac{1}{(s-1) \cdot s} = \frac{1}{(-1-1) \cdot (-1)} = \frac{1}{2}$$

$$B = (s-1) \cdot \left(\frac{1}{(s+1)(s-1)s} \right) \Big|_{s=1} = \frac{1}{(s+1) \cdot s} = \frac{1}{(1+1) \cdot 1} = \frac{1}{2}$$

$$C = s \cdot \left(\frac{1}{(s+1)(s-1)s} \right) \Big|_{s=0} = \frac{1}{(s+1)(s-1)} = \frac{1}{1 \cdot (-1)} = -1$$

$$\Rightarrow Y(s) = \frac{\frac{1}{2}}{s+1} + \frac{\frac{1}{2}}{s-1} + \frac{1}{s} = \frac{1}{2} \cdot \frac{1}{s-1} + \frac{1}{2} \cdot \frac{1}{s+1} + \frac{1}{s}$$

$$\Rightarrow y(t) = \frac{1}{2} \cdot e^{t} + \frac{1}{2} \cdot e^{-t} + 1 = \frac{1}{2}\left(e^{t} + e^{-t}\right) + 1$$