Opgave 7.1

Design et båndpasfilter med maksimal flad pasbånd, der overholder følgende specifikation

- Centerfrekvens $f_c = 1,5$ kHz.
- Pasbåndsbredde (-3 dB) $\Delta f_3 = 250$ Hz.
- Stopbåndsbredde (-40 dB) $\Delta f_{40} \leq 1,1$ kHz.

Det digitale filter skal findes ved brug af bilineær z-transformation og have en samplefrekvens på 10 kHz.

$f_s = 10000$ Hz

1. Prewarping konstanten bestemmes

$$C = \cot\left(\frac{w_c T}{2}\right) = \cot\left(\frac{1500 \cdot \frac{1}{10000} \cdot 2\pi}{2}\right) \approx 1.9626$$

2. Prewarpede stopbåndsfrekvens

Find stopbåndsfrekvenser og afskæringsfrekvenser

$$Q = \frac{1500}{250} = 6$$

$$Q = \frac{f_c}{\Delta f}$$

$$f_1 = f_c \cdot \left(\sqrt{1 + \frac{1}{4Q^2}} - \frac{1}{2Q}\right)$$

$$f_2 = f_c \cdot \left(\sqrt{1 + \frac{1}{4Q^2}} + \frac{1}{2Q}\right)$$

$$\Rightarrow \begin{cases} f_{a1} = 1380 \text{ Hz}, & f_{a2} = 1630 \text{ Hz} \\ f_{s1} = 1047 \text{ Hz}, & f_{s2} = 2147 \text{ Hz} \end{cases}$$

Prewarp them bitcheeeesssssssssssssss!

$$\Omega_a = C \cdot \tan\left(\frac{w_a T}{2}\right)$$

$\Omega_{a1} =$ 0.908512

$\Omega_{a2} =$ 1.10316

$\Omega_{s1} =$ 0.669882

$\Omega_{s2} =$ 1.5693

Finder formfaktoren for at finde filter ordenen

$$W_a = \frac{\Delta f_a}{f_c} \qquad W_s = \frac{\Delta f_s}{f_c}, \qquad F = \frac{W_s}{W_a}$$

$\underline{f_c = 1}$

$$W_a = \frac{\Omega_{a2} - \Omega_{a1}}{1 \cdot 2\pi}, \quad W_s = \frac{\Omega_{s2} - \Omega_{s1}}{1 \cdot 2\pi}$$

$$\underline{\underline{F = 4{,}62071}}$$

Butterworth filter bruges da det har den mest konstante forstærkning

Dette er et lavpas filter

$$H_4(s) = \frac{1}{(s^2 + 0.7654s + 1)(s^2 + 1.8478s + 1)}$$

Transformer til båndpas

$$s \to \frac{1}{W_a} \cdot \left(s + \frac{1}{s}\right)$$

$$H_{bp}(s) = H_{4lp}(s) \Big|_{s = \frac{1}{W_a} \cdot \left(s + \frac{1}{s}\right)}$$

$$= \frac{1}{1 + 84.3524 \left(\frac{1}{s} + s\right) + 3557.56 \left(\frac{1}{s} + s\right)^2 + 87\,891.4 \left(\frac{1}{s} + s\right)^3 + 1.08567 \times 10^6 \left(\frac{1}{s} + s\right)^4}$$

Vi denormerer filtret

$$s \to \frac{s}{\omega_c}$$

$$\left(1.10768 \times 10^{-18}\, s^4\right) / \left(9488.53 + 0.0815034\, s + 0.000427634\, s^2 + 2.75356 \times 10^{-9}\, s^3 + 7.22337 \times 10^{-12}\, s^4 + 3.09993 \times 10^{-17}\, s^5 + 5.41986 \times 10^{-20}\, s^6 + 1.16292 \times 10^{-25}\, s^7 + 1.52416 \times 10^{-28}\, s^8\right)$$

z-tranformation af filter

$$s \to c \cdot \frac{z-1}{z+1}$$

$$H(z) = \left(1.73196 \times 10^{-21} \left(-1. + z^2\right)^4\right) / \left(0.999966 + 7.99976\, z + 27.9993\, z^2 + 55.9988\, z^3 + 69.9988\, z^4 + 55.9993\, z^5 + 27.9998\, z^6 + 7.99997\, z^7 + 1.\, z^8\right)$$

Create a HIGH-pass filter with fs = 8000 and a cutoff at 2500 Hz and a stop band at 3500 Hz with at least -40db.

1. Find prewarping constant

$$C = \cot\left(\frac{w_a T}{2}\right) = \cot\left(\frac{2500 \cdot 2\pi \cdot \frac{1}{8000}}{2}\right) \approx 0{,}668$$

2. Prewarp and normalized frequencies

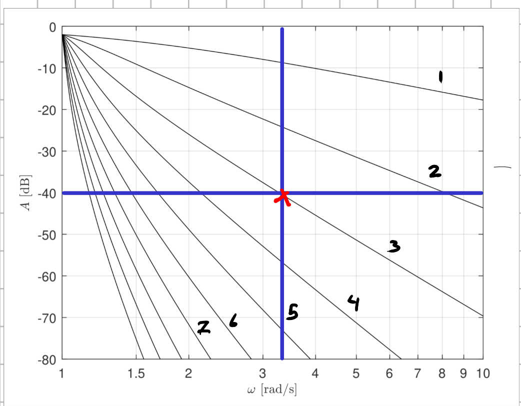
$$\Omega = C \cdot \tan\left(\frac{wT}{2}\right) \Rightarrow \begin{cases} \Omega_a = 1 \\ \Omega_s = C \cdot \tan\left(\frac{f_s \cdot 2\pi \cdot 1}{2}\right) = C \cdot \tan\left(3500\pi \cdot \frac{1}{8000}\right) \approx 3{,}359 \end{cases}$$

We are normalizing around the cutoff frequency

3. Choose filter with order

I choose a Chebyshev for fun!

$$F = \frac{\Omega_s}{\Omega_a} = \Omega_s = 3{,}359$$



$$N = 3$$

4. We lookup the transfer function in a table

n	Chebyshev Polynomials
1	$s + 1.965$
2	$s^2 + 1.097s + 1.102$
3	$s^3 + 0.7378s^2 + 1.0222s + 0.3269$
4	$s^4 + 0.952s^3 + 1.453s^2 + 0.742s + 0.275$
5	$s^5 + 0.7064s^4 + 1.4995s^3 + 0.6935s^2 + 0.4594s + 0.0817$

$$\Rightarrow H_n(s) = s^3 + 0{,}7378s^2 + 1{,}0222s + 0{,}3269$$

5. Convert to high-pass: $H_{hp}(s) = H_n(s)\big|_{s=\frac{1}{s}} = 0.3269 + \frac{1}{s^3} + \frac{0.7378}{s^2} + \frac{1.0222}{s}$

6. De-normalize filter: $H(s) = H_{hp}(s)\big|_{s=\frac{s}{w_a}} = 0.3269 + \frac{343\,000\,000\,000\,\pi^3}{s^3} + \frac{3.56808 \times 10^8}{s^2} + \frac{22\,479.4}{s}$

7. Do the z-transformation

$$H(z) = H(s)\Big|_{s = C\cdot\frac{z-1}{z+1}} = 0.3269 + \frac{33642.7\,(1+z)}{-1+z} + \frac{7.99189\times10^{8}\,(1+z)^2}{(-1+z)^2} + \frac{3.56505\times10^{13}\,(1+z)^3}{(-1+z)^3}$$

Not sure if this is correct...