Opgave 1.1 (Fourierrækker)

Find Fourierkoefficienterne og Fourierrækken for firkant-signalet defineret som

$$f(x) = \begin{cases} 0 & \text{hvis } -1 \leq x < 0 \\ 1 & \text{hvis } 0 \leq x < 1 \end{cases}$$

og

$$f(x+2) = f(x)$$

Løsning:

$$f(x) = \frac{1}{2} + \sum_{n=1,3,5,\ldots} \frac{2}{n\pi} \sin(n\pi x)$$

$-1 \leq x < 1 \Rightarrow T = 2 \Rightarrow L = 1$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cdot \cos\left(\frac{n\pi x}{L}\right) dx = \int_{-1}^{0} \underbrace{f(x)}_{0} \cdot \cos(n\pi x)\, dx + \int_{0}^{1} \underbrace{f(x)}_{1} \cdot \cos(n\pi x)\, dx$$

$$= \int_0^1 \cos(n\pi x)\, dx = \int_{x=0}^{x=1} \cos(u) \frac{1}{n\pi} du = \left[\frac{1}{n\pi} \cdot \sin(u)\right]_{x=0}^{x=1} = \left[\frac{1}{n\pi} \cdot \sin(n\pi x)\right]_0^1$$

$u = n\pi x \Rightarrow \frac{du}{dx} = n\pi \Rightarrow dx = \frac{1}{n\pi} du$

$$= \frac{2}{n\pi} \cdot \underbrace{\sin(n\pi)}_{0} - \frac{2}{n\pi} \cdot \underbrace{\sin(0)}_{0} = 0$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \cdot \sin\left(\frac{n\pi x}{L}\right) dx = \int_{-1}^{1} f(x) \cdot \sin(n\pi x)\, dx$$

$$= \cancel{\int_{-1}^{0} \underbrace{f(x)}_{0} \cdot \sin(n\pi x)\, dx} + \int_0^1 \underbrace{f(x)}_{1} \cdot \sin(n\pi x)\, dx = \int_0^1 \sin(\underbrace{n\pi x}_{u})\, dx$$

$\frac{du}{dx} = n\pi \Rightarrow dx = \frac{1}{n\pi}$

$$= \int_{x=0}^{x=1} \sin(u) \cdot \frac{1}{n\pi} du = \frac{1}{n\pi} \cdot \left[-\cos(u)\right]_{x=0}^{x=1} = \frac{1}{\pi n} \cdot \left[-\cos(n\pi)\right]_0^1 = \frac{1}{\pi n} \cdot \left(\underbrace{-\cos(\pi)}_{1} + \underbrace{\cos(0)}_{1}\right) = \frac{2}{\pi n}$$

$$a_0 = \int_0^1 f(x) \cdot \cos(0)\, dx = \int_0^1 f(x)\, dx = \int_0^1 1\, dx = [x]_0^1 = 1$$

$$g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cdot \cos\left(\frac{n\pi x}{L}\right) + b_n \cdot \sin\left(\frac{n\pi x}{L}\right)\right) = \underline{\underline{\frac{1}{2} + \sum_{n=1}^{\infty} \left(\frac{2}{\pi n} \cdot \sin(n\pi x)\right)}}$$

Opgave 1.2 (Fouriertransformation)

Enhedstrinfunktionen defineret som

$$u(t-a) = \begin{cases} 1 \text{ for } t-a > 0 \\ 0 \text{ for } t-a < 0 \end{cases}$$

benyttes til at definere en firkantimpuls

$$x(t) = u(t-a) - u(t-b)$$

hvor $a < b$.

- Tegn grafen for firkantimpulsen $x(t)$
- Udregn den Fouriertransformerede af $x(t)$

Løsning:

$$X(\omega) = \frac{e^{-j\omega a} - e^{-j\omega b}}{j\omega}$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \mathrm{dt}$$

$$X(w) = \int_{-\infty}^{\infty} x(t) \cdot e^{-jwt} \, dt = \int_{-\infty}^{a} \cancel{\underbrace{x(t)}_{0} \cdot e^{-jwt}} \, dt + \int_{a}^{b} \underbrace{x(t)}_{1} \cdot e^{-jwt} \, dt + \int_{b}^{\infty} \cancel{\underbrace{x(t)}_{0} \cdot e^{-jwt}} \, dt$$

$$= \int_{a}^{b} e^{-jwt} \, dt = \int_{t=a}^{t=b} e^{u} \cdot \frac{-1}{jw} \, du = \left[e^{u} \cdot \frac{-1}{jw} \right]_{t=a}^{t=b} = \left[e^{-jwt} \cdot \frac{-1}{jw} \right]_{a}^{b}$$

$$u = -jwt \Rightarrow \frac{du}{dt} = -jw \Rightarrow dt = \frac{-1}{jw} du$$

$$\int_{a} \underbrace{x(t)}_{1} \cdot e^{-jwt} \, dt$$

$$= \frac{-e^{-jwb} + e^{-jwa}}{jw} = \frac{e^{-jwa} - e^{-jwb}}{jw}$$