430.457

Introduction to Intelligent Systems

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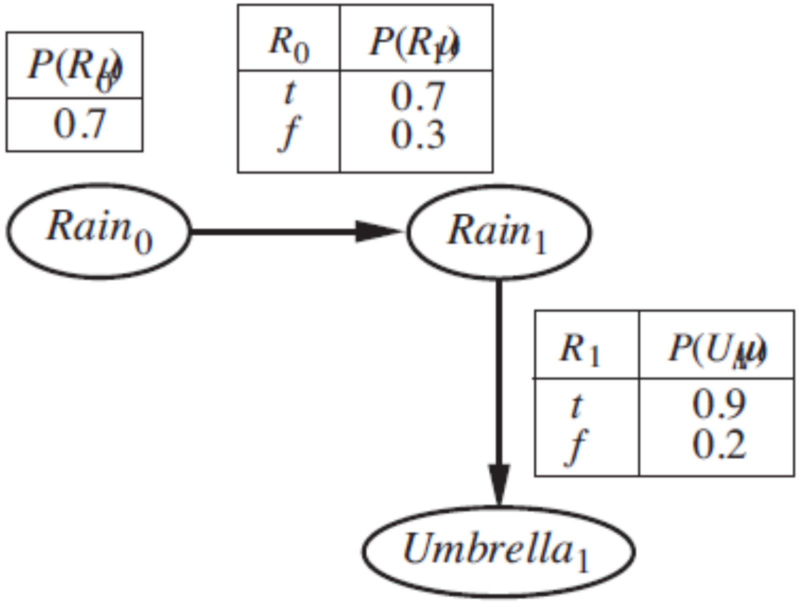
Dynamic Bayesian Networks

Dynamic Bayesian Networks (DBNs)

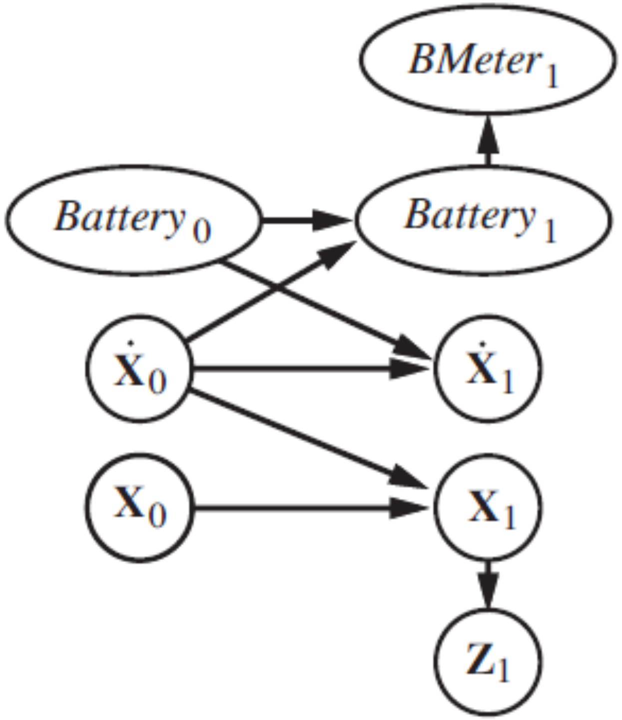
- Bayesian network for representing a temporal probability model
  - Includes hidden Markov Models and Kalman filters
- Decomposition of hidden variables
  - Example:
    - A DBN with 20 Boolean variables; each variable has three parents in the previous time; The transition model has $20 \times 2^3 = 160$ probabilities.
    - A HMM will require 2^{40} probabilities in its transition model (there are 2^{20} states).

Constructing DBNs

- Initial state distribution: $P(\mathbf{X}_0)$
- Transition model: $P(\mathbf{X}_{t+1}|\mathbf{X}_t)$
- Sensor model: $P(\mathbf{E}_t|\mathbf{X}_t)$
- Stationary process



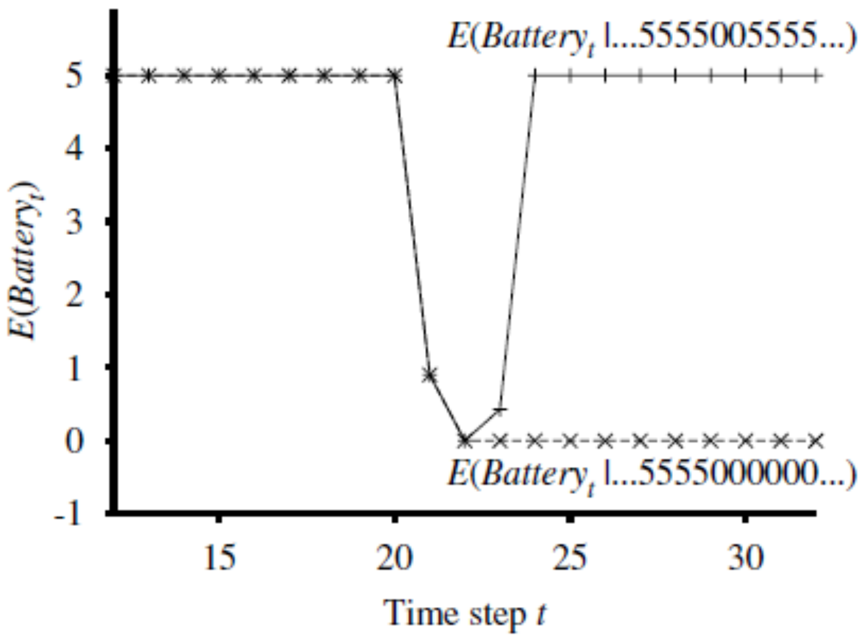
Example: Mobile Robot



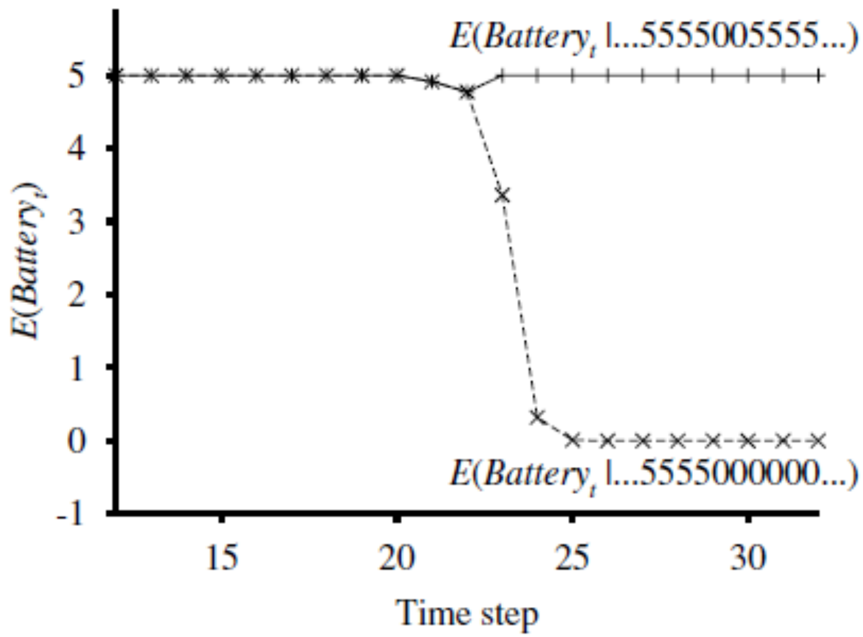
1. Sensor noise
2. Sensor failures
   - Transient failure
   - Persistent failure

Transient Failure Model

Transient failure model: $P(BMeter_t = 0 \mid Battery_t = 5) = 0.03$

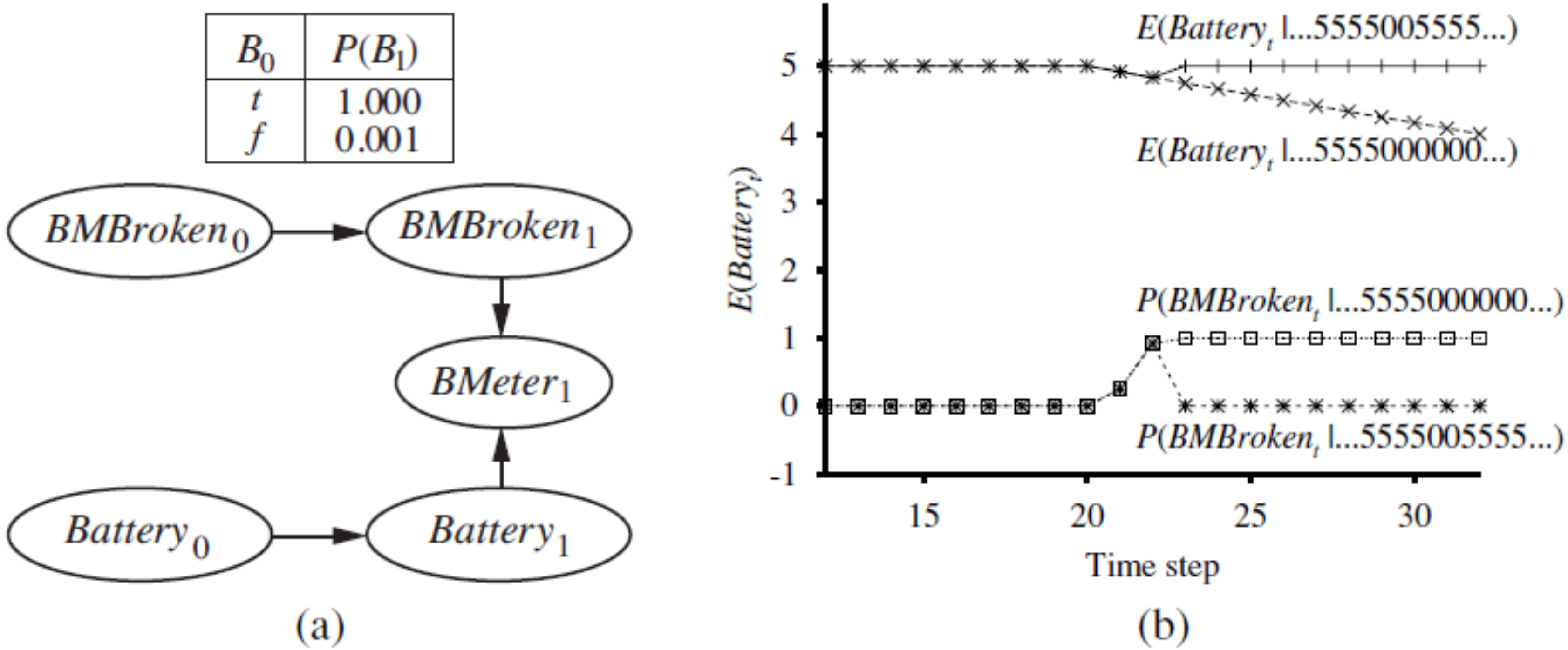


without the transient failure model



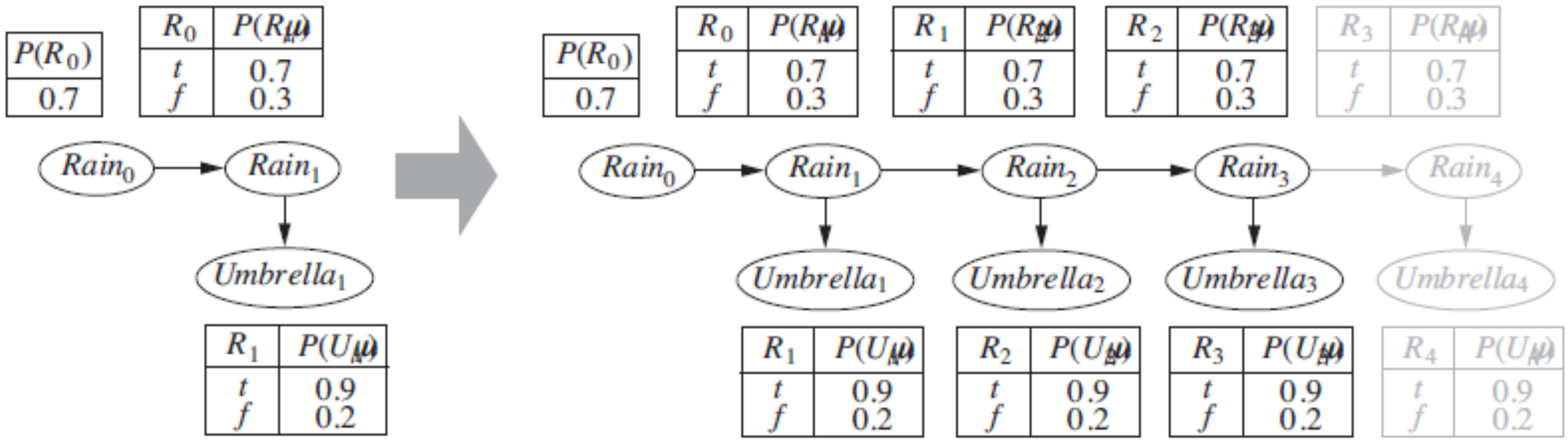
with the transient failure model

Persistent Failure Model



Exact Inference in DBNs

Unrolling



- We can apply any exact inference algorithm to the unrolled dynamic Bayesian network
  - E.g., variable elimination, clustering, …
- But, in general, the complexity is exponential in the number of states.
- Infeasible for large number of variables

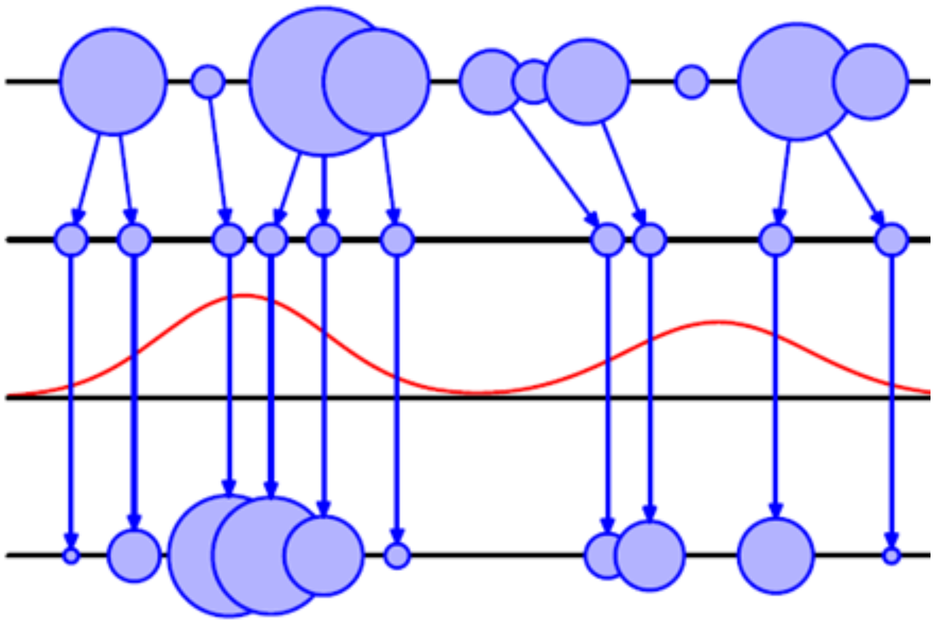
Particle Filtering: Approximate Inference in DBNs

Also known as sequential Monte Carlo (SMC):

- Sequential importance sampling (SIS)
- Sequential importance resamping (SIR)

Other names:

- bootstrap filtering
- condensation (computer vision)
- interacting particle approximation
- survival of the fittest



Particle Filtering

Sample N particles from $P(\mathbf{X}_0)$.

1. Each sample is propagated forward by sampling the next state value $\mathbf{x}_{t+1}$ given the current value $\mathbf{x}_t$ for the sample, based on the transition model $\mathbf{P}(\mathbf{X}_{t+1} \mid \mathbf{x}_t)$.
2. Each sample is weighted by the likelihood it assigns to the new evidence, $P(\mathbf{e}_{t+1} \mid \mathbf{x}_{t+1})$.
3. The population is *resampled* to generate a new population of N samples. Each new sample is selected from the current population; the probability that a particular sample is selected is proportional to its weight. The new samples are unweighted.

function PARTICLE-FILTERING($\mathbf{e}$, N, dbn) **returns** a set of samples for the next time step
  inputs: $\mathbf{e}$, the new incoming evidence
        N, the number of samples to be maintained
        dbn, a DBN with prior $\mathbf{P}(\mathbf{X}_0)$, transition model $\mathbf{P}(\mathbf{X}_1|\mathbf{X}_0)$, sensor model $\mathbf{P}(\mathbf{E}_1|\mathbf{X}_1)$
  persistent: S, a vector of samples of size N, initially generated from $\mathbf{P}(\mathbf{X}_0)$
  local variables: W, a vector of weights of size N

  for i = 1 to N **do**
      $S[i] \leftarrow$ sample from $\mathbf{P}(\mathbf{X}_1 \mid \mathbf{X}_0 = S[i])$ /* step 1 */
      $W[i] \leftarrow \mathbf{P}(\mathbf{e} \mid \mathbf{X}_1 = S[i])$ /* step 2 */
  $S \leftarrow$ WEIGHTED-SAMPLE-WITH-REPLACEMENT(N, S, W)   /* step 3 */
  return S

Example

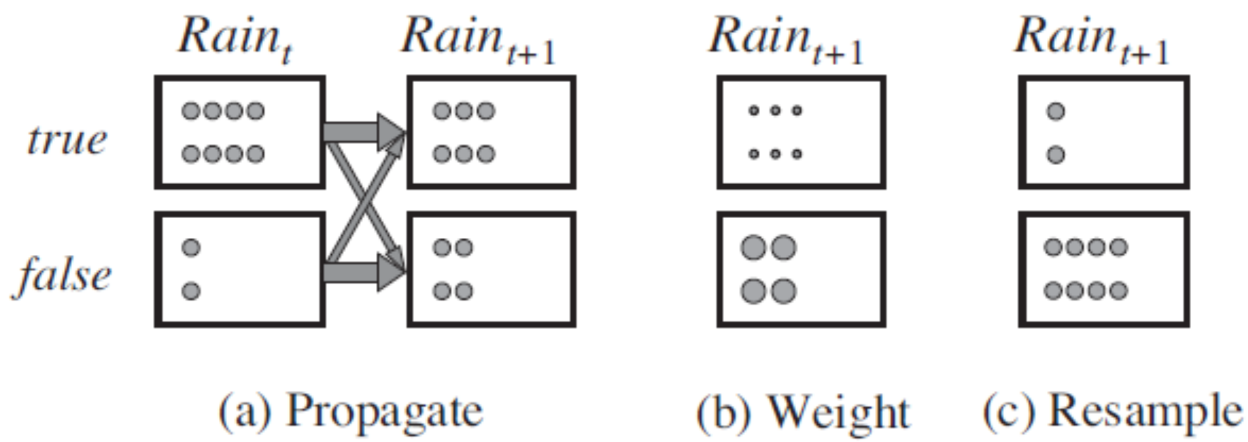


Figure 15.18 The particle filtering update cycle for the umbrella DBN with $N = 10$, showing the sample populations of each state. (a) At time t, 8 samples indicate $rain$ and 2 indicate $\neg rain$. Each is propagated forward by sampling the next state through the transition model. At time $t + 1$, 6 samples indicate $rain$ and 4 indicate $\neg rain$. (b) $\neg umbrella$ is observed at $t + 1$. Each sample is weighted by its likelihood for the observation, as indicated by the size of the circles. (c) A new set of 10 samples is generated by weighted random selection from the current set, resulting in 2 samples that indicate $rain$ and 8 that indicate $\neg rain$.

Consistency of Particle Filtering

- Let $N(\mathbf{x}_t|\mathbf{e}_{1:t})$ be the number of samples occupying state $\mathbf{x}_t$ given $\mathbf{e}_{1:t}$.
- Then, $\frac{1}{N}N(\mathbf{x}_t|\mathbf{e}_{1:t}) \approx P(\mathbf{x}_t|\mathbf{e}_{1:t})$ for large N.
- Sampling from the transition model:

$$N(\mathbf{x}_{t+1}|\mathbf{e}_{1:t}) = \sum_{\mathbf{x}_t} P(\mathbf{x}_{t+1}|\mathbf{x}_t)N(\mathbf{x}_t|\mathbf{e}_{1:t})$$

- Weighting by the likelihood:

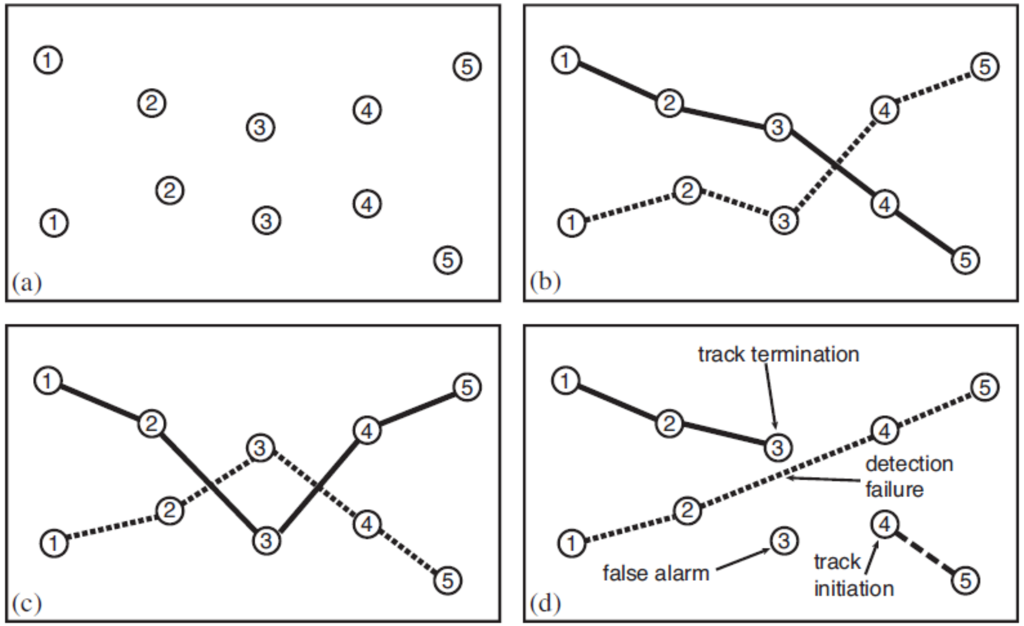
$$W(\mathbf{x}_{t+1}|\mathbf{e}_{1:t+1}) = P(\mathbf{e}_{t+1}|\mathbf{x}_{t+1})N(\mathbf{x}_{t+1}|\mathbf{e}_{1:t})$$

Hence,

$$\begin{aligned}
\frac{1}{N}N(\mathbf{x}_{t+1}|\mathbf{e}_{1:t+1}) &= \alpha W(\mathbf{x}_{t+1}|\mathbf{e}_{1:t+1}) = \alpha P(\mathbf{e}_{t+1}|\mathbf{x}_{t+1})N(\mathbf{x}_{t+1}|\mathbf{e}_{1:t}) \\
&= \alpha P(\mathbf{e}_{t+1}|\mathbf{x}_{t+1}) \sum_{\mathbf{x}_t} P(\mathbf{x}_{t+1}|\mathbf{x}_t)N(\mathbf{x}_t|\mathbf{e}_{1:t}) \\
&\approx \alpha N P(\mathbf{e}_{t+1}|\mathbf{x}_{t+1}) \sum_{\mathbf{x}_t} P(\mathbf{x}_{t+1}|\mathbf{x}_t)P(\mathbf{x}_t|\mathbf{e}_{1:t}) \\
&= \alpha' P(\mathbf{e}_{t+1}|\mathbf{x}_{t+1}) \sum_{\mathbf{x}_t} P(\mathbf{x}_{t+1}|\mathbf{x}_t)P(\mathbf{x}_t|\mathbf{e}_{1:t}) \\
&= P(\mathbf{x}_{t+1}|\mathbf{e}_{1:t+1})
\end{aligned}$$

Tracking Many Objects

- When there are many objects and many measurements, we may not know which object generated which observation.
- This problem is known as the **identity management** or **data association** problem.



$$P(x^A_{0:t}, x^B_{0:t}, e^1_{1:t}, e^2_{1:t}) =$$
$$P(x^A_0)P(x^B_0) \prod_{i=1}^{t} P(x^A_i \mid x^A_{i-1})P(x^B_i \mid x^B_{i-1})\, P(e^1_i, e^2_i \mid x^A_i, x^B_i)$$

$$P(x^A_{0:t}, x^B_{0:t}, e^1_{1:t}, e^2_{1:t}) = P(x^A_0)P(x^B_0)\prod_{i=1}^{t} P(x^A_i \mid x^A_{i-1})P(x^B_i \mid x^B_{i-1})\, P(e^1_i, e^2_i \mid x^A_i, x^B_i)$$

$$\begin{aligned} P(e^1_i, e^2_i \mid x^A_i, x^B_i) &= \sum_{\omega_i} P(e^1_i, e^2_i \mid x^A_i, x^B_i, \omega_i) P(\omega_i \mid x^A_i, x^B_i) \\ &= \sum_{\omega_i} P(e^{\omega_i(A)}_i \mid x^A_i) P(e^{\omega_i(B)}_i \mid x^B_i) P(\omega_i \mid x^A_i, x^B_i) \\ &= \frac{1}{2} \sum_{\omega_i} P(e^{\omega_i(A)}_i \mid x^A_i) P(e^{\omega_i(B)}_i \mid x^B_i)\ . \end{aligned}$$

- ω_t is the assignment between objects and observations. For n objects, there are $n!$ possible values for ω_t.
- **Hungarian algorithm** finds the best assignment.
- For handling data association problems for multiple time steps, more sophisticated algorithms exist.