

Opgave 1

Beregn volumen af det område, der er beskrevet ved ulighederne:

$$x \leq 0, y \geq 0, z \geq 0, x^2 + y^2 \leq 4 \text{ og } z \leq -x^2 + 4$$

To cylindrical coordinates

$$x = r \cdot \cos \theta \quad y = r \sin \theta \quad z = z \quad dx dy dz = r \cdot d\theta dr dz$$

$$x^2 + y^2 = a^2 \Rightarrow r = a$$

$$\begin{cases} x \leq 0 \text{ and } 0 \leq y \Rightarrow 0 \leq \theta \leq \frac{\pi}{2} \\ x^2 + y^2 \leq 4 \Rightarrow 0 \leq r \leq 2 \\ z \leq -x^2 + 4 \Rightarrow z \leq -(r \cdot \cos(\theta))^2 + 4 = r^2 \cdot \cos^2(\theta) + 4 \end{cases}$$

$$\Rightarrow \int_0^4 \int_0^{\frac{\pi}{2}} \int_0^{r^2 \cdot \cos^2(\theta) + 4} r \, dz d\theta dr = \int_0^4 \int_0^{\frac{\pi}{2}} [rz]_0^{r^2 \cdot \cos^2(\theta) + 4} d\theta dr$$

$$= \int_0^4 \int_0^{\frac{\pi}{2}} r^3 \cdot \cos^2(\theta) + 4r - \cancel{r \cdot 0} d\theta dr = \int_0^4 \left[r^3 \left(\frac{\theta}{2} + \frac{1}{4} \sin(2\theta) \right) + 4r\theta \right]_0^{\frac{\pi}{2}} d\theta dr$$

$$= \int_0^4 \left[r^3 \left(\frac{\theta}{2} + \frac{1}{4} \sin(2\theta) \right) + 4r\theta \right]_0^{\frac{\pi}{2}} d\theta dr = \int_0^4 r^3 \left(\frac{\pi}{4} + \frac{1}{4} \sin\left(2 \cdot \frac{\pi}{2}\right) \right) + 4 \cdot \frac{\pi}{2} \cdot r d\theta dr$$

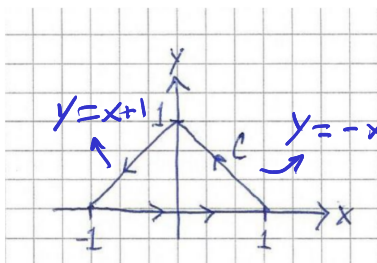
$$= \int_0^4 r^3 \cdot \frac{\pi}{4} + 2\pi r = \left[\frac{\pi}{4} \cdot r^4 + \pi r^2 \right]_0^4 = \frac{\pi}{4} \cdot 4^4 + \pi 4^2 = 64\pi + 16\pi = 80\pi$$

Opgave 2

Betragt vektorfeltet

$$\mathbf{F} = \begin{pmatrix} xy^2 \\ xy^2 - xy \end{pmatrix} = \begin{matrix} f_1 \\ f_2 \end{matrix}$$

og kurven C , der er beskrevet ved skitsen herunder.



Beregn

$$\oint_C \mathbf{F} \cdot d\mathbf{r}$$

$$\Rightarrow \oint_C \vec{F} d\mathbf{r} = \iint_R y^2 - y - xy \, dA$$

$$\left. \begin{aligned} y = x + 1 &\Rightarrow x = y - 1 \\ y = -x + 1 &\Rightarrow x = -y + 1 \end{aligned} \right\} y - 1 \leq x \leq -y + 1$$

$$\Rightarrow \int_0^1 \int_{y-1}^{-y+1} y^2 - y - xy \, dx \, dy = \int_0^1 \left[y^2 x - yx - \frac{yx^2}{2} \right]_{y-1}^{-y+1} dy$$

$$= \int_0^1 y^2(-y+1) - y(-y+1) - \frac{y(-y+1)^2}{2} - y^2(y-1) + y(y-1) + \frac{y(y-1)^2}{2} dy$$

$$= \int_0^1 -y^3 + y^2 + y^2 - y + \frac{1}{2}y^2 - \frac{1}{2}y - y^3 + y^2 + y^2 - y + \frac{1}{2}y^2 + \frac{1}{2}y dy$$

$$= \int_0^1 -2y^3 + 5y^2 dy$$