Opgave 1

Beregn volumen af det område, der er beskrevet ved ulighederne:

$x \leq 0,\ y \geq 0,\ z \geq 0,\ x^2 + y^2 \leq 4$ og $z \leq -x^2 + 4$

To cylindrical coordinates

$x = r \cdot \cos\theta \quad y = r\sin\theta \quad z = z \quad \mathrm{dxdydz} = r \cdot \mathrm{d}\theta\mathrm{drdz}$

$$x^2 + y^2 = a^2 \Rightarrow r = a$$

$$\begin{cases} x \leq 0 \text{ and } 0 \leq y \Rightarrow 0 \leq \theta \leq \frac{\pi}{2} \\ x^2 + y^2 \leq 4 \Rightarrow 0 \leq a \leq 4 \\ z \leq -x^2 + 4 \Rightarrow z \leq -\left(r \cdot \cos(\theta)\right)^2 + 4 = r^2 \cdot \cos^2(\theta) + 4 \end{cases}$$

$$\Rightarrow \int_0^4 \int_0^{\frac{\pi}{2}} \int_0^{r^2 \cdot \cos(\theta) + 4} r \, dz \, d\theta \, dr = \int_0^4 \int_0^{\frac{\pi}{2}} \left[rz\right]_0^{r^2 \cdot \cos^2(\theta) + 4} d\theta \, dr$$

$$= \int_0^4 \int_0^{\frac{\pi}{2}} r^3 \cdot \cos^2(\theta) + 4r - \cancel{r \cdot 0} \, d\theta \, dr = \int_0^4 \left[r^3 \cdot \left(\frac{\theta}{2} + \frac{1}{4}\sin(2\theta)\right) + 4r\theta\right]_0^{\frac{\pi}{2}} dr$$

$$= \int_0^4 \left[r^3 \cdot \left(\frac{\theta}{2} + \frac{1}{4}\sin(2\theta)\right) + 4r\theta\right]_0^{\frac{\pi}{2}} dr = \int_0^4 r^3 \cdot \left(\frac{\pi}{4} + \frac{1}{4}\underbrace{\cancel{\sin\left(2 \cdot \frac{\pi}{2}\right)}}_{0}\right) + 4 \cdot \frac{\pi}{2} \cdot r \, dr$$

$$= \int_0^4 r^3 \cdot \frac{\pi}{4} + 2\pi r = \left[\frac{\pi}{4} \cdot r^4 + \pi r^2\right]_0^4 = \frac{\pi}{4} \cdot 4^4 + \pi 4^2 = 64\pi + 16\pi = 80\pi$$

Opgave 2

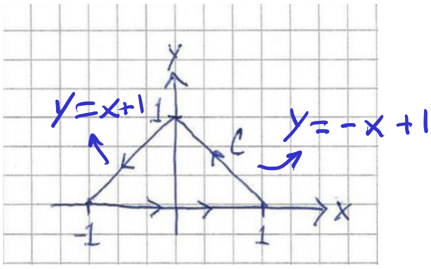
Betragt vektorfeltet

$$\mathbf{F} = \begin{pmatrix} xy^2 \\ xy^2 - xy \end{pmatrix} \begin{matrix} = f_1 \\ = f_2 \end{matrix}$$

og kurven C, der er beskrevet ved skitsen herunder.

$$\iint_R \frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y} \, \mathrm{dA}$$

$$\frac{\partial f_2}{\partial x} - \frac{\partial \cdot f_1}{y} = y^2 - y - xy$$



Beregn

$$\oint_C \mathbf{F} \cdot \mathbf{dr}$$

$$\Rightarrow \oint \vec{F} \, dr = \iint_R y^2 - y - xy \, dR$$

$$\left.\begin{matrix} y = x+1 \Rightarrow x = y-1 \\ y = -x+1 \Rightarrow x = -y+1 \end{matrix}\right\} \; y-1 \leq x \leq -y+1$$

$$\Rightarrow \int_0^1 \int_{y-1}^{-y+1} y^2 - y - xy \, dx \, dy = \int_0^1 \left[y^2 x - yx - \frac{yx^2}{2} \right]_{y-1}^{-y+1} dy$$

$$= \int_0^1 y^2(-y+1) - y(-y+1) - \frac{y \cdot (-y+1)^2}{2} - y^2(y-1) + y(y-1) + \frac{y(y+1)^2}{2} \, dy$$

$$= \int_0^1 -y^3 + y^2 + y^2 - y + \frac{1}{2}y^2 - \frac{1}{2}y - y^3 + y^2 + y^2 - y + \frac{1}{2}y^2 + \frac{1}{2}y \, dy$$

$$= \int_0^1 -2y^3 + 5y^2$$