Opgave 1

Beregn volumen af det område, der er beskrevet ved ulighederne:

$$\left.\begin{aligned} &z \le x + y + 10 \\ &z \ge x^2 + y^2 \\ &x^2 + y^2 \le 4 \end{aligned}\right\} \text{ Very cylinder}$$
$$x \ge 0$$
$$y \le 0$$

$x = r \cdot \cos\theta \qquad y = r\sin\theta \qquad z = z \qquad \mathrm{dxdydz} = r \cdot \mathrm{d}\theta\mathrm{drdz}$

$x^2 + y^2 = r^2$

$z \le x + y + 10 \Rightarrow z \le r \cdot \cos(\theta) + r \cdot \sin(\theta) + 10$

$z \ge r^2$

$r^2 \le 4 \Rightarrow 0 \le r \le 2$

$$\left.\begin{aligned} &r \cdot \cos(\theta) \ge 0 \Rightarrow \cos(\theta) \ge 0 \Rightarrow \theta \ge \tfrac{\pi}{2} + n\pi \\ &r \cdot \sin(\theta) \le 0 \Rightarrow \sin(\theta) \le 0 \Rightarrow \theta \le 0 + n\pi \end{aligned}\right\} \tfrac{-\pi}{2} \le \theta \le 0$$

$$\int_0^2 \int_{-\frac{\pi}{2}}^0 \int_{r^2}^{r \cdot (\cos(\theta) + \sin(\theta)) + 10} r \, dz \, d\theta \, dr = \int_0^2 \int_{-\frac{\pi}{2}}^0 \Big[r \cdot z\Big]_{r^2}^{r \cdot (\cos(\theta) + \sin(\theta)) + 10} d\theta \, dr$$

$$\Rightarrow \int_0^2 \int_{-\frac{\pi}{2}}^0 r \cdot \cos(\theta) + r \cdot \sin(\theta) + 10 - r^3 \, d\theta \, dr = \int_0^2 \Big[r \cdot \sin(\theta) - r \cdot \cos(\theta) + 10\theta - r^3\theta\Big]_{-\frac{\pi}{2}}^0 dr$$

$$\Rightarrow \int_0^2 r \cdot \cancel{\sin}(0) - r \cdot \overset{1}{\cos(0)} + 1\cancel{0} \cdot 0 - r\cancel{^3} \cdot 0 - r \cdot \overset{-1}{\sin\left(\tfrac{-\pi}{2}\right)} + r \cdot \cancel{\cos}\left(\tfrac{-\pi}{2}\right) - 10 \cdot \tfrac{-\pi}{2} + r^3 \cdot \tfrac{-\pi}{3}$$

$$= \int_0^2 -r + r + 5\pi - \frac{\pi r^3}{2} \, dr = \int_0^2 5\pi - \frac{\pi r^3}{2} \, dr = \left[5\pi r - \frac{\pi r^4}{8}\right]_0^2 = 10\pi - \frac{16\pi}{8} = 10\pi - 2\pi$$

$$\underline{\underline{= 8\pi}}$$

Opgave 2

Betragt vektorfeltet

$$\mathbf{F} = \begin{pmatrix} 4y + 2z \\ 4x + 2yz \\ 2x + y^2 \end{pmatrix}$$

a)

Beregn

$$\int_C \mathbf{F} \cdot \mathbf{dr}$$

hvor C er en ret linje fra $(1, 1, 1)$ til $(5, 7, 9)$.

b)

Beregn fluxen af $\mathbf{F}$ op gennem disken D, der ligger i xy-planet med centrum i $(0, 0)$ og med radius 2.

$$\vec{r} = \begin{bmatrix} 5-1 \\ 7-1 \\ 9-1 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 8 \end{bmatrix}$$

The line from point (1, 1, 1) to (5, 7, 9) if t=0..1

$$\vec{r}(t) = r \cdot t + P_0 = \begin{bmatrix} 4 \\ 6 \\ 8 \end{bmatrix} \cdot t + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4t+1 \\ 6t+1 \\ 8t+1 \end{bmatrix} \Rightarrow \begin{cases} x = 4t+1 \\ y = 6t+1 \\ z = 8t+1 \end{cases} \qquad r'(t) = \begin{bmatrix} 4 \\ 6 \\ 8 \end{bmatrix}$$

$$\oint_C F(x,y,z)\, d\vec{r} = \int_0^1 \begin{pmatrix} 4(6t+1) + 2(8t+1) \\ 4(4t+1) + 2(6t+1)(8t+1) \\ 2(4t+1) + (6t+1)^2 \end{pmatrix} \begin{pmatrix} 4 \\ 6 \\ 8 \end{pmatrix} dt = \int_0^1 \begin{bmatrix} 16(6t+1) + 8(8t+1) \\ 24(4t+1) + 12(6t+1)(8t+1) \\ 16(4t+1) + 8(6t+1)^2 \end{bmatrix} dt$$

$$\oint_C \vec{F}(x,y)\, \mathrm{d}\vec{r} = \int_a^b \vec{F}(x(t), y(t)) \cdot \vec{r}'(t) \mathrm{d}t$$

Add them up (magic??)

$$\Rightarrow \int_0^1 16(6t+1) + 8(8t+1) + 24(4t+1) + 12(6t+1)(8t+1) + 16(4t+1) + 8(6t+1)^2 \; dt$$

$$= \int_0^1 40(4t+1) + 16(6t+1) + 8(8t+1) + 12(48t^2+14t+1) + 8(36t^2+12t+1) \; dt$$

$$= \int_0^1 160t + 40 + 96t + 16 + 64t + 8 + 576t^2 + 168t + 12 + 288t^2 + 96t + 8 \; dt$$

$$= \int_0^1 864t^2 + 584t + 84 \; dt = \left[288t^3 + 294t^2 + 84t \right]_0^1 = \underline{\underline{288 + 292 + 84 = 664}}$$

b)

$\vec{n} = \begin{bmatrix}0\\0\\1\end{bmatrix}$

Beregn fluxen af **F** op gennem disken D, der ligger i xy-planet med centrum i $(0,0)$ og med radius 2.

$\to x^2+y^2=4$

$$\mathbf{F} = \begin{pmatrix} 4y+2z \\ 4x+2yz \\ 2x+y^2 \end{pmatrix}$$

$$\text{flux} = \iint_S \vec{F} \bullet \vec{n}\, \mathrm{dS} \Rightarrow \iint_S \begin{bmatrix} 4y+2z \\ 4x+2yz \\ 2x+y^2 \end{bmatrix} \bullet \begin{bmatrix}0\\0\\1\end{bmatrix} ds = \iint_S 2x+y^2\, ds$$

Convert to polar coordinates

$$x = r\cdot\cos(\theta) \qquad y = r\cdot\sin(\theta) \qquad \mathrm{dA} = r\,\mathrm{d}\theta\,\mathrm{dr} \qquad x^2+y^2=a^2 \Rightarrow a$$

$$\iint 2x+y^2\, dx\, dy \Rightarrow \int_0^2\int_0^{2\pi} 2\cdot r\cdot\cos(\theta) + r^2\cdot\sin(\theta)^2\cdot r\; d\theta\, dr$$

$$= \int_0^2 \left[\cancel{2r^2\cdot\sin(\theta)}^{\,0} + r^3\cdot\left(\frac{\theta}{2} - \cancel{\frac{1}{4}\cdot\sin(2\theta)}^{\,0}\right)\right]_0^{2\pi} dr$$

$$= \int_0^2 r^3\cdot\frac{2\pi}{2}\, dr = \int_0^2 \pi r^3\, dr = \left[\frac{\pi r^4}{4}\right]_0^2 = \frac{16\pi}{4} = \underline{\underline{4\pi}}$$

Opgave 3

Betragt kurven C, der er parametriseret ved

$$\mathbf{r}(t) = \begin{pmatrix} \frac{1}{\sqrt{2}}t^2 \\ t \\ \frac{1}{3}t^3 \end{pmatrix}$$

hvor $0 \leq t \leq 2$.

$$\vec{r}(t) = \begin{bmatrix} \frac{1}{\sqrt{2}} t^2 \\ t \\ \frac{1}{3} t^3 \end{bmatrix} \Rightarrow \vec{r}'(t) = \begin{bmatrix} \frac{2}{\sqrt{2}} t \\ 1 \\ t^2 \end{bmatrix} \Rightarrow |\vec{r}'(t)| = \sqrt{\tfrac{4}{2}t^2 + 1^2 + t^4}$$

$$= \sqrt{t^4 + 2t^2 + 1}$$

$$= \sqrt{(t^2+1)^2}$$

$$= t^2 + 1$$

a)

Beregn længden af kurven C.

$$\Rightarrow \int_0^2 t^2 + 1 \, dt = \left[\frac{t^3}{3} + t\right]_0^2 = \frac{8}{3} + 2 = \underline{\underline{\frac{14}{3}}}$$

b)

Beregn

$$\int_C z \, ds$$

$$\Rightarrow \vec{F}(x, y, z) = \begin{bmatrix} 0 \\ 0 \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{t^3}{3} \end{bmatrix}$$

$$\oint_C \vec{F} \, d\vec{r} = \int_0^2 \begin{bmatrix} 0 \\ 0 \\ \frac{t^3}{3} \end{bmatrix} \bullet \begin{bmatrix} \frac{2}{\sqrt{2}} t \\ 1 \\ t^2 \end{bmatrix} dt = \int_0^2 \frac{1}{3} \cdot t^5 \, dt = \left[\frac{1}{18} \cdot t^6\right]_0^2 = \underline{\underline{\frac{64}{18}}}$$

why this not correct??

Opgave 4

Betragt differentialligningen

$$y''(t) + 4y(t) = \exp(-2t) = e^{-2t}$$

hvor $y(0) = 0$ og $y'(0) = 0$.
Brug nu Laplacetransformation til at løse ligningen og således bestemme $y(t)$.

$$\Rightarrow Y(s) \cdot s^2 + 4 \cdot Y(s) = \frac{1}{s+2} \Rightarrow Y(s) \cdot (s^2+4) = \frac{1}{s+2} \Rightarrow Y(s) = \frac{1}{s+2} \cdot \frac{1}{s^2+4}$$

$$\Rightarrow Y(s) = \frac{A}{s+2} + \frac{B \cdot s + C}{s^2+4}$$

$$\Rightarrow \frac{1}{s+2} \cdot \frac{1}{s^2+4} = \frac{A}{s+2} + \frac{B \cdot s + C}{s^2+4} \Rightarrow 1 \cdot 1 = A \cdot (s^2+4) + (B \cdot s + C)(s+2)$$

$$\Rightarrow 1 = As^2 + 4A + B \cdot s^2 + 2Bs + Cs + 2C$$

$$\Rightarrow 1 = s^2(A+B) + s(2B+C) + 4A + 2C$$

$$\begin{cases} A + B = 0 \Rightarrow A = -B \\ 2B + C = 0 \Rightarrow -2B = C \Rightarrow 2A = C \\ 4A + 2C = 1 \Rightarrow 2A + C = \frac{1}{2} \Rightarrow 2A = \frac{1}{2} - C \end{cases} \quad \begin{aligned} &\Rightarrow C = \frac{1}{2} - C \Rightarrow 2C = \frac{1}{2} = \frac{1}{4} \\ &\Rightarrow 2A = \frac{1}{4} \Rightarrow A = \frac{1}{8} \\ &\Rightarrow B = \frac{-1}{8} \end{aligned}$$

$$Y(s) = \frac{\frac{1}{8}}{s+2} + \frac{\frac{-1}{8} \cdot s + \frac{1}{4}}{s^2+4} = \frac{1}{8} \cdot \frac{1}{s+2} - \frac{1}{8} \cdot \frac{s}{s^2+4} + \frac{1}{8} \cdot \frac{2}{s^2+2^2}$$

$$y(t) = \frac{1}{8} \cdot e^{-2t} + \frac{1}{8} \cdot \cos(2t) + \frac{1}{8} \cdot \sin(2t)$$

$$\underline{\underline{y(t) = \frac{1}{8}\left(e^{-2t} + \cos(2t) + \sin(2t)\right)}}$$