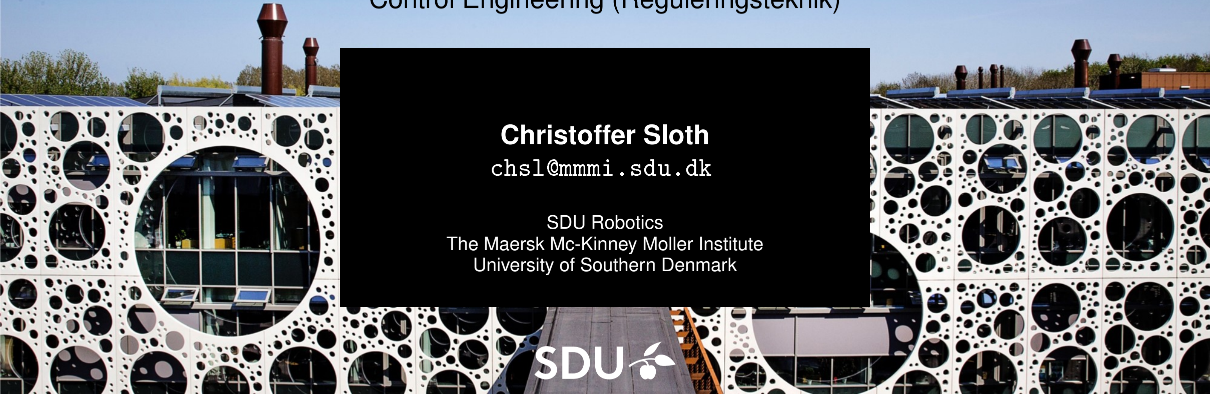
The Root Locus Method

Control Engineering (Reguleringsteknik)



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The Maersk Mc-Kinney Moller Institute
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Agenda

Introduction

Curriculum for Reguleringsteknik (REG)

Matematiske og grafiske metoder til syntese af lineære tidsinvariante systemer:[1]

- diskret og kontinuert tilstandsbeskrivelse
- analyse i tid og frekvens
- stabilitet, reguleringshastighed, følsomhed og fejl
- digitale PI, PID, LEAD og LAG regulatorer (serieregulatorer)
- tilstandsregulering, pole-placement og tilstands-estimering (observer)
- optimal regulering (least squares) og optimal tilstands-estimation (Kalman-filter)

Færdigheder:
Efter gennemførelse af kurset kan den succesfulde studerende:

- kunne analysere, dimensionere og implementere såvel kontinuert som tidsdiskret regulering af lineære tidsinvariante og stokastiske systemer

Kompetencer:
Efter gennemførelse af kurset kan den succesfulde studerende:

- anvende og implementere klassiske og moderne reguleringsteknikker for at kunne styre og regulere en robot hurtig og præcist

[1] Based on https://fagbesk.sam.sdu.dk/?fag_id=39673

Introduction

Motivation (Tuning of Controller)

Consider a system with an unknown parameter K

$$u(s) \longrightarrow \boxed{G(s, K)} \longrightarrow y(s)$$

The ***root locus method*** is used for describing how the poles of $G(s)$ move around in the s-plane, when the parameter K changes.

The Ziegler-Nichols tuning method presented in the previous lecture is useful, when a model of the plant is not available. If a transfer function $G(s)$ of the plant is available, then other methods can be used to design the control (find the controller gains).

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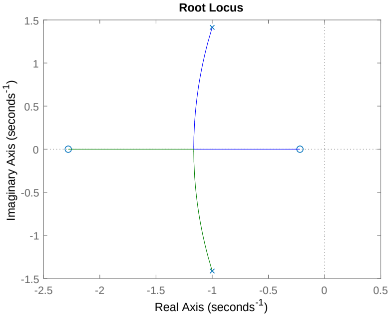
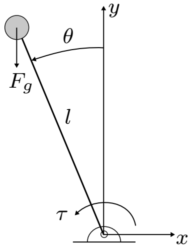


A linearized model of the inverted pendulum is given by the transfer function

$$G(s) = \frac{1}{s^2 - s - 1}$$



that has poles in $\frac{1}{2} \pm \frac{\sqrt{5}}{2}$.

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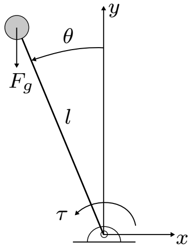

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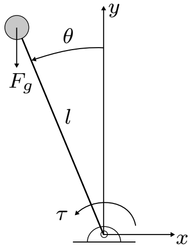

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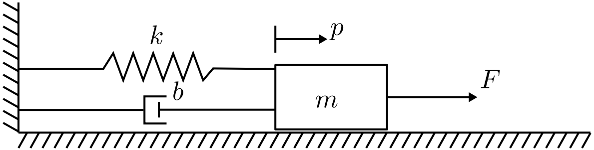
Problem: What controller structure (P/PI/PID) should be chosen to stabilize the system?

We will see that a P controller or PI controller cannot stabilize the system, since an extra zero (D-term) is needed.

Consider the mass-spring-damper system



with transfer function

$$G(s) = \frac{1/m}{s^2 + b/ms + k/m}$$

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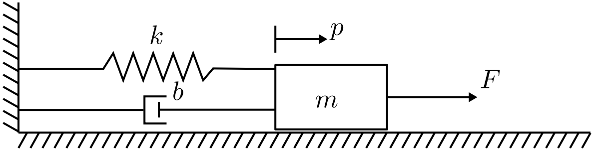

with transfer function

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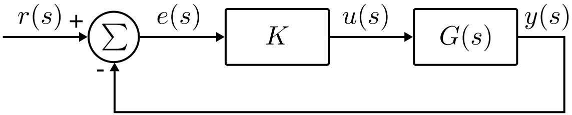
What happens to the poles of the system when the stiffness of the spring changes? (the can happen due to temperature changes or production inaccuracies)

Our systems should be robust towards such changes.

Consider the closed-loop system shown below



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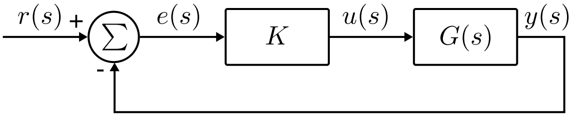

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$$\frac{KG(s)}{1 + KG(s)}$$

characteristic equation

$$1 + KG(s) = 0$$

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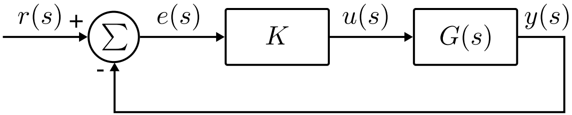

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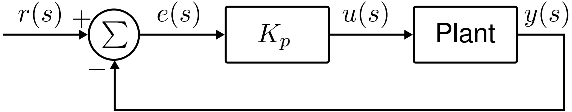
characteristic equation

$$1+KG(s)=0$$

Problem: How does K affect the location of the roots of the ***characteristic equation***, when it is changed from 0 to ∞?

Consider a proportional controller with gain K_p that controls a plant with transfer function $G(s) = \frac{1}{\tau s+1}$.



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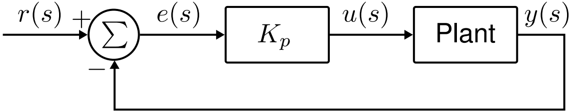
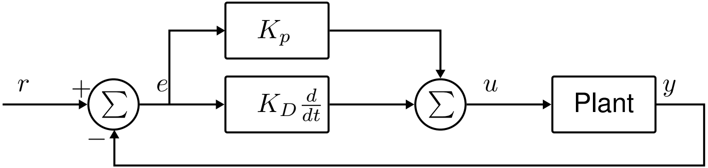
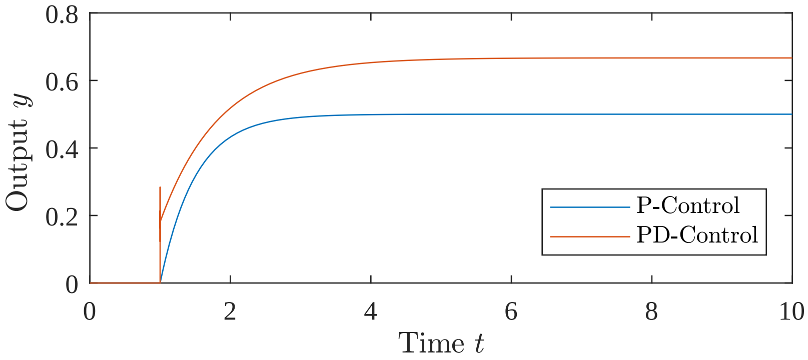


Consider a PD-controller with gains K_p and K_d that controls a plant with transfer function $G(s) = \frac{1}{\tau s+1}$.



Recall that the closed-loop poles do not completely describe the behavior of a system.



Both the P-control and the PD-control has a closed-loop pole at $s = -2$.

The damping ratio ζ of an underdamped second-order system is smaller than one; thus, it has a complex pole pair with poles at

$$-\underbrace{\zeta\omega_n}_{\sigma} \pm j\,\underbrace{\omega_n\sqrt{1-\zeta^2}}_{\omega_d}.$$

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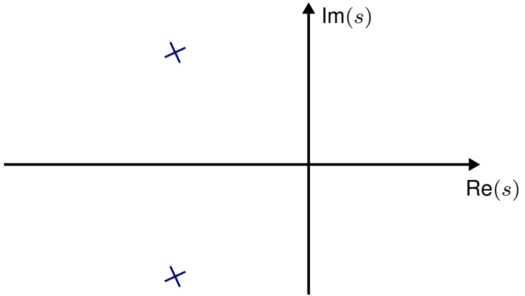

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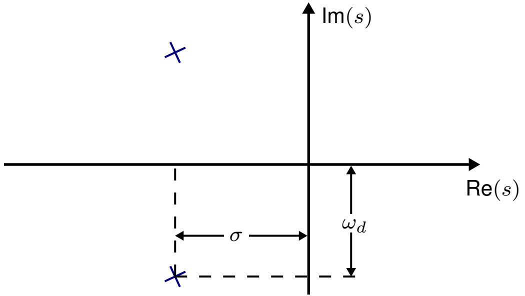

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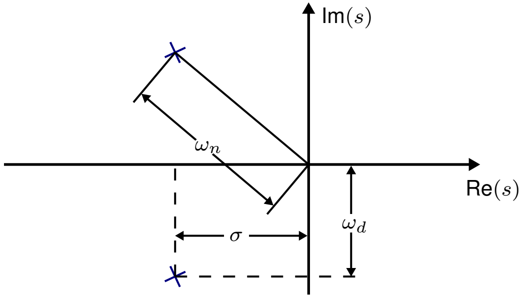

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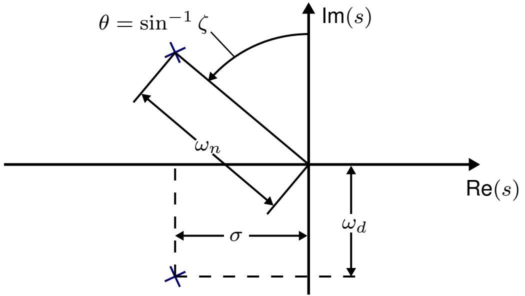
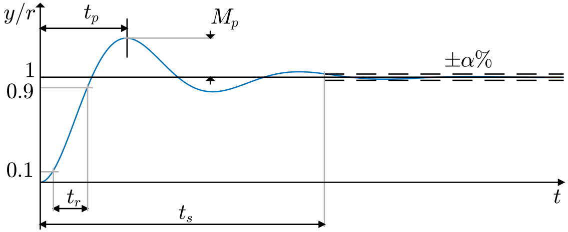


We consider four different performance measures of dynamical systems

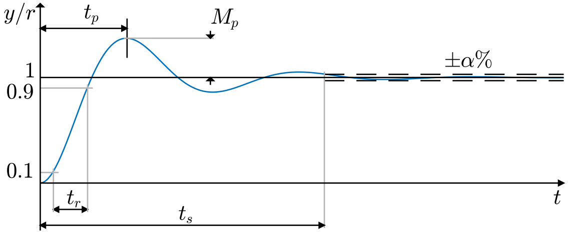
1. The **rise time** t_r.
2. The **settling time** t_s.
3. The **peak time** t_p.
4. The **overshoot** M_p.



Performance Specification

Rise Time

The rise time of a dynamical system is the time it takes to come from 0.1 times the setpoint value to reach 0.9 times the setpoint value.



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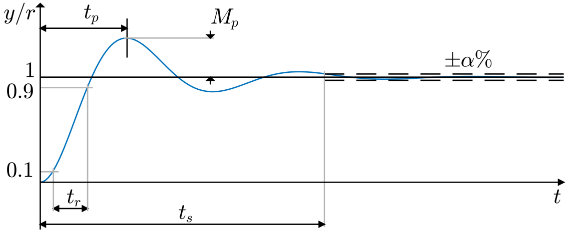


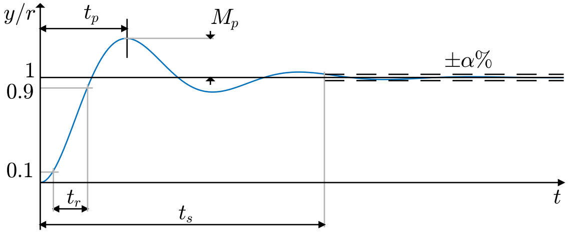
For a second-order system the rise time can be approximated as

$$t_r = \frac{1.8}{\omega_n}$$

Performance Specification

Settling Time

The $\alpha\%$-settling time is the time it takes for the step response to reach $\pm\alpha\%$ of the setpoint value without leaving it again.



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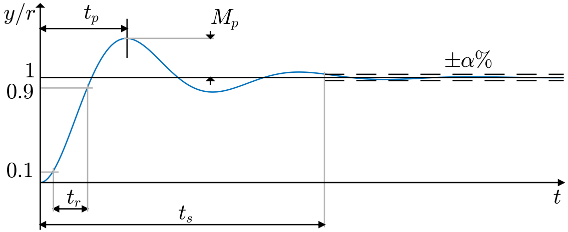

For a second-order system the settling time can be approximated as

$$t_s = \frac{-\log(\alpha/100)}{\omega_n \zeta}$$

The peak time t_p can be found from the step response of the system

$$y(t) = 1 - e^{-\sigma t}\left(\cos(\omega_d t) + \frac{\sigma}{\omega_d}\sin(\omega_d t)\right)$$

At the maximum of $y(t)$, we have $\dot{y}(t) = 0$

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Thus, $\dot{y}(t) = 0$ for $\sin(\omega_d t)$, i.e.,

$$t_p = \frac{\pi}{\omega_d}.$$

The overshoot is computed from the step response at the peak time

$$
\begin{aligned}
y(t_p) &= 1 - e^{-\sigma\pi/\omega_d}\left(\cos(\pi) + \frac{\sigma}{\omega_d}\sin(\pi)\right) \\
&= 1 + M_p
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$$

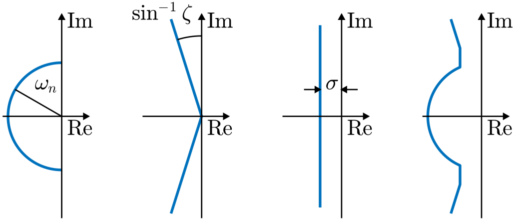
This gives an expression for the overshoot

$$
M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}
$$

for $0 \leq \zeta \leq 1$.

Performance Specification

Frequency and Time Specifications



To obtain a rise rise shorter than t_r

$$\omega_n \geq \frac{1.8}{t_r}$$

To obtain an overshoot that is smaller than M_p

$$\zeta \geq \sqrt{\frac{\left(\frac{\log(M_p)}{-\pi}\right)^2}{1+\left(\frac{\log(M_p)}{-\pi}\right)^2}}$$

To obtain a settling time shorter than t_s

$$\sigma \geq \frac{4.6}{t_s}$$

Consider a system with input $u(t)$ and output $y(t)$

Transfer Functions

Introduction

Consider a system with input $u(t)$ and output $y(t)$

A transfer function

$$H(s) = \frac{Q(s)}{P(s)}$$

describes the input-output relation of the system in frequency domain

Consider a system with input $u(t)$ and output $y(t)$

$$\xrightarrow{u(t)} \boxed{\text{System}} \xrightarrow{y(t)}$$

A transfer function

$$H(s) = \frac{Q(s)}{P(s)}$$

describes the input-output relation of the system in frequency domain, i.e.

$$H(s) = \frac{Y(s)}{U(s)}$$

where $U(s)$ is the Laplace transform of the input $u(t)$ and $Y(s)$ is the Laplace transform of the output $y(t)$.

$$\xrightarrow{U(s)} \boxed{H(s)} \xrightarrow{Y(s)}$$

Consider the transfer function

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$z \in \mathbb{C}$ is a ***zero*** of $H(s)$ if

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$z \in \mathbb{C}$ is a ***zero*** of $H(s)$ if

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$p \in \mathbb{C}$ is a ***pole*** of $H(s)$ if

$$P(p) = 0$$

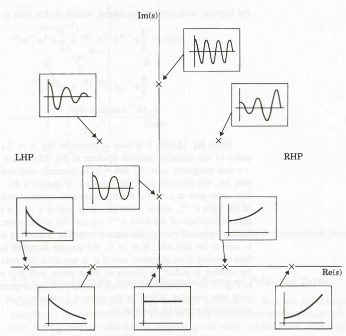
A transfer function

$$H(s) = \frac{Q(s)}{P(s)}$$

is ***stable*** if all its poles $p_1, \ldots, p_k \in \mathbb{C}$ are located in the left half of the s-plane, i.e., if

$$\mathrm{Re}(p_i) \leq 0 \quad \text{for } i = 1, \ldots, k.$$

Otherwise, the transfer function is ***unstable***.



The waveform at a given point in the s-plane is given by e^{st} which means that

1. The oscillation frequency increases when $|\mathrm{Im}(s)|$ increases.
2. The decay rate increases when $\mathrm{Re}(s)$ decreases.

We consider a transfer function on the following form

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where $K \in \mathbb{R}$ is the free parameter.

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The following notation is exploited in the following. Let

$$G(s) = \frac{Q(s)}{P(s)}$$

where m (n) is the degree of Q (P).

Consider a mass-spring-damper system (with uncertain stiffness k) controlled by a P-controller with gain $K_p = 1$ and system parameters $m = 1$ kg, $b = 1$ N/(rad/s). This closed-loop system has characteristic equation

$$1 + \frac{1}{s^2 + s + k}$$

How can this system be put on the standard form

$$1 + kG(s) = 0?$$

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1. Group all k-terms

$$(s^2 + s + 1) + k = 0$$

2. Divide by non-k terms

$$1 + k\frac{1}{s^2 + s + 1} = 0$$

Consider the characteristic equation

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Lemma. A univariate polynomial of degree d has d roots in $\mathbb{C}$.

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Lemma. A univariate polynomial of degree d has d roots in $\mathbb{C}$.

RULE 1: There are N lines (loci) where $N = \max(m, n)$.

Similar to previously, the characteristic equation is rewritten as

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RULE 2: As K increases from 0 to ∞, the roots move from the poles of $G(s)$ to the zeros of $G(s)$.

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Complex conjugate root theorem. Let P be a univariate polynomial with real coefficients, i.e.,

$$P(x) = \sum_{\alpha=0}^{d} a_\alpha x^\alpha$$

where $a_\alpha \in \mathbb{R}$, and $x \in \mathbb{C}$. If $a + jb$ is a root of P with $a, b \in \mathbb{R}$ then its complex conjugate $a - jb$ is also a root of P.

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RULE 3: When roots are complex they occur in conjugate pairs.

We study the rewritten characteristic equation on the form

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and see that the phase of $\frac{Q(s)}{P(s)}$ should be $180°$ to satisfy the equation.

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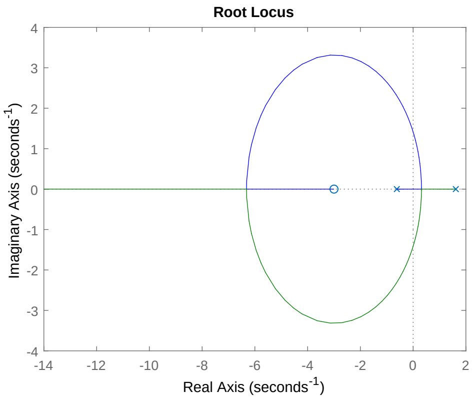
Let $z \in \mathbb{C}$ then $zz^* = |z|^2$; hence, complex pole pairs and pairs of complex conjugated zeros do not affect the phase of $T(s)$ for $s \in \mathbb{R}$.

The phase of $(s - z_m)$ when $s, z_m \in \mathbb{R}$ is

$$\angle(s - z_m) = \begin{cases} 180^\circ & \text{If } s < z_m \\ 0^\circ & \text{otherwise} \end{cases}$$

RULE 4: The portion of the real axis to the left of an odd number of open loop poles and zeros are part of the loci.

Root Locus
Rule 5



Root Locus

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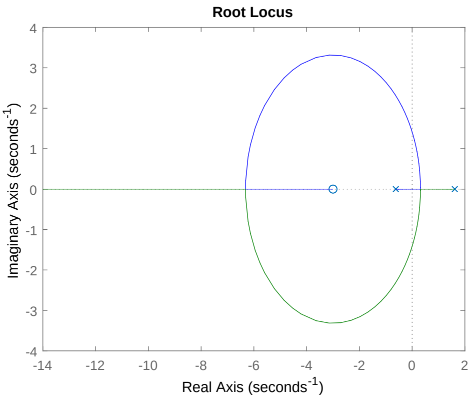

RULE 5: Lines leave and enter the real axis at $90°$.

For very large values of s the equation

$$1 + K\frac{s^m + b_1 s^{m-1} + \cdots + b_m}{s^n + a_1 s^{n-1} + \cdots + a_n}$$

can be approximated by

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and finally

$$\phi_l = \frac{180^\circ + 360^\circ(l-1)}{n-m} \quad \text{for } l = 1, \ldots, n-m$$

Root Locus
Rule 6

Let $m < n - 1$ then

$$-\sum r_i = -\sum p_i$$

where r_i is the closed-loop poles and p_i is the open-loop poles.

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Thus,

$$\alpha = \frac{\sum p_i - \sum z_i}{n - m}$$

Consider the following characteristic equation $1 + K\frac{Q(s)}{P(s)} = 0$, where $G(s) = Q(s)/P(s)$, m (n) is the degree of Q (P).
The following is a subset of the rules used for drawing a root locus

1. There are N lines (loci) where $N = \max(n, m)$.
2. As K increases from 0 to ∞, the roots move from the poles of $G(s)$ to the zeros of $G(s)$.
3. When roots are complex they occur in conjugate pairs.
4. The portion of the real axis to the left of an odd number of open loop poles and zeros are part of the loci.
5. Lines leave and enter the real axis at $90°$.

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6. Lines go to infinity along asymptotes.
   The angles of the asymptotes are
   $$\phi_l = \frac{180^\circ + 360^\circ(l-1)}{n-m} \qquad \text{for } l = 1, \ldots, n-m$$
   The centroid of the asymptotes is
   $$\frac{\sum_{i=1}^{n} p_i - \sum_{j=1}^{m} z_j}{n-m}$$

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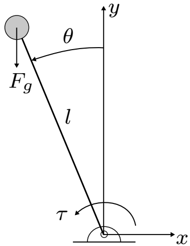
7. The angle(s) of departure of a branch of the locus from a pole of multiplicity q is given by
$$\phi_{l,\text{dep}} = \frac{\sum \psi_i - \sum_{i \neq l} \phi_i - 180° - 360°(l-1)}{q}$$
were $\sum \psi_i$ is the sum of the angles to all the zeros and $\sum_{i \neq l} \phi_i$ is the sum of the angles to all but the lth pole.
8. At no time will the same root cross over its own path.
9. If there there are not enough poles or zeros to make a pair then the extra lines go to or come from infinity.

Recall the linearized model of an inverted pendulum given by the transfer function

$$G(s) = \frac{1}{s^2 - s - 1}$$



that has poles in $\frac{1}{2} \pm \frac{\sqrt{5}}{2}$.

Determine a controller that can stabilize the system.