430.457

Introduction to Intelligent Systems

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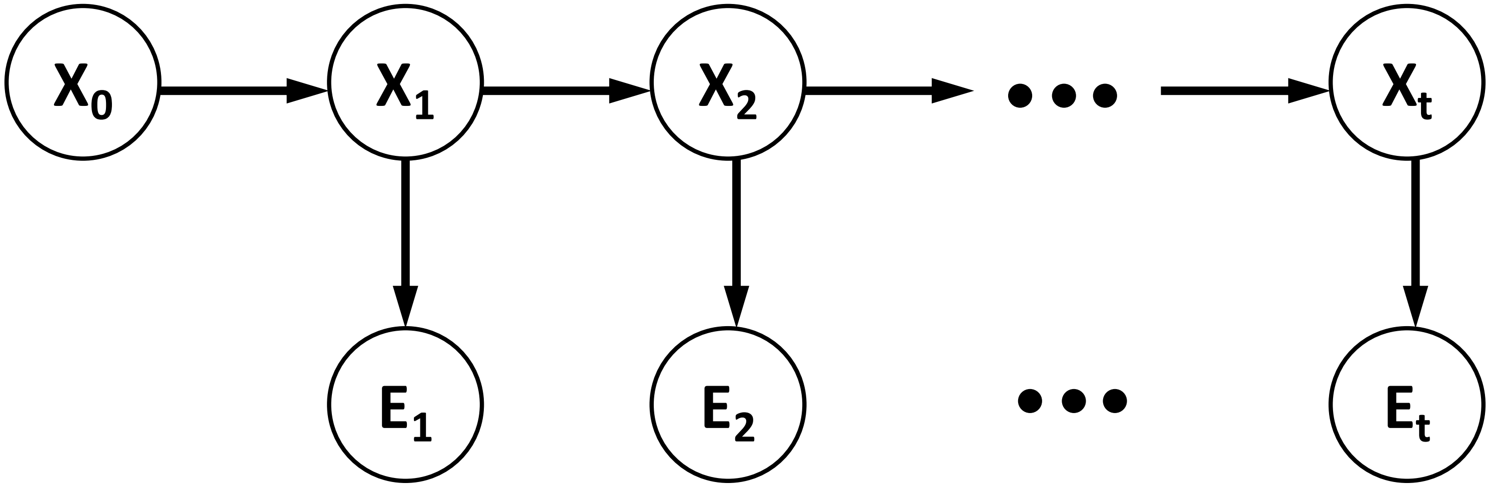
HIDDEN MARKOV MODEL (HMM)

Hidden Markov Model (HMM)



A **hidden Markov model** is fully specified by the following three components:

1. **Transition model**: $P(X_t|X_{t-1})$
2. **Observation model**: $P(E_t|X_t)$
3. **Initial state distribution**: $P(X_0)$

Matrix Algorithms for HMMs

- $\mathbf{X}_t \in \{1, \ldots, S\}$, where S is the number of states.
- **Transition matrix** $\mathbf{T} \in \mathbb{R}^{S \times S}$, such that $[\mathbf{T}]_{ij} = P(X_t = j | X_{t-1} = i)$.
- Observation e_t is represented by a diagonal matrix $\mathbf{O}_t \in \mathbb{R}^{S \times S}$, such that $[\mathbf{O}_t]_{ii} = P(e_t | X_t = i)$.
- **Forward update:**

$$P(\mathbf{X}_{t+1} | \mathbf{e}_{1:t+1}) = \alpha P(\mathbf{e}_{t+1} | \mathbf{X}_{t+1}) \sum_{\mathbf{x}_t} P(\mathbf{X}_{t+1} | \mathbf{x}_t) P(\mathbf{x}_t | \mathbf{e}_{1:t})$$

$$\boxed{\mathbf{f}_{1:t+1} = \alpha \mathbf{O}_{t+1} \mathbf{T}^T \mathbf{f}_{1:t}}$$

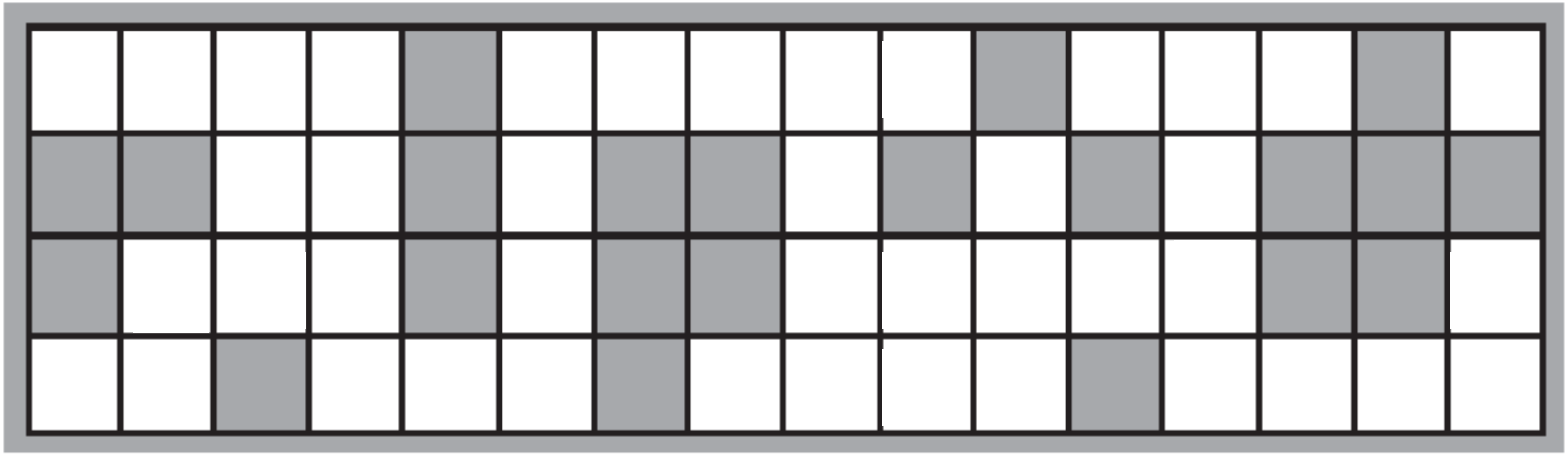
- **Backward update:**

$$P(\mathbf{e}_{k+1:t} | \mathbf{X}_k) = \sum_{\mathbf{x}_{k+1}} P(\mathbf{e}_{k+1} | \mathbf{x}_{k+1}) P(\mathbf{e}_{k+2:t} | \mathbf{x}_{k+1}) P(\mathbf{x}_{k+1} | \mathbf{X}_k)$$

$$\boxed{\mathbf{b}_{k+1:t} = \mathbf{T} \mathbf{O}_{k+1} \mathbf{b}_{k+2:t}}$$

- Time complexity: $O(S^2 t)$; Space complexity: $O(St)$

Example: Localization

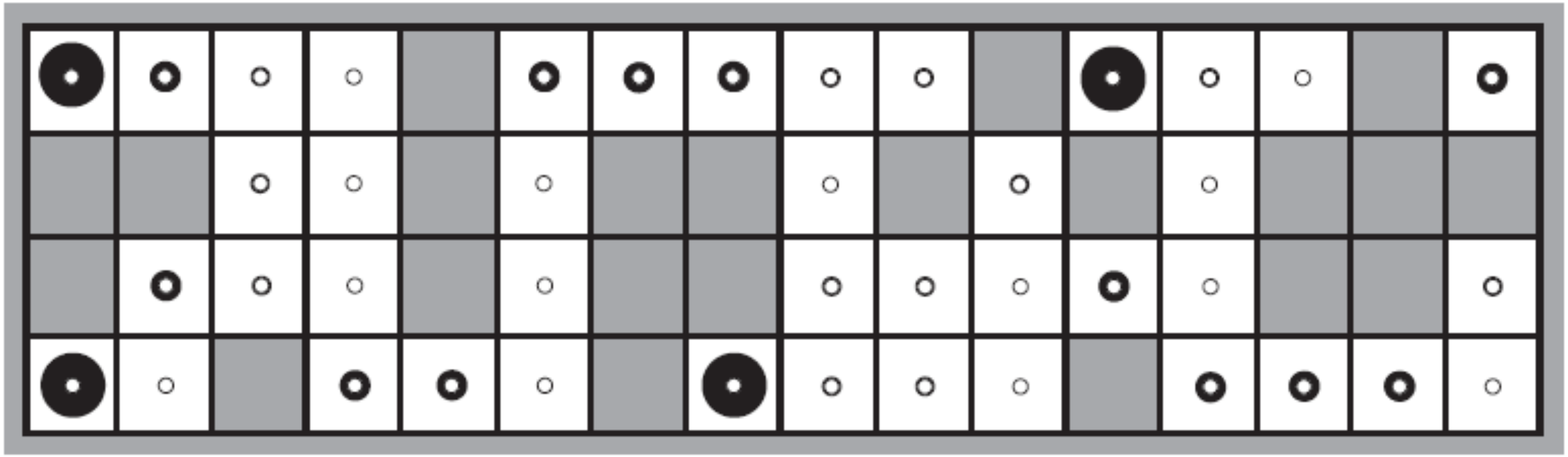


- $\{s_1, \ldots, s_n\}$: set of empty squares. $X_t \in \{1, \ldots, n\}$.
- $E_t \in \{0000, \ldots, 1111\}$: presence/absence of an obstacle in four directions
- ϵ: sensor error rate
- d_{it}: number of bits that are different between the true values for square i and the actual reading e_t

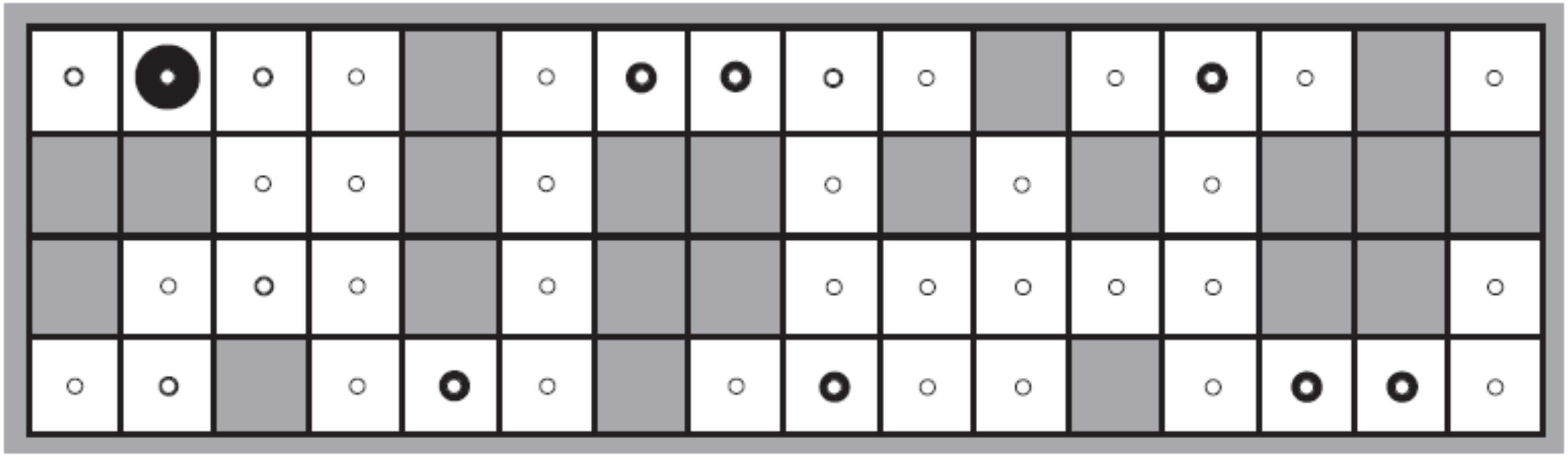
$$P(X_{t+1} = j \mid X_t = i) = \mathbf{T}_{ij} = (1/N(i) \text{ if } j \in \text{NEIGHBORS}(i) \text{ else } 0)$$

$$P(E_t = e_t \mid X_t = i) = \mathbf{O}_{t_{ii}} = (1 - \epsilon)^{4 - d_{it}} \epsilon^{d_{it}}$$

$\mathbf{P}(X_1 \mid E_1 = NSW)$



$\mathbf{P}(X_2 \mid E_1 = NSW, E_2 = NS)$



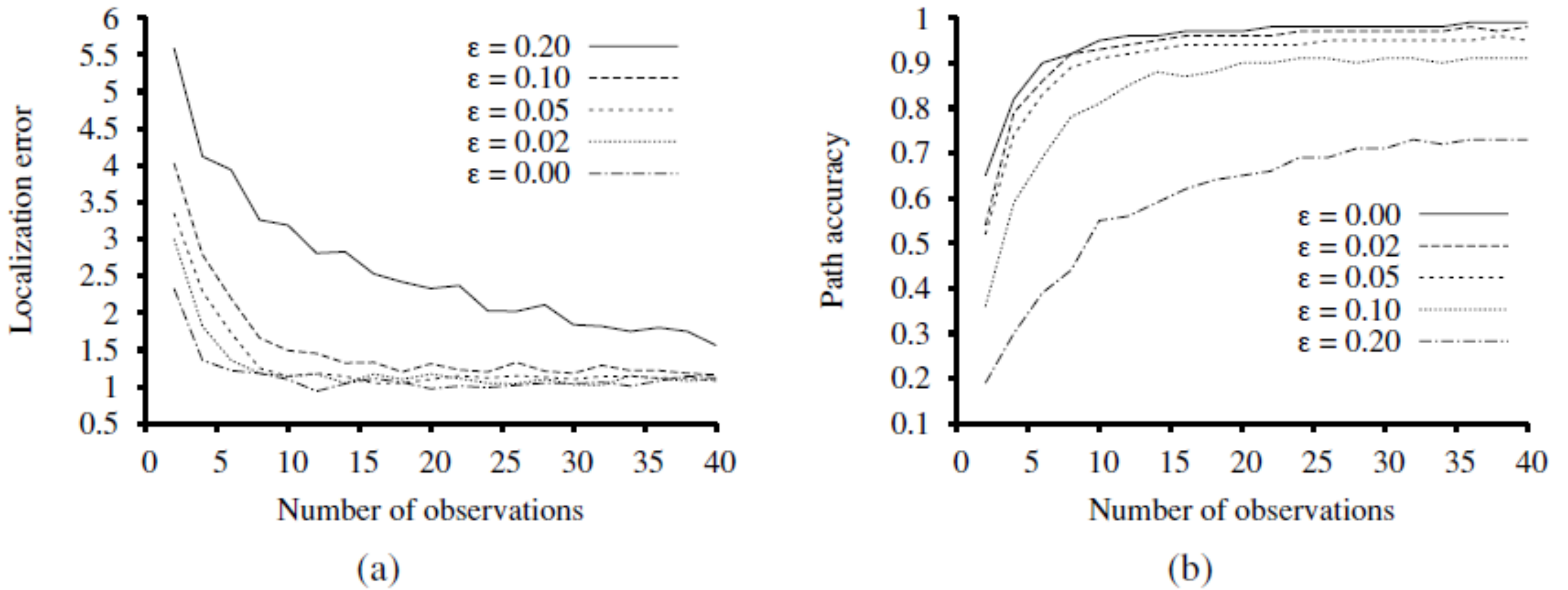
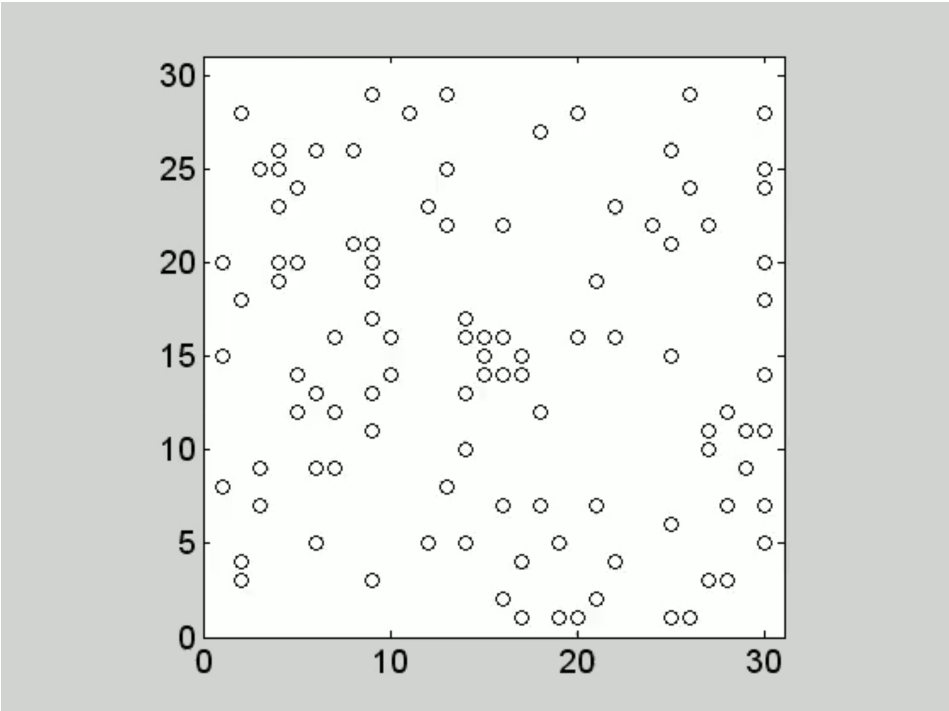
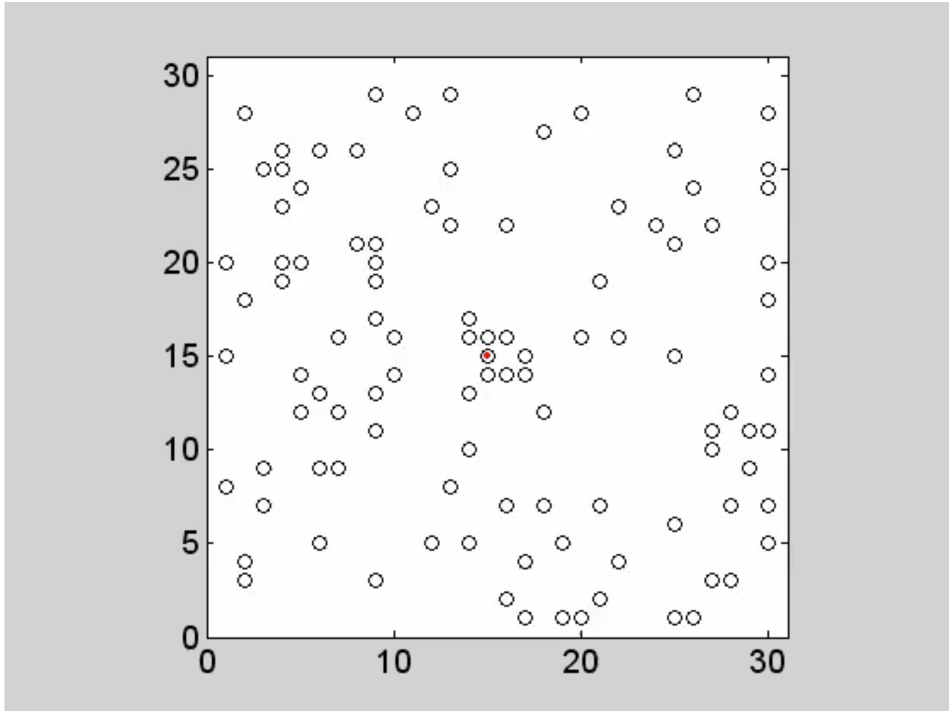


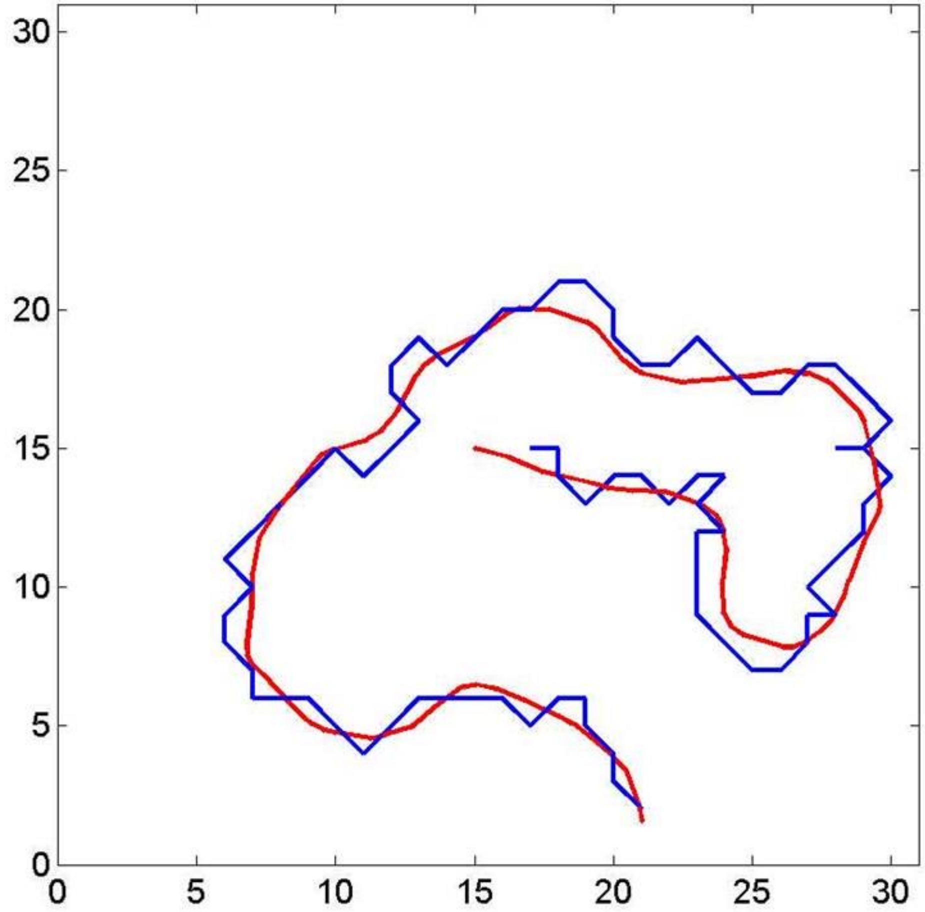
Figure 15.8 Performance of HMM localization as a function of the length of the observation sequence for various different values of the sensor error probability ϵ; data averaged over 400 runs. (a) The localization error, defined as the Manhattan distance from the true location. (b) The Viterbi path accuracy, defined as the fraction of correct states on the Viterbi path.

Example: Tracking

Songhwai Oh and Shankar Sastry, **"Tracking on a Graph,"** in Proc. of the ACM/IEEE International Conference on Information Processing in Sensor Networks (IPSN), Apr. 2005.







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