Problem 3:

Determine whether the given vector field is conservative, and find a potential if it is

$$\mathbf{F}(x, y, z) = x\mathbf{i} - 2y\mathbf{j} + 3z\mathbf{k}$$

Solution:

$$\phi(x, y, z) = \frac{x^2}{2} - y^2 + \frac{3z^2}{2}$$

Check if conservative

$$\frac{\partial \lambda}{\partial \ell^1} = \frac{\partial x}{\partial \ell^2} \Rightarrow 0 = 0 \quad \checkmark$$

$$\frac{\partial f_1}{\partial z} = \frac{\partial f_3}{\partial x} \Rightarrow 0 = 0$$

Conservative!

$$\frac{\partial z}{\partial t^2} = \frac{\partial \lambda}{\partial t^3} \Rightarrow 0 = 0 \quad \lambda$$

$$\Rightarrow f = \frac{1}{2}x^2 + A(y,z) = -y^2 + B(x,z) = \frac{3}{2}z^2 + C(x,y)$$

$$\Rightarrow f = \frac{1}{2} x^2 - y^2 + \frac{3}{2} Z^2$$

Problem 4:

Determine whether the given vector field is conservative, and find a potential if it is

$$\mathbf{F}(x, y, z) = y\mathbf{i} + x\mathbf{j} + z^2\mathbf{k}$$

Solution:

$$\phi(x, y, z) = xy + \frac{z^3}{3}$$

$$f_1 = y, \quad f_2 = x, \quad f_3 = z^2$$

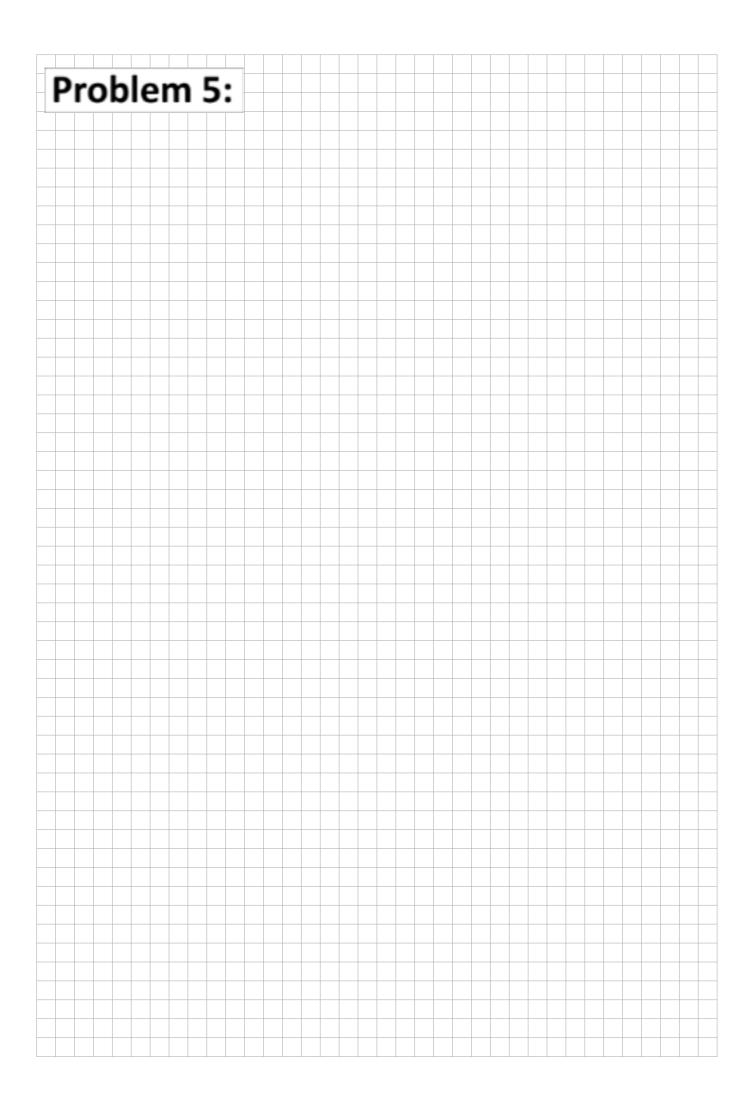
$$\frac{\partial f_1}{\partial y} = \frac{\partial f_2}{\partial x} \Rightarrow 1 = 1$$

$$\frac{\partial f_1}{\partial z} = \frac{\partial f_3}{\partial x} \Rightarrow 0 = 0$$

$$\frac{\partial f_2}{\partial z} = \frac{\partial f_3}{\partial y} \Rightarrow 0 = 0$$
CONSERVATIVE!

$$\Rightarrow \qquad \begin{cases} f_{x} \\ f_{y} \\ f_{z} \end{cases} = \begin{cases} f = xy + A(y, z) \\ f = xy + B(x, z) \\ f = xy + B(x, z) \end{cases}$$

All three equations are true if: $A(y, z) = B(x, z) = \frac{1}{3}z^3$, C(x, y) = xy



Problem 6:

Evaluate the given line integral over the specified curve $\mathcal C$

$$\int_{\mathcal{C}} (x+y) ds$$
, $\mathbf{r} = at\mathbf{i} + bt\mathbf{j} + ct\mathbf{k}$, $0 \le t \le m$

Solution:

$$\int_{C} (x+y) \, ds = \frac{(a+b)\sqrt{a^2+b^2+c^2}}{2} m^2$$

$$f(x, y, z) = x + y$$

$$x(t) = at, y(t) = bt, z(t) = ct$$

$$\int_{t=a}^{t=b} f(x(t), y(t)) dS$$

$$f(x(t), y(t), z(t)) = at + bt$$

$$\int_{t=a}^{t=b} f(x(t),y(t)) \mathrm{d}S$$

$$\mathrm{dS} = \sqrt{\left(rac{\partial x}{\partial t}
ight)^2 + \left(rac{\partial y}{\partial t}
ight)^2}\mathrm{dt}$$

$$\int_{b=0}^{t=m} f(x(b), y(b), z(b)) cls = \int_{b=0}^{t=m} at + bt cls$$

$$CLS = \sqrt{\chi_{6}^{2} + \chi_{6}^{2} + \xi_{6}^{2}}$$
 $CLE = \sqrt{\alpha_{5}^{2} + b_{5}^{2} + c_{5}^{2}}$ cLE

$$= \sqrt{a^2 + b^2 + \zeta^2} \cdot \left[\frac{a}{2} t^2 + \frac{b}{2} t^2 \right]_0^m = \sqrt{a^2 + b^2 + \zeta^2} \cdot \left(\frac{a m^2}{2} + \frac{b m^2}{2} \right)$$

$$= \sqrt{a^2 + b^2 + c^2} \cdot \frac{1}{2} \cdot (a+b) \cdot m^2 = \frac{\sqrt{a^2 + b^2 + c^2} \cdot (a+b)m^2}{2}$$

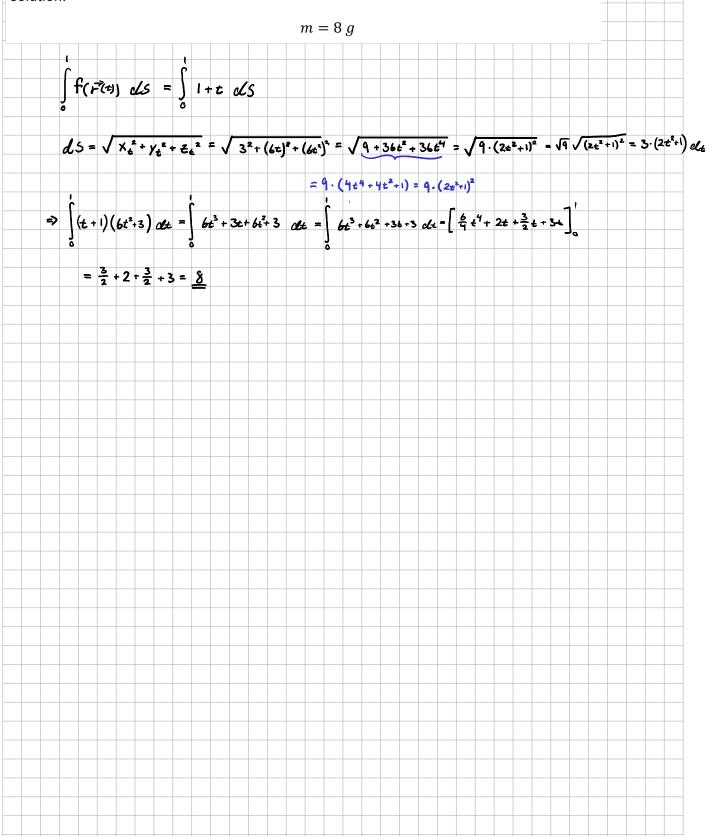
Problem 7:

Find the mass of a wire along the curve

$$\mathbf{r} = 3t\mathbf{i} + 3t^2\mathbf{j} + 2t^3\mathbf{k}, \ (0 \le t \le 1)$$

if the density at $\mathbf{r}(t)$ is 1 + t g/unit length.

Solution:





Problem 8:

Evaluate the line integral of the tangential component of the given vector field along the given curve

$$\mathbf{F}(x, y) = xy\mathbf{i} - x^2\mathbf{j}$$
 along $y = x^2$ from (0, 0) to (1, 1)

Solution:

$$-\frac{1}{4}$$

$$\times (t) = t, \quad y(t) = t^2 \Rightarrow \hat{r}(t) = t\hat{i} + t^2\hat{j}, \quad 0 \le t \le 1$$

Project the vector field onto r(t) to get the tangential component

Projection af
$$\vec{u}$$
 på \vec{v} . Dvs. \vec{u} i \vec{v} 's retning.

$$s = ec{u} ullet rac{ec{v}}{|ec{v}|} = |ec{u}| \cdot \cos heta$$

 $S = \{(x(y), y(y)) \mid \overrightarrow{f}(y) \mid \Rightarrow (x(y), y(y)) \mid$