## **Problem 1:**

$$\iiint xy^2 \cos(z) \ dydzdx \ R: \left\{ x, y, z \, \middle| \, \begin{array}{c} 0 \le x \le 2 \\ 0 \le y \le 3 \ and \ 0 \le z \le \pi/2 \end{array} \right.$$

Answer: 18

$$\int\int\int_{0}^{2\pi} Xy^{2} \cdot \cos(z) dy dz dx = \int\int_{0}^{2\pi} \left[ x \cdot \cos(z) \cdot \frac{y^{3}}{3} \right]_{0}^{3} dz dx$$

$$= \int_{0}^{2\pi} x \cdot \cos(z) \cdot \frac{3}{3} dz dx = 9 \cdot \int_{0}^{2\pi} x \cdot \cos(z) dz dx$$

$$= q \cdot \int_{0}^{2} \left[ x \cdot \sin(z) \right]^{\frac{\pi}{2}} dx = q \cdot \int_{0}^{2} x dx = q \cdot \left[ \frac{x^{2}}{2} \right]_{0}^{2} = q \cdot \frac{4}{2} = 18$$

$$\iiint 2\sqrt{y}e^{-x^2} \, dz dx dy \quad R: \left\{ x, y, z \, \middle| \, \begin{array}{c} 0 \le x \le 1 \\ 0 \le y \le 4 \, and \, 0 \le z \le x \end{array} \right.$$
 Answer:  $\frac{16}{3}(1 - e^{-1})$ 

$$\iiint_{0} 2 \sqrt{y} e^{-x^{2}} dz dx dy = Z \cdot \iint_{0} \left[ e^{-x^{2}} \int_{0}^{1} dz dx dy \right]$$

$$= Z \cdot \iint_{0} \left[ e^{-x^{2}} \left[ Z \right]_{0}^{x} dx dy = Z \cdot \iint_{0} \left[ e^{-x^{2}} \cdot x dx dy \right] \right]$$

$$u = -x^{2} \Rightarrow \frac{du}{dx} = -2x \Rightarrow dx = \frac{-1}{2x} du$$

$$= Z \cdot \int \sqrt{y} \int e^{\mu} \cdot \chi \cdot \frac{1}{Z\chi} du dy = - \int \sqrt{y} \int e^{\mu} du dy$$

$$= Z \cdot \int \sqrt{y} \int e^{\mu} \cdot \chi \cdot \frac{1}{Z\chi} du dy = - \int \sqrt{y} \int e^{\mu} du dy$$

$$= -\int_{0}^{4} \sqrt{y} \cdot \left[e^{a}\right]_{X=c}^{\chi=1} dy = -\int_{0}^{4} \sqrt{y} \cdot \left[e^{-x^{2}}\right]_{0}^{4} dy = -\int_{0}^{4} \sqrt{y} \cdot \left[e^{-(x^{2})}\right]_{0}^{4} dy$$

$$= -(e^{-1}-1) \cdot \int_{0}^{4} \sqrt{y} \, dy = (1-e^{-1}) \cdot \int_{0}^{4} y^{\frac{1}{2}} \, dy = (1-e^{-1}) \cdot \int_{0}^{4} y^{\frac{1}{2}} \, dy$$

$$= (1 - e^{-1}) \cdot \left[ \frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_{0}^{4} = (1 - e^{-1}) \cdot \left[ \frac{z}{3} \cdot y^{\frac{3}{2}} \right]_{0}^{4} = (1 - e^{-1}) \cdot \frac{z}{3} \left[ (\sqrt{y})^{3} \right]_{0}^{4}$$

$$= (1 - e^{-1}) \cdot \frac{2}{3} \cdot ((\sqrt{4})^3 - (\sqrt{6})^3) = (1 - e^{-1}) \cdot \frac{2}{3} \cdot 2^3 = (1 - e^{-1}) \cdot \frac{16}{3}$$

**Problem 3:** 

$$\iiint 1 + 2x - 3y \, dx dy dz$$

$$R: \begin{cases} x, y, z & -a \le x \le a \\ -b \le y \le b \text{ and } -c \le z \le c \end{cases}$$

Answer: 8abc

$$\int_{-c}^{c} \int_{-b}^{a} \int_{-a}^{a} \left[ + Z_{x} - 3y \right]_{-a}^{a} dx dy dz = \int_{-c}^{c} \left[ x + x^{2} - 3yx \right]_{-a}^{a} dy dz$$

$$= \int_{-6}^{6} \int_{-6}^{6} a + a^{2} - 3ya - \left(-a + (-a)^{2} - 3y(-a)\right) dy dz$$

$$= \int_{-c-b}^{c-b} a + a^2 - 3ya + a - a^2 - 3ay \ dy dz = \int_{-c-b}^{c-b} 2a - bay \ dy dz$$

$$= \int_{-c}^{c} \left[ 2ay - 3ay^{2} \right]_{-b}^{b} dz = \int_{-c}^{c} 2ab - 3ab^{2} - \left( 2a(-b) - 3a(-b)^{2} \right) dz$$

$$= \int_{-c}^{c} 2ab - 3ab^{2} + 2ab + 3ab^{2} \quad clz = \int_{-c}^{c} 4ab \quad dz = \left[4abz\right]_{-c}^{c}$$

## **Problem 4:**

Find triple integral of function  $\frac{1}{(x+y+z)^3} dx dy dz$  when R is bounded by 6 planes z=1, z=2, y=0, y=z, x=0, and x=y+z.

Answer:  $\frac{3}{16}$ ln 2.

$$\int \int \frac{1}{(x+y+z)^3} dx dy dz = \int \int \frac{1}{(x+y+z)^2} \left(-(x+y+z)^2\right) du dy dx$$

$$\int \int \frac{1}{(x+y+z)^2} dx dy dz = \int \int \frac{1}{(x+y+z)^2} \left(-(x+y+z)^2\right) du dy dx$$

$$= \iiint_{0}^{2} \frac{z}{u^{2}} \frac{y+z}{2}$$

$$= \iiint_{0}^{2} \frac{u^{2}}{u^{2}} \frac{y+z}{2}$$

$$= \frac{-1}{2} \int \int \left[ u^2 \right]^{y+2} dy dx = \frac{-1}{2} \int \int \left[ \left( \frac{1}{x+y+2} \right)^2 \right]^{y+2} dy dz = \frac{-1}{2} \int \int \left[ \frac{1}{(x+y+2)^2} \right]^{y+2} dy dz$$

$$= \frac{-1}{2} \int_{1}^{2} \int_{(y+z+y+z)^{2}}^{2} \frac{dy}{(y+z)^{2}} dy dz = \frac{-1}{2} \int_{1}^{2} \int_{(2y+2z)^{2}}^{2} \frac{dy}{(y+z)^{2}} dy dz$$

$$= \frac{-1}{2} \int_{1}^{2} \int_{(y+z+y+z)^{2}}^{2} \frac{dy}{(y+z)^{2}} dy dz = \frac{-1}{2} \int_{1}^{2} \int_{(2y+2z)^{2}}^{2} \frac{dy}{(y+z)^{2}} dy dz$$

$$= \frac{-1}{2} \int_{1}^{2} \int_{(y+z+y+z)^{2}}^{2} \frac{dy}{(y+z+z)^{2}} dy dz = \frac{-1}{2} \int_{1}^{2} \int_{(2y+2z)^{2}}^{2} \frac{dy}{(y+z+z)^{2}} dy dz$$

$$= \frac{-1}{2} \iint_{1}^{2} \frac{1}{(y+\xi)^{2}} \frac{1}{(y+\xi)^{2}} dy d\xi$$

$$= \frac{-1}{2} \iint_{-3}^{-3} \frac{1}{4} \cdot \frac{1}{(y+z)^2} dy dz = \frac{3}{8} \iint_{-3}^{2} \frac{1}{(y+z)^2} dy dz = \frac{3}{8} \iint_{-3}^{2} u^{-2} du dz$$

$$u = y - 2 \Rightarrow cluedy$$

$$= \frac{3}{8} \int_{12}^{2} \left[ -u^{-1} \right]_{y=0}^{\sqrt{2}} dz = \frac{3}{8} \int_{16}^{2} \left[ (y+z)^{-1} \right]_{0}^{2} dz = \frac{3}{8} \int_{16}^{2} \left[ (y+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{8} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{8} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{8} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{8} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{8} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{8} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{8} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{8} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{8} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{8} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{8} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{8} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{8} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{8} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{8} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{16} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{16} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{16} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{16} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{16} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{16} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{16} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{16} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{16} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{16} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{16} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{16} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{16} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{16} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{16} \int_{16}^{2} \left[ (z+z)^{-1} - (c+z)^{-1} \right]_{0}^{2} dz = \frac{3}{$$

almetst right...



